R. & M. No. 2857
(13,639)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

The Rolling Power of an Elastic Swept Wing

By
E. G. BROADBENT, M.A., A.F.R.Ae.S.

LONDON: HER MAJESTY'S STATIONERY OFFICE
1954
PRICE 5s 6d NET

The Rolling Power of an Elastic Swept Wing

By

E. G. BROADBENT, M.A., A.F.R.Ae.S.

COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR),
MINISTRY OF SUPPLY

Reports and Memoranda No. 2857*

July, 1950

Summary.—An iterative method of solution is given for the problem of loss in rolling power due to wing deformation. The method is applicable at subsonic or supersonic speeds, and compressibility effects are allowed for, provided the variation of the aerodynamic derivatives with Mach number is known. The numerical labour involved in the solution is not great and the accuracy is considerably greater than can be achieved by the semi-rigid method.

1. *Introduction*.—The loss in rolling power due to wing deformation often provides the design criterion for the torsional stiffness of a wing. A number of reports examine the theory of this problem for unswept wings. At first, the semi-rigid theories of Pugsley and Roxbee-Cox[1], and Hirst[2] proved sufficiently reliable in practice, and the great rapidity with which these theories could be applied led to their almost universal adoption. Eventually, however, it became apparent that on some occasions a more exact method was desirable, and an iteration process was devised.

There are indications that work on swept wings is following the same course as that on unswept wings in the past. Simple semi-rigid methods are becoming no longer adequate and there has arisen a pressing need for an iterative process giving greater accuracy. The present paper offers an iterative method of solution developed on similar lines to that for unswept wings[3], although it has been found convenient to use matrix notation throughout. The matrices are introduced purely as a shorthand notation, and no advanced matrix theory is used ; a brief explanation is given in the Appendix.

The physical basis of the method of solution is the same as that of Collar and Broadbent[3]. A specific Mach number is assumed and the appropriate aerodynamic derivatives a_1, a_2 and m are obtained for this Mach number, either from theory or wind-tunnel tests or a combination of both. The derivatives are determined for an arbitrary mode of deformation of the wing and are assumed to be constant.

The next step is to assume a particular value of the rolling power, which is defined in terms of X, where X is the ratio of the rolling velocity of the aircraft considered to that of an otherwise identical aircraft in the same condition but with infinitely stiff wings. When a value has been assigned to X it is only necessary to obtain the mode of deformation of the wings in order to solve the rolling equation in terms of the dynamic pressure q. From this, since the Mach number is prescribed, the height appropriate to the assigned value of X can be evaluated. To determine the mode of deformation of the wing, it is necessary first to decide what is the most useful form

* R.A.E. Report Structures 85, received 2nd January, 1951.

in which to express its elastic properties. In the method suggested this is done by means of two square matrices of flexibility coefficients, related to the application of a vertical load and pure torque about an axis perpendicular to the centre-line respectively. A matrix iteration process, somewhat different from that of Collar and Broadbent[3], is then used to determine the mode of deformation. Further iterations are performed for different values of X, and a graph of rolling power against height is obtained for the chosen Mach number.

The method is more laborious to apply than the semi-rigid approach of Broadbent and Mansfield[4], but on the other hand it is not subject to the error caused by the structural assumption of semi-rigidity. The error introduced by the aerodynamic assumptions depends upon the accuracy with which the derivatives are determined for the assumed mode and the extent to which they then apply to the final mode. Allowance for the change of derivatives with mode, though not precluded in principle, would increase the labour considerably, particularly if the derivatives were to be determined theoretically. The iterations suggested in the present paper appear to converge very rapidly (although no proof of convergence is attempted) and the time taken to do the calculations is about the same as for the iterative method of R. & M. 2186[3] for unswept wings.

2. *Assumptions and Presentation of Initial Data.*—2.1. *Aerodynamic Assumptions.*—The basic assumption here is that the aerodynamic forces on a swept wing with aileron can at all times be expressed in terms of the values, over a number of fore-and-aft strips, of the derivatives

$$\frac{\partial C_L}{\partial \alpha} = a_1$$

$$\frac{\partial C_L}{\partial \xi} = a_2$$

and

$$\left(\frac{\partial C_m}{\partial \xi}\right)_{C_L \text{ const}} = -m.$$

This assumption implies not only that the derivatives remain constant over a wide range of incidence and aileron angle, but also that they are independent of the mode of deformation*. Since the calculations are made at constant Mach number it is most convenient to express the derivatives in the form of spanwise functions for a series of Mach numbers, which can cover both subsonic and supersonic speeds if needed.

One simplifying assumption is made, which differs from that of R. & M. 2186[3], that the aileron angle is constant over the span. In R. & M. 2186 the aileron is assumed to be torsionally rigid so that the aileron angle varies over the span due to the wing twist. With a swept wing, however, the changes of incidence along the span are in part due to bending and in part due to torsion. Of these the bending component at least cannot induce any local changes of aileron angle as the aileron will be forced to bend with the wing, and since the method of the present report is to deal with flexibilities that involve both bending and torsion without separating them, the assumption of a torsionally rigid aileron would represent a major complication.

No account is taken of sideslip effects.

2.2. *Elastic Assumptions.*—Other than the assumption of constant aileron angle, which has already been referred to, there are no restrictions imposed on the elastic deformation of the structure. The main problem is to express the elastic properties in the required matrix form.

* The variation of a_1 against span will normally be known from experiment for a mode of constant pitch. A somewhat better answer may be given if a_1 is known for, say, a linear mode of twist, as this will not be so far removed from the true mode. If extreme accuracy is necessary the variation of a_1 could be included in the iteration, but then the increase in computational effort would be very great and would destroy the chief attraction of the method.

Consider the swept-back wing shown in Fig. 1 with a fore-and-aft section (parallel to the aircraft centre-line) given by XX′ cutting the leading edge at X_L and the trailing edge at X_T. A contour board is fitted to the wing in the vertical plane through XX′ such that a vertical load can be applied to the wing on this line, and if necessary outside the section X_LX_T. A vertical load is now applied at a point Q in this section with the aircraft held at its centre-line. In general the section X_LX_T will be deflected vertically, but it will also rotate either nose-up or nose-down according as the load is applied well aft or well forward in XX′. There will be one point in the section, Q_0 say, at which a vertical load produces no rotation of X_LX_T. By repeating the test at a number of different spanwise stations a locus of points similar to Q_0 will be obtained (AB for example in Fig. 1) such that a single vertical load applied at a point on the Q_0 line* will produce no rotation of the fore-and-aft section through that point. This does not, of course, mean that there will be no rotation of any other fore-and-aft section, and in general for a swept wing all the sections inboard of that loaded will rotate through a finite angle.

Thus the fore-and-aft section through a point R on the Q_0 line may rotate through an angle θ when a vertical load is applied at a point P on the Q_0 line outside the section through R.

Let the nose-up rotation under unit down-load at P be written

$$\text{rotation at R due to load } P = \Theta_{RP}.$$

Then the rotation of each section of the wing under any known loading along the Q_0 line can be obtained from the square flexibility matrix $[\Theta_{RP}]$, where the order of the matrix is determined by the number of strips considered. For the distributed load along the Q_0 line can be considered as a number of concentrated loads, one being applied in each strip, when the rotations of the sections are given by the matrix product

$$\{\theta\} = [\Theta]\{Z\} \qquad \text{.. .. ..} \qquad (1)$$

where $\{\theta\}$ is the column matrix of the required rotations,

$\{Z\}$ is the column matrix of the applied concentrated loads,

and $[\Theta]$ is the square flexibility matrix defined above.

We may note certain properties of the matrix $[\Theta]$. By definition all the terms in the principal diagonal are zero. Also, to first-order theory, all the terms to the left of the principal diagonal would be zero, since if a load produces no rotation of the section in which it is applied it will produce no first-order rotations of the sections outboard of itself. If $[\Theta]$ is obtained by means of a series of stiffness tests, however, any measured values of Θ_{RP}, $R > P$ should be included.

A matrix similar to $[\Theta]$ must now be defined under unit torque loading instead of the linear vertical load. Let the nose-up rotation of the section through R due to unit applied nose-up moment in the section through P be written:—

$$\text{rotation at R due to moment at P} = \bar{\Theta}_{RP}.$$

The moment to be applied in order to determine Θ_{RP} must of course be about an axis perpendicular to the aircraft centre-line. An equation similar to (1) may now be used to express the rotations of the fore-and-aft strips under a series of fore-and-aft moments (*i.e.*, about axes perpendicular to the aircraft centre-line) applied one to each strip. Thus

$$\{\theta\} = [\bar{\Theta}]\{\bar{M}\} \qquad \text{.. .. ..} \qquad (2)$$

where $\{\bar{M}\}$ is the column matrix of the applied nose-up moments (the bar is used to differentiate from the symbol for Mach number),

and $[\bar{\Theta}]$ is the square flexibility matrix.

---

* The line AB of Fig. 1 obviously bears considerable similarity to the flexural axis of an unswept wing. To avoid confusion, however, it will be referred to as the Q_0 line throughout the present paper.

The matrix $[\bar{\Theta}]$ should be symmetric and to first order should also obey the relationship

$$\bar{\Theta}_{RP} = \bar{\Theta}_{RR} \, (= \bar{\Theta}_{PR}), \; P > R. \quad \ldots \quad (3)$$

A further first-order relationship may be derived between the elements of the matrices $[\Theta]$ and $[\bar{\Theta}]$. It can be expressed

$$\Theta_{RP} = (x_P - x_R) \, \bar{\Theta}_{RR}, \; P > R \quad \ldots \quad (4)$$

where x_P is the distance of the point P aft of A (*see* Fig. 1),

and x_R is the distance of R aft of A.

The suffixes relating to the various strips P, R, etc., are defined to increase consecutively from wing root to tip; *e.g.*, equations (3) and (4) relate to the rotation of a strip R due to loads applied at the strip P which is outboard of strip R.

The two matrices $[\Theta]$ and $[\bar{\Theta}]$ together with the location of the Q_0 line provide all the elastic information required in a form useful for the iterations. Their application will be illustrated by the example of section 4.

3. *The Theoretical Consideration.*—3.1. *Derivation of the Basic Equation.*—Consider the swept wing of Fig. 2. The elastic properties defined in section 2.2 are known, and the Q_0 line (AB) is drawn on Fig. 2. The locus of aerodynamic centres must also be known, and it is indicated in its typical quarter-chord position on Fig. 2, although it is not essential that the locus should be a straight line, and of course it may vary with Mach number. The line as drawn will be supposed to refer to the particular Mach number under consideration and will correspond to that used in evaluating a_1, a_2 and m. A fore-and-aft strip of chord c and width dy is shown at a distance y from the centre-line. In the strip dy the Q_0 line is aft of the aerodynamic centre by a distance ec. The wing semi-span is s. The aircraft is flying with forward speed V and is free to roll about its centre-line.

Application of aileron will produce a redistribution of lifting pressures over the wing that can be expressed as lift forces along the aerodynamic centre and pitching moments about this line. When the aircraft has reached a steady rolling velocity p this redistribution will be due partly to the direct effect of the aileron, but also partly due to the wing deformations and due to the aerodynamic damping in roll. The forces are converted into a series of lift forces (one to each strip, of which there will normally be about 6 or 8 taken over the span, some over the aileron sections and some not) acting along the Q_0 line, and pitching moments about this line. These forces and moments can be written as column matrices.

Let $\{L\}$ be the matrix of lift forces under the application of down aileron angle,

and $\{\bar{M}\}$ be the matrix of nose-up pitching moments about the Q_0 line.

We may write

$$\{L\} = \{L_\theta\}\theta_0 - \{L_y\}p/V + \{L_\xi\}\xi \quad \ldots \quad (5)$$

where L_θ is the component due to deformation,

L_y ,, ,, component due to damping,

L_ξ ,, ,, component due to the aileron application,

θ_0 ,, ,, rotation of a reference section (say the tip strip),

ξ ,, ,, aileron angle (assumed constant).

In a similar way the moments can be written

$$\{\bar{M}\} = \{M_\theta\}\theta_0 - \{M_y\}p/V + \{M_\xi\}\xi \quad \ldots \quad (6)$$

The condition of steady rolling implies that the net rolling moment is zero, and this is expressed by the equation

$$\lfloor y \rfloor \{L\} = 0 \quad \ldots \quad (7)$$

where $\lfloor y \rfloor$ is the row matrix of the dimension y applied to each strip. Substitution of equation (5) gives

$$\lfloor y \rfloor \Big(\{L_\theta\}\theta_0 - \{L_y\}p/V + \{L_\xi\}\xi \Big) = 0 \, . \quad \ldots \quad (8)$$

The corresponding equation for an otherwise identical aircraft with rigid wings would be

$$\lfloor y \rfloor \Big(\{L_\xi\}\xi - \{L_y\}\frac{p_r}{V} \Big) = 0 \quad \ldots \quad (9)$$

where p_r is the rolling velocity of the rigid aircraft. A measure of the rolling effectiveness of the elastic aircraft is given by p/p_r; let this ratio be X. Then equation (9) may be rewritten

$$\lfloor y \rfloor \{L_\xi\}\xi = \lfloor y \rfloor \{L_y\}\frac{p}{XV} \, . \quad \ldots \quad (10)$$

This equation expresses the rolling velocity p in terms of the aileron angle ξ, and by means of equations (8) and (10) a similar relation is obtained between θ_0 and ξ. These two equations may be written

$$\left.\begin{aligned} \frac{p}{V} &= X \frac{\lfloor y \rfloor \{L_\xi\}}{\lfloor y \rfloor \{L_y\}} \xi \\ \text{and} \qquad \theta_0 &= -(1 - X) \frac{\lfloor y \rfloor \{L_\xi\}}{\lfloor y \rfloor \{L_\theta\}} \xi \end{aligned}\right| \quad \ldots \quad (11)$$

It now remains to relate the forces acting on the wing to the displacements by means of the flexibility matrices. By means of section 2 the relation is

$$\{\theta\} = -[\Theta]\{L\} + [\bar{\Theta}]\{\bar{M}\} \quad \ldots \quad (12)$$

where $\{\theta\}$ is the column matrix of rotations in the condition of equilibrium.

Since the columns $\{L\}$ and $\{\bar{M}\}$ as given by equation (5) and (6) are in terms of θ_0, p/V and ξ, the equation (12) can be written in terms of ξ only by use of the relations (11). Before proceeding with these substitutions it will be convenient to write the forces L and $\bar{M}$ non-dimensionally by introducing the aerodynamic derivatives a_1, a_2 and m. Consider, for example, the moment term M_θ. The distribution of lift (a_1) is known for the Mach number considered, so that

$$M_\theta = qc\, dy\, a_1(\theta/\theta_0)ec \quad \ldots \quad (13)$$

since the term ec represents the required moment arm of the lift force. Here q is again the dynamic pressure, and dy the width of the strip considered (*see* Fig. 2). We write

$$M_\theta = qc_r^2 s \bar{m}_\theta \quad \ldots \quad (14)$$

where

$$\bar{m}_\theta = ea_1(c/c_r)^2 f\, d\eta \quad \ldots \quad (15)$$

is the non-dimensional aerodynamic moment*, and c_r is a reference chord. The function θ/θ_0 representing the variation of acquired incidence over the span is replaced by f, which is a spanwise twist function made unity at the reference section where $\theta = \theta_0$. The word twist is used here in the sense of rotations of the fore-and-aft strips. The symbol η in equation (15) is defined by the relation

$$\eta = y/s \, . \quad \ldots \quad (16)$$

* The bar is inserted over $\bar{m}_\theta$ to avoid confusion with the standard symbol for torsional stiffness.

Finally, since the definition (15) applies to all the strips, it may be written in matrix form

$$\{\overline{m}_\theta\} = \{(c/c_r)^2 ea_1 f\, d\eta\} \quad \ldots \quad (17)$$

where the brackets { } enclose column matrices. The expression corresponding to equation (14) is now

$$\{M_\theta\} = qc_r^2 s\{\overline{m}_\theta\}\,. \quad \ldots \quad (18)$$

The other non-dimensional coefficients are defined by expressions similar to (17), and the six may be summarised

$$\left.\begin{aligned} \{l_\theta\} &= \{a_1(c/c_r)f\, d\eta\} \\ \{l_\eta\} &= \{a_1(c/c_r)\eta\, d\eta\} \\ \{l_\xi\} &= \{a_2 c/c_r\, d\eta\} \\ \{\overline{m}_\theta\} &= \{ea_1(c/c_r)^2 f\, d\eta\} \\ \{m_\eta\} &= \{ea_1(c/c_r)^2\eta\, d\eta\} \\ \{m_\xi\} &= \{(ea_2 - m)(c/c_r)^2\, d\eta\} \end{aligned}\right\} \quad \ldots \quad (19)$$

so that the equations corresponding to (18) are

$$\left.\begin{aligned} \{L_\theta\} &= qc_r s\{l_\theta\} \\ \{L_y\} &= qc_r s^2\{l_\eta\} \\ \{L_\xi\} &= qc_r s\{l_\xi\} \\ \{M_\theta\} &= qc_r^2 s\{\overline{m}_\theta\} \\ \{M_y\} &= qc_r^2 s^2\{m_y\} \\ \{M_\xi\} &= qc_r^2 s\{m_\xi\} \end{aligned}\right\} \quad \ldots \quad (20)$$

The equations (20) are now substituted in (5) and (6) to give the aerodynamic forces in terms of the non-dimensional coefficients (19). These in their turn are now substituted in the basic equation (12) which relates the aerodynamic forces to the elastic displacements. This, with $\{\theta\}$ written in the form $\{f\}\theta_0$ gives

$$\frac{\{f\}\theta_0}{qc_r s} = -\,[\Theta]\left(\{l_\theta\}\theta_0 - \{l_\eta\}\frac{ps}{V} + \{l_\xi\}\xi\right) + c_r[\overline{\Theta}]\left(\{\overline{m}_\theta\}\theta_0 - \{m_\eta\}\frac{ps}{V} + \{m_\xi\}\xi\right). \quad \ldots \quad (21)$$

It only remains to substitute for θ_0 and p/V from (11), which gives on cancellation of ξ

$$\frac{\{f\}A(1-X)}{qc_r s} = -\,[\Theta]\Big(\{l_\theta\}A(1-X) + \{l_\eta\}X - \{l_\xi\}B\Big)$$

$$+\, c_r[\overline{\Theta}]\Big(A(1-X)\{\overline{m}_\theta\} + X\{m_\eta\} - B\{m_\xi\}\Big) \quad \ldots \quad (22)$$

where A and B scalar quantities defined by

$$\left.\begin{aligned} A &= \frac{\lfloor\eta\rfloor\{l_\eta\}}{\lfloor\eta\rfloor\{l_\theta\}} \\ \text{and}\quad B &= \frac{\lfloor\eta\rfloor\{l_\eta\}}{\lfloor\eta\rfloor\{l_\xi\}} \end{aligned}\right\} \quad \ldots \quad (23)$$

It should be noted that in substituting in equation (21) some re-arrangements have been made to give the form (22) (the equation has been multiplied through by B). The equation (22) is now solved for each Mach number required by the iteration process described in section 3.2 and illustrated by the example in section 4.

3.2. *The Method of Solution.*—The wing is divided into a number of fore-and-aft strips. To make the method worthwhile from the point of view of accuracy the total number of strips should be not less than five, with at least three over the aileron span; in practice between six and eight strips is usual. The elastic data appropriate to the number of strips chosen are now required, and are best obtained directly from stiffness tests at the required sections. If the test results are only available for three or four intermediate sections, the full matrices cannot be obtained directly by means of interpolation. Probably the best plan in such a case is to obtain the principal diagonal terms of the matrix $[\bar{\Theta}]$ by interpolation, from which by use of the properties mentioned in section 2.2 the two matrices $[\Theta]$, and $[\bar{\Theta}]$ can be built up on first-order theory. Correction factors may then be deduced on the basis of the test results already available: for example

$$\bar{\Theta}_{PR} = \bar{\Theta}_{RP} = 1 \cdot 02\bar{\Theta}_{RR},\ P > R\ ;$$

Θ_{RP} increased by 1 per cent from its first-order value, $P > R$;

$$\Theta_{PR} = -\ 0 \cdot 01\Theta_{RP},\ P > R;$$

where the factors of 2 per cent, 1 per cent and -1 per cent are obtained from the test results. If the aircraft is in the design stage, the stiffness matrices may be calculated.

A particular Mach number is now assumed at which the calculations are to be made, and preferably one at which the spanwise variation of the derivatives a_1, a_2 and m is known. All the required data are now available. Matrix columns are now set up to give the four invariable coefficients l_η, l_ξ, m_η and m_ξ, from which the constant B (equation (23)) is evaluated. An initial modal function is now assumed; if no special guidance is available it is probably best to choose f proportional to η. A value of X is assigned, and the values of A (equation (23)) and $A(1 - X)$ worked out. The two factors of the stiffness matrices in equation (22) are now worked out as column matrices, and the matrix multiplication carried out as indicated. Summation of the two columns now leads to a column which as indicated by equation (22) should be proportional to f. In general this will not be so and a second mode is derived with which the iteration is repeated. The process is continued until the mode obtained repeats that assumed to the required accuracy.

Suppose that when the iterations have converged the value of the last column (corresponding to the right-hand side of equation (22)) is n times that of the function f (normally n will be of the order 10^{-6} due to the presence of this factor in the flexibility matrices). Then, from equation (22)

$$\rho a^2 = \frac{2A(1 - X)}{M^2 c_r s n} \qquad \ldots \qquad (24)$$

where a is the speed of sound,

and M is the Mach number.

By repeating the iteration with different values of X (normally only one iteration is needed for each new value) a graph of X against ρa^2 (and therefore against altitude) is obtained. Further calculations at different Mach numbers will yield the complete picture from which a cross-plot will show the variation of rolling power (X) against Mach number for any chosen height.

The solution in terms of X is readily converted into a solution in terms of $ps/\xi V$ or $ps/\xi a$, for by equation (11)

$$\frac{ps}{\xi V} = X \frac{\lfloor \eta \rfloor \{l_\xi\}}{\lfloor \eta \rfloor \{l_\eta\}} . \qquad \ldots \qquad (25)$$

This may be written

$$\frac{ps}{\xi V} = X/B \qquad \ldots \qquad (26)$$

which can be compared with equation (22) of R. & M. 2186[5]. Also

$$\frac{ps}{\xi a} = M \frac{ps}{\xi V} . \qquad \ldots \qquad (27)$$

It is evident that by use of equations (26) and (27) the results of the calculation can be presented in any desired form.

4. *An Example of the Method.*—The example which follows is intended only to illustrate the method of solution described in section 3.2. The wing considered is hypothetical and is not intended to approximate to any known design, and the aerodynamic and elastic data are not based on any detailed design; they are, however, believed to be reasonable.

4.1. *The Initial Data.*—The wing considered is shown in Fig. 3. The sweepback over the root portion is 45 deg and over the tip portion 30 deg at the quarter-chord. The aileron spans the portion of reduced sweepback with a chord ratio of 20 per cent. Six strips are considered of which the outer three cover the aileron span.

The elastic data are assumed to have been already converted to the form required either from stiffness tests (with or without the method of interpolation suggested in section 3.2) or from theory. The matrix $[\bar{\Theta}]$ has been multiplied by the reference chord c_r which in this example is taken as the centre-line chord (obtained by producing the leading and trailing edges to the centre-line); this saves an additional multiplication in evaluating the right-hand side of equation (22). The Q_0 line is shown on Fig. 3; it is quite possible that the shape in which it has been drawn (which is, in fact, parabolic) may be unrealistic, though the tendency for it to move ahead of the leading edge at the tip is correct for a swept-back wing. The two matrices are

$$[\Theta] = \begin{bmatrix} 0 & 0\cdot08 & 0\cdot29 & 0\cdot51 & 0\cdot80 & 1\cdot16 \\ 0 & 0 & 0\cdot52 & 1\cdot02 & 1\cdot72 & 2\cdot56 \\ -0\cdot01 & -0\cdot03 & 0 & 0\cdot99 & 2\cdot31 & 3\cdot91 \\ -0\cdot02 & -0\cdot04 & -0\cdot04 & 0 & 2\cdot33 & 5\cdot11 \\ -0\cdot03 & -0\cdot07 & -0\cdot10 & -0\cdot10 & 0 & 5\cdot93 \\ -0\cdot04 & -0\cdot09 & -0\cdot13 & -0\cdot18 & -0\cdot18 & 0 \end{bmatrix} \times 10^{-6} \qquad \ldots \qquad (28)$$

$$c_r[\bar{\Theta}] = \begin{bmatrix} 2\cdot19 & 2\cdot26 & 2\cdot26 & 2\cdot26 & 2\cdot26 & 2\cdot26 \\ 2\cdot26 & 5\cdot16 & 5\cdot28 & 5\cdot28 & 5\cdot28 & 5\cdot28 \\ 2\cdot26 & 5\cdot28 & 9\cdot80 & 10\cdot05 & 10\cdot05 & 10\cdot05 \\ 2\cdot26 & 5\cdot28 & 10\cdot05 & 17\cdot0 & 17\cdot4 & 17\cdot4 \\ 2\cdot26 & 5\cdot28 & 10\cdot05 & 17\cdot4 & 36\cdot2 & 37\cdot1 \\ 2\cdot26 & 5\cdot28 & 10\cdot05 & 17\cdot4 & 37\cdot1 & 81\cdot5 \end{bmatrix} \times 10^{-6} \qquad \ldots \qquad (29)$$

both in radians per pound.

The calculation will be made for a Mach number of $0\cdot8$, and for this value the aerodynamic derivatives are given by the graphs of Fig. 4. (Values of a_1 are appropriate to a mode of constant pitch.) These graphs show the spanwise variation of the derivatives and are typical of high-speed tunnel results. They do not, however, relate specifically to the wing considered and since they are quite arbitrary it has not been thought worthwhile to carry through calculations for a series of Mach numbers as would be done in practice.

4.2. *The Iteration Process.*—Table 1 gives a typical iteration and is explained below; the value of X assumed is $0 \cdot 4$.

TABLE 1

General dimensions: $s = 20$ ft Let $X = 0 \cdot 4$

$c_r = 12 \cdot 89$ ft

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Strip	η	$d\eta$	c/c_r	$e(c/c_r)$	a_1	a_2/a_1	m	a_2
1	0·18	0·16	0·876	0·192	4·0	0·02	0·02	0·08
2	0·35	0·18	0·759	—0·037	4·3	0·06	0·04	0·26
3	0·52	0·16	0·641	—0·205	4·7	0·13	0·10	0·61
4	0·66	0·12	0·545	—0·282	5·1	0·50	0·56	2·55
5	0·8	0·16	0·448	—0·276	5·5	0·63	0·71	3·46
6	0·94	0·12	0·352	—0·236	3·9	0·63	0·71	2·46

	(10)	(11)	(12)	(13)	(14)
	$(ea_2—m)c/c_r$	l_η	l_ξ	m_η	m_ξ
1	—0·00216	0·101	0·0112	0·0194	—0·000302
2	—0·0400	0·206	0·0356	—0·00765	—0·00548
3	—0·189	0·252	0·0628	—0·517	—0·0195
4	—1·024	0·220	0·167	—0·0622	—0·0670
5	—1·273	0·316	0·248	—0·0869	—0·0913
6	—0·830	0·155	0·104	—0·00365	—0·0350

$$\text{whence } B = \frac{0 \cdot 76502}{0 \cdot 45351} = 1 \cdot 687$$

	(15)	(16)	(17)	(18)
	$a_1 . c/c_r . d\eta$	$ea_1 . (c/c_r)^2 . d\eta$	f_3	l_θ
1	0·560	0·108	0·0694	0·0389
2	0·589	—0·0219	0·158	0·0931
3	0·484	—0·0992	0·294	0·142
4	0·334	—0·0938	0·480	0·160
5	0·394	—0·1089	0·793	0·312
6	0·165	—0·0388	1·0	0·165

TABLE 1—*continued*

$$\text{whence } A_3 = \frac{0\cdot76502}{0\cdot62373} = 1\cdot227 \; ; \quad A_3(1 - X) = 0\cdot7359$$

	(19)	(20)	(21)	(22)	(23)	(24)	(25)
	$\overline{m}_0$	Matrix factors		Matrix products		sum	f_4
1	0·0075	−0·0501	0·0138	0·0433	0·3035	0·3468	0·0811
2	−0·0035	−0·0909	0·00361	0·113	0·669	0·782	0·183
3	−0·0292	−0·0994	−0·00927	0·191	1·232	1·423	0·333
4	−0·0450	0·076	0·0550	0·113	2·138	2·251	0·527
5	−0·0864	0·0624	0·0557	−0·0371	3·520	3·483	0·815
6	−0·0388	−0·00798	0·0159	−0·00180	4·276	4·274	1·0

Note.—The factor 10^{-6} is omitted from columns (22) to (24) inclusive.

Columns (1) to (4) give the geometric data of the wing strips and column (5) gives the position of the Q_0 line taken from Fig. 3. Columns (6), (7) and (8) are the aerodynamic derivatives (the mid-strip values) taken from Fig. 4. Column (9) is obtained by the product* (6) × (7), and column (10) is [(5) × (9)] − [(4) × (8)]. The coefficients l_η, l_ξ, m_η and m_ξ can now be worked out and tabulated in columns (11) to (14); the relations are

$$\begin{aligned}(11) &= (2) \times (3) \times (4) \times (6)\\ (12) &= (3) \times (4) \times (9)\\ (13) &= (2) \times (3) \times (4) \times (5) \times (6)\\ (14) &= (3) \times (4) \times (10)\end{aligned}$$

The constant B is now evaluated:

$$B = \frac{(2) \times (11)}{(2) \times (12)} .$$

Columns (15) and (16) should be worked out at this stage in order to derive the columns appropriate to l_θ and $\overline{m}_\theta$; column (15) is given by (3) × (4) × (6) and column (16) by (5) × (15). A mode of distortion must now be chosen, and as mentioned earlier a good first attempt in lieu of any direct suggestion as to the correct mode is to choose f proportional to η. In the present calculation, however, a test of the rapidity of the convergence of the process was desired, and the mode initially chosen was deliberately made quite artificial (*see* Table 2). Consequently the iteration actually shown in Table 1 is that performed on the third mode; this mode is given in column (17). The coefficient l_θ is now obtained by (15) and (17) and given in column (18). The constant A_3 is obtained:

$$A_3 = \frac{(2) \times (11)}{(2) \times (18)} .$$

It will be noted that A assumes a different value with each value of f, and hence the suffix $_3$ denotes that the value of A obtained corresponds to f_3. Column (19) gives $\overline{m}_\theta$, which is obtained by the product (16) × (17). The matrix factors can now be calculated; thus column (20) is [− (18) × 0·7359] − [(11) × 0·4] + [(12) × 1·687] where 0·7359 is $A_3(1 - X)$, 0·4 is X and 1·687 is B. In a similar way column (21) is [(19) × 0·7359] + [(13) × 0·4] − [(14) × 1·687].

* In this description the product of two columns is to be taken as the column obtained by multiplying the elements of the first column by the corresponding elements of the second column.

The matrix $[\Theta]$ given by equation (28) is now post-multiplied by the column (20) to give (22) and the matrix $c_r[\bar{\Theta}]$ given by equation (29) is multiplied by (21) to give (23). The sum of the columns (22) and (23) gives column (24) which is proportional to the new mode. This mode, f_4, is given in column (25) by dividing column (24) by the value at the sixth strip. The iteration from column (17) onwards is now repeated until column (25) repeats column (17); the columns corresponding to (17) and (24) are now given in Table 2 for the set of iterations until convergence was achieved.

TABLE 2

	$(17)_1$	$(24)_1$	$(17)_2$	$(24)_2$	$(17)_3$	$(24)_3$	$(17)_4$
	f_1		f_2		f_3		f_4
1	0	0·266	0·858	0·422	0·0694	0·3468	0·0811
2	0	0·581	1·87	0·959	0·158	0·782	0·183
3	0	0·949	3·06	1·789	0·294	1·423	0·333
4	0	1·468	4·74	2·917	0·480	2·251	0·527
5	0	1·173	3·78	4·822	0·793	3·483	0·815
6	1	0·310	1·0	6·079	1·0	4·274	1·0

	$(24)_4$	$(17)_5$	$(24)_5$	$(17)_6$
		f_5		f_6
1	0·3547	0·0802	0·354	0·0802
2	0·800	0·181	0·799	0·181
3	1·46	0·330	1·457	0·330
4	2·314	0·524	2·31	$0{\cdot}523_5$
5	3·598	0·814	3·593	0·814
6	4·420	1·0	4·413	1·0

The mode given under $(17)_1$ was that originally assumed, and in this case the impossible mode given (zero everywhere except at the tip) was chosen to test the rapidity of the convergence. The first iteration leads to an equally improbable mode, but the second iteration gives f_3 which is already fairly close to the final mode. The error in f_3 is about 15 per cent in the worst case near the root, in f_4 is nowhere greater than about 1 per cent and in f_5 is zero to three-figure accuracy. The final mode (for which $A(1 - X) = 0{\cdot}6996$) is plotted in Fig. 5. Reference to equation (24) shows that the height is now obtained from the ambient pressure p_a given by

$$\gamma p_a = \rho a^2 = \frac{2 \times 0{\cdot}6996 \times 10^6}{4{\cdot}413 \times 0{\cdot}64 \times 20 \times 12{\cdot}89}$$

$$= 1{,}921$$

from which the height $h = 14{,}300$ ft.

As regards the convergence of the overall process, the critical parameter is $A(1 - X)$ divided by the number in the sixth strip of column (24). The values of this parameter, which is directly proportional to ρa^2, converge in the following manner:—

9·55, 0·0208, 0·172, 0·158, 0·158,

or expressed as ratios to the final value of $0 \cdot 158$ in such a way as to be always greater than unity (' octaves ' up or down),

$$60 \cdot 4,\ 7 \cdot 60,\ 1 \cdot 09,\ 1 \cdot 00,$$

which is exceedingly rapid in view of the large initial error.

4.3. *The complete Calculation.*—The iterations described in section 4.2 have sufficed to give the air density ρ, and consequently the height, at which the aircraft considered will have a rolling performance given by $X = 0 \cdot 4$ at a Mach number of $0 \cdot 8$. To cover the variation with height the calculations must be repeated from column (17) onwards for different values of X. This process is not so laborious as it may seem since the mode changes very little with X from that already found, and for the calculations relating to the example of this section one iteration proved sufficient for each new value of X. Table 3 gives the result of these calculations, and the curve of X against height is plotted in Fig. 6.

TABLE 3

X	ρa^2	h
0	3,564	−6,000
0·1	3,117	−1,500
0·2	2,697	2,700
0·3	2,302	6,900
0·4	1,921	11,600
0·6	1,220	22,700
0·8	582·5	38,900

In Table 3, ρa^2 is in pounds per square foot and h in feet above sea-level. Table 3 (Fig. 6) shows that complete aileron reversal does not occur at a Mach number of $0 \cdot 8$ for any real height (*i.e.*, above sea-level), but it also shows that the value of X has fallen to $0 \cdot 13$ at sea-level which represents very poor rolling performance.

For reasons stated earlier the calculations have not been made for any other Mach number. Nevertheless the sort of variation to be expected with Mach number is indicated by the dotted curves of Fig. 6; it is apparent that for $M = 0$, $X = 1$ at all heights, and further that all the curves are asymptotic to the line $X = 1$ as the height becomes indefinitely large. If Fig. 6 is plotted for sufficient Mach numbers then it presents the rolling effectiveness under all conditions, although the cross-plot of X against Mach number for different heights is also useful from some aspects.

To plot the actual rolling performance, it is necessary to transform X into the wing-tip helix angle $ps/\xi V$ given by equation (26). In a similar way the expression $ps/\xi a$, which is proportional to the rolling velocity per unit aileron angle, is obtained from equation (27), and the variation of these two parameters with height at $M = 0 \cdot 8$ is given by Table 4.

TABLE 4

X	h	$ps/\xi V$	$ps/\xi a$
0	−6,000	0	0
0·1	−1,500	0·059	0·047
0·2	2,700	0·119	0·095
0·3	6,900	0·178	0·142
0·4	11,600	0·237	0·190
0·6	22,700	0·356	0·285
0·8	38,900	0·474	0·379
1·0	∞	0·593	0·474

Since the calculation is made for constant Mach number, the two columns $ps/\xi V$ and $ps/\xi a$ are directly proportional to each other, but the constant of proportionality will be different at different Mach numbers. In Fig. 7 $ps/\xi V$ is plotted against height for $M = 0\cdot8$ and the trend for other Mach numbers shown dotted; $ps/\xi a$ is treated similarly in Fig. 8.

5. *Concluding Remarks.*—The method outlined in this paper is not restricted in application to subsonic speeds. It was in fact first developed to assist in the interpretation of aerodynamic tests on rocket models at both subsonic and supersonic speeds, and provided the aerodynamic data are available it can be used over any range of Mach numbers.

Since the example of section 4 was worked out for a purely hypothetical case the results have no great quantitative significance. It is, however, interesting to compare the value of the familiar stiffness criterion for wing flutter achieved by the hypothetical wing of section 4 with the standard requirements. The criterion is

$$K = \frac{1}{V_d}\left(\frac{m_\theta}{dc_m^2}\right)^{1/2} \quad \ldots \quad (30)$$

where m_θ is the symmetric 'torsional' stiffness measured at $\eta = 0\cdot7$ in respect of a torque applied about an axis perpendicular to the centre-line

d is approximately $0\cdot9s$

c_m is the geometric mean chord

and V_d is the design diving speed (E.A.S.) of the aircraft.

If the design diving speed of the aircraft is taken as that corresponding to $M = 0\cdot8$ at sea level, then the achieved value of K (using slug, foot, second units) is $0\cdot025$*. This is considerably less than the usual requirement for wings without wing engines of $0\cdot04$, although not very different from the requirement of $0\cdot028$ for wings carrying engines. Two major factors, as well as a minor one, should be mentioned as relevant to the numerical value of $0\cdot025$ achieved. First it is clear from Figs. 6 to 8 that the rolling power achieved at $M = 0\cdot8$ at sea level (the hypothetical design diving conditions) is quite inadequate. Moreover little change in the values of the derivatives would be needed to reduce the margin of positive rolling power under these conditions to zero. A more realistic value of the design diving speed might be that appropriate to $M = 0\cdot8$ at 10,000 ft, in which case $K = 0\cdot029$. Secondly the 'torsional' stiffness is that obtained by applying a torque about an axis perpendicular to the centre-line, and the flexibility therefore is not purely torsional in the sense of rotation about the spar axis. The magnitude and direction of this effect will depend on the relative bending and torsional stiffnesses, and on the degree of sweep; for example, if the wing is very flexible in bending and of appreciable sweepback no amount of purely torsional stiffening will have much effect on the achieved value of K. This effect is shown in Ref. 4. The third minor comment which must be made on the value of $0\cdot025$ for K for the hypothetical wing is that K should be based (as regards comparison with the criterion used in the past in flutter work) on the symmetric stiffness, which would perhaps give a value for K some 10 per cent higher than that quoted.

---

* This value is based on the stiffness obtained from the example of section 4, which is antisymmetric (*see* below).

LIST OF PRINCIPAL SYMBOLS

$\left.\begin{matrix}A\\B\end{matrix}\right\}$	Constants defined by equation (23)
C_L	Aerodynamic section lift coefficient
C_m	Aerodynamic section moment coefficient
K	Torsional stiffness criterion defined by equation (30)
L	Aerodynamic lift force
L_y	Component of lift due to rolling velocity
L_θ	Component of lift due to distortion
L_ξ	Component of lift due to aileron
M	Mach number
$\bar{M}$	Aerodynamic pitching moment
M_y	Component of moment due to rolling velocity
M_θ	Component of moment due to distortion
M_ξ	Component of moment due to aileron
V	Forward speed
X	Aileron effectiveness given by p/p_r
a	Speed of sound
a_1	Aerodynamic derivative $\partial C_L/\partial\alpha$
a_2	Aerodynamic derivative $\partial C_L/\partial\xi$
c	Wing chord
c_r	Value of c at a reference section
e	An elastic parameter (*see* section 2.2 and Fig. 1)
f	Distortion function θ/θ_0
h	Height
l_θ, l_η, l_ξ	Non-dimensional counterparts of L_θ, L_y and L_ξ respectively
m	Aerodynamic derivative $-(\partial C_m/\partial\xi)C_{L\,\mathrm{const}}$
m_0	Torsional stiffness of wing
$\overline{m}_\theta, m_\eta, m_\xi$	Non-dimensional counterparts of M_θ, M_y and M_ξ
p	Rolling velocity
p_r	Rolling velocity of hypothetical rigid aircraft
q	Dynamic pressure
s	Wing semi-span (measured perpendicular to centre-line)
x	Dimension parallel to aircraft centre-line
y	Dimension perpendicular to centre-line
$[\Theta]$	Flexibility matrix (*see* equation (1))
$[\bar{\Theta}]$	Flexibility matrix (*see* equation (2))
α	Wing section incidence
θ	Angle of rotation of fore-and-aft sections
θ_0	Value of θ at the tip section
η	Non-dimensional counterpart of y ($\eta = y/s$)
ρ	Air density
ξ	Aileron angle relative to wing

REFERENCES

No.	Author	Title, etc.
1	A. G. Pugsley and H. Roxbee-Cox ..	The Aileron Power of a Monoplane. R. & M. 1640. April, 1934.
2	D. M. Hirst	The Calculation of the Critical Reversed Speeds of Wings. R. & M. 1568. September, 1933.
3	A. R. Collar and E. G. Broadbent ..	The Rolling Power of an Elastic Wing. R. & M. 2186. October, 1945.
4	E. G. Broadbent and Ola Mansfield ..	Aileron Reversal and Wing Divergence of Swept Wings. R. & M. 2817. September, 1947.
5	R. A. Frazer, W. J. Duncan and A. R. Collar.	*Elementary Matrices.* Cambridge University Press, 1938.

APPENDIX

The Matrix Notation

The notation used is taken from Ref. 5. Square brackets, [ ], are used generally to denote matrices, and brackets { } and ⌊ ⌋ denote column matrices and row matrices respectively. In practice the only matrices denoted by square brackets in the present report are square matrices. Thus equation (1)

$$\{\theta\} = [\Theta]\{Z\}$$

may be written

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \theta_n \end{bmatrix} = \begin{bmatrix} \Theta_{11}, & \Theta_{12}, & \Theta_{13} & \ldots & \Theta_{1n} \\ \Theta_{21}, & \Theta_{22}, & \ldots & \ldots & \Theta_{2n} \\ \Theta_{31}, & \Theta_{32}, & \ldots & \ldots & \Theta_{3n} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \Theta_{n1} & \ldots & \ldots & \ldots & \Theta_{nn} \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ Z_n \end{bmatrix}$$

which is equivalent to the n linear equations

$$\begin{aligned} \theta_1 &= \Theta_{11}Z_1 + \Theta_{12}Z_2 + \Theta_{13}Z_3 + \ldots + \Theta_{1n}Z_n \\ \theta_2 &= \Theta_{21}Z_1 + \Theta_{22}Z_2 + \Theta_{23}Z_3 + \ldots + \Theta_{2n}Z_n \\ &\cdot \\ &\cdot \\ &\cdot \\ &\cdot \\ \theta_n &= \Theta_{n1}Z_1 + \Theta_{n2}Z_2 + \Theta_{n3}Z_3 + \ldots + \Theta_{nn}Z_n \,. \end{aligned}$$

A row matrix post-multiplied by a column matrix gives a scalar quantity. Thus equation (10)

$$\lfloor y \rfloor \{L_\xi\}\, \xi = \lfloor y \rfloor \{L_y\} \frac{p}{XV}$$

is the same as

$$[y_1, y_2, y_3 \ldots y_n] \begin{bmatrix} L_{\xi\,1} \\ L_{\xi\,2} \\ L_{\xi\,3} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ L_{\xi\,n} \end{bmatrix} \xi = [y_1, y_2, y_3 \ldots y_n] \begin{bmatrix} L_{y\,1} \\ L_{y\,2} \\ L_{y\,3} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ L_{y\,n} \end{bmatrix} \frac{p}{XV}$$

which is equivalent to the single equation

$$(y_1 L_{\xi\,1} + y_2 L_{\xi\,2} + y_3 L_{\xi\,3} + \ldots + y_n L_{\xi\,n})\xi$$
$$= (y_1 L_{y\,1} + y_2 L_{y\,2} + y_3 L_{y\,3} + \ldots + y_n L_{y\,n}) \frac{p}{XV}$$

The more involved equations of the report, such as equation (22), are built up from simple products of the type illustrated in this Appendix.

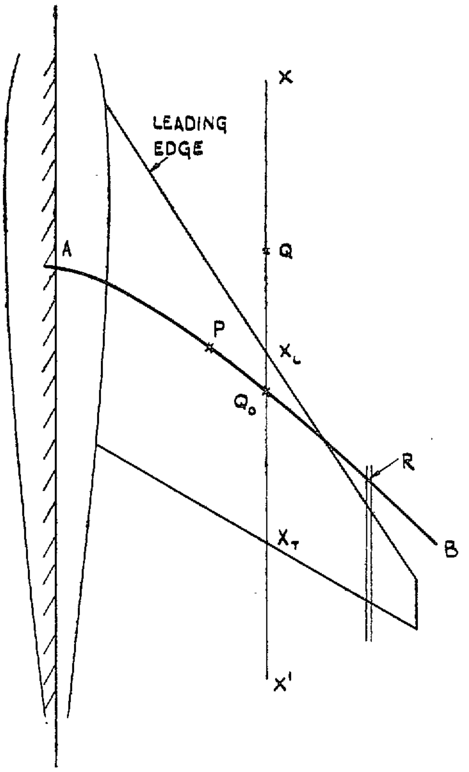

FIG. 1. Reference diagram for the elastic properties of a swept wing.

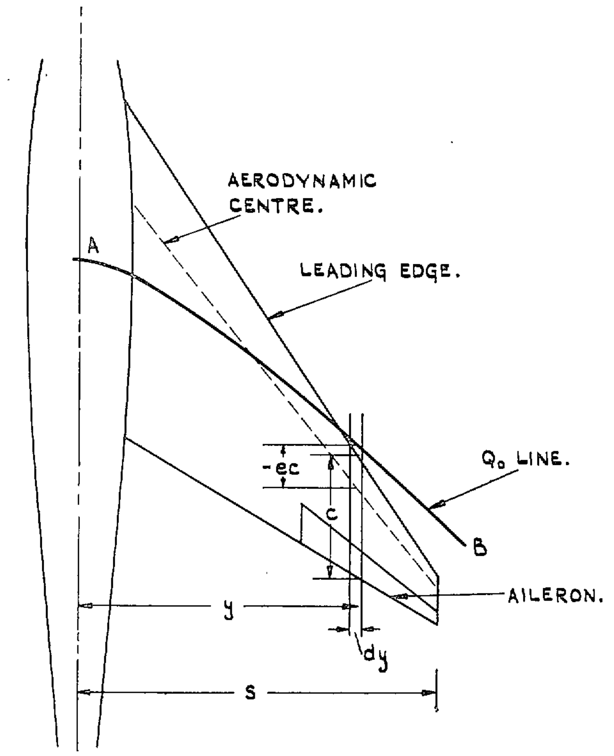

FIG. 2. Reference diagram for the aero-elastic properties of a swept wing.

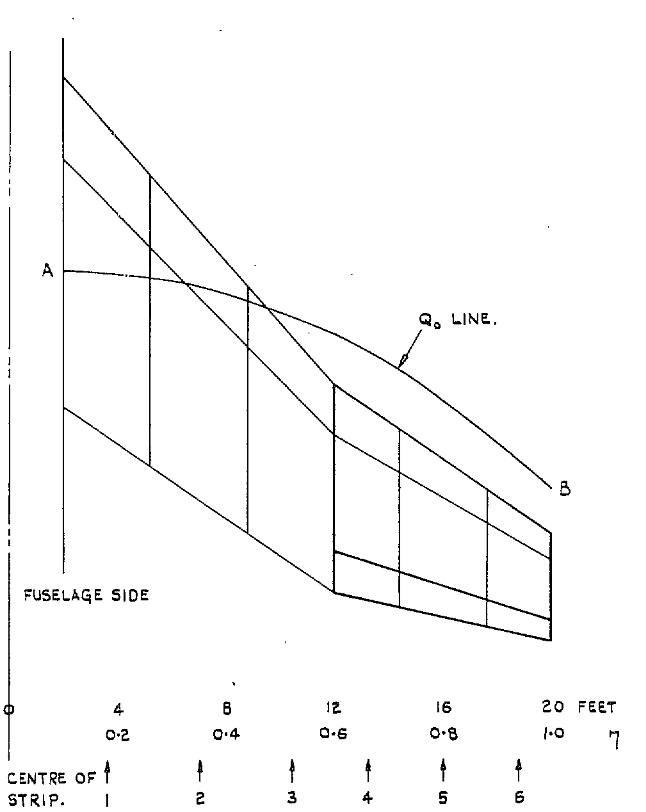

FIG. 3. Diagram of wing for example of section 4.

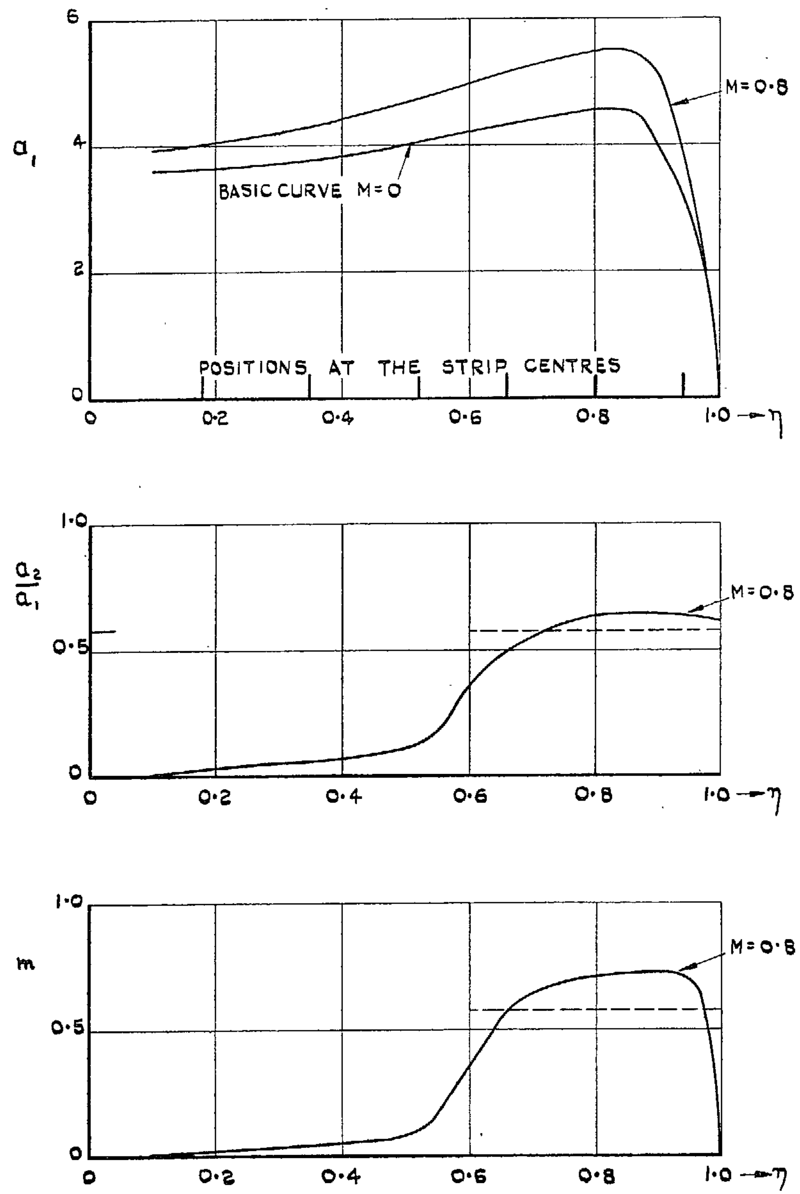

FIG. 4. Aerodynamic data for example of section 4.

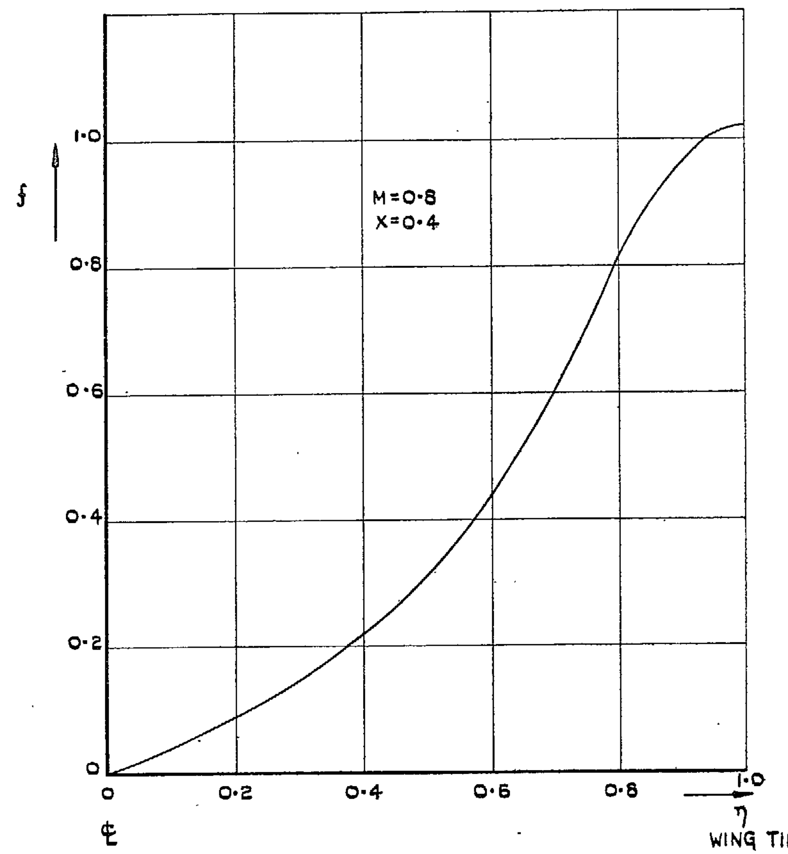

FIG. 5. Mode of twist obtained for the wing of section 4.

(61796) Wt. 17/680 K.9 5/54 Hw.

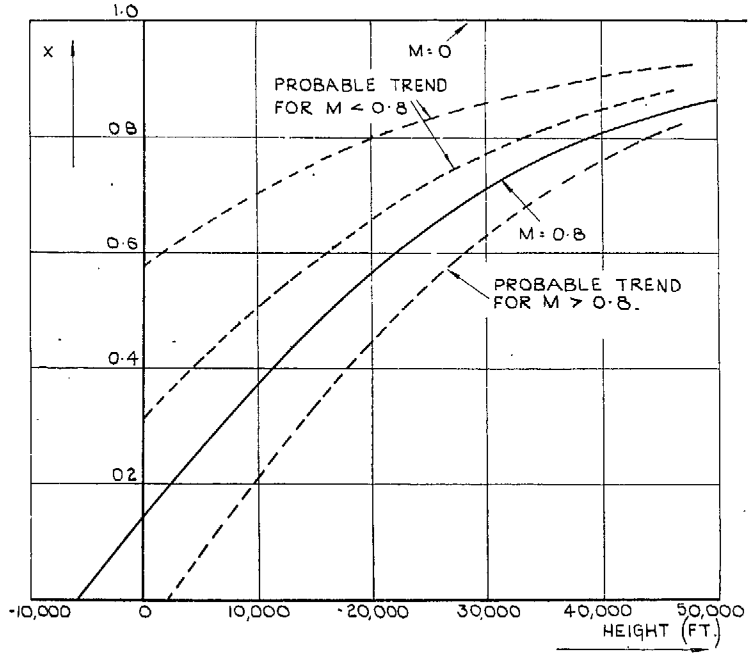

FIG. 6. X against height plotted from the example of section 4.

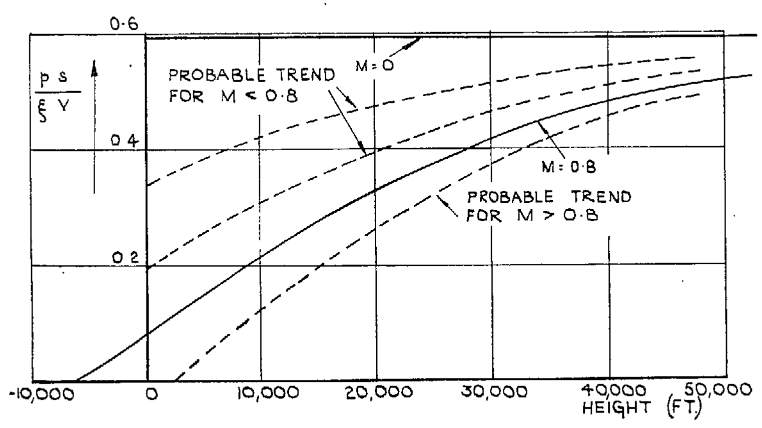

FIG. 7. $ps/\xi V$ against height from the example of section 4.

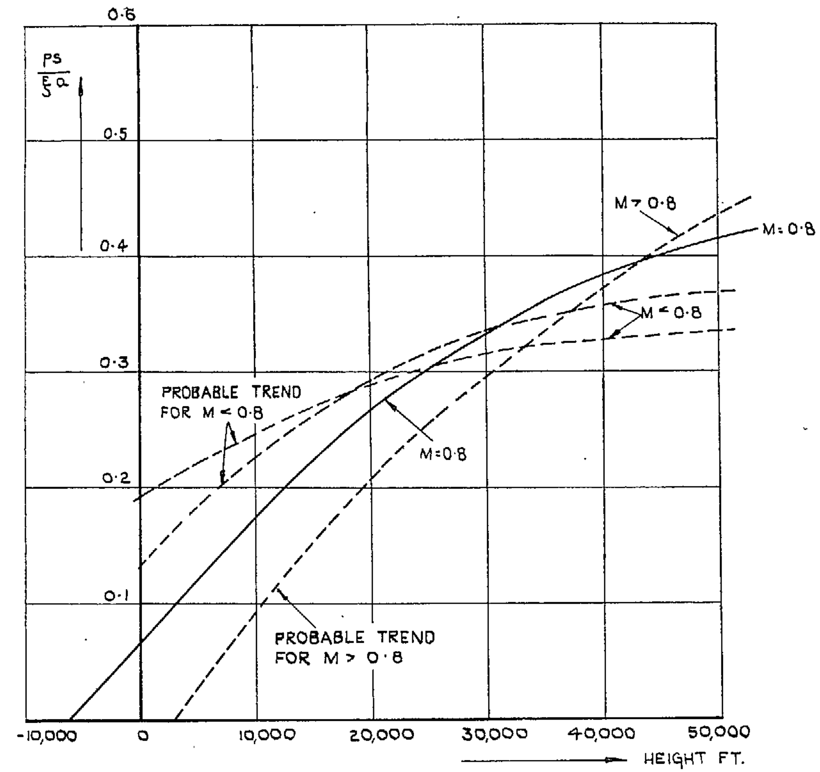

FIG. 8. $ps/\xi a$ against height from the example of section 4.

PRINTED IN GREAT BRITAIN

Publications of the Aeronautical Research Council

ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)

1936 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40*s*. (40*s*. 9*d*.)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 50*s*. (50*s*. 10*d*.)

1937 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40*s*. (40*s*. 10*d*.)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 60*s*. (61*s*.)

1938 Vol. I. Aerodynamics General, Performance, Airscrews. 50*s*. (51*s*.)
Vol. II. Stability and Control, Flutter, Structures, Seaplanes, Wind Tunnels, Materials. 30*s*. (30*s*. 9*d*.)

1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50*s*. (50*s*. 11*d*.)
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63*s*. (64*s*. 2*d*.)

1940 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Icing, Stability and Control, Structures, and a miscellaneous section. 50*s*. (51*s*.)

1941 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Stability and Control, Structures. 63*s*. (64*s*. 2*d*.)

1942 Vol. I. Aero and Hydrodynamics, Aerofoils, Airscrews, Engines. 75*s*. (76*s*. 3*d*.)
Vol. II. Noise, Parachutes, Stability and Control, Structures, Vibration, Wind Tunnels. 47*s*. 6*d*. (48*s*. 5*d*.)

1943 Vol. I. (*In the press.*)
Vol. II. (*In the press.*)

ANNUAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

1933–34	1*s*. 6*d*. (1*s*. 8*d*.)	1937	2*s*. (2*s*. 2*d*.)
1934–35	1*s*. 6*d*. (1*s*. 8*d*.)	1938	1*s*. 6*d*. (1*s*. 8*d*.)
April 1, 1935 to Dec. 31, 1936.	4*s*. (4*s*. 4*d*.)	1939–48	3*s*. (3*s*. 2*d*.)

INDEX TO ALL REPORTS AND MEMORANDA PUBLISHED IN THE ANNUAL TECHNICAL REPORTS, AND SEPARATELY—

April, 1950 - - - - R. & M. No 2600. 2*s*. 6*d*. (2*s*. 7½*d*.)

AUTHOR INDEX TO ALL REPORTS AND MEMORANDA OF THE AERONAUTICAL RESEARCH COUNCIL—

1909–1949. R. & M. No. 2570. 15*s*. (15*s*. 3*d*.)

INDEXES TO THE TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

December 1, 1936 — June 30, 1939.	R. & M. No. 1850.	1*s*. 3*d*. (1*s*. 4½*d*.)
July 1, 1939 — June 30, 1945.	R. & M. No. 1950.	1*s*. (1*s*. 1½*d*.)
July 1, 1945 — June 30, 1946.	R. & M. No. 2050.	1*s*. (1*s*. 1½*d*.)
July 1, 1946 — December 31, 1946.	R. & M. No. 2150.	1*s*. 3*d*. (1*s*. 4½*d*.)
January 1, 1947 — June 30, 1947.	R. & M. No. 2250.	1*s*. 3*d* (1*s*. 4½*d*.)
July, 1951.	R. & M. No. 2350.	1*s*. 9*d*. (1*s*. 10½*d*.)

Prices in brackets include postage.

Obtainable from

HER MAJESTY'S STATIONERY OFFICE

York House, Kingsway, London, W.C.2; 423 Oxford Street, London, W.1 (Post Orders: P.O. Box 569, London, S.E.1); 13a Castle Street, Edinburgh 2; 39, King Street, Manchester, 2; 2 Edmund Street, Birmingham 3; 1 St. Andrew's Crescent, Cardiff; Tower Lane, Bristol 1; 80 Chichester Street, Belfast, or through any bookseller

S.O. Code No. 23-2857