

N.A.E.

R. & M. No. 2854

(13,917)

A.R.C. Technical Report



NATIONAL AERONAUTICAL  
ESTABLISHMENT.  
7 MAR 1955  
LIBRARY

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL  
REPORTS AND MEMORANDA

NATIONAL AERONAUTICAL  
ESTABLISHMENT  
7 MAR 1955  
LIBRARY

# The Critical Whirling Speeds and Natural Vibrations of a Shaft Carrying a Symmetrical Rotor

By

E. DOWNHAM, B.Sc. (Eng.), A.F.R.Ae.S.

*Crown Copyright Reserved*

LONDON: HER MAJESTY'S STATIONERY OFFICE

1954

FOUR SHILLINGS NET

# The Critical Whirling Speeds and Natural Vibrations of a Shaft Carrying a Symmetrical Rotor

By

E. DOWNHAM, B.Sc. (Eng.), A.F.R.Ae.S.

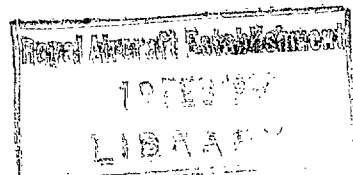
COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR),  
MINISTRY OF SUPPLY

---

*Reports and Memoranda No. 2854\**

*December, 1950*

---



*Summary.*—The experiments described in this report are part of a programme of model experiments designed to establish an accurate method for calculating the critical whirling speeds of complex systems.

The critical whirling speeds and natural vibrations of a single shaft flexibly supported and carrying a flexible rotor of appreciable moment of inertia have been investigated and good agreement has been obtained between experimental and calculated results for the rotating system. There is some discrepancy between calculated and experimental results for the vibration of the non-rotating system, which is thought to be due to the operational characteristics of the flexible bearing.

1. *Introduction.*—The problems associated with shaft whirling have already been discussed by the author in a previous report<sup>1</sup>, and preliminary experiments have been carried out on a model rig<sup>2</sup>, confirming the fundamental theory of shaft whirling.

The experiments described in the report are part of a programme of model experiments<sup>1</sup> designed to establish an accurate method for calculating the critical whirling speeds of complex systems. The particular system investigated is essentially a rotor mounted on a cantilever shaft having an additional lateral support, the whole having degrees of flexibility introduced additional to that of the shaft itself. Flexibility of known value is introduced symmetrically at the lateral support. Flexibility is also provided in the rotor, which has four flexible arms enabling it to distort out of the plane of rotation. The whole system has kinetic symmetry, and the moment of inertia of the rotor can be varied without changing its mass.

The dynamics of the system, when vibrating and whirling, were investigated theoretically using methods suggested by Morris<sup>3,4</sup> and based on the Jeffcott theory of shaft whirling<sup>2,5</sup> (1919), the forms of vibration considered involving only shaft bending without torsion. It is shown that for this particular system there will be one critical whirling speed which increases as the inertia of the rotor increases. The lower natural frequency of vibration of the non-rotating system is shown to decrease as the inertia of the rotor increases. For a particular rotational speed of the rotating system the natural vibrations will be circular, there being four resonant frequencies which vary with rotational speed. These circular vibrations are exponentially stable and a critical whirling speed occurs only when the frequency of the forced vibration due to rotor unbalance coincides with a natural frequency of vibration of the rotating system.

The effect of the inertia couples due to the rotor, on the deflection forms for the whirling system, and for the non-rotating system vibrating naturally, was obtained experimentally; and these deflection forms were compared with that of the non-rotating system due to a static load at the point of attachment of the rotor.

---

\* R.A.E. Report Structures 97, received 16th April, 1951.

The natural frequencies of the non-rotating system and the critical whirling speeds were calculated for different amounts of rotor inertia, and the results compared with measured critical whirling speeds and natural frequencies. The comparison gives good agreement between calculated and measured critical speeds but the calculated natural frequencies of the non-rotating system were approximately 10 per cent below those measured. This is considered to be due to the outrigger bearing imposing a greater constraint on the non-rotating system when vibrating transversely than on the rotating system when whirling, thus causing a change in the mode of vibration and an increase in the natural frequency.

Following these investigations the natural frequencies of the rotating system were calculated over a range of shaft speeds. Two sets of calculations were made, one assuming the rotor arms to be rigid, and another allowing for the flexibility of the arms. These frequencies were then measured experimentally. A comparison between calculated and measured frequencies gave good agreement, and it was shown that the allowance for flexibility in the rotor arms increased the accuracy of the calculations, especially when the measured natural frequencies of the system were remote from the natural frequency of the rotor arms. It is concluded therefore, that providing the constraints in a system are known, accurate calculations of the critical whirling speed and natural frequencies can be made using the theoretical treatment described.

2. *Theoretical Investigations.*—2.1. *Analytical Treatment.*—2.1.1. *The whirling condition.*—In this case the dynamics of a shaft (Fig. 1) supported in bearings of symmetrical stiffness carrying a load  $W$  of mass  $m$  and of appreciable moments of inertia  $a, a, c$  ( $c$  being the polar moment and  $a$  the moment about a diameter in lb in. sec<sup>2</sup> units) are considered.

In Fig. 1 the line  $OZ$  is drawn through  $O$  the point of attachment between the rotor and the shaft parallel to the centre-line of the bearings. The point  $O'$  represents the displaced position of  $O$  having co-ordinates  $X$  and  $Y$  relative to the major (fixed) axes  $OX$  and  $OY$ .

The point  $G$  represents the c.g. of the rotor assumed to be displaced an amount  $h$  from  $O'$  in the plane  $XOY$ . The motion of the system is considered relative to the three axes  $Gx, Gy$  and  $Gz$  drawn parallel to the major axes  $OX, OY$  and  $OZ$ .

The positions of the principle axes of the rotor in a general displacement of the system are represented by  $GA, GB$  and  $GC$ , the inertias relative to these axes being  $a, a$  and  $c$  respectively. The positions of these axes is determined by imagining first a small rotation  $\Phi$  in the  $xz$ -plane about the  $y$ -axis, then a small rotation  $\theta$  about the displaced  $x$ -axis and finally a rotation  $\Omega t$  about the axis  $GC$ .

The rotations  $\Phi$  and  $\theta$  are assumed to be small which would be the case in practice and  $\Omega$  is the angular speed of the shaft.

The instantaneous angular velocities about the axes  $AG, BG$  and  $CG$  are represented by  $\omega_1, \omega_2, \omega_3$  respectively and are obtained by summing the components of the angular velocities  $\dot{\Phi}, \dot{\theta}$  and  $\Omega$  in the planes  $BCG, ACG$  and  $ABG$  respectively.

$$\text{Thus } \left. \begin{aligned} \omega_1 &= \dot{\Phi} \sin \Omega t - \dot{\theta} \cos \Omega t \\ \omega_2 &= \dot{\Phi} \cos \Omega t + \dot{\theta} \sin \Omega t \\ \omega_3 &= \Omega \end{aligned} \right\} \dots \dots \dots \dots \dots \dots \dots (1)$$

The angular momenta due to the angular velocities of equations (1) are respectively  $a\omega_1, a\omega_2$  and  $c\omega_3$  and therefore the component angular momenta in the directions  $y$  to  $z$  and  $z$  to  $x$  are given by

$$\text{and } \left. \begin{aligned} h_x &= a\omega_1 \cos \Omega t - a\omega_2 \sin \Omega t + c\omega_3 \dot{\Phi} \\ h_y &= a\omega_1 \sin \Omega t + a\omega_2 \cos \Omega t + c\omega_3 \dot{\theta} \end{aligned} \right\} \dots \dots \dots \dots \dots \dots \dots (2)$$



2.1.2. *The natural vibrations of the rotating system.*—The natural vibrations of the rotating system are next considered, appropriate to the case where the forcing due to the out of balance  $h$  is absent. For these there will be a solution of the form

$$X = X_0 \cos(\omega t + \eta) \qquad Y = X_0 \sin(\omega t + \eta)$$

$$\Phi = \Phi_0 \cos(\omega t + \eta) \qquad \theta = \Phi_0 \sin(\omega t + \eta)$$

which constitute circular vibrations of amplitude  $X_0$  and  $\Phi_0$  and frequency  $\omega/2\pi$ . Inserting these values in equations (9), (10), (11) and (12) with  $h$  put equal to zero gives

$$(-my_{11}\omega^2 + 1)X_0 + (-a\omega^2 + c\Omega\omega)z_{11}\Phi_0 = 0$$

and

$$-mz_{11}\omega^2 X_0 + [(-a\omega^2 + c\Omega\omega)\phi_{11} + 1]\Phi_0 = 0.$$

Eliminating  $X_0$  and  $\Phi_0$  from these two equations gives the frequency equation

$$ma\Delta\omega^4 - mc\Delta\Omega\omega^3 - (a\phi_{11} + my_{11})\omega^2 + c\phi_{11}\Omega\omega + 1 = 0 \quad \dots \dots \dots (14)$$

$y_{11}$ ,  $\phi_{11}$ , and  $\Delta$  being positive and  $c > a$ .

It may be shown that there are four real roots to this equation two of which are positive and two negative. This means that for any value of  $\Omega$  there will be four natural free vibrations which are circular. The negative roots correspond to circular vibrations in the opposite direction to the rotation of the shaft and the positive roots to circular vibrations in the same direction.

The whirling speed occurs when  $\omega = \Omega$ , i.e., when the forced circular vibration due to out-of-balance has the same frequency as a positive natural vibration of the rotating shaft.

2.1.3. *The natural vibrations of the non-rotating system.*—The frequencies of vibration of the non-rotating system are obtained by substituting  $\Omega = 0$  in equation (14) which gives

$$ma\Delta\omega^4 - (a\phi_{11} + my_{11})\omega^2 + 1 = 0$$

from which

$$\frac{1}{\omega^2} = \frac{(my_{11} + a\phi_{11}) \pm \sqrt{[(my_{11} + a\phi_{11})^2 - 4ma\Delta]}}{2} \quad \dots \dots \dots (15)$$

The lower of the two frequencies obtained from equation (15) is considered to be the most important for the purpose of these investigations and is given approximately by

$$\frac{1}{\omega_1^2} = my_{11} + \frac{az_{11}^2}{y_{11}} + a^2\Delta \frac{z_{11}^2}{my_{11}^3} \quad \dots \dots \dots (16)$$

The solution given by equation (16) for a particular case has been found to be within 0.1 per cent of the more accurate solution obtained from equation (15).

2.2. *Calculated Results for the Experimental System.*—2.2.1. *The effect of rotor inertia on the natural vibrations of the non-rotating system.*—Using equation (16) section 2.1.3 in conjunction with the flexibility coefficients obtained from Appendix I, the natural frequencies of the non-rotating system were calculated for the particular range of rotor inertias investigated in the first series of experiments (section 3.2.1).

2.2.2. *The effect of rotor inertia on the critical whirling speed of the system.*—Using equation (13) section 2.1.1 in conjunction with the flexibility coefficients obtained from Appendix I the critical whirling speeds were calculated as in 2.2.1.



The lower natural frequency of the non-rotating system was measured for the same values of rotor inertias used in the whirling experiments and is shown in Fig. 3 compared with the calculated frequencies of section 2.2.1.

The deflection forms of the shaft when statically loaded at the rotor end, when vibrating transversely (non-rotating system) and when whirling were recorded and are compared in Fig. 4. It was not practicable to record the three deflection forms at the same tip amplitudes and for the purpose of comparison the measured forms were adjusted accordingly. The vibrating and whirling deflection forms were recorded with maximum possible rotor inertias.

3.2.2. *The natural vibrations of the rotating system.*—For these investigations the rotor inertias were set at the maximum possible values ( $c = 0.0526$  lb in. sec<sup>2</sup>,  $a = 0.026$  lb in. sec<sup>2</sup>) and the rotor was accurately balanced. The rotating shaft was excited by means of a motor-driven crank elastically coupled to the shaft. Although the excitation was in a lateral direction, because of the inertia coupling it was effective in exciting the circular vibrations. An inductance-type proximity pick-up was used to investigate the resonant frequencies of the shaft, the signal from the pick-ups being recorded from a cathode-ray tube by means of a continuous-film camera and the frequencies measured against a fifty-cycle timing trace.

The frequencies of the natural vibrations of the rotating shaft were obtained at various speeds of shaft rotation by running the shaft at a particular speed and exciting it over the frequency range of the exciter; the resonant frequencies were observed visually on a monitor cathode-ray tube and then recorded to give an exact frequency reading. The range of shaft speeds over which investigations were carried out was chosen to cover speeds from zero r.p.m. to a value well above the critical whirling speed of the shaft. The results of these experiments are shown plotted in Figs. 5 and 6.

4. *Discussion of the Results.*—4.1. *The Critical Whirling Speeds of the System and the Natural Vibrations of the Non-rotating System.*—From Fig. 3 it is seen that the calculated values of critical whirling speeds agree closely with those measured, but there is some discrepancy between measured and calculated natural frequencies of the non-rotating system, the difference being approximately 10 per cent over the range of rotor inertias investigated. This confirms the results of previous experiments on this rig<sup>2</sup>. It is considered that the outrigger bearing imposes an extra constraint on the non-rotating system when vibrating transversely which is not present when the shaft is whirling thus changing the mode of vibration and increasing the natural frequency.

The deflection forms of Fig. 4 show clearly the effect of the inertia couples arising from the appreciable inertia of the rotor. Theoretically when the shaft is vibrating transversely the inertia couple will act in phase with the displacement of the shaft and have the same effect on the system as an increase in the flexibility coefficient  $y_{11}$ , i.e., reducing the natural frequency. This is shown in Fig. 4 by the increased curvature of the transverse vibration form compared with the static deflection form. When the shaft whirls the inertia couple theoretically acts in opposite phase to the displacement of the shaft thereby increasing the critical whirling speed: again this is shown in Fig. 4 by the decrease in curvature of the whirl deflection form, compared with the static form.

4.2. *The Natural Vibrations of the Rotating System.*—Figs. 5 and 6 show the measured natural frequencies of the rotating shaft.

In Fig. 5 the measured frequencies have been plotted as they actually appeared, i.e., as definite frequencies at particular shaft rotational speeds, either in a forward or reverse sense with respect to the direction of rotation of the shaft. At  $\Omega = 0$  there are the two natural frequencies appropriate to the non-rotating system (see section 2.1.3). When the system is rotating, four frequencies are obtained in divergent pairs from the original two. The theoretical explanation of this phenomenon may be obtained from equation (14) (section 2.1.2) which gives the frequencies of the free vibrations ( $\omega$ ) for any rotational speed ( $\Omega$ ). When  $\Omega$  is zero there are four roots

of equation (14) given by  $\omega = \pm \alpha$  and  $\omega = \pm \gamma$ : the negative roots of  $\omega$  correspond to the reverse whirls and the positive roots correspond to forward whirls. When the shaft is rotating the roots of equation (14) are no longer symmetrical, the negative roots decrease in frequency with increase of shaft speed and the positive roots increase in frequency, the negative roots are asymptotic to  $\omega = 0$  and  $\omega = -\sqrt{(\phi_{11}/m\Delta)}$  respectively and the positive roots are asymptotic to  $\omega = +\sqrt{(\phi_{11}/m\Delta)}$  and  $\omega = c(\Omega/a)$  respectively.

In Fig. 6 the natural frequencies of Fig. 5 are plotted algebraically, positive and negative appropriate to forward and reverse motions respectively. Three sets of results have been plotted ( $\omega$  against  $\Omega$ ), two calculated (one assuming rigid rotor arms and one allowing for flexibility in the rotor arms) and a set of measured results obtained by exciting the rotating system. It is seen that the lower frequencies are not appreciably affected by the flexibility of the rotor arms; the effect on the critical whirling speed of introducing arm flexibility into the calculations was found to be of the order of 1 per cent. However, the higher frequencies are affected to a greater degree, which is to be expected as the natural frequency of the rotor arms, was 58 c.p.s. and the measured value of  $\gamma$  was 60 c.p.s. The calculated roots of the negative frequency originating in  $-\gamma$  are higher than those measured when the flexibility of the rotor arms is ignored, whereas on correcting for this flexibility the frequencies obtained are lower than those measured although the discrepancy between theory and experiment is less than in the latter case. In the case of the root originating in  $+\gamma$  again the correction for rotor arm flexibility is too great for the lower rotational shaft speeds but as the shaft speed increases the two curves of calculated and measured results quickly converge as the frequency of the circular vibration becomes appreciably greater than the natural frequencies of the rotor arms.

5. *Conclusions.*—The experiments described show that, providing the constraints in a system are known precisely, it is possible to predict accurately the critical whirling speeds and the natural frequencies of the rotating system for a rotor of appreciable moment of inertia. The allowance for flexibility of the rotor arms increases the accuracy of the calculations, especially when the frequencies are appreciably different from the natural non-rotary frequencies. The method has the additional advantage that it is easily applied to the basic theory since only a change in the flexibility coefficient  $\phi_{11}$  is required.

The self-aligning outrigger bearing used in the experiments is shown to operate satisfactorily when the shaft is whirling, though the restraint on bending increases the natural frequency of vibration when the shaft is not rotating.

6. *Further Developments.*—The effect of bearings on the critical whirling speeds of shaft systems, having regard to the constraints imposed on the system, and also the nature of the support given to a system when plain bearings are used, forms an important aspect in design calculations. A programme of tests on a large model rig is at present being carried out at Royal Aircraft Establishment to investigate the constraints in plain bearings of different lengths and clearance. Where possible, measurements are being taken of similar constraints in full-scale test rigs and aircraft power plant installations.

## REFERENCES

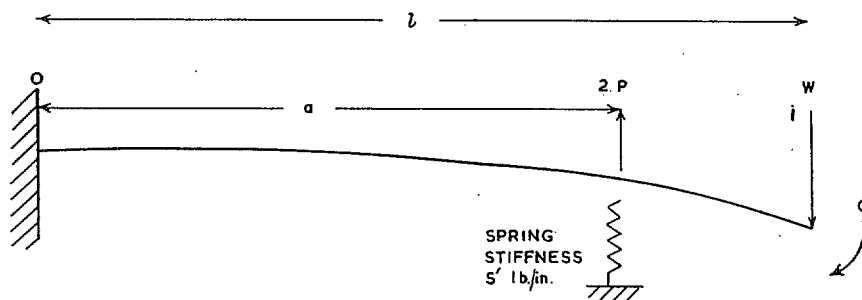
<i>No.</i>	<i>Author</i>	<i>Title, etc.</i>
1	E. Downham .. .. .	The Experimental Approach to the Problems of Shaft Whirling. C.P.55. June, 1950.
2	E. Downham .. .. .	Some Preliminary Model Experiments on the Whirling of Shafts. R. & M. 2768. June, 1950.
3	J. Morris .. .. .	<i>The Escalator Method in Engineering Vibration Problems.</i> Chapman and Hall. 1947.
4	J. Morris .. .. .	<i>The Strength of Shafts in Vibration.</i> Crosby, Lockwood. 1929.
5	H. H. Jeffcott .. .. .	The Lateral Vibration of Loaded Shafts in the Neighbourhood of a Whirling Speed. The Effect of Want of Balance. <i>Phil. Mag.</i> Vol. XXXVII, pp. 304 to 314. March, 1919.



## APPENDIX I

### *The Calculation of the Flexibility Coefficients $y_{11}$ , $\phi_{11}$ and $z_{11}$*

The flexibility coefficients are best calculated by means of the energy method.



The installation to be considered consists of a cantilever shaft 01 carrying a load  $W$  at 1 and supported by a flexible prop at point 2. A couple  $C$  is assumed to act about point 1.

Let  $P$  be the force exerted by the flexible prop.

Let  $S$  be stiffness (lb/in.) of the prop.

Let  $M_{02}$  be the bending moment at any section of the shaft between 0 and 2 distance  $x$  from 0.

Let  $M_{12}$  be the bending moment at any section of the shaft between 1 and 2 distance  $x$  from 0.

Then  $M_{02} = W(l - x) + C - P(a - x)$   
 and  $M_{12} = W(l - x) + C.$

Between 0 and 2

$$EI \frac{d^2y}{dx^2} = M_{02} = W(l - x) + C - P(a - x)$$

$$EI \frac{dy}{dx} = W\left(lx - \frac{x^2}{2}\right) + Cx - P\left(ax - \frac{x^2}{2}\right)$$

$$EI y = W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + \frac{Cx^2}{2} - P\left(\frac{ax^2}{2} - \frac{x^3}{6}\right).$$

At point 2,  $y = y_2$ ,  $P = Sy_2$ ,  $x = a.$

Therefore  $EI y_2 = W\left(\frac{la^2}{2} - \frac{a^3}{6}\right) + \frac{Ca^2}{2} - Sy_2 \frac{a^3}{3}.$

Therefore  $\left[EI + \frac{Sa^3}{3}\right]y_2 = W\left(\frac{la^2}{2} - \frac{a^3}{6}\right) + \frac{Ca^2}{2}.$

For the actual system  $a = 10$  in.,  $l = 12$  in.,  $EI = 5446$  lb in.<sup>2</sup>,  $S = 32.89$  lb/in.

Therefore  $y_2 = \frac{W(600 - 167) + 50C}{5446 + 10952} = 0.0264W + 0.00305C.$

Therefore  $P = Sy_2 = 0.8683W + 0.1003C.$

The work done in bending the shaft is given by

$$V = \frac{1}{2EI} \int_0^a M_{02}^2 dx + \frac{1}{2EI} \int_a^l M_{12}^2 dx + \frac{P}{2} y_2$$

$$= \frac{1}{2}[0.0367W^2 + 0.01050WC + 0.0013C^2]. \quad \dots \dots \dots (1)$$

Let  $y_{11}$  be the linear deflection at 1 due to unit load at 1

$z_{11}$  be the linear deflection at 1 due to unit couple at 1

= angular deflection at 1 due to unit load at 1

$\phi_{11}$  be the angular deflection at 1 due to unit couple at 1.

Then due to the load  $W$  and couple  $C$ , the linear deflection at point 1

$$= Wy_{11} + Cz_{11}.$$

Angular deflection at point 1 =  $Wz_{11} + C\phi_{11}$

and the work done by couple  $C$  and load  $W$

$$= \frac{1}{2}W[Wy_{11} + Cz_{11}] + \frac{1}{2}C[Wz_{11} + C\phi_{11}]$$

$$= \frac{1}{2}[W^2y_{11} + 2WCz_{11} + C^2\phi_{11}]. \quad \dots \dots \dots (2)$$

Equating the expressions on the right-hand sides of equations (1) and (2) gives

$$\frac{1}{2}[0.0367W^2 + 0.01050WC + 0.0013C^2]$$

$$= \frac{1}{2}[W^2y_{11} + 2WCz_{11} + C^2\phi_{11}]$$

whence  $y_{11} = 0.0367 \text{ in./lb}$

$z_{11} = 0.00525 \text{ in./lb}$

and  $\phi_{11} = 0.0013 \text{ radn/lb in.}$

## APPENDIX II

### *Dimensional Details of Model Rig*

Effective length of shaft	12 in.
Distance of outrigger bearing from the fixed end of the shaft	10 in.
Equivalent weight of shaft	0.0443 lb
Weight of rotor	0.888 lb
Hence equivalent end load	0.9323 lb
Measured spring rate of outrigger bearing support	32.89 lb/in.
Measured stiffness of shaft alone	9.47 lb/in.
Measured stiffness of shaft and outrigger bearing support combined	27.3 lb/in.

The deflection coefficients, calculated in Appendix II, are :—

$$y_{11} = 0.0367 \text{ in./lb}$$

$$\phi_{11} = 0.0013 \text{ radn/lb in.}$$

$$z_{11} = 0.00525 \text{ in./lb in.}$$

From measurements taken on the rotor

$$u_{aa}/l_a^2 = 0.00033.$$

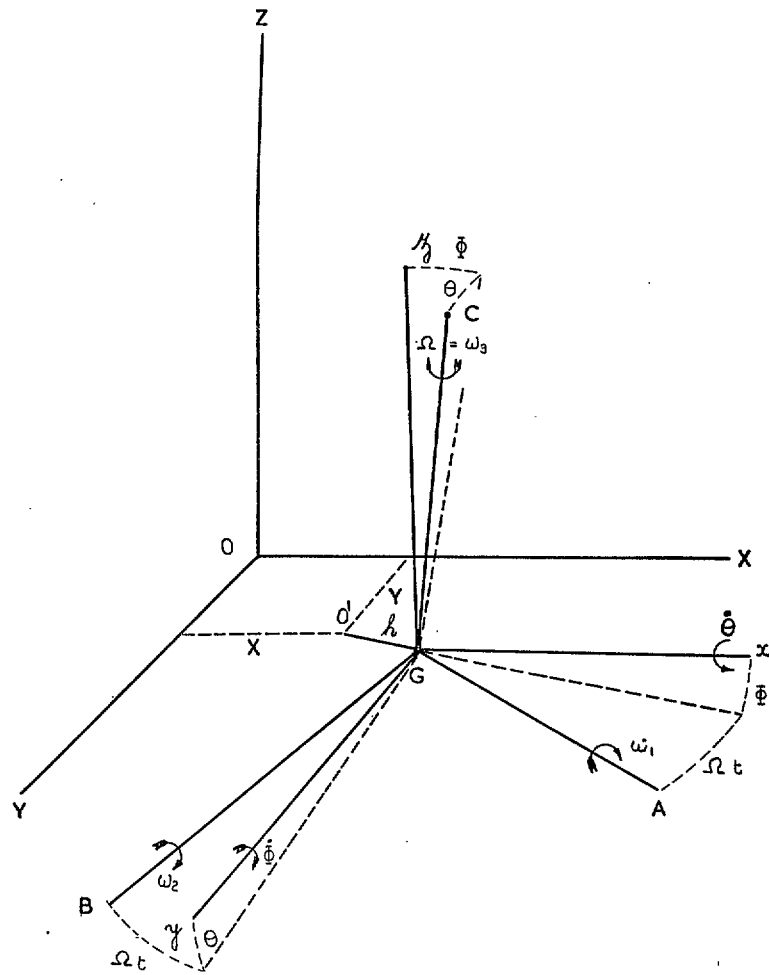


FIG. 1. The whirling condition.

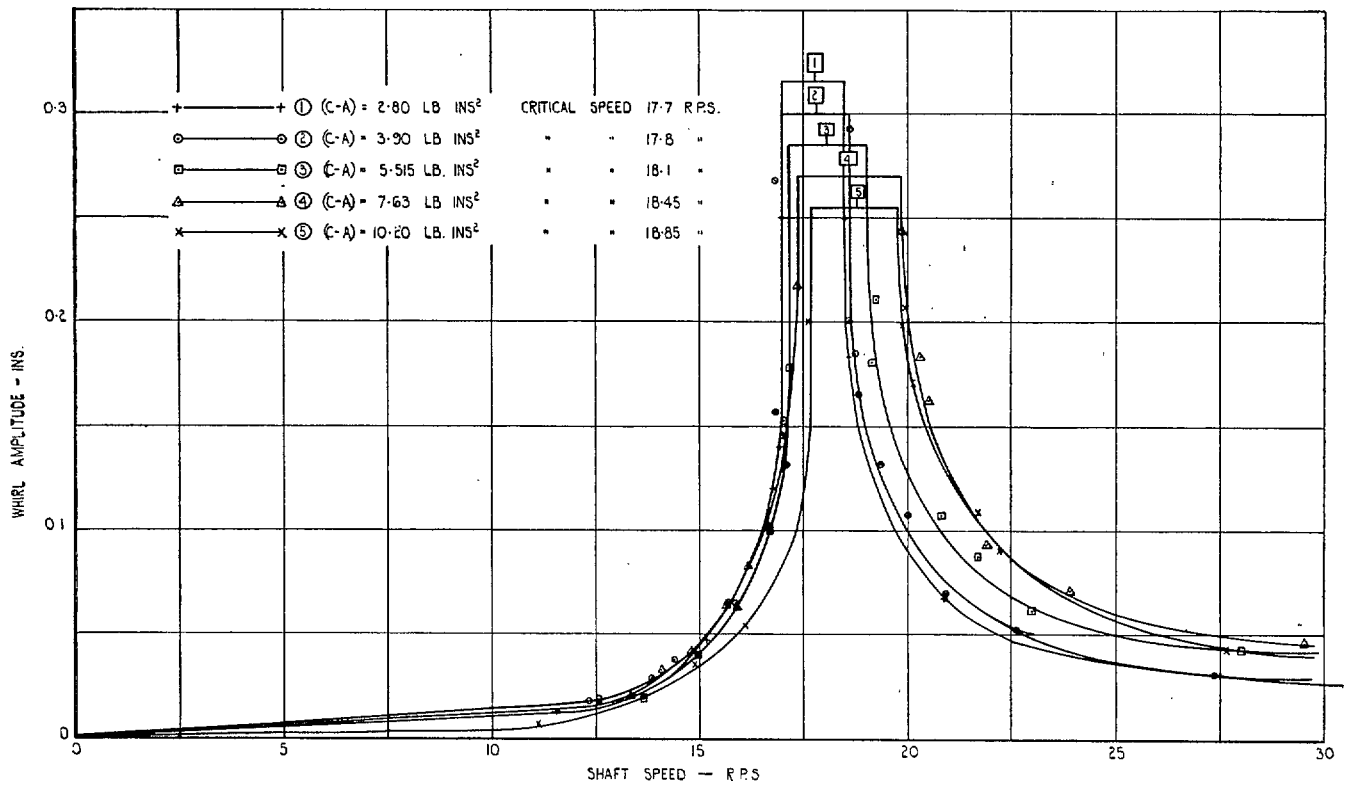


FIG. 2. The effect of rotor inertia on the critical whirling speed of a rotating shaft.

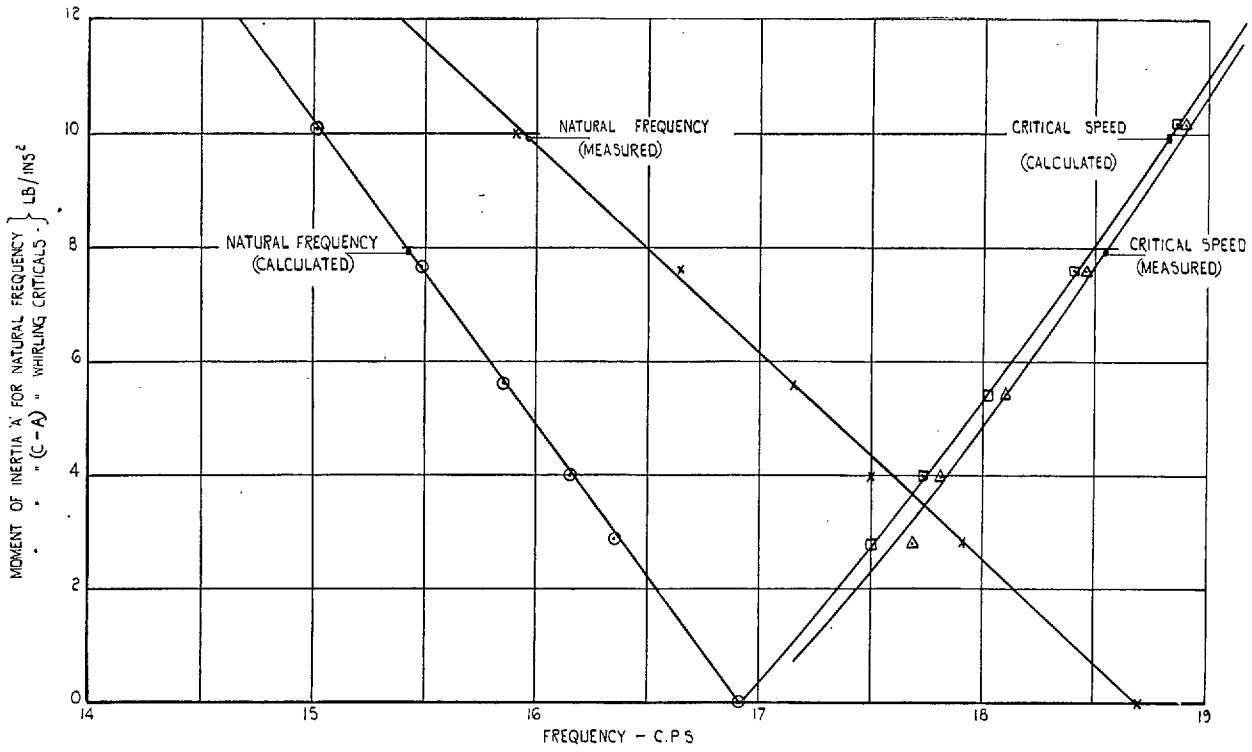


FIG. 3. The effect of rotor inertia on the natural vibrations of the non-rotating system and on the critical whirling speed.

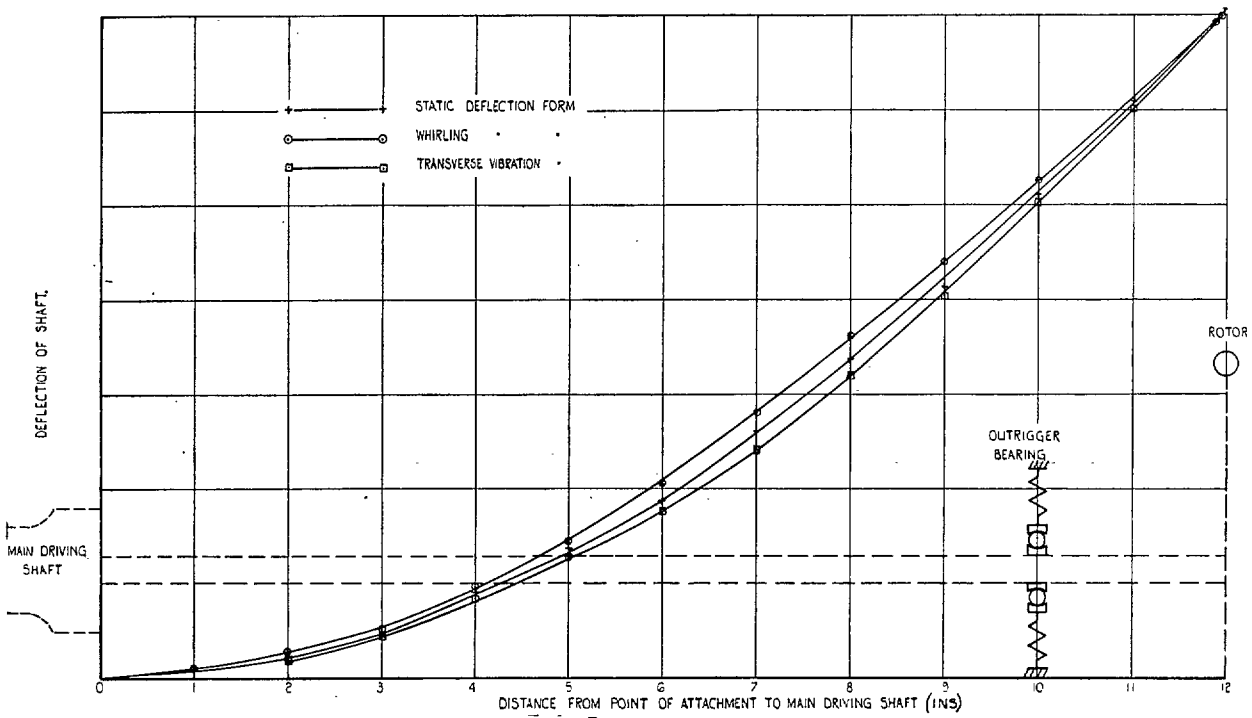


FIG. 4. Comparison between deflection forms of the non-rotating system when loaded statically at the rotor and vibrating naturally and of the rotating system when whirling.

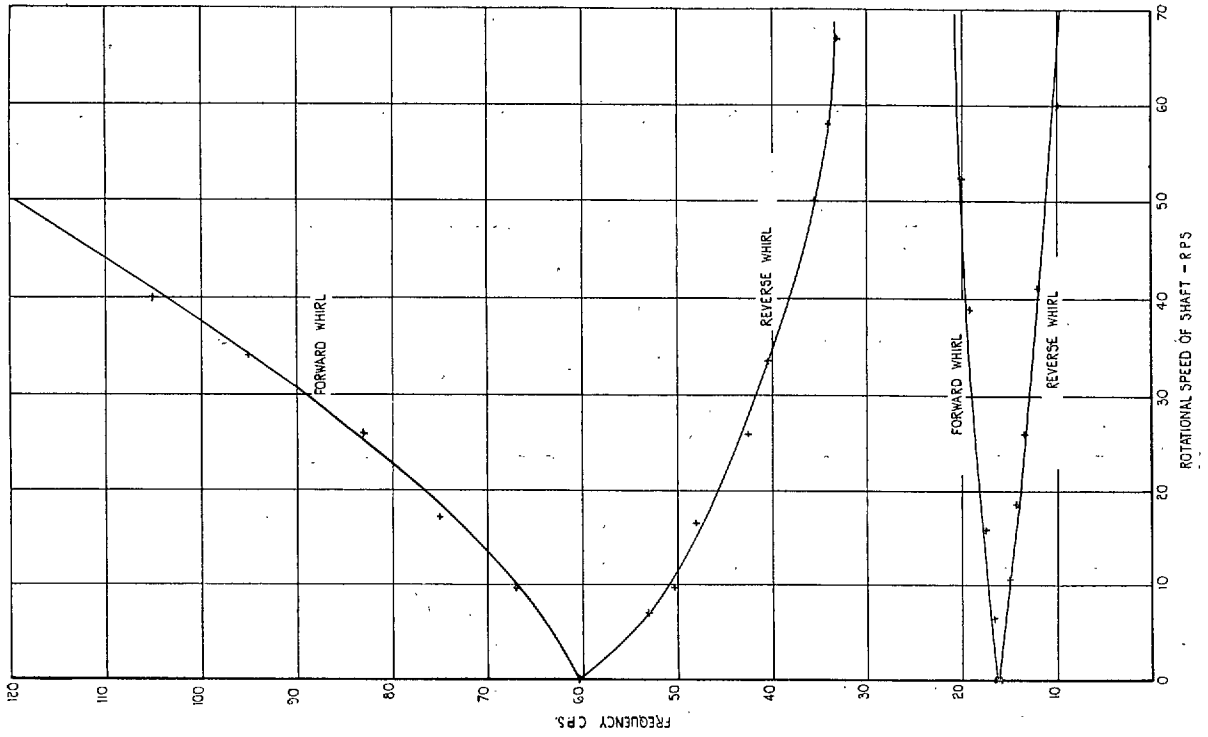


FIG. 5. The measured natural vibrations of a rotating shaft carrying a rotor of appreciable symmetric moments of inertia.

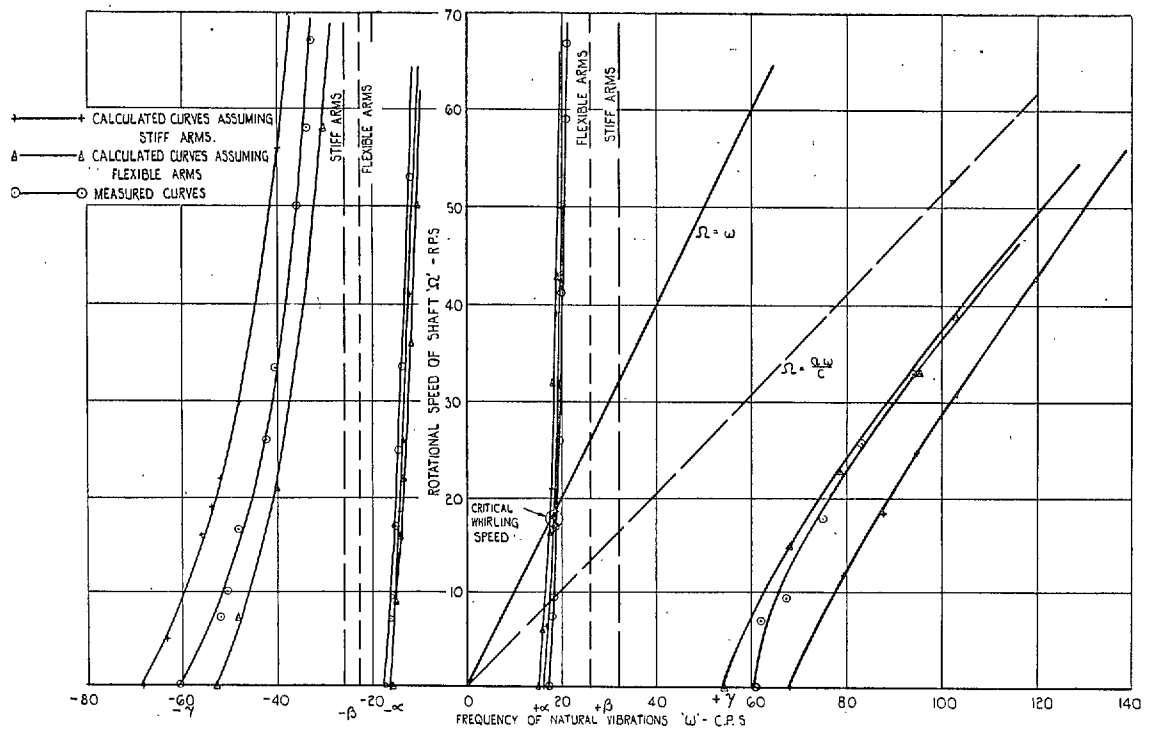


FIG. 6. The natural vibrations of a rotating shaft carrying a rotor of appreciable moments of inertia (measured and calculated).

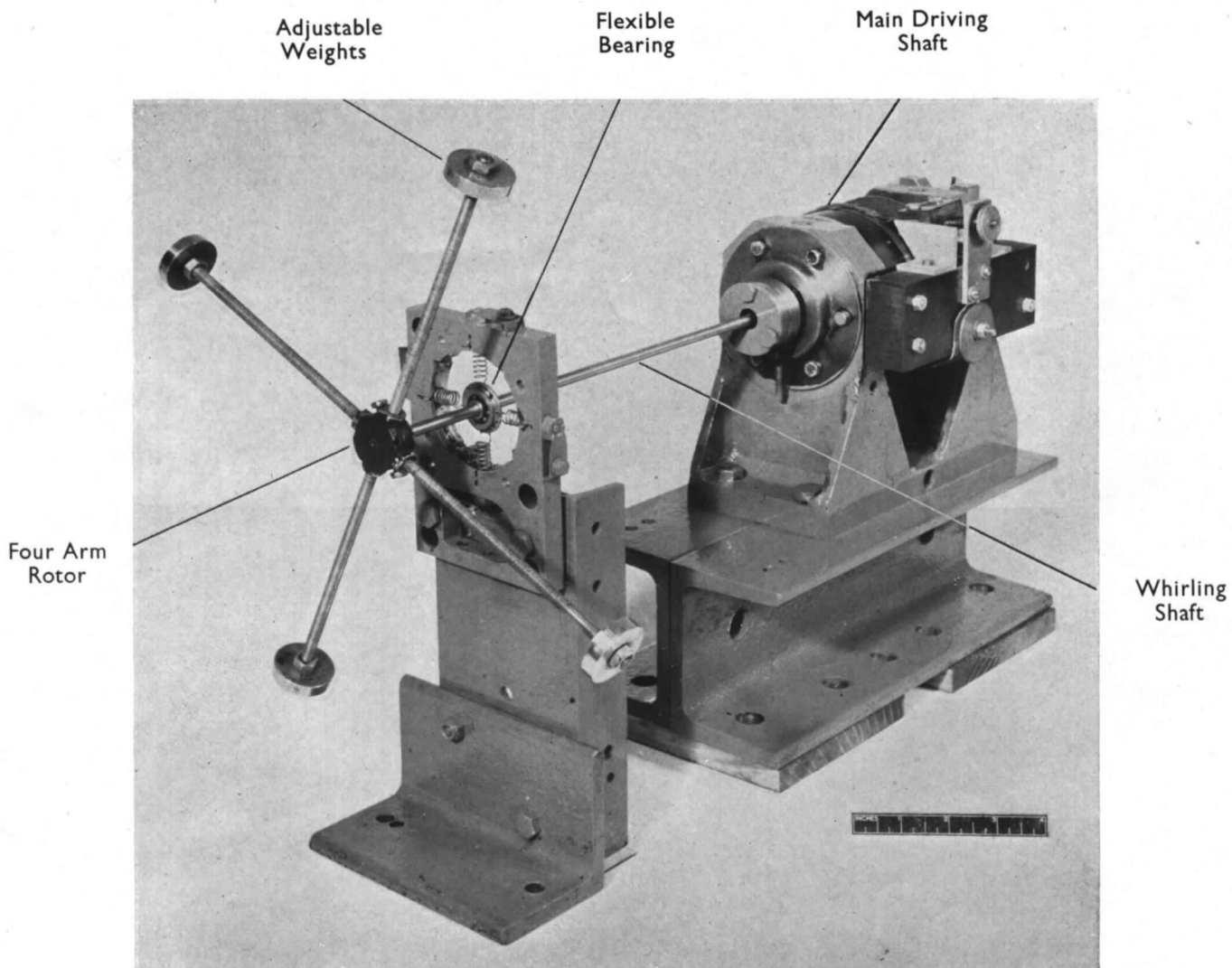


FIG. 7. Details of the model shaft rotor system.

# Publications of the Aeronautical Research Council

## ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)

- 1936 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40s. (41s. 1d.).  
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 50s. (51s. 1d.)
- 1937 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40s. (41s. 1d.).  
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 60s. (61s. 1d.)
- 1938 Vol. I. Aerodynamics General, Performance, Airscrews. 50s. (51s. 1d.)  
Vol. II. Stability and Control, Flutter, Structures, Seaplanes, Wind Tunnels, Materials. 30s. (31s. 1d.)
- 1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50s. (51s. 1d.)  
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63s. (64s. 2d.)
- 1940 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Icing, Stability and Control, Structures, and a miscellaneous section. 50s. (51s. 1d.)
- 1941 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Stability and Control, Structures. 63s. (64s. 2d.)
- 1942 Vol. I. Aero and Hydrodynamics, Aerofoils, Airscrews, Engines. 75s. (76s. 3d.)  
Vol. II. Noise, Parachutes, Stability and Control, Structures, Vibration, Wind Tunnels. 47s. 6d. (48s. 7d.)
- 1943 Vol. I. Aerodynamics, Aerofoils, Airscrews, 80s. (81s. 4d.)  
Vol. II. Engines, Flutter, Materials, Parachutes, Performance, Stability and Control, Structures. 90s. (91s. 6d.)
- 1944 Vol. I. Aero and Hydrodynamics, Aerofoils, Aircraft, Airscrews, Controls. 84s. (85s. 8d.)  
Vol. II. Flutter and Vibration, Materials, Miscellaneous, Navigation, Parachutes, Performance, Plates, and Panels, Stability, Structures, Test Equipment, Wind Tunnels. 84s. (85s. 8d.)

## ANNUAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

1933-34	1s. 6d. (1s. 8d.)	1937	2s. (2s. 2d.)
1934-35	1s. 6d. (1s. 8d.)	1938	1s. 6d. (1s. 8d.)
April 1, 1935 to Dec. 31, 1936.	4s. (4s. 4d.)	1939-48	3s. (3s. 2d.)

## INDEX TO ALL REPORTS AND MEMORANDA PUBLISHED IN THE ANNUAL TECHNICAL REPORTS, AND SEPARATELY—

April, 1950 - - - - R. & M. No. 2600. 2s. 6d. (2s. 7½d.)

## AUTHOR INDEX TO ALL REPORTS AND MEMORANDA OF THE AERONAUTICAL RESEARCH COUNCIL—

1909-1949 - - - - R. & M. No. 2570. 15s. (15s. 3d.)

## INDEXES TO THE TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

December 1, 1936 — June 30, 1939.	R. & M. No. 1850.	1s. 3d. (1s. 4½d.)
July 1, 1939 — June 30, 1945.	R. & M. No. 1950.	1s. (1s. 1½d.)
July 1, 1945 — June 30, 1946.	R. & M. No. 2050.	1s. (1s. 1½d.)
July 1, 1946 — December 31, 1946.	R. & M. No. 2150.	1s. 3d. (1s. 4½d.)
January 1, 1947 — June 30, 1947.	R. & M. No. 2250.	1s. 3d. (1s. 4½d.)
July, 1951 - - - -	R. & M. No. 2350.	1s. 9d. (1s. 10½d.)

*Prices in brackets include postage.*

Obtainable from

**HER MAJESTY'S STATIONERY OFFICE**

York House, Kingsway, London W.C.2 ; 423 Oxford Street, London W.1 (Post Orders : P.O. Box No. 569, London S.E.1) ;  
13A Castle Street, Edinburgh 2 ; 39 King Street, Manchester 2 ; 2 Edmund Street, Birmingham 3 ; 1 St. Andrew's  
Crescent, Cardiff ; Tower Lane, Bristol 1 ; 80 Chichester Street, Belfast OR THROUGH ANY BOOKSELLER

S.O. Code No. 23-2854

**R. & M. No. 2854**