R. & M. No. 2853
(11,666)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

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LONDON: HER MAJESTY'S STATIONERY OFFICE
1954
PRICE 3s 6d NET

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COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR), MINISTRY OF SUPPLY

*Reports and Memoranda No. 2853**
April, 1948

Summary.—Generalised curves have been constructed from which estimates can be made of those dynamic characteristics of the servo-tab-type of control which are of chief interest to the designer, *viz.*,

(i) the magnitude of the first overshoot of the main flying control beyond its equilibrium position,

(ii) the lag of the main control surface behind the tab movement,

(iii) the damping of the main control surface oscillation,

(iv) the angular velocity possessed by the main control when it first passes through its equilibrium position.

The characteristics evaluated for two specific cases, a 50,000-lb and a 300,000-lb aircraft, indicate no special problems to the designer or pilot except with regard to overshoot of the control at low flying speeds. Elastic stops are considered to be the most promising solution to this.

1. *Introduction.*—It has long been suggested that one way of overcoming the problem of the aerodynamic balancing of controls, especially on medium and heavy aircraft where this problem is acute, is to drive the main controls by the servo action of a tab; that is, the pilot's controls are connected directly with tabs on the main control surfaces, and the main controls themselves are driven by the hinge moments exerted on them by aerodynamic forces on the deflected tabs. In Ref. 1 it has been shown that there should be no difficulty in achieving light control forces with this type of control with aircraft of up to 250,000 lb all-up weight, and by slightly more careful balancing of either the tab or main control, or both, this figure could possibly be raised to 500,000 or even 1,000,000 lb. Furthermore the practicability of this system of control has been thoroughly demonstrated in flight by a number of large aircraft; in Germany by the large flying boats, the Blohm and Voss 222 and 238 of 100,000 lb and 200,000 lb respectively, and in America by the Douglas B19 and B36 of 165,000 and 280,000 lb respectively. In Great Britain some examples of this control were flown before the flutter problem was sufficiently understood, and because of the troubles encountered it was rightly shelved; but even though adequate knowledge of the flutter prevention methods is available at the present time, there still seems to be a prejudice against its use here. This prejudice arises primarily because designers class the servo-tab control as 'aerodynamic', and thus subconsciously feel that it will be erratic in performance and bad from the 'repeatability' viewpoint, this feeling arising from bitter experience with highly balanced controls having fixed aerodynamic balance. We cannot stress too strongly that, in the case

* R.A.E. Report Aero. 2263, received 24th July, 1948.

of both the spring-tab and the servo-tab, 'repeatability' troubles must fundamentally and automatically be greatly eased as compared with the fixed-balance control.

A further objection to the servo-tab has been centred round the dynamics of the control system, that is on such questions as the lag of the main control behind the tab movement initiated by the pilot, and the overshoot and damping of the main control movement. It is to help dispel any doubts about these points that this report has been prepared. A short graphical method of predicting the principal dynamic characteristics has been developed, which is sufficiently accurate to form a basis for the assessment of these properties in the design stage.

2. *Range of Investigation.*—A generalised calculation has been made of the motion of the main control surface following a specified displacement of the pilot's controls. If the follow-up ratio between tab and control surface is zero, this condition corresponds to a certain tab deflection; if, however, the follow-up ratio is other than zero, the tab motion depends on the main control deflection as well as the pilot's control movements (the follow-up ratio is the apparent gearing between the main control surface and the tab, when the main control is moved with the pilot's control held fixed, *i.e.*,

$$\left(\frac{\partial \beta}{\partial \xi}\right)_x$$

where β is tab angle

ξ is control surface angle

x is pilot's control deflection).

The derivation of the equations of motion of the control-tab system is given in the Appendix, but for the benefit of the general reader the chief simplifying assumptions made in the bulk of the calculations are listed below:—

(*a*) the aerodynamic forces arising out of the accelerated motion of the moving control, *i.e.*, the virtual inertia forces, have been ignored. This assumption is to be justified on the ground that the frequency of oscillation of a servo-tab type of control is quite low;

(*b*) in our calculations we have neglected to take into account the response effect of the aircraft. In effect this assumption means that b_1 has been assumed zero; this will not seriously affect the dynamic characteristics for cases with b_1 other than zero, especially for the fast applications at low flight speeds in which we are mainly interested;

(*c*) the pilot moves his control (*i.e.*, wheel, stick or pedal) at a linear rate through a certain displacement and then holds his control fixed;

(*d*) the stick gearing ratio and the follow-up ratio are assumed to be constant over the entire range of tab and control deflection;

(*e*) the control hinge moments due both to tab and control deflection are assumed to be linear with displacement;

(*f*) in arriving at the final form of the equation of motion, certain terms were considered to be sufficiently small to be disregarded. These are indicated in the Appendix.

Using these assumptions the final form of the equation of motion of the main control surface becomes:

$$i_f \frac{d^2\bar{\xi}}{d\tau^2} + h_{\dot{\xi}} \frac{d\bar{\xi}}{d\tau} - \tfrac{1}{2}(b_2 + Nb_3)\bar{\xi} = -\tfrac{1}{2}(b_2 + Nb_3)\bar{x}$$

where

$$\bar{\xi} = \frac{\text{the instantaneous deflection of the main control surface}}{\text{final steady control deflection corresponding to the given stick displacement}}$$

$$\bar{x} = \frac{\text{the instantaneous displacement of the pilot's control}}{\text{final stick displacement}}$$

$$i_f = \frac{\text{the moment of inertia of the control plus tab about the control hinge}}{\rho S_\xi C_\xi^{\ 3}}$$

$h_{\dot\xi}$ the control surface damping coefficient $= \dfrac{\text{total damping moment}}{\rho V S_\xi C_\xi^{\ 2} \dot\xi}$

$b_2 = \left(\dfrac{\partial C_H}{\partial \xi}\right)_\beta$

$b_3 = \left(\dfrac{\partial C_H}{\partial \beta}\right)_\xi$

N the follow-up ratio (i.e., $\left(\dfrac{\partial \beta}{\partial \xi}\right)_x$)

τ a non-dimensional form of time $\left(= \dfrac{V}{C_\xi} t\right)$.

This equation has been interpreted graphically, in terms of the main quantites we are interested in (expressed in non-dimensional form) namely,

(i) the periodic time of oscillation of the control system without damping (T). This has been taken for convenience in evaluation; the difference between this and the true periodic time *with* damping is small;

(ii) the time for the resultant control oscillation to damp to half-amplitude;

(iii) the value of the overshoot;

(iv) the lag of the control behind the tab;

(v) the angular velocity possessed by the control as it passes through its equilibrium position for the first time.

The quantity (iv) is of great interest in determining if the control system has sufficient response to satisfy the pilot; the quantities (i), (ii) and (iii) are of most interest in considering the stability and damping of the system; the quantity (v) is of most interest if the main control is not to be allowed to overswing beyond its maximum steady deflection, and therefore any energy which it possesses when it reaches the maximum deflection dynamically has to be absorbed by some kind of stop.

Further calculations have been made in two specific cases, one for the ailerons of a 50,000-lb aircraft and one for the ailerons of a 300,000-lb aircraft, both of normal wing loading (about 50 lb/sq ft), of the complete motion of the control following a given tab application, with the follow-up ratio zero. This has been done for different forward speeds, paying especial attention to the low speeds where the response of the main control to tab movement is worst; also as a check on the validity of the assumption regarding the manner of applying tab, which has been made in deriving our generalised curves, in one case the tab has been assumed to be applied sinusoidally instead of at a linear rate.

These specific calculations give a complete picture of the motion of the control surface and also afford an arithmetical check on the generalised method.

3. *Determination of Coefficients.*—Before going on to the results of the calculations, it is as well to discuss briefly the determination of the coefficients used in the evaluation of the dynamic characteristics of the servo-tab system.

Apart from the purely geometric parameters, there are three aerodynamic coefficients that need to be estimated, namely b_2, b_3 and $h_{\dot\xi}$. The most satisfactory of published methods of estimating b_1 and b_2 are given in Ref. 2, provided a reasonable value of b_2 is being worked to (*i.e.*, about $-0{\cdot}2$ to $-0{\cdot}3$). The value of $h_{\dot\xi}$ is difficult to establish. Many theoretical studies have been made of this last coefficient, Refs. 3 and 4, but they do not agree too well with the scanty experimental data which is available. For the purpose of this note therefore a brief study has been made of the experimental data from Ref. 5 and the following empirical law found, which seems to fit the data reasonably well:—

$$h_{\dot{\xi}} = 0 \cdot 8E^{0 \cdot 4}\left(1 + \frac{B}{100}\right)$$

where

$$E = \frac{\text{control chord}}{\text{total surface chord}} \text{ (measured over the control span)}$$

and

B is the percentage balance of the main control.

It must be emphasised, however, that very little information exists, and it would be unwise to use the formula for any work in which an accurate knowledge of the damping is essential. The effect of balance on the control is particularly dubious, as it is clear that it must vary with the type of balance, *e.g.*, an internally sealed balanced control should have more damping than a set-back hinge balanced control.

4. *Results*.—4.1. *Generalised Investigation*.—Fig. 1 serves to define the characteristics involved with the exception of T, the periodic time of oscillation of the control system in the absence of damping:—

ξ_r is the magnitude of the first overshoot

$t_{1/2}$ is the time taken for the oscillation to damp to half amplitude

$\dot{\xi}_0$ is the angular velocity possessed by the control when it first reaches the equilibrium position

t_L is the lag of the control behind the tab when the control first reaches the equilibrium position.

In Figs. 2 and 3 the generalised results are plotted. In Fig. 2, $t_{1/2}/T$ is plotted against T/t_0, where t_0 is the time of application of the pilot's control, the curves being drawn for constant values of $t_L \cdot 2\pi/T$ and constant values of ξ_r/ξ_0.

The values of T and $t_{1/2}$ can be obtained from the following equations:

$$T = 2\pi \frac{C_{\xi}}{V} \sqrt{-\left(\frac{2i_f}{b_2 + Nb_3}\right)}$$

$$t_{1/2} = 1 \cdot 386 \frac{C_{\xi}}{V} \frac{i_f}{h_{\dot{\xi}}} .$$

Following numerical evaluation of T and $t_{1/2}$, the other characteristics can be obtained from Figs. 2 and 3.

4.2. *Specific Calculations*.—The data assumed in evaluating the time history of the motion of the ailerons of the 50,000-lb aircraft are listed in Table 1 (in the Appendix); in the main they are based on an actual servo-tab design for the *Lancaster* ailerons, which has been made at the Royal Aircraft Establishment and will shortly be flight tested. Some alterations have been made, however, bearing in mind that this design was a modification of an existing control system and contained features that would have been different if the servo-tab system had been envisaged from the outset. The data assumed for the 300,000-lb aircraft are in fact applicable to the *Brabazon* aircraft. When, however, data were not available at the time of making the calculations, the corresponding dimensions appropriate to the *Lancaster* were scaled up.

For the 50,000-lb aircraft three cases were considered, all for zero follow-up ratio, and linear rate of tab application:

(a) a time of tab application of $0 \cdot 25$ sec, forward speed 50 m.p.h.

(b) ,, ,, ,, ,, ,, ,, $0 \cdot 25$,, ,, ,, 100 ,,

(c) ,, ,, ,, ,, ,, ,, $0 \cdot 50$,, ,, ,, 100 ,,

The results are shown in Fig. 4a. It is seen that the overshoot at very low speeds can amount to about 20 per cent if rapid control movements are made, but increased flight speed and slower control application is beneficial in cutting down these overshoots. The lag of the main control behind the tab is small as soon as the aircraft is in flight but can be about $0 \cdot 2$ sec when the controls are first beginning to be used accurately (about 50 m.p.h.).

For the 300,000-lb aircraft two cases were considered, both for zero follow-up ratio, $0 \cdot 25$ sec time of application of the pilot's control, and forward speed of 100 m.p.h.:

(i) linear application of tab

(ii) sinusoidal application of tab.

It is clear at once that little difference is caused in the result by varying the mode of application of tab, and that the assumption of linear application made in the rest of the calculations is justified. Further, the overshoot is slightly reduced by the increase in size for the same conditions, though the lag is increased.

Once the aircraft is in flight (say $V > 100$ m.p.h.), the lag of the control behind the tab even when the tab is operated extremely fast ($t \leqslant 0 \cdot 25$ sec) is small compared with the response time of the aircraft; the lag is about $0 \cdot 05$ sec for the 50,000-lb aircraft and $0 \cdot 15$ sec for the 300,000-lb aircraft at 100 m.p.h. It is not expected that any difficulty would be introduced in flight by lags of these orders, though the pilot may possibly remark on the 'sponginess' of feel due to the light forces, and the poor response of the aircraft to the main controls at low speeds. Experience with spring-tabs to date, however, suggests that this complaint of 'sponginess' is one that often disappears as soon as the pilots become acclimatised to the new type of control.

The overshoot, which may be up to $0 \cdot 2$ of the maximum control deflection, with associated velocities at the first passage through the equilibrium position of up to 30 deg/sec, must be dealt with. There have been several methods suggested amongst which the most plausible are:—

(1) allowing the control to overswing by suitable clearance on the main surfaces,

(2) damping the motion of the main control surface,

(3) damping the motion of the tab (*i.e.*, the main control run),

(4) providing elastic stops to retard the control as it reaches its maximum position.

Solution (1) is not a good one, for it does not meet the case of threshing of the controls on the ground in the wind, and also complicates considerably the balance design and precludes, from a practical point of view, the use of internally balanced controls; (2) and (3) both have the objection that the restriction is felt during the whole of the flight, and (2) especially adversely affects the lag of main control surface behind the tab, while (3) does not help the ground threshing case. It is felt that (4) is the most promising solution, and experiments using this system are now under way at the R.A.E. on the servo-tab rudder of a special *Lancaster*. This last solution will also cope with the case of the control threshing in the wind on the ground while at rest, or taxi-ing. It is considered that the ground case will give the most severe design case for the stops, *i.e.*, the kinetic energy requiring to be absorbed from the control surface by the stops will be greater in this case than in flight. Part of the testing of these stops on the *Lancaster* at the R.A.E. will consist of an investigation of this point, which should enable us, therefore, to give some guidance to designers on the maximum control angular velocity which should be assumed for the design case, if the ground case does prove to be the most severe.

5. *Worked Example.*—For the purpose of illustrating the method of using the generalised curves to evaluate dynamic characteristics of a system we have set out below a worked example.

Taking the case of the 50,000-lb aircraft of section 4.2, the particular conditions are:—

(*a*) a forward speed of 50 m.p.h. E.A.S.

(*b*) a follow-up ratio of 0 : 1

(*c*) a time of pilot's control application of 0·25 sec at a linear rate.

The relevant data from Table I is:—

(i) aileron surface area $S_\xi = 41 \cdot 0$ sq ft

(ii) aileron mean chord $C_\xi = 2 \cdot 37$ ft

(iii) moment of inertia of aileron-tab system about aileron hinge axis $= 3 \cdot 26$ slugs ft^2

(iv) $b_2 \left(= \dfrac{\partial C_H}{\partial \xi}\right) = -0 \cdot 3$

(v) $h_{\dot\xi} = 0 \cdot 55$

$$i_f = \frac{I_f}{\rho S_\xi C_\xi{}^3} = \frac{3 \cdot 26}{0 \cdot 002378 \times 41 \cdot 0 \times 2 \cdot 37^3} = 2 \cdot 51$$

$$T = 2\pi \frac{C_\xi}{V} \sqrt{-\left(\frac{2 i_f}{b_2 + N b_3}\right)} = 2\pi \times \frac{2 \cdot 37}{73 \cdot 3} \sqrt{\left(\frac{2 \times 2 \cdot 51}{0 \cdot 3}\right)} = 0 \cdot 83 \text{ sec}$$

$$t_{1/2} = 1 \cdot 386 \frac{C_\xi}{V} \frac{i_f}{h_{\dot\xi}} = 0 \cdot 205 \text{ sec}$$

and

$$\frac{t_{1/2}}{T} = \frac{0 \cdot 205}{0 \cdot 83} = 0 \cdot 246 \, ; \quad \frac{T}{t_0} = \frac{0 \cdot 83}{0 \cdot 25} = 3 \cdot 32 \, .$$

From Fig. 2,

$$\frac{\xi_r}{\xi_0} = 0 \cdot 185$$

$$\frac{2\pi t_L}{T} = 1 \cdot 46$$

therefore

$$t_L = 0 \cdot 193 \text{ sec.}$$

From Fig. 3,

$$\frac{T}{2\pi} \frac{\dot\xi_0}{\xi_0} = 0 \cdot 3$$

$$\frac{\dot\xi_0}{\xi_0} = 2 \cdot 27 \, .$$

Summarising the results:—

Quantity	Value from generalised method
$\dfrac{\text{Aileron overshoot}}{\text{Steady aileron displacement}}$	0·185
Time for oscillation to damp to half-amplitude	0·205 sec
Lag of main control behind tab at time it first reaches its equilibrium position	0·193 sec
Angular velocity possessed by main control surface as it passes through its equilibrium position for the first time	$2 \cdot 27 \xi_0$ deg/sec*

* ξ_0 = final steady aileron deflection measured in degrees.

6. *Conclusions.*—(i) A satisfactory generalised graphical method of calculating the dynamic characteristics of a servo-tab system of control has been obtained. This method will enable design estimates to be made rapidly, and will facilitate the examination of the numerical effects of variation in the control system, *i.e.*, the effect of alterations in inertia or follow-up ratio.

(ii) The main characteristics of the servo-tab system, as illustrated by the two specific cases investigated completely, present no special problem to the pilot or designer. It is concluded that the most promising means of dealing with overshoot is to provide elastic stops at the maximum deflection position; this solution will also cope with the threshing of the controls on the ground in the wind. It is expected that the ground case will be the most severe, *i.e.*, the stop will have to absorb the maximum kinetic energy from the control in the ground threshing case, and practical experiments to test this case are at present in progress at the R.A.E.

Notation

S_ξ Control area

C_ξ Control mean chord

ξ Control deflection

β Control tab deflection

x Displacement of pilot's control column

ξ_0 Final steady control deflection

β_0 Final steady tab deflection

x_0 Final steady displacement of pilot's control column

$\bar{\xi} = \xi/\xi_0$

$\bar{\beta} = \beta/\beta_0$

$\bar{x} = x/x_0$

ξ_r Angular overshoot of main control

m Stick gearing $= \left(\frac{\partial \beta}{\partial x}\right)_\xi$

N Follow-up ratio $= \left(\frac{\partial \beta}{\partial \xi}\right)_x$

$$b_2 = \frac{\partial(\text{control hinge-movement coefficient})}{\partial(\text{control angle in radians})}$$

$$b_3 = \frac{\partial(\text{control hinge-moment coefficient})}{\partial(\text{tab angle in radians})}$$

I Inertia of control-tab system about control hinge axis

$$i_f = \frac{I}{\rho S_\xi C_\xi^{\,3}}$$

$h_{\dot{\xi}}$ Control damping coefficient ($\rho V S_\xi C_\xi^{\,2} h_{\dot{\xi}} \dot{\xi}$ = control damping moment)

T Period of free vibration of system, ignoring damping

$$= 2\pi \frac{C_\xi}{V} \sqrt{-\left(\frac{2i_f}{b_2 + Nb_3}\right)}$$

t_0 Time lag of main control } Defined in Fig. 1.

$t_{1/2}$ Time to damp to half-amplitude } Defined in Fig. 1.

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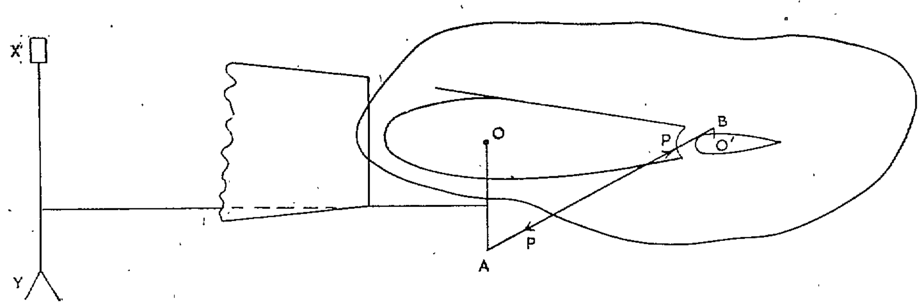
APPENDIX I

Derivation of the Equations of Motion



The sketch above shows a servo-tab system with follow-up. XY representing the pilot's stick is coupled, via the control run, to a link OA freely pivoted about the control hinge axis. The extremity A of this link is connected to the end B of the arm O'B which forms part of the control tab.

The tab deflection β is, in general, a function of both stick position and main control deflection, *i.e.*,

$$\beta = f(x, \xi)$$

where x measures the stick movement relative to some fixed datum and ξ denotes angular deflection of main control.

Thus tab movement

$$\beta = \left(\frac{\partial \beta}{\partial x}\right)_\xi x + \left(\frac{\partial \beta}{\partial \xi}\right)_x \xi = mx + N\xi \quad \ldots \quad (1)$$

where

$$m = \left(\frac{\partial \beta}{\partial x}\right)_\xi, \text{ stick-tab gearing}$$

and

$$N = \left(\frac{\partial \beta}{\partial \xi}\right)_x, \text{ follow-up ratio.}$$

Both m and N are assumed to remain constant over the whole range of control deflection.

At any instant during the control operation let the compressive force induced in the member AB be P.

Consider now the dynamics of that portion of the system enclosed within the envelope.

Kinetic energy $T = \frac{1}{2}I\dot{\xi}^2 + \frac{1}{2}m_t h^2\dot{\xi}^2 + \frac{1}{2}I_t(\dot{\xi} + \dot{\beta})^2$

where

- I is moment of inertia of control (less tab) about its hinge line
- I_t is moment of inertia of tab about its hinge line
- m_t is mass of tab
- h is distance between hinge axis of main control and its tab.

In practice I_t is very small and hence the expression for kinetic energy of the system can be rewritten $T = \frac{1}{2}I_f\dot{\xi}^2$

where

I_f is moment of inertia of control + tab, about the control axis.

The external forces applied to the main control are of three kinds:—

(i) force transmitted through the member AB

(ii) aerodynamic moments resulting from deflection of main control and its tab

(iii) aerodynamic damping moments.

Thus our equation of motion takes the form

$$I_f\ddot{\xi} = qS_\xi C_\xi(b_0 + b_1\alpha + b_2\xi + b_3\beta) - (H_{\dot{\xi}}\dot{\xi} + H_{\dot{\beta}}\dot{\beta}) - Pa \,. \qquad (2)$$

In the practical case both the terms $H_{\dot{\beta}}\dot{\beta}$ and Pa can be ignored without introducing an appreciable error, thus

$$I_f\ddot{\xi} + \rho VS_\xi C_\xi{}^2 h_{\dot{\xi}}\dot{\xi} - qS_\xi C_\xi(b_2 + Nb_3)\xi = qS_\xi C_\xi(b_0 + b_1\alpha) + qS_\xi C_\xi b_3 mx \qquad (3)$$

where

$$\rho VS_\xi C_\xi{}^2 h_{\dot{\xi}} = H_{\dot{\xi}} \,.$$

Introducing non-dimensional time variable $\tau = (V/C_\xi)t$ and replacing I_f by its non-dimensional measure i_f $(= I_f/\rho S_\xi C_\xi{}^3)$, and if at the same time we measure ξ and x from their initial equilibrium positions, the term $qS_\xi C_\xi(b_0 + b_1\alpha)$ will drop out of (3) and the equation will reduce to the form

$$i_f\frac{d^2\xi}{d\tau^2} + h_{\dot{\xi}}\frac{d\xi}{d\tau} - \tfrac{1}{2}(b_2 + Nb_3)\xi = \tfrac{1}{2}b_3 mx \,. \qquad (4)$$

If we denote the final steady values of ξ, β and x by ξ_0, β_0 and x_0 respectively then we obtain directly from (4) the equation:—

$$-(b_2 + Nb_3)\xi_0 = b_3 mx_0 \,. \qquad (5)$$

Using this equation and replacing

$$\xi,\ \beta \text{ and } x \text{ by } \bar{\xi}\left(= \frac{\xi}{\xi_0}\right),\ \bar{\beta}\left(= \frac{\beta}{\beta_0}\right) \text{ and } \bar{x}\left(= \frac{x}{x_0}\right)$$

we finally arrive at the equation:—

$$i_f\frac{d^2\bar{\xi}}{d\tau^2} + h_{\dot{\xi}}\frac{d\bar{\xi}}{d\tau} - \tfrac{1}{2}(b_2 + Nb_3)\bar{\xi} = -\tfrac{1}{2}(b_2 + Nb_3)\bar{x} \,.$$

Period of free vibration of system assuming no damping $= 2\pi\dfrac{C_\xi}{V}\sqrt{-\left(\dfrac{2i_f}{b_2 + Nb_3}\right)}$.

Half-amplitude damping time $= 2\log_e 2\,\dfrac{C_\xi}{V}\,\dfrac{i_f}{h_{\dot{\xi}}}$

TABLE I

Values used in Calculation

(Data applicable to all curves given in Figs. 4a, 4b)

	50,000-lb aircraft	300,000-lb aircraft
Moment of inertia of aileron about aileron hinge axis	$1\cdot091$ slugs ft^2	$16\cdot60$ slugs ft^2
Moment of inertia of tab about tab hinge axis	$0\cdot013$ slugs ft^2	$0\cdot20$ slugs ft^2
Mass of tab	$0\cdot775$ slugs	$3\cdot63$ slugs
Distance between hinge axes of tab and aileron	$1\cdot67$ ft	$3\cdot0$ ft
Wing chord (mean over aileron span)	$11\cdot20$ ft	$18\cdot5$ ft
Aileron mean chord	$2\cdot37$ ft	$4\cdot26$ ft
Aileron span	$17\cdot3$ ft	$45\cdot0$ ft
Tab mean chord	$0\cdot7$ ft	$1\cdot26$ ft
$b_0 = b_1$	0	0
b_2	$-0\cdot3$	$-0\cdot3$
Maximum aileron deflection	25 deg	25 deg

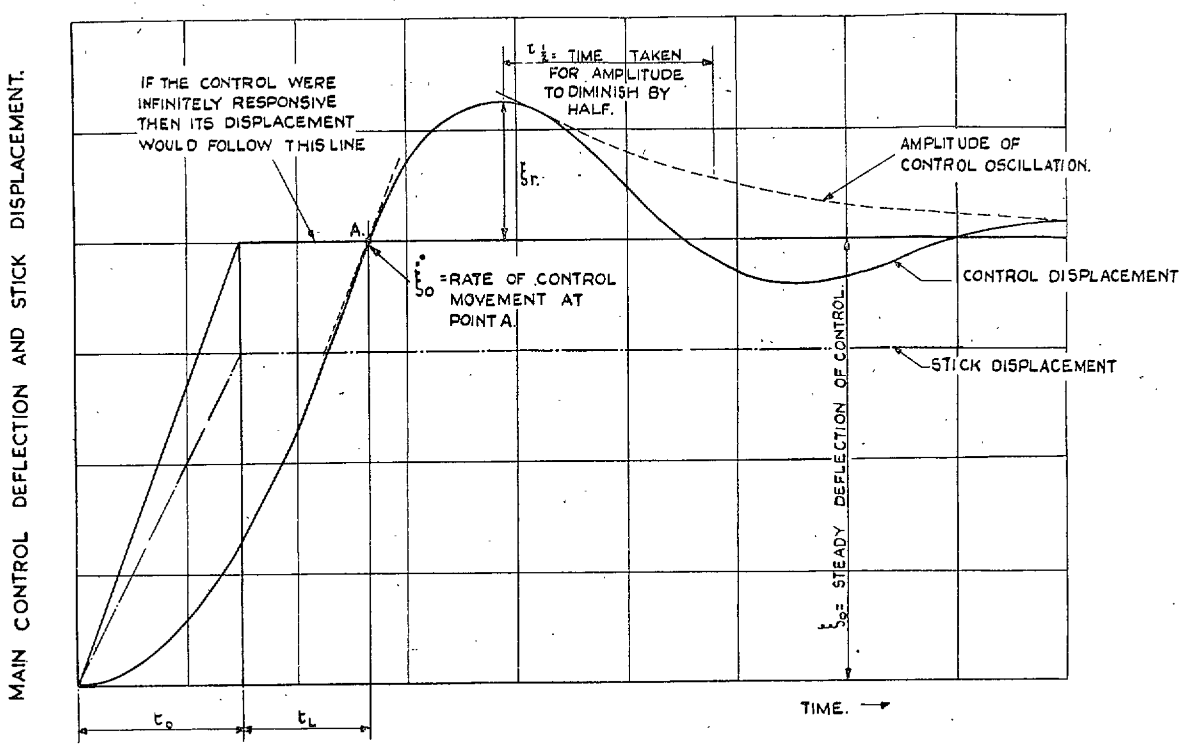


FIG. 1. Sketch giving definition of variables concerned.

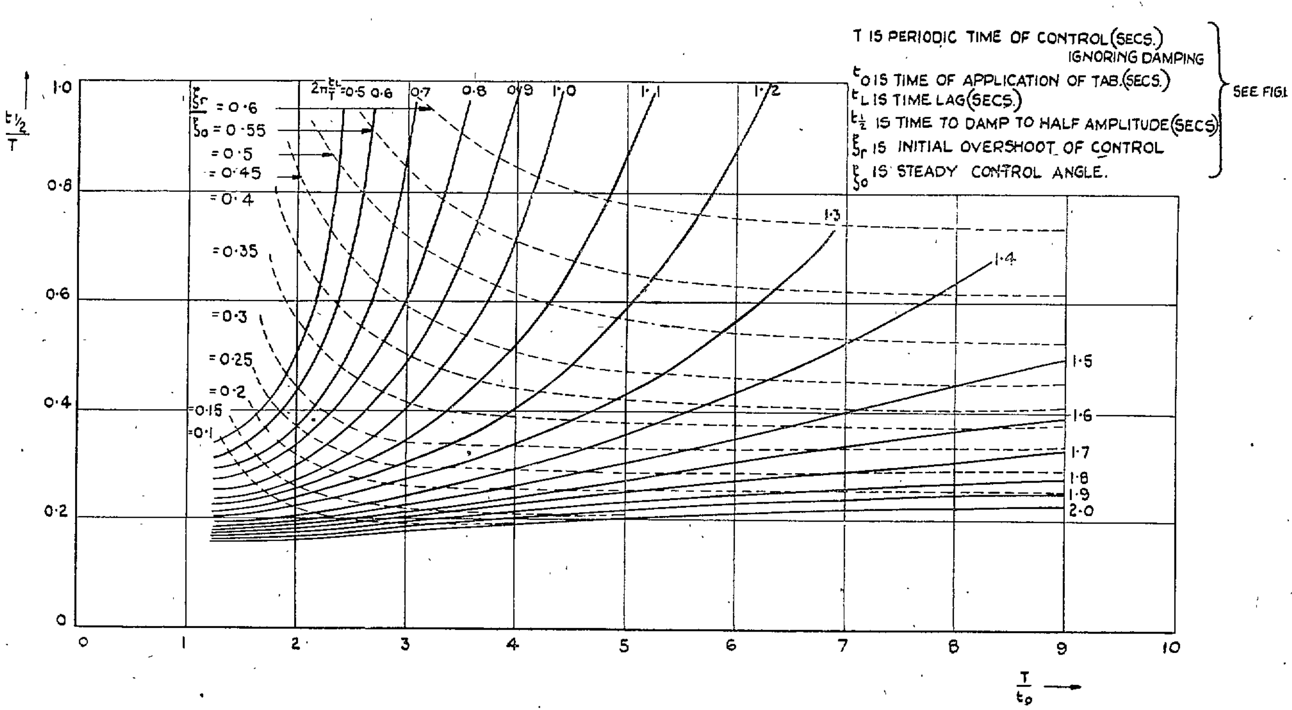


FIG. 2. Oscillation of servo-tab control system: Time lag and overshoot expressed as functions of half-amplitude damping time and time of application of tab.

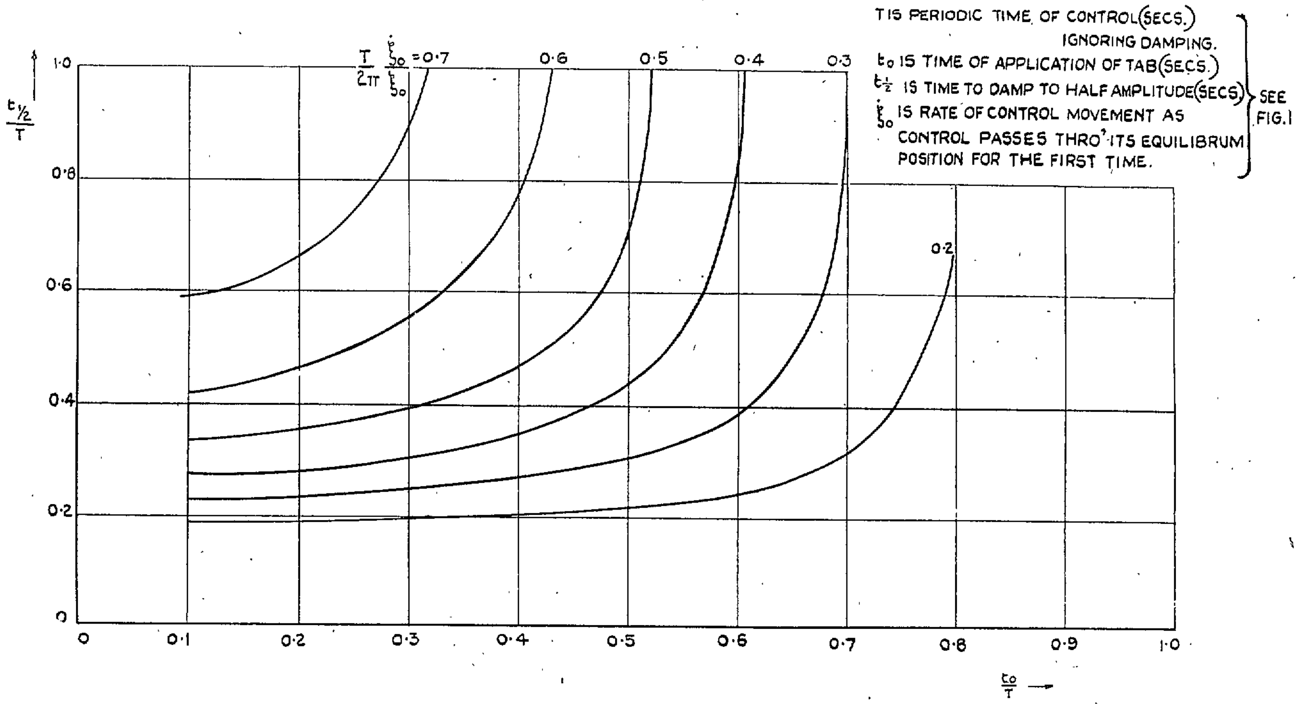


FIG. 3. Oscillation of servo-tab control system: Angular velocity of control as it passes through equilibrium position for the first time.

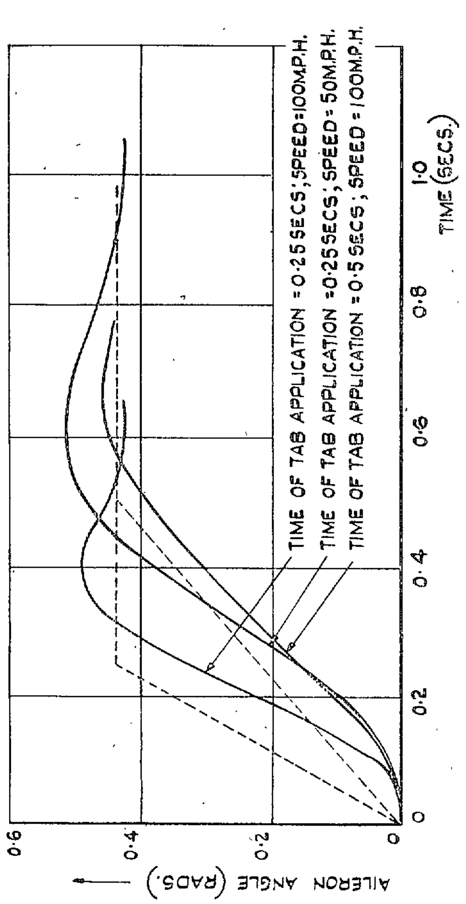


FIG. 4a. 100-ft span aircraft (50,000 lb A.U.W.).

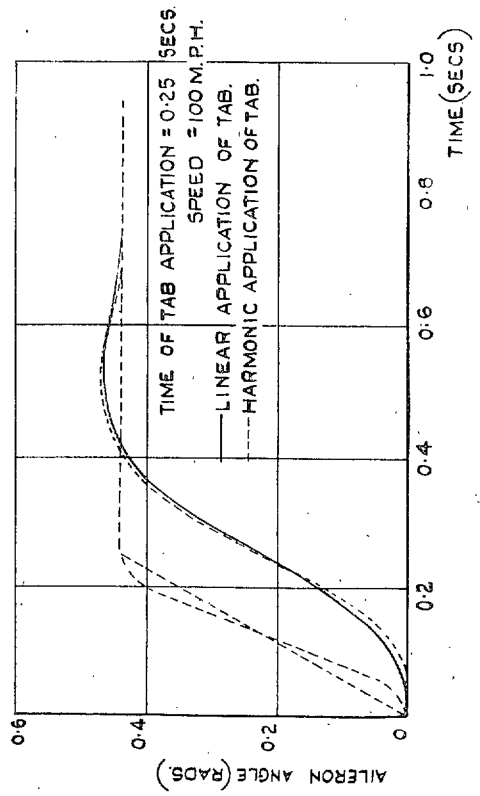


FIG. 4b. 230-ft span aircraft (300,000 lb A.U.W.).

FIG. 4 (a and b). Oscillation of servo-tab controls (ailerons) on two sizes of aircraft (at sea-level).

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