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The Geared Elevator Tab and Tail-Unit Stiffness Requirements

By

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Summary.—The effect of a flexible, geared, elevator tab upon the validity of the stiffness criteria for tail-units and rear fuselages is examined. To this end, the distortions of a hypothetical, semi-rigid tail-unit under the air loads induced when the elevator is displaced are calculated for various arrangements of tab and forward aerodynamic balance of the elevator. Notice is also taken of the loss of control effectiveness and change in elevator hinge moment resulting from these distortions.

It is found that a geared elevator tab covering only a fraction of the elevator span may lead to large tip distortions and appreciable reduction of control effectiveness of the elevator if it is placed near the inboard end. From consideration of the distortions of the tab and the effect upon elevator hinge moment, a torsional-stiffness criterion for tabs is proposed.

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1. *Introduction.*—This report is an extension of some previous work of Collar and Victory[1] in which minimum values for the conventional stiffness criteria for tailplanes, elevators and fuselages were proposed. These values were based partly upon general statistical evidence and partly upon investigation of the structural deformations and the resultant loss of control effectiveness. One of the assumptions made in this previous work was that the elevator was aerodynamically balanced along its whole length to the extent of reducing the hinge-moment coefficient $-b_2$ to $0 \cdot 2$. Whilst it was suggested that the stiffness requirement for a horn-balanced elevator should be more severe than that for other elevators, no special mention was made of the case where all the balance is provided by a geared tab covering only a portion of the elevator span and thus giving rise to local over-balancing resembling in some degree that produced by a horn-balance.

The present report records a limited theoretical investigation into this case. The effect of tab torsional flexibility is also examined in order to provide some basis for a torsional-stiffness criterion for tabs. Such a criterion appears desirable in view of the requirement that tabs should be as light as possible for flutter reasons[3] and might be used in their preliminary design.

The investigation refers to a simplified tail-unit; calculations being performed for tabs covering one-third, two-thirds and the whole of the elevator span. The hinge-moment coefficients for a plain elevator depend on many factors, such as the shape of the nose of the elevator, and can lie between fairly wide limits. It is thought that the values of the coefficients chosen are representative; but the characteristics of individual tail-units may vary appreciably.

In section 3 of the report the equations which give the structural distortion, loss of control effectiveness and increase in elevator hinge moment are derived for the general case. The approach to the problem is the same as that given in Ref. 1. The tail-unit is assumed to distort in identical arbitrary modes in the normal stiffness tests and under the increments in aerodynamic loading due to elevator displacement. The magnitudes of the distortions are obtained by equating the work done by the aerodynamic forces and the increases in the strain energies of the tail unit. In section 4 values of the aerodynamic coefficients and modal functions are substituted in the

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equations and quantitative results obtained for various proportions of tab and forward aerodynamic balance. The distortions of an elevator with a fractional-span tab at its inboard end differ considerably from those of an elevator with an equivalent amount of only forward balance and emphasise the value of correct positioning of the tab as a means of reducing the distortions of an elevator and, consequently, increasing its effectiveness. From the results obtained it is also deduced that the form of a tab torsional-stiffness criterion is mainly conditioned by the increase of elevator hinge moment with increase of tab flexibility and it is upon this basis that the criterion has been formulated.

A subsidiary calculation has been carried out to find the effect of using different modal assumptions and the results are given in an Appendix.

2. *Assumptions.*—The tail-unit considered (Fig. 1) consists of a straight-tapered tailplane and elevator with a tip-chord equal to half the root-chord; the root being taken to be at the aircraft centre-line. There is no shielding of the tailplane by the fuselage. The elevators extend over the whole of the tailplane span and their chord is 40 per cent of the total chord. Four tab sizes are used: tabs of one-third, two-thirds and full elevator span having a chord of 10 per cent of the total chord, and a 4-per cent chord tab of full span. All the tabs have their inboard ends at the inboard end of the elevator, where the hinge moments are reacted, and have no aerodynamic balance.

It is assumed that:

(*a*) the flexural axis is straight and lies at 30 per cent of the total chord behind the tailplane leading edge

(*b*) the tailplane, elevator and tab twist linearly and the fuselage distorts in a parabolic mode (except in Appendix)

(*c*) the control circuit is rigid

(*d*) sections in the line of flight do not distort

(*e*) the tailplane pitching moment makes a negligible contribution to the fuselage distortion compared with tailplane lift

(*f*) the whole of the pitching moment of the tailplane-elevator combination is effective in producing tailplane twist

(*g*) the aerodynamic coefficients are constant over the span

(*h*) the forward aerodynamic balance of the elevator affects the elevator coefficients only

(*i*) the elevator hinge-moment coefficients b_1, b_3 are linear functions of the coefficient b_2.

3. *Analysis for General Case with Flexible Tab.*—3.1. *Structural Distortions.*—For the tail-unit of Fig. 1, the stiffness criteria of A.P. 970[2] can be expressed in the forms:

$$K_t = \frac{1}{V_D}\left(\frac{T_t}{\sigma s \bar{c}^2}\right)^{1/2} \quad \ldots \quad (1)$$

$$K_e = \frac{1}{V_D}\left(\frac{T_e}{\sigma 2s(E\bar{c})^2}\right)^{1/2} \quad \ldots \quad (2)$$

$$K_f = \frac{1}{V_D}\left(\frac{F_f}{\sigma 2s\bar{c}d}\right)^{1/2} \quad \ldots \quad (3)$$

where

K_t and K_e are the criteria of the torsional stiffnesses of the tailplane and elevator respectively, and K_f is the criterion of the fuselage vertical stiffness

$2s$ is the span and $\bar{c}$ is the mean chord of the tailplane, $E\bar{c}$ is the mean chord of the elevator

d is the distance between the wing quarter-chord point and the elevator hinge-line

$V_D\sigma^{1/2}$ is the design diving-speed of the aircraft in ft/sec E.A.S.

T_t is the torsional stiffness of the tailplane measured at a section $0 \cdot 8s$ from the centre-line

T_e is the torsional stiffness of the elevator measured between two sections each $0{\cdot}9s$ from the centre-line

F_f is the fuselage vertical stiffness between the wing quarter-chord point and the elevator hinge-line.

In (1), (2) and (3) above the compressibility factor $(1 - M^2)^{1/4}$ has been omitted for convenience. This factor can always be re-introduced at any stage.

$$K_g = \frac{1}{V_D}\left(\frac{T_g}{\sigma s_g(\varepsilon\bar{c})^2}\right)^{1/2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (4)$$

is suggested as a possible form of tab torsional-stiffness criterion where

s_g is the distance of the furthest section of the tab from the control lever

$\varepsilon\bar{c}$ is the mean chord of the tab

T_g is the torsional stiffness of the tab between its furthest section and the control lever.

Let u be the vertical deflection of the elevator hinge-line (positive downwards) and $f_t\theta$, $f_e\psi$, $f_g\tau$ be the current twists of the tailplane, elevator and tab respectively, induced by an arbitrary elevator deflection. θ, ψ, τ are the deflections (positive nose-up) of the reference sections, where the modal functions f_t, f_e, f_g are unity.

If the fuselage distorts in a mode given by $u_y = u(y/d)^n$, where y is the distance of any section downwind of the wing quarter-chord point, the slope at the elevator hinge-line is $n.u/d = n\phi$ (say).

Hence

$$\left.\begin{aligned} \alpha &= \alpha_0 + n\phi + f_t\theta \\ \eta &= \eta_0 - f_t\theta + f_e\psi \\ \beta &= -K\eta_0 - f_e\psi + f_g\tau \end{aligned}\right\}, \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (5)$$

where

α_0 is the mean tailplane incidence of the undistorted system

η_0 is the applied elevator angle at the root.

Also the local coefficients for the surfaces are given by:

$$\left.\begin{aligned} C_L &= a_1\alpha + a_2\eta + a_3\beta \\ C_M &= ea_1\alpha + (ea_2 - m)\eta + (ea_3 - m_g)\beta \\ C_H &= b_1\alpha + b_2\eta + b_3\beta \\ C_T &= c\alpha + c_2\eta + c_3\beta \end{aligned}\right\}, \quad \ldots \ldots \ldots \ldots \quad (6)$$

the lift being at the quarter-chord and the pitching moment being about the flexural axis.

Putting $q = \frac{1}{2}\rho V^2$, the lift on the tailplane is

$$2\int_0^s C_L qc\,dx = -\phi\,\frac{F_f}{d} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (7)$$

and the torque of a narrow fore-and-aft strip about the flexural axis is

$$\delta M = C_M qc^2\,\delta x.$$

The equivalent torque at the reference section is

$$\delta M' = f_t C_M qc^2\,\delta x.$$

The total equivalent hinge moment at the reference section is then

$$M' = \int_0^s f_t C_M qc^2\,dx = m_\theta\theta \text{ (say)} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (8)$$

Similarly the total elevator hinge moment at its reference section is

$$H' = \int_0^s f_e C_H q(Ec)^2\,dx = h_\psi\psi \text{ (say)} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (9)$$

and for the tab

$$G' = \int_0^{s_g} f_g C_T\,(\varepsilon c)^2\,dx = h_\tau \tau \text{ (say)} \qquad \ldots \qquad (10)$$

If we take the tip as our reference section except for the tabs where we take their outermost sections, assuming semi-rigidity, we have from (2), (3) and (4)

$$m_\theta = (f_t)_{0\cdot 8}{}^2 T_t = (f_t)_{0\cdot 8}{}^2 \sigma\,.\,V_D{}^2 s^2 \bar{c} K_t{}^2 \qquad \ldots \qquad (11)$$

$$h_\psi = 2(f_e)_{0\cdot 9}{}^2 T_e = 4(f_e)_{0\cdot 9}{}^2 \sigma V_D{}^2 s(E\bar{c})^2 K_e{}^2 \qquad \ldots \qquad (12)$$

$$h_\tau = T_g = \sigma V_D{}^2 s_g(\varepsilon\bar{c})^2 K_g{}^2. \qquad \ldots \qquad (13)$$

Also from (1),

$$F_f = 2\sigma V_D{}^2 s\bar{c}K_f{}^2. \qquad \ldots \qquad (14)$$

Writing $(V/V_D)^2 = \lambda$, substituting from equations (5), (6), (11) to (14) in (7) to (10) and differentiating with respect to η_0 we have

$$\left.\begin{aligned}
&\left(\frac{K_f{}^2}{\frac{1}{2}\rho_0\lambda} + \gamma_{11}\right)\phi' + \gamma_{12}\theta' + \gamma_{13}\psi' + \gamma_{14}\tau' = A_1\,,\\
&\gamma_{21}\phi' + \left(\frac{(f_t)_{0\cdot 8}{}^2 K_t{}^2}{\frac{1}{2}\rho_0\lambda} + \gamma_{22}\right)\theta' + \gamma_{23}\psi' + \gamma_{24}\tau' = A_2\,,\\
&\gamma_{31}\phi' + \gamma_{32}\theta' + \left(\frac{4(f_e)_{0\cdot 9}{}^2 K_e{}^2}{\frac{1}{2}\rho_0\lambda} + \gamma_{33}\right)\psi' + \gamma_{34}\tau' = A_3\,,\\
&\gamma_{41}\phi' + \gamma_{42}\theta' + \gamma_{43}\psi' + \left(\frac{K_g{}^2}{\frac{1}{2}\rho_0\lambda} + \gamma_{44}\right)\tau' = A_4\,,
\end{aligned}\right\}. \qquad \ldots \qquad (15)$$

where

$$\phi' = \frac{\partial\phi}{\partial\eta_0}\,, \qquad \theta' = \frac{\partial\theta}{\partial\eta_0}\,, \quad \text{etc.,}$$

and

$$\left.\begin{array}{ll}
\gamma_{11} = \int_0^1 a_1 n\left(\frac{c}{\bar{c}}\right) d\xi & -\gamma_{31} = \int_0^1 b_1\, n f_e\left(\frac{c}{\bar{c}}\right)^2 d\xi \\
\gamma_{12} = \int_0^1 (a_1 - a_2) f_t\left(\frac{c}{\bar{c}}\right) d\xi & -\gamma_{32} = \int_0^1 (b_1 - b_2) f_t f_e\left(\frac{c}{\bar{c}}\right)^2 d\xi \\
\gamma_{13} = \int_0^1 (a_2 - a_3) f_e\left(\frac{c}{\bar{c}}\right) d\xi & -\gamma_{33} = \int_0^1 (b_2 - b_3) f_e{}^2\left(\frac{c}{\bar{c}}\right)^2 d\xi \\
\gamma_{14} = \int_0^{s_g/s} a_3 f_g\left(\frac{c}{\bar{c}}\right) d\xi & -\gamma_{34} = \int_0^{s_g/s} b_3 f_e f_g\left(\frac{c}{\bar{c}}\right)^2 d\xi \\
-A_1 = \int_0^1 (a_2 - Ka_3)\left(\frac{c}{\bar{c}}\right) d\xi & A_3 = \int_0^1 (b_2 - Kb_3) f_e\left(\frac{c}{\bar{c}}\right)^2 d\xi \\
-\gamma_{21} = \int_0^1 e a_1 n f_t\left(\frac{c}{\bar{c}}\right)^2 d\xi & -\gamma_{41} = \int_0^{s_g/s} c_1 n f_g\left(\frac{c}{\bar{c}}\right) d\xi \\
-\gamma_{22} = \int_0^1 (ea_1 - ea_2 + m) f_t{}^2\left(\frac{c}{\bar{c}}\right)^2 d\xi & -\gamma_{42} = \int_0^{s_g/s} (c_1 - c_2) f_t f_g\left(\frac{c}{\bar{c}}\right)^2 d\xi \\
-\gamma_{23} = \int_0^1 (ea_2 - ea_3 - m + m_g) f_t f_e\left(\frac{c}{\bar{c}}\right)^2 d\xi & -\gamma_{43} = \int_0^{s_g/s} (c_2 - c_3) f_e f_g\left(\frac{c}{\bar{c}}\right)^2 d\xi \\
-\gamma_{24} = \int_0^{s_g/s} (ea_3 - m_g) f_t f_g\left(\frac{c}{\bar{c}}\right)^2 d\xi & -\gamma_{44} = \int_0^{s_g/s} c_3 f_g{}^2\left(\frac{c}{\bar{c}}\right)^2 d\xi \\
A_2 = \int_0^1 (ea_2 - eKa_3 - m + Km_g) f_t\left(\frac{c}{\bar{c}}\right)^2 d\xi & A_4 = \int_0^{s_g/s} (c_2 - Kc_3) f_g\left(\frac{c}{\bar{c}}\right)^2 d\xi
\end{array}\right\}. \quad (16)$$

The four equations (15) define the distortions of the tail-unit in terms of the values of the stiffness criteria achieved, the modes assumed and the aerodynamic coefficients. It is evident that for a rigid tab each coefficient with a 4 in its suffix will be zero.

3.2. *Loss of Control Effectiveness.*—The pitching moment of the tail about the wing quarter-chord point $= F_f\phi = P$ (say).

Therefore

$$\begin{aligned}\frac{\partial P}{\partial \eta_0} &= \phi' F_f \\ &= \rho\lambda V_D{}^2 s\bar{c}(A_1 - \gamma_{11}\phi' - \gamma_{12}\theta' - \gamma_{13}\psi' - \gamma_{14}\tau'), \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (17)\end{aligned}$$

from (14) and (15).

If the tail-unit is rigid, $\phi' = \theta' = \psi' = \tau' = 0$ and

$$\frac{\partial P}{\partial \eta_0} = \rho\lambda V_D{}^2 s\bar{c}A_1.$$

The elevator effectiveness is, therefore,

$$1 - \frac{\gamma_{11}}{A_1}\phi' - \frac{\gamma_{12}}{A_1}\theta' - \frac{\gamma_{13}}{A_1}\psi' - \frac{\gamma_{14}}{A_1}\tau' \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (18)$$

and the losses in elevator effectiveness are

$$\left.\begin{aligned} L_f &= \frac{\gamma_{11}}{A_1}\phi' \quad \text{due to fuselage distortion} \\ L_t &= \frac{\gamma_{12}}{A_1}\theta' \quad \text{due to tailplane distortion} \\ L_e &= \frac{\gamma_{13}}{A_1}\psi' \quad \text{due to elevator distortion} \\ L_g &= \frac{\gamma_{14}}{A_1}\tau' \quad \text{due to tab distortion} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (19)$$

Substituting L_f, L_t, etc., for $(\gamma_{11}/A_1)\phi'$, $(\gamma_{12}/A_1)\theta'$, etc., in equation (15), we get

$$\left.\begin{aligned} &\left(\frac{K_f{}^2}{\frac{1}{2}\rho_0\lambda} + \gamma_{11}\right)\frac{L_f}{\gamma_{11}} + L_t + L_e + L_g - 1 = 0 \\ &\frac{\gamma_{21}}{\gamma_{11}}L_f + \left(\frac{(f_t)_{0\cdot 8}{}^2K_t{}^2}{\frac{1}{2}\rho_0\lambda} + \gamma_{22}\right)\frac{L_t}{\gamma_{12}} + \frac{\gamma_{23}}{\gamma_{13}}L_e + \frac{\gamma_{24}}{\gamma_{14}}L_g - \frac{A_2}{A_1} = 0 \\ &\frac{\gamma_{31}}{\gamma_{11}}L_f + \frac{\gamma_{32}}{\gamma_{12}}L_t + \left(\frac{4(f_e)_{0\cdot 9}{}^2K_e{}^2}{\frac{1}{2}\rho_0\lambda} + \gamma_{33}\right)\frac{L_e}{\gamma_{13}} + \frac{\gamma_{34}}{\gamma_{14}}L_g - \frac{A_3}{A_1} = 0 \\ &\frac{\gamma_{41}}{\gamma_{11}}L_f + \frac{\gamma_{42}}{\gamma_{12}}L_t + \frac{\gamma_{43}}{\gamma_{13}}L_e + \left(\frac{K_g{}^2}{\frac{1}{2}\rho_0\lambda} + \gamma_{44}\right)\frac{L_g}{\gamma_{14}} - \frac{A_4}{A_1} = 0 \end{aligned}\right\}, \quad \ldots \quad \ldots \qquad (20)$$

which give the losses of control effectiveness corresponding to the distortions given by equations (15).

3.3. *Elevator Hinge Moment.*—The total hinge moment reacted by the control lever is

$$H = 2\int_0^s qC_H(Ec)^2\,dx.$$

$$\frac{\partial H}{\partial \eta_0} = 2\int_0^1 \left\{ b_1(n\phi' + f_t\theta') + b_2(1 - f_t\theta' + f_e\psi') + b_3(-K - f_e\psi' + f_g\tau') \right\} qs(Ec)^2\, d\xi, \qquad (21)$$

from (5) and (6).

Putting

$$\frac{\partial H}{\partial \eta_0} = 2\hat{b}_2 \int_0^1 qs(Ec)^2\, d\xi, \quad \ldots \qquad (22)$$

we have

$$\begin{aligned} \hat{b}_2 = (b_1 n\phi' + b_2) + \frac{1}{\int_0^1 (Ec)^2\, d\xi} \Bigg\{ & -Kb_3 \int_0^{s_g/s} (Ec)^2\, d\xi \\ & + (b_1 - b_2)\theta' \int_0^1 f_t(Ec)^2\, d\xi + b_2\psi' \int_0^1 f_e(Ec)^2\, d\xi \\ & - b_3\psi' \int_0^{s_g/s} f_e(Ec)^2\, d\xi + b_3\tau' \frac{S}{S_g} \int_0^{s_g/s} f_g(Ec)^2\, d\xi \Bigg\} . \quad \ldots \end{aligned} \qquad (23)$$

With a rigid tail-unit ($\phi' = \theta' = \psi' = \tau' = 0$) the overall hinge-moment coefficient becomes

$$\hat{b}_2\,(\text{rigid tail-unit}) = b_2 - Kb_3 \int_0^{s_g/s} (Ec)^2\, d\xi \Big/ \int_0^1 (Ec)^2\, d\xi = \bar{b}_2\ (\text{say}). \quad \ldots \qquad (23a)$$

4. *Calculated Results.*—4.1. *Structural Distortions and Loss of Control Effectiveness with Rigid Tab.*—Using the current minimum values of the stiffness criteria[2] *i.e.*, $K_f = 0 \cdot 12$, $K_t = 0 \cdot 036$, $K_e = 0 \cdot 012$, equations (15) were solved for the case of an elevator without a tab but with forward balance sufficient to reduce the hinge-moment coefficient $-b_2$ to $0 \cdot 2$. The rates of reference-section displacement obtained from the solution were regarded as 'standards' and by substituting them in equations (15) independent expressions were obtained connecting the stiffness criteria with the A and γ coefficients.

The values of the stiffness criteria necessary to maintain these standard rates of displacement were calculated for elevators with different amounts of forward balance and with full-span, 2/3-span and 1/3-span, 10-per cent chord, rigid tabs with various gear ratios. It was found that the elevator hinge-moment coefficients b_1 and b_3 could be represented as linear functions of the coefficient b_2 only; and, as it was assumed that the other coefficients were independent of the amount of elevator balance, that the stiffness criteria could be expressed in terms of the coefficient b_2 and the tab gear ratio K.

The calculations were for the design diving-speed ($\lambda = 1$) and the values of the aerodynamic coefficients used were

$$\begin{aligned} a_1 &= 4 \cdot 0, & b_1 &= -0 \cdot 05 + 0 \cdot 025 b_2, & m &= 0 \cdot 5, \\ a_2 &= 2 \cdot 5, & b_2 &= \text{from } 0 \text{ to } -0 \cdot 6, & m_g &= 0 \cdot 45, \\ a_3 &= 1 \cdot 3, & b_3 &= -0 \cdot 065 + 0 \cdot 25 b_2, & e &= 0 \cdot 05. \end{aligned}$$

The value $b_2 = -0 \cdot 6$ is appropriate to the case with no forward aerodynamic balance.

The results are given as graphs of criterion versus b_2 for constant $\bar{b}_2$, the overall elevator hinge-moment coefficient if the tail-unit were rigid (*see* (23a)).

The full-span tab case can be taken to be the ideal tab arrangement for a rigid tail-unit, in which the elevator is uniformly balanced along its whole length. A much less favourable case is that with the third-span tab placed at the elevator root; this approaches the case in which all the hinge moment of the plain elevator is reacted by a control lever at the root.

From Figs. 2 to 4 it will be seen that for all tabs the values of K_f and K_t are reduced as the proportion of geared balance increases. This is due to the total lift and pitching moment of the tail being reduced by the tab's contribution to them. There is more variety in the curves of the elevator stiffness criterion. For the full-span tab the value of the stiffness criterion necessary to maintain the distortions constant varies little when the method of balancing is changed; but with the third-span tab it can become much larger than the present minimum, especially if there is little or no forward aerodynamic balance. In the latter case the tab is at the elevator root only and has no effect on the torque on the two-thirds of the elevator which are outboard of it. Broadly speaking, if the tab is moved further outboard it affects the torque on a larger part of the elevator than before, reducing the area of the elevator torque/torsional-rigidity diagram, the size of which is a measure of the tip deflection. It should be possible to reduce the distortion of most elevators to a reasonable magnitude by moving the tab to a position far enough out on the elevator.

It will be noticed that a rigid tab modifies the elevator distortion even when its gear ratio is zero; for if we put the hinge-moment coefficients $\bar{b}_2 = b_2 = -0 \cdot 2$ so that, except for the presence of a rigid tab, each tab case corresponds to the first case taken, an elevator without a tab, we find that for the same elevator distortion a greater value of the elevator stiffness criterion is needed in the with-tab case than in the without-tab case. The difference is as much as 18 per cent for the full-span tab case. This effect of the rigid tab with zero gearing is of course due to the fact that as the elevator twists the tab acquires an angle relative to it; an effect which is represented in equation (15) by the presence in the coefficients γ_{13}, γ_{23}, γ_{33} of tab terms dissociated from the gear ratio K.

Calculations similar to those for the distortions were performed to find the loss in control effectiveness due to a third-span, 10-per cent chord, rigid tab. The losses of control effectiveness due to the distortions used above were found for the elevator-without-tab case and substituted in equations (20). The values of the stiffness criteria necessary to maintain these losses of control effectiveness constant were then found, the forward balance of the elevator and the tab gear ratio being varied as before. The calculations were again made for the design diving speed.

It will be seen from the first of equations (20) that the value of the fuselage criterion necessary for constant losses of control effectiveness is unaffected by the method of balancing, since γ_{11} is independent of the aerodynamic coefficients save a_1. For the tailplane, the replacement of a fully-balanced elevator with only forward balance by one with only geared-tab balance leads to an increase in the value of the tailplane criterion of less than $1\frac{1}{2}$ per cent. Curves of elevator criterion versus b_2 for constant $\bar{b}_2$ are given in Fig. 5. It will be seen that they are similar to the corresponding curves of criterion for constant distortion.

4.2. *Structural Distortions and Losses of Control Effectiveness with Flexible Tab.*—The tab was then allowed to distort and the tail-unit distortions were calculated over a range of values of the tab stiffness criterion, using the current values of the other criteria. This was done for three sizes of tab, namely, full-span 10-per cent chord, full-span 4-per cent chord and one-third-span 10-per cent chord. The hinge-moment coefficient $-\bar{b}_2$ was reduced to $0 \cdot 2$ solely by the tab, there being no forward aerodynamic balance. For the 4-per cent tab case the aerodynamic coefficients changed and their new values were:—

$$a_3 = 0 \cdot 5, \quad b_3 = -0 \cdot 55, \quad m_g = 0 \cdot 32, \quad c_1 = c_2 = -0 \cdot 05, \quad c_3 = -0 \cdot 2.$$

The results are given in Figs. 6 to 8 as graphs of distortion (at the design diving speed) *vs.* the proposed tab stiffness criterion (25), which differs from the criterion assumed previously (4) in that it depends upon the elevator span and not the tab span. The tab distortion tends to zero for large values of the stiffness criterion but increases rapidly when the stiffness is low. The distortions of the other surfaces vary similarly with the value of the tab stiffness criterion; for stiff tabs they approach their values for a rigid tab but again show a fairly sudden increase when the tab stiffness is reduced.

Calculations were performed to find the difference in loss of control effectiveness between elevators with rigid and flexible tabs; the calculations covering variation of both speed and tab stiffness criterion.

The calculations giving the loss of control-effectiveness *vs.* stiffness criterion were for the design diving speed. The results are presented as curves of the ratio of the loss of control effectiveness with a flexible tab to that for the corresponding rigid tab versus the proposed tab stiffness criterion (Fig. 9). It is seen that the stiffness of the tab has little effect on the total loss of control effectiveness since although the elevator distortion is larger when the tab is flexible the tendency of the tab distortion is to counteract the resulting increased loss of lift.

Only one value of the tab stiffness criterion was taken for each size of flexible tab in the loss of control-effectiveness *vs.* speed calculations. The values chosen were such that the ratio of the tab reference section distortion to the gear ratio was approximately the same in each case. The actual ratios were 16·0 per cent, 16·0 per cent and 18·3 per cent for the full-span 10-per cent chord, full-span 4-per cent chord and one-third-span 10-per cent-chord tabs respectively. The losses of control effectiveness up to the control-reversal speed are given in Figs. 10 to 12 whence it is seen that there is little difference between the elevators with rigid and flexible tabs up to the design diving speed.

4.3. *Elevator Hinge Moment with Rigid and Flexible Tabs.*—The elevator hinge moments with rigid and flexible tabs were calculated for the three elevator-tab arrangements used in section 4.2 (*i.e.*, full-span 10-per cent chord, full-span 4-per cent chord and one-third-span 10-per cent chord tabs, $\bar{b}_2 = -0 \cdot 2$, no forward aerodynamic balance). The calculations covered variation of both stiffness criterion and speed.

The results of the calculation in which the tab stiffness was varied are presented in Fig. 13 as curves of the ratio of elevator hinge moment with flexible tab to that with the corresponding rigid tab *vs.* the proposed stiffness criterion. As in the case of the tab distortions the increases in the elevator hinge moments are small for large values of the stiffness criterion but increase rapidly when the stiffness is low. The calculations pertain to the design diving speed.

Fig. 14 gives the results of the calculation by which the increments in the overall elevator hinge-moment coefficient due to aero-elastic distortion $(\hat{b}_2 - \bar{b}_2)$ were found for different air speeds. The values of the stiffness criterion for the flexible tab are those used in the corresponding calculation involving loss of control effectiveness described in section 4.2.

5. *Proposed Tab Stiffness Criterion.*—Little help in the formulation of a tab stiffness criterion can be gained from consideration of the flutter aspect of the problem as this is conditioned to a large extent by the inertia characteristics of the tab. As a general rule however, the tab's torsional stiffness should be as high as considerations of its inertia will allow. Also the flexibility of the tab has been seen to have hardly any effect on the total loss of control effectiveness.

We are thus left with considerations of the structural distortions and the elevator hinge moment on which to base our tab torsional-stiffness criterion. With regard to the structural aspect of the distortions it is suggested that the distortion per unit tab-span is of more account than the tip deflection of the tab and therefore that the tip deflection of a fractional-span tab should be proportionately less than that of a full-span tab. On this basis a suitable criterion is

$$\hat{K}_g = \frac{1}{V_D}\left(\frac{T_g s_g}{\sigma s_e^{\,2}\,(\varepsilon \bar{c})^2}\right)^{1/2} . \qquad \ldots \quad (24)$$

It will be noted that this reduces to the criterion assumed initially (4) when the tab covers the whole of the elevator span. The tip deflections of the full-span tabs are almost identical for values of $\hat{K}_g$ which are greater than 0·02 (Figs. 6 and 7). For the one-third-span tab in the same range the tip deflection is about a third of that of the full-span tabs.

However, this expression for the stiffness criterion leads to relatively large elevator hinge moments with full-span tabs compared with those with one-third-span tabs. The criterion which suggests itself on comparison of the elevator hinge moments is

$$\bar{K}_g = \frac{1}{V_D}\left(\frac{T_g}{\sigma s_e\,(\varepsilon\bar{c})^2}\right)^{1/2} . \qquad (25)$$

It will be seen from Fig. 13 that a stiffness criterion of this form would lead to increases in the elevator hinge moments which would be approximate to each other for values of $\bar{K}_g$ greater than 0·01. Although this criterion increases the tab reference section displacement possible with the one-third-span tab to just below that for the full-span tab, it is preferred to that given in equation (24) as it is thought that the estimated increases in the elevator hinge moment are more serious than the structural distortions involved.

Whilst it is realized that extensive statistical data should be collected before a minimum value for the stiffness criterion is finally agreed upon, a temporary value might be of some use. From Fig. 13 it would seem that a suitable value might be 0·02, which results in an increase in elevator hinge moment of between 10 per cent and 14 per cent for the three cases considered compared with the hinge moments with a rigid tab. The corresponding calculated tab reference section displacements will be 7·5 per cent, 8·7 per cent and 8·3 per cent of the applied elevator angle for the full-span 10-per cent, full-span 4-per cent and one-third-span 10-per cent tabs respectively.

6. *Conclusions.*—It is concluded that a geared tab which covers only a part of the elevator span and which is located at the elevator root may lead to comparatively large elevator tip distortions and loss of control effectiveness. It is thought however that these can be reduced by moving the tab to a position further outboard on the elevator.

The flexibility of the tab has little effect on the loss of control effectiveness but if it is large it may lead to large tab distortions and elevator hinge moments. It is proposed that a torsional-stiffness criterion of the form $K_g = \{1/V_D\}\{T_g/[\sigma s_e(\varepsilon\bar{c})^2]\}^{1/2}$ should be used for tabs.

7. *List of Symbols.*—

a_1, a_2, a_3	=	$\partial C_L/\partial\alpha$, $\partial C_L/\partial\eta$, $\partial C_L/\partial\beta$ respectively
b_1, b_2, b_3	=	$\partial C_H/\partial\alpha$, $\partial C_H/\partial\eta$, $\partial C_H/\partial\beta$ respectively
$\hat{b}_2$		Overall elevator hinge-moment coefficient $= H/\left(\eta_0 \,.\, 2qs\int_0^1 (Ec)^2\,d\xi\right)$
$\bar{b}_2$		Value of $\hat{b}_2$ with rigid tail-unit
c_1, c_2, c_3	=	$\partial C_T/\partial\alpha$, $\partial C_T/\partial\eta$, $\partial C_T/\partial\beta$ respectively
c, (Ec), (εc)		Local chord of whole surface, elevator and tab respectively
$\bar{c}$, $(E\bar{c})$, $(\varepsilon\bar{c})$		Mean chord of whole surface, elevator and tab respectively
d		Distance between wing quarter-chord point and elevator hinge-line
ec		Distance of flexural axis behind aerodynamic axis
f_t, f_e, f_g		Modes of distortion of tailplane, elevator and tab respectively (*see* 3.1)
h_ψ, h_τ		Tip-to-root stiffnesses of elevator and tab respectively
m_θ		Tip-to-root stiffness of tailplane
n		*See* equation (5) and preceding paragraph
q		Dynamic head $(\frac{1}{2}\rho V^2)$
s		Semi-span of tailplane (and elevator)
s_g		Length of tab
x		Distance from centre-line of aircraft
A_1, A_2, A_3, A_4		Defined in equations (16)

C_L, C_M, C_H, C_T	Section lift and pitching-moment coefficients for the tailplane, and elevator and tab hinge-moment coefficients respectively
F_f	Fuselage vertical stiffness between wing quarter-chord point and elevator hinge-line
G'	Total equivalent hinge moment of tab at its reference section (*see* equation (10))
H'	Total equivalent hinge moment of elevator at its reference section (*see* equation (9))
H	Total elevator hinge moment reacted at the control lever
K	Gear ratio of tab
K_f	Fuselage vertical stiffness criterion
K_t, K_e K_g, $\bar{K}_g$, $\hat{K}_g$	Tailplane, elevator and three tab torsional-stiffness criteria
L_f, L_t, L_e, L_g,	Losses of elevator effectiveness due to distortions of fuselage, tailplane, elevator and tab respectively
M'	Total equivalent pitching moment of tailplane at its reference section (*see* equation (8))
P	Pitching moment of tail-unit about the wing quarter-chord point
T_t, T_e, T_g	Torsional stiffnesses of tailplane, elevator and tab respectively defined in section 3.1
V, V_D	Air speed and design diving speed respectively
α	Local angle of incidence of tailplane
α_0	Mean angle of incidence of rigid tailplane
β	Local tab angle
γ_{rs}	Defined in equations (16)
η, η_0	Local elevator angle and angle at control lever respectively
ϕ, θ, ψ, τ	Reference section deflections of fuselage, tailplane, elevator and tab respectively (*see* equation (5) and preceding paragraphs)
ϕ', θ', ψ', τ'	First differential coefficients of above with respect to η_0
λ	$= (V/V_D)^2$
ρ, ρ_0	Density of air at height and at sea-level respectively
σ	$= \rho/\rho_0$
ξ	$= x/s$

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APPENDIX I

Effect of Modal Assumptions

The assumptions, made in the main body of the report, that the tailplane and elevator torsion modes are linear and that the fuselage flexural mode is parabolic have some empirical support. Some calculations were performed however to find the effect on the tail-unit of alterations in the modes. This was done only for the elevator-without-tab case at the design diving speed. Each of the modes was altered separately; the tailplane and elevator modes being changed to parabolic and the fuselage to cubic. The stiffnesses of the tailplane and elevator between the root and tip were kept constant with change of mode.

The effect of variations of the modes on the reference section displacements and losses of control effectiveness are given in Tables 1 and 2 below.

TABLE 1

Tip Values of	$n=2, f_t=\xi, f_e=\xi$	$n=3, f_t=\xi, f_e=\xi$	$n=2, f_t=\xi^2, f_e=\xi$	$n=2, f_t=\xi, f_e=\xi^2$
$-\phi'$	0·105	0·088	0·112	0·111
$-\theta'$	0·272	0·282	0·142	0·292
$-\psi'$	0·181	0·176	0·170	0·113

TABLE 2

Loss of control effectiveness due to	$n=2, f_t=\xi, f_e=\xi$	$n=3, f_t=\xi, f_e=\xi$	$n=2, f_t=\xi^2, f_e=\xi$	$n=2, f_t=\xi, f_e=\xi^2$
ϕ'	0·337	0·421	0·358	0·354
θ'	0·073	0·075	0·024	0·078
ψ'	0·080	0·078	0·075	0·031
$\phi'+\theta'+\psi'$	0·490	0·575	0·457	0·464

It will be seen that in each case the largest changes from the 'standard' values occur in the deflection and loss of control effectiveness pertinent to the varied mode. In most cases the deflection or loss of control effectiveness is reduced, sometimes considerably as in the cases of the elevator and tailplane. The only exception is the fuselage loss of control effectiveness which increases 25 per cent due to the large increase in the slope per unit deflection at any point on changing from a parabolic to cubic mode. Since the loss due to fuselage distortion is a major part of the total loss the latter increases by about 17·5 per cent. It is concluded that the results given by the tailplane and elevator modes used in the main body of the work are conservative if, for the modes obtaining in practice, the signs of f and $\partial^2 f/\partial\xi^2$ are the same and there is no distortion at the root.

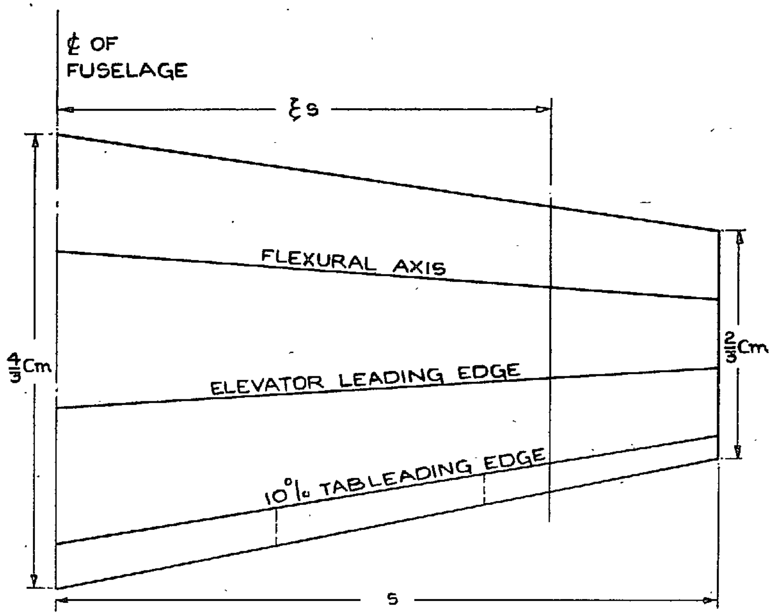
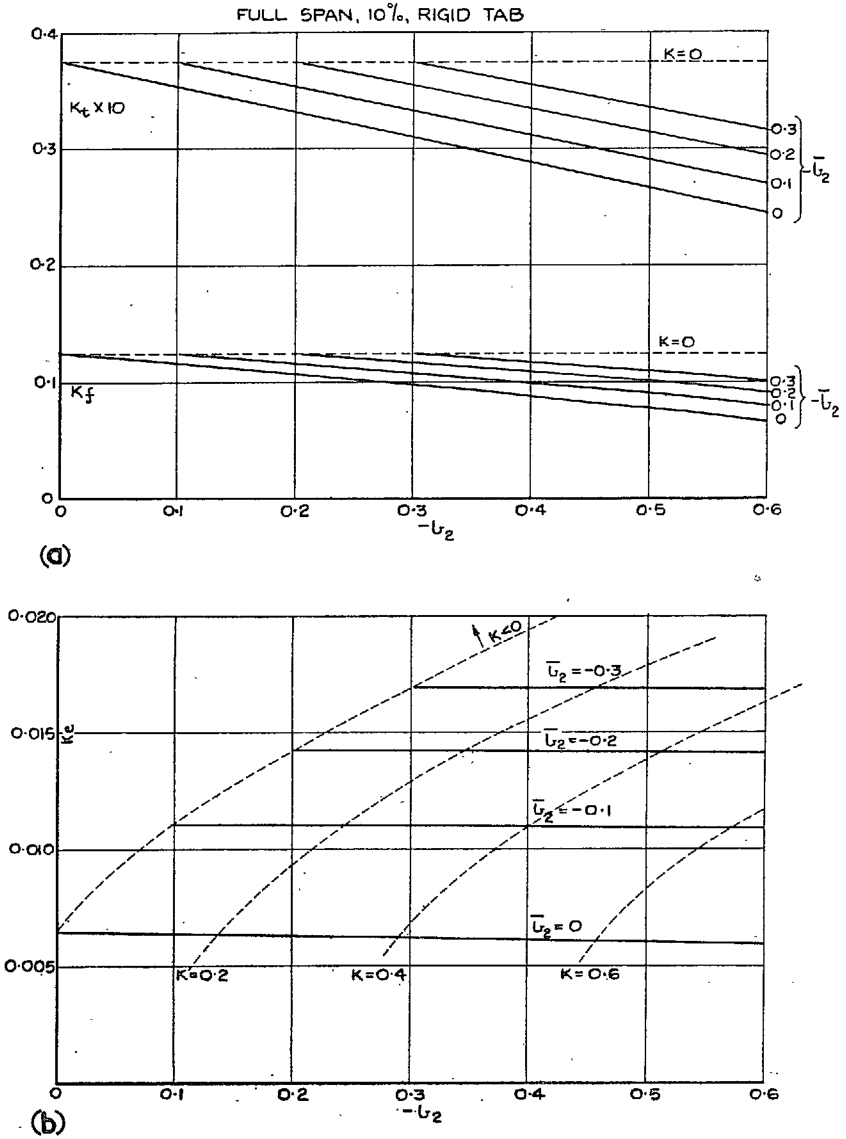
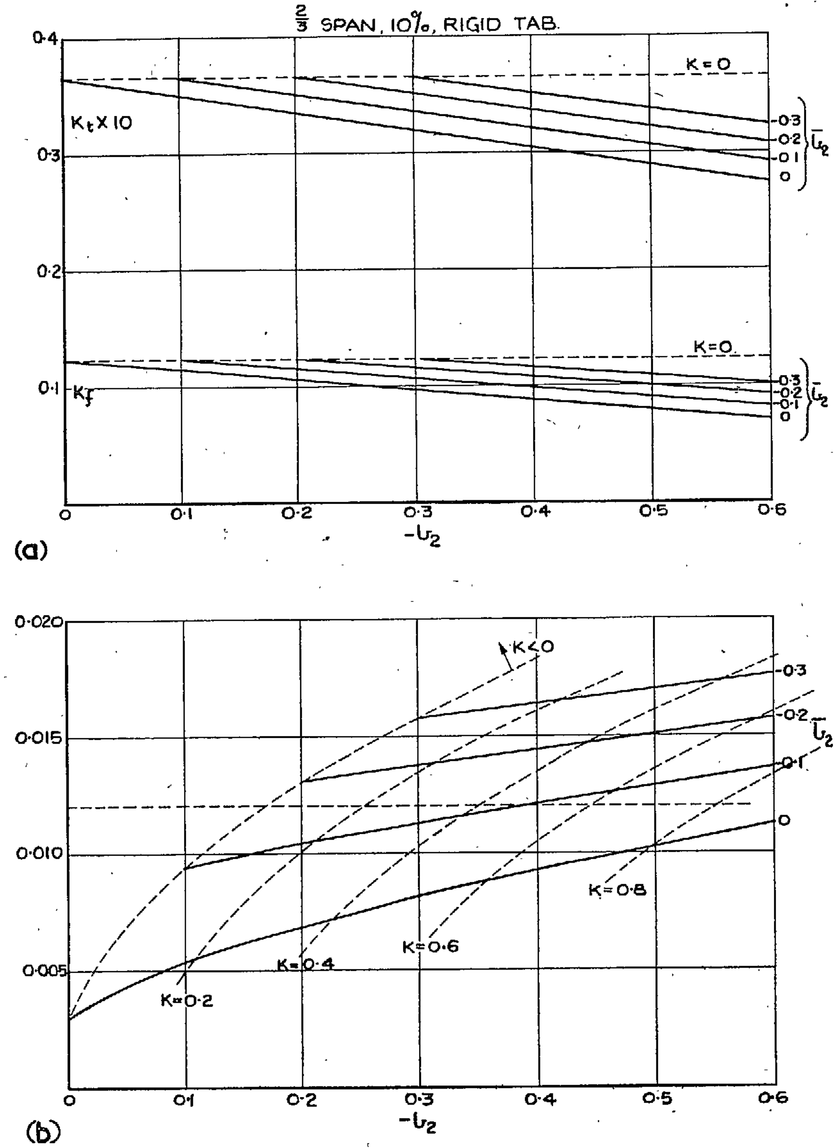


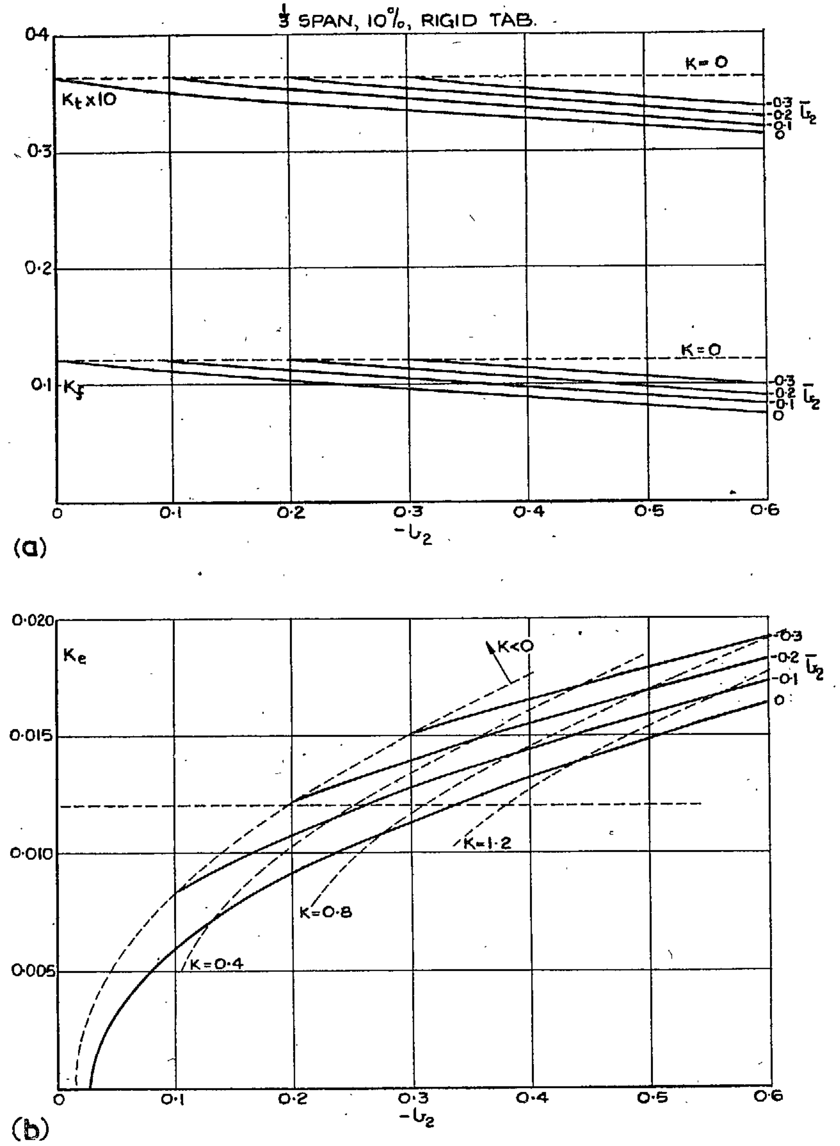
FIG. 1. Tailplane geometry.



FIGS. 2a and 2b. Values of stiffness criteria vs. b_2 for constant reference section distortions at the design diving speed.



FIGS. 3a and 3b. Values of stiffness criteria vs. b_2 for constant reference section distortions at the design diving speed.



FIGS. 4a and 4b. Values of stiffness criteria vs. b_2 for constant reference section distortions at the design diving speed.

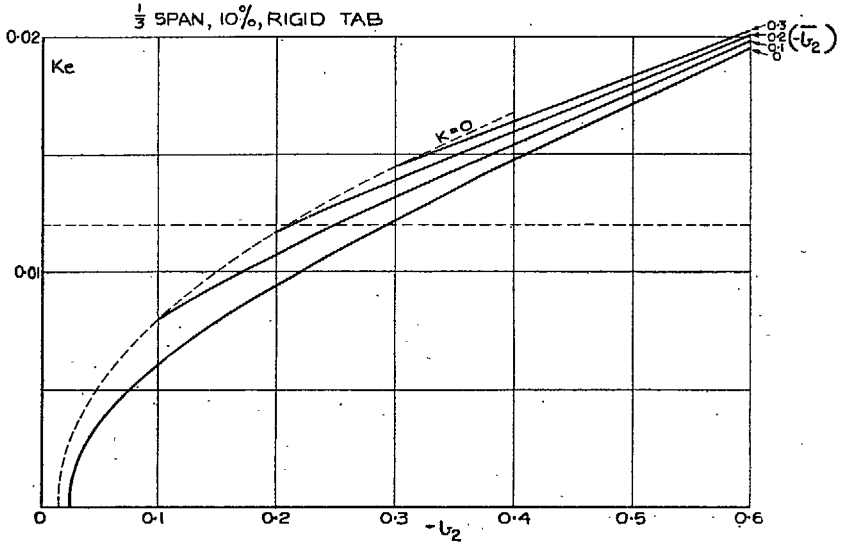


FIG. 5. Value of stiffness criterion vs. b_2 for constant loss of control effectiveness at the design diving speed.

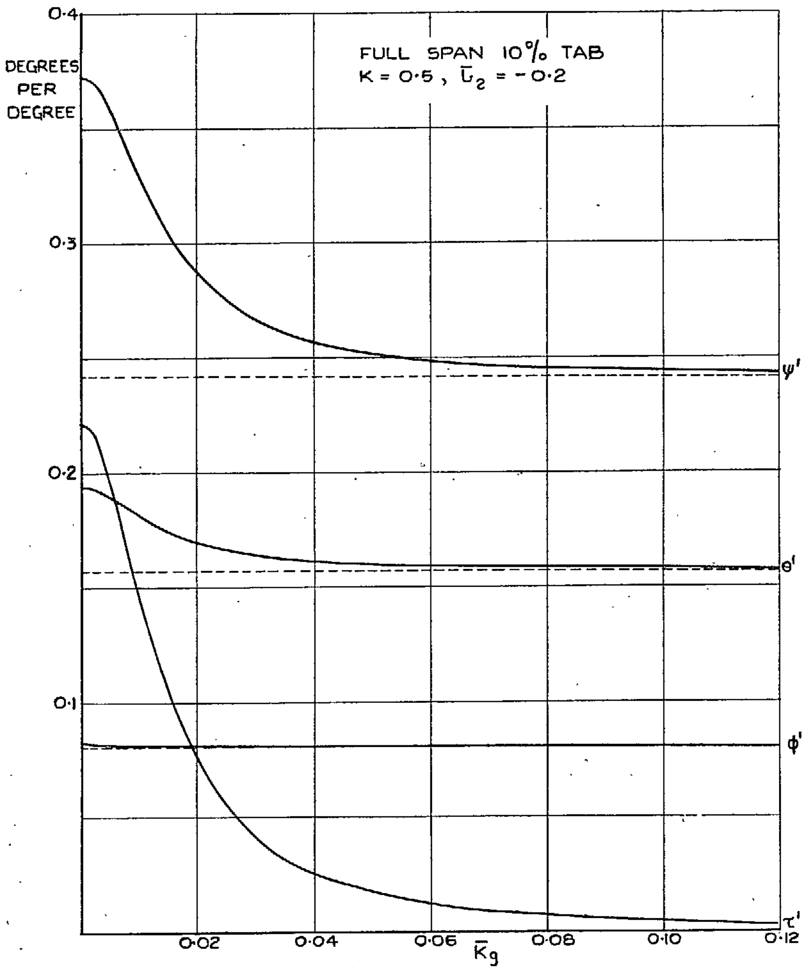


FIG. 6. Effect of tab flexibility on distortions at the design diving speed.

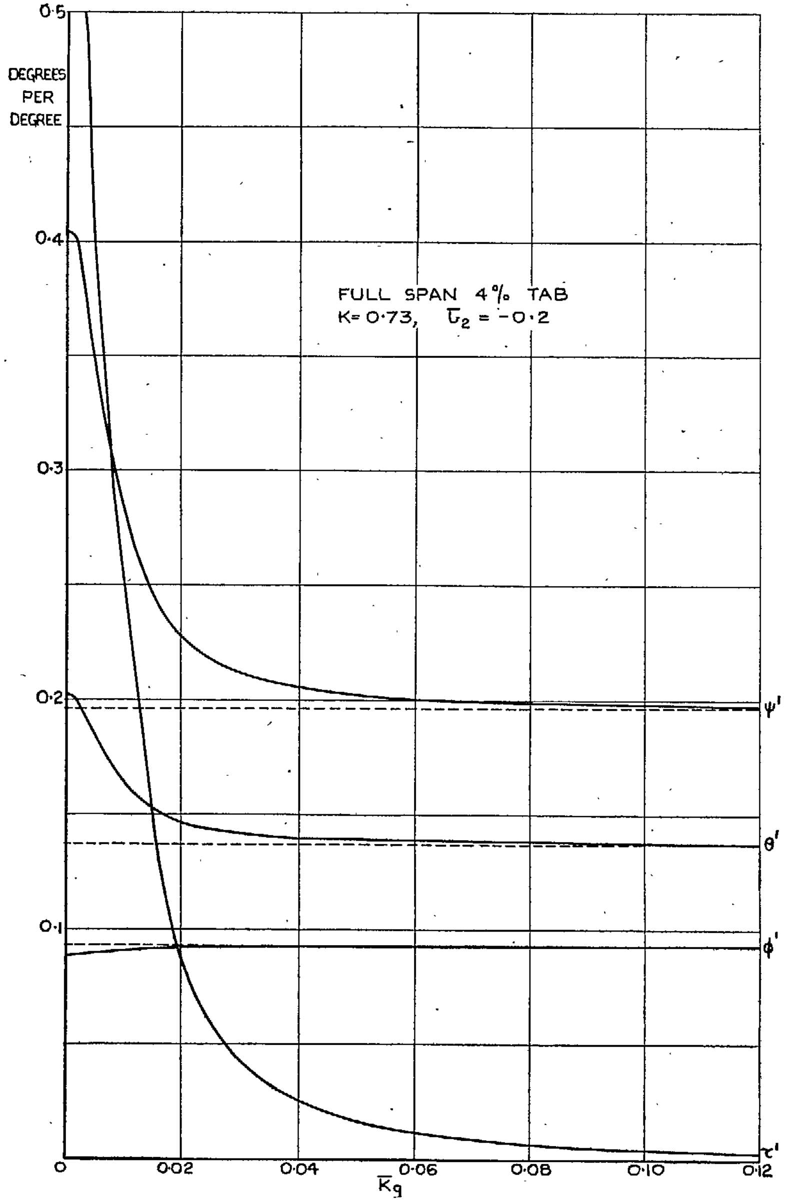


FIG. 7. Effect of tab flexibility on distortions at the design diving speed.

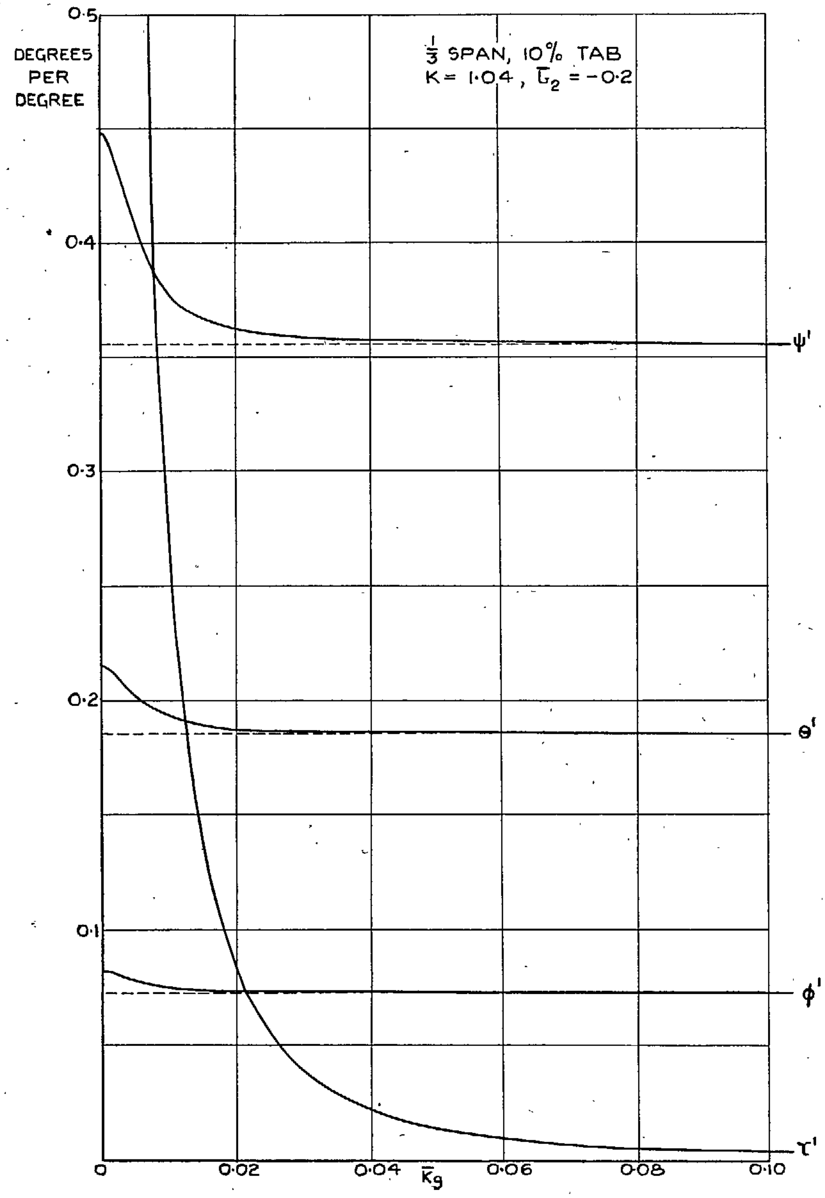


FIG. 8. Effect of tab flexibility on distortions at the design diving speed.

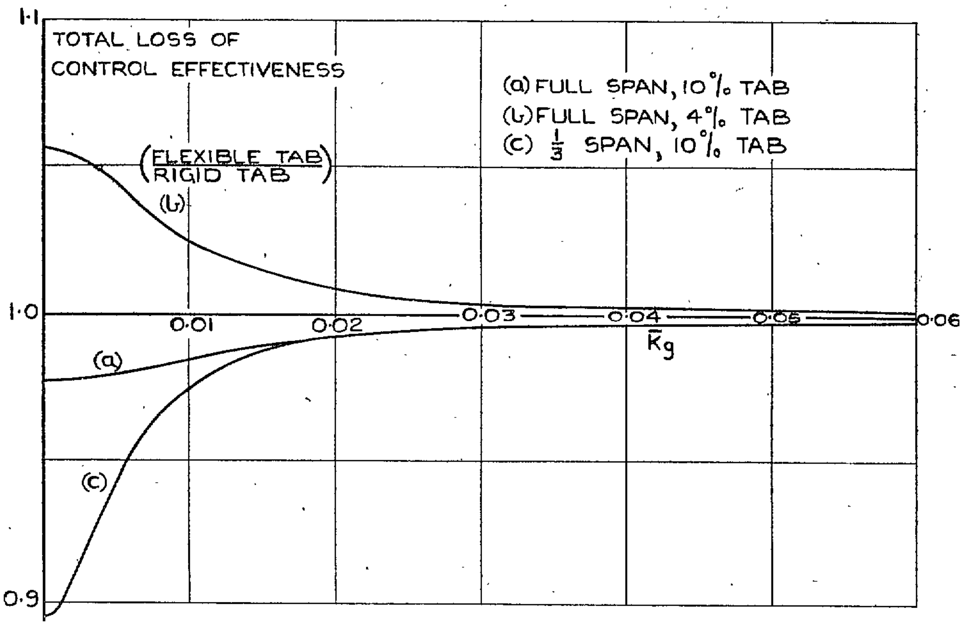


FIG. 9. Effect of tab flexibility on total loss of control effectiveness at the design diving speed.

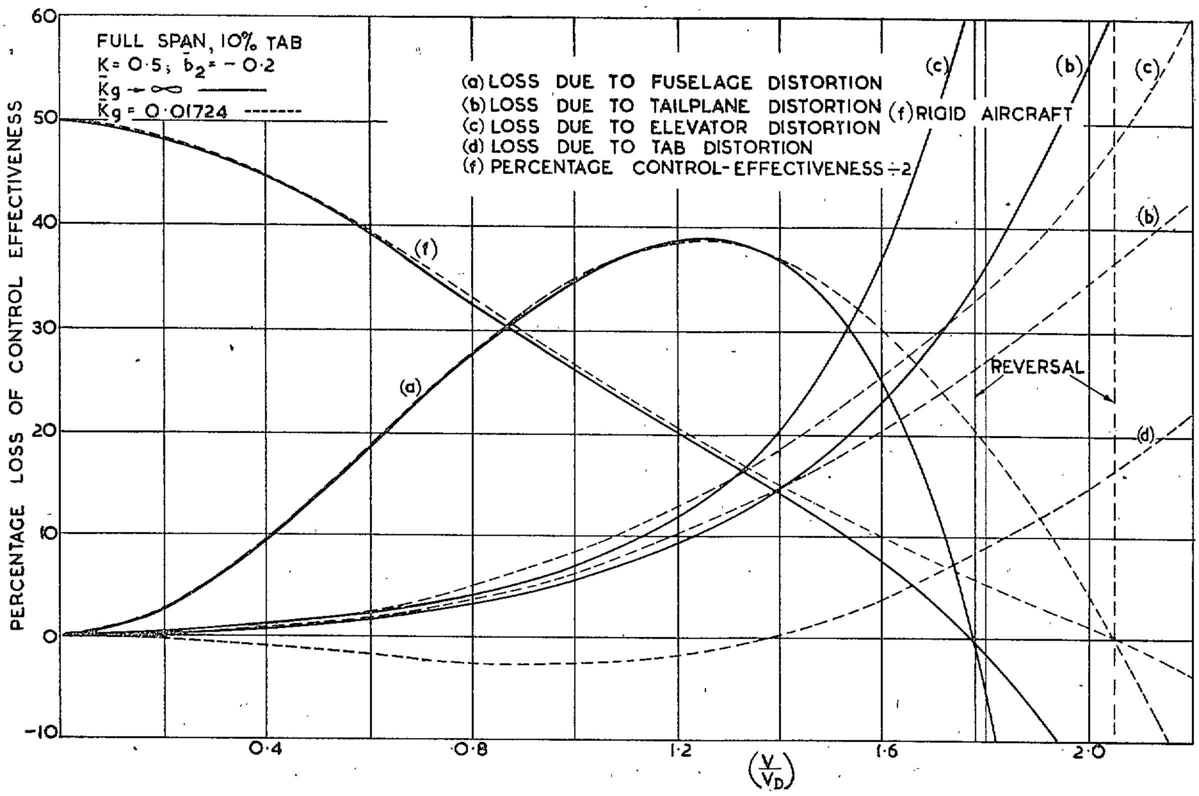


FIG. 10. Effect of tab flexibility on losses of control effectiveness.

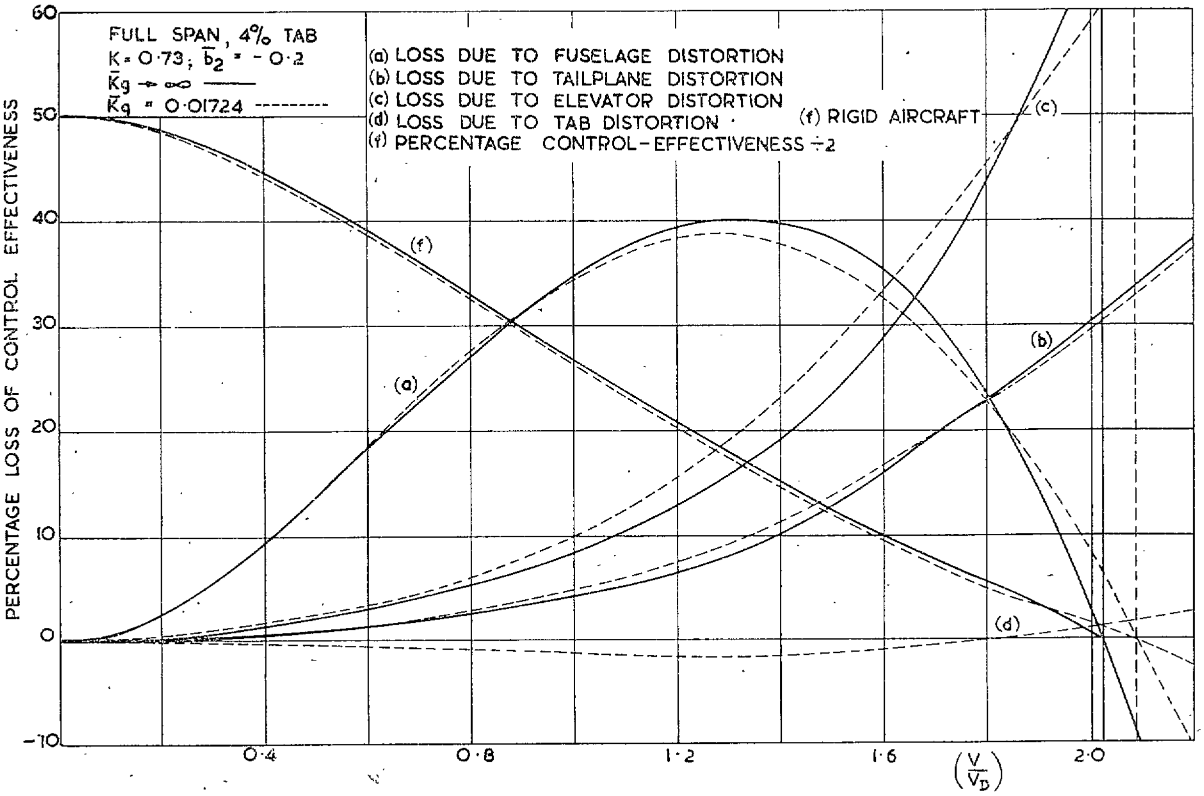


FIG. 11. Effect of tab flexibility on losses of control effectiveness.

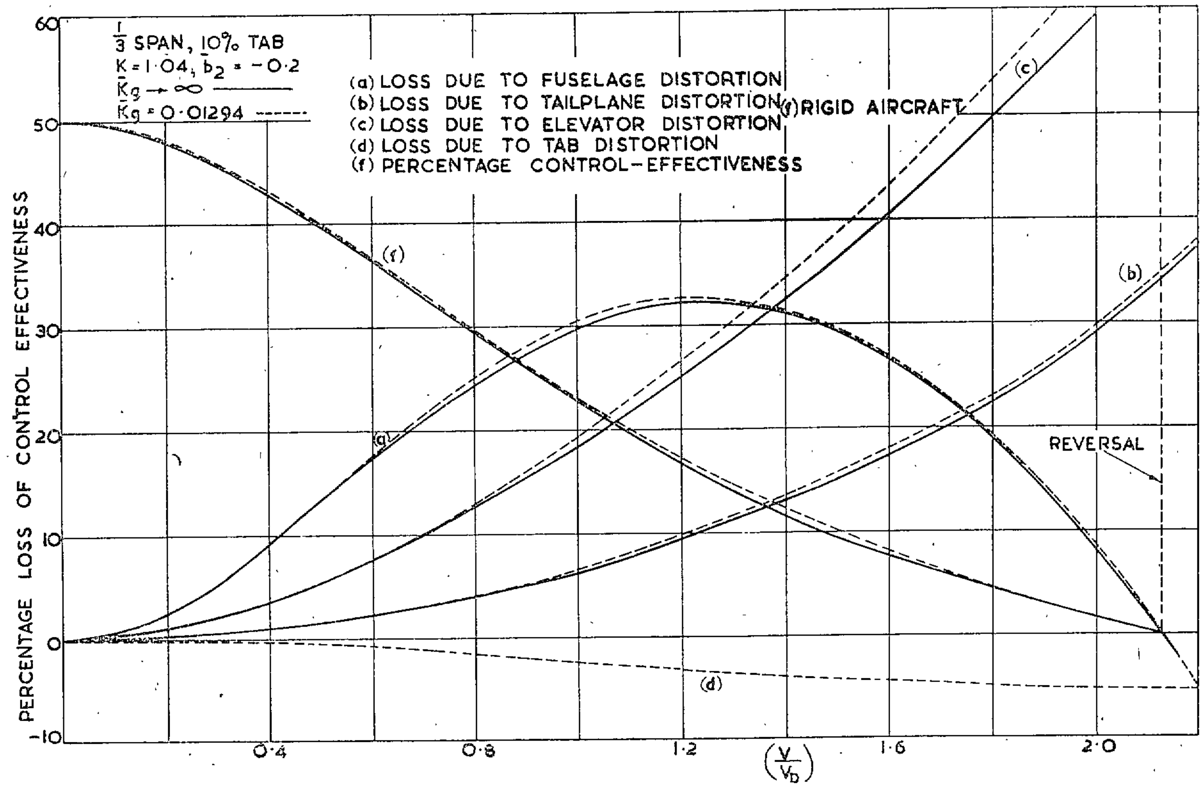


FIG. 12. Effect of tab flexibility on losses of control effectiveness.

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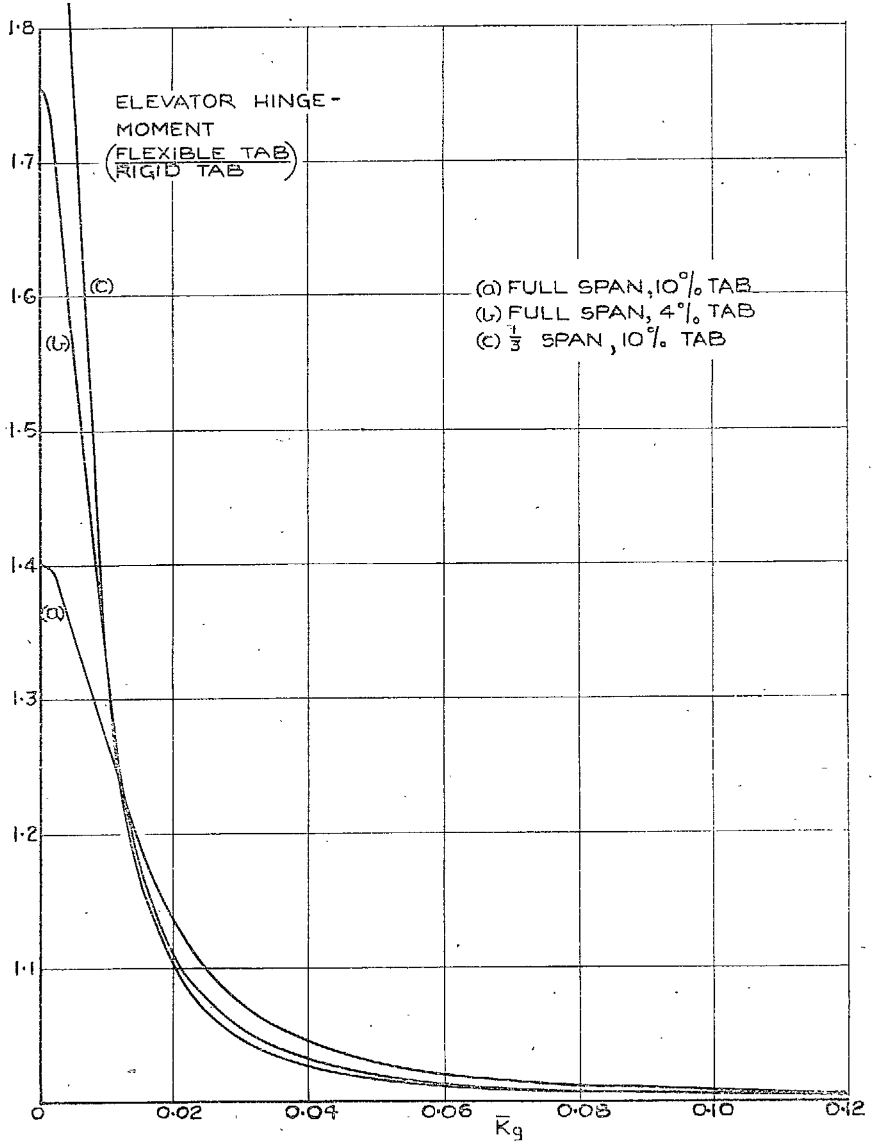

FIG. 13. Effect of tab flexibility on elevator hinge moment at the design diving speed.

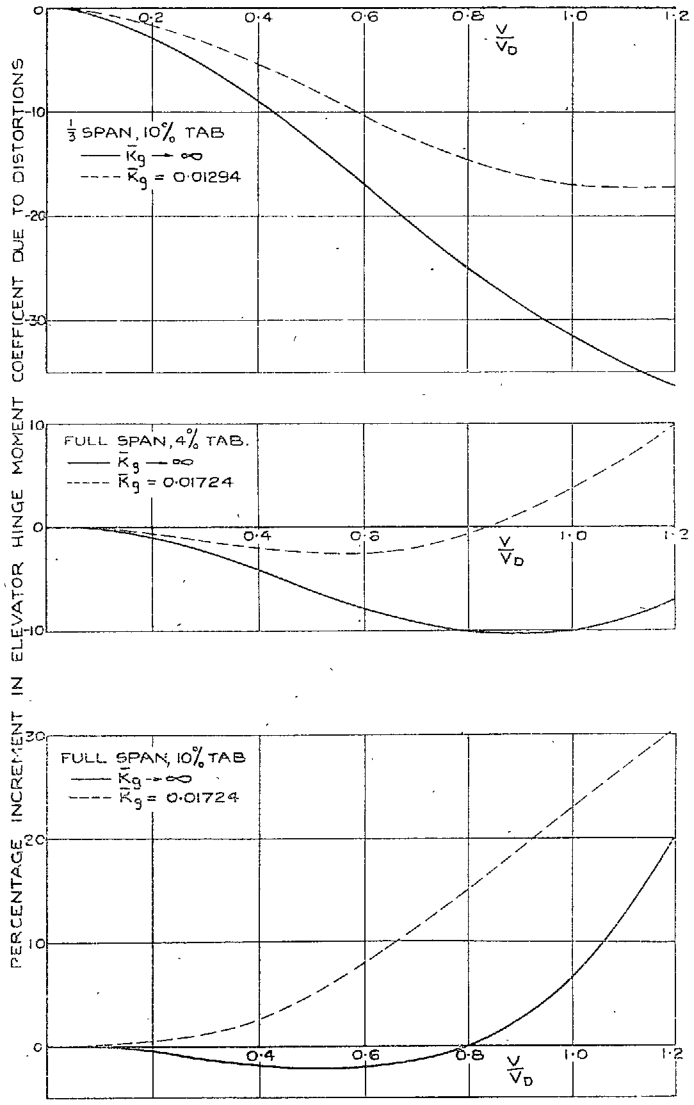

FIG. 14. Effect of air speed on elevator hinge moment.

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