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R. & M. No. 2839
(13,378)
(A.R.C. Technical Report)

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

Symmetric Flutter Characteristics of a Hypothetical Delta Wing

By

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LONDON: HER MAJESTY'S STATIONERY OFFICE
1954
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Communicated by the Principal Director of Scientific Research (Air), Ministry of Supply

Reports and Memoranda No. 2839*

May, 1950

Summary.—This report considers the flutter characteristics of a hypothetical Delta wing. It details the results of quaternary calculations showing the effect on the reduced critical speed of the shapes and relative natural frequencies of the first two normal modes of the aircraft. From these results the stiffnesses necessary to avoid flutter are deduced for two forms of wing structure. The aerodynamic forces have been obtained by using two-dimensional derivatives multiplied by the cosine of the quarter-chord sweepback in conjunction with strip theory applied to fore-and-aft strips. This procedure is of doubtful validity for the low aspect-ratio wing considered. With this reservation, however, the results confirm the adequacy of the present Ministry of Supply wing-stiffness requirement.

1. *Introduction.*—This investigation was undertaken to obtain knowledge of the flutter characteristics of a Delta wing sufficient to determine whether the existing general requirements[1] for the prevention of wing flutter are likely to be still satisfactory for this type of wing. Some of the information has already been issued in connection with certain project work at the Royal Aircraft Establishment. The present report details the complete results of the investigation.

The obvious method of attacking the problem is to obtain the critical flutter boundary between stability and instability as a relation between the standard static flexural and torsional wing stiffnesses (l_ϕ and m_θ). Before this could be done the stiffnesses of the modes used in the calculations would have to be expressed in terms of l_ϕ and m_θ; some assumptions about the wing structure would then have to be made. With wings of greater aspect ratio it is usually sufficient to consider the structure as a straight beam along the axis of the flexural centres (*i.e.*, the straight line which is the best approximation to the locus of the flexural centres), with rigid fore-and-aft ribs connected to it and otherwise independent of each other. However, this representation may be far from the truth for a Delta wing owing to the effect of the root. It might be better to assume the inner portion of the beam curved so as to cross the aircraft centre-line at right-angles.

In view of these doubts a different approach was employed; the flutter boundaries were obtained in the first place as functions of the two parameters t_1 and f_2/f_1 defining the shapes and relative frequencies of the first two normal modes (Figs. 5 to 19). From these results the curve of l_ϕ against m_θ for any form of wing structure can be obtained. This has been done for the two cases referred to in the previous paragraph (Fig. 28).

* R.A.E. Report Structures 68, received 13th September, 1950.

2. *The Problem.*—Instabilities of an aircraft involving at least one elastic mode as a primary degree of freedom are known as flutter or divergence according as the motion is oscillatory or not. These instabilities will only be present over a certain speed range. The limiting speeds are the critical flutter speeds and the divergence speeds respectively.

An aircraft has an infinite number of degrees of freedom. The appropriate equations of motion in the flight condition are, in general, impossible of solution. It is therefore necessary to consider the equations for a finite number of degrees of freedom, and for practical reasons the number must be kept small. These have to be selected so as to give a good approximation to the true solution for the type of flutter being investigated ; they must be such that a close approximation to the true mode of distortion at the critical flutter speed can be obtained by a suitable combination of them.

The condition investigated in the present case is that of symmetric wing flutter, with the following four degrees of freedom :—

(i) Parabolic flexure of the mid-chord line with no change of incidence in the flight direction .. .. (q_1)

(ii) Linear twist of sections in the flight direction about the mid-chord line, with no camber of the sections .. .. (q_2)

(iii) Pitching of the aircraft about the aircraft c.g. .. .. (q_3)

(iv) Vertical translation of the aircraft .. .. (q_4)

Freedoms (i) and (ii) are the elastic freedoms and are chosen to represent the wing motion that is directly significant from an aerodynamic point of view. They are of course different from the basic flexural and torsional freedoms (referred to the flexural axis) that have been commonly used for higher aspect ratio unswept wings.

The flutter equation then is[2]

$$\left[a\mu^2 + b\mu + c + \frac{e}{f_1^{\,2}}\left(\frac{f_1 l}{V}\right)^2\right] q = 0 \qquad \ldots \qquad (1)$$

where a is matrix of the inertia coefficients (aerodynamics and structural)

b is matrix of the aerodynamic damping coefficients

c is matrix of the aerodynamic stiffness coefficients

e is matrix of the structural stiffness coefficients

q is column vector of the amplitudes, *i.e.*, $\{q_1, q_2, q_3, q_4\}$

$\mu = i \times$ the mean frequency parameter $= i\omega c_m/V$.

It was assumed that the actual first two normal modes could be accurately obtained from a calculation with these four degrees of freedom. Hence a flutter calculation with these four degrees of freedom would be the same as one with the first two normal modes and the two body freedoms q_3 and q_4. Thus, as is shown in Appendix I, if we assume a value for the ratio of the amplitudes of the two elastic modes present ($t_1 = q_2/q_1$) in the first normal mode, then the shape of the first two normal modes is completely determined. The further assumption of the ratio f_2/f_1 of the frequencies of the two normal modes then determines the three stiffness coefficients (e_{11}, e_{12}, e_{22}) as functions of the fundamental natural frequency f_1.

A hypothetical wing was selected which, it was thought, would be similar to those likely to be encountered in practice. The plan of the wing is shown in Fig. 1 and the rest of the data is given at the end of the report.

3. *The Coefficients.*—The method of determining the coefficients a_{rs}, b_{rs}, c_{rs} is adequately described in numerous papers (*e.g.*, Ref. 2) and only those details peculiar to this investigation are related here.

3.1. *Aerodynamic Coefficients*.—The estimation of the oscillatory aerodynamic forces on a Delta wing involves some unusual difficulties. The sweepback of the leading edge, the high taper and the small aspect ratio all imply that a two-dimensional theory cannot be very satisfactory. A three-dimensional calculation of the forces however would have involved more time than was available. It was therefore decided to use Dietze's two-dimensional compressible theory[3] applied to strips in the line of flight. The derivatives were multiplied by cos β to allow for sweepback, where β is the angle of sweepback of the quarter-chord axis of the wing (*i.e.*, 45 deg in the present case). This correction can be shown to be theoretically correct for a rigid, untapered, swept wing of infinite span.

Account was also taken of the spanwise variation of local frequency parameter. The coefficients were calculated for a mean frequency parameter of 0·4 and a Mach number of 0·7 and these values were used throughout.

3.2. *The Structural Stiffness Coefficients*.—Since only two of the four degrees of freedom are elastic modes; all the stiffness coefficients except e_{11}, e_{12} and e_{22} are zero. The assumption of a value of t_1 determines these three stiffnesses as functions of the two frequencies of the normal modes. The further assumption of the ratio of these two frequencies gives the stiffnesses in the form e_{rs}/f_1^2. Formulae for them are obtained in Appendix I.

4. *Solutions and Results*.—The quaternary flutter equations were solved, as described in Appendix II, for f_1l/V and the frequency parameter. This was done for a wide range of values of the two parameters t_1 and f_2/f_1 which respectively defined the shape and ratio of the natural frequencies of the first two normal modes. The results are plotted in Figs. 5 to 19. The two branches are :—

(a) The lower frequency branch—' body freedom flutter.'

(b) The higher frequency branch—' flexure-torsion flutter.'

The flutter equations were also solved for $\mu = 0$ to give the divergence speeds which are shown in the same figures. These cannot be very accurate, as the assumed aerodynamic forces were not appropriate to a frequency parameter of zero as they should be at the critical divergence speed.

The flutter modes were determined for a few cases in order to illustrate the two types of flutter obtained. The amplitudes and phases of the mid-chord downward displacement and of the incidence have been determined for the tip section and have been plotted against t_1 in Figs. 20 to 27. The amplitudes of the downward displacements are divided by the tip chord for comparison with the incidence amplitudes. The amplitudes and phases of the displacements due to vertical translation, pitch about the aircraft c.g. and elastic deformation are given in addition to those of the total displacement. The total downward displacement of the mid-chord point at the tip was used as a reference. Thus a phase of θ indicates that the displacement in question leads the reference displacement by an angle θ.

Figs. 20 to 23 give the results for $(f_2/f_1)^2 = 1\cdot3$, and Figs. 24 to 27 those for $(f_2/f_1)^2 = 5\cdot0$ (*cf*. Figs. 15 to 17). It will be noted that the body-freedom flutter in every case consisted almost entirely of vertical translations of the whole aircraft. The flexure-torsion flutter modes in general had comparable amounts of elastic and rigid body movement, though in certain cases they were almost entirely elastic.

The shapes of the two normal modes for a given value of t_1 are shown in Figs. 2 to 4. Fig. 3 gives the corresponding value of t_2, Fig. 2 shows the position of the nodal lines for various values of t_r (*i.e.*, either t_1 or t_2), and Fig. 4 gives the amplitudes at the wing tips in the normal modes.

The assumption of the form of the wing structure determines t_1, $f_1^2 l^5/m_\theta$ and l_ϕ/m_θ as functions of f_2/f_1. Hence, using these relationships, curves of $(1/V^2)(m_0/0\cdot9s.c_m^2)$ against $(1/V^2)(l_\phi/0\cdot9s.c_m^2)$ (V being the critical flutter speed) can be deduced from the other results. This was done for two possible representations of a Delta wing structure. The curves are shown in Fig. 28. The representations chosen were :—

(i) 'Straight-beam theory.' The wing was considered to be a straight beam located at the mid-chord line with rigid fore-and-aft ribs interconnected only by the beam. The flexural and torsional stiffnesses per unit length of the beam were assumed to vary as the cube of the chord.

(ii) 'Curved-beam theory.' The wing was considered to be a beam located on the line ABC (Fig. 1) with rigid fore-and-aft ribs. The flexural and torsional stiffnesses per unit length of the beam were assumed to be constant over the curved portion of the beam and to vary as the cube of the chord over the straight portion. The curves of t_1, $f_1^2 l^5/m_0$ and l_ϕ/m_θ for the two of types structure are shown in Figs. 29, 30 and 31. It will be noticed that only the straight-beam theory gave body-freedom flutter.

5. *Conclusions.*—Before drawing any conclusions, it is well to note the limitations of this work. Firstly, the whole investigation is for an aircraft with one particular plan form, wing density, and mass distribution. Secondly, the aerodynamics used cannot be very satisfactory since a two-dimensional theory was used, outside its region of applicability, modified only by a somewhat arbitrary correction factor. Thirdly, the aerodynamic forces were calculated for a Mach number of $0\cdot7$ and a frequency parameter of $0\cdot4$, and were assumed to be correct for all Mach numbers and frequency parameters. Previous experience has shown that this will probably produce little error provided (*a*) the airflow, at the resultant critical speed, is completely sub-sonic ; and (*b*) the resultant frequency parameter is between $0\cdot3$ and $0\cdot6$. There may therefore be some error in the lower frequency branch.

However, within these limitations it does appear (Fig. 28) that the present wing stiffness criterion[1] $(1/V)\sqrt{(m_0/0\cdot9s.c_m^2)} > 0\cdot04$ at the design diving speed) should be an adequate safeguard against flutter for Delta wings. A criterion of a different form relating the frequencies and nodal lines of the normal modes could be obtained from the results shown in Figs. 5 to 19. The possibility of body-freedom flutter is to be noted, especially as, when present, it usually occurs at a lower critical speed than the flexure-torsion flutter.

For the investigation of the flutter of an actual Delta wing aircraft, it will be necessary for the calculations to involve at least four degrees of freedom—two suitable normal modes obtained from resonance tests and the two body freedoms—pitch and vertical translation. The use of arbitrary modes does not seem advisable unless an adequate set of influence coefficients can be measured. The accuracy of theoretical stiffnesses is doubtful, and the effect of an apparently unimportant wrong assumption may be great, as for example is shown in Fig. 28.

There is plenty of scope for further work on the subject—variation of other parameters, investigation of antisymmetric flutter, use of three-dimensional aerodynamic theory, comparison with practical results, etc.

LIST OF SYMBOLS

l	Length of the mid-chord axis of the wing
c	Wing chord in the line of flight (c_0 root chord, c_t tip chord)
c_m	Mean value of c
s	Semi-span
l_ϕ	Flexural stiffness at $0\cdot7$ span

m_θ		Torsional stiffness at 0·7 span *i.e.*, $l_\phi = 0 \cdot 49\ W\ l^2/Z$ where Z is the downward displacement, at 0·7 span, of the mid-chord axis under a load W applied at the same point and $m_\theta = M/\alpha$ where α is the nose-up twist of a section, at 0·7 span, perpendicular to the mid-chord axis, under a nose-up torque M applied at the same section in the plane of the section.
q_i		Amplitude of the i'th degree of freedom
f		Frequency in c.p.s.
V		True air speed
f_r		Frequency of the r'th normal mode
t_r		Ratio of the amplitudes q_2/q_1 in the r'th normal mode
ω	=	$2\pi f$, frequency in radians per second
$\omega c_m/V$		Frequency parameter
μ	=	$i\omega c_m/V$

Data of the Hypothetical Delta

Aspect ratio 2·5

Taper ratio (tip chord to root chord) 0·15

Sweepback on the quarter-chord line 45 deg

Aircraft c.g. at mid-root chord

Wing density 0·95 lb/ft^3

Inertia axis, at 0·4 chord

Wing mass distribution proportional to the cube of the chord

Radius of gyration of wing section in the line of flight $= c^2/4c_0$ about lateral axis through mid-chord

Fuselage mass = one quarter of the mass

Fuselage radius of gyration about aircraft c.g. $= 0 \cdot 36c_0$

The expression $c^2/4c_0$ for the wing sectional radius of gyration was unfortunately used in error. It had been intended to use a value of $c/4$. The inertia coefficients used however have approximately the same value as those obtained taking the wing sectional radius of gyration $= 0 \cdot 185c$ and using a different fuselage radius of gyration $= 0 \cdot 43c_0$.

REFERENCES

No.	*Author*	*Title, etc.*
1	—	Design requirements for aeroplanes for the Royal Air Force and Royal Navy. Ministry of Supply publication A.P.970. Part 5.
2	H. Templeton	The technique of flutter calculations. R.A.E. Report Structures 142. April, 1953.
3	F. Dietze	The air forces for the harmonically oscillating aerofoil in a compressible medium at subsonic speeds (two-dimensional problem.) Translated from the German: ZWB Reports 1733 and 1733/2. January 1943. A.R.C. 10,219. December, 1946.

APPENDIX I

Determination of the Stiffness Coefficients

With the four degrees of freedom q_1, q_2, q_3, q_4, the equation for the normal modes will be, from (1)

$$[-a(2\pi f c_m)^2 + el^2][q] = 0. \qquad \text{(A1)}$$

Assuming, in either normal mode

$$q_2/q_1 = t \qquad \text{(A2)}$$

then in the same normal mode

$$\frac{q_3}{q_1} = -\left\{\frac{Y_{14} + Y_{24}t}{Y_{34}}\right\}. \qquad \text{(A3)}$$

and

$$\frac{q_4}{q_1} = \frac{Y_{13} + Y_{23}t}{Y_{34}} \qquad \text{(A4)}$$

where

$$Y_{rs} = \begin{vmatrix} -a_{3r}(2\pi f c_m)^2 + e_{3r}l^2 \,; & -a_{3s}(2\pi f c_m)^2 + e_{3s}l^2 \\ -a_{4r}(2\pi f c_m)^2 + e_{4r}l^2 \,; & -a_{4s}(2\pi f c_m)^2 + e_{4s}l^2 \end{vmatrix} \qquad \text{(A5)}$$

But, since the modes 3 and 4 are non-elastic modes,

$$e_{3m} = e_{4m} = 0 \text{ for all values of } m. \qquad \text{(A6)}$$

Hence q_3/q_1 and q_4/q_1 will only depend on t and the inertia coefficients (which have been kept constant throughout the greater part of this investigation),

i.e.,

$$\frac{q_3}{q_1} = -\left\{\begin{vmatrix} a_{31} & a_{34} \\ a_{41} & a_{44} \end{vmatrix} + t\begin{vmatrix} a_{32} & a_{34} \\ a_{42} & a_{44} \end{vmatrix}\right\} \Bigg/ \begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix} \qquad \text{(A7)}$$

$$\frac{q_4}{q_1} = \left\{\begin{vmatrix} a_{31} & a_{33} \\ a_{41} & a_{43} \end{vmatrix} + t\begin{vmatrix} a_{32} & a_{33} \\ a_{42} & a_{43} \end{vmatrix}\right\} \Bigg/ \begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix}. \qquad \text{(A8)}$$

Thus substituting in the first two rows of (A1) we get

$$e_{11} = -\left(\frac{2\pi f c_m}{l}\right)^2 \left\{\frac{|A_{22}| - t|A_{21}|}{\begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix}}\right\} - e_{12}t \qquad \text{(A9)}$$

and

$$e_{22}t = -\left(\frac{2\pi f c_m}{l}\right)^2 \left\{\frac{-|A_{12}| + t|A_{11}|}{\begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix}}\right\} - e_{12} \qquad \text{(A10)}$$

where $|A_{rs}|$ is the co-factor of a_{rs} in $|a|$.

Hence if t_1 and t_2 are the values of t for the first two normal modes and f_1 and f_2 the corresponding frequencies, then

$$e_{12} = -\frac{\left(\frac{2\pi f_1 c_m}{l}\right)^2}{\begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix}} \left\{\frac{\left(1 - \left(\frac{f_2}{f_1}\right)^2\right)\left|A_{22}\right| - \left(t_1 - \left(\frac{f_2}{f_1}\right)^2 t_2\right)\left|A_{21}\right|}{t_1 - t_2}\right\} \qquad \text{(A11)}$$

and hence $f_1^2\{|A_{22}| - t_1|A_{21}| - t_2|A_{12}| + t_1t_2|A_{11}|\} = f_2^2\{|A_{22}| - t_1|A_{12}| - t_2|A_{21}| + t_1t_2|A_{11}|\}$. (A12)

But $|A_{12}| = |A_{21}|$ since $[a]$ is a symmetrical matrix.

Therefore $(f_1^2 - f_2^2)\{|A_{22}| - (t_1 + t_2)|A_{12}| + t_1t_2|A_{11}|\} = 0$. .. .. (A13)

Since $f_1 \neq f_2$ for all values of t_1 and t_2,

we have $|A_{22}| - (t_1 + t_2)|A_{12}| + t_1t_2|A_{11}| = 0$, ... (A14)

i.e., $$t_2 = \frac{|A_{22}| - t_1|A_{12}|}{|A_{12}| - t_1|A_{11}|} \quad \text{(A15)}$$

Thus the assumption of a value for t_1 determines completely the first two normal modes. The further assumption of a value for f_2/f_1, enables us to calculate the three stiffness coefficients, as proportions of f_1^2(or f_2^2) from equations (A9), (A10) and (A11).

Also

$$\frac{e_{11}e_{22} - e_{12}^2}{f_1^4} = \left(\frac{2\pi c_m}{l}\right)^4 \left(\frac{f_2}{f_1}\right)^2 \left\{ \frac{|A_{11}|\,|A_{22}| - |A_{12}|^2}{\begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix}^2} \right\} \quad \text{(A16)}$$

which is independent of t_1 and serves as a useful check in the numerical computation of the stiffnesses.

APPENDIX II

Solution of the Flutter Equation

From (1) we have

$$\left| a\mu^2 + b\mu + c + \frac{e}{f_1^2}\left(\frac{f_1 l}{V}\right)^2 \right| = 0 \quad \text{(A17)}$$

The expansion of this determinant gives a polynomial of the form :—

$$p_0\mu^8 + p_1\mu^7 + p_2\mu^6 + p_3\mu^5 + p_4\mu^4 + p_5\mu^3 + p_6\mu^2 + p_7\mu + p_8 = 0 \quad \text{(A18)}$$

where the p coefficients are functions of $(f_1 l/V)^2$.

The roots of (A18) which are purely imaginary (*i.e.*, the frequency parameter $= - i\mu$ is real) give the critical flutter speeds ; and those which are zero give the divergence speeds.

Hence at the critical flutter speed, splitting (A18) into real and imaginary parts, we have

$$p_0\mu^8 + p_2\mu^6 + p_4\mu^4 + p_6\mu^2 + p_8 = 0 \quad \text{(A19)}$$

and

$$p_1\mu^6 + p_3\mu^4 + p_5\mu^2 + p_7 = 0 \quad \text{(A20)}$$

each of which, because of (A6), will be a quadratic in $(f_1 l/V)^2$.

These equations are then solved by assuming a purely imaginary value of μ, substituting in (A19) and (A20), solving one of the equations for $(f_1 l/V)^2$, substituting the two roots in the other equation and finding the corresponding values of the left-hand side R_1 and R_2 and then repeating the process with different values of μ until a value is found for which either R_1 or R_2 is zero. A thorough examination of the whole range of frequency parameter values from 0 to ∞ is necessary to ensure that no roots are overlooked.

From (A18) it follows that the divergence speeds are given by the solution of

$$p_8 = 0 \quad \text{(A21)}$$

which is a quadratic equation in $(f_1 l/V)^2$.

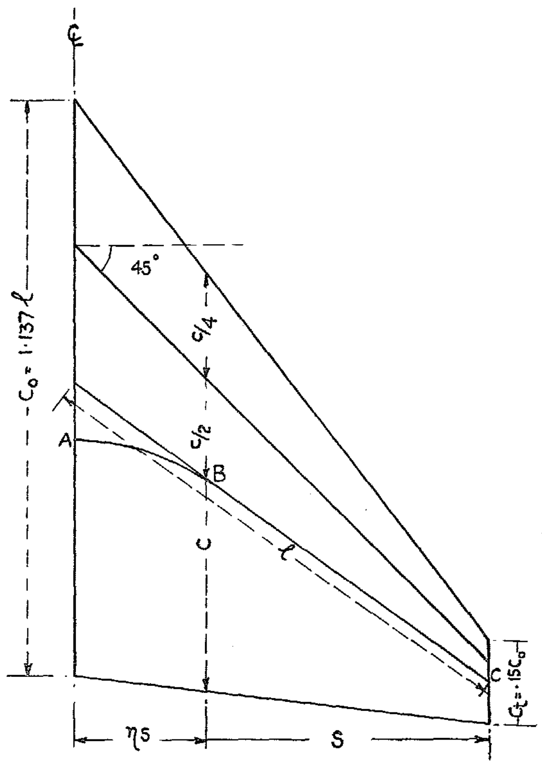

FIG. 1. Plan of the hypothetical wing.

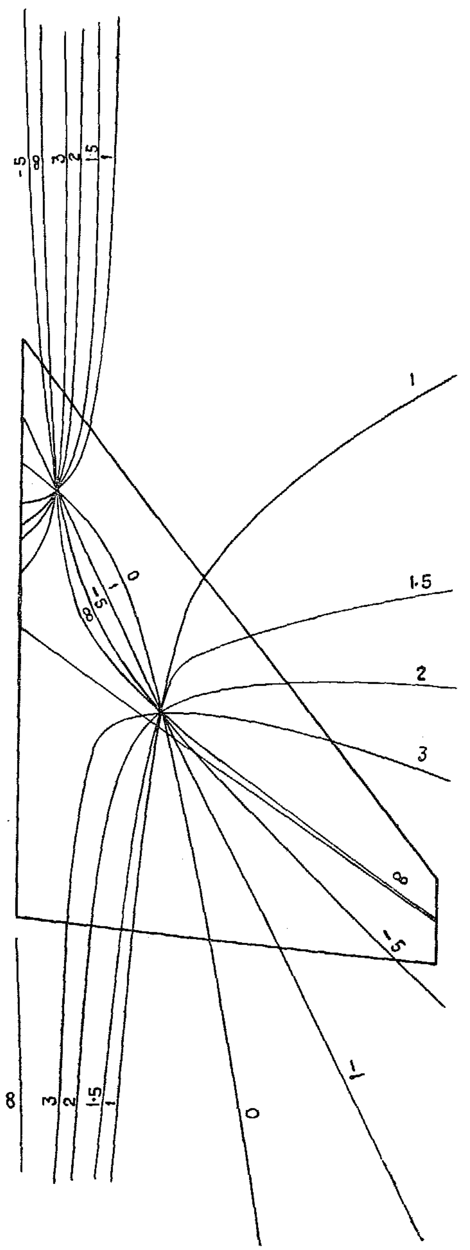

FIG. 2. Nodal lines for various values of the parameter t_r.

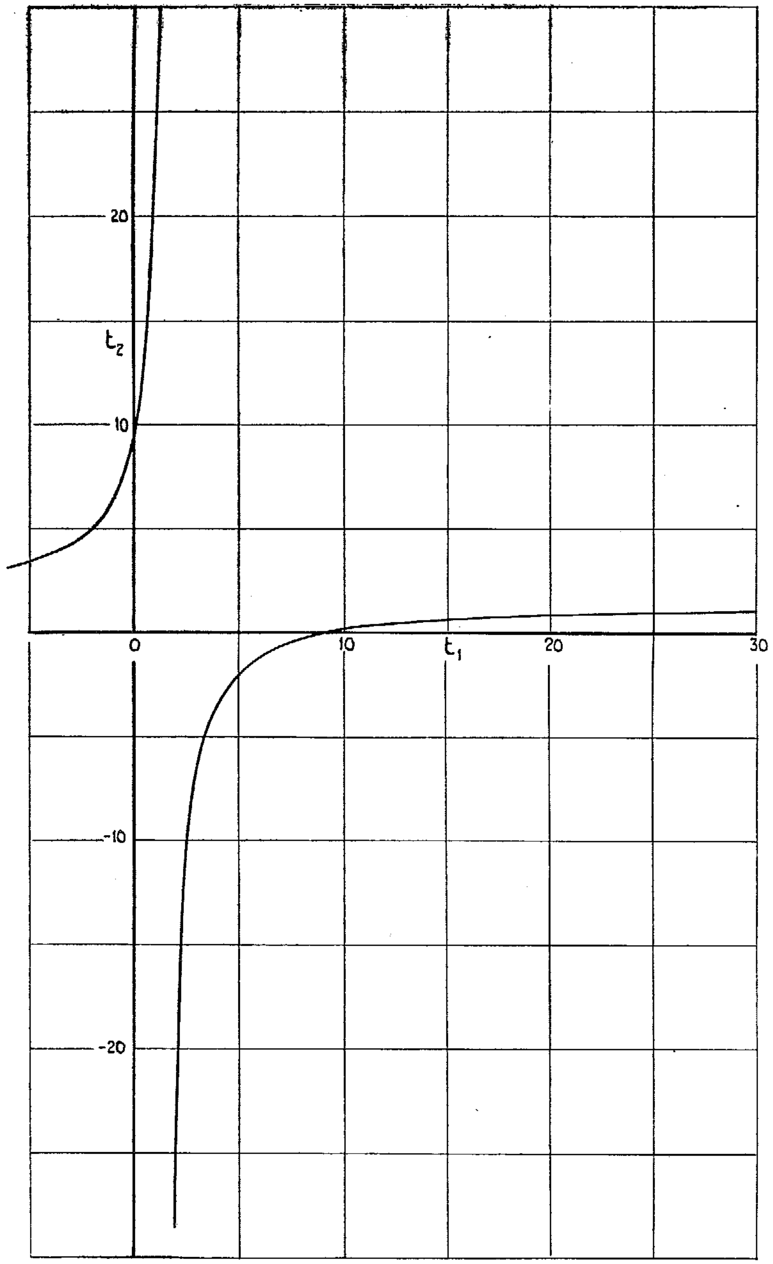

FIG. 3. Relationship between t_1 and t_2, the modal parameters.

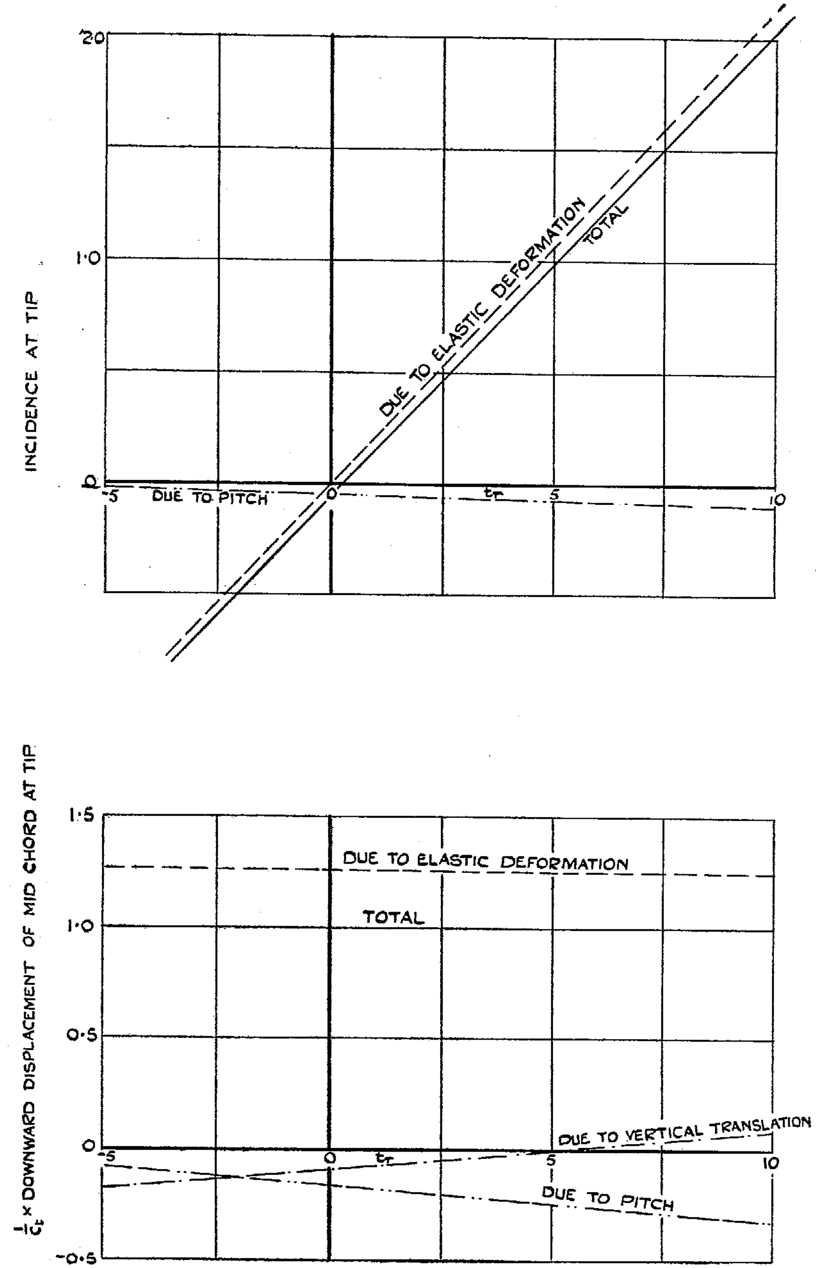

FIG. 4. Amplitudes of normal modes at tip.

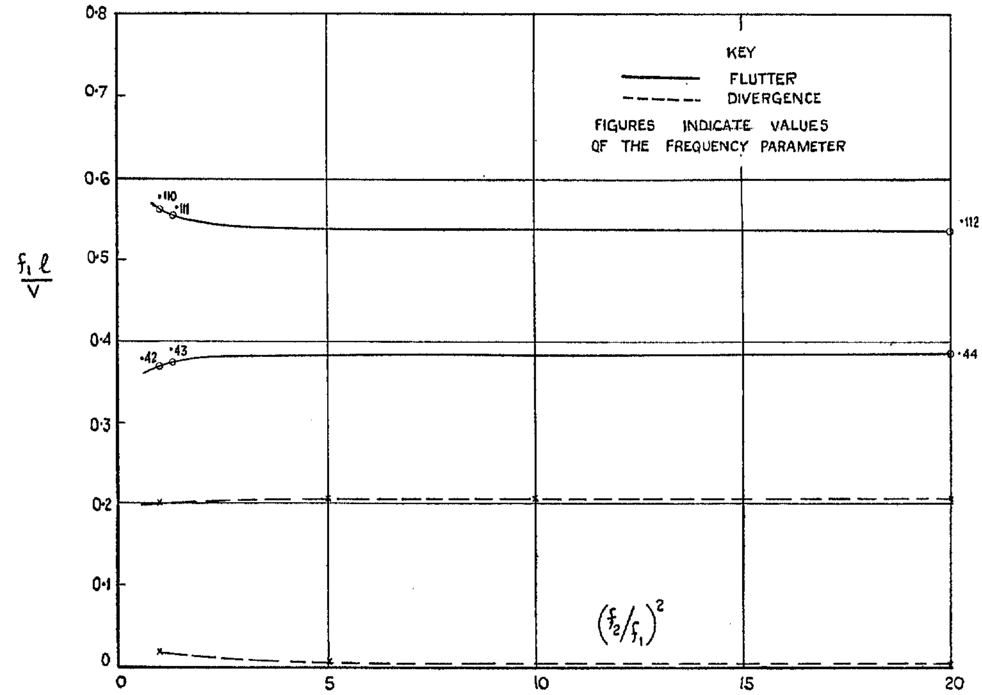

FIG. 5. Critical speed curves for $t_1 = -5 \cdot 0$.

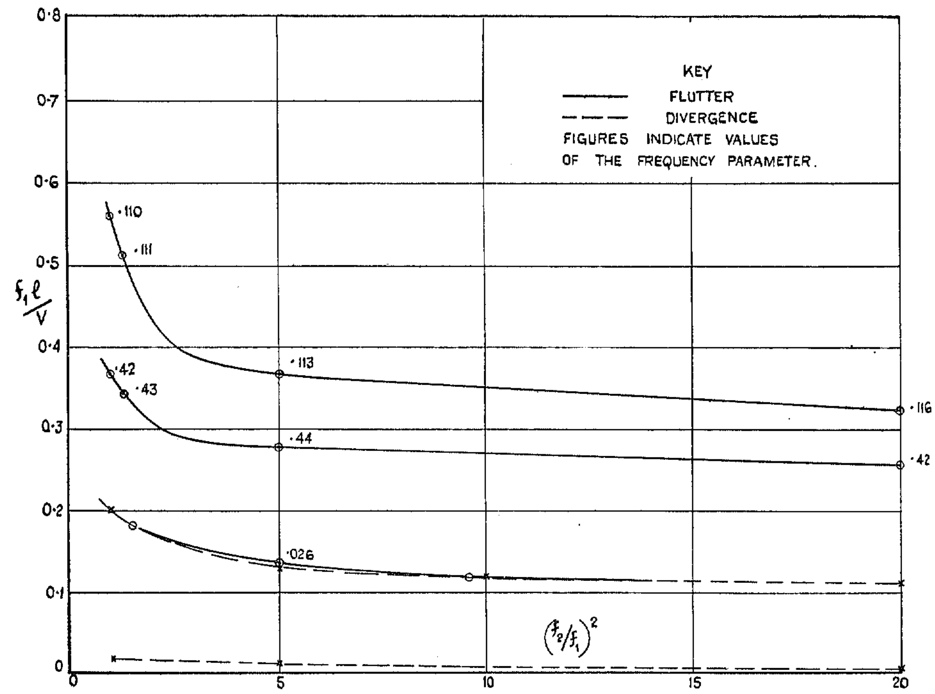

FIG. 6. Critical speed curves for $t_1 = -1 \cdot 0$.

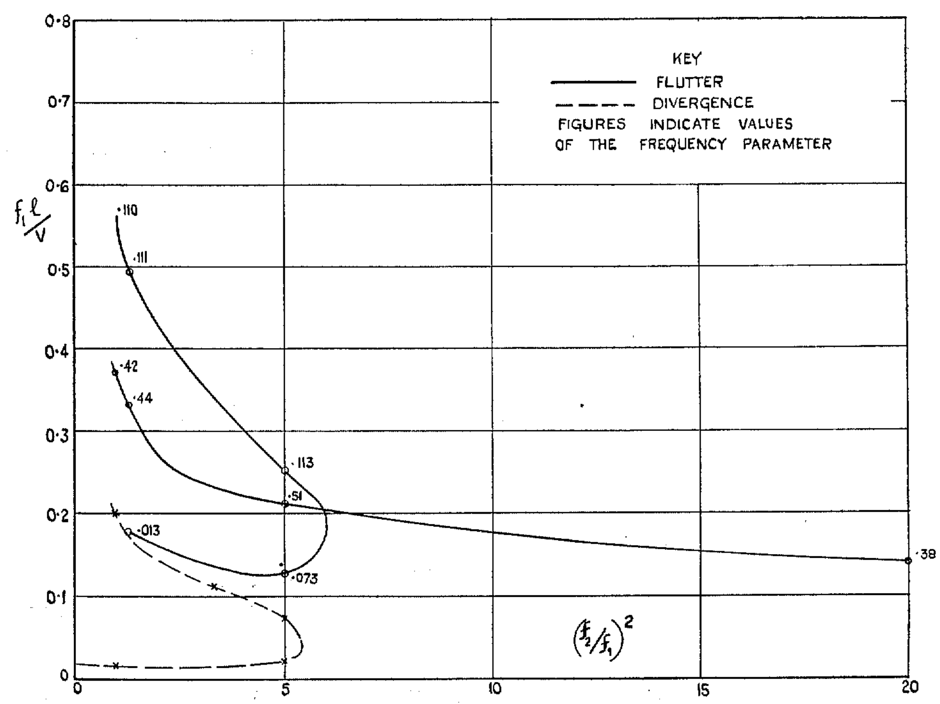

FIG. 7. Critical speed curves for $t_1 = 0$.

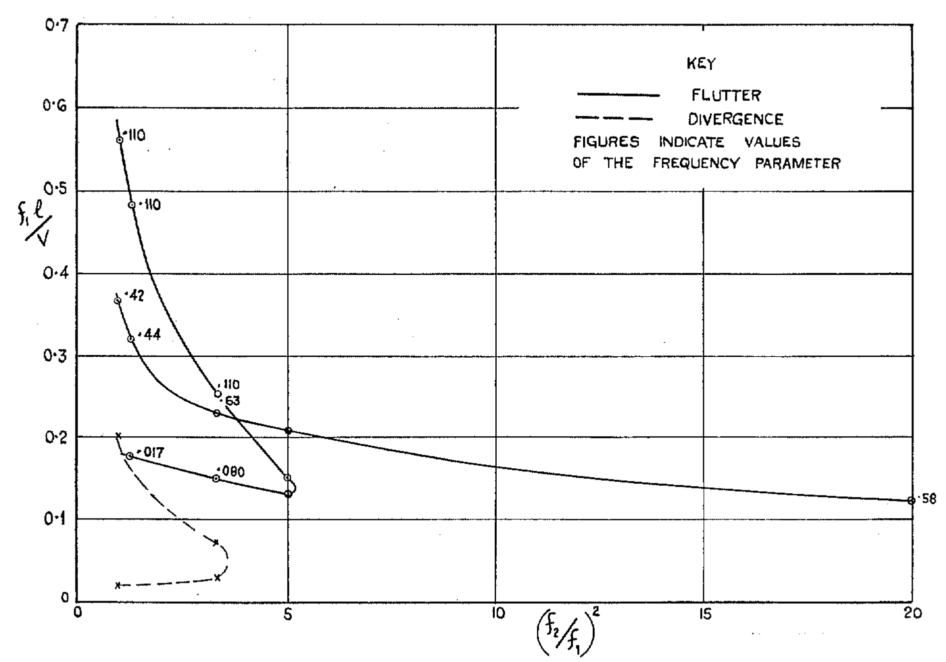

FIG. 8. Critical speed curves for $t_1 = 0{\cdot}5$.

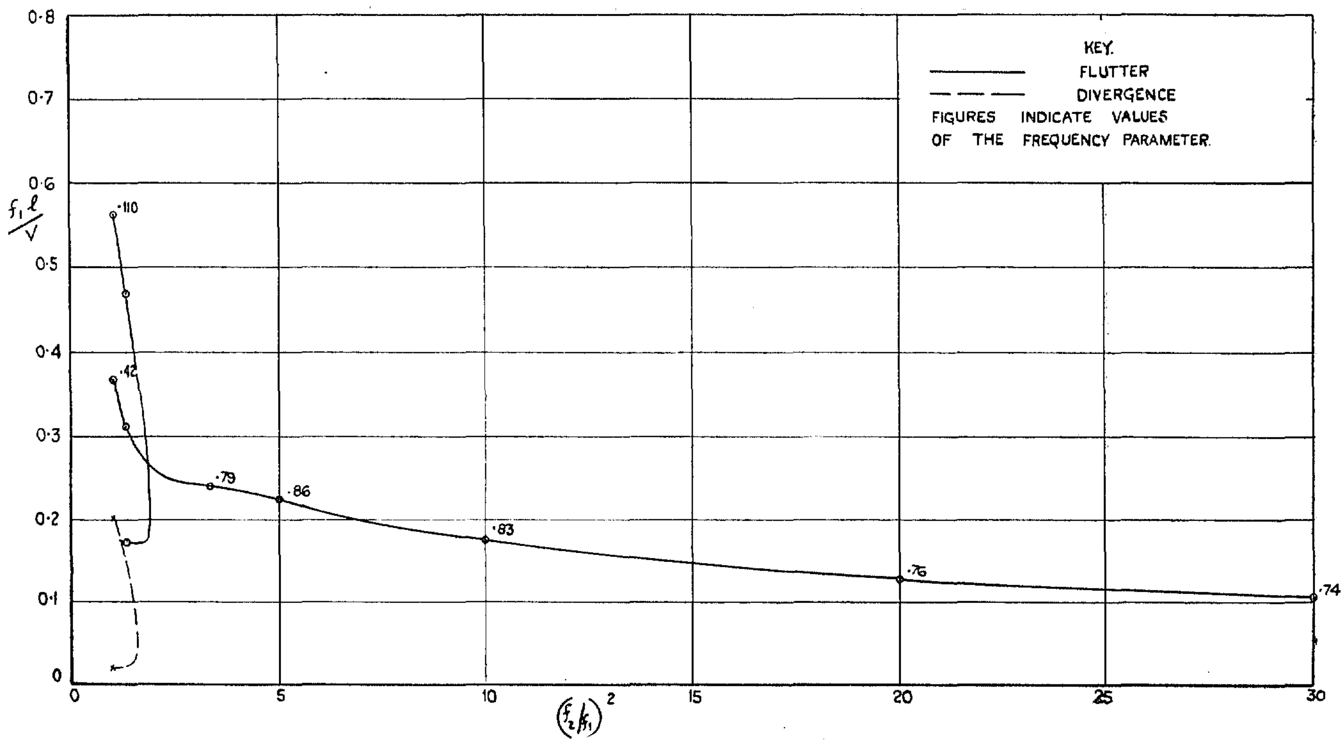

FIG. 9. Critical speed curves for $t_1 = 1 \cdot 0$.

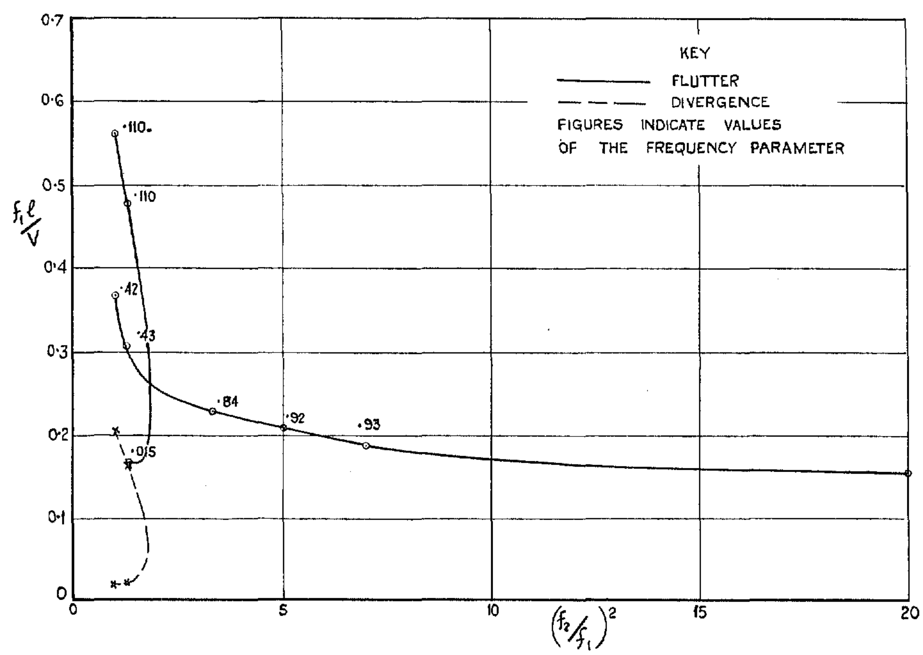

FIG. 10. Critical speed curves for $t_1 = 2 \cdot 0$.

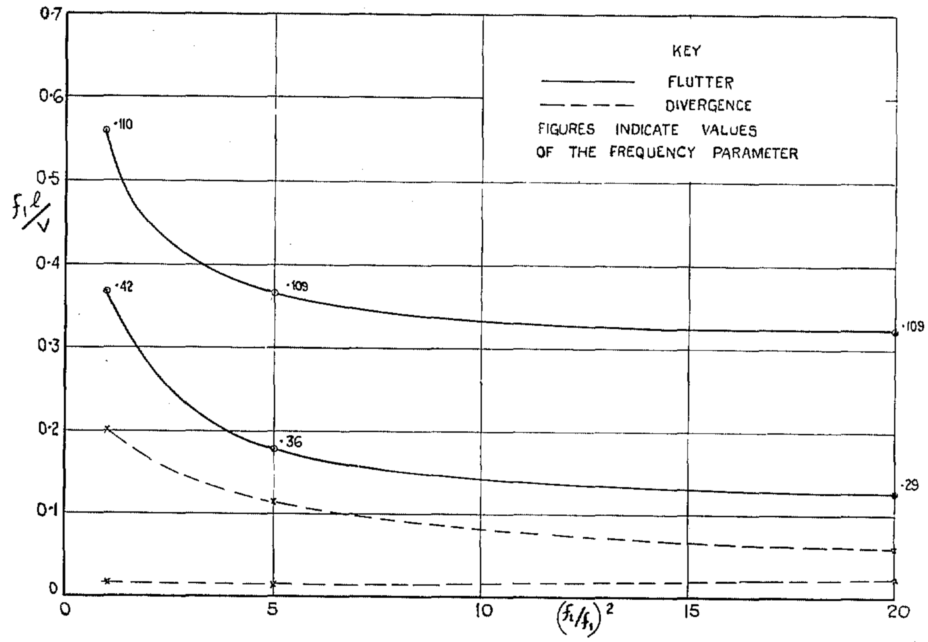

FIG. 11. Critical speed curves for $t_1 = 4 \cdot 0$.

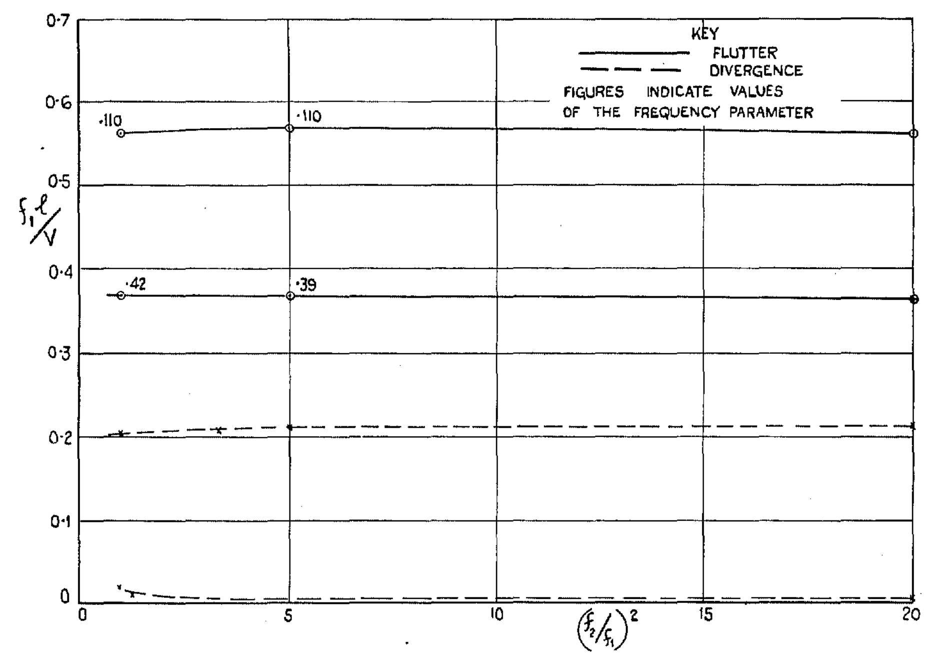

FIG. 12. Critical speed curves for $t_1 = 10 \cdot 0$.

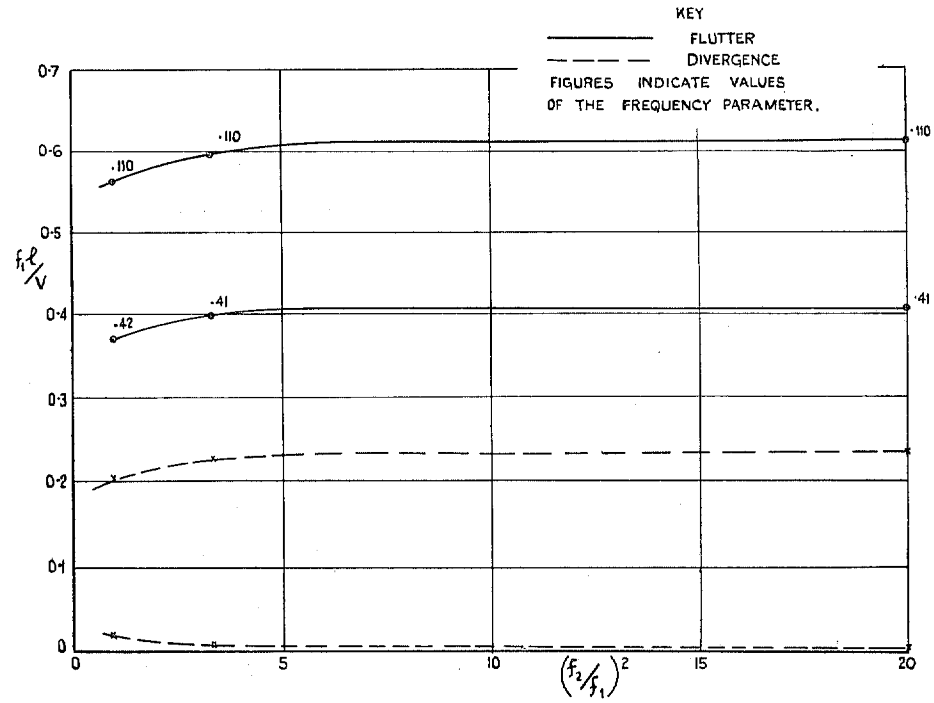

FIG. 13. Critical speed curves for $t_1 = 23 \cdot 38$.

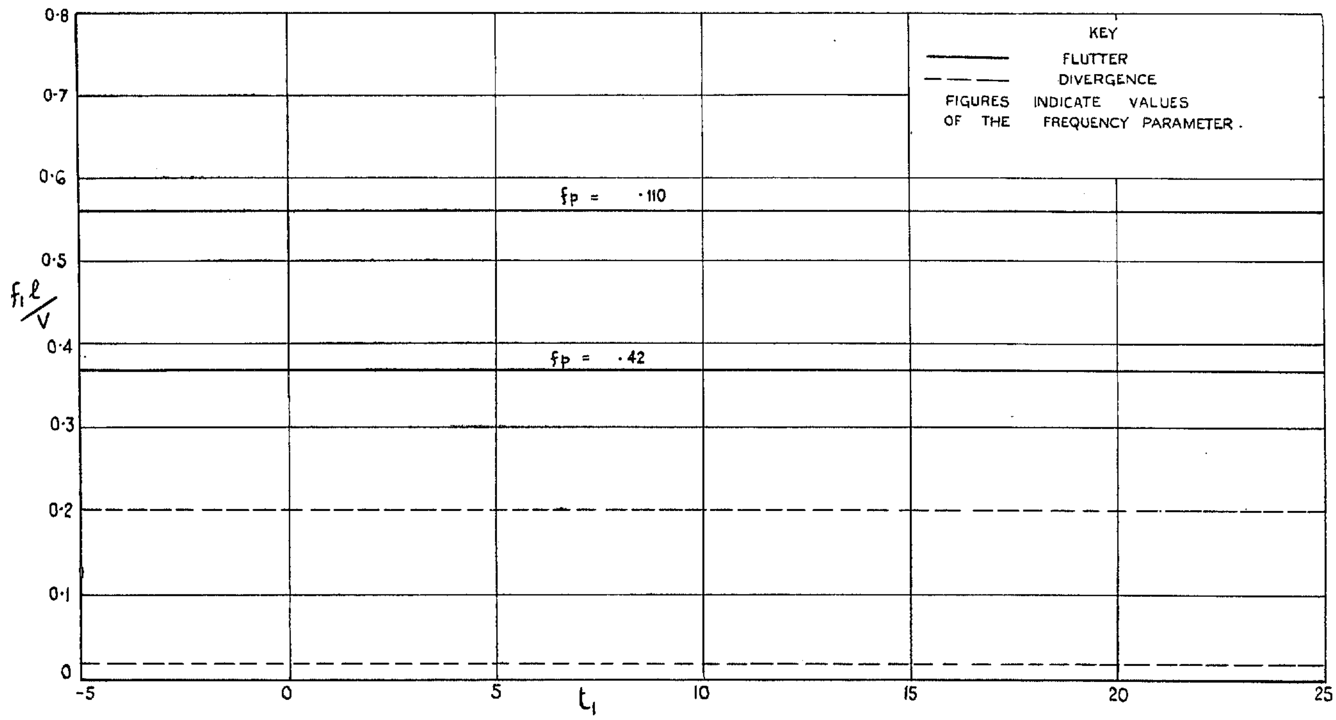

FIG. 14. Critical speed curves for $(f_2/f_1)^2 = 1 \cdot 0$.

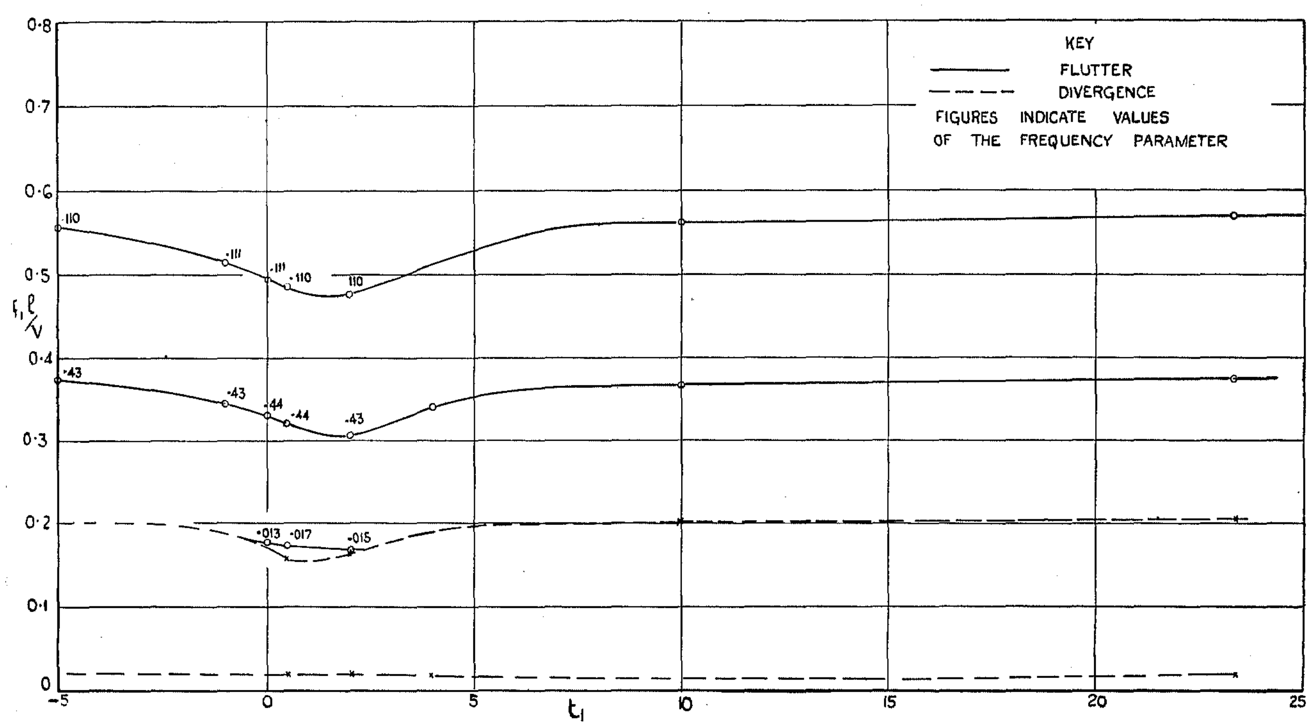

FIG. 15. Critical speed curves for $(f_2/f_1)^2 = 1 \cdot 3$.

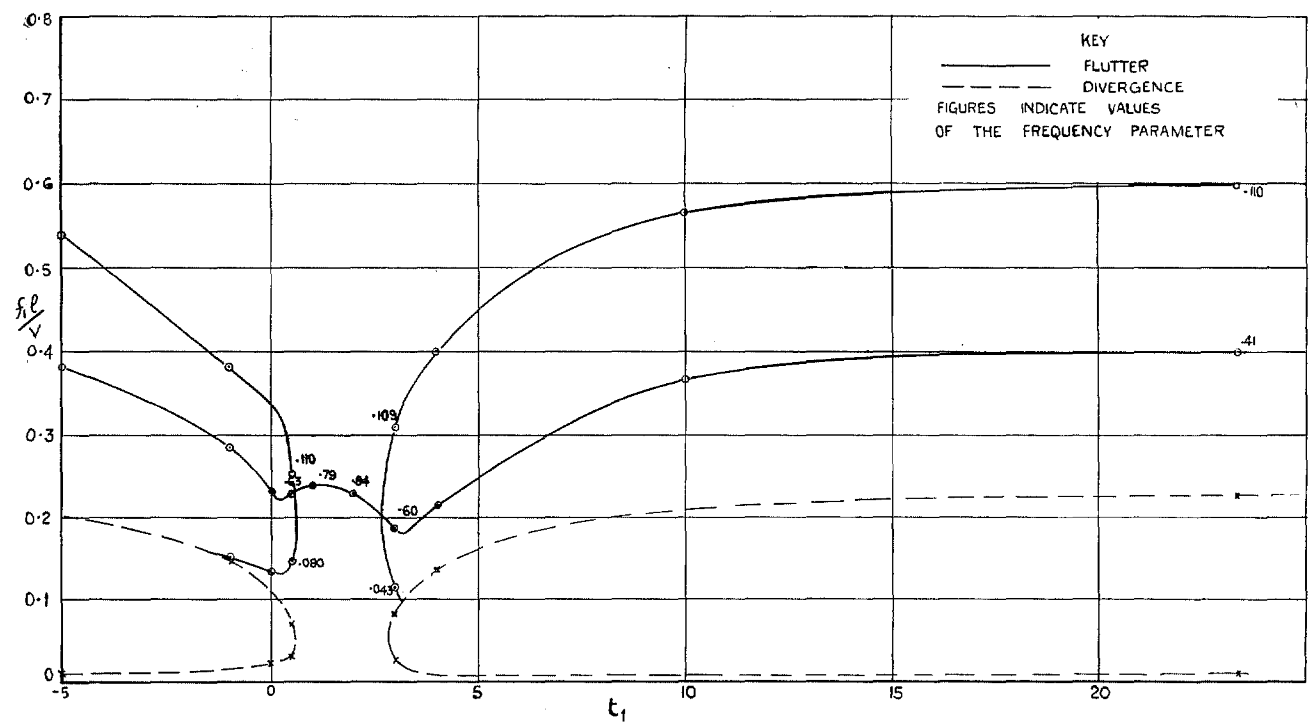

FIG. 16. Critical speed curves for $(f_2/f_1)^2 = 3 \cdot 33$.

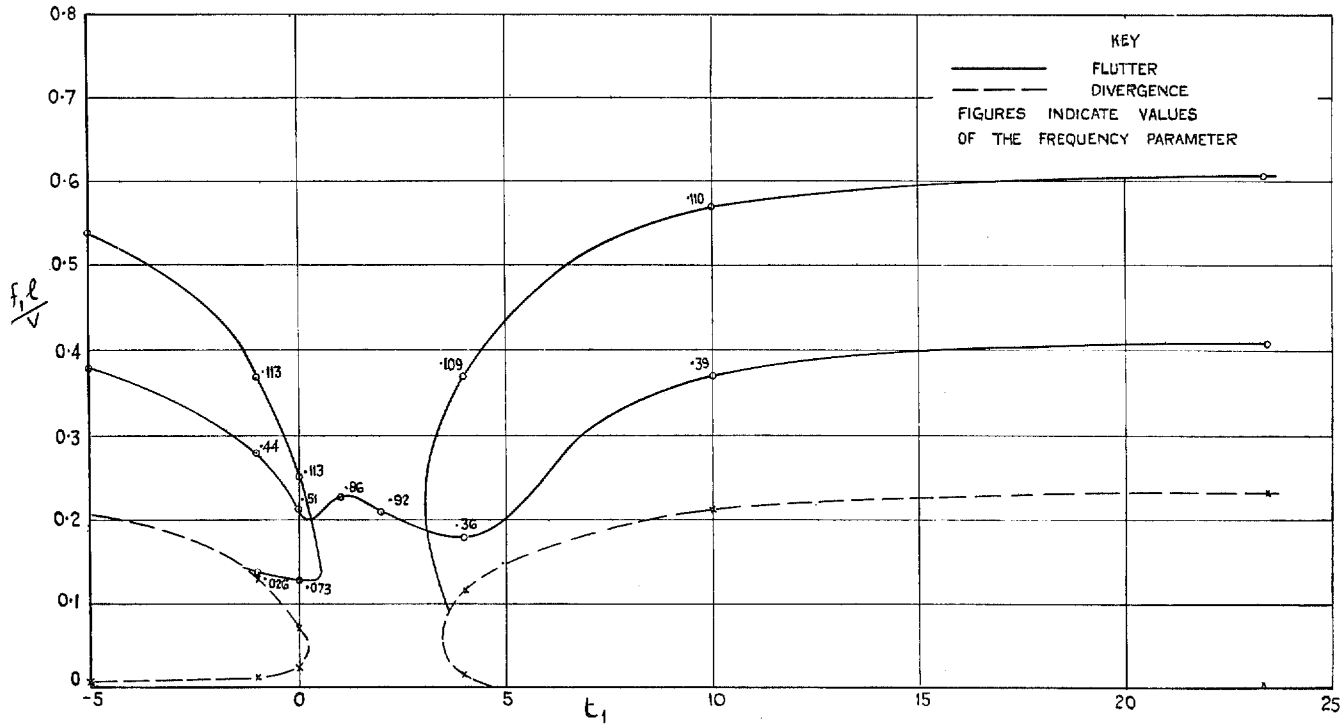

FIG. 17. Critical speed curves for $(f_2/f_1)^2 = 5 \cdot 0$.

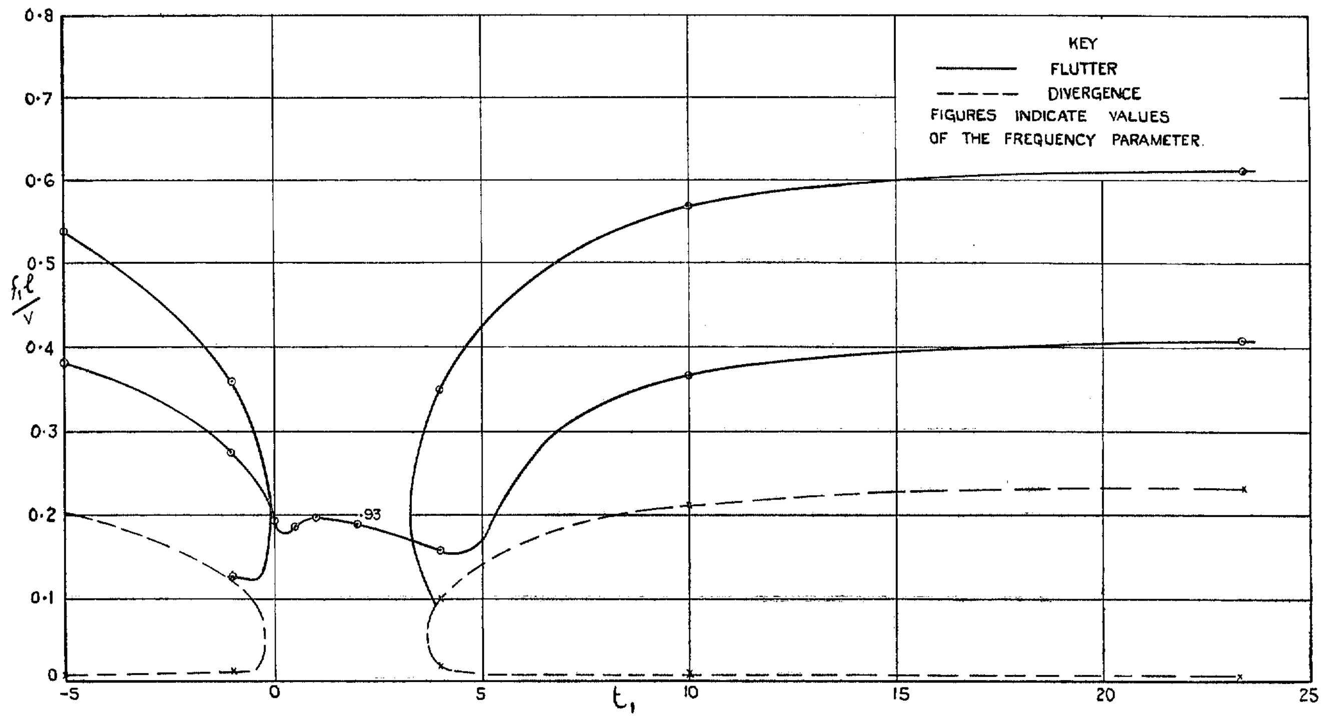

FIG. 18. Critical speed curves for $(f_2/f_1)^2 = 7 \cdot 0$.

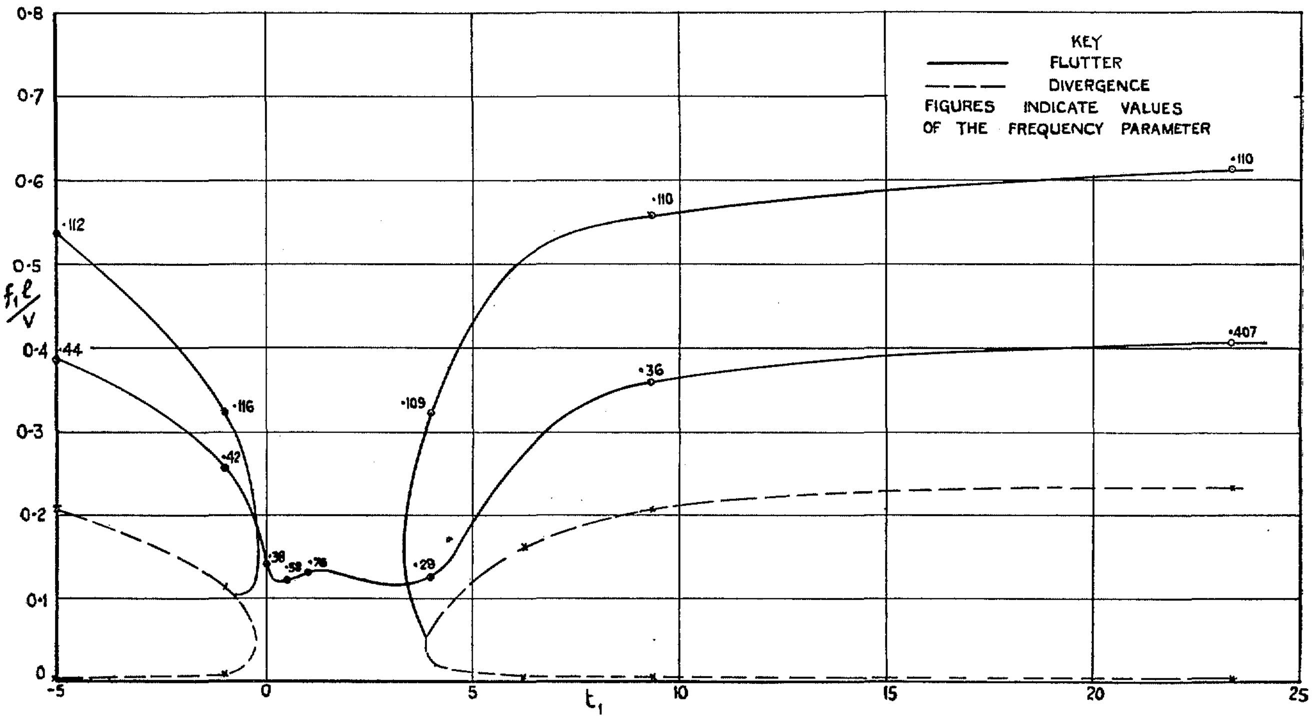

FIG. 19. Critical speed curves for $(f_2/f_1)^2 = 20\cdot0$.

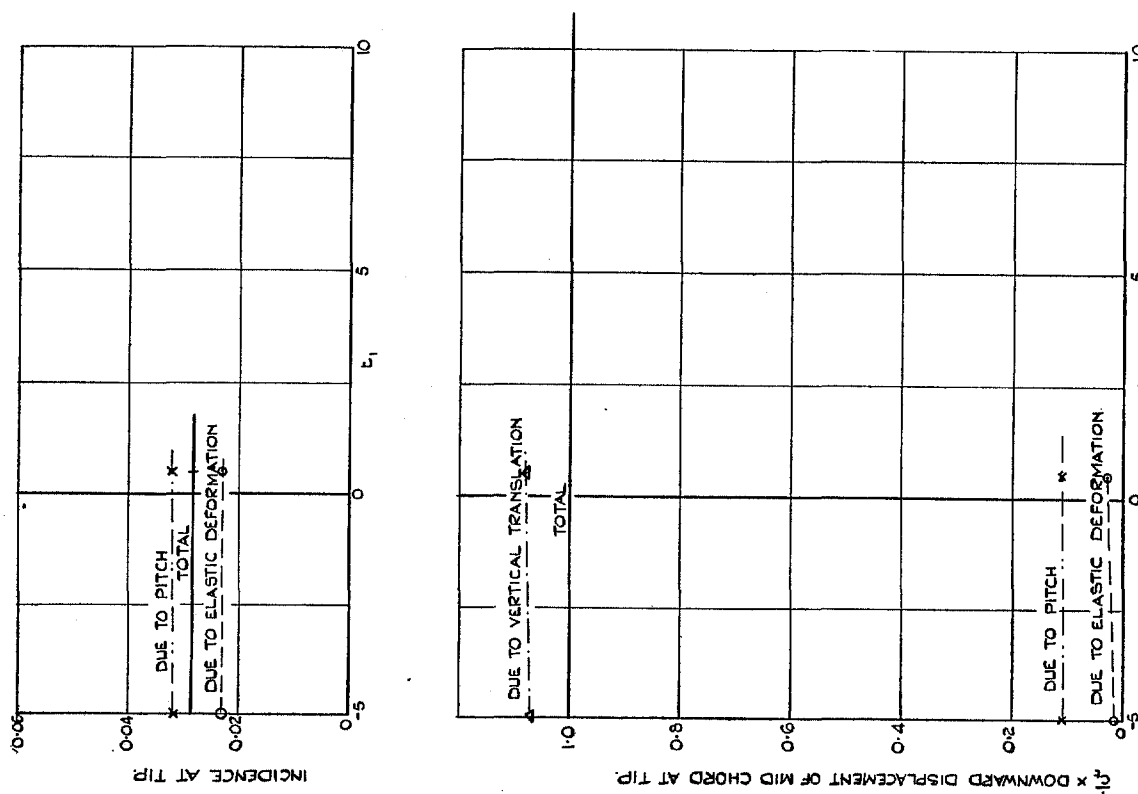

FIG. 20. Body-freedom flutter modes at lower critical speed for $(f_2/f_1)^2 = 1\cdot3$: amplitudes.

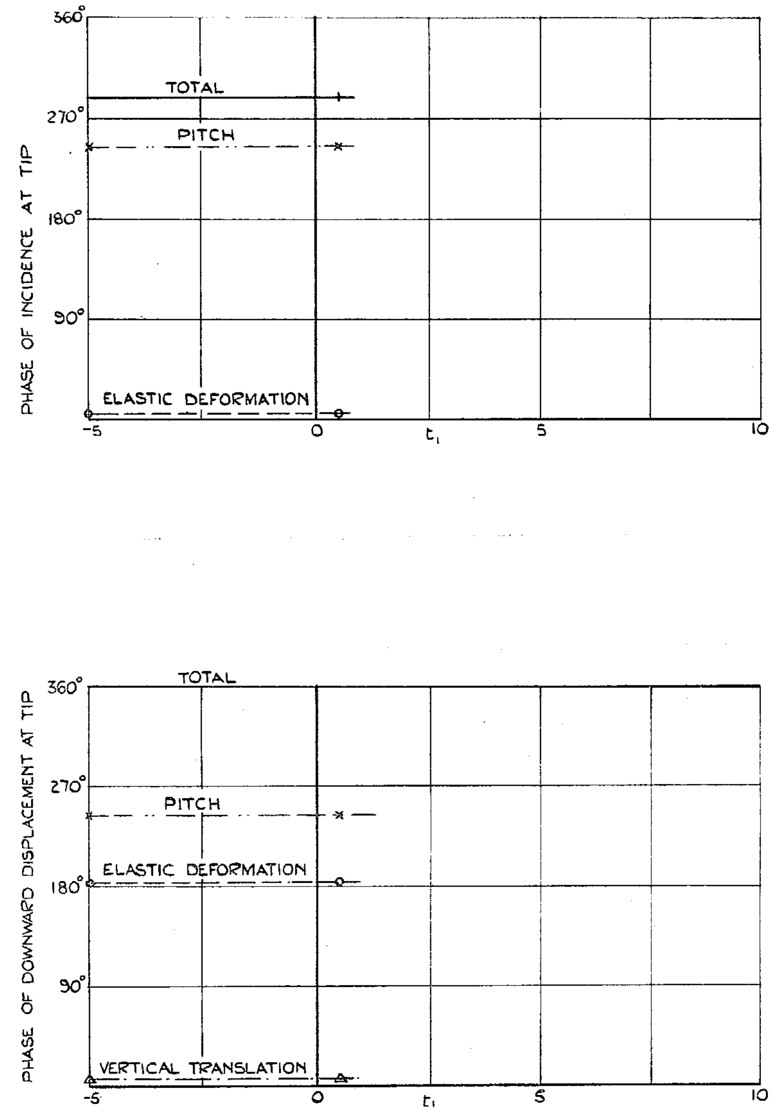

FIG. 21. Body-freedom flutter modes at lower critical speed for $(f_2/f_1)^2 = 1 \cdot 3$: phases.

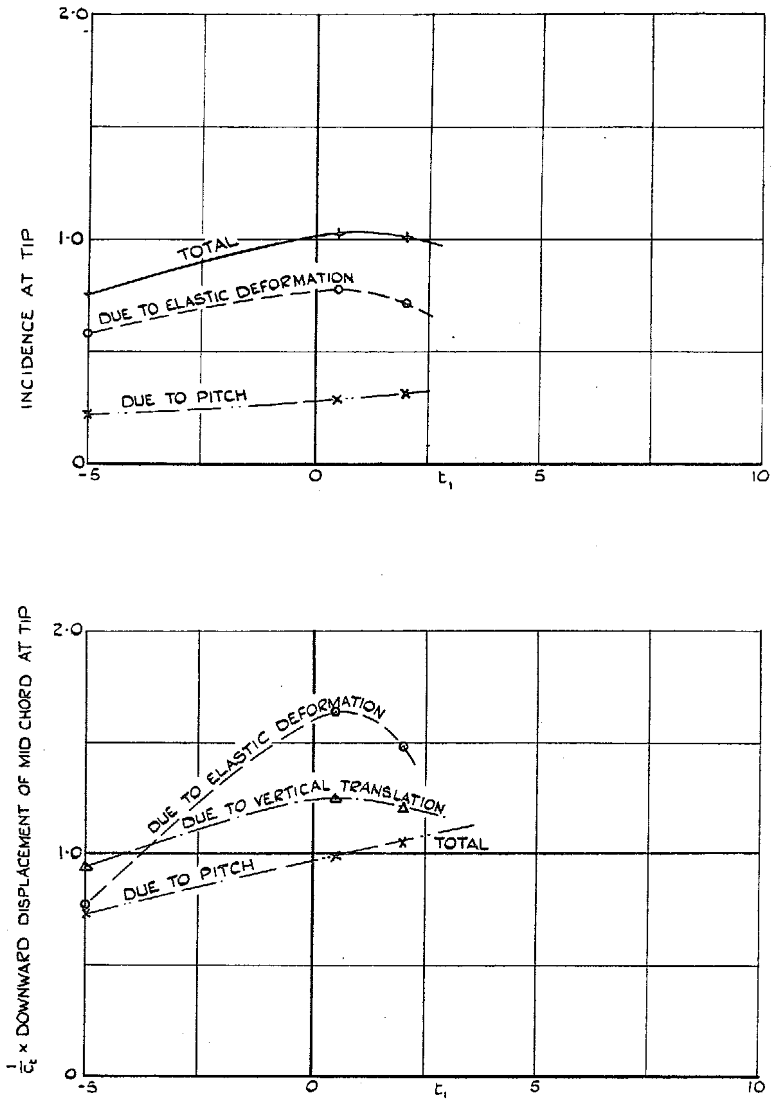

FIG. 22. Flexure-torsion flutter modes for $(f_2/f_1)^2 = 1 \cdot 3$: amplitudes.

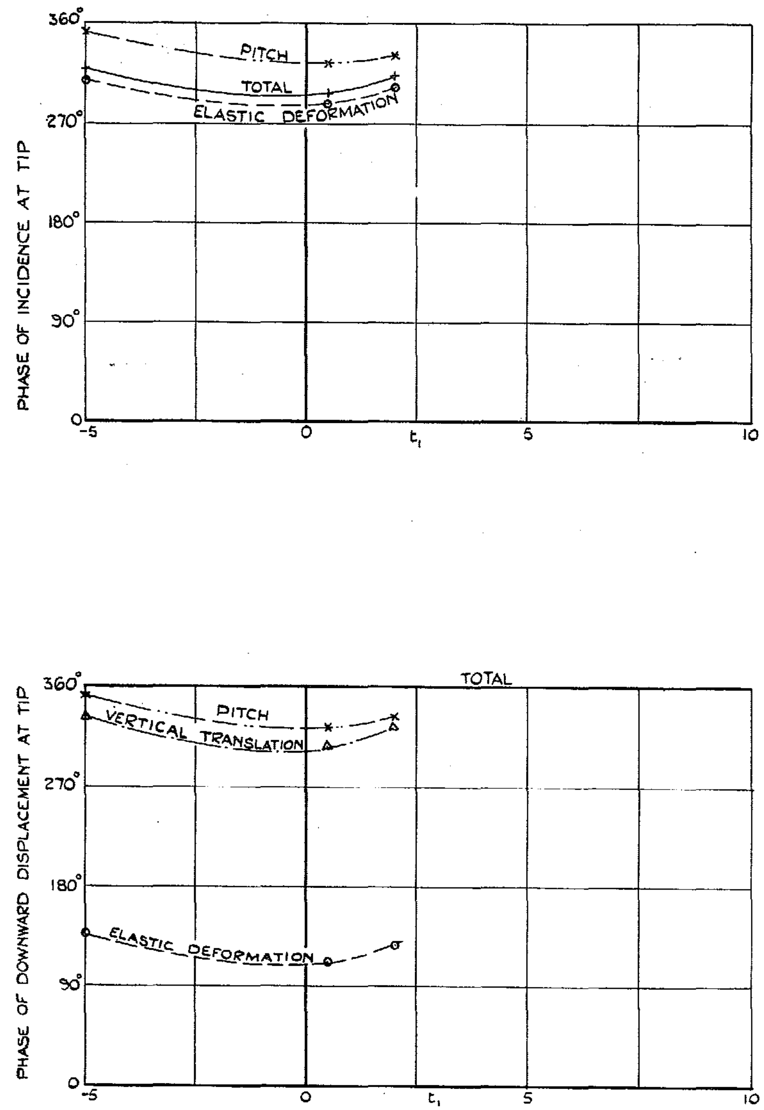

FIG. 23. Flexure—torsion flutter modes for $(f_2/f_1)^2 = 1 \cdot 3$: phases.

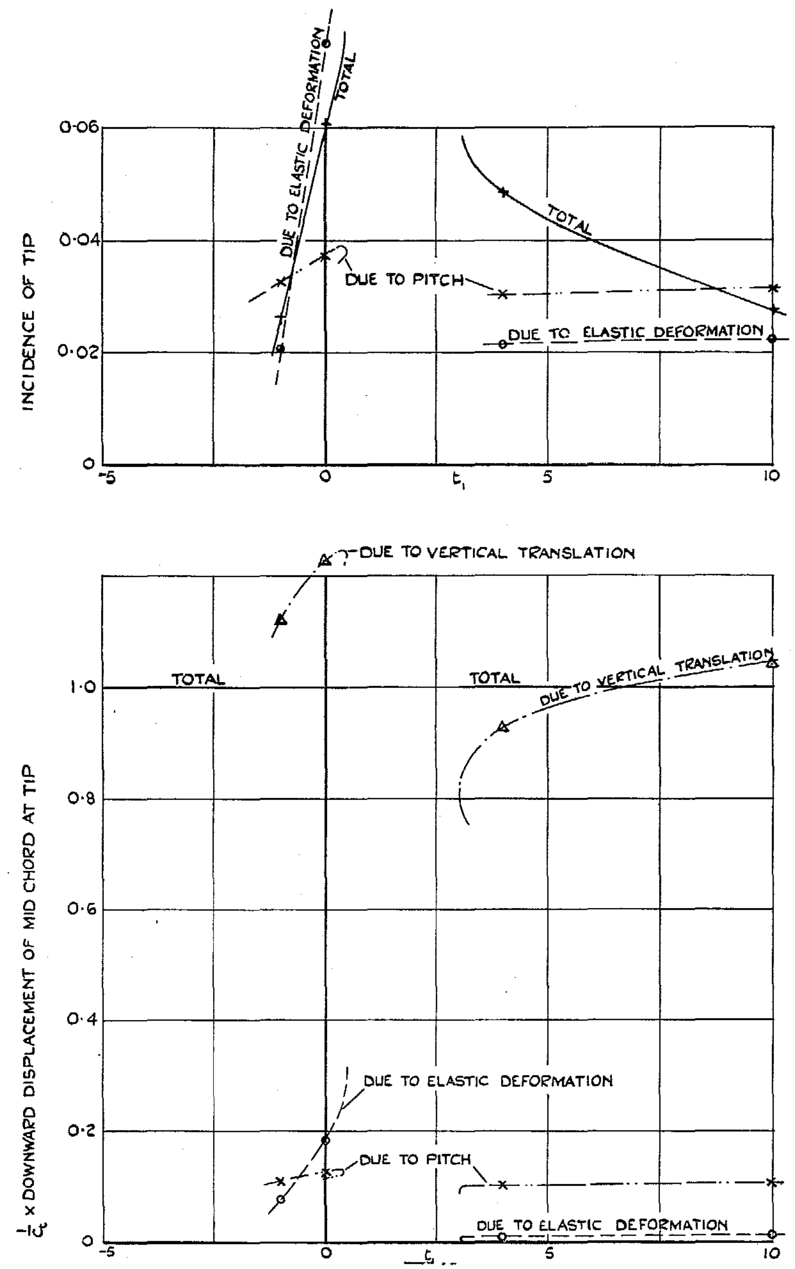

FIG. 24. Body-freedom flutter modes at lower critical speed for $(f_2/f_1)^2 = 5 \cdot 0$: amplitudes.

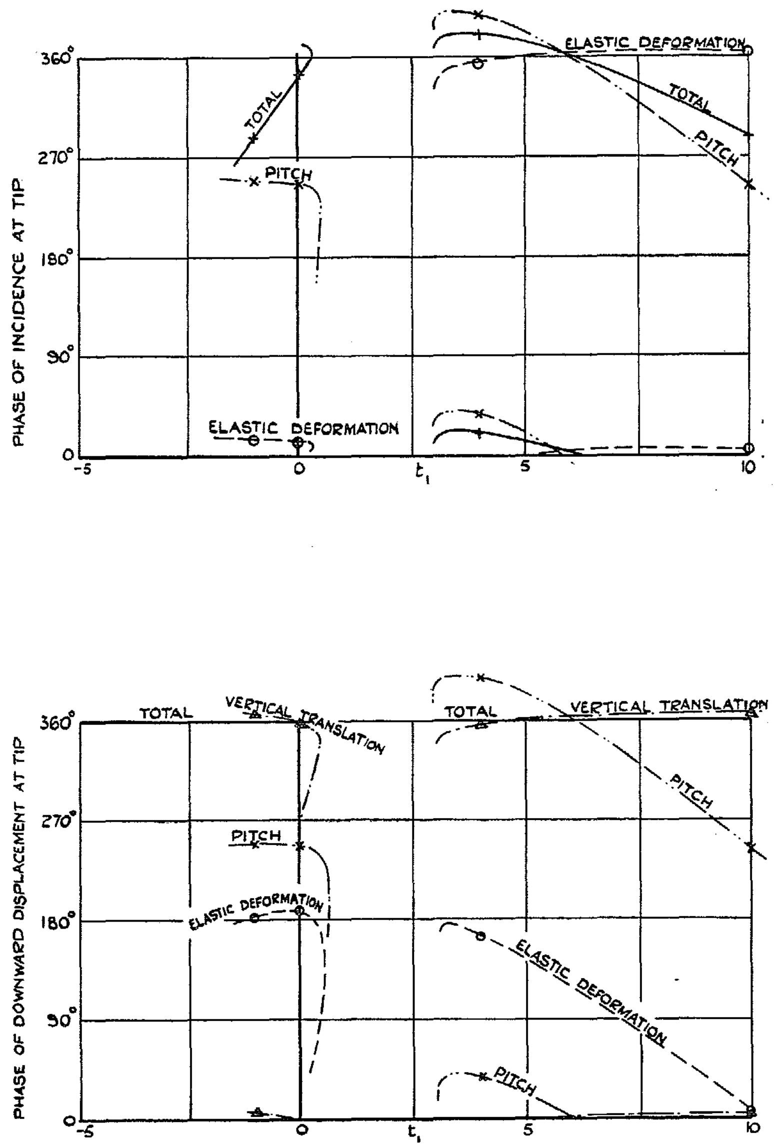

FIG. 25. Body-freedom flutter modes at lower critical speed for $(f_2/f_1)^2 = 5 \cdot 0$: phases.

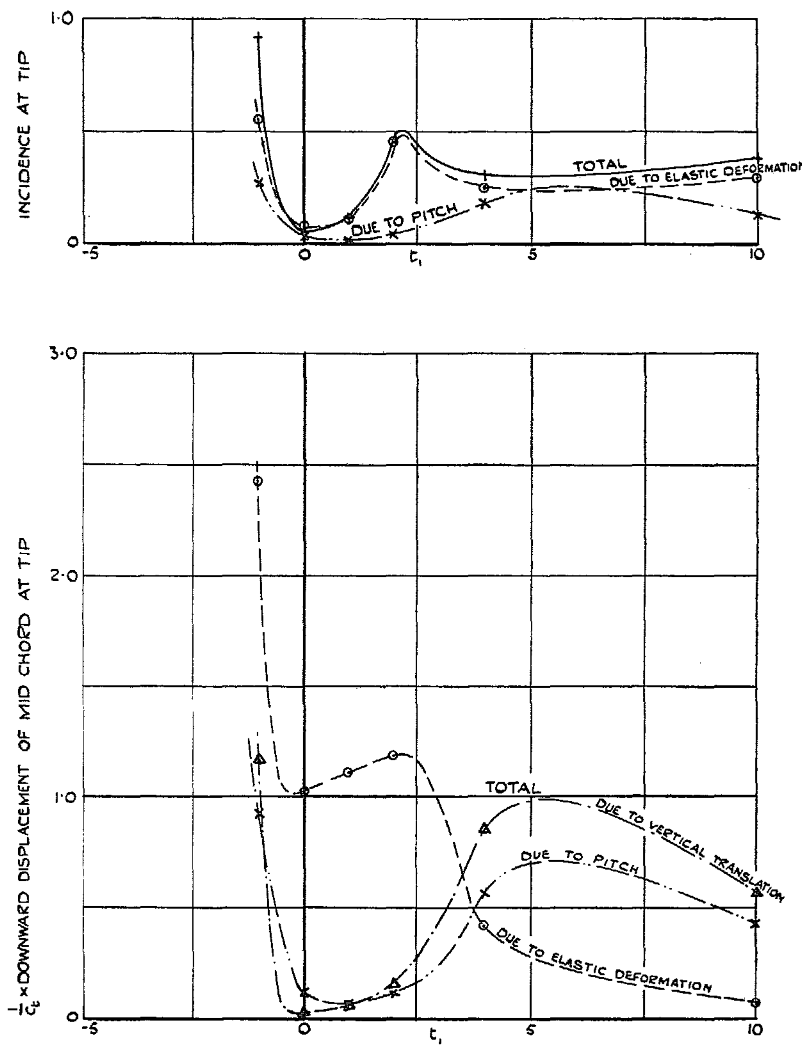

FIG. 26. Flexure-torsion flutter modes for $(f_2/f_1)^2 = 5 \cdot 0$: amplitudes.

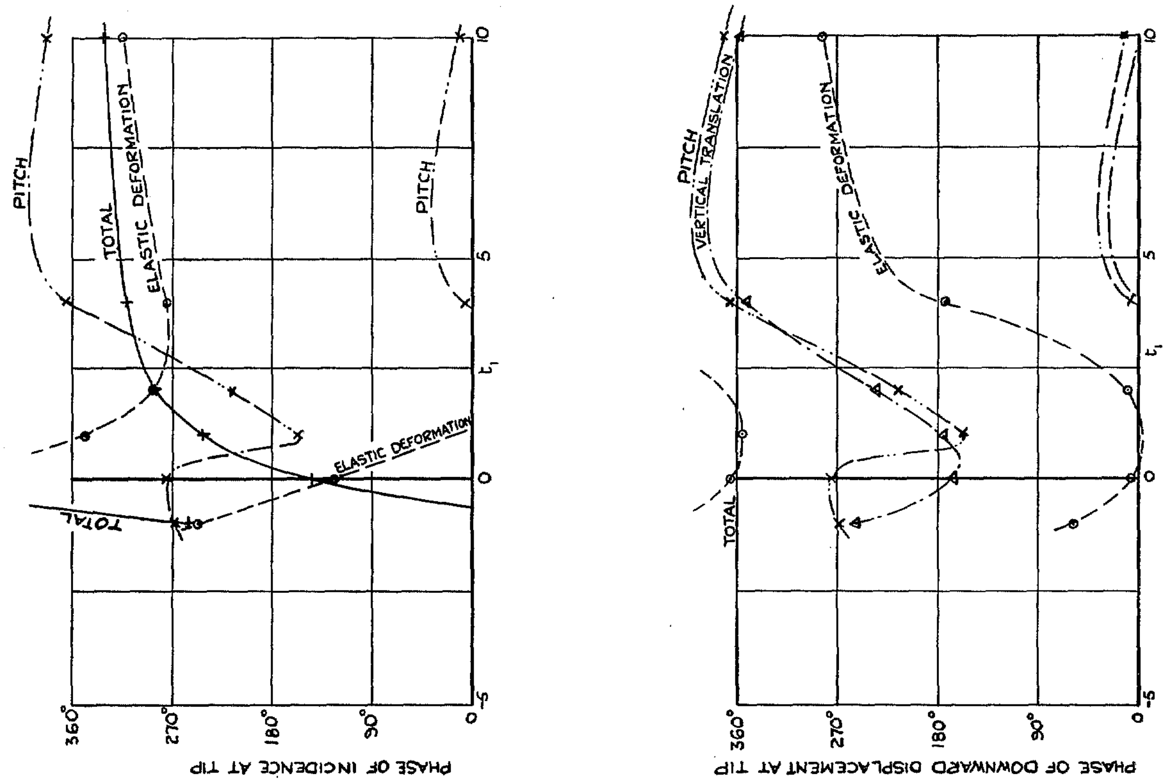

FIG. 27. Flexure-torsion flutter modes for $(f_2/f_1)^2 = 5\cdot0$: phases.

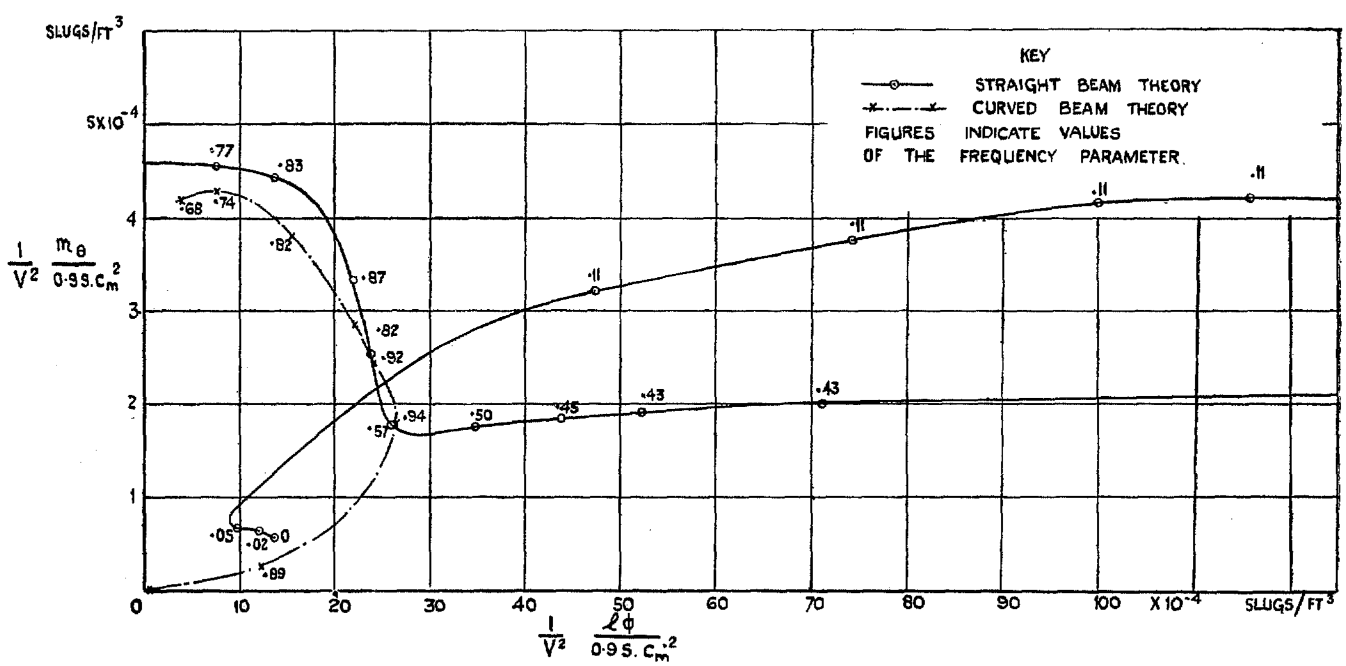

FIG. 28. Relationship between the wing stiffnesses at the critical flutter speeds.

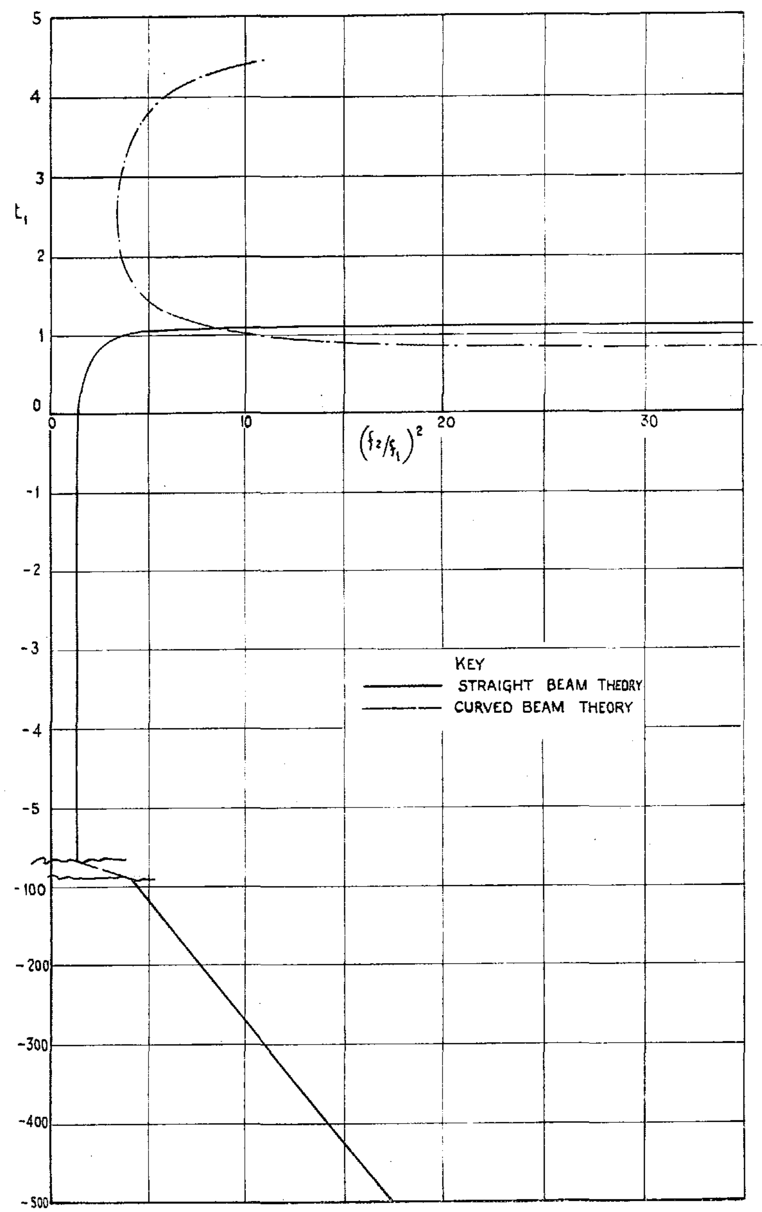

FIG. 29. Relationship between the modal parameter t_1 and the frequency ratio for two assumed structural forms.

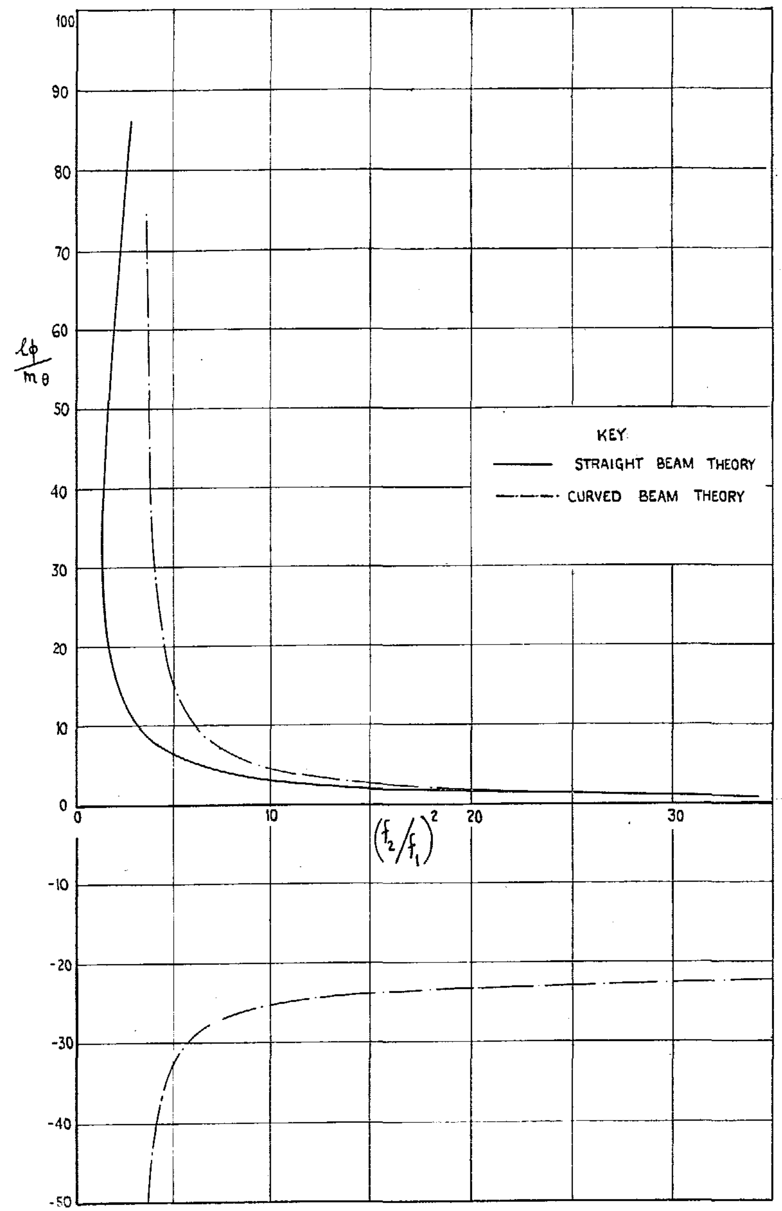

FIG. 30. Relationship between the stiffness ratio and the frequency ratio for two assumed structural forms.

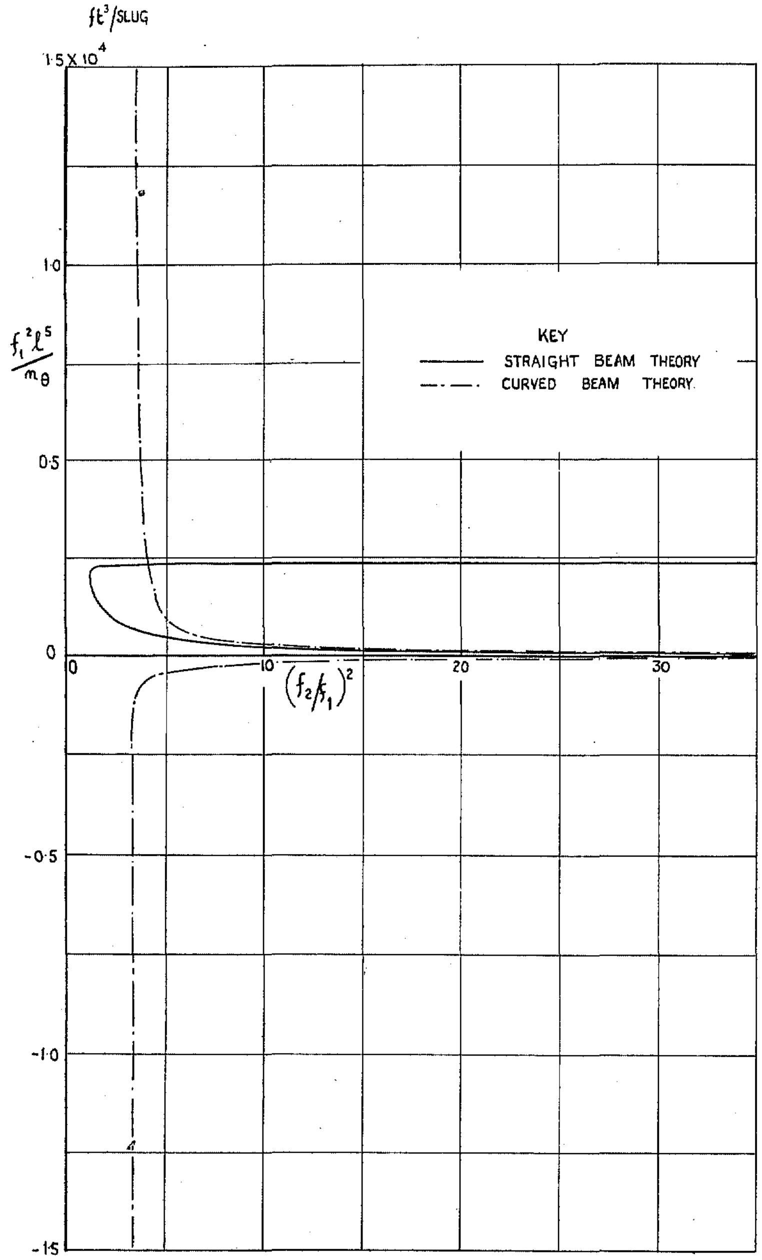

FIG. 31. Relationship between the frequency-stiffness ratio and the frequency ratio for two assumed structural forms.

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