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Summary.—The flutter problems of high-speed aircraft are considered generally and specific consideration is then given to the new problems introduced by the use of new wing plan forms.

The theoretical and experimental results on the coupled ('classical') symmetrical flutter of swept (including 'barbed' and cranked forms) and delta wings is reviewed and presented to show the effect and importance of the body freedoms of the aircraft on the critical flutter speed and frequency.

A criterion is proposed for deciding the 'dangerous type' of fundamental normal mode to be considered in flutter calculations. The danger here is that the fundamental normal mode can combine with the body freedoms and give rise to a form of flutter which is independent of the wing torsional stiffness. It is suggested that the deciding feature is the shape of the nodal line in the fundamental mode. If it is such as to indicate rotation of the fore-and-aft wing sections near the tip, then the mode is considered to be dangerous.

1. *Introduction.*—In classical† symmetrical flutter of conventional wings the flutter motion is predominantly that of the elastic modes of deformation of the wing in flexure and torsion, occurring at a frequency between the natural frequencies of these modes (usually nearer to the higher torsional frequency) and the wing torsional stiffness is the main criterion for design. For wing plan forms involving sweepback, such as are now being used for high-speed aircraft, the normal modes contain greater bodily movements in pitch and vertical translation than for unswept wings, and flutter can arise in a form consisting of a single elastic mode and the body freedoms with a frequency below that of the elastic mode. If the elastic mode is the first normal mode consisting primarily of wing flexure then torsional stiffness is unimportant in this type of flutter and the flexural stiffness is critical.

The theoretical and experimental results of published and unpublished work have been reviewed and are presented to show the effect and importance of the body freedoms on the critical flutter speed and frequency. The reduction of the critical speed for one representative swept-back wing was of the order of 30 per cent. Three different forms of symmetric flutter are described all of which can arise from the four degrees of freedom obtained from two normal modes and two body freedoms and particular attention is paid to flutter which is insensitive to the torsional stiffness of the wing. A criterion is proposed for deciding whether the fundamental normal mode to be considered in flutter calculations is dangerous.

The direct effects of compressibility and shock-wave phenomena in modifying the aerodynamic derivatives of classical flutter and in producing new types of flutter are outside the scope of this report, but a brief description is given of the work in hand.

* R.A.E. Report Structures 37, received 9th June, 1949.

† In the present paper the term 'classical flutter' is used to describe the growing oscillation produced by combination of two or more stable degrees of freedom. Aileron compressibility flutter does not fall into this class.

2. *General Review of Flutter Problems of High-Speed Aircraft.*—The most obvious effect of the present increase in speed of aircraft is to bring in a range where abrupt changes of the aerodynamic forces are to be expected, which will modify the aerodynamic derivatives of classical flutter and produce new types of flutter. Unstable oscillations of the control surface associated with an oscillating shock-wave pattern have been experienced in the U.S.A.[1] but so far no similar incidents have been encountered in this country.

The indirect effects of high speed are revealed in changes in control systems and main-surface plan forms. Controls have always been a source of flutter incidents and accidents and lengthy investigations have been needed to ensure adequate safety; moreover the present trend of modifications to existing systems and the introduction of new systems does not simplify the flutter problems. The changes in spring tabs in the form of preloading the main spring or the inclusion of an auxiliary spring are examples of the former, whilst the use of power operation illustrates the latter. The introduction of powered flying controls has, in some cases, lent itself to the idea of deleting the control-surface mass-balance, which is theoretically unnecessary for a control which can be made irreversible. It is in any case possible to argue that balance masses are only of use in combating classical flutter, and since it may be necessary to take steps to avoid compressibility flutter (*e.g.*, by means of artificial damping, or irreversible operation) the mass-balance weights may become redundant. But although this scheme of control design appears very attractive at first sight, the requirements are by no means easy to achieve in practice. A discussion of the merits and difficulties of powered flying controls together with the allied flutter problem is very relevant to the design of high-speed aircraft (*see* Ref. 2); indeed, Collar[3] has suggested that the time is near at hand when the use of irreversible powered flying controls will eliminate the control-surface flutter problem from this class of aircraft. On the other hand it is still felt in some quarters that aerodynamic troubles (such as compressibility flutter) can be cured by aerodynamic means, and that classical flutter can be cured by mass-balancing, and neither of these run the risk of breakdown in service use that must be faced by irreversible powered controls. But the problems here are mainly of a practical nature, and their solution, one way or the other, must be left to the future. The changes in main-surface plan forms for high-speed aircraft and in particular the use of swept-back wings have very important effects on the flutter problem. One serious difficulty is the lack of reliable information about the aerodynamic forces which act on the swept-back wing during the flutter motion. The unknowns in this field present a great source of difficulty to the theoretician and it is clearly necessary to carry out as much wind-tunnel testing as possible in parallel with any theoretical work; this applies equally to the actual calculation of critical speeds and to the evaluation of flutter derivatives. At high subsonic Mach numbers the difficulties are acute; the derivative information is scanty and not always reassuring (Bratt[4] has measured negative damping at the National Physical Laboratory), the current small aspect ratios render strip theory unreliable and the technique of actual flutter testing is extremely difficult. It is nevertheless necessary to do something to meet the demand of the aircraft designer for information as to how stiff the aircraft wings should be, what is the optimum mass loading, and so on.

It may be thought that the old established methods of theoretical estimation of wing flutter speeds have completed their span of usefulness, and that new methods must be sought to attack the flutter problems of high Mach numbers. But in spite of the handicap just mentioned, it is the writer's opinion that the standard methods are still capable of yielding valuable information by which the required wing stiffness can be determined. It cannot be claimed that the prediction of critical flutter speeds in any specific case will be very accurate, but the importance of the various parameters can be investigated with considerable reliability, and provided intelligent use is made of wind-tunnel results, even though at low speeds, the general flutter picture of an aircraft can be obtained with reasonable accuracy. On the experimental side the work cannot be confined to wind-tunnel tests, although the Lockheed 'ump' technique[5] shows how much can be dealt with in the tunnel, and it becomes important to develop a technique of testing rocket-powered missiles for Mach numbers near unity. It is believed that the combination of these methods will eventually guide us safely through the transonic speeds, and not the least important part of the work is the investigation of the basic flutter characteristics of an aircraft by means of the theoretical flutter calculations.

Since much of the wing flutter work in Great Britain has been based on binary calculations with the degrees of freedom wing bending and wing torsion, it is natural that the first investigations should deal with this type of flutter. The results of much of this work are now well known, but for completeness the matter is discussed in the next section. These preliminary investigations of the flutter of a built-in wing were followed by a consideration of the importance of the aircraft body freedoms. The body freedoms are known not to have much effect on the flutter characteristics of an unswept wing though their importance in other types of flutter was realised (Ref. 6). Lambourne, however, in some model tests on a flying wing of cranked shape[7] (*see* Fig. 1) had shown that the bodily degrees of freedom could have an important effect on wing flutter. On general grounds moreover, it seemed likely that the body freedoms might be of more importance for a swept-back aircraft than for an unswept design, partly because of the greater bodily motion in the fundamental normal mode, and partly because of the greater contribution to the overall pitching inertia which comes from the wings.

The results of these investigations showed that the bodily degrees of freedom are indeed important. Moreover it was found that the critical flutter frequency in certain types of flutter could have a value almost as low as half the frequency of the fundamental normal mode. For a fighter aircraft the frequency of the fundamental normal mode could well be of the order of 7 c.p.s. which might yield a flutter frequency of about 4 c.p.s. which is of the same order as the high-frequency pitching oscillation encountered in longitudinal stability work. The establishment of this fact provides an interesting fulfilment of Collar's remarks in 'The Expanding Domain of Aeroelasticity'[8] where he emphasises the importance of closing the gap between the study of aeroelastic effects (including flutter) and of rigid-body stability problems.

3. *The Swept-Back Wing Fixed at the Root.*—As the flutter of swept-back wings was first considered since the war some encouragement was found in the German results which became available, for it was shown that quite good agreement would be obtained between theory and practice by still using the methods of strip theory. It remained to carry out the work of estimating critical speeds.

3.1 *Theoretical Work.*—One of the difficulties early envisaged from the point of view of the theoretical aerodynamics was the effect of the change in camber of the aerodynamic section as the wing bent. Minhinnick has suggested, in unpublished papers, that the difficulty could be overcome by considering the downwash at a point and comparing the results with those for an unswept wing. It is assumed (Fig. 2) that sections perpendicular to the wing remain undistorted, then if the leading-edge deflection is denoted by z_0 and the rotation by θ we may write for the unswept wing

$$z = z_0 + \theta x ,$$

whence downwash $= w = \dfrac{\partial z}{\partial t} + \dfrac{V \partial z}{\partial x}$

or
$$\frac{w}{V} = i\nu \frac{z_0}{c} + \theta + \frac{i\nu x\theta}{c} , \qquad (1)$$

where
$$\nu = \frac{\omega c}{V}$$

Similarly for the swept wing

downwash:
$$w = \frac{\partial z}{\partial t} + V \cos \varepsilon \frac{\partial z}{\partial x} + V \sin \varepsilon \frac{\partial z}{\partial y}$$

whence
$$\frac{w}{V \cos \varepsilon} = i\nu \frac{z_0}{c} + \theta + \frac{i\nu x\theta}{c} + \left(\frac{\partial z_0}{\partial y} + \frac{x\, \partial\theta}{\partial y}\right) \tan \varepsilon ,$$

where $$\nu = \frac{\omega c}{V \cos \varepsilon} \quad \text{for the swept wing.}$$

By comparing (1) and (2) Minhinnick was able to deduce the substitutions

$$\left.\begin{array}{ll} V \cos \varepsilon & \text{for } V \\ \dfrac{z_0}{c} - \dfrac{i}{\nu}\dfrac{\partial z_0}{\partial y} \tan \varepsilon + \dfrac{c}{\nu^2} \cdot \dfrac{\partial \theta}{\partial y} \tan \varepsilon & \text{for } \dfrac{z_0}{c} \\ \theta - \dfrac{ic}{\nu}\dfrac{\partial \theta}{\partial y} \tan \varepsilon & \text{for } \theta \end{array}\right\} \quad \ldots \quad \ldots \quad (2)$$

from which he modified the standard vortex-sheet derivative theory (*e.g.*, that of Küssner and Schwarz[9]) and obtained corresponding expressions for the swept-back case.

Similar results had already been obtained with a different (unpublished) method by Jahn, but Minhinnick's were the more readily adapted to existing methods of calculation and were therefore adopted in a number of subsequent calculations. It has been found, however, that results which agree very closely with those given by Minhinnick's method can be obtained by factoring all the standard aerodynamic derivatives taken in the line of flight by cos ε, and this semi-empirical rule has also been much employed in *ad hoc* calculations.

If this approximate cos ε rule is employed the flutter equations have the same form for a swept as for an unswept wing. As an example, consider the wing sketched in Fig. 5. In a practical flutter calculation it is usual to choose modes of vibration of the form bending and torsion along and about the swept-back axis respectively. In this way the aerodynamic coefficients become modified to a considerable extent in some cases from their relative values for the unswept wing, but it may simplify consideration of the problem here to choose modes of the same aerodynamic form as for the unswept wing. We therefore adopt modes of bending with no aerodynamic twist, and of pure twist, given by

$$z = z_r f(\eta) ,$$
$$\theta = \theta_r F(\eta) ,$$

where z is the vertical deflection of the flexural axis (taken as the nodal line of the second mode) and the suffix r refers to a reference section; η is a non-dimensional spanwise co-ordinate made unity at the tip, and f and F are modal functions.

The equations for the aerodynamic lift and pitching moment on a strip may be written

$$\frac{dL}{\rho V^2 sc\, d\eta \cos \varepsilon} = \left\{\lambda^2\left(\frac{c}{c_r}\right)^2 l_{\ddot{z}} + \lambda\left(\frac{c}{c_r}\right) l_{\dot{z}} + l_z\right\} \frac{z_l}{c} + \left\{\lambda^2\left(\frac{c}{c_r}\right)^2 l_{\ddot{\alpha}} + \lambda\left(\frac{c}{c_r}\right) l_{\dot{\alpha}} + l_\alpha\right\} \alpha ,$$

and

$$\frac{dM}{\rho V^2 sc^2\, d\eta \cos \varepsilon} = \left\{\lambda^2\left(\frac{c}{c_r}\right)^2 m_{\ddot{z}} + \lambda\left(\frac{c}{c_r}\right) m_{\dot{z}} + m_z\right\} \frac{z_l}{c} + \left\{\lambda^2\left(\frac{c}{c_r}\right)^2 m_{\ddot{\alpha}} + \lambda\left(\frac{c}{c_r}\right) m_{\dot{\alpha}} + m_\alpha\right\} \alpha ,$$

where $$\lambda = i\nu_r = \frac{i\omega c_r}{V_c}$$

ω is the flutter frequency
z_l is the value of z at leading edge
s is the semi-span
$l_{\ddot{z}}$ etc., are aerodynamic derivatives (for an unswept wing).

In the expansion for the work done in the Lagrangian Equations ($= - L\,\delta z + M\,\delta\alpha$) we write the coefficient of $-\rho V^2 s c_r^2\,\delta q_s\,q_t \cos\varepsilon$ as $(a_{st}\lambda^2 + b_{st}\lambda + c_{st})$ where the generalised co-ordinates q_1 and q_2 are given by

$$q_1 = \frac{z_r}{c_r} \text{ for the first mode,}$$

$$q_2 = \theta_r \text{ for the second mode.}$$

It follows that

$$c_{11} = \int_0^1 l_z f^2\,d\eta\,,$$

and similarly for the other coefficients. The dimensional inertial coefficients are readily derived and may be written

$$A_{11} = c_r^2 s \int_0^1 m f^2\,d\eta\,,$$

$$A_{12} = c_r^2 s \int_0^1 m j\left(\frac{c}{c_r}\right) fF\,d\eta\,,$$

$$A_{22} = c_r^2 s \int_0^1 m K^2\left(\frac{c}{c_r}\right)^2 F^2\,d\eta\,,$$

where

m is the mass/unit span

jc is the distance between the inertia and flexural axes (with the former aft)

Kc is the section radius of gyration.

To make these coefficients compatible with the non-dimensional aerodynamic coefficients, they must be divided by the factor $\rho s c_r^4 \cos\varepsilon$,

whence
$$a_{11} = \frac{1}{\varrho c_r^2}\int m f^2 \sec\varepsilon\,d\eta\,, \qquad \qquad (4)$$

and similarly for a_{12} and a_{22}.

It will be seen that, neglecting end effects, the aerodynamic and inertia matrices are numerically unchanged as sweepback is increased by rotating a wing about its root (as distinct from shearing it back), for in a_{11}, for example, c_r and m will both be proportional to sec ε*. Any dependence of the critical speed on the angle of sweepback must therefore come from the elastic matrix which on the assumptions, $f = \eta^2$ and $F = \eta$ can be written

$$e = y\begin{bmatrix} \left(\frac{c_r}{s}\right)^2\left(\frac{l_\phi}{m_\theta} + 4\sin^2\varepsilon\right)\cos^2\varepsilon, & -2\left(\frac{c_r}{s}\right)\sin\varepsilon\cos\varepsilon, \\ -2\left(\frac{c_r}{s}\right)\sin\varepsilon\cos\varepsilon, & 1 \end{bmatrix} \qquad \qquad (5)$$

where m_θ is the torsional stiffness

l_ϕ is the flexural stiffness

and
$$y = \frac{m_\theta \sec^2\varepsilon}{\rho V^2 s c_r^2 \cos\varepsilon}\,.$$

At first sight the terms within the brackets of (5) differ considerably from those for an unswept wing ($\varepsilon = 0$), but in practice, for normal stiffness ratios and for moderate to large aspect ratios,

* There will, in general, be some change in the aerodynamic derivatives for the new frequency parameter ($= \nu = \omega c/V$); however, if the law given by (6) is obeyed exactly the frequency parameter will be unchanged for a constant flutter frequency. In any case the effect of a small change of frequency parameter on the derivatives is likely to be negligible.

the corrections are not large, and in particular the critical speed is known to be insensitive to the value of the flexural stiffness (*i.e.*, to e_{11}). It might therefore be expected that sweepback (by rotation about the root) would affect the flutter speed chiefly in accordance with a constant value of the parameter y. Since sc_r^2 is proportional to sec ε, this result implies that

$$V_c \propto \sec \varepsilon \quad \ldots \qquad (6)$$

This broad argument helps to give a physical picture to the general effect of sweepback on the simplest form of wing flutter, and it is borne out to the extent that binary calculations show a fairly small departure from the law (6). A more detailed examination of the results obtained will now be given in comparison with the results obtained from experiment.

3.2 *The Experimental Results and Comparison with Theory.*—At the time when the importance of sweepback was first realised a series of tests on tapered wings were in hand at the Royal Aircraft Establishment with a view to checking the latest proposal for a wing stiffness criterion[10]. The proposed criterion was given in the form

$$\frac{m_\theta}{\rho V^2 s c_m^2} > C\left(\frac{g - 0\cdot 1}{1 - 0\cdot 8k + 0\cdot 4k^2}\right)^2 \phi(M) \, , \quad \ldots \qquad (7)$$

where s is the semi-span

c_m is the mean chord

m_θ is the wing torsional stiffness between $0\cdot 7s$ and root

V is the design speed

ρ is the air density

gc is the distance of inertia axis from leading edge

k is the ratio (tip chord)/(root chord)

M is the Mach number

and $\phi(M) = (1 - M^2)^{-1/2}, \qquad 0 < M < 0\cdot 8 \, ,$

$\qquad = 1\cdot 67 \qquad\qquad 0\cdot 8 < M.$

C is a constant, the value suggested being $1\cdot 2$.

The check wind-tunnel tests were being carried out in a low-speed open-jet tunnel and could not, therefore, check the dependence of the criterion on Mach number. The other important parameters of inertia axis position and wing taper, were, however, catered for by four wings each of different taper with the inertia axis variable between $0\cdot 4$ and $0\cdot 5$ of the chord. The flexural axis was at $0\cdot 35$ chord.

These four wings provided a good means of checking the theoretical strip-theory calculations for the swept-back case as their rig was readily modified to permit sweepback up to angles of 50 deg. To avoid the trouble of constantly changing the tip it was cut off parallel to the tunnel stream for a sweepback of 35 deg and was consequently out of true for the other angles of sweepback (*see* Fig. 3). From these wind-tunnel tests the sixty different critical speeds were measured corresponding to four different tapers, three different inertia-axis positions and five different angles of sweepback. A typical selection of the results is given in Fig. 4. It will be seen that the critical speed increases roughly with sec ε, as was suggested by the theoretical argument in the last section, but that there is an initial lag where the critical speed actually decreases slightly for very small angles of sweepback. It will be noted that the theoretical results (obtained by Minhinnick's method) which are also plotted on the same diagram agree closely in shape with the experimental values, though they are pessimistic to the extent of about 10 per cent, probably as a result of using unfactored theoretical two-dimensional derivatives. It is noteworthy also that a change of inertia-axis position has much the same effect on the flutter speed of the fixed-root wing whether it is swept back or not. This fact is also true of the wing taper.

It is hoped that similar results will be obtained eventually for delta-shaped wings, but to the time of writing* only one isolated specimen has been tested. The most interesting feature of this test was the observation of the mode in which the model fluttered. This appeared to be a combination of the first two normal modes in still air, but each of these modes included bending and torsion in not very different ratios of amplitude, and as a result the flutter mode appeared to exhibit appreciable torsion about half way along the wing but none at all at the wing tip. The model was of very simple construction which may not have been adequately representative.

4. *The Free Aircraft with Swept-Back Wings.*—The work which is described above gave results more favourable to the swept-back wing than had originally been hoped, in comparison for example with similar work on aileron reversal[11]. However, Lambourne's work[7] had clearly shown that to solve the flutter problem for a wing with fixed root was not sufficient for dealing with a complete aircraft. Further work was therefore carried out in which flutter calculations were made (mostly in *ad hoc* cases) with the bodily degrees of freedom taken into account as well as with various combinations of normal, or sometimes arbitrary, modes. The results were found to be of great importance. It was shown that a quaternary calculation taking into account the following symmetric degrees of freedom.

(1) Fundamental normal mode (wing bending)

(2) Overtone normal mode (wing torsion)

(3) Aircraft pitch

(4) Aircraft vertical translation,

could in certain circumstances, yield three distinct types of flutter at about the same critical speed. The three types may be distinguished by the different amplitude ratios of the flutter motion, when if the unimportant motions are ignored the three forms are:

(*a*) 1–2
(*b*) 1–3–4
(*c*) 2–3–4.

The letters (*a*), (*b*) and (*c*) are used throughout this paper to distinguish between the three types of flutter.

The bodily freedoms play little part in type (*a*) which is basically unaltered from the flutter described in section 2. On the other hand types (*b*) and (*c*) are quite unrelated to this form of flutter, and the former is even independent of the wing torsional stiffness. Experimental confirmation of these results is not easy as the experimental technique needed for carrying out flutter tests on a free aircraft model is in itself quite a difficult matter[7]. It has been demonstrated in the tunnel, however, that freeing the body can in some cases make a considerable reduction in the critical speed of a swept-back wing. The results will now be examined in more detail.

4.1. *Theoretical Results.*—4.1.1. *General discussion.*—It is perhaps worth while introducing the subject of 'body-freedom flutter' by a brief physical consideration. As it is inconvenient in such a consideration to deal with spanwise integrals, the aircraft wing will be supposed concentrated in a typical section of unit span (*see* Fig. 5). This section is carried on a light flexible arm from a massive centre-section; by way of compensation it may be supposed that if the true semi-span is s units, the wing and air density over the section are increased s times. Probably the section should be situated at about 60 per cent of the span.

4.1.1.1. *Flutter involving wing bending.*—The type of flutter designated (*b*) above is the only form in which wing bending is of prime importance. It is safe to assume for all practical purposes that the fundamental normal mode consists of pure wing bending (which is taken along the line

* April 1949.

whose sweepback in Fig. 5 is ε) combined with some aircraft pitch and translation. But since the three degrees of freedom to be considered are:

(1) Normal mode
(2) Pitch
(3) Vertical translation

it matters little whether any pitch or translation is included in (1) or not. For the purpose of this illustration it is necessary, as before, to use an artifice which preserves the same aerodynamic terms (neglecting changes in frequency parameter) as the sweepback is changed. The co-ordinates chosen are:

(1) Vertical motion of the section with no change of incidence. The actual bending mode is assumed parabolic, so this degree of freedom involves a complicated motion of the fuselage
(2) Aircraft pitch about the section quarter-chord
(3) Aircraft vertical translation.

With these degrees of freedom the air forces for constant frequency parameter vary with sweepback in accordance with the factor $sc^2 \cos \varepsilon$ which is constant if the wing is swept back by rotation about the root.

Some simplifying assumptions are now made:

(i) The line of bending is at the quarter-chord (sweep ε)
(ii) The section c.g. and also the fuselage c.g. is on this line
(iii) The section mass (m) equals the half-fuselage mass
(iv) The sectional radius of gyration is half the fuselage radius of gyration and equals $\frac{1}{4}c$ where c is the section chord.
(v) l (*see* Fig. 5) $= 2c$.
(vi) The co-ordinates (1) and (3) (denoted in general by q) are made non-dimensional by dividing vertical displacements by the section chord c.

It is now a simple matter to derive the inertia terms in the Lagrangian flutter equations. In non-dimensional form the inertia matrix (which is of course symmetrical) may be written

$$[a] = \begin{bmatrix} 1 + \sin^2\varepsilon\,(4\sin^2\varepsilon + \frac{1}{4}), & -\sin\varepsilon\,(4\sin^2\varepsilon + \frac{1}{4}), & 1 + 2\sin^2\varepsilon \\ & \frac{5}{16} + 4\sin^2\varepsilon, & -2\sin\varepsilon \\ & & 2 \end{bmatrix} \quad \ldots \quad \ldots \quad (8)$$

With ε equated to zero (8) reduces to the form

$$[a] = \begin{bmatrix} 1, & 0, & 1 \\ & \frac{5}{16}, & 0 \\ & & 2 \end{bmatrix} \quad \ldots \quad \ldots \quad (9)$$

and in this condition the co-ordinates chosen above are quite usual, for it only needs some vertical translation to turn (1) into a normal mode. As the wing is swept back however, the equations (for constant frequency parameter) remain unchanged except in so far as the inertia matrix $[a]$ changes according to (8), and it will be seen that the coupling terms a_{12} and a_{23} have been introduced and grow with increasing sweepback. Both these coupling terms have negative signs and from this it appears that the part played by the pitching degree of freedom in any flutter which may occur from (8) is not quite analagous to that of torsion in flexure-torsion flutter, when (with the usual location of the inertia axis aft of the flexural axis) the coefficient a_{12} would be positive. Nevertheless, the growth of the coupling terms given by (8) may promote flutter. As an indication of the circumstances in which this is so, we may examine the problem in the light of normal modes.

4.1.1.2. *Consideration of the normal mode.*—The three degrees of freedom used in the last section corresponded to a parabolic mode of wing bending, pitch and vertical translation of the aircraft as a whole. From these three degrees of freedom a normal mode can be derived, and as it is a matter of experience that the fundamental normal mode of an aircraft consists of parabolic wing bending and body freedoms alone, this normal mode will be fairly representative. If this normal mode is now used in place of degree of freedom (1) in the ternary calculation, the result will, of course, be unaltered. In passing it may be noted that physical considerations necessitate that in general some aircraft pitch will be introduced into the normal mode.

It is known that flutter is well-nigh impossible unless one of the constituent degrees of freedom involves a change of wing incidence—as opposed to translation[12]. Now for an unswept wing the ternary:

(1) normal mode

(2) pitch

(3) vertical translation

is stable, so it is a reasonable deduction to suppose that the possibility of this type of flutter with a swept-back wing must be dependent on the amount of incidence change in the normal mode. Or with the modified co-ordinates introduced above, it must depend on the amplitude of q_2 in the normal mode. The equation to the normal mode is

$$K[a][q] = [e][q] \,, \qquad (10)$$

where K is a scalar constant and q is the modal column. Equation (10) may be written, since there is only one elastic term,

$$K \begin{bmatrix} a_{11} - e, & a_{12}, & a_{13} \\ a_{21}, & a_{22}, & a_{23} \\ a_{31}, & a_{32}, & a_{33} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = 0 \,, \qquad (11)$$

from which the amplitude ratios are

$$q_1 : q_2 : q_3 = \begin{vmatrix} a_{22}, & a_{23} \\ a_{32}, & a_{33} \end{vmatrix} : - \begin{vmatrix} a_{21}, & a_{23} \\ a_{31}, & a_{33} \end{vmatrix} : \begin{vmatrix} a_{21}, & a_{22} \\ a_{31}, & a_{32} \end{vmatrix} . \qquad (12)$$

By equation (8) it follows that the amplitude q_2 is proportional to

$$-\left(-4 \sin^3 \varepsilon + \tfrac{3}{2} \sin \varepsilon\right), \qquad (13)$$

and it is to be noted with interest that its value is zero not only when ε is zero, but also when ε has the value $\sin^{-1}\sqrt{\frac{3}{8}}$, or about 38 deg. Furthermore the expression (13) has its maximum (negative) value for a sweepback given by

$$\varepsilon = \sin^{-1}\sqrt{\tfrac{1}{8}} \qquad (14)$$

$$= 21 \text{ deg approximately.}$$

We may therefore argue that flutter of this type is most likely to be experienced with angles of sweepback of about 20 deg, and conversely that when the sweep reaches about 40 deg the likelihood is passed.

It will be interesting for the purpose of comparison with results to be given later to investigate the position of the node in the vibration mode both on the centre-line and at the current section of the idealised aircraft. The amplitude ratios for 20-deg sweepback are

$$q_1 : q_2 : q_3 = 1 \cdot 09 : -0 \cdot 352 : -0 \cdot 794. \qquad (15)$$

This gives nodal points (shown in Fig. 6a) situated about $0 \cdot 4c$ ahead of O on the centre-line and $0 \cdot 85c$ aft of A in the section. With sweepback 40 deg (Fig. 6b) the node on the centre-line is somewhat further forward, and, of course, the section node is nearly at infinity (forward in this case); the amplitude ratios are

$$q_1 : q_2 : q_3 = 2 \cdot 28 : 0 \cdot 08 : -2 \cdot 03. \qquad \ldots \qquad (16)$$

The probable shapes of the nodal lines are shown dotted on Fig. 6. For the critical case the nodal line assumes roughly the shape of a parabola with its axis on the aircraft centre-line and its vertex forward of the aircraft c.g. in such a way that it passes through the rear of the typical section. It will be seen later that this type of nodal line in the fundamental mode is almost always associated with flutter of the type fundamental mode combined with body freedoms.

4.1.1.3. *Types of flutter involving wing torsion.*—The forms of flutter which depend primarily on wing torsion are less difficult to deal with than that depending primarily on bending, since it is quite usual for the torsional stiffness of a wing to be prescribed by flutter considerations, whereas the bending stiffness is determined by strength considerations. In the case of flutter involving equally the bending and torsional stiffnesses (type (*a*) above) it is equally true for the swept wing as for the unswept wing that the torsional stiffness is (for normal stiffness ratios) the important parameter, and so it is also in type (*c*) as well as in antisymmetric flutter.

Comparatively little attention has been paid to antisymmetric flutter for it was expected that sweepback would have a greater effect on the symmetric modes of vibration than on the anti-symmetric. Although this has in general proved to be true, the first flutter test of a tip-to-tip swept-back model in a tunnel at the R.A.E. showed that anti-symmetric flutter could not be neglected, for as the sweepback was increased the type of flutter changed from symmetric to antisymmetric. As a result of this a series of flutter calculations has been started on this form of flutter, and critical speeds comparable with those of the symmetric modes from the combination of wing torsion and aircraft roll have been obtained. These two degrees of freedom are found to be the important ones in the consideration of antisymmetric wing flutter without sweepback, although a certain amount of wing bending is always associated with the aircraft roll. It appears, however, that for a swept-back wing the critical speed can in some circumstances be relatively less than for the corresponding unswept wing.

The symmetric forms of flutter in which wing torsion is most important all bear a close resemblance to the familiar flexure-torsion flutter of an unswept wing. Type (*a*) has been found to occur at some speed on wings of all plan forms for suitable positions of the flexural and inertial axes. The flutter is curable by 'mass-balancing', *i.e.*, by locating the inertia axis well forward. It also exhibits the familiar characteristic that for normal stiffness ratios a decrease in flexural stiffness is slightly beneficial. Finally, flutter of type (*c*) involving torsion and body freedoms only can also be prevented by moving the inertial axis to a forward position in the wing.

4.1.2. *Theoretical calculations on a hypothetical swept wing.*—The plan form of the wing considered is shown in Fig. 7. The aspect ratio is 8 and the ratio of the root chord to the tip chord is 2. The calculations were made primarily with a view to finding in what circumstances the body freedoms became important in the flutter. Accordingly the positions of the inertial and flexural axes, which are usually regarded as important parameters, were kept fixed and coincident at the half-chord, and instead the sweepback and the fuselage mass were varied. A further important part of the investigation (which is not yet complete) was to provide a check on two theoretical points: one is the question of deciding whether the simple factor of $\cos \varepsilon$ applied to the aerodynamic derivatives is satisfactory, and the other is whether normal modes should be used in preference to arbitrary modes.

The derivative question was soon disposed of. Comparative calculations were made between Minhinnick's method and the simple factor of cos ε on the derivatives. In all cases the derivatives used were the acceleration potential derivatives of Dietze[13] for a Mach number of 0·7. The method of calculation, as in the calculation of the next section, was to assume a mean frequency parameter (ν) and then to work out aerodynamic stiffness, damping and inertia coefficients which were then treated as independent of frequency parameter. This assumes that the derivatives are insensitive to frequency parameter, which is true only for relatively small variations in the range where body freedoms are important (ν between about 0·2 and 0·6). A few results for 30-deg sweepback are given in Table 1.

TABLE 1

Type of flutter	Speed parameter		ν		Assumed ν
	Minhinnick	cos ε	Minhinnick	cos ε	
b	8·0	8·9	0·21	0·22	0·2
c	7·1	7·3	0·24	0·22	0·2
b	20·0	27·4	0·17	0·16	0·2
c	7·3	7·2	0·23	0·22	0·2

The speed parameter of the second and third columns is directly proportional to the critical speed. The two methods agree quite well in most cases; a margin of 10 per cent would cover the difference except in one or two isolated cases, *e.g.*, that of the third row of Table 1. The larger discrepancies, however, seem to be associated either with very high critical speeds or with somewhat unreal physical conditions.

The desired check on the relative merits of the method of calculation using respectively normal and arbitrary assumed modes of vibration was not so easy to carry out. The desire to obtain this check arose from the following considerations. It is clearly necessary to make some form of flutter assessment in the early design stages of a high-speed aircraft, and often the only satisfactory way of doing so will be by means of flutter calculations. At this stage normal modes will not readily be available and a considerable saving in time will be possible if a calculation using arbitrary modes is satisfactory. In considering the flutter problems of unswept wings we were fortunate in that experience showed little to choose between the two methods[12], but early calculations on swept-back wings suggested that this state of affairs was no longer true.

The scheme of the check was to prescribe an aircraft whose flexibility was restricted to six 'semi-rigid' degrees of freedom: the two body freedoms, two modes of flexural displacement and two modes of torsional displacement. The four elastic modes thus prescribed were treated as the arbitrary modes, and the four comparable normal modes were worked out. The programme which followed, and is by no means complete at the time of writing, was to solve all the relevant binaries with arbitrary modes followed by the relevant ternaries, quaternaries and so on up to the senary and then to repeat this process with the normal modes. A comparison between the two methods would then show which most quickly approximated to the full solution. This process has the merit that it also shows how rapidly in an absolute sense it converges, and therefore how many degrees of freedom need be taken into account for a complete aircraft—provided that the six degrees of freedom prescribed are sufficient. Unfortunately all the results are not yet available, and all that can be said at present is that the method using arbitrary modes seems to introduce some very pessimistic results—even in a ternary solution—which are not present to the same extent in the normal mode calculations. A section of the results for arbitrary modes is given in Table 2.

TABLE 2

Sweepback—30 deg

F is parabolic wing flexure — V is vertical translation
T is linear wing torsion — P is pitching.

Critical speed is proportional to upper figure.
Critical frequency is proportional to lower figure.

Binaries:	FT	FV	FP	TP	TV
	4·58	—	3·38	2·86	7·05
	1·17	—	0·54	0·50	0·86
Ternaries:	FTV	FVP	FTP	TVP	
	4·65	—	2·24	5·26	
	1·24	—	0·38	0·91	
Quaternary:	FTVP				
	6·68				
	1·32				

It is evident from this diagram that quaternary solutions at least are necessary if the calculation is to be made using arbitrary modes. Moreover the arbitrary modes used in this test case possess a much closer relationship to the modes which are possible in this hypothetical aircraft than they would for a real aircraft in which an infinite number of modes is possible. Unfortunately the final answer for the senary is not yet available so it remains to be seen whether the quaternary gives a sufficiently close approximation to it.

In the parallel investigation using normal modes the lowest critical speed number obtained was for the binary of second normal mode and pitch (similar to TP above) and had the value 4·76. In both cases a noteworthy feature is the absence of flutter of type (*b*)—FVP.

Normal mode calculations have been made on the hypothetical wing to determine how the flutter characteristics of those forms of flutter which involve body freedoms are affected by the shape of the relevant normal mode. The parameter used to vary the shape of the normal mode was the relative mass of the fuselage and the wing for a sweepback of 30 deg. The results are given in Fig. 7. The first five diagrams relate to the fundamental normal mode, and each one shows the position of the nodal line for a ratio of fuselage-mass to wing-mass that varies from 0 to 15/7. With each diagram is a statement of the calculated critical speed obtained from the combination of the normal mode with the body freedoms in terms of the same units used in Tables 1 and 2. It will be seen that the critical speed increases with the relative mass of the fuselage and becomes imaginary for some value of the mass between 3/14 and 3/7 of the wing mass. At the same time the nodal line in the fundamental normal mode changes its shape from roughly parabolic, thereby involving considerable change of incidence, to a straight line running across the wing and giving practically no change of incidence. For the heaviest fuselage-mass considered the nodal line again takes up a somewhat parabolic shape but carries with it no change of incidence over the outer half of the wing. The mass ratio of 3/7 was used in the calculation leading to Table 2, and explains the absence of flutter of type (*b*).

The other three diagrams relate to the first overtone, *i.e.*, the first mode of a primarily torsional character. In this case increase of the fuselage-mass from zero rapidly promotes the flutter, which now belongs to class (*c*). After quite a small increase of fuselage-mass the normal mode settles down to a shape which is practically unchanged even with a very heavy fuselage, and consists of simple torsion of the wing.

This series of diagrams has led to an attempt to provide a simple criterion for the purpose of deciding whether a given normal mode is likely to be dangerous (*i.e.*, likely to yield a real critical flutter speed on combination with the body freedoms) or not. In the case of the fundamental normal mode the critical factor appears to be whether or not there is a change of incidence over the outer half of the wing—a similar result to that used in the illustration at the beginning of the paragraph (section 3). That this is a necessary condition seems physically obvious, but diagram (f) of Fig. 7 shows that it is not sufficient, and Minhinnick has put forward the proposal illustrated by Fig. 8. He takes a point A half-way out along the wing span and at mid-chord and defines a quadrant BAC by drawing AB perpendicular to and AC parallel to the aircraft centre-line in the sense indicated by Fig. 8. If now the nodal line in the normal mode runs into the sector in a roughly radial direction from A, then the mode may be classed as dangerous.

This test has been used successfully in a number of cases, and is sound enough even to give a rough idea of the order of magnitude of the associated critical speed*. The lowest speeds for a given frequency are associated with those modes whose nodal lines run along the middle of the wing (as in Fig. 7h), whereas if the nodal line runs nearly parallel to AB or AC the critical speed, if it exists at all, will be comparatively high. For a given modal shape and for a given size of aeroplane the critical flutter speed is directly proportional to the natural frequency of the mode†.

Finally a set of calculations have been made to determine the effect of sweepback on flutter of type (b)—the fundamental mode associated with body freedoms. The results bear out those which were suggested by the investigation of section 3.1.1. With the fuselage-mass zero (apparently the worst condition for this type of flutter) a real critical speed is obtained for sweepback of less than 10 deg, and in this case the lowest critical speed is associated with about 10 deg sweepback, but the speed remains quite low up to 30-deg sweep, beyond which that particular set of calculations was not extended. This type of flutter has not yet been found in calculation for a sweepback of more than about 40 deg.

4.1.3. *Theoretical calculations on a hypothetical delta.*—One problem which immediately suggests itself in the consideration of wing flutter of a delta aircraft is the question of whether distortion of the wing aerodynamic sections will prove to be important. However, as it was desired to acquire a certain amount of basic data as quickly as possible, this problem was left in abeyance while the early calculations were carried out.

The wing geometry assumed is illustrated in Fig. 9. The aspect ratio is $2 \cdot 5$, the taper ratio of the tip chord to the root chord is $0 \cdot 15$ and the sweepback of the quarter-chord line is 45 deg. The fuselage-mass was assumed to be one-quarter of the wing-mass and the wing-inertia axis was taken to be at 40 per cent of the chord, with the overall c.g. at the middle of the root chord. Two-dimensional derivatives were used appropriate to a Mach number of $0 \cdot 7$. The frequency parameter ($\nu = \omega c/V$) was varied over the span with an assumed mean value of $0 \cdot 4$, but an approximation common in British work was made in that it was assumed that the aerodynamic forces could be expressed as coefficients of q, $\dot{q}$ and $\ddot{q}$‡, and that the variation of these coefficients with frequency parameter could be neglected for values of ν not very different from $0 \cdot 4$.

Four degrees of freedom were considered:

(1) Parabolic flexure of the mid-chord line with no change of incidence in the line of flight

(2) Linear twist of sections in the line of flight about a lateral axis through the mid-chord of the section

(3) Pitch of the aircraft about its c.g.

(4) Vertical translation of the aircraft.

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* The success of a similar attack for an unswept wing was demonstrated by Küssner[14, 15, 16] and led to the simple formula

$$V_c \propto nc$$

where n is the lowest 'dangerous' resonance frequency of the wing (for most cantilever wings with mass-balanced ailerons n = natural frequency of wing in torsion, approximately).
c is the mean chord of 'oscillating part' of wing.

† This is only strictly true if the inertias are kept constant, but the error is probably not great for practical variations from this. In all the calculations referred to in this section the wing mass is distributed in proportion to the square of the wing chord. The radius of gyration of the wing is equal to one-quarter of the local chord and of the fuselage is mostly $0 \cdot 3l$ where l is the length of the wing.

‡ q is a generalised co-ordinate.

Modes (1) and (2) are semi-rigid modes chosen for their convenient aerodynamic properties, and as a result there is an elastic cross-stiffness between the two. For the purpose of presenting the results the two normal modes given by these four degrees of freedom were worked out, and the ratio of the natural frequencies of these two modes ($=f_2/f_1$) was used as one basic parameter. The second basic paramter (t_1) is the amplitude ratio q_2/q_1 in the fundamental normal mode. The whole of the elastic matrix can now be expressed in terms of a single typical stiffness parameter and in terms of the variables t_1 and f_2/f_1.

The stiffness parameter actually used was in terms of the frequency f_1, i.e., $(f_1 l/V)^2$ as this is the most logical way of expressing the stiffness. (An alternative would have been $m_\theta/\rho V^2 s c_m^2$ where m_θ is the wing torsional stiffness measured in any specified manner, V is the critical speed and $s c_m^2$ makes the parameter non-dimensional.)

The quaternary calculations which followed might be expected to yield three solutions in some cases corresponding to the forms (*a*), (*b*) and (*c*) quoted at the beginning of this section, but so far flutter of type (*c*) has not yet been found in this series of calculations. A typical set of solutions is given in Table 3.

TABLE 3

t_1	$(f_2/f_1)^2$	First solution		Second solution		Location of nodal line in fundamental normal mode relative to critical region
		$f_1 l/V$	ν	$f_1 l/V$	ν	
−5	20	0·386	0·445	0·537	0·112	inside
−1 −1	20 5	0·258 0·278	0·421 0·443	0·324 0·368	0·116 0·113	inside
0 0	20 5	0·141 0·212	0·376 0·506	 0·251	 0·113	just inside
1 1 1	20 10 5	0·129 0·176 0·224	0·765 0·828 0·861			outside

The absolute magnitudes of the numbers in the first column are appropriate to the generalised co-ordinates $q_1 =$ (vertical tip deflection)$/l$, and $q_2 =$ tip rotation, where l is the length of the quarter-chord line. Each assumed value of t_1 defines the shape of the two normal modes and those giving shapes corresponding to practical experience are -1, 0 and 1. The critical speeds are given in terms of the parameter $f_1 l/V$ and a blank indicates that the critical speed for that case is imaginary. The values of ν (the frequency paramter) are those which were obtained by the calculation and bear comparison with the assumed value of 0·4. The last column indicates the general shape of the nodal line with reference to the critical region just described (section 4.1.2).

The two solutions given relate to different types of flutter, and although the amplitude ratios have not been worked out, the general character of the results together with the frequencies indicate that the first solution corresponds to flexure torsion flutter (*a*), and the second solution to flexure–body-freedom flutter (*b*). Evidently the case $t_1 = 0$ represents a transition case between the dangerous type of nodal line and the safe type, and this being so the results bear out the nodal line criterion very well. Another noteworthy feature of the results is that where a second solution exists at all the associated critical speed is lower than that for the first solution, although it is possible that if the results were corrected for the true frequency parameter this conclusion might be modified. The flutter frequency in the second solution is considerably lower than that in the first solution, and is in fact less than the frequency of the fundamental normal mode f_1. This fact is in accordance with other experience of flexure–body-freedom flutter.

The critical speed parameter $m_\theta/\rho V^2 s c_m^2$ has been much used in Great Britain in the form of the design criterion

$$K_d = \frac{1}{V_d}\left(\frac{m_\theta}{d c_m^2}\right)^{1/2}, \qquad \ldots \qquad (17)$$

where V_d is the design diving speed. For aircraft of high wing density the stiffness requirement has been specified as $K_d \geqslant 0 \cdot 04$ for wings without engines. This value may be compared for interest with those obtained in one or two cases for the delta wing for the corresponding critical speed criterion

$$K_c = \frac{1}{V_c}\left(\frac{m_\theta}{d c_m^2}\right)^{1/2}, \qquad \ldots \qquad (18)$$

where V_c is the critical speed. To avoid flutter K_d must exceed K_c, normally by a margin of about 25 per cent. Values for K_c in the delta calculations are about $0 \cdot 02$ or slightly larger as based on the torsional stiffness of mode 2 alone (these are taken only from the first solutions of Table 1), so that the criterion (17) with $K_d = 0 \cdot 04$ appears unnecessarily severe for a delta wing; $K_d = 0 \cdot 03$ would be better.

4.1.4. *The cranked wing.*—No systematic series of calculations has been made on wings of this plan form (*see* Fig. 1) but one specific case has been investigated by normal mode flutter calculations. This shape of wing is adopted chiefly in flying wing designs, and as the sweepback is not intended primarily to delay the drag rise, the type of design is not required specifically for high speeds. Nevertheless the results are of interest, because the low fuselage-mass and moderate degree of sweepback render this plan form one of the most likely to encounter flutter of type (*b*).

In the case considered, the four degrees of freedom were:

(1) first normal mode (frequency 1·26)

(2) second normal mode (frequency 3·14)

(3) aircraft pitch about the c.g.

(4) vertical translation,

where the frequencies are given in terms of the final flutter frequency. The nodal line in the fundamental mode was of the dangerous type, similar to that of Fig. 7a, but in the first overtone was not. The calculations gave the following results:—

TABLE 4

	1–4	All other binaries	1–2–4	1–3–4	1–2–3–4
Flutter speed ..	0·71	∞	0·72	1·08	1·00
Flutter frequency ..	0·73		0·61	0·74	1·00

On this occasion the ternary solution including the normal mode and the two body freedoms gives a rather optimistic answer, and the general effect of aircraft pitch as a separate degree of freedom is apparently to raise the flutter speed.

4.1.5. *The barbed wing.*—This shape of wing has certain advantages from an aerodynamic point of view in that the tip-stalling characteristics are improved. One specific case has been investigated theoretically, but the fuselage-mass, unlike that of the cranked wing, is not negligible, and the mean sweepback though moderate is greater than for the cranked wing. The nodal line in the fundamental mode of vibration was just outside the danger area of Fig. 8, running roughly parallel to AC. The second mode, which represented an overtone in bending rather than a fundamental torsional mode, was also of the safe type, but the third mode had a torsional shape similar to that of Fig. 7g of the dangerous type. Table 5 gives the results:—

TABLE 5

1—fundamental mode
2—first overtone
3—second overtone
4—pitch about c.g.
5—vertical translation

	1–4–5	2–4–5	3–4–5	1–3–4	1–3–4–5
Critical speed ..	—	—	0·99	1·04	1·00
Critical frequency ..	—	—	0·52	0·98	1·00

As would be expected the calculations involving the two lowest modes only are stable. The third mode yields flutter when combined either with the body freedoms (type (*c*)) or with the fundamental flexural mode (type (*a*)), but in the quaternary only flutter of type (*a*) is encountered. The lower frequency flutter of type (*c*) is only just avoided in the quaternary, and a slight increase in flexural stiffness would be sufficient to promote it. It is not easy to see why this particular effect exists, as the curve of e_{11} against e_{33} (the stiffnesses of the first and third modes respectively) is very complicated, but an increase of about 30 per cent in the stiffness ratio would be enough to introduce the second type of flutter at higher speeds (beyond the speed range of the aircraft).

4.2. *Experimental Results with Body Free.*—Unfortunately no quantitative results can be quoted in this section because the difficulties of the experimental technique have not yet been mastered.

4.2.1. *Cranked wing.*—The importance of the bodily freedoms on the flutter of 'flying wings' was stressed by Frazer[7a] and borne out in the subsequent experimental work by Lambourne[7].

A considerable amount of work was carried out on a tip-to-tip model of a cranked wing, but the author[7] expressed dissatisfaction with the experimental technique used. Qualitatively there is no great disagreement between his results and those of the present paper, but he made the interesting observation that as the mass of the fuselage is increased indefinitely the critical speed tends to an asymptotic value different from and less than that with the body locked while the critical frequency tends to zero.

4.2.2. *Swept wing.*—The tests which were made at the R.A.E. on the series of tapered model wings with fixed root have now been extended to cover the case of a free body. As in Lambourne's work a variable fuselage inertia is provided, but instead of the aircraft being allowed to pitch about two parallel axes it is supported in such a way as to give complete freedom for symmetrical motion. The wing is supported by three rigid vertical rods each of which is fixed to the tunnel floor by a universal joint and also to a plate carrying the wing by a universal joint. The system is sketched in Fig. 10; light centring springs are now shown in the diagram. Fore-and-aft motion of the plate is prevented by a roller which rolls on a fixed lateral beam—this device is not shown in Fig. 10. The critical speeds which have been measured up to the time of writing are in many cases less than the corresponding speeds obtained with the body locked, and in some circumstances the reduction is considerable (30 to 40 per cent). It is confirmed that in most of these cases the flutter mode consists primarily of wing bending and pitch combined as in the fundamental normal mode; the axis of pitching appears to describe an elliptical motion, thus indicating some out of phase vertical translation. The frequency of the flutter, as would be expected is much less than with the body fixed. It is also shown that the body-freedom forms of flutter occur readily at low angles of sweepback. For a sweepback of 50 deg it is difficult to obtain the body-freedom flutter.

In addition to this work which is being continued, certain very simple tunnel tests have been carried out. A diagrammatic illustration of a typical model of this sort is given in Fig. 11. The construction is a simple wooden spar with variable sweepback; the ribs are of balsa and the skin is silk doped in Vaseline. This type of model is used only for qualitative visual observations, but in view of the extreme ease with which it can be built and tested (the overall span is about 2 ft) it has proved of considerable help in a physical sense. As will be seen from Fig. 11 the model actually flies in the tunnel and is reasonably free in all body motion, though the swinging link obviously imposes some sort of restraint on the motion. The first such model to be tested behaved very well and provided flutter without body freedoms at zero sweep and flutter with body freedoms for 30-deg sweep at about the same critical speed; the latter form was, however, anti-symmetric. This result led to theoretical investigations on flutter of the form torsion-roll mentioned earlier.

Although the experimental results mentioned in this section must be accepted with some reserve, yet their emphasis on the importance of the body freedoms is unmistakable. The investigations are still in an early stage and it will be some time before a clear idea can be obtained of the effects of all the relevant parameters.

5. *Aerodynamic Derivatives.*—It is quite beyond the scope of this paper to give any detailed report on the progress of our flutter derivative knowledge, but a few general remarks may not be out of place. In Great Britain recent progress on the theoretical side has been due almost entirely to the work of W. P. Jones at the N.P.L., and some of the more important aspects of his recent work are indicated in the Reference list (17 to 21). It is, however, clear that although the supersonic and subsonic fields can be studied with some confidence when shock-waves are absent, the intervening region of Mach numbers near unity is intractable mathematically.

Bratt has carried out two-dimensional derivative measurements both at supersonic and at high subsonic Mach numbers[4] in the course of which he encountered negative aerodynamic damping, but these results represent a very small fraction of what is required for the confident prediction of flutter speeds at high Mach numbers.

6. *Flutter Experiments at High Speed.*—For *ad hoc* work of the immediate future, a more hopeful line seems to be the measurement of critical speeds on flight models. In this type of experimental work the model may either be dropped—in which case its own weight provides the acceleration necessary to achieve high speeds—or propelled by rocket power. In these circumstances the ultimate speed and Mach number can be regulated to have any desired value, and telemetering equipment mounted in the body can be used to transmit the flutter motions (critical speed, frequency, and some indication of the mode) back to a recording base. The experimental difficulties associated with the body freedoms are still present, however, in the testing of dropped bodies. For the mass of the body itself is so great in order to obtain the required acceleration that it is not comparable with that of an aircraft fuselage. Even with the rocket-propelled models the necessity of installing telemetering apparatus leads to a body of unrepresentative size and weight. Various schemes are, however, being considered for overcoming these difficulties as it is felt that the correct provision of body freedoms is essential if a direct relation to full-scale is desired.

Another form of flight testing which has been considered is that of carrying small models on high-speed parent aircraft. For this series of tests built-in models were envisaged, and the tests were intended to be used as a check on aerodynamic derivative theory rather than for direct application to full-scale. One of the chief drawbacks to the scheme appeared to be the doubt as to the exact influence of body intereference on the aerodynamic flow.

7. *Prevention of Flutter of High-Speed Aircraft.*—The important effects on wing flutter of the bodily degrees of freedom of the aircraft have been clearly demonstrated, but although considerable effort has been concentrated on the problem at the R.A.E. the investigations are still in a comparatively preliminary state due to the length of the calculations and many results quoted have had to be taken from incomplete researches. In particular the experimental side of the programme

is only just beginning to yield results and as yet none of these relate to high Mach numbers. On the other hand the implications of the theoretical work cannot be overlooked and it becomes an important matter for the designer of an aircraft of swept-back or delta plan form to satisfy himself not only that the wing torsional stiffness is adequate to prevent flexure-torsion flutter, but also that his design is equally immune from the other forms of flutter discussed in this paper. This task is difficult to carry out in a logical manner, for the types of flutter dependent on body freedoms may prove to be sensitive to parameters which are not known in the early stages of design. To determine the necessary flutter precautions, for example, the strictly logical approach would be to make a provisional design; normal mode calculations would then be made and on the basis of these flutter calculations would be carried out including the two body freedoms. The next step would be to determine what modifications would be necessary to avoid flutter, and then to repeat the calculations from the beginning. In this way, by a process of successive approximation, the efficient design should eventually be reached, but the cost in time and labour would be very severe, in addition to the risk of modifications from other sources throwing the train of calculations out of gear.

Unfortunately, once having decided to cut short this process, it is still difficult to see what method provides the soundest basis for doing so. On the one hand the calculation of normal modes might be omitted and reliance placed on direct flutter calculations using arbitrary modes. At the opposite extreme it may be thought more profitable to calculate the normal modes but omit the flutter calculations, in which case criteria or experience would have to decide whether the results were satisfactory. Another line of attack, which has much to recommend it, is to build an elastic-inertia model of the aircraft (this can be of simplified construction) and then to rely on the model for obtaining normal modes and if necessary the effect of mass variations; in this case flutter calculations would be carried out on the modes obtained from the model but it would be quite reasonable first to try and improve, say, the nodal line positions, by some adjustment of design. The use of flutter models for specific aircraft using the techniques outlined in section 6 is a supplementary or alternative approach.

But whatever method is decided upon, there can be little doubt of the important part to be played by the ground resonance test results obtained on the completed aircraft. These must give the final criterion of the basic flutter picture, and if they show up bad features from the point of view of flutter, elaborate investigations (calculations and/or experiments) may have to be undertaken to clear the aircraft. In view of the importance of these ground resonance tests, it is essential that they should be carried out both accurately and efficiently; to this end any improvements which can be made in the measuring or vibrating apparatus will be very valuable.

8. *Conclusions.*—The work reviewed and presented in this report shows the important effects of the bodily freedoms of the aircraft on the flutter of the wings of high-speed aircraft and the need for thorough theoretical and experimental investigations to prevent the flutter of swept and delta aircraft.

LIST OF SYMBOLS

$F(\eta)$	Torsional mode
M	Mach number
V	Forward speed
a_{rs}	Non-dimensional structural inertia coefficient
b_{rs}	Non-dimensional aerodynamic damping coefficient
c, c_m	Wing chord and mean value
c_{rs}	Non-dimensional aerodynamic stiffness coefficient
e_{rs}	Non-dimensional elastic coefficient
$f(\eta)$	Bending mode
f	Natural frequency of normal mode
g	Fraction of the chord of inertia axis aft of leading edge
k	Ratio of tip chord to root chord
l	Length of wing along swept-back axis
$l\phi$	Flexural elastic stiffness
$\left.\begin{matrix} l_{\ddot{z}}\ l_{\dot{z}}\ l_z \\ l_{\ddot{\alpha}}\ l_{\dot{\alpha}}\ l_\alpha \end{matrix}\right\}$	Aerodynamic derivatives (lift)
m_θ	Torsional elastic stiffness
$\left.\begin{matrix} m_{\ddot{z}}\ m_{\dot{z}}\ m_z \\ m_{\ddot{\alpha}}\ m_{\dot{\alpha}}\ m_\alpha \end{matrix}\right\}$	Aerodynamic derivatives (pitching moment)
q	Generalised co-ordinate
s	Semi-span
w	Downwash
x	Chordwise dimension
y	Spanwise dimension also in equations (3), (4) and (5) non-dimensional stiffness-speed parameter
z	Vertical dimension
γ_{rs}	Non-dimensional aerodynamic inertia coefficient
ε	Angle of sweepback
θ	Angle of twist or incidence
ν	Frequency parameter $= \omega c/V$
λ	$i\nu_r$
ρ	Air density
ω	Critical frequency

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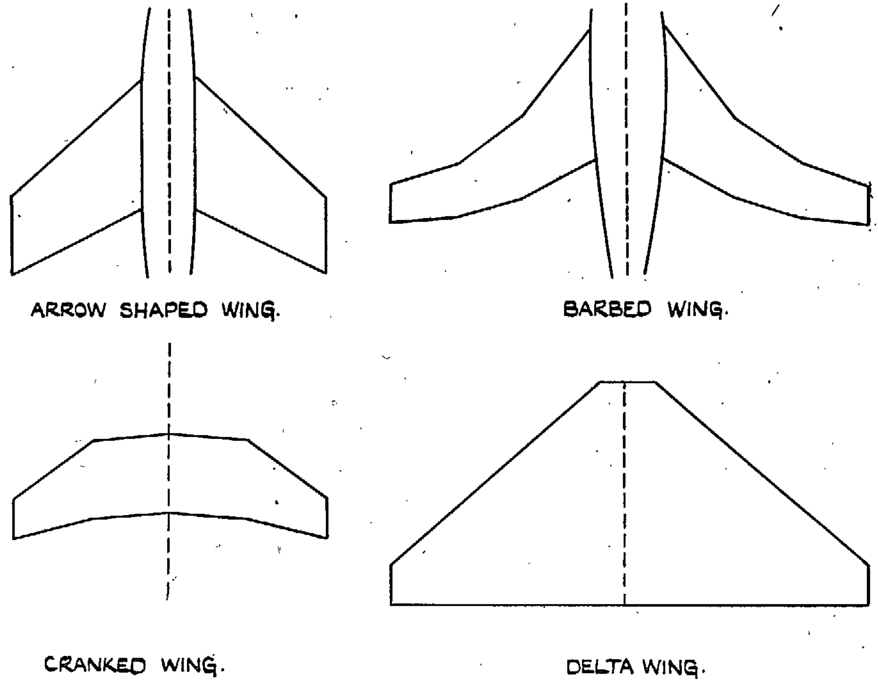

FIG. 1. Different types of wing plan form.

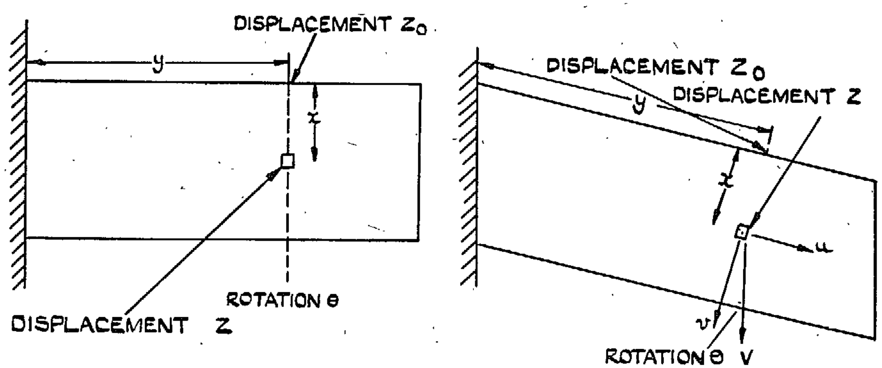

FIG. 2. Diagram for comparing downwash in the swept and unswept cases.

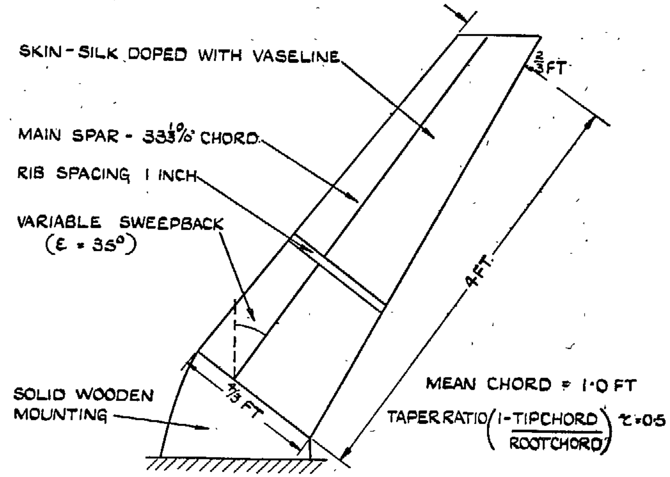

FIG. 3. Model wing used in flutter tests.

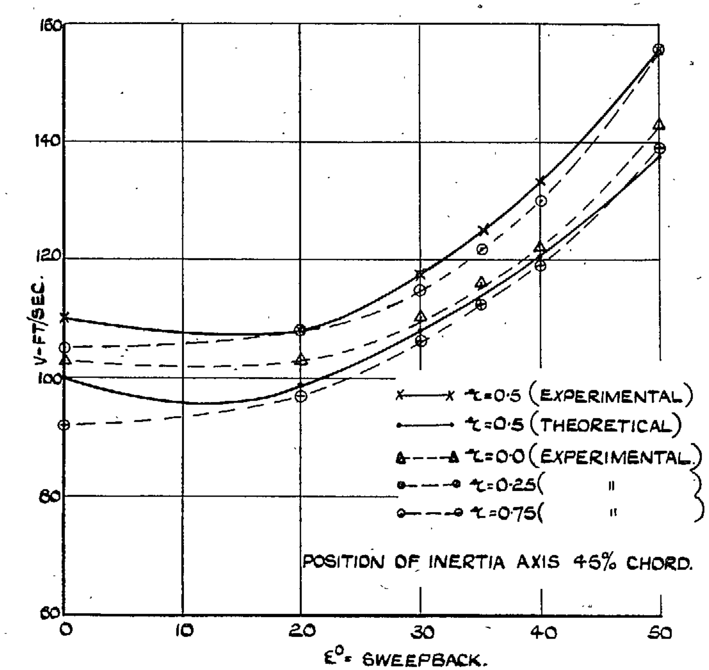

FIG. 4. Critical flutter speed *vs.* sweepback for fixed wing root.

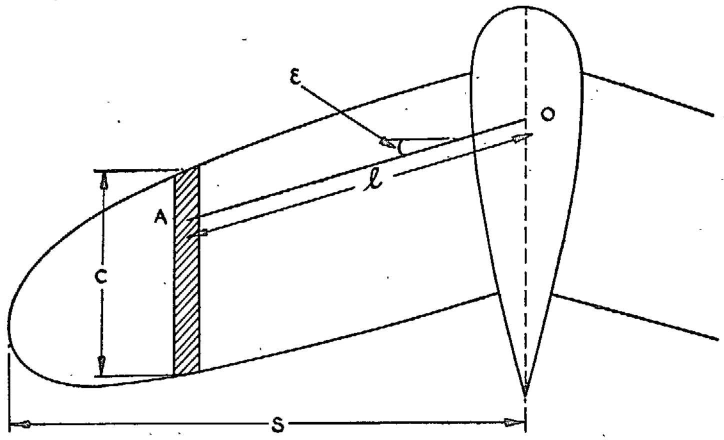

FIG. 5. Wing for example.

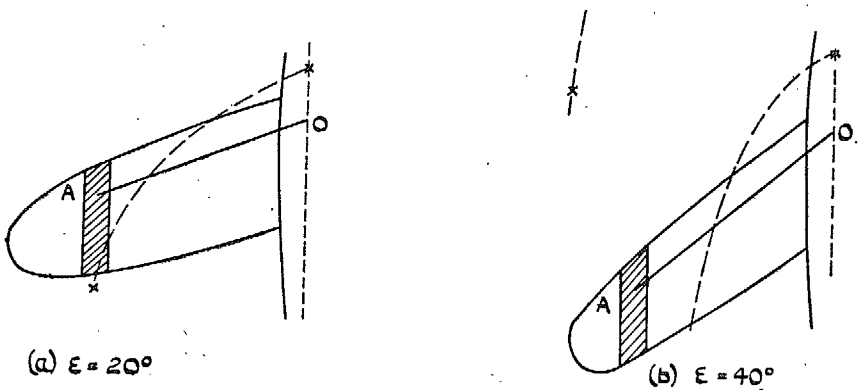

FIG. 6. Nodal line position.

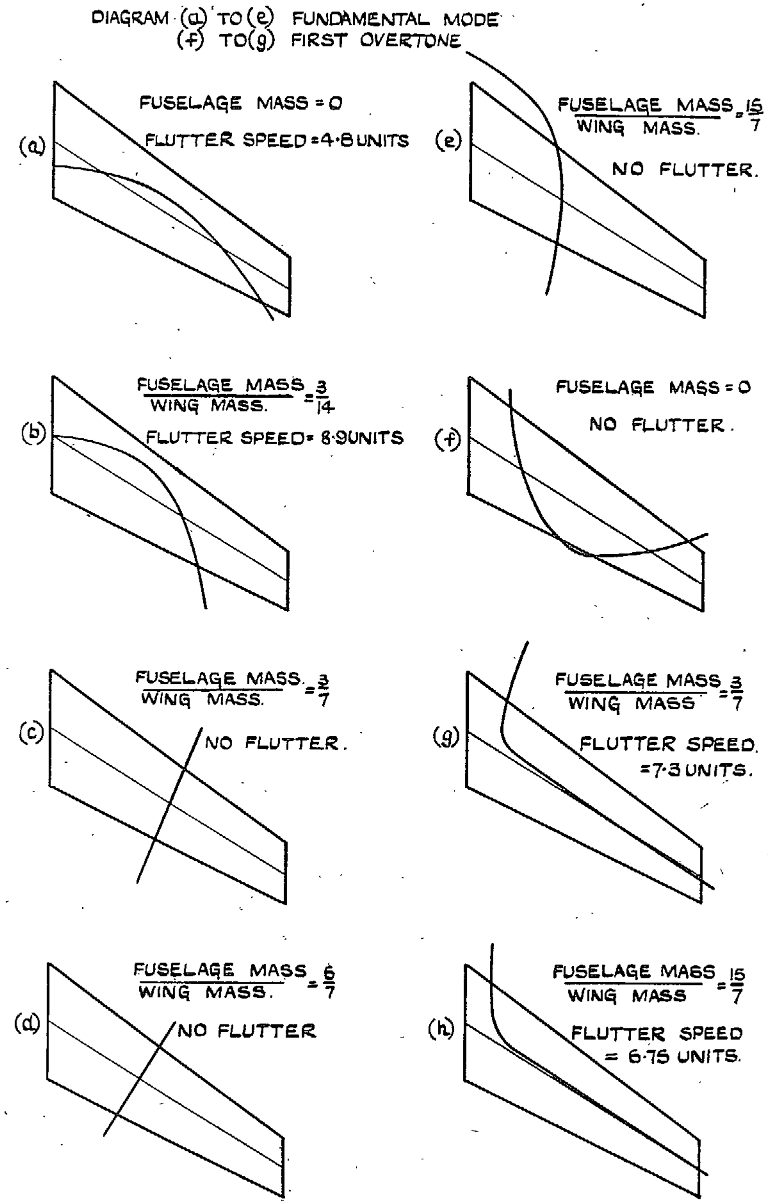

FIG. 7. Diagram showing nodal lines of fundamental and first overtone normal modes of the hypothetical wing.

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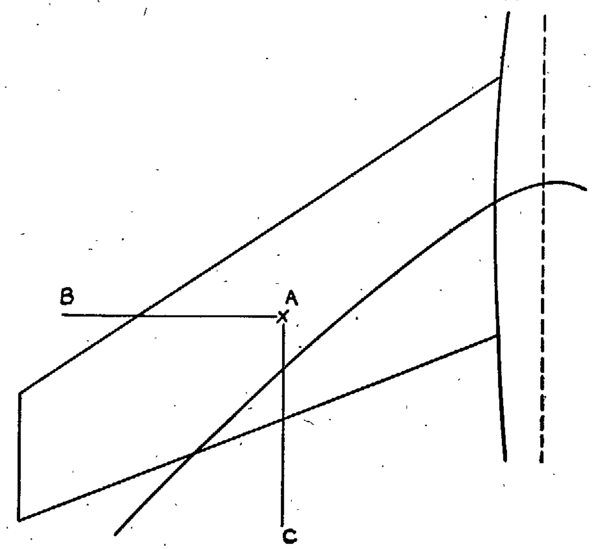

FIG. 8. Criterion for dangerous normal mode.

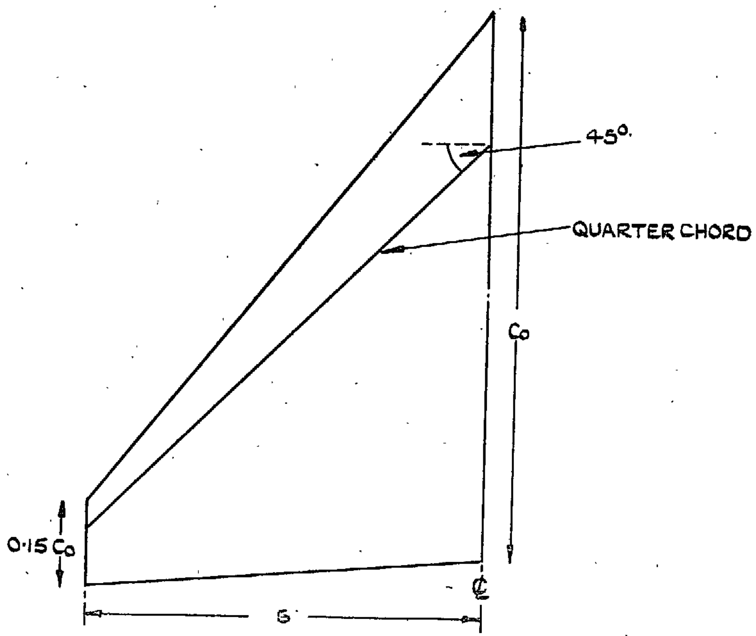

FIG. 9. Diagram of hypothetical delta wing.

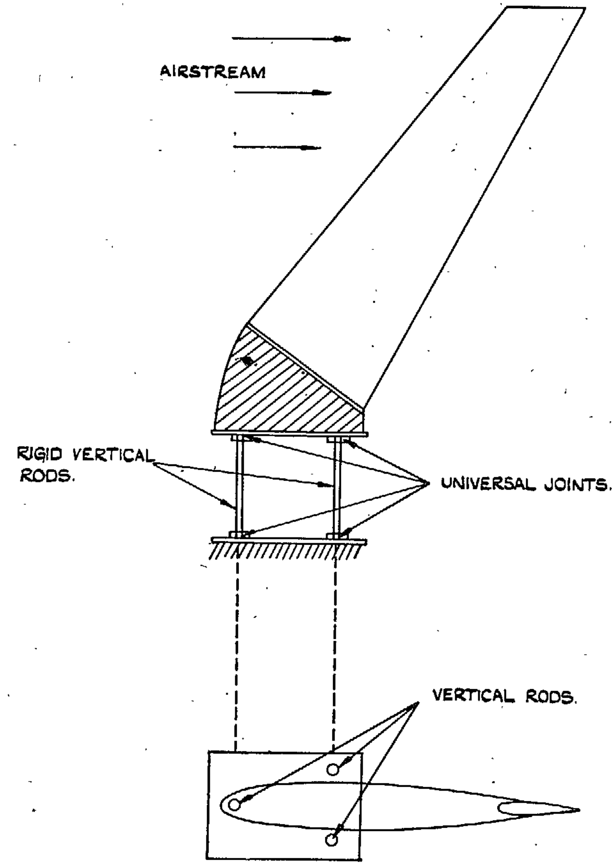

FIG. 10. Diagrammatic representation of support for model wings with body free.

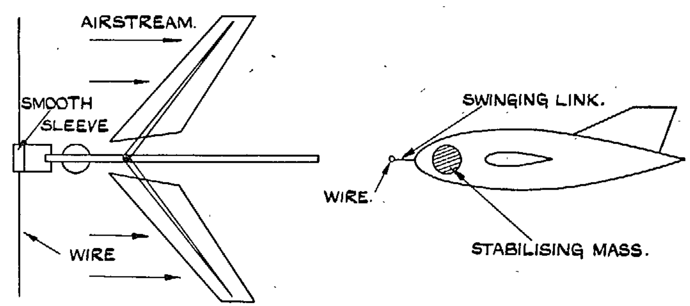

FIG. 11. Diagram of simple body-free model.

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