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Note on Aerodynamic Camber

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Summary.—The following note has been written at the suggestion of the Chairman of the Stability and Control Sub-Committee. It is intended to explain the theoretical significance of camber derivatives, and to assess the various available methods of making experimental measurements with particular reference to the use of a curved-flow tunnel. The note amplifies the arguments put forward by the writer in Ref. 1 (1950), that there is particular need for sytematic information about the influence of curvature of flow on control hinge-moments as a step towards the understanding of three-dimensional viscous flow.

After a definition of aerodynamic camber and a historical account of the development of the idea and its importance, the present state of knowledge of its aerodynamic derivatives is described. Camber derivatives are required for evaluating tunnel interference corrections and are useful for estimating corrections for aspect ratio and scale effect, in so far as the flow at a section of a finite wing can be represented as an equivalent two-dimensional flow. This quasi two-dimensional approach to the problem of control surfaces should be combined with experimental checks on the aerodynamic derivatives of various wings with flaps and also with a study of three-dimensional boundary layers.

Formulae for the camber derivatives of lift and pitching moment need confirmation. The derivatives of lift at the stall and of hinge moments over the whole range of incidence are virtually unknown and in consequence the determination of $(C_L)_{\max}$ and C_H is seriously limited. The significance of these two-dimensional camber derivatives is illustrated by the quantitative uncertainties that may arise. It is suggested that these might be removed by establishing formulae for the unknown derivatives from a series of tests of uncambered aerofoils with a range of flaps in the curved-flow tunnel at the Langley Aeronautical Laboratory, U.S.A., by simulating a uniform rate of change of pitch. The uncertain characteristics of the curved flow would make necessary a check between results so obtained and those deduced from tests in a straight tunnel of aerofoils with various amounts of parabolic camber. There appears to be no other satisfactory technique for measuring camber derivatives.

1. *Introduction.*—In the general form in which it appears in two-dimensional aerofoil design, camber is given by algebraic formulae or by a closely spaced set of ordinates defining the mean line of an aerofoil. Aerodynamic camber however is a single parameter which arises specifically from theoretical considerations. A first approximation to the load distribution on a thin finite wing does not satisfy the exact boundary condition

$$\text{downwash angle} = \text{local incidence}$$

at all points of the wing surface. When this condition has virtually been satisfied at each mid-chord position, there remains an excess downwash angle

$$(\text{downwash angle} - \text{local incidence})$$

which to a sufficient approximation is linear along the wing chord.

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It is convenient to define aerodynamic camber as that corresponding to a small downwash angle, which vanishes at mid-chord and varies linearly from leading edge to trailing edge. Thus aerodynamic camber is equivalent to a parabolic camber line with its vertex and maximum ordinate at mid-chord and is given in magnitude by the ratio

$$\gamma = \frac{\text{maximum ordinate}}{\text{aerofoil chord}} .$$

Aerodynamic camber may be described as a fictitious curvature of flow, that is useful in determining the incremental correction to wing loading which will cancel a given excess downwash angle. However in potential flow the principle of superposition may be applied to wing loading, defined as a velocity potential difference, and to downwash angle. Under such conditions aerodynamic camber amounts to a special case of geometric camber.

By a suitable approximation the three-dimensional problem of a cambered wing may be treated theoretically so that the chordwise pressure distribution at each section is expressed in terms of the two-dimensional distributions corresponding to uniform incidence and aerodynamic camber. It is therefore profitable to study the effects of aerofoil thickness and boundary layers on these two-dimensional distributions in order to explain some of the corresponding effects in three dimensions. Since aerodynamic camber is often of great importance in potential flow, this suggested approach to an understanding of viscous flow is at present limited by the lack of basic two-dimensional data.

2. *Historical Development of Theory*.—The first use of aerodynamic camber appears to be due to Prandtl[2] (1920) in his original paper on wind-tunnel interference, the change in upwash in a chordwise direction being an essential idea in the two-dimensional theory of tunnel interference. Its greater importance in the three-dimensional theory was recognised by Glauert and Hartshorn (R. & M. 947[3], 1924), who obtained the interference correction to downwash angle at the tail of a model of small span. These tunnel corrections were expressed as an effective change of angle of incidence, regarded as uniform over a wing or tailplane.

By this time the quantitative significance of camber as a means of producing lift had been explained by the theory of a thin aerofoil in two dimensions. The method of solution, first proposed by Munk[4] (1922), was developed by Glauert (R. & M. 910[5], 1924), who showed that the incidence and pitching moment at zero lift could be computed in fair agreement with experiment. The analysis of R. & M. 910 was then extended by Glauert (R. & M. 1095[6], 1927) in an improved manner to cover hinge moments.

The basis of the theory of aerodynamic camber is a special case of Glauert's thin-aerofoil theory, in which wing thickness is neglected altogether. Theodorsen and Garrick[7] (1933) gave an exact treatment of the general problem of the potential flow past two-dimensional aerofoils of any shape. A particular solution is obtained for a section consisting of a circular arc without thickness. It is shown that the lift slope and aerodynamic centre are unaffected by first-order changes in camber*. By an approximation, Goldstein has simplified the method of Ref. 7 to take account of first-order effects of wing thickness. It appears from C.P. 69[8] (Goldstein, 1942) that the incidence and pitching moment at zero lift are unaffected by first-order changes in thickness. There can however be first-order effects of thickness on the optimum incidence and the optimum lift coefficient, which arise from a change in lift slope. Apart from this the design of mean camber lines is essentially based on Glauert's thin-aerofoil theory.

The vital step in the development of ideas is that the theory of cambered wings is of great significance in connection with tunnel interference and the distribution of load on uncambered wings.

Once again the lead was given in the field of tunnel interference in R. & M. 1566[9], p. 41 (Glauert, 1933), where induced curvature of flow is discussed in some detail. Although the theory had advanced since the early days of Ref. 2, it was not until 1941 that Preston and

* There may however be a first-order effect of camber on the hinge-moment derivative $b_1 = \partial C_H/\partial\alpha$, but this is likely to be small.

Manwell (R. & M. 2465[10]) showed that the interference corrections to the two-dimensional derivatives m_1 ($= \partial C_m/\partial\alpha$) and especially b_1 ($=\partial C_H/\partial\alpha$) were much greater than had been supposed. Indeed by comparison with the simple theory of Ref. 2, which assumed a constant correction to incidence given by the induced upwash at mid-chord, the interference correction to b_1 is increased four or five-fold when aerodynamic camber is taken fully into account. Miss Lyon[11] (1942) then provided the means of estimating the tunnel interference on three-dimensional wings with control surfaces, including the effect of a set-back hinge. Again it was found that a considerably larger effective change of incidence was required to correct hinge moments to the free stream values than was needed to correct lift.

The importance of aerodynamic camber in the determination of the load distribution on finite wings emerged from the lifting-surface theory. It became apparent from the work of Wieghardt[12] (1939) and others that inaccuracies of the classical lifting-line theory became pronounced at low aspect ratios. The general treatment of the lifting-surface theory by W. P. Jones (R. & M. 2145[13], 1943) reduced the complicated mathematical problem to a form suitable for evaluation and numerical solution. With the aid of approximations the writer[14] (1946) has presented the theory of R. & M. 2145[13], so that the loading on a wing of arbitrary plan form is obtained by a comparatively simple process. The local chordwise pressure distribution at each wing section is found to be a linear combination of the two-dimensional distributions appropriate to uniform incidence, aerodynamic camber and, if necessary, deflected flap. It is thus possible to relate the theoretical forces and moments on a finite thin wing with partial span controls to the two-dimensional values (R. & M. 1095[6]) for a thin wing.

However, at this stage no consideration had been given to the effects of viscosity as introduced by aerofoil boundary layers. The writer[15] (1947) has extended the work of Miss Lyon[11] on tunnel interference to control surfaces of partial span. Ref. 15 also includes certain tentative formulae for the practical two-dimensional derivatives of lift, pitching moment and hinge moment with respect to aerodynamic camber. Swanson and Crandall[16] (1947) have produced semi-empirical charts for the aspect ratio corrections to lift and hinge moments as indicated by lifting-surface theory and experimental data. As pointed out in Ref. 1, Ref. 16 gives a suggested viscous correction to the hinge-moment camber-derivative, which differs widely from the corresponding formula of Ref. 15. This discrepancy has caused concern about the accuracy with which the three-dimensional $\partial C_H/\partial\alpha$ can be estimated.

3. *Existing Knowledge of Derivatives.*—As explained in section 1, the magnitude of aerodynamic camber is defined by the ratio

$$\gamma = \frac{\text{maximum ordinate}}{\text{aerofoil chord}} .$$

In practice γ is a fairly small quantity and does not usually exceed a value $0 \cdot 1$. Cambers of such magnitude are treated with sufficient accuracy by a linear theory. To this approximation it can be shown that a parabolic and circular camber are equivalent. But it is important to realise that aerodynamic camber must not be regarded as just any camber that is symmetrical about the mid-chord position. This is well illustrated in Ref. 17 (Abbott, von Doenhoff and Stivers, 1945), which includes aerofoils with the N.A.C.A. mean line 65, which is parabolic, and with the N.A.C.A. mean line $a = 1 \cdot 0$. These correspond respectively to elliptic and uniform chordwise loadings and both have maximum camber at mid-chord. For the same maximum camber, $\gamma = 0 \cdot 06$, the theoretical angle of zero lift of the mean line $a = 1 \cdot 0$ is 44 per cent greater than that of the mean line 65. It follows that most cambered aerofoils cannot be used to simulate aerodynamic camber. It would for example be incorrect to use experiments on an aerofoil with N.A.C.A. mean line $a = 1 \cdot 0$ to deduce a correction factor* for the effect of boundary layers on aerodynamic camber derivatives.

* According to Ref. 17, errors of about 20 per cent in no-lift angle would be expected.

It is necessary to consider the derivations of lift, pitching moment and hinge moment, which are defined respectively to be

$$a' = \frac{\partial C_L}{\partial \gamma} ;$$

$$m' = \frac{\partial C_m}{\partial \gamma} \text{ about the quarter-chord axis} ;$$

$$b' = \frac{\partial C_H}{\partial \gamma} .$$

For a thin aerofoil these derivatives are calculated theoretically from Ref. 6. The ordinate y of a parabolic camber line (Ref. 9, p. 41 *et seq.*) satisfies

$$\frac{dy}{dx} = 8\gamma \left(\tfrac{1}{2} - \frac{x}{c} \right) = 4\gamma \cos \theta ,$$

where

$$x = \tfrac{1}{2}c(1 - \cos \theta)$$

is the distance downstream of the leading edge. In the notation of R. & M. 1095[6], it follows that the lift per unit area is

$$\left.\begin{aligned} \rho V k &= 2\rho V^2 A_1 \sin \theta \\ &= 8\rho V^2 \gamma \sin \theta \end{aligned}\right\} \qquad (1)$$

Simple chordwise integrations give the theoretical derivatives

$$\left.\begin{aligned} (a')_T &= 4\pi \\ (m')_T &= -\pi \end{aligned}\right\} \qquad (2)$$

For two-dimensional control surfaces, let

$\theta = \theta_1$ denote the position of the hinge line,

$\theta = \theta_2$ denote the position of the nose of the control,

$Ec = \tfrac{1}{2}c(1 + \cos \theta_1)$ denote the control chord measured from the hinge.

Then the hinge moment corresponding to equation (1) gives

$$\begin{aligned} (b')_T &= -\frac{4}{E^2} \int_{\theta_2}^{\pi} \sin^2 \theta \, (\cos \theta_1 - \cos \theta) d\theta \\ &= -\frac{1}{E^2} \left[2(\pi - \theta_2) \cos \theta_1 + \sin 2\theta_2 \cos \theta_1 + \frac{4}{3} \sin^3 \theta_2 \right] . \end{aligned} \qquad (3)$$

Without nose balance, $\theta_1 = \theta_2$ and equation (3) gives

$$(b')_T = -\frac{1}{E^2} \left[2(\pi - \theta_1) \cos \theta_1 + \frac{3}{2} \sin \theta_1 + \frac{1}{6} \sin 3\theta_1 \right] . \qquad (4)$$

As discussed in section 2, α_0 and C_{m0}, the incidence and pitching-moment coefficient at zero lift are unaffected by first-order changes in thickness (C.P.69[8]), and the lift slope, a_1, and aerodynamic centre are unaffected by first order changes in camber (Ref. 7).
In accordance with a linear theory

$$\left.\begin{aligned} a' &= -a_1 \frac{\partial \alpha_0}{\partial \gamma} \\ m' &= \frac{\partial C_{m0}}{\partial \gamma} - m_1 \frac{\partial \alpha_0}{\partial \gamma} \end{aligned}\right\} \qquad (5)$$

where

$$\frac{\partial \alpha_0}{\partial \gamma} = -2 .$$

But there are first order effects of thickness on a_1 and m_1 ($= \partial C_m/\partial\alpha$), which may be deduced from Refs. 18 and 19 (Bryant, 1950)*. In the case of a 15 per cent thick wing, the theoretical corrections for thickness to the values in equation (2) are about

$$+ 12 \text{ per cent to } (a')_T ;$$

$$+ \ 8 \text{ per cent to } (m')_T .$$

If it is assumed that the hinge moment at zero lift is unaffected by first-order changes in thickness and that the derivative b_1 is unaffected by first-order changes in camber, then there would be a corresponding correction to $(b')_T$. On this basis

$$(b')_T - 2(b_1)_T$$

would be independent of thickness. On estimating the effect of thickness on $(b_1)_T$ from the curves given by Bryant in Ref. 18, Fig. 19a (1950), it is found that for a 15 per cent thick wing there would be a correction for thickness of roughly

$$- 7 \text{ per cent to } (b')_T .$$

The writer[15] has expressed the effect of viscosity on these derivatives in the following way:

$$\left. \begin{aligned} a' &= 4\pi \frac{a_1}{(a_1)_T} \\ m' &= -\pi \frac{a_1}{(a_1)_T} \end{aligned} \right\} \qquad \ldots \qquad (6)$$

$$b' = (b')_T \frac{b_1}{(b_1)_T} , \qquad \ldots \qquad (7)$$

where $(b_1)_T$ and $(b')_T$ denote the theoretical values of b_1 and b', as given by thin-aerofoil theory in equations (3) or (4), and the ratios $(b')_T/(b_1)_T$ are tabulated in Ref. 15.

Equations (6) are based entirely on the evidence of Jacobs, Ward and Pinkerton[20] (1933), who give experimental data for a series of rectangular wings of aspect ratio $A = 6$ and thickness/chord ratios $0\cdot06 \leqslant t/c \leqslant 0\cdot21$, each without camber and with parabolic N.A.C.A. mean lines 25, 45 and 65. The two-dimensional derivatives a' and m' were deduced by applying corrections for aspect ratio based on the approximate lifting-surface theory (Ref. 14). The writer is unaware of any more recent data that seriously conflict with equations (6). It is concluded from Ref. 17, Fig. 37, that experimentally wing thickness has little effect on α_0. Moreover for N.A.C.A. four-digit aerofoils, which include sections with parabolic camber, α_0 is approximately $0\cdot93$ times the value given by thin-aerofoil theory (Ref. 5). From equations (5) and (6),

$$-\frac{\alpha_0}{2\gamma} = \frac{2\pi}{(a_1)_T} , \qquad \ldots \qquad (8)$$

which, for aerofoils of thickness/chord ratio $0\cdot06 < t/c < 0\cdot13$, lies within 3 per cent of $0\cdot93$. It is recommended that equation (6) should be used provided that there is no separation of the boundary layers.

Equation (7) is based on the combined evidence of Kirk[21] (1943) and Preston, Sweeting and Cox (R. & M. 2007[22], 1944). In Ref. 21, a two-dimensional aerofoil with three types of control surface of chord ratio $E = 0\cdot3$ was tested in a closed rectangular tunnel of variable height, and measurements of lift and hinge moment were made. For each control surface the measured values of the derivatives a_1, a_2, b_1, b^2 were obtained for two ratios of

$$\frac{c}{h} = \frac{\text{aerofoil chord}}{\text{tunnel height}} .$$

* Refs. 18 and 19 are combined in R. & M. No. 2730.

Tunnel interference was expressed as an equivalent change of incidence and aerodynamic camber (Ref. 11), viz.:

$$\left.\begin{aligned}(\Delta\alpha) &= \frac{\pi}{96}\left(\frac{c}{h}\right)^2 (C_L + 4C_m{}^*)\\ (\Delta\beta) &= \frac{\pi}{192}\left(\frac{c}{h}\right)^2 C_L.\end{aligned}\right\} \quad \ldots \ldots \ldots \quad (9)$$

For a given control, one estimate of b' can be deduced from the two values of b_1 corresponding to the two ratios of c/h and another estimate from the two values of b_2. The two estimates of b' so obtained were not consistently in agreement, but suggested the empirical relation (7) even for overbalanced controls. In R. & M. 2007[22], a two-dimensional Piercy section was pressure-plotted in a closed rectangular tunnel, in which the roof and floor could be adjusted to follow the free streamlines and thus remove the tunnel interference, given approximately in (9). These experiments did not yield a satisfactory estimate of b', but within wide limits of accuracy tended to substantiate equation (7). However, more data are needed before this can be accepted as an established relation.

Swanson and Crandall[16] (1947) have suggested a tentative empirical formula

$$\frac{b'}{(b')_T} = 1 - 0\cdot0005\phi^2\,, \quad \ldots \ldots \ldots \quad (10)$$

where ϕ is the trailing-edge angle measured in degrees. It will be shown in section 4 that the uncertainty in b', as defined by the discrepancy between equations (7) and (10), is of considerable importance.

To summarise,

(i) the formula (6) for a' seems well-founded for low incidences;

(ii) the formula (6) for m' needs confirmation;

(iii) the formula (7) for b' is insecurely based and differs widely from formula (10), when b_1 is small.

4. *Practical Significance of Derivatives.*—Aerodynamic camber is essentially a theoretical device. Practically speaking, camber derivatives are useful in so far as the flow at a section of a finite wing can be represented as an equivalent two-dimensional flow. It is very unlikely that a quasi-two-dimensional approach to the general problem of wing loading will be a complete resource, as the three-dimensional boundary layer is subject to lateral pressure gradients and velocities which culminate in the formation of trailing vorticity at the trailing edge. Especially towards the wing tips a full appreciation of the fluid motion must be pioneered by direct experiment.

But it is reasonable to suppose that the growth of a boundary layer is primarily governed by the Reynolds number and the longitudinal pressure distribution. Preston (R. & M. 2725[23], 1949) has clearly shown that two-dimensional quantitative analysis of the effect of boundary layers is possible, given accurate knowledge of their profiles in a considerable region containing the trailing edge. Great advance has been made towards the correlation between aerofoil shape and the effects of boundary layers on two-dimensional derivatives[18,19] (Bryant, 1950). This basic information has opened up possibilities of extending the scope of semi-empirical calculations to include three-dimensional viscous effects. In Ref. 24 and subsequently the writer has found an encouraging measure of agreement between experimental and computed characteristics of

* It was necessary to estimate C_m as pitching moments were not measured.

ailerons, which suggests that a closer knowledge of camber derivatives would be profitable and might explain the discrepancies.

It will be helpful to illustrate the sort of uncertainties that exist, when the principles of aerodynamic camber are applied.

Pankhurst and Pearcey (C.P.28[25], 1950) have pointed out that the free-stream maximum lift, as estimated from two-dimensional tunnel experiments, is often uncertain within $\pm 1\frac{1}{2}$ per cent. The interference correction to $(C_L)_{\max}$ is $a'(\Delta\gamma)$, where $(\Delta\gamma)$ is given in equation (9). At maximum lift $a_1 = 0$, and, according to equation (6), $a' = 0$. It can safely be asserted that a' lies somewhere between zero and its theoretical value 4π. If a value $a' = 2\pi$ is used, there is an element of uncertainty

$$(\Delta C_{L\max}) = \pm \frac{\pi^2}{96}\left(\frac{c}{h}\right)^2 (C_L)_{\max} \qquad \text{(11)}$$

$$= \pm\, 0{\cdot}016_5\, (C_L)_{\max}, \text{ when } c/h = 0{\cdot}4\,.$$

The corresponding uncertainty in pitching moment is

$$(\Delta C_m) = \pm\, 0{\cdot}004 (C_L)_{\max},$$

which is not quite so important, as it only represents a margin of error

$$\pm\, 0{\cdot}004 \times \text{chord}$$

in the centre of pressure.

The discrepancy between equations (7) and (10) is illustrated in the case of the two-dimensional aerofoil section 1541 (Ref. 18) with $\phi = 15$ deg and plain controls of chord ratios $E = 0{\cdot}2$ and $0{\cdot}4$. From experiments with forward transition the respective values

$$b_1 = -\,0{\cdot}180 \text{ and } -\,0{\cdot}275$$

were obtained. Then

formula (7) gives $b' = -\,1{\cdot}31_5$ and $-\,1{\cdot}81$ respectively;

formula (10) gives $b' = -\,3{\cdot}24$ and $-\,4{\cdot}35$ respectively. (12)

In two-dimensional control testing the interference correction to C_H from equation (9) is

$$(\Delta C_H) = \frac{\pi}{192}\left(\frac{c}{h}\right)^2 C_L \,.\, b'\,,$$

where, say, the measured $C_L = 2\pi\alpha$ and $c/h = 0{\cdot}4$. Thus

$$(\Delta b_1) = -\,0{\cdot}016_5\, b'$$

By choosing a value of b' midway between those given in equation (12), the uncertainty in the estimated free-stream b_1 would be about

$$\left.\begin{array}{l} \pm\, 9 \text{ per cent when } E = 0{\cdot}2 \\ \pm\, 7\frac{1}{2} \text{ per cent when } E = 0{\cdot}4 \end{array}\right\} \qquad \text{(13)}$$

These magnitudes would be larger in the three-dimensional case. Such margins of error may not always matter in ad hoc research, but are important in experiments of a fundamental nature.

From calculations by the approximate lifting-surface theory of Ref. 14 for an unswept wing of medium taper and aspect ratio $A = 5 \cdot 83$, it is found identically for flaps of any constant E, that

$$\left.\begin{array}{l} (a)\ \text{for full span } 0 < \eta < 1\,, \\ \qquad \dfrac{\partial C_H}{\partial \alpha} = 0 \cdot 744(b_1)_T - 0 \cdot 0166(b')_T \\ (b)\ \text{for outer half-span } \tfrac{1}{2} < \eta < 1\,, \\ \qquad \dfrac{\partial C_H}{\partial \alpha} = 0 \cdot 703(b_1)_T - 0 \cdot 0247(b')_T\,. \end{array}\right\} \quad \ldots \quad (14)$$

By substituting the practical values of b_1 and b' in equation (12) for the theoretical values in equation (14), the following values of $\partial C_H/\partial \alpha$ are obtained:

	(a) full span		(b) $\frac{1}{2} < \eta < 1$	
	$E = 0 \cdot 2$	$E = 0 \cdot 4$	$E = 0 \cdot 2$	$E = 0 \cdot 4$
Using b' from (7)	$-0 \cdot 112$	$-0 \cdot 174_5$	$-0 \cdot 094$	$-0 \cdot 149$
Using b' from (10)	$-0 \cdot 080$	$-0 \cdot 132$	$-0 \cdot 046_5$	$-0 \cdot 086$

The magnitude of the coefficient of b' in equation (14) increases as aspect ratio decreases and is very roughly proportional to $1/A$. It can reasonably be deduced that for wings of low aspect ratio, say $A < 3$, with ailerons of section 1541 the present uncertainty in $\partial C_H/\partial \alpha$ would exceed $\pm 0 \cdot 05$.

It is desirable to know whether the derivative b' is significant in scale effect. R. & M. 2730[18] indicates that for medium values of ϕ, two-dimensional scale effect on b_1 is not large. It is worth noting that neither formula for b' suggests important changes with Reynolds number.

5. *Techniques of Measurement.*—There are four techniques for simulating aerodynamic camber, which will now be considered.

(a) cambered models,

(b) principle of tunnel interference,

(c) whirling arm,

(d) curved-flow tunnel.

(a) *Cambered models.*—Tests of a series of two-dimensional models varying in camber and trailing-edge angle would be a possible way of estimating the derivatives a', m' and b'. But, unless the tests were carried out on the scale of those reported in Ref. 20, the results by themselves would be of limited value. An investigation on such a large scale would require a large number of models and be very costly in labour. It should be borne in mind that in viscous flow the principle of superposition no longer applies and that with boundary layers present aerodynamic and geometric cambers may not be equivalent, even though the geometric camber is parabolic. However in two-dimensional flow it is difficult to imagine that there is much difference, since in both cases the changes in boundary layer with camber are small changes from a given state.

Though indirect this is probably the most accurate method of measuring the derivatives, the chief limitation being the difficulty in setting an aerofoil at an absolute incidence. This technique is clearly useful as a check on any proposed formula.

(*b*) *Principle of tunnel interference.*—The use of this principle is exemplified in Refs. 21 and 22. By testing a given aerofoil in a given tunnel and varying the magnitude of the tunnel interference, forces and moments can be measured under the influence of a variable aerodynamic camber. By applying the interference corrections under the different conditions, each free-stream derivative may be equated and an estimate of the corresponding camber derivative may be obtained. This is the ideal approach to the problem as it returns to the basic definition of aerodynamic camber and is not subject to any of the uncertainties in (i). Kirk [21] used a closed rectangular tunnel whose height could be varied, and Preston (R. & M. 2007[22]) attempted to remove the tunnel interference altogether by adjusting the roof and floor of a closed tunnel to follow the free streamlines.

However the results of Refs. 21 and 22 were inconclusive in that the estimates of b' were erratic. The technique demands a high degree of accuracy in all the aerodynamic measurements and tunnel calibrations; and small inaccuracies due to a reduced length of working-section or approximate streamlined wall settings cannot be overlooked.

(*c*) *Whirling arm.*—The use of a whirling arm to determine camber derivatives is based on the conception that to the first order a rate of change of pitch is equivalent to an aerodynamic camber. As far as the boundary conditions at the surface of a thin two-dimensional aerofoil are concerned, a camber line given by

$$\frac{d^2y}{dx^2} = \frac{q}{V},$$

where q is the rate of change of angle of pitch, corresponds to an aerodynamic camber (*see* section 3)

$$\gamma = \frac{cq}{8V}. \qquad \ldots \quad (15)$$

Thus the pitching derivatives

$$\left.\begin{aligned} m_q &= \tfrac{1}{2}\frac{\partial C_m}{\partial(qc/V)} \\ h_q &= \tfrac{1}{2}\frac{\partial C_H}{\partial(qc/V)} \end{aligned}\right\} \qquad \ldots \quad (16)$$

could be measured on a whirling arm and, if $\frac{qc}{V}$ is small enough,

$$\left.\begin{aligned} m' &= 16m_q \\ b' &= 16h_q \end{aligned}\right\}. \qquad \ldots \quad (17)$$

The advantage of this technique is that of direct measurement, but there seem to be three practical difficulties:

(i) the difficulty of simulating two-dimensional flow,

(ii) the low Reynolds number (about $0{\cdot}5 \times 10^6$),

(iii) that qc/V is fixed for a given wing in a given whirling arm and the corresponding aerodynamic cambers* are rather small.

Added to these there is the complication that the fluid in the boundary layer will tend to follow the circular path of the model and will be under the action of an appreciable outward centrifugal force. This feature of circular flight will cause a possible extraneous effect on the aerodynamic moments, not required for the purposes of aerodynamic camber.

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* $\gamma = \frac{cq}{8V} = \frac{c}{8R}$, where R is the radius of the arm

$=$ about $0{\cdot}006$ for the N.P.L. Whirling Arm.

(*d*) *Curved-flow tunnel*.—Some experiments on lateral stability have been conducted in the Langley 6-ft Square Tunnel with a curved-flow test section[26] (Bird, Jaquet and Cowan, 1948). The measurement of longitudinal derivatives due to rate of change of pitch was being contemplated. As suggested by the writer in Ref. 1, this raises the possibility of measuring m_q and h_q without the outward centrifugal force on the boundary layer, which arises on a whirling arm. The Langley tunnel is suitable for two-dimensional tests at Reynolds numbers somewhat greater than 10^6. Furthermore the curvature of the tunnel can be varied, so that with an aerofoil of 2-ft chord it would be possible to cover a range of aerodynamic camber given as in equation (15) by

$$0 < \gamma = \frac{cq}{8V} < 0\cdot015\,.$$

Thus all the difficulties associated with the use of a whirling arm for camber derivatives would not enter into the curved-flow technique.

The aerofoil must be placed so that the chord line is parallel to the direction of flow at the mid-chord. This introduces the difficulty of setting an aerofoil at an absolute incidence. If, for example, the incidence can be set to an accuracy of

$$\pm\,0\cdot1 \text{ deg} = \pm\,0\cdot0017_5 \text{ radians,}$$

then corresponding to the maximum camber, $\gamma = 0\cdot015$, the accuracy in lift, *i.e.*, a', would be about $\pm\,6$ per cent, which is not serious. The error in m', the camber derivative of pitching moment about the quarter-chord, would be trivial: and the error in b' on this account would be less than $\pm\,2$ per cent. Thus for the purposes of determining m' and b' a tolerance of $\pm\,\frac{1}{4}$ deg in absolute incidence could be allowed, but a closer setting is necessary in the case of a'.

It is concluded that in so far as longitudinal stability derivatives can be measured in a curved-flow tunnel, this technique is ideal for estimating derivatives of aerodynamic camber. The usefulness of the Langley tunnel will now be critically examined from this standpoint.

6. *Langley Curved-Flow Tunnel*.—The conditions in the Langley curved-flow tunnel are described in Ref. 26. The tunnel is designed to give the motion of the air relative to a body in steady curved flight. The required fluid velocity is directly proportional to the distance from the centre of rotation, and is reproduced by means of curved side walls combined with drag screens of variable density,spaced close together towards the inner wall and far apart towards the outer wall. The desired relative motion is accompanied by gradients of static pressure and total head normal to the streamlines. The resulting curved flow past an aerofoil differs from that required to simulate an aerodynamic camber in respect of

(*a*) the normal pressure gradient,

(*b*) the centrifugal force on the boundary layers,

(*c*) the partially curved wake,

(*d*) the high degree of turbulence.

From the following paragraphs it will appear that (*a*) is fully accounted for by a known buoyancy correction, unless there is a further small correction due to (*b*), that the effect of (*c*) is negligible, but that there may be artificial boundary layer transitions and small errors due to an oscillatory wake associated with (*d*).

The radius of curvature corresponding to a small rate of change of pitch, q, is V/q, It is easily shown that the corresponding normal gradient of static pressure is

$$\frac{\partial p}{\partial y} = \rho V q$$

This gradient produces a buoyancy, giving a correction to the lift force, which can be calculated quite accurately (R. & M. 1166[27]). It seems that the buoyancy corrections to pitching and hinge moments are likewise calculable. By way of a check these corrections could perhaps be compared with measured moments on a thin aerofoil of circular camber of radius V/q at zero incidence.

The force of buoyancy balances the centrifugal force on the undisturbed curved stream and exceeds the centrifugal force on the retarded air in the aerofoil boundary layers, which therefore tends to move towards the centre of rotation. This effect is in the opposite sense to that found on the whirling arm and in actual flight. Although it is considered in Ref. 26, that this effect is probably of second order, the corrections to aerodynamic measurements on this account are uncertain and might be deduced from a comparison of equivalent experiments on a whirling arm and in a curved-flow tunnel.

Provided that the flow is steady, there is no appreciable effect due to a curvature of the wake in two-dimensional flow, since there is no total vorticity at any section of the wake. It is important to bear in mind that Glauert (R. & M. 1242[28], 1929) showed that the oscillatory damping derivative about a pitching axis at quarter-chord is double the corresponding derivative in steady circular motion, *viz.*:

$$m_{\dot{\alpha}} = 2m_q \, . \qquad \qquad (18)$$

But the curved flow is presumably steady enough to preclude any large effect of this nature. For a thin aerofoil the value of m_q given in (R. & M. 1216[29]), which corresponds to a curved wake, is related to the camber derivative m' by equation (17), *viz.*:

$$m' = 16m_q \, .$$

There is, however, a high degree of turbulence, particularly towards the inner side of the tunnel, where the drag screens are closely spaced. Again it is believed that the corrections to aerodynamic characteristics are small (Ref. 26). This is more likely to be true for longitudinal derivatives such as m_q and h_q, defined in equation (16), than for lateral rolling and yawing derivatives. There may be artificial effects on boundary-layer transitions due to increased turbulence. But comparative tests on a whirling arm using fixed positions of transition on both aerofoil surfaces should show up any serious discrepancies.

7. *Concluding Remarks.*—The problem of predicting three-dimensional control characteristics can be tackled in three ways:—

(*a*) an extended series of *ad hoc* experiments,

(*b*) a quasi-two-dimensional approach

(*c*) a systematic investigation of three-dimensional boundary layers.

The first of these provides usable information throughout, but as a direct attack on the general problem is impracticable in view of the excessive number of geometric and aerodynamic variables. The second is an analytical approach in which the problem is subdivided into stages, all the variables entering into at least one stage and the number of variables in any particular stage being kept small. This method is unlikely to give a full explanation of three-dimensional viscous flow. The third approach amounts to a study of viscous fluid motion, vital in the fundamental development of the subject, but in itself too indirect. It seems to the writer that (*c*) should be pursued whenever possible, but that the main effort should be concentrated on (*b*). It is also important to undertake a wide range of checks between (*a*) and (*b*) and an explanation of any discrepancies by means of (*c*).

This note has set out to show that a quantitative analysis of viscous flow on these lines requires more detailed knowledge of two-dimensional camber derivatives. It is stated that the theoretical forces and moments on a finite thin wing with controls of partial span can be approximately related to the corresponding two-dimensional theoretical derivatives with respect to uniform incidence, deflected flap and aerodynamic camber. The quasi-two-dimensional approach involves the substitution of the practical values of these derivatives. The derivatives with respect to incidence and flap are usually available, but those of lift, pitching moment and hinge moment with respect to aerodynamic camber, viz. a', m' and b', are far from certain. It is considered that

(i) the formula (6) for a' is well-founded for small incidences only;

(ii) the formula (6) for m' needs confirmation;

(iii) the two tentative formula (7) and (10) for b' differ seriously, when b_1 is small.

At low incidences it seems desirable to know a' and m' within $\pm$ 5 per cent. and b' within $\pm$ 10 per cent. Near the stall it is desirable to know a' within, say, $\pm$15 per cent. Knowledge to this accuracy is necessary to ensure

(1) a reliable estimation of tunnel interference on $(C_L)_{max}$ and C_H;

(2) a useful evaluation of aerodynamic derivatives, particularly $\partial C_H/\partial \alpha$, for any given finite wing;

(3) estimation of scale effect on $(C_L)_{max}$ and control derivatives.

The two most promising techniques for measuring camber derivatives are

(A) tests on a series of two-dimensional aerofoils varying in parabolic camber and trailing-edge angle;

(B) direct tests of uncambered aerofoils in a curved-flow tunnel simulating a uniform rate of change of pitch.

The former technique is rather costly in model construction and involves more experimental time; the latter could only be carried out in the United States of America, as the Langley curved-flow tunnel is the only equipment of its kind. The latter method is the more economical, but is subject to uncertainties in the condition of flow. These uncertainties could be checked in three ways

(C) by a check between a', as measured at low incidences, and the fairly reliable value given in equation (6);

(D) by a comparison of camber derivatives of a given aerofoil by both techniques, or

(E) by comparative measurements of longitudinal stability derivatives on a whirling arm and in the curved-flow tunnel.

Until more trustworthy estimates of b' can be made, much of the accumulated data on two-dimensional hinge moments is of limited use. Until m' can be estimated with certainty the aerodynamic centre of finite wings can only be predicted in a tentative way. Until a' is known at the stall the determination of the free stream $(C_L)_{max}$ from tunnel experiments will not be satisfactory, nor will be the estimation of scale effect on $(C_L)_{max}$. This lack of fundamental knowledge is retarding the understanding of three-dimensional viscous flow.

It is urged that there is particular need for systematic information about the influence of curvature of flow on the hinge moment experienced by a control surface and on the aerodynamic centre and maximum lift of a wing. It is recommended that the Langley Aeronautical Laboratory should be approached to ascertain the available results from which such information could be deduced and to promote further tests on two-dimensional aerofoils varying in thickness and trailing-edge angle with a range of control flaps.

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