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Summary.—This report contains design formulae and curves for estimating the maximum forces, together with the times and drafts associated with these forces, in main-step landings of seaplanes provided there is neither rotation nor chine immersion. Good agreement is found with the results of model tests made under controlled conditions.

The basic formulae and curves presented are considered to be the most satisfactory and accurate of the many proposed in recent years. They involve the use of a new basic parameter $(1/y_0)$ which is a measure of the effect of forward velocity; a new formula for associated mass, {(area)²/perimeter} and a new method of plotting which is considered to be the most useful for the analysis of experimental data. The first is defined by

$$\frac{1}{y_0} = \frac{V_T \tan \tau}{V_{n\,0}} \qquad \text{if } V_T \text{ is constant}$$

$$= \frac{V_H \sin \tau}{V_{n\,0}} \qquad \text{if } V_H \text{ is constant}$$

where τ is the attitude,

$V_{n\,0}$ is the velocity component normal to the keel at first impact,

V_T and V_H are the velocity components parallel to the keel and undisturbed water surface respectively.

Introduction.—This report presents formulae and curves for estimating the maximum forces, together with the times and drafts associated with these forces, in main-step landings of seaplanes, provided that there is no rotation and that the chines do not become immersed. It also compares the values estimated by these formulae with the results[1] of model tests made by the N.A.C.A. under controlled conditions in their Impact Basin, when good agreement is found.

The basic formulae and curves given are considered to be the simplest and most accurate which can be evolved at present from the many proposed in various reports in recent years and reviewed in R. & M. 2720[6]. They involve the use of a new basic 'impact parameter' and a new estimate for associated mass, which is based on three- rather than on two-dimensional concepts.

It was convenient to split the report into two parts. Part I contains a statement of the formulae recommended for use in design estimates, together with numerical examples. Part II contains the comparison with experimental data. A simplified theoretical treatment and all mathematical details relevant to both parts is given in Appendix I.

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† Of the Hydrodynamics Department, Messrs. Saunders-Roe, Ltd. The report was prepared by collaboration between the two Establishments.

The method of plotting adopted in the figures of the report is new and is chosen as being the most useful for the analysis of experimental data.

Tentative conclusions are drawn concerning the qualitative effects of chine immersion, but quantitative estimates would best be obtained from systematic experimental evidence. The theory can be extended to cover this case, but at present its exact evaluation would be laborious.

Similarly a theory is available for the evaluation (by an iteration process) of the effects of rotation but here again systematic experimental evidence is needed, since the calculations involved would be extremely laborious. The same considerations apply to bow and rear-step impact cases.

The only theoretical pressure distribution at present available is that obtained by Wagner[2] for the two-dimensional impact case (vertical drop of an infinitely long wedge at zero attitude). It has not been included here because of doubts as to its validity in the general impact case. Experiments being made should help to define a suitable distribution. The same experiments should also give information on pitching inertia reliefs.

However, within the limits of their range of application (main-step landings without rotation or chine immersion) the formulae of this report should give sufficiently accurate estimates for design use.

This report is part of a series giving the results of investigations of water impact forces and pressures.

PART I. DESIGN FORMULAE

This part of the report contains formulae for estimating

1. maximum acceleration,
2. time to maximum acceleration,
3. draft at maximum acceleration, and
4. maximum draft,

in main-step landings. It is assumed that there is no rotation, that the chines do not become immersed during the impact, and that the wing lift equals the aircraft weight.

1. *The Basic Impact Parameter.*—Under full-scale landing conditions, it may be assumed that, up to the instant of maximum acceleration, the velocity component parallel to the keel remains sensibly constant. If so, then the magnitude of all the impact effects may be shown (Appendix I) to depend on a basic impact parameter

$$y_0 = \frac{V_{n\,0}}{V_T \tan \tau} \quad \ldots \quad (1)$$

$$= \frac{\tan (\gamma_0 + \tau)}{\tan \tau} \quad \ldots \quad (2)$$

where (*see* Fig. 1),

$V_{n\,0}$ is velocity normal to keel at touchdown,

V_T is velocity parallel to keel (constant),

τ is attitude of keel relative to water surface,

γ_0 is flight path angle (at touchdown), relative to water surface.

The physical significance of this parameter is that it represents the relative magnitude of the contributions of the initial velocities normal to the keel ($V_{n\,0}$) and parallel to the keel (V_T) to the impact motion normal to the keel.

In normal landings, the flight-path angle is kept small and, to give emphasis to the smaller flight-path angles, all the curves have been plotted against $(1/y_0)$. In pure planing $(\gamma = 0)$ motions, $(1/y_0) = 1$, while in pure impact motions $(1/y_0) = 0$. In the majority of cases the values of $(1/y_0)$ will lie between 0·8 and 0·3.

Thus the basic parameter is taken as

$$\frac{1}{y_0} = \frac{V_T \tan \tau}{V_{n\,0}} \quad \ldots \quad (3)$$

$$\frac{\tan \tau}{\tan (\gamma_0 + \tau)} \,. \quad \ldots \quad (4)$$

2. *Maximum Acceleration.*—The maximum acceleration in a main-step landing without rotation is given in terms of g by

$$\frac{(dV_n/dt)_{\max}}{g} = - A_0 \frac{K^{1/3}(V_{n\,0}{}^2/g)}{(W/\varrho g)^{1/3}} \quad \ldots \quad (5)$$

$$= - A_0 K^{1/3} \frac{C_{Vn\,0}{}^2}{C_{\Delta\,0}{}^{1/3}} \quad \ldots \quad (6)$$

where A_0 is the maximum acceleration factor (Table 1, Fig. 4),

K is the associated mass factor (Table 3, Figs. 2 and 3),

$V_{n\,0}$ is the velocity component normal to the keel at touchdown (ft/sec).

W is the weight of the aircraft (lb),

ϱ is the density of the water (slugs/cu ft),

g is the acceleration due to gravity (ft/sec^2),

$$C_{Vn\,0} = \frac{V_{n\,0}}{\sqrt{(gb)}} \,, \qquad C_{\Delta\,0} = \frac{W}{\varrho g b^3}$$

and b is the beam (ft).

2.1. *The Maximum Acceleration Factor A_0.*—This factor is obtained theoretically from the equation of motion in Appendix I as a function only of the touchdown conditions and is completely determined by the value of the basic parameter y_0. Values are given in Table 1 and are plotted in Fig. 4.

2.2. *The Associated-Mass Factor K.*—The associated-mass factor K is determined by the geometry and attitude of the hull or float. Its derivation is given fairly fully because of its importance in any design estimate and because the form now proposed is new. It is defined by

$$\mu M = \varrho K (h \sec \tau)^n \quad \ldots \quad (7)$$

where μM is the associated mass of water,

M is the mass of the aircraft $= W/g$,

and h is the draft.

The form of equation 7 depends on the cross-section of the hull or float bottom. For a vee-bottom with straight transverse step it can be taken that $n = 3$ and therefore

$$\mu M = \varrho K (h \sec \tau)^3 \,. \quad \ldots \quad (8)$$

This is the case considered in the present report in formulating the equation of motion in Appendix I.

The $(Area)^2/Perimeter$ Formula and the Deadrise Correction ξ_1.—The most satisfactory estimate for the value of K is given by the $(\text{Area})^2/\text{Perimeter}$ formula obtained by Crewe. The formula is

$$\mu M = \varrho \frac{8}{3\pi} \frac{(\text{Area})^2}{\text{Perimeter}} \xi_1 \qquad \ldots \qquad (9)$$

where 'Area' and 'Perimeter' refer to the projection on the keel plane of the total pressure-bearing wetted area, and ξ_1 is a correction for deadrise. Brief details of its derivation are given in Appendix II.

The most logical value for ξ_1 is that given by Kreps[3] (R. & M. 2681[4]),

$$\xi_1 = (1 - \theta/\pi) \qquad \ldots \qquad (10)$$

where θ is the deadrise angle in radians (Fig. 1).

For a triangular projected wetted area it can easily be shown that,

$$\frac{(\text{Area})^2}{\text{Perimeter}} = \frac{\lambda_0^2 l^3}{2\{\lambda_0 + \sqrt{(1 + \lambda_0^2)}\}} \qquad \ldots \qquad (11)$$

where l = wetted length,

$$\frac{1}{\lambda_0} = \frac{l^2}{\text{wetted area}} \qquad \ldots \qquad (12)$$

$$= l/c,$$

and $2c$ is the wetted width.

λ_0 is therefore the reciprocal of the aspect ratio. Substituting from equations (10) and (11) in (9),

$$\mu M = \varrho \frac{4}{3\pi} \frac{\lambda_0^2}{\lambda_0 + \sqrt{(1 + \lambda_0^2)}} (1 - \theta/\pi) l^3$$

$$= \varrho \frac{4}{3\pi} \frac{\lambda_0^2 \cot^3 \tau}{\lambda_0 + \sqrt{(1 + \lambda_0^2)}} (1 - \theta/\pi)(h \sec \tau)^3 \qquad \ldots \qquad (13)$$

since $l = h \operatorname{cosec} \tau$, $\qquad \ldots \qquad (14)$

assuming no splash forward.

Comparison of equations 13 with 8 then gives,

$$K = \frac{4}{3\pi} \frac{\lambda_0^2 \cot^3 \tau}{\lambda_0 + \sqrt{(1 + \lambda_0^2)}} (1 - \theta/\pi) . \qquad \ldots \qquad (15)$$

Table 3 and Fig. 3 give values of $\dfrac{K^{1/3} \tan \tau}{(1 - \theta/\pi)^{1/3}}$ in terms of $1/(1 + \lambda_0)$. The parameter $1/(1 + \lambda_0)$ was chosen (in preference to λ_0) for ease of interpolation, since, as Table 3 shows, there are constant differences over a large portion of a range which includes the majority of design cases.

Water Surface Conditions.—It remains to determine λ_0 in any particular case. From equation 12,

$$\lambda_0 = c/l , \qquad \ldots \qquad (16)$$

and if there is no splash forward then

$$l = h \operatorname{cosec} \tau . \qquad \ldots \qquad (14)$$

The value of c (*see* Fig. 1) depends on the amount of 'splash-up' (or, rise of displaced water along the sides of the float). The theoretical value obtained by Wagner[2] for wedges of very small deadrise angle in a pure impact motion ($V_T = 0$) is,

$$c = \pi/2 \,.\, (h \sec \tau) \cot \theta \,, \qquad \qquad (17)$$

while in R. & M. 2681[4] the theoretical value,

$$c = \pi/2 \,.\, (1 - \theta/\pi) h \sec \tau \cot \theta \qquad \qquad (18)$$

has been obtained for wedges of finite deadrise angle in the same motion.

The former value (17) has arbitrarily been chosen as standard (*see* Part II) in the present report. Hence, from equations 14, 16 and 17,

$$\lambda_0 = \pi/2 \,.\, \tan \tau \cot \theta \,, \qquad \qquad (19)$$

and, using this formula, values of $1/(1 + \lambda_0)$ are given in Table 3 and Fig. 2 in terms of τ and θ.

Non-straight Transverse Step.—The values of K given in this report are strictly true only for triangular wetted areas, which implies that the vee-bottom has a straight transverse step and that the keel line is straight.

If the step is faired in plan form then a working approximation to K can be obtained by assuming a straight transverse step at the maximum chord line of the wetted area and then proceeding as before. Otherwise the associated mass may be determined from equation 9, but when it is reduced to the form of equation 7 the index n will not necessarily be equal to three. A different value of n will modify the equation of motion and result in different values of A_0 to those given here.

2.3. *Numerical Example.*—Suppose it is required to determine the maximum acceleration when a seaplane of weight 80,000 lb lands at a speed of 80 knots and a rate of descent of 5 ft/sec. The deadrise angle of the hull-bottom is 25 deg, the beam is 10 ft and the attitude can be taken as 8 deg.

From equation 5 above, the maximum acceleration will be given by,

$$\frac{(dV_n/dt)_{\max}}{g} = - A_0 K^{1/3} \frac{(V_{n0}^2/g)}{(W/\varrho g)^{1/3}} \,. \qquad \qquad (5)$$

Taking $g = 32 \cdot 19$ ft/sec^2 and $\varrho = 2$ slugs/cu ft (sea-water), then

$$\left(\frac{W}{\varrho g}\right)^{1/3} = 10 \cdot 75 \,.$$

Since $V = 80$ knots $\simeq 135$ ft/sec (given)

and $V_{v0} = 5$ ft/sec (given)

then $\gamma_0 = 2 \cdot 1$ deg.

Hence,

$$\frac{1}{y_0} = \frac{\tan \tau}{\tan (\gamma_0 + \tau)} = 0 \cdot 789$$

and therefore, from Fig. 4,

$$A_0 = 0 \cdot 174 \,.$$

From Fig. 2, when

$$\tau = 8 \text{ deg and } \theta = 25 \text{ deg},$$

$$\frac{1}{1 + \lambda_0} = 0 \cdot 679 ,$$

and Fig. 3 then gives,

$$\frac{K^{1/3} \tan \tau}{(1 - \theta/\pi)^{1/3}} = 0 \cdot 393 ,$$

from which,

$$K^{1/3} = 2 \cdot 66 .$$

Finally,

$$V_{n\,0} = V \sin (\gamma_0 + \tau)$$

$$= 23 \cdot 7 .$$

Substituting these values in (5), the maximum deceleration normal to the keel is $0 \cdot 75g$.

If equation (6) were used, the relevant values of $C_{Vn\,0}$ and $C_{\Delta\,0}$ would be,

$$C_{Vn\,0} = 1 \cdot 32$$

$$C_{\Delta\,0} = 1 \cdot 24 .$$

3. *Time to the Instant of Maximum Acceleration.*—This is given in seconds by

$$t_m = B_0 \frac{(W/\varrho g)^{1/3} \cos^{1/3} \tau}{K^{1/3} V_{v\,0}} , \qquad \qquad (20)$$

where B_0 will be called the ' time to maximum acceleration factor ' and the other symbols have the same meaning as before.

Theoretical values of B_0, obtained in Appendix I from the equation of motion, are given in Table I and Fig. 5.

Taking the same landing case as in section 2.3, we have

$$1/y_0 = 0 \cdot 789$$

hence $\quad B_0 = 0 \cdot 366$ (from Fig. 5)

$$\cos^{1/3} \tau = 0 \cdot 997 .$$

Hence

$$t_m = \frac{0 \cdot 366 \times 10 \cdot 75 \times 0 \cdot 997}{2 \cdot 66 \times 5}$$

$$= 0 \cdot 295 \text{ sec.}$$

4. *Draft at the Instant of Maximum Acceleration.*—Equation 8 reads,

$$\mu M = \varrho K (h \sec \tau)^3 \qquad \qquad (8)$$

where h is the draft below the undisturbed water level at any time.

Hence,

$$h = \mu^{1/3} \left(\frac{W}{\varrho g} \right)^{1/3} \frac{\cos \tau}{K^{1/3}} \qquad \qquad (21)$$

and, in particular, the draft at the instant of maximum deceleration is

$$h_m = \mu_m{}^{1/3} \left(\frac{W}{\varrho g} \right)^{1/3} \frac{\cos \tau}{K^{1/3}} . \qquad \qquad (22)$$

More generally and to cover the launching tank case $V_H =$ const. as well as the present full-scale case $V_T =$ const., this can be written,

$$h_m = x_m^{1/3}\left(\frac{W}{\varrho g}\right)^{1/3}\frac{\cos\tau}{K^{1/3}}, \qquad \ldots \qquad (23)$$

where x_m will be called the 'draft at maximum acceleration factor'. $\{x_m = \mu_m$ when V_T is constant, but $x_m = \mu_m \cos^2 \tau$ when V_H is constant.$\}$

Values of $x_m^{1/3}$, obtained theoretically from the equation of motion in Appendix 1, are given in Table 1 and Fig. 6.

For the same case as in section 2.3, $1/y_0 = 0{\cdot}789$ and hence, from Fig. 6,

$$x_m^{1/3} = 0{\cdot}286.$$

Therefore,

$$h_m = \frac{0{\cdot}286 \times 10{\cdot}75 \times 0{\cdot}990}{2{\cdot}66}$$

$$= 1{\cdot}144 \text{ ft.}$$

5. *Maximum Draft.*—This is given by,

$$h_n = x_n^{1/3}\left(\frac{W}{\varrho g}\right)^{1/3}\frac{\cos\tau}{K^{1/3}}, \qquad \ldots \qquad (24)$$

where values of $x_n^{1/3}$, derived from the equation of motion, are given in Table 1 and Fig. 6.

For the same case as in section 2.3, $1/y_0 = 0{\cdot}789$ and hence, from Fig. 6,

$$x_n^{1/3} = 0{\cdot}295 .$$

Therefore,

$$h_n = \frac{0{\cdot}295 \times 10{\cdot}75 \times 0{\cdot}990}{2{\cdot}66}$$

$$= 1{\cdot}184 \text{ ft.}$$

6. *Chine Immersion.*—The formulae of sections 2 to 4 no longer apply if the chines become immersed before the instant of maximum acceleration and the formula of section 5 no longer applies if the chines become immersed before maximum draft is reached.

The drafts given in sections 4 and 5 are referred to the undisturbed water level. The wetted width at this level would be given by (*see* Fig. 1)

$$2c_0 = 2h \cot\theta \sec\tau . \qquad \ldots \qquad (25)$$

The actual wetted width is greater than this because of splash-up and, as mentioned in section 2.2, the factor of $\pi/2$ is supported by experimental evidence. Thus the wetted width is (as in equation 17),

$$2c = \pi h \sec\tau \cot\theta . \qquad \ldots \qquad (26)$$

Chine immersion occurs when $2c = b$, i.e.,

$$h = \frac{b\cos\tau\tan\theta}{\pi} \qquad \ldots \qquad (27)$$

or,

$$x^{1/3} = \frac{K^{1/3}}{(W/\varrho g)^{1/3}}\,\frac{b\tan\theta}{\pi}$$

$$= \frac{K^{1/3}\tan\theta}{\pi C_{\Delta\,0}^{1/3}} . \qquad \ldots \qquad (28)$$

For the numerical example considered, this happens when

$$x^{1/3} = \frac{2 \cdot 66 \times 0 \cdot 466}{\pi \times 1 \cdot 074} = 0 \cdot 368 ,$$

so that both maximum acceleration and maximum draft occur before chine immersion.

The full effects of chine immersion on maximum acceleration have not yet been determined. Theoretically they would seem to depend on the formula assumed for the associated mass and further experimental evidence is needed to justify the choice of any particular theory.

Use of the (area)²/perimeter formula gives maximum accelerations when the chines are immersed which are either greater or smaller than those predicted by equation 5 or 6, depending on the values of the various parameters. In particular, the ratio

$$\frac{\text{Maximum acceleration taking account of chine immersion}}{\text{Maximum acceleration assuming an infinitely wide bottom (i.e., eqn. 5)}}$$

increases with decreasing attitude, provided the other parameters are equal. It also increases with decreasing y_0.

In most cases, however, the ratio is less than unity and seldom increases more than slightly above unity.

This being so, chine immersion can generally be assumed to give a relief on the value of maximum acceleration as calculated by equation 5 or 6. Draft and time would most probably be increased above the values given by equations 23 and 20, but there seems to be no effect on the values of maximum draft.

PART II. COMPARISON OF THE FORMULAE OF PART I WITH EXPERIMENTAL RESULTS

1. *Method of Comparison.*—The various impact formulae of Part I give results which are appropriate to the full-scale conditions τ = const. and V_T = const. Model-scale tests in launching tanks are made under the condition V_H = const. Comparison between the two is best made by transforming the results of the latter (made under the additional condition τ = const.) to full-scale conditions. This can best be done as follows

(*a*) The experimental results are reduced to the form of the various impact factors listed in column 1 of Table 2. This involves assuming values for the associated-mass factor *K*.

(*b*) These impact factors are then multiplied by the correction factors of column 2 of Table 2.

(*c*) The results are then the full-scale values corresponding to values of the basic impact parameter given by

$$\frac{1}{y_0} = \frac{V_H \sin \tau}{V_{n0}} \qquad \text{(29)}$$

$$= \frac{\cos \gamma_0 \sin \tau}{\sin (\gamma_0 + \tau)} \qquad \text{(30)}$$

and can be compared directly with the curves of Figs. 4 to 6.

Note that when V_H is constant, $1/y_0$ is given by (29) or (30), whereas when V_T is constant equations (3) or (4) apply, *i.e.*,

$$\frac{1}{y_0} = \frac{V_T \tan \tau}{V_{n0}} \qquad \text{(3)}$$

$$= \frac{\tan \tau}{\tan (\gamma_0 + \tau)} \cdot \qquad \text{(4)}$$

This means that a given value of $1/y_0$ will correspond to different initial values of γ and τ in the two cases. In particular, if $\gamma_0 = \pi/2$ full-scale and $\tau \neq 0$, then from (4),

$$1/y_0 = -\tan^2 \tau , \qquad \qquad (31)$$

and this condition can only be realised in a launching tank (equation 29 or 30) by making V_H or τ negative. The latter course would introduce unwanted bow effects, so that if vertical drops with finite attitude full-scale are required, then the model-scale V_H should be slightly negative as given by (29) and (31).

$$1/y_0 = 0 \text{ corresponds full-scale to } (\gamma + \tau) = \pi/2.$$

Figs. 7 to 10 give a comparison on the above basis of the theoretical results of Part I with experimental results obtained under controlled conditions. The latter have been taken from various N.A.C.A. reports[1] on tests made in their impact basin. These tests were made with three floats, of deadrise angles 22·5 deg, 30 deg and 40 deg, at various attitudes and weights. Their bottoms approximated closely to the plane-faced wedge shape with a straight transverse step. The test technique was such that during the impact the horizontal velocity and attitude were nearly constant.

Measurements were made of maximum acceleration, draft at and time to this maximum acceleration, and of maximum draft. In the present report these quantities have been reduced as described above.

The objects of the comparison were both to check the assumptions made about the associated mass and to check the validity of final formulae given in Part I.

2. *Validity of Assumptions about the Associated Mass.*—The assumptions made about the associated mass are described in section 2.2 of Part I. It shows that there is a choice between two splash-up factors, $\pi/2$ and $\pi/2 \,.\, (1 - \theta/\pi)$, which determine the wetted area to be assumed in equation 9.

The experimental results have therefore been reduced separately on the bases of these two factors. Figs. 7a, 8a, 9a and 10a give the comparison with theory assuming a splash-up factor of $\pi/2$. Figs. 7b, 8b, 9b and 10b give the comparison assuming a splash-up factor of $\pi/2 \,.\, (1 - \theta/\pi)$.

The general agreement is good with either factor and it is difficult to choose between them. The factor $\pi/2 \,.\, (1 - \theta/\pi)$ is theoretically the more justifiable (R. & M. 2681[4]), but some experimental measurements (unpublished) of wetted areas in planing tests tend to support the factor $\pi/2$ and it might be said that this factor gives slightly the better agreement in Figs. 7 to 10.

For the present, the factor $\pi/2$ has been chosen as standard and Fig. 2 has been plotted accordingly. Change to another factor would only involve alteration of this figure.

3. *Validity of the Formulae of Part I.*—The accuracy of the design formulae of Part I is now evident from Figs. 7a, 8a, 9a and 10a.

Fig. 7a gives a comparison between theoretical and experimental maximum acceleration factors (A_0). The theoretical curve is a good mean for the experimental results and some of the scatter at small values of $1/y_0$ can be attributed to the effects of either chine or bow immersion. Table 4 gives the values of $1/y_0$ (based on $\pi/2$ splash-up factor) below which these effects might be expected at the various attitudes and loadings represented by the symbols in Fig. 7a, and reasonable correlation can be found. From inspection of this figure the qualitative estimates given in section 6 of Part I of the effects of chine immersion on maximum acceleration have been made. No estimates can be made of bow effect.

Fig. 8a gives a comparison between theoretical and experimental 'draft at maximum acceleration factors'. Here the theoretical curve is in good agreement with experiment prior to chine immersion as evidenced by reference to Table 4. With the exception of the case $\theta = 22\frac{1}{2}$ deg, $\tau = 12$ deg, $W = 1100$ lb represented by the vee-symbol, chine immersion increases the draft above the chines-out theoretical value.

Fig. 9a gives a comparison between theoretical and experimental maximum draft factors. In this case there is good agreement over the whole range and no chine immersion effects can be found.

Fig. 10a gives a comparison between theoretical and experimental 'time to maximum acceleration' factors. Less experimental evidence is available than for the other factors, but, with what there is, the agreement is good.

All of these comparisons are with results obtained model scale under controlled conditions. Some full-scale landing tests (R. & M. 2629[7]) made under operational conditions (when, in general, rotation was present) have given a fair measure of agreement with the theory except that the times to maximum acceleration are considerably longer. Further evidence is needed on this point and tests are being made.

4. *Conclusions.*—

(*a*) The formulae of Part I give very good agreement with the results of impact tests made under controlled conditions without rotation when the chines are not immersed.

(*b*) Chine immersion seems, in general, to have the effect of decreasing the maximum accelerations and increasing the associated times and drafts from those which would be estimated by the formulae of Part I. No effect is evident on maximum draft.

(*c*) Quantitative estimates of the effects of chine immersion and rotation are best obtained from systematic tests under controlled conditions. Extensions of the theory are available to assist in the reduction of the results of such tests.

LIST OF SYMBOLS

(a) *Geometry of hull or float bottom and of impact*

θ	Deadrise angle
τ	Attitude measured relative to horizontal at tangent to keel at step
γ	Flight-path angle measured relative to horizontal at tangent to keel at step
l	Wetted length
h	Draft with respect to undisturbed water level
$2c_0$	Wetted width at undisturbed water level
$2c$	Wetted width
b	Beam

(b) *Velocities*

V	Resultant velocity at time t
V_T, V_n	Velocity components parallel to and perpendicular to the keel
V_H, V_v	Velocity components parallel to and perpendicular to undisturbed water surface (horizontal and vertical if water is calm)

Subscript '$_0$' refers to velocities at first impact

(c) *Weights and Masses*

W	Weight of aircraft
M	Mass of aircraft ($= W/g$)
μM	Associated mass of water
μ'	$\mu \cos^2 \tau$ (used in calculations where $V_H =$ const.)

(d) *Factors*

$\dfrac{1}{y_0}$ Basic impact parameter,

$$= \frac{V_T \tan \tau}{V_{n0}} \text{ if } V_T = \text{const.}$$

$$= \frac{V_H \sin \tau}{V_{n0}} \text{ if } V_H = \text{const.}$$

K Associated-mass factor $= \dfrac{\mu M}{\varrho (h \sec \tau)^n}$

A_0 Maximum acceleration factor, $V_T = \text{const.}$

$$= \frac{(dV_n/dt)_{\max}}{g} \frac{(W/\varrho g)^{1/3}}{K^{1/3}(V_{n0}^2/g)}$$

A Time to maximum acceleration factor $V_H = \text{const.}$

$$= \frac{(dV_n/dt)_{\max}}{g} \frac{(W/\varrho g)^{1/3} \cos^{1/3} \tau}{K^{1/3}(V_{n0}^2/g)}$$

B_0 Time to maximum acceleration factor $V_T = \text{const.}$

$$= t_m V_{v0} \left(\frac{\varrho g K}{W \cos \tau} \right)^{1/3}$$

where subscript m refers to conditions at maximum acceleration

$x_m^{1/3}$ Draft at maximum acceleration factor

$$= \left(\frac{\varrho g K}{W} \right)^{1/3} \sec \tau h_m \text{ if } V_T = \text{const.}$$

$$= \left(\frac{\varrho g K}{W} \right)^{1/3} \sec^{1/3} \tau h_m \text{ if } V_H = \text{const.}$$

$x_n^{1/3}$ Maximum draft factor

$$= \left(\frac{\varrho g K}{W} \right)^{1/3} \sec \tau h_n \text{ if } V_T = \text{const.}$$

$$= \left(\frac{\varrho g K}{W} \right)^{1/3} \sec^{1/3} \tau h_n \text{ if } V_H = \text{const.}$$

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3	R. Kreps	Experimental investigation of impact in landing on water. N.A.C.A. Tech. Memo. 1046. 1943.
4	R. J. Monaghan	Theoretical examination of effect of deadrise in seaplane-water impacts. R. & M. 2681. March, 1949.
5	P. R. Crewe	A proposed theory to cover water impacts of seaplanes in which the craft has constant attitude and a tangential to keel velocity relative to the water. R. & M. 2513. December, 1946.
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APPENDIX I

The Mathematical Theory of the Report

The following theory represents a simplified approach, by associated-mass methods, to the problem and was used in determining the formulae of Part I and the correction factors of Part II. Justification of the various assumptions made will be found in other reports (R. & M. 2681[4], R. & M. 2513[5]) and the results are supported by the experimental evidence presented in Part II.

It is assumed throughout the work that the attitude remains constant during the impact. Two different equations of motion can then be found, depending on whether the velocity component (V_T) parallel to the keel or the velocity component (V_H) parallel to the free surface is assumed to remain constant during the impact. The former assumption corresponds approximately to full-scale conditions and will be called Case 1. The latter assumption corresponds to model-scale test conditions and will be called Case 2. Case 1 will be taken as standard and correction factors will be derived for reducing Case 2 experimental results to forms suitable for direct comparison with it.

1. *The Theory of Case* 1.—1.1. *Equation of Motion.*—V_T is constant and therefore the resultant water force on the hull will be perpendicular to the keel,

i.e.,
$$F = -M\frac{dV_n}{dt}, \qquad \text{(I.1)}$$

where M is the mass of the body $= (W/g)$

and V_n is the velocity component perpendicular to the keel (*see* Fig. 1).

Also, F is assumed to be composed of a 'pure impact' force

$$F_1 = \frac{d}{dt}(\mu M \,.\, V_n)\,, \qquad \text{(I.2)}$$

where μM is defined as the associated mass of water and of an 'impact planing' force which takes the form

$$F_2 = \mu M \,.\, \frac{a}{l}\, V_n V_T\,, \qquad \text{(I.3)}$$

where l is the wetted length

and a is an empirical factor.

Prior to the moment of chine immersion, the value of the factor a can be taken as 3.

Combining equations (I.1), (I.2) and (I.3), and taking $a = 3$, we obtain,

$$-\frac{dV_n}{dt} = \frac{d}{dt}(\mu V_n) + 3\frac{\mu}{l}\, V_n V_T\,. \qquad \text{(I.4)}$$

Now if μM can be expressed in the form

$$\mu M = \varrho K(h \sec \tau)^3\,, \qquad \text{(I.5)}$$

where h is the draft,

and since

$$l = h \operatorname{cosec} \tau\,, \quad (\textit{see}\ \text{Fig. 1}) \qquad \text{(I.6)}$$

then

$$3\frac{\mu}{l} = \frac{d\mu}{dh} \sin \tau\,. \qquad \text{(I.7)}$$

Substituting from (I.7) in (I.4) and since $dh/dt = V_v$, we can obtain

$$d\{(1 + \mu)V_n\} + \frac{V_n V_T \sin \tau}{V_v}\, d\mu = 0\,. \qquad \text{(I.8)}$$

Now,

$$V_v = V_n \cos \tau - V_T \sin \tau$$

and equation (I.8) can then be put in the non-dimensional form

$$\frac{d\mu}{1 + \mu} + \frac{w_T - 1}{w_T^{\,2}}\, dw_T = 0 \qquad \text{(I.9)}$$

where

$$w_T = \frac{V_n}{V_T \tan \tau}\,, \qquad \text{(I.10)}$$

i.e., the basic equation is

$$\frac{dx}{1 + x} + \frac{y - 1}{y^2}\, dy = 0\,, \qquad \text{(I.11)}$$

with $x = \mu$ and $y = w_T$.

1.2. *Value of Maximum Acceleration.*—

$$\frac{dV_n}{dt} = \frac{dV_n}{dw_T} \cdot \frac{dw_T}{d\mu} \cdot \frac{d\mu}{dh} \cdot \frac{dh}{dt} \, .$$

Hence, substituting from (I.10), (I.9) and (I.5) we obtain

$$\frac{dV_n}{dt} = \frac{K^{1/3} \, V_{n0}^2}{(W/\varrho g)^{1/3}} \left[\frac{3\mu^{2/3}}{1+\mu} \left(\frac{w_T}{w_0} \right)^2 \right] . \qquad \text{(I.12)}$$

This has a maximum when

$$d \left[\frac{3\mu^{2/3}}{1+\mu} \left(\frac{w_T}{w_0} \right)^2 \right] = 0 \, ,$$

i.e., when

$$\mu_m = \frac{2(w_{T\,m} - 1)}{7(w_{T\,m} - 1) + 6} , \qquad \text{(I.13)}$$

where suffix m denotes conditions at the instant of maximum acceleration.

Also, by integrating(I.9) we obtain (since, when $t = 0$, $\mu = 0$ and $w_T = w_{T\,0}$)

$$\log\,(1 + \mu) + \log\, w_T + \frac{1}{w_T} = \log\, w_{T\,0} + \frac{1}{w_{T\,0}} \, . \qquad \text{(I.14)}$$

Equations (I.13) and (I.14) can then be solved graphically to give values of $w_{T\,m}$ and μ_m in terms of $w_{T\,0}$. The results are given in Table 1. ($y_m = w_{T\,m}$, $x_m = \mu_m$.)

Substituting these values in (I.12) we obtain,

$$\left(\frac{dV_n}{dt} \right)_m = - \, A_0 \frac{K^{1/3} \, V_{n0}^2}{(W/\varrho g)^{1/3}} \qquad \text{(I.15)}$$

where

$$A_0 = \frac{3\mu_m^{2/3}}{1 + \mu_m} \left(\frac{w_{T\,m}}{w_{T\,0}} \right)^2 , \qquad \text{(I.16)}$$

and values of A_0 are given in Table 1.

Note that (I.15) is equivalent to

$$\frac{(dV_n/dt)_m}{g} = - \, A_0 K^{1/3} \frac{C V_{n0}^2}{C_{\Delta 0}^{1/3}} , \qquad \text{(I.15a)}$$

where $C_{V_{n0}}^2 = V_{n0}/\sqrt{gb}$ and $C_{\Delta 0} = W/\varrho g b^3$ where b is the beam.

1.3. *Draft at Instant of Maximum Acceleration.*—By definition

$$\mu M = \varrho K (h \sec \tau)^3 \qquad \text{(I.5)}$$

where h is the draft.

Therefore, corresponding to any value of μ, we have

$$h = \frac{\mu^{1/3}}{K^{1/3}} \left(\frac{W}{\varrho g} \right)^{1/3} \cos \tau \, , \qquad \text{(I.17)}$$

and

$$\mu^{1/3} = K^{1/3} \left(\frac{\varrho g}{W} \right)^{1/3} h \sec \tau \qquad \text{(I.18)}$$

will be taken as the draft factor. At the instant of maximum deceleration μ takes the value μ_m and values of $\mu_m^{1/3} (= x_m^{1/3})$ are given in Table 1.

1.4. *Maximum Draft.*—At maximum draft, $w_T = 1$ and therefore (from (I.14))

$$\log(1 + \mu_n) = \log w_{T0} + \frac{1}{w_{T0}} - 1, \qquad \text{(I.19)}$$

where suffix n denotes conditions at this instant. Values of $\mu_n^{1/3}$ (obtained from (I.19)) for substitution in (I.18) are given in Table 1. $(\mu_n = x_n)$.

1.5. *Time to Maximum Acceleration.*—

Since

$$V_v = dh/dt,$$

we have,

$$t = \int_0^t dt = \int_0^h \frac{dh}{V_v}$$

and therefore,

$$V_{v0} t = \int_0^h \frac{V_{v0}}{V_v}\, dh. \qquad \text{(I.20)}$$

Now,

$$\frac{V_{v0}}{V_v} = \frac{w_{T0} - 1}{w_T - 1} \qquad \text{(I.21)}$$

and

$$dh = \left(\frac{W \cos\tau}{\varrho g K}\right)^{1/3} d\mu'^{1/3} \qquad \text{(I.22)}$$

where $\mu' = \mu \cos^2 \tau$.

Hence,

$$V_{v0} t = \left(\frac{W \cos\tau}{\varrho g K}\right)^{1/3} \int_0^{\mu'} \frac{w_{T0} - 1}{w_T - 1}\, d\mu'^{1/3}. \qquad \text{(I.23)}$$

From this we can define a 'time to maximum acceleration factor' B_0 by

$$B_0 = V_{v0} t_m \left(\frac{\varrho g K}{W \cos\tau}\right)^{1/3} \qquad \text{(I.24)}$$

$$= x_m^{1/3} \int_0^1 \frac{y_0 - 1}{y - 1}\, d\left(\frac{x}{x_m}\right)^{1/3}, \qquad \text{(I.25)}$$

with $y = w_T$ and $x = \mu'$

and values of B_0 are given in Table 1 (obtained by graphical integration of (I.25)).

2. *Theory of Case 2 and Derivation of Correction Factors.*—2.1. *Equation of Motion.*—V_H is held constant and therefore the measured force is normal to the free surface. Using similar considerations to those of Case 1, we obtain

$$-\frac{dV_v}{dt} = \cos\tau \left\{\frac{d}{dt}(\mu V_n) + V_n V_T \sin\tau \frac{d\mu}{dh}\right\}. \qquad \text{(I.26)}$$

V_H is constant, hence

$$dV_v = \sec\tau \,.\, dV_n\,.$$

therefore,

$$(1 + \mu\cos^2\tau)\, dV_n + V_n \cos^2\tau \left\{1 + \frac{V_T \sin\tau}{V_v}\right\} d\mu = 0\,. \qquad \text{(I.27)}$$

Put

$$\mu \cos^2 \tau = \mu' \quad \ldots \quad (I.28)$$

and

$$w_H = \frac{V_n}{V_H \sin \tau}, \quad \ldots \quad (I.29)$$

then (I.27) becomes

$$\frac{\cos^2 \tau \, d\mu'}{1 + \mu'} + \frac{w_H - 1}{w_H^2} dw_H = 0 . \quad \ldots \quad (I.30)$$

The basic equation is therefore,

$$\frac{s \, dx}{1 + x} + \frac{y - 1}{y^2} dy = 0 \quad \ldots \quad (I.31)$$

which differs from (I.11) only by the inclusion of the factor $s = \cos^2 \tau$.

2.2. *Maximum Acceleration.*—In this case we obtain, by similar processes to section 1.2 above

$$\left(\frac{dV_v}{dt}\right)_m = - A \sec^{1/3} \tau \frac{K^{1/3} V_{n0}^2}{(W/\varrho g)^{1/3}} \quad \ldots \quad (I.32)$$

where

$$A = \frac{3\mu_m'^{2/3}}{1 + \mu_m'} \left(\frac{w_{Hm}}{w_{H0}}\right)^2 \quad \ldots \quad (I.33)$$

and

$$\mu_m' = \frac{2(w_{Hm} - 1)}{w_{Hm} - 1 + 6sw_{Hm}} . \quad \ldots \quad (I.34)$$

We now derive a correction factor for A to make it directly comparable with A_0 of (I.16) and Table 1 (at the same value of y_0).

2.2.1. *Correction Factor from Case* 2 *to Case* 1.—When $s = 1$, $A_0 = A$.

When $\quad s \neq 1,\ A_0 = \left(\frac{A_{s=1}}{A_{s \neq 1}}\right) A$

$$= f(s) \,.\, A . \quad \ldots \quad (I.35)$$

In normal seaplane landings the attitude τ is small (usually less than 12 deg) so that $f(s)$ might be expected to be nearly unity in practical cases. The following method for determining $f(s)$ was therefore considered sufficiently accurate. It consisted in determining $f(s)$ in the two limiting cases $V_H \to 0$ and $V_{v0} \to 0$ and then determining its variation with $(1/w_{H0})$ for a single value of $s = 0{\cdot}95$ (corresponding to $\tau = 12$ deg). When this is done, we obtain

$$1/w_{H0} \to 0 , \qquad f(s) \to s^{1/2}$$

$$1/w_{H0} \to 1 , \qquad f(s) \to s^{2/3}$$

and for intermediate values of $1/w_{H0}$, Fig. 11 shows the variation of $f(s)$ with $1/w_{H0}$ for $s = 0{\cdot}95$. It shows that a good approximation to $f(s)$ is

$$f(s) = s^{7/12} \quad \ldots \quad (I.36)$$

i.e., $\quad f(s) = (\cos \tau)^{7/6}$

i.e., $\quad A_0 = A(\cos \tau)^{7/6} . \quad \ldots \quad (I.37)$

2.3. *Draft at Instant of Maximum Acceleration.*

$$\mu M = \varrho K(h \sec \tau)^3 \quad \ldots \quad (I.5)$$

therefore,

$$\mu_m{}' = \frac{\varrho K}{M} h^3 \sec \tau .$$

We require the ratio $\mu_m/\mu_m{}'$.

As an approximation we can take the arithmetic mean of the limiting values in the cases $V_H \rightarrow 0$ and $V_{v0} \rightarrow 0$.

$$\frac{\mu_m}{\mu_m{}'} = s + \frac{1-s}{14} . \quad \ldots \quad (I.38)$$

For $s = 0{\cdot}95$, (I.47) gives

$$\frac{\mu_m}{\mu_m{}'} = - s + 0{\cdot}0036 .$$

Therefore, if we take

$$\mu_m/\mu_m{}' = s = \cos^2 \tau \quad \ldots \quad (I.39)$$

the results should be within the limits of experimental error.

2.4. *Maximum Draft.*—At maximum draft, $w_T = w_H = 1$, and therefore from the integrals of (I.9) and (I.30)

$$\log (1 + \mu_n) = \log w_{T0} + \frac{1}{w_{T0}} - 1 , \quad \ldots \quad (I.19)$$

and $$s \log (1 + \mu_n{}') = \log w_{H0} + \frac{1}{w_{H0}} - 1 , \quad \ldots \quad (I.40)$$

where subscript n denotes conditions at this instant.

h_n can then be determined from μ_n and $\mu_n{}'$ by means of (I.5).

We require the factor $\mu_n/\mu_n{}'$.

Fig. 11 shows its value for the extreme case $s = 0{\cdot}95$, corresponding approximately to $\tau = 12$ deg. Denoting μ' or μ by x, and w_{T0} or w_{H0} by y_0,

then as $$\frac{1}{y_0} \rightarrow 1 , \quad \frac{x_{s=1}}{x_{s=0\cdot95}} \rightarrow s ,$$

and as $$\frac{1}{y_0} \rightarrow 0 , \quad \frac{x_{s=1}}{x_{s=0\cdot95}} \rightarrow 0.$$

Values of $1/y_0$ occurring in practice are approximately from $0{\cdot}3$ to $0{\cdot}8$, so that taking the ratio as s should be sufficiently accurate.

2.5. *Time to Maximum Acceleration.*—As in (I.5) we can obtain a 'time to maximum acceleration factor' B given by,

$$B = t_m V_{v0} \left(\frac{\varrho g K}{W \cos \tau} \right)^{1/3}$$

$$= x_m{}^{1/3} \int_0^1 \left(\frac{y_0 - 1}{y - 1} \right) d\left(\frac{x}{x_m} \right)^{1/3} \quad \ldots \quad (I.41)$$

with $y = w_H$ and $x = \mu'$.

In the limiting cases it can be shown that as

$$\text{(a)} \qquad \frac{1}{y_0} \to 0\,, \qquad B \simeq \int_0^{xm} \frac{1+sx}{x^{2/3}}\,dx$$

and evaluation of this integral to three figures for $s = 0{\cdot}95$ gave $B_0/B = 1$

$$\text{(b)} \qquad \frac{1}{y_0} \to 1, \qquad B \text{ varies as } s^{-1/3}$$

and therefore, $B_0/B = s^{1/3}$.

Taking the arithmetic mean gives

$$B_0/B \simeq s^{1/6}$$
$$= \cos^{1/3} \tau\,. \qquad \text{(I.42)}$$

APPENDIX II

Deviation of the (Area)²/Perimeter Formula for Associated Mass

The (Area)²/Perimeter formula for associated mass

$$\mu M = \varrho \frac{8}{3\pi} \frac{(\text{Area})^2}{\text{Perimeter}}$$

is derived from the exact potential flow solution for an elliptic plate (as given in *Hydrodynamics* by H. Lamb).

On physical grounds $(\text{Area})^{1/2}$/Perimeter can be considered as a measure of escapement relief, since it is a measure of the perimeter of the surface relative to the area it encloses and hence of the ease with which fluid can escape over the edges. Thus if the length of the perimeter is large relative to the $(\text{Area})^{1/2}$, then escape will be easy and it is reasonable to suppose that the associated mass will be reduced.

$(\text{Area})^{3/2}$ can be considered as a measure of the associated volume of water, and its combination with $(\text{Area})^{1/2}$/Perimeter seemed suitable for generalisation. As a trial, the constant $(8/3\pi)$ from the elliptic plate solution was retained.

Applying the formula to rectangular plates gives close agreement with Pabst's empirical formula[8] for $b/l > 0{\cdot}1$. The maximum error is 8 per cent at $b/l = 0$ (by comparison with the two-dimensional flat-plate potential-flow solution).

Good agreement was also found with the results of tests made by Kreps[3] on a variety of shapes, so that the (Area)²/Perimeter formula seems a suitable choice for determining the impact associated mass.

A theoretical examination of the effect of deadrise on associated mass is made in R. & M. 2681[4], which supports the use of Kreps[3] correction factor $\xi_1 = 1 - \theta/\pi$ so that the final formula chosen for associated mass is

$$\mu M = \varrho \frac{8}{3\pi} \frac{(\text{Area})^2}{\text{Perimeter}} (1 - \theta/\pi)\,.$$

TABLE 1

Theoretical Values of the Various Impact Factors

$\frac{1}{y_0}$	x_m	x_n	A_0	y_m	$x_m^{1/3}$	$x_n^{1/3}$	B_0
1·0	0	0	0	1·0			0
0·95	0·00129	0·00129	0·03229	1·00388	0·1089	0·1090	0·124
0·90	0·00525	0·00536	0·07539	1·01604	0·1738	0·1750	0·235
0·85	0·01193	0·01255	0·12029	1·03734	0·2285	0·2324	
0·80	0·02120	0·02341	0·16449	1·06870	0·2768	0·2854	0·354
0·75	0·03283	0·03840	0·20688	1·11128	0·3202	0·3374	
0·70	0·04646	0·05831	0·24700	1·16643	0·3595	0·3878	0·441
0·65	0·06168		0·28472	1·23599	0·3951		
0·60	0·07808	0·1172	0·32000	1·32234	0·4274	0·4894	0·505
0·55	0·09537		0·35337	1·42945	0·4569		
0·50	0·1131	0·2131	0·38401	1·56140	0·4836	0·5973	0·559
0·45							
0·40	0·1492	0·3720	0·44057	1·93639	0·5304	0·7192	0·599
0·35							
0·30	0·1850	0·6553	0·49066	2·57540	0·5698	0·8686	0·634
0·25							
0·20	0·2199	1·247	0·53532	3·86486	0·6036	1·077	0·664
0·15							
0·10	0·2535	3·066	0·57565	7·74915	0·6329	1·453	0·689
0·50							
0	0·2857	∞	0·61231	∞	0·6586		0·707
—0·50	0·3013		0·6294	—15·588			

TABLE 2

Correction Factors from Model Scale Test Results with V_H = Constant to Standard Conditions (V_T = const.)

Reduced experimental quantities	Correction factor Multiply by	Quantity obtained
Maximum acceleration factor $A = \dfrac{\left(\dfrac{dV_v}{dt}\right)}{g} \dfrac{\left(\dfrac{Wg^2}{\varrho}\right)^{1/3} \cos^{1/3}\tau}{K^{1/3}V_{n\,0}^{\,2}}$	$\cos^{7/6}\tau$	A_0
Time to maximum acceleration factor $B = (\text{experimental } t_m) V_{v\,0} \left(\dfrac{\varrho g K}{W \cos\tau}\right)^{1/3}$	$\cos^{1/3}\tau$	B_0
Draft at maximum acceleration factor $x_m^{1/3} = \left(\dfrac{\varrho g K}{W}\right)^{1/3} \sec^{1/3}\tau\, h_m$	$\cos^{2/3}\tau$	$x_m^{1/3}$ V_T = const.
Maximum draft factor $x_n^{1/3} = \left(\dfrac{\varrho g K}{W}\right)^{1/3} \sec^{1/3}\tau\, h_n$	$\cos^{2/3}\tau$	$x_n^{1/3}$ V_T = const.

TABLE 3

Associated-Mass Factor for Triangular Wetted Areas

$$\frac{K^{1/3}\tan\tau}{(1-\theta/\pi)^{1/3}} = \left[\frac{4}{3\pi}\,\frac{\lambda_0^2}{\lambda_0+\sqrt{(1+\lambda_0^2)}}\right]^{1/3}$$

where $\lambda_0 = \frac{\text{half wetted width}}{\text{wetted length}}$.

---

1. $\dfrac{K^{1/3}\tan\tau}{(1-\theta/\pi)^{1/3}}$ as a function of $\dfrac{1}{1+\lambda_0}$

$\frac{1}{1+\lambda_0}$	$\frac{K^{1/3}\tan\tau}{(1-\theta/\pi)^{1/3}}$	$\frac{1}{1+\lambda_0}$	$\frac{K^{1/3}\tan\tau}{(1-\theta/\pi)^{1/3}}$
0·34	0·729	0·68	0·391
0·36	0·706		
0·38	0·683	0·70	0·372
		0·72	0·353
0·40	0·661	0·74	0·334
0·42	0·640	0·76	0·314
0·44	0·620	0·78	0·295
0·46	0·600		
0·48	0·580	0·80	0·275
		0·82	0·255
0·50	0·560	0·84	0·233
0·52	0·541	0·86	0·212
0·54	0·522	0·88	0·190
0·56	0·503		
0·58	0·484	0·90	0·167
		0·92	0·144
0·60	0·466	0·94	0·118
0·62	0·447	0·96	0·088
0·64	0·428	0·98	0·055
0·66	0·410	1·00	0·000

2. $\dfrac{1}{1+\lambda_0}$ with $\dfrac{\pi}{2}$ splash-up. $\left(\lambda_0 = \dfrac{\pi}{2}\,.\,\tan\tau\cot\theta\,.\right)$

τ \ $\theta =$	10	15	20	25	30
2	0·7627	0·8301	0·8690	0·8947	0·9132
4	0·6162	0·7092	0·7682	0·8093	0·8389
6	0·5165	0·6188	0·6880	0·7385	0·7776
8	0·4441	0·5483	0·6225	0·6787	0·7234
10	0·3890	0·4917	0·5679	0·6274	0·6758
12	0·3456	0·4452	0·5216	0·5827	0·6336

TABLE 3—*continued*

3. $\frac{1}{1+\lambda_0}$ with $\frac{\pi}{2}(1-\theta/\pi)$ splash-up. $\left(\lambda_0 = \frac{\pi}{2}(1-\theta/\pi)\tan\tau\cot\theta\,.\right)$

τ \ $\theta =$	10	15	20	25	30
2	0·7729	0·8420	0·8819	0·9080	0·9266
4	0·6296	0·7269	0·7885	0·8314	0·8631
6	0·5307	0·6391	0·7127	0·7664	0·8076
8	0·4582	0·5697	0·6497	0·7104	0·7584
10	0·4027	0·5135	0·5965	0·6616	0·7144
12	0·3586	0·4668	0·5508	0·6186	0·6748

TABLE 4

Values of $\frac{1}{y_0}$ (assuming $\frac{\pi}{2}$ splash-up factor) at which

(a) bow effect

(b) chine immersion

would occur at the instant of maximum acceleration in the N.A.C.A. tests[1]

(a) *Bow effect*

θ (deg)	τ	W lb	$1/y_0$
22½	3	1100	0·293
	3	1416	0·399
	3	1716	0·480
	3	2416	0·576

(*b*) *Chine immersion*

θ (deg)	τ	W lb	$1/y_0$
22½	3	1716	0·146
	3	2416	0·342
	6	1040, 1100	0·400
	9	1100	0·599
	12	1100	0·687
30	6	1230	0·263
	15	1230	0·695

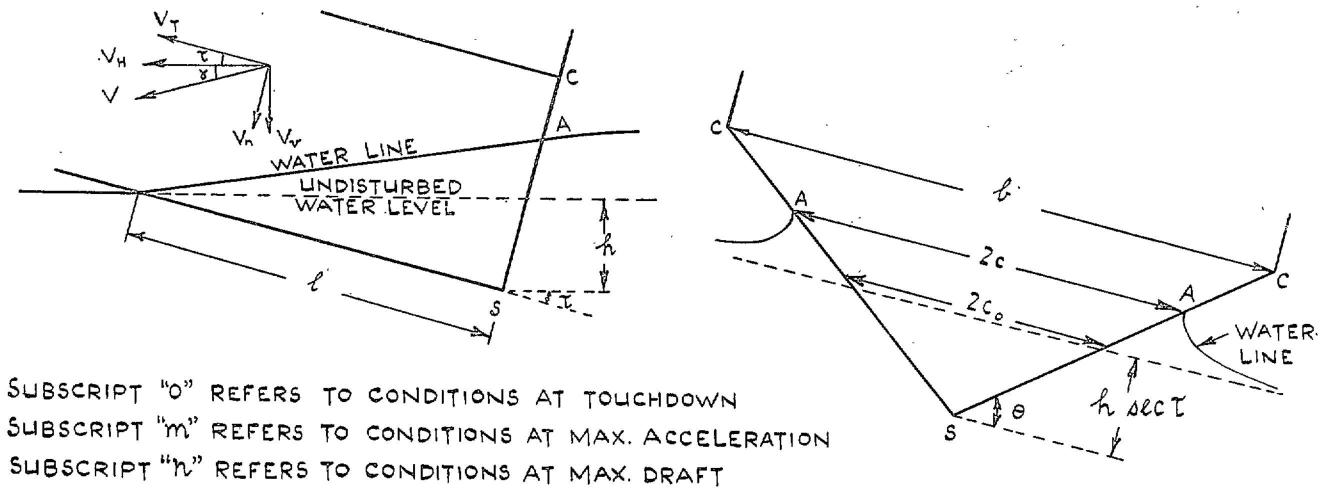

FIG. 1. Conditions during the impact of a plane-faced wedge.

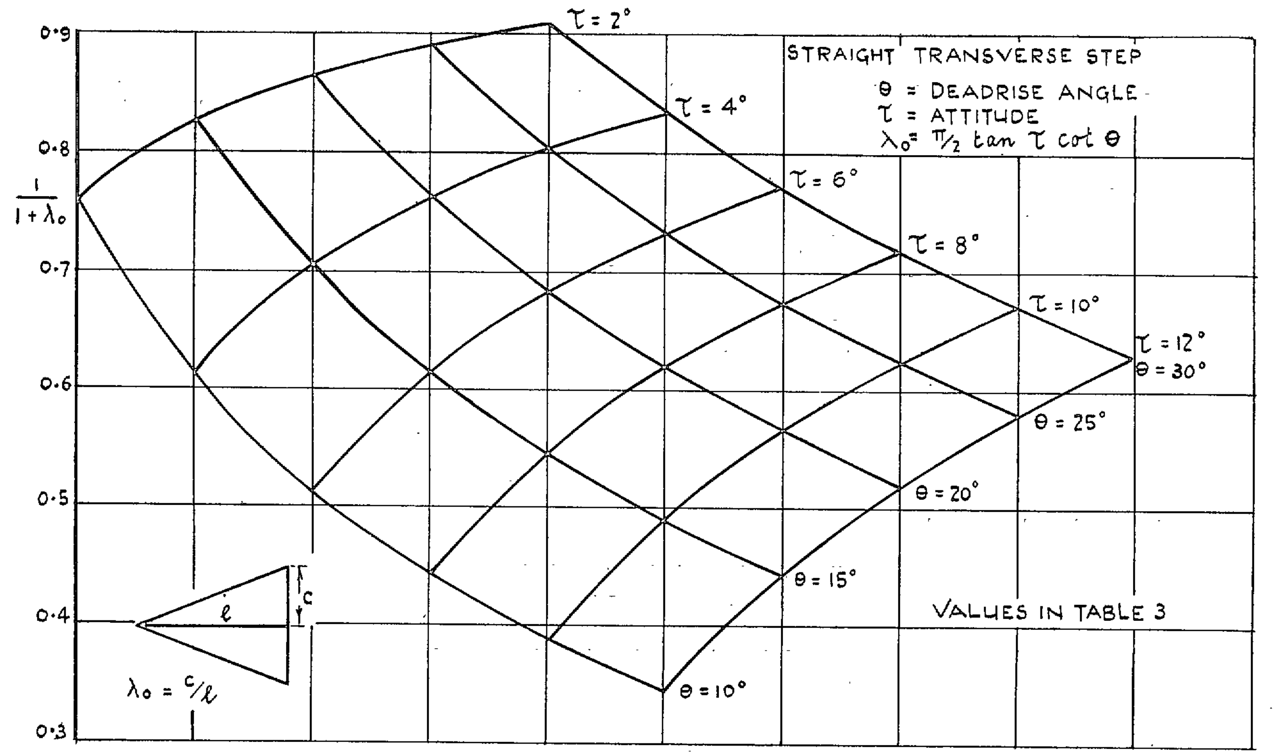

FIG. 2. Determination of associated-mass factor. Chines not immersed.
(1) Aspect ratio parameter $1/(1 + \lambda_0)$, assuming $\pi/2$ splash-up factor.

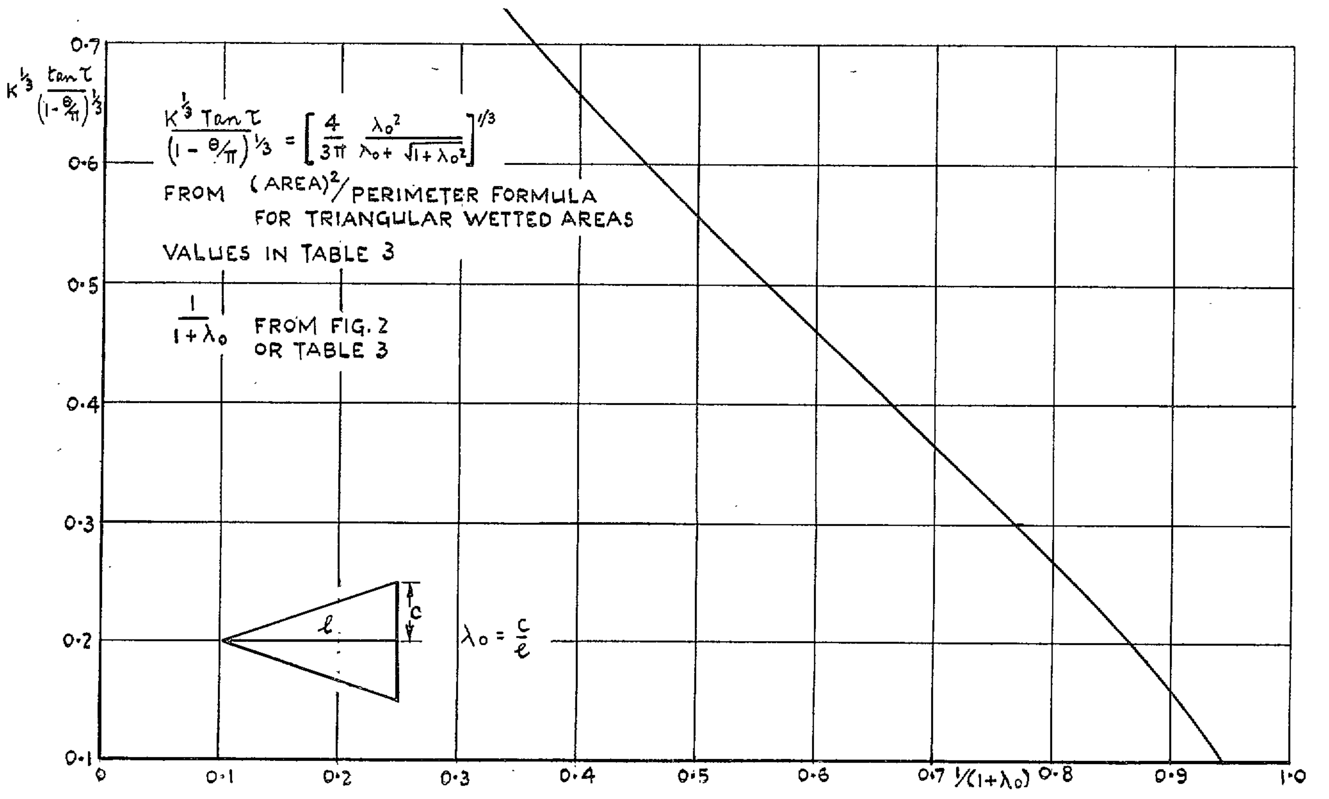

FIG. 3. Determination of associated-mass factor. Chines not immersed.
(2) $K^{1/3}$ as a function of $1/(1 + \lambda_0)$.

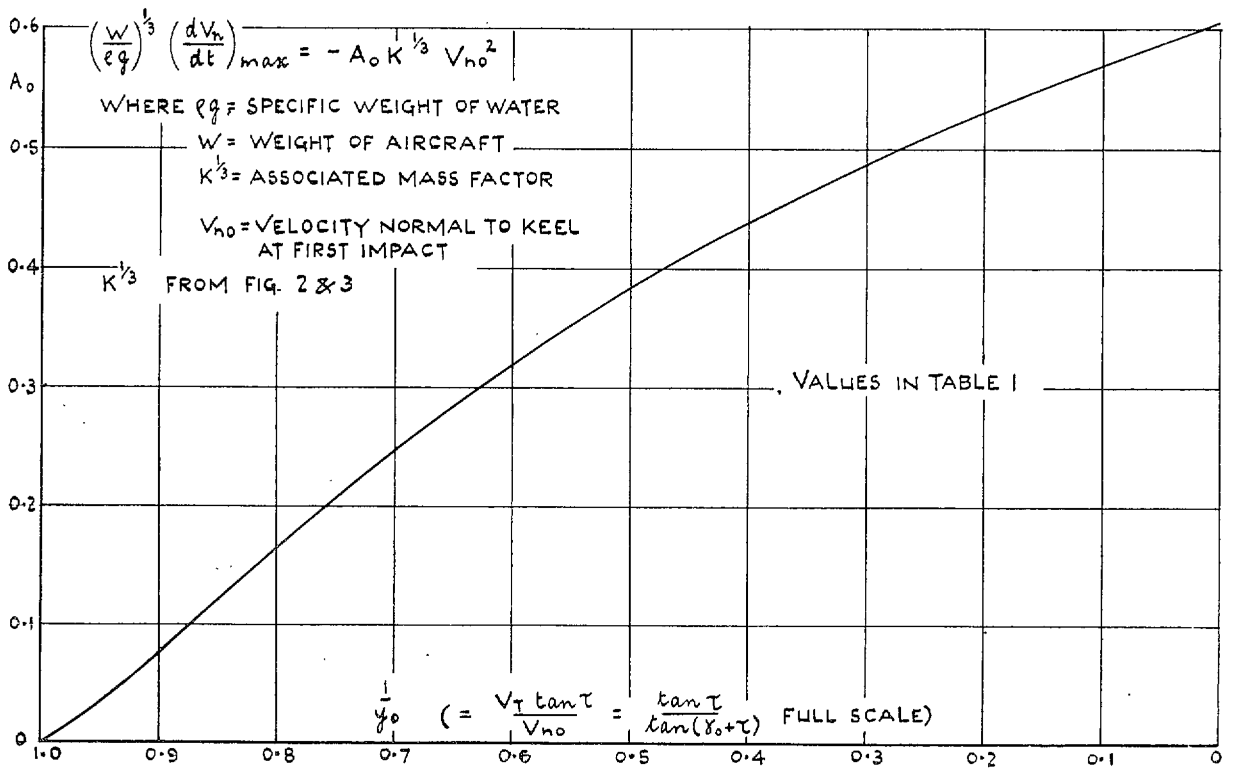

FIG. 4. Maximum acceleration factor for main-step landings.

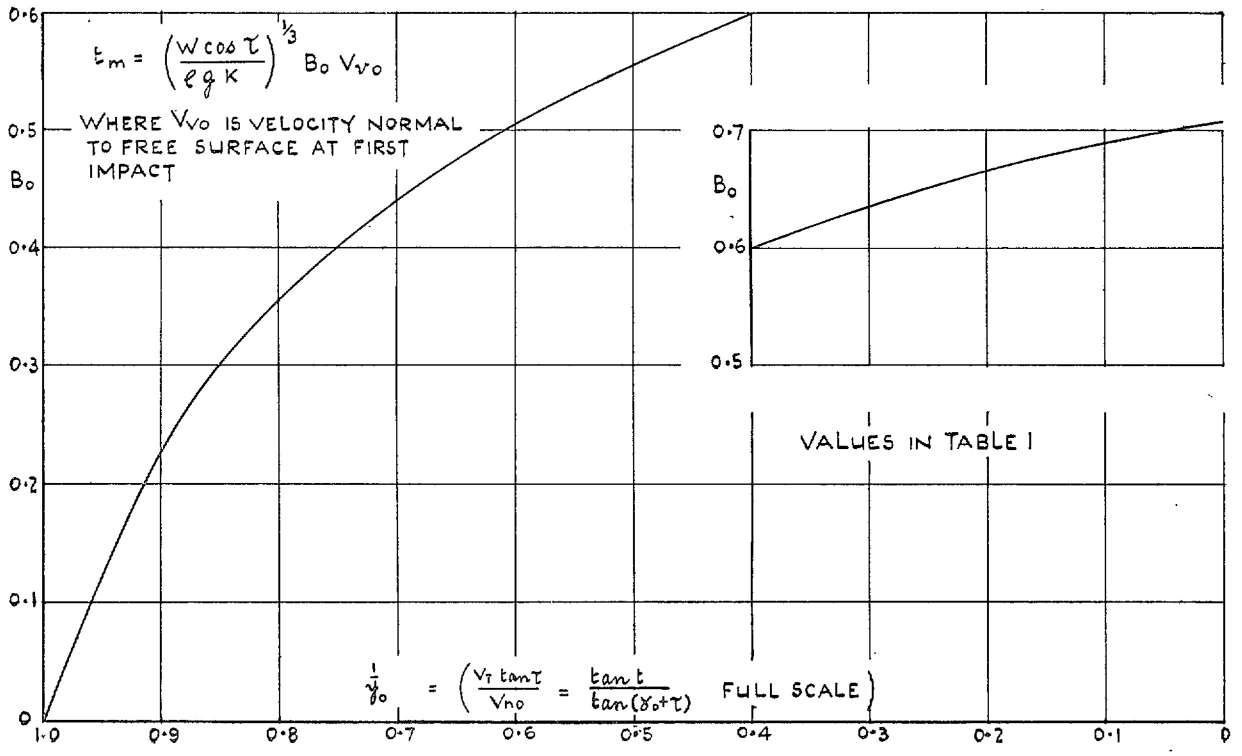

FIG. 5. Time to maximum acceleration factor B_0.

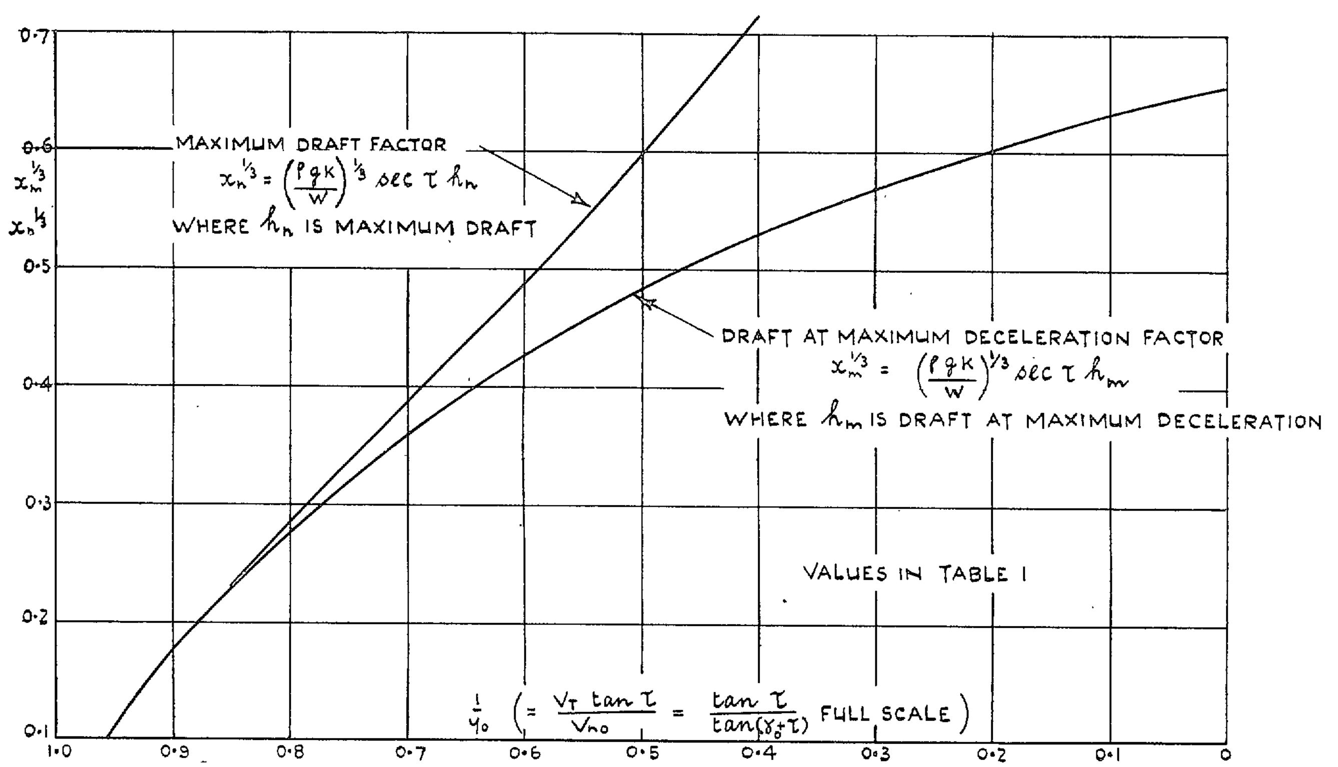

FIG. 6. Draft at maximum acceleration and maximum draft factors.

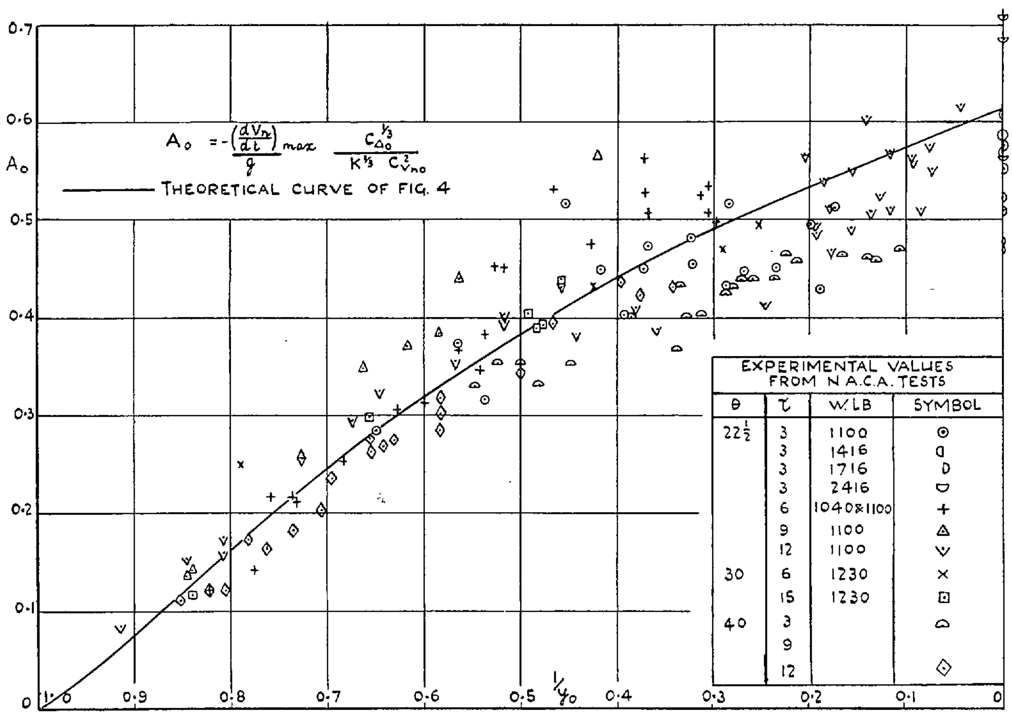

FIG. 7a. Comparison between experiment and theory for maximum acceleration factor, assuming $\pi/2$ splash-up.

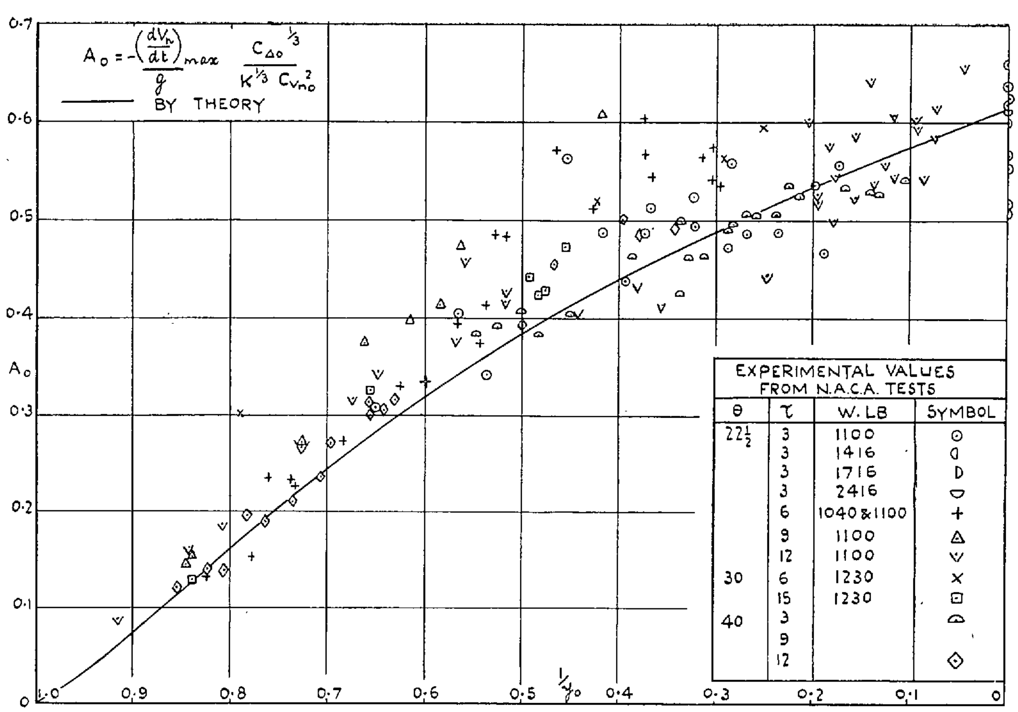

FIG. 7b. Comparison between experiment and theory for maximum acceleration factor, assuming $\pi/2 \, . \, (1 - \theta/\pi)$ splash-up.

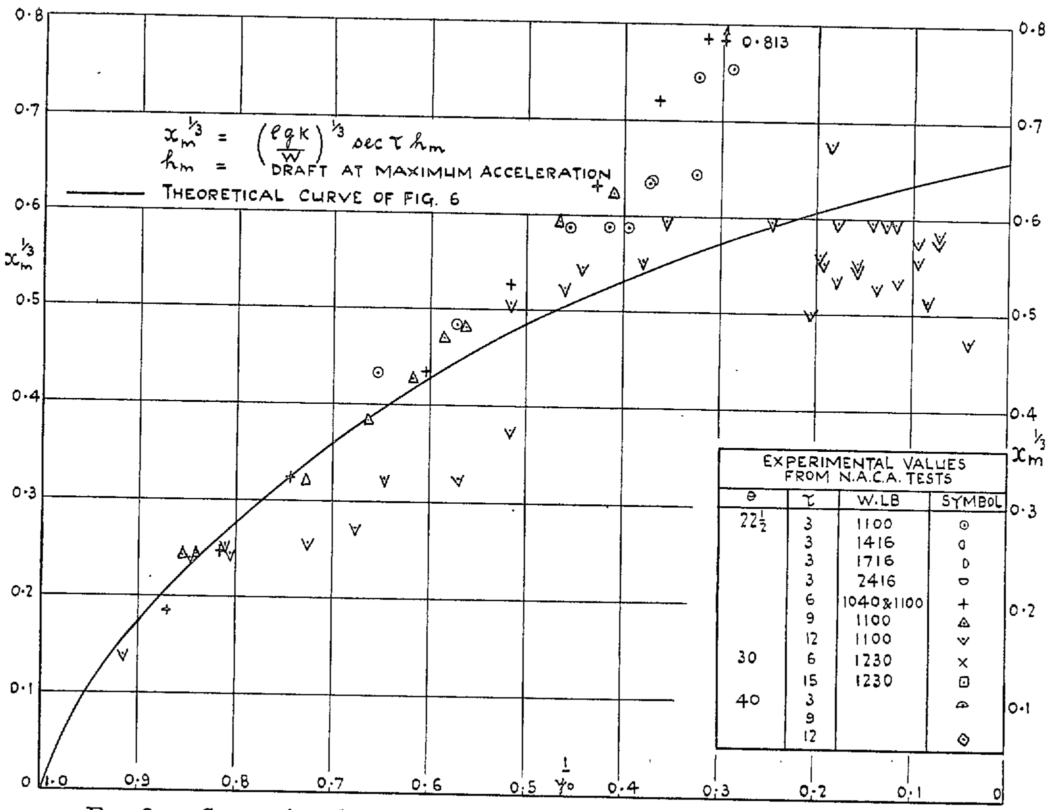

FIG. 8a. Comparison between experiment and theory for draft at maximum acceleration, assuming $\pi/2$ splash-up.

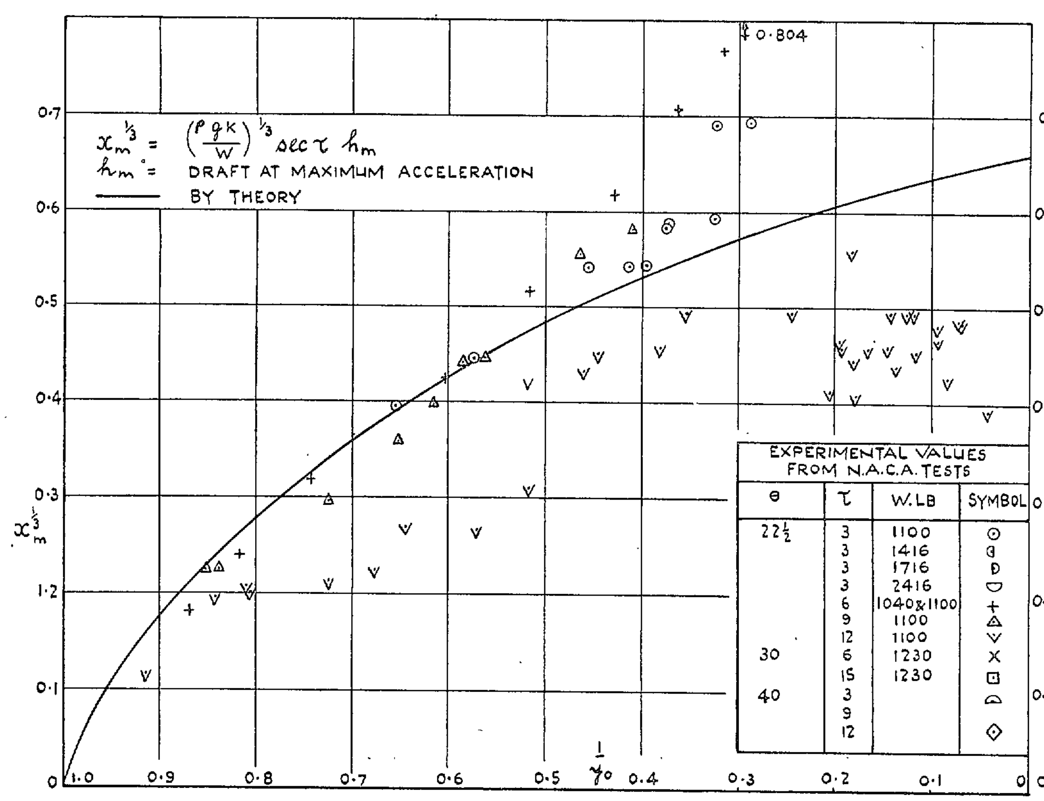

FIG. 8b. Comparison between experiment and theory for draft at maximum acceleration, assuming $\pi/2 \cdot (1 - \theta/\pi)$ splash-up.

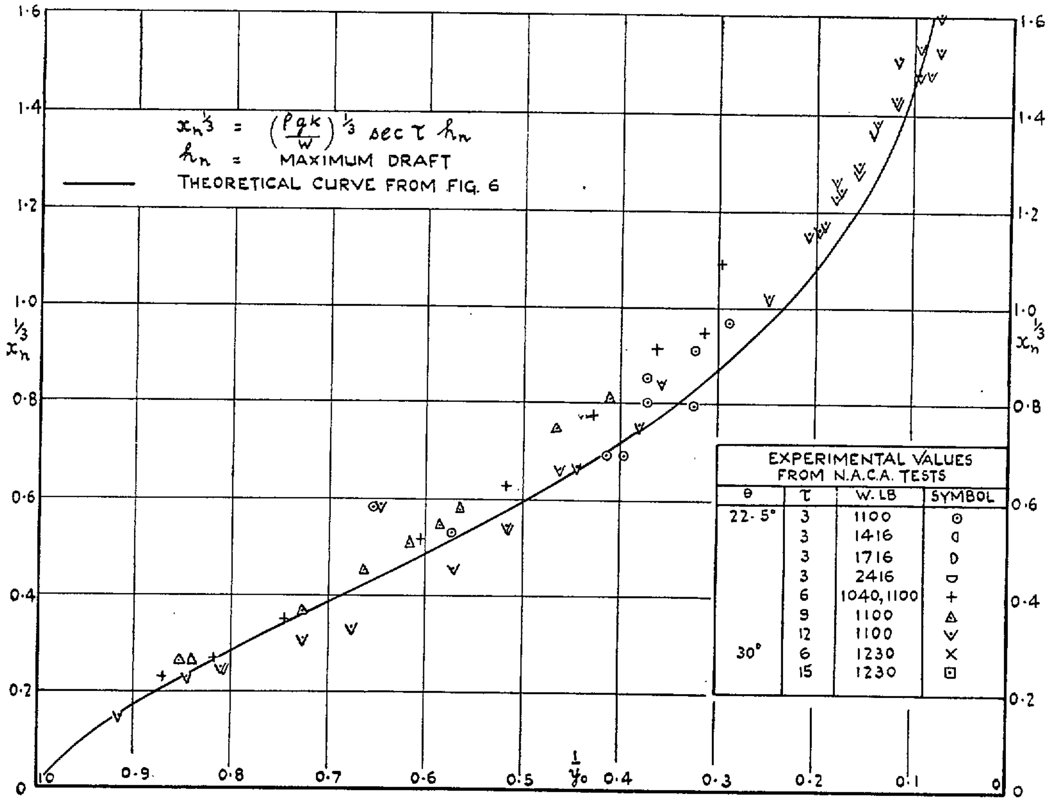

FIG. 9a. Comparison between experiment and theory for maximum draft factor, assuming $\pi/2$ splash-up.

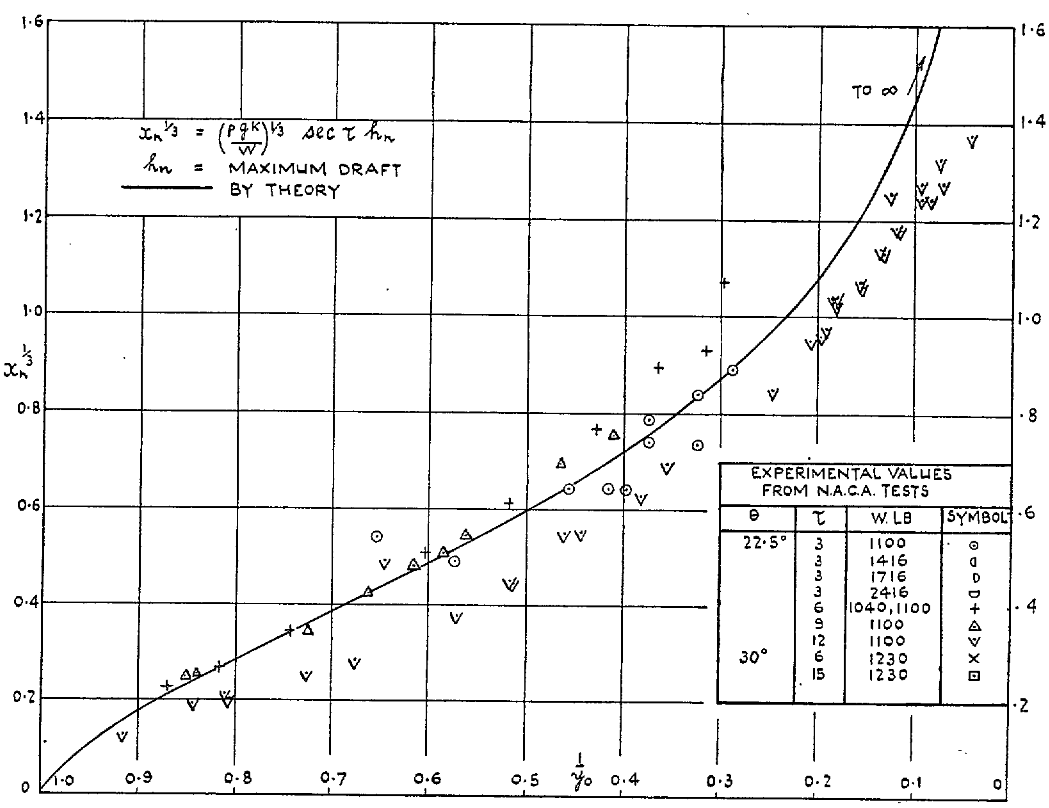

FIG. 9b. Comparison between experiment and theory for maximum draft factor, assuming $\pi/2 \cdot (1 - \theta/\pi)$ splash-up.

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