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The Performance after Power Failure of a Helicopter with Blade Pitch Control Parts I and II

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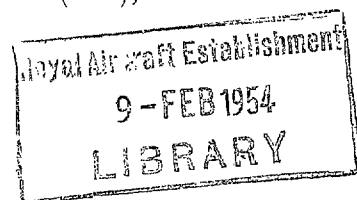# The Performance after Power Failure of a Helicopter with Blade Pitch Control

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Summary.—In Part I a review is made of helicopter performance after engine failure. The transition from powered operation to autorotation is discussed and a theoretical analysis of the motion is given for a single-rotor helicopter with blade-pitch control. The technique of landing from a steady autorotative glide is dealt with briefly; the possibility is indicated of making a safe landing before the transition to steady autorotation has been completed. Reference is also made to the case of engine failure so near the ground that a safe landing may be made by increasing the blade pitch to make immediate use of the rotor energy.

In Part II, tests made to investigate the performance of a *Hoverfly* I in the transition to autorotation following power cut in level flight are described; particular attention is given to the minimum rotor speed attained and to the height lost during the transition. Tests were made to investigate the performance for immediate reduction of pitch only; the need for quick pitch reduction however is stressed because of the rapid fall-off in rotor speed following power-cut at high pitch.

Part I

The Performance after Power Failure of a Helicopter with Blade-Pitch Control

1. *Introduction.*—It is well known that in the event of power failure on current types of helicopters it is possible, given sufficient height, to change over to autorotation of the rotor and then make a power-off glide landing. A certain amount has been written (Refs. 1 and 2) about helicopter performance in an engine-off landing, but little has yet been published about the transition from powered operation to autorotation following sudden power failure. One aspect of this transition performance of obvious practical importance, and on which there is little information, is the height required to reach steady autorotation; it is also important to know how the rotor speed varies and, in particular, whether the speed tends to fall to a dangerously low value early in the transition.

In this part of the report a general review is made of the performance after engine failure in the light of knowledge available from various sources, but mainly from flight and theoretical work at the Airborne Forces Experimental Establishment. A theoretical analysis is given of the motion of the helicopter following a change in the operating conditions of the rotor, both in forward and vertical flight; the theory is applicable both to the transition to autorotation, and to an engine-off landing.

2. *The Transition to Steady Autorotation.*—2.1. *In Forward Flight.*—The general nature of the performance in the transition to autorotation in forward flight on a single-rotor helicopter is illustrated in Fig. 1; the records given were obtained on the *Hoverfly* I. Immediately

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following the simulated sudden engine failure the blade pitch was reduced quickly to the lowest setting. The rotor speed decreased considerably while this was being done and continued to fall for a short time with the pitch at the lowest value. The variation of rotor speed depends on the way in which the change of flow through the rotor takes place. In powered level flight the flow through the rotor is downwards, while in steady power-off flight the resultant flow is upwards. Until the upwards flow is sufficiently established after engine cut, the rotor speed tends to decrease.

The combination of pitch and rotor speed reduction results in a large loss of lift and in a high initial downwards acceleration. The height lost in reaching steady autorotation also depends on the way in which the new rotor flow (and therefore the thrust) builds up. In the example in Fig. 1, the motion does not approach steady conditions directly, the rotor speed increasing beyond the steady value and then decreasing towards it. It appears however that the rate of descent became approximately constant after about only 30 ft while the rotor speed was building up from the minimum value.

The height required to make the transition appears to depend in general on the relationships between the initial and final values of the flight and rotor speeds. From a relatively high speed, for example, it is possible by tilting the rotor disc back (subject to aircraft control) and thereby reducing the flight speed, to build up the flow quickly. From low speed, on the other hand, it may be necessary to tilt the disc forward to gain speed and to await the increase in the disc incidence resulting from the steepening of the glide path. From the point of view of rotor speed it is evidently advantageous to have a relatively high initial rotor speed because of the larger reserve of energy. Further, at lower rotor speeds, the blade pitch in powered operation is higher and a greater change is involved in going in to autorotation; there may be a relatively greater initial loss of rotor speed before the reduction of pitch is completed because of higher blade drag at high pitch. Serious consequences could follow engine failure with the rotor speed at its normal minimum operating value.

Flight tests are being made on the *Hoverfly* I to investigate the variation with the initial flight speed, of the height to reach a steady glide. Owing to the rate at which the transition takes place it is not easy for the pilot to control accurately the helicopter during the manoeuvre, nor to repeat a given performance; the difficulty is increased by the fact that, in addition, the control is less powerful while the rotor lift is reduced than in normal flight. From the results obtained it appears that the height to reach steady autorotation at a speed giving the lowest gliding angle varies from about 170 ft at 40ft/sec to 60 ft at 100 ft/sec. Details of the flight-test results are given in Part II of this report. Further flight work is proceeding on the *Bristol* 171 helicopter which, having a comparatively heavy rotor, gives a slower rate of change of rotor conditions and is well suited for investigating engine-off performance. An investigation is being made of the performance following engine failure for a full range of initial flight and rotor speeds.

A theoretical analysis of the transition to autorotation for a helicopter with blade-pitch control has been given by Hohenemser (Ref. 3). The theory has been developed only for constant flight velocity and with the assumptions that the vertical velocity (in comparison with the flight velocity), and the disturbance of the rotor speed, are small. Estimates given in Ref. 3 for the performance of the *Flettner* helicopter show that at 70 ft/sec the height lost in reaching steady autorotation is about 140 ft. It is assumed that the rotor speed decreases by 10 per cent before the pitch is reduced to the final autorotative value; thereafter the rotor speed falls by about a further 10 per cent before starting to build up again. From other results given by Hohenemser it appears that if the flight velocity is constant during the change to steady autorotation, the height lost in the process increases slightly with speed.

For a fuller investigation of the motion in the transition and, in particular, for an indication of the optimum manner of attaining steady autorotation, it is necessary to permit variation of the flight velocity during the manoeuvre. An extended form of theoretical analys is in which this is done is given in Appendix I. As in Ref. 3 it is still assumed that the helicopter pitching

attitude and the disc attitude to the horizontal are constant. The effects of unsteady aerodynamic conditions are not considered; it is assumed that the rotor thrust is given by the normal formulae for steady conditions.

The motion of the helicopter is considered relative to axes fixed in space Ox horizontal forwards and Oz vertical downwards. Assuming the fuselage drag to be D_1V^2, and the rotor drag in the approximate form $H_1V\omega R$ (Ref. 4), then (with the notation given in the list of symbols at the end of the Report), the equations of motion are

$$ M\frac{dV_x}{dt} = -T\sin\alpha - D_1VV_x - H_1V\omega R\cos\alpha $$

$$ M\frac{dV_z}{dt} = Mg - T\cos\alpha - D_1VV_z + H_1V\omega R\sin\alpha $$

$$ J\frac{d\omega}{dt} = -\frac{Tu}{\omega} - \frac{\pi\rho\sigma R^5}{8}C_D\omega^2 . $$

It is shown in Appendix I that $T/2\pi R\rho^2V^2$ and u/V are functions of i and μ. The equations are reduced therefore to a non-dimensional form in i, μ, and ν ($= V_0/V$), where $V_0 = \sqrt{(Mg/2\pi\rho R^2)}$.

The disc attitude and the blade pitch are assumed constant at their final value in the transition in this analysis but the motion can evidently be determined when these parameters are varied in a step-by-step manner. The numerical work involved however in applying the analysis is heavy. Though it does not seem possible, while including variation of flight velocity, to reduce the equations to a form giving a general analytical solution, certain approximations possible without much loss of accuracy, can be made for some simplification of the analysis; in the initial stages of the transition to autorotation, for example, when there is a great loss of lift, the rotor and fuselage drag forces are comparatively unimportant and may be neglected. Considerably more simplification is possible at the later stage, however, when the rate of descent approximates to the steady value (the rotor speed then possibly being little more than its minimum value). It may then be assumed that V_z is constant, that the thrust is approximately equal to the weight, and only two equations are required to determine the motion. The theory for this case is also given in Appendix I.

Estimates have been made for a modern type helicopter* with the rotor disc at different attitudes (— 5, 0, 7 deg) to show the effect of increasing or decreasing the flight speed in the early stages of the transitional motion (Fig. 2). The initial speed is the same (112·6 ft/sec) in all cases. With a disc attitude $\alpha = -5$ deg, the flight speed first decreases slightly and then increases steadily; for $\alpha = 0$ deg the speed decreases slightly, and for $\alpha = 7$ deg it decreases more quickly. The fall off in rotor speed is in no case large; the greatest reduction is 7 per cent with $\alpha = -5$ deg after about 2·6 sec; for $\alpha = 0$ deg the reduction is 5 per cent after 1·95 sec and for $\alpha = 7$ deg the reduction is little more than 2 per cent after 1·9 sec†. In the first two cases the rate of descent approaches a constant value while the rotor speed is passing its minimum; for $\alpha = -5$ deg the rate of descent is then about 30 ft/sec, and for $\alpha = 0$ it is 20 ft/sec. With a disc attitude of $\alpha = 7$ deg, however, the picture is slightly more complicated, for although the rate of descent appears to be approaching a constant value while the rotor speed passes its minimum, it shortly afterwards starts to increase again. It appears that the change occurs in the neighbourhood of the speed giving the lowest glide angle in steady conditions; although the connection between the speeds is not obvious analytically, it appears reasonable to find

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* The main characteristics of the helicopter for the purposes of this report are: $Mg = 4,500$ lb, $R = 23\cdot5$ ft, $\varkappa = 52\cdot6$, $\sigma = 0\cdot0504$, $d_c = 0\cdot0024$, $h_c = 5\cdot67 \times 10^{-5}$, $\theta = 0\cdot016$ radn.

† It will be noted that in the present analysis the initial rotor speed is that with the blade pitch at the autorotative setting; it is assumed that this rotor speed has already been reduced to allow for the loss in speed occurring while the pitch is being reduced.

that the relative gain (from the point of view of the height lost) in utilising the translational energy should decrease as the speed varies away from a speed corresponding to the best gliding speed.

For the first two cases the rotor speed builds up again beyond the original initial value; for $\alpha = -5$ deg it reaches the initial value in 7·5 sec and for $\alpha = 0$ deg in 6·0 sec. The height lost during this stage is 200 ft for $\alpha = -5$ deg and 110 ft for $\alpha = 0$ deg. For $\alpha = 7$ deg the rotor speed builds up partly towards the initial value and then remains roughly constant. The height lost until the rotor speed reaches the steady value however, is only 17 ft which is considerably less than the 150 ft and 75 ft required to reach a corresponding rotor speed with $\alpha = -5$ deg and $\alpha = 0$ deg respectively.

The advantages of utilising the translational energy in the transitional motion are obvious from this analysis. It may not be possible in practice however to make as much use of the energy as appears desirable because of the effect of the rotor-disc attitude on the overall control of the helicopter. It will be noted too that the different rotor-disc positions do not necessarily lead to reasonable steady flight conditions; some change in the rotor attitude will in fact normally be necessary to produce a desired steady flight condition. The performance obtained from the analysis should probably be regarded therefore only as an indication of the trend in the performance for different attitudes of the rotor.

2.2. *In Vertical Flight.*—The transition from powered vertical flight to steady autorotation in forward flight is similar to the forward flight case. The transition to vertical autorotative descent may however involve greater loss of height and greater reduction of rotor speed if the rotor descends in the downwash stream which persists from the power-driven condition of the rotor. For autorotation, it is necessary to achieve rotor velocity downwards relative to this stream, and although the downwash gradually dissipates, the initial stage at least will involve some extra loss of height.

No flight tests have yet been made to investigate this type of transition. It is of some importance because it appears that the greatest height is required to reach steady autorotation in this way; it is not likely to occur much in practice however. since normally the flight path can deviate at least sufficiently from the vertical to avoid the downwash stream. Some data were obtained on a transition with slight displacement from a vertical path after an actual engine cut in hovering flight on an S.51 in Holland (Ref. 5); automatic observer records showed that the rotor speed fell from 195 r.p.m. to 130 r.p.m. (well below the normal minimum of 170 r.p.m.) and that the height lost in reaching steady rotor autorotation was about 300 ft.

A theoretical analysis of the transitional motion in vertical flight is given in Appendix II. The basic equations of motion are:

$$M \frac{d^2z}{dt^2} = -T + Mg$$

$$J \frac{d\omega}{dt} = -\frac{Tu}{\omega} - \frac{\pi\rho\sigma R^5}{8} C_D \omega^2.$$

If, following power cut in hovering, the helicopter is descending in the downwash stream $v_0(z)$, the rotor thrust is determined by the velocity of the air relative to the rotor, which is $V = v_0 - dz/dt$. It is assumed that the formulae for the thrust in steady conditions apply, so

$$T = 2\pi\rho R^2 V^2 f_1^2 = 2\pi\rho R^2 u^2 F_1^2$$

where f_1, F_1 are rotor coefficients* with the empirical relationship given in Fig. 3.

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* The coefficients $f_1 (= \pm \sqrt{f})$ and $F_1 (= \pm \sqrt{F})$ are used in preference to the original coefficients f and F—in accordance with recent practice. f_1 and F_1 give a more definite representation of the rotor operating condition and avoid the ambiguity of sign which may arise in analysis if $\sqrt{f}$ and $\sqrt{F}$ occur.

Using in addition the blade element formula for the thrust a relationship is derived in Appendix II between f_1, F_1 and μ, in the form,

$$\mu = \frac{\phi}{2f_1} \left[\left(\frac{1}{F_1^2} + \frac{8}{3} \frac{\theta}{\phi} \right)^{1/2} - \frac{1}{F_1} \right]$$

where $\phi = a\sigma/8$.

The equations of motion are reduced to a non-dimensional form in μ and $V_1 = V/V_0$; because v_0 is a function of z it is convenient to take $z_1 = zg/V_0^2$ as independent variable.

The usefulness of the above analysis is limited because of lack of information about v_0. The theory for the case of vertical autorotation in still air $(v_0 = 0)$ is therefore also given in Appendix II. The same form of relationship as before obtains between f_1, F_1 and μ; the equations of motion in μ and V are rather simpler and are given with $\tau = gt/V_0$ as independent variable.

An estimate of the performance of a modern-type helicopter for transition in still air is given in Fig. 4. The height lost in reaching steady autorotation is about 300 ft; the fall-off in rotor speed is not large, so that the rotor speed is always well above the minimum practicable. For the helicopter descending in a downwash stream decreasing linearly to zero in about 300 ft during the descent, a rough estimate suggests that at least 450 ft will be required to attain steady autorotation.

3. *Engine-off Landing.*—An investigation of the engine-off landing performance of a helicopter with blade-pitch control has been made on the *Hoverfly* I (Ref. 2). The tests have shown that landings can be made satisfactorily from approach glides of varying steepness. The technique used is similar in all cases; the disc attitude is increased to slow down the forward speed and the blade pitch is raised to reduce the rate of descent; use of the rotor energy in this way is limited either by blade stalling or by excessive upward coning of the blades. The motion in a forward speed landing can be analysed by the theory for transitional motion outlined in section 2.1.

The analysis can be used to determine the landing technique to adopt for the best performance in the ordinary sense, that is, for the shortest landing distance from 100 ft. From the point of view of the height required to make an emergency landing however, it is more important to determine the technique for the least height loss in the pull-out and landing.

Similarly the theory for the vertical transitional motion outlined in section 2.2 can be used to analyse the vertical engine-off landing performance. It is of interest in this case to determine, for different initial rotor speeds, the least height at which a pull-up to the maximum acceptable landing velocity can be made. The neglect of ground effect might lead to considerable errors in estimates of the vertical landing performance. It is suggested in Appendix II that allowance for this effect can be made using an approximate formula, based on experimental data in Ref. 6, giving the thrust in the neighbourhood of the ground,

$$\frac{T_g}{T} = 0 \cdot 95 + \frac{0 \cdot 2R}{z_0 - z} \cdot$$

4. *Possible Types of Emergency Landing.*—4.1. *After Power Failure Near the Ground.*— Following engine cut on a helicopter, the rotor speed tends to decrease and a loss of rotor lift results. If the failure occurs sufficiently near the ground it is possible, by increasing the blade pitch, to maintain the rotor lift for a period long enough to permit a safe landing. It has been stated (Ref. 7) that a landing of this type is possible on the *Hoverfly* I after power failure in hovering from heights up to 30 ft. It is likely that the height from which this type of landing can be made increases with forward speed because the translational energy may be used to maintain the rotor speed.

The simpler form of analysis of Appendix I with constant rate of descent is suitable for investigating the effect of forward speed on the height from which a landing can be made when

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immediate use is made of the rotor energy. The rate of descent can be taken to be the maximum permissible for landing. The vertical flight performance is determined by the theory for the motion in still air; the rotor continues to operate in the power-driven state utilising the rotor energy as the source of power.

Estimates have been made of the performance of a modern-type helicopter, assuming a rate of descent of 12 ft/sec. In vertical flight, making some allowance for ground effect, the rotor speed is estimated to decrease from the initial (maximum permissible) value to the minimum value in 3·7 sec; an emergency landing should therefore be possible following engine failure from about 45 ft. The performance in forward flight depends on the way in which the flight speed is varied. It is estimated in one case that the rotor speed decreases from the maximum to the minimum value, while the flight speed decreases from 60 ft/sec to about 30 ft/sec, in 8 sec; an emergency landing should therefore be possible following engine failure at 60 ft/sec from about 95 ft.

4.2. *After Power Failure Well Away from the Ground.*—Above the safe height band near the ground it is necessary in general to reduce the blade pitch to minimise the tendency for the rotor speed to decrease. With sufficient height it is possible to reach a steady autorotative glide and make a normal engine-off landing. There is therefore a curve of height, decreasing as speed increases (since the height lost in the transition to autorotation decreases as the speed increases) from which an emergency landing can definitely be made in the event of power failure.

It remains to be considered whether a safe landing can be made before steady autorotation has been attained. The vertical flight case is in some ways the easier to consider from this point of view. It can be seen from Fig. 4 that the rate of sink in the vertical transition can build up very quickly to near the final steady value, while the rotor speed is only beginning to increase again. The downwards acceleration is practically zero therefore beyond this stage and the thrust roughly equal to the aircraft weight. If the rotor speed is above the minimum flying value it appears possible to use some of the remaining available rotor energy to reduce the rate of descent by raising the pitch and so increasing the thrust. The stage of the transition from which sufficient deceleration can be achieved to make a safe landing can be determined by the transitional theory for vertical flight.

Similarly in forward flight it should be possible to make a safe landing before steady autorotation is reached. An estimate of the possible types of landing in this case can be obtained by continuing the analysis of the transitional motion at the increased disc attitude and blade pitch to be used in the landing. The possibilities of landing from an early stage of the transition may be more favourable than in vertical flight since some of the translational energy may also be used in the pull-out.

PART II

Performance of Hoverfly I *during Transition from Powered
Flight to Autorotation*

1. *Introduction.*—Flight tests have been made on a *Hoverfly* I to investigate the performance during the transition from powered level flight to autorotation.

2. *Description of Aircraft.*—The *Hoverfly* I has one 38-ft diameter 3-bladed main rotor and a small tail rotor for torque compensation. The seating arrangement is for two persons side by side. The main rotor blades have drag and flapping hinges and their pitch can be varied cyclically and collectively for control purposes. The collective pitch of the tail-rotor blades is controlled by pedals for yawing control.

The *Hoverfly* I KK.989 used in the tests was a standard aircraft with the addition of nose-wheels. These consisted of a pair of freely rotating Spitfire tail-wheels mounted one on each side of the aircraft on an axle attached to the door-step brackets. The nose-wheels were fitted to prevent damage should the aircraft tend to nose over in an engine-off landing.

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An automatic observer was fitted in the radio compartment behind the pilot; this included the following instruments:—A.S.I., altimeter, stopwatch, attitude gyro, rotor and engine tachometers, Desynns indicating collective pitch and longitudinal stick positions, and accelerometer measuring in the normal plane.

The tests were made at an all-up weight of 2,700 lb, the c.g. position being 8·05 ft aft of the datum.

3. *Tests Made.*—In the tests, while the aircraft was flying on a steady level course with a rotor speed of 235 r.p.m., the engine was throttled back as quickly as possible and the collective pitch reduced to the minimum value. The motion during the transition to a steady autorotative glide was recorded on the auto-observer.

As it was not known to what extent the rotor speed would drop following the power cut, in the first tests the reduction of pitch to the autorotative setting was commenced prior to engine cut being simulated. In the final tests, however, the pitch was not changed until after the engine cut, but it was then reduced as quickly as possible to the minimum value.

Attempts were made also to investigate the effect on the performance of gaining or losing forward speed during the transition; accurate control of the manoeuvre was not easy, however, because of the rapidity of the change.

The tests were made from initial speeds in level flight of 25 to 60 knots I.A.S. and in calm conditions.

4. *Results of Tests.*—Typical records showing the variation of air speed, rotor speed, collective pitch, longitudinal stick position and height lost during a transition to autorotation are given in Fig. 1. It will be seen that the collective pitch was reduced to the autorotative setting within a second of the engine cut. The rotor speed fell off sharply to 218 r.p.m. in just over a second, remained constant for a short period and then built up to 248 r.p.m. finally stabilising at 240 r.p.m.

As the initial air speed was high the azimuth control was pulled back on engine cut to reduce the air speed and speed up the change of flow through the disc by tilting the disc back. Subsequent corrective control action had to be taken to prevent the nose of the aircraft rising excessively. The air speed fell off from 57 knots to 35 knots in approximately six seconds.

The height lost in re-attaining the initial value of the rotor speed, 235 r.p.m., was 60 ft. It appears, however, that the rate of descent became approximately constant before steady rotor conditions had been attained; a height of only 30 ft was needed to reach this stage at which the rotor lift must again have been approximately equal to the weight. The height lost before re-attaining the initial rotor speed after engine cut in level flight at different speeds is shown in Fig. 5. This was found to vary from 170 ft at 25 knots to 40 ft at 60 knots initial air speed. The figure also shows the height lost to the point where the rate of descent becomes approximately constant, varying from 105 ft at 25 knots to 25 ft at 60 knots initial air speed.

Fig. 6 shows the minimum value to which the rotor speed fell in each test, and also the rotor speed where the lift is assumed to be again approximately equal to the weight. Lower minimum rotor speeds were recorded during the early part of the tests because a longer time was taken to reduce the collective pitch. The results given are considered to be the best obtainable with this aircraft, *i.e.*, they represent the minimum fall-off of rotor speed following engine cut. This is discussed further in section 5.

The effects of longitudinal control movements during the transition were difficult to analyse as it proved very difficult to repeat a manoeuvre exactly. In general, however, it appeared advantageous to try and obtain the air speed (40 to 45 knots) corresponding to the least rate of descent as soon as possible. This means stick movement forward at lower initial speeds, and stick back to lose air speed at higher initial speeds.

5. *Discussion.*—In these tests the collective pitch was reduced immediately engine failure was simulated, and reached the autorotative setting within a second of the engine cut. This is considered to be quicker than would be achieved by a pilot not expecting sudden engine

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failure. It is probable that at least one second would elapse before the pilot starts to reduce the pitch, so that it can be two seconds or more before the pitch is reduced to the autorotative value.

Rough estimates for the *Hoverfly* I suggests that if reduction of pitch is delayed after engine cut in flight at high pitch, the rotor speed can fall from the maximum value to approximately the normal minimum flying value in less than one second.

It may of course be possible to build up the rotor speed even though it falls below the minimum flying value; an example of this has occurred on a Sikorsky S.51 helicopter and is described in Ref. 5.

It is considered, however, that the longest period for which pitch reduction can be delayed would be about two seconds, and only so when the initial flight and collective pitch conditions are favourable; after this delay very rapid reduction of pitch would be required.

It may be noted also that the height lost in attaining a steady glide also increases with the time taken to reduce pitch.

The tests described in Ref. 2 show that about 90 ft is required to effect a safe engine-off landing, with little ground run, from a steady autorotative glide. Therefore it appears that an upper boundary to the danger height band can be drawn from 260 ft at 25 knots to 130 ft at 60 knots airspeed. If there is this amount of height available on engine failure the pilot can be sure of having time to apply the controls and make a safe landing without damaging the aircraft.

However as discussed in Part I a safe landing can almost certainly be made from lesser heights. Fig. 1 shows that downward acceleration ceases while the rotor speed is near the minimum value, and it may be possible to make a reasonably safe landing from this point of the transition. This will depend to a large extent on whether the terrain is suitable for a landing with forward speed, and may demand exceptional skill on the part of the pilot.

6. *Conclusions.*—For the *Hoverfly* I the height lost between engine failure in level flight and the establishment of an autorotative glide at the original rotor speed has been found to vary from 170 ft at 25 knots to 40 ft at 60 knots initial air speed. The optimum performance from the point of view of the height lost appeared to be obtained by taking control action to approach the best gliding speed as quickly as possible.

An upper boundary to the danger height band, in which it is undesirable to remain, can be drawn for this aircraft from 260 ft at 25 knots initial airspeed to 130 ft at 60 knots.

The performance given is for immediate reduction of pitch following power failure. Because of the rapid fall-off in rotor speed, the necessity for quick action following power failure at high pitch is stressed. Rough estimates suggest that for safety the delay before pitch reduction must not exceed two seconds.

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LIST OF SYMBOLS

a Slope of blade-lift coefficient curve

b Number of blades

c Rotor blade chord at $0 \cdot 7R$

C_D Blade profile-drag coefficient at the mean effective lift coefficient

$d_c = \dfrac{D_1}{2\pi\rho R^2}$

$D_1 = \dfrac{\text{fuselage drag}}{V^2}$

$\left.\begin{aligned} f &= \dfrac{T}{2\pi\rho R^2 V^2} \\[4pt] f_1 &= \pm\sqrt{f} \\[4pt] F &= \dfrac{T}{2\pi\rho R^2 u^2} \\[4pt] F_1 &= \pm\sqrt{F} \end{aligned}\right\}$ Vertical flight coefficients

$h_c = \dfrac{C_D \sigma}{8}$

$H_1 = \tfrac{1}{4}C_D bc\rho R$

i Rotor disc incidence to flight path, positive upwards

J Rotor moment of inertia

$m = \dfrac{\pi\rho\sigma R^5 C_D}{8J}$

M Helicopter mass

$n = \left(\dfrac{8Vf_1}{F_1}\dfrac{Mg}{\pi\rho\sigma R^5 C_D}\right)^{1/3}$

R Rotor radius

t Time

T Rotor thrust

T_g Rotor thrust within ground-effect region

$T' = \dfrac{T}{2\pi\rho R^2 V^2}$, expressed as a function of i and μ, for forward flight analysis

u Resultant airflow normal to the rotor

$u' = u/V$ expressed as a function of i and μ.

$u_1 = u/V_0$

v Induced flow through the rotor

v_0 Downwash stream through which the rotor descends after power cut in hovering flight.

$V_D = v_0/V_0$

V Flight speed

V_x, V_z Components of the flight speed

$$V_0 = \left(\frac{Mg}{2\pi\rho R^2}\right)^{1/2}$$

$$V_1 = V/V_0$$

$$V_{x1}, V_{z1} = V_x/V_0, \ V_z/V_0$$

x, z Space co-ordinates, x horizontal forwards, z vertical downwards

z_0 Initial height of the rotor above the ground

$z_1 = zg/V_0^2$

α Rotor disc attitude to the horizontal, positive upwards

θ Blade pitch

$\phi = a\sigma/8$

$\varkappa = MR^2/J$

$\mu = V/\omega R$

$\nu = V_0/V$

ρ Air density

$\sigma = bc/\pi R$

$\tau = gt/V_0$

ω Rotor speed

APPENDIX I
Analysis of Transitional Motion in Forward Flight

The motion of a single-rotor helicopter with a torque reaction tail rotor is considered. The tail rotor is geared to the main rotor, and it is assumed that its effect on the performance is covered by making suitable additions to the inertia and drag of the main rotor (Ref. 1).

The analysis is made with reference to axes fixed in space, Ox horizontal forwards and Oz vertical downwards. The flight velocity is V, with components V_x, V_z; the mean induced flow at the rotor disc is v and the mean resultant downwards flow, u. The fuselage drag is D_1V^2 and the rotor blade profile drag coefficient C_D; the transverse rotor drag is assumed in the approximate form $H_1V\omega R$, where $H_1 = \frac{1}{4}C_D bc\rho R$ (Ref. 4). The rotor moment of inertia about the rotational axis is J and the rotor speed, ω. It is assumed that the helicopter pitching attitude and the disc attitude to the horizontal, α, are maintained constant; α is measured positive upwards. The equations of motion are

$$M\frac{dV_x}{dt} = -T\sin\alpha - D_1VV_x - H_1V\omega R\cos\alpha$$

$$M\frac{dV_z}{dt} = Mg - T\cos\alpha - D_1VV_z + H_1V\omega R\sin\alpha$$

$$J\frac{d\omega}{dt} = -\frac{Tu}{\omega} - \frac{\pi\rho\sigma R^5}{8}C_D\omega^2.$$

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Neglecting the effect of unsteady aerodynamic conditions it is assumed that the normal formulae for the thrust in steady conditions obtain. Thus $T = 2\pi\rho R^2 V_R v$, where

V_R, the resultant flow at the rotor $= \{V^2 + v^2 + 2v(V_x \sin\alpha + V_z \cos\alpha)\}^{1/2}$.

Then, since $u = v - (V_x \sin\alpha + V_z \cos\alpha)$,

$$\frac{T}{2\pi\rho R^2 V^2} = \frac{v}{V}\left\{1 - \frac{v^2}{V^2} + 2\frac{v}{V}\frac{u}{V}\right\}^{1/2} . \quad .. \qquad .. \qquad .. \qquad .. \qquad .. \qquad .. \qquad (I.1)$$

Also, from the blade element theory

$$T = \tfrac{1}{4}\pi\rho a\sigma R^4\left(\tfrac{2}{3}\theta - \frac{u}{\omega R}\right)\omega^2$$

or

$$\frac{T}{2\pi\rho R^2 V^2} = \frac{\phi}{\mu^2}\left(\tfrac{2}{3}\theta - \frac{\mu}{V}u\right) \qquad .. \qquad .. \qquad .. \qquad .. \qquad .. \qquad .. \qquad .. \qquad (I.2)$$

where $\phi = a\sigma/8$.

It is possible to determine graphically from (I.1) and (I.2) for selected values of α and θ, v/V as a function of u/V and μ. Then since $u = v - (V_x \sin\alpha + V_z \cos\alpha)$, u/V and $T/2\pi\rho R^2 V^2$ can be determined as functions of μ and $(V_x \sin\alpha + V_z \cos\alpha)/V$.

It may be seen that if i is the rotor-disc incidence to the flight path, then $\sin i = (V_x \sin\alpha + V_z \cos\alpha)/V$; we may therefore write

$$\frac{u}{V} = u'(i, \mu) \text{ or } u'; \qquad .. \qquad .. \qquad .. \qquad .. \qquad .. \qquad .. \qquad .. \qquad (I.3)$$

and

$$\frac{T}{2\pi\rho R^2 V^2} = T'(i, \mu) \text{ or } T'. \qquad .. \qquad .. \qquad .. \qquad .. \qquad .. \qquad .. \qquad .. \qquad (I.4)$$

A reasonable approximation to these functions can be made by neglecting terms above the first order in v' ($= v/V$). Then $T' = v'(1 - v'\sin i)$ approximately, and v' can be determined directly from this equation and (I.2). It then follows that

$$u' = \frac{1 + \dfrac{\phi}{\mu} - \left\{\left(1 + \dfrac{\phi}{\mu}\right)^2 - \dfrac{4\phi\sin i}{\mu}\left(\dfrac{2}{3}\dfrac{\theta}{\mu} + \sin i\right)\right\}^{1/2}}{2\sin i} - \sin i \quad .. \qquad .. \qquad .. \qquad (I.5)$$

and

$$T' = \frac{\mu}{\phi}\left[\frac{2}{3}\frac{\theta}{\mu} + \sin i - \frac{1 + \dfrac{\phi}{\mu} - \left\{1 + \dfrac{\phi^2}{\mu} - 4\dfrac{\phi}{\mu}\sin i\left(\dfrac{2}{3}\dfrac{\theta}{\mu} + \sin i\right)\right\}^{1/2}}{2\sin i}\right].$$

It is convenient to put the equations in a non-dimensional form in terms of i and μ; the third dependent variable is $\nu = V_0/V$ where $V_0^2 = Mg/2\pi\rho R^2$. It will be noted that $V_x/V_0 = \cos(i - \alpha)/\nu$ and $V_z/V_0 = \sin(i - \alpha)/\nu$. Then, with $\tau = gt/V_0$, $d_c = D_1/2\pi\rho R^2$, $h_c = C_D\sigma/8$, and $\varkappa = MR^2/J$, the equations of motion become

$$\nu\frac{di}{d\tau} = -T'\cos i + \nu^2\cos(i - \alpha) + \frac{h_c \sin i \cos i}{\mu}$$

$$\frac{d\nu}{d\tau} = T'\sin i - \nu^2\sin(i - \alpha) + d_c + \frac{h_c \cos i}{\mu}$$

$$\nu\frac{d\mu}{d\tau} = -T'\mu(\sin i - \varkappa\mu^2 u') + \nu^2\mu\sin(i - \alpha) - d_c\mu + h_c(\tfrac{1}{2}\varkappa - \cos i).$$

These equations, with T' and u' from (I.3) and (I.4), determine the transitional motion for selected values of α and θ. Application of the analysis involves fairly heavy numerical work. Some simplification results if Hohenemser's assumption of constant flight velocity is adopted, but the effect of utilising the translational energy by converting it to rotational energy in the rotor can not then be investigated. This still remains possible if it is assumed that the rate of descent, V_z, is constant. In this case the thrust is approximately equal to the weight, and if, for further simplification, the rotor drag is neglected, the equations of motion can be reduced, with $V_{x1} = V_x/V_0$, $V_1 = V/V_0$ and $u_1 = u/V_0$, to the form

$$\frac{dV_{x1}}{d\tau} = -\sin\alpha - d_c V_{x1} V_1 \qquad .$$

$$\frac{d\mu}{d\tau} = -\mu\frac{V_{x1}}{V_1^2}\sin\alpha - d_c\mu\frac{V_{x1}^2}{V_1} + \frac{\varkappa u_1\mu^3}{V_1^2} + \tfrac{1}{2}h\varkappa V_1.$$

From the momentum thrust equation with $T = Mg$ it follows, with $v_1 = v/V_0$, that

$$1 = v_1\{V_{x1}^2 + V_{z1}^2 + v_1^2 - 2v_1(V_{x1}\sin\alpha + V_{z1}\cos\alpha)\}^{1/2}.$$

v_1 can be determined graphically from this relationship in terms of V_{x1}, for given values of V_{z1} and α. u_1 then follows as a function of V_{x1} from

$$u_1 = v_1 - (V_{x1}\sin\alpha + V_{z1}\cos\alpha).$$

APPENDIX II

Analysis of Transitional Motion in Vertical Flight

The downwash stream created by the rotor in powered vertical flight persists for some time after the blade pitch is reduced following power failure; if the helicopter then descends vertically it does so in down-flowing air. The downwash velocity depends on the distance below the initial position of the rotor and, because of dissipation, on time also. For simplicity in the analysis power cut in hovering flight only is considered at this stage, and it is assumed that the downwash velocity is a function only of the distance below the origin, $v_0(z)$. By successive approximation this function can be taken to accord roughly with the conditions at a given height when the rotor passes there.

The rate of descent of the helicopter is dz/dt; due to the downwash the inflow relative to the rotor, V, is downwards at a rate $(v_0 - dz/dt)$. The induced flow being v, the mean flow downwards through the disc is $u = v + V$. The equations of motion are

$$M\frac{d^2z}{dt^2} = -T + Mg \qquad .$$

$$J\frac{d\omega}{dt} = -\frac{Tu}{\omega} - \frac{\pi\rho\sigma R^5}{8}C_D\omega^2. \qquad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (\text{II.1})$$

It is assumed as in Appendix I, that the thrust formulae for steady conditions can be used. Hence

$$T = 2\pi\rho R^2 V^2 f_1^2 = 2\pi\rho R^2 u^2 F_1^2 \qquad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (\text{II.2})$$

where f_1, F_1 are rotor operating constants with the relation shown in Fig. 3. Also, from the blade element theory

$$T = \frac{abcR}{4}\rho\omega^2 R^2\left(\tfrac{2}{3}\theta - \frac{u}{\omega R}\right). \qquad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (\text{II.3})$$

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A relationship can be determined from (II.2) and (II.3) between f_1, F_1 and μ, in the form

$$\mu = \frac{\phi}{2f_1}\left\{\left(\frac{1}{F_1^2} + \frac{8}{3}\frac{\theta}{\phi}\right)^{1/2} - \frac{1}{F_1}\right\}. \quad \text{..} \quad \text{..} \quad \text{..} \quad \text{..} \quad \text{..} \quad \text{(II.4)}$$

The equations of motion can be put in a non-dimensional form in μ and $V_1 = V/V_0$; because v_0 is a function of z, it is simplest to use as independent variable $z_1 = zg/V_0^2$. Then putting $v_D = v_0/V_0$, the equations become

$$\frac{dV_1}{dz_1} = \frac{dv_D}{dz_1} - \frac{1}{v_D - V_1} + \frac{f_1^2 V_1^2}{v_D - V_1}$$

$$\frac{d\mu}{dz_1} = \frac{\mu}{V_1}\frac{dv_D}{dz_1} - \frac{\mu}{V_1(v_D - V_1)} + \frac{f_1^2 \mu V_1}{v_D - V_1} + \frac{\varkappa f_1^3}{F_1}\frac{\mu^3 V_1}{v_D - V_1} + \frac{\varkappa h_c}{2}\frac{V_1}{v_D - V_1}.$$

These equations can be used with (II.4) and Fig. 3 to analyse the vertical transitional motion for a selected value of θ.

The helicopter may also descend vertically but not in a downwash stream. In this case $v_0 = 0$ and $V = -dz/dt$. The equations of motion are then applicable not only to an initial hovering condition, but with the z-origin at the initial rotor position, to any initial vertical flight condition. Using $\tau = gt/V_0$ as the independent variable the equations become

$$\frac{dV_1}{d\tau} = V_1^2 f_1^2 - 1$$

$$V_1\frac{d\mu}{d\tau} = -\mu + f_1^2 \mu V_1^2 + \frac{\varkappa f_1^3}{F_1}\mu^3 V_1^2 + \frac{\varkappa h_c}{2}V_1^2.$$

For the transition from hovering, the initial conditions need special consideration because $1/f_1 = 0$ when $V = 0$. Terms in the equations may be grouped however into (Vf_1) and (μf_1) which are finite initially with values

$$(Vf_1)_0 = \frac{\omega_0 R\phi}{2}\left[\left(\frac{1}{F_1^2} + \frac{8}{3}\frac{\theta}{\phi}\right)^{1/2} - \frac{1}{F_1}\right]_0$$

$$(\mu f_1)_0 = \frac{\phi}{2}\left[\left(\frac{1}{F_1^2} + \frac{8}{3}\frac{\theta}{\phi}\right)^{1/2} - \frac{1}{F_1}\right]_0.$$

The initial values of $(dV_1/d\tau)$ and of $(d\mu/d\tau)$ are zero; to find the variation of (Vf_1) in the first interval in step-by-step integration it is necessary to use the variation of ω which is given by

$$\frac{d\omega}{d\tau} = -\frac{\varkappa \omega^2 R}{V_0}\left[\frac{\mu^3 f_1^3}{F_1} + \frac{h_c}{2}\right].$$

As for forward flight, the analysis is considerably simplified if the rate of descent $(-V)$ is assumed constant. With the thrust equal to the weight

$$f_1 = -\left(\frac{Mg}{2\pi\rho R^2 V^2}\right)^{1/2}$$

$$u = \frac{f_1}{F_1}V.$$

The rotor motion is determined by equation (II.1) which reduces to

$$\omega\frac{d\omega}{dt} = -m(n^3 + \omega^3)$$

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where

$$m = \frac{\pi \rho \sigma R^5 C_D}{8J}$$

$$n = \left(\frac{8Vf_1}{F_1} \frac{Mg}{\pi \rho \sigma R^5 C_D} \right)^{1/3}.$$

This equation has the integral

$$(3mn)t = \log \frac{(n^2 - n\omega + \omega^2)^{1/2}}{n + \omega} + \sqrt{3} \arctan \frac{\omega - n/2}{\sqrt{3}n/2} + \text{constant.}$$

This determines the variation of rotor speed with time; the height lost is given by $(-V)t$.

This solution can be applied to analysis of the vertical performance following power failure so near the ground that immediate use can be made of the rotor energy to control the rate of sink to the ground. In this case, ground effect may have some influence on the performance. Approximate allowance can be made for this effect on the basis of the results given in Ref. 6. In Fig. 3 of that report T_g/T is given as a function of a blade loading coefficient and of $(z_0 - z)/2R$, where $(z_0 - z)$ is the distance from the ground. An algebraic function giving a reasonable approximation to the curves, for a mean value of the blade loading coefficient, is

$$\frac{T_g}{T} = 0 \cdot 95 + \frac{0 \cdot 2R}{z_0 - z}.$$

This formula applies for values of $(z_0 - z)$ up to $4R$ at which the thrust becomes equal to the value away from the ground; above this height, ground effect vanishes in practice.

For application to the present analysis it is assumed that the airflow speeds at the rotor disc are related to T in the normal way; hence if $T_g = Mg$,

$$\frac{Mg}{0 \cdot 95 + 0 \cdot 2R/(z_0 - z)} = 2\pi \rho R^2 V^2 f_1^2 = 2\pi \rho R^2 u^2 F_1^2$$

Since the rate of descent is constant, $z = Vt$, and the motion is determined by using the above formulae in (II.1). To obtain a simply integrable form of equation, however, it is necessary to assume a mean value for z; this should give a sufficiently accurate indication of the difference in the performance due to ground effect.

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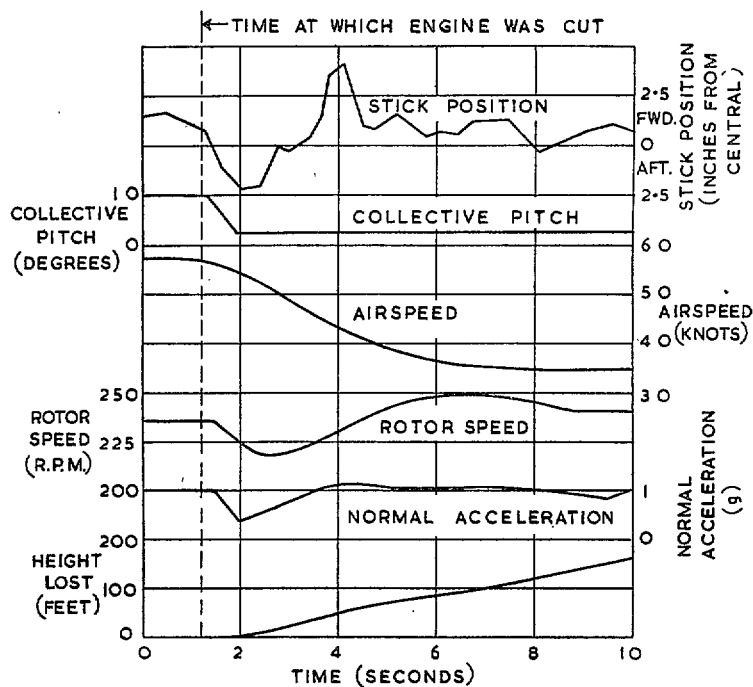
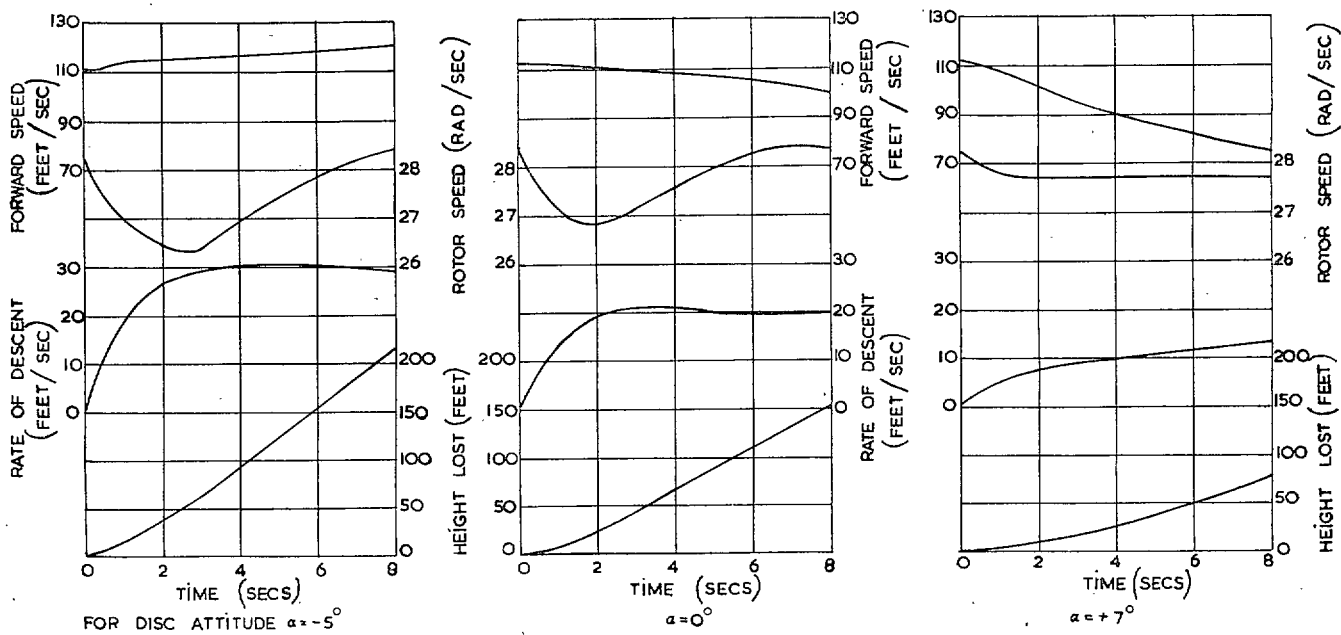FIG. 1. *Hoverfly* 1. Transition to autorotation after engine cut in level flight.

FIG. 2. Estimated performance for the transition to autorotation in forward flight of a modern-type helicopter with different rotor-disc attitudes, showing the effect of speed variation on the performance.
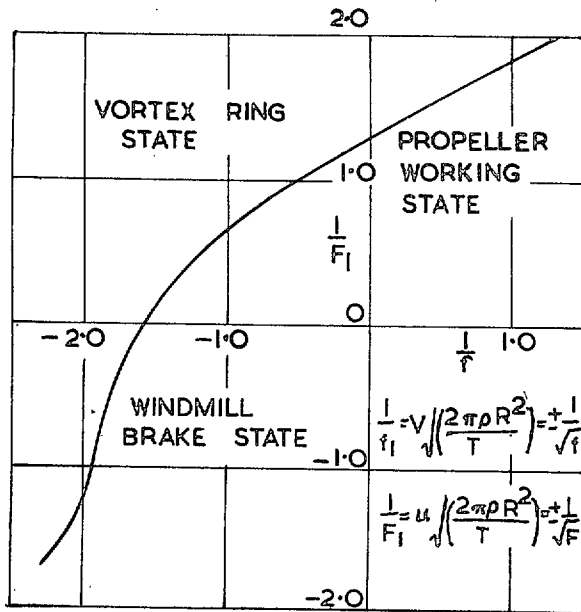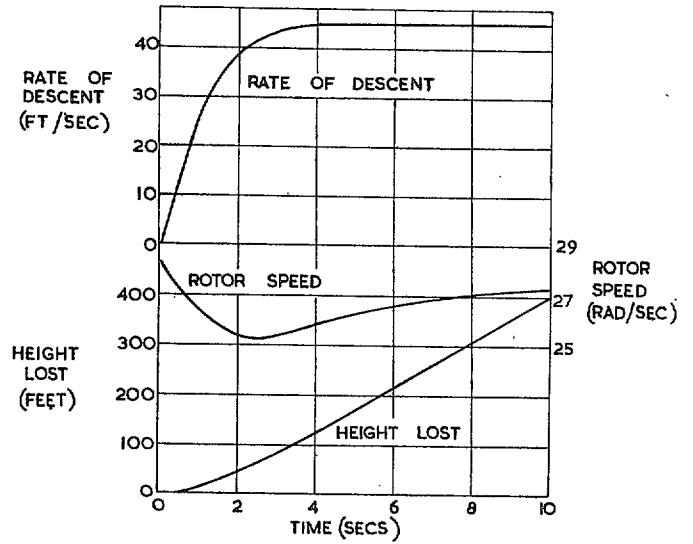
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FIG. 3. Relationship of f_1 and F_1.

FIG. 4. Estimated performance for the transition from hovering flight to vertical autorotation of a modern-type helicopter; the effect of the powered condition rotor downwash is not included.
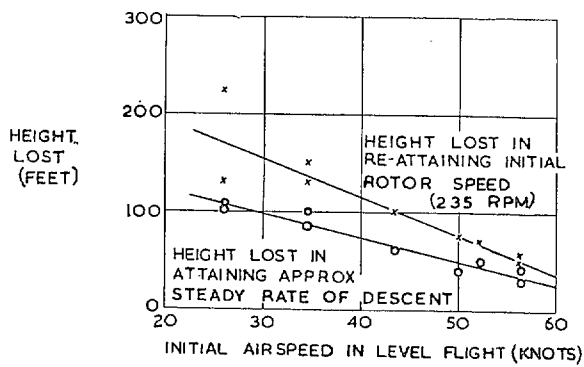
FIG. 5. Height lost in transition.
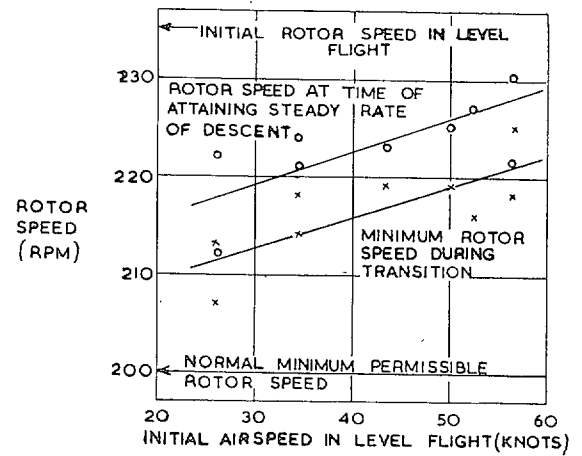
FIG. 6. Variation of rotor speed in transition.

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