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The Effect of Compressibility on the Attitude of Aircraft in Rectilinear Flight

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Summary.—Purpose of Investigation.—The attitude of aircraft (*i.e.*, the angle between the aircraft datum and the flight path) is of considerable importance in the aiming of certain airborne armament. An investigation was therefore made of the effect of compressibility on the attitude of aircraft in flight in a straight path.

Scope of Investigation.—The application of the results of linear perturbation theory to the problem was examined, and the deductions made compared with the results of attitude measurements on a *Spitfire* IX over a wide range of altitude and air speed.

Conclusions.—As is well known, linear perturbation theory indicates a reduction of the slope of the curve of attitude against lift coefficient with increase in Mach number. The theory indicates, however, that in straight flight at a constant ratio of wing lift to air pressure the variation of Mach number with lift coefficient is such that to a first approximation the slope of the curve of attitude against lift coefficient remains unchanged at the low Mach number value, only the intercept, or apparent no-lift angle, being altered (Fig. 1). This reduction in no-lift angle is proportional to the ratio of the lift to the air pressure but is not directly affected by Mach number or air speed.

The experiments show a reduction in no-lift angle which agrees with that predicted by theory for the aspect ratio of wing tested.

The change in apparent no-lift angle is of the order of half a degree between sea level and 40,000 ft.

The above conclusions should not be applied to wings over 15 per cent thick.

1. *Introduction.*—Theory and wind-tunnel tests indicate[1] that the compressibility of air may be expected to affect the lift-curve slope of thin wings. The effects of this on the variation of aircraft attitude with air speed and height has therefore been examined theoretically and experimentally.

2. *Scope of Investigations.*—A theoretical examination of the problem was made by reference to the results of linear perturbation theory[1], and the results compared with those of attitude measurements on a *Spitfire* IX in level flight at altitudes of approximately 5,000 ft, 20,000 ft and 30,000 ft.

3. *Theoretical Investigation.*—Linear perturbation theory indicates[1] that the first order effect of compressibility on the lift of an aerofoil is to increase the lift-curve slope at constant Mach number, *i.e.*, the partial derivative of the lift coefficient with respect to incidence, while leaving

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the no-lift angle unchanged. This change in slope increases with increase in Mach number; Ref. 1 gives the formula:—

$$\frac{A}{a} = \frac{1 + \frac{a_\infty}{\pi A}}{(1 - M^2)^{1/2} + \frac{a_\infty}{\pi A}} \qquad \ldots \quad (1)$$

where $A = \frac{\partial C_L}{\partial \alpha}$ in compressible flow at a Mach number M

$a = \frac{\partial C_L}{\partial \alpha}$ in incompressible flow

$a_\infty = \frac{\partial C_L}{\partial \alpha}$ in incompressible flow for infinite aspect ratio

A is the aspect ratio of the aerofoil.

With this notation the attitude-lift equation may be written

$$\alpha = \alpha_0 + \frac{1}{A} C_L \qquad \ldots \quad (2)$$

where α is the attitude of aircraft to flight path

and α_0 the attitude of aircraft to flight path at no-lift.

Hence, combining equations (1) and (2) we have

$$\alpha = \alpha_0 + \left(\frac{(1 - M^2)^{1/2} + K}{1 + K}\right) \frac{1}{a} C_L \qquad \ldots \quad (3)$$

where K is equal to $a_\infty/\pi A$ and is approximately $0 \cdot 34$ for an aspect ratio of $5 \cdot 6$ and an a_∞ of about 6, as for the aircraft tested. (For infinite aspect ratio K is zero and equation (3) is reduced to the simple Glauert relation.)

Expanding the term within the bracket in terms of M and neglecting the fourth and higher powers we have

$$\alpha = \alpha_0 + \frac{1}{a} C_L - \frac{1}{2a(1 + K)} C_L M^2. \qquad \ldots \quad (4)$$

(The neglect of the remaining terms in M seems permissible up to a Mach number of about $0 \cdot 8$, when the term in M^4 becomes one-sixth of that in M^2; the ratio of these two terms is in fact $\frac{1}{4}M^2$).

But $C_L M^2 = \frac{L}{\frac{1}{2}\rho c_0{}^2 S p}$ $\qquad \ldots \quad (5)$

where c_0 is the speed of sound at I.C.A.N. sea level and p is absolute-air-pressure/standard-absolute-air-pressure at sea level and hence we may write

$$\alpha = \left\{\alpha_0 - \frac{1}{a(1 + K)} \; \frac{1}{\rho_0 c_0{}^2 S} \; \frac{L}{p}\right\} + \frac{1}{a} C_L \qquad \ldots \quad (6)$$

For a given aircraft at constant L/p—*e.g.*, in level flight at constant weight and pressure altitude—the second term of the right-hand side, which represents the first order effect of compressibility, is constant.

Under these conditions, therefore, the first order effect of compressibility is to reduce the attitude by an amount which is independent of air speed (and Mach number), the amount of this reduction being proportional to the ratio of lift to air pressure. The flight curve of altitude against lift coefficient at constant L/p crosses the curves at constant Mach number in the manner sketched in Fig. 1.

Thus attitude tests made in flight at constant weight and various altitudes would be expected to result in a series of parallel lines of attitude against lift coefficient, the displacement of the lines being proportional to the ratio of lift to air pressure. This shift will be referred to below, for convenience, as the ' apparent reduction in no-lift angle.'

4. *Experimental Investigation.*—4.1. *Apparatus.*—The investigation was made on a standard *Spitfire* IX, (Service No. BS.352) whose wing area was 242 sq ft and span 36·8 ft. The wing section was NACA 2200 series, 13·2 per cent thick at the root and 6·6 per cent thick at the tip. The aircraft was equipped with an automatic observer containing a pendulum inclinometer, an airspeed indicator, an altimeter and a watch.

4.2. *Technique.*—Readings were taken by means of the automatic observer with the aircraft in steady level flight, at air speeds covering as large a range as was practicable, at pressure altitudes of 5,000 ft, 20,000 ft and 30,000 ft. During each run readings were taken approximately every ten seconds for about five minutes and mean values taken. The zero correction to the inclinometer was measured by taking a reading with the aircraft on the ground and simultaneously measuring the angle of the aircraft datum to the horizontal.

4.3. *Results.*—The observed attitudes (of the aircraft datum to the flight path), together with the corresponding pressure altitudes, aircraft weights and equivalent air speeds are presented in Table 1 and are plotted against lift coefficient in Fig. 2. The corresponding values of the Mach number (M), the lift coefficient (C_L) and of $C_L M^2$ are also tabulated.

4.4. *Analysis of Results.*—A relation of the form $\alpha = \alpha_0 + d_1 C_L + d_2(C_L M^2)$ was assumed (*cf.* section 3) and the values of d_1 and d_2 which fitted the experimental results best were estimated by the method of least squares[2]. The ranges of d_1 and d_2 found to correspond to a 95 per cent level of probability were:—

$$d_1 \qquad 12 \cdot 1 \pm 0 \cdot 1$$
$$d_2 \qquad -3 \cdot 8 \pm 1 \cdot 7$$

If results for which the lift coefficient exceeded 0·5 were excluded the corresponding ranges were

$$d_1 \qquad 12 \cdot 0 \pm 0 \cdot 8$$
$$d_2 \qquad -6 \cdot 5 \pm 5 \cdot 5$$

It may be noted that the highest value of M among the results excluded was 0·32.

5. *Comparison of Theory with Experiment.*—Young's formula (eqn. 1) would give $\frac{1}{2}(1 + k)$ for the ratio of d_2 to d_1 (*see* section 3) which is about 0·37 for the aspect ratio wing tested (5·6). The Glauert relation, which has sometimes been applied to finite aspect ratios although not strictly applicable, gives $-$ 0·5. For a d_1 of 12 the corresponding values of d_2 are $-$ 4·4 and $-$ 6·0.

Thus the estimate ($-3 \cdot 8$) of the value of d_2 made from the test results as a whole agrees with Young's formula within the limits of experimental error, but differs significantly both from zero and from the Glauert formula. The estimate ($-6 \cdot 5$) made from the results obtained at lift coefficients less than $0 \cdot 5$ agreed rather better with Glauert than Young but not significantly so. It differed significantly from zero.

The variation in apparent no-lift attitude is illustrated in Fig. 3, on which the experimental results, corrected to no-lift by means of the first value of d_1 quoted in section 4.4, are plotted against $C_L M^2$, together with the line corresponding to $d_2 = -3 \cdot 8$.

6. *Variation of Apparent No-lift Attitude with Altitude and Wing Loading.*—The change of apparent no-lift attitude in level flight is illustrated by Fig. 4, which is based on the results of the present investigation. In this figure the reduction in attitude for the *Spitfire* IX is plotted against height for a number of wing loadings, to a rather open scale, for $d_2 = -3 \cdot 8$.

It will be seen that a reduction in apparent no-lift attitude of about half a degree may be met with on going from sea level to 40,000 ft.

7. *Applicability of Results.*—Linear perturbation theory is only applicable to 'thin wings,' and should not be applied to wings with thickness to chord ratios greater than about 15 per cent. (The average thickness to chord ratio of the wings of the *Spitfire* IX is about 10 per cent (section 4.1)).

The conclusions drawn from the present investigation should not, therefore, be applied to wings over about 15 per cent thick.

8. *Conclusions.*—Linear perturbation theory indicates[1] that the first order effect of compressibility on the attitude of an aircraft relative to its flight path is an apparent reduction in no-lift angle (Fig. 1). This depends only, for a given aircraft, on the ratio of the lift (or, in level flight, weight) to the air pressure. It does not depend directly on air speed or Mach number.

Attitude measurements have been made on a *Spitfire* IX in level flight over a range of altitudes and analysed by the method of least squares. The results show a decrease in apparent zero lift attitude which agrees with the theoretical decrease for the aspect ratio of wing tested.

The change in apparent no-lift attitude is about half a degree on going from sea level to 40,000 ft.

The above conclusions should not be applied to wings over 15 per cent thick.

9. *Further Developments.*—None proposed.

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No.	*Author*	*Title, etc.*
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2	Fisher	*Statistical Methods for Research Workers.* Ch. V. para. 29. Oliver and Boyd.

TABLE 1

Pressure Altitude (ft)	Weight (lb)	E.A.S. (knots)	Attitude to horizontal (deg)	Mach Number (M)	Lift Coefficient (C_L)	$C_L M^2$
4790	6593	254·2	− 0·90	0·420	0·124	0·022
5125	6572	204·0	− 0·23	0·339	0·193	0·022
4965	6551	178·0	+ 0·81	0·294	0·254	0·022
4965	6533	162·9	1·28	0·269	0·301	0·022
4885	6515	149·0	1·93	0·246	0·358	0·022
4990	6497	137·0	2·58	0·227	0·423	0·022
5045	6470	129·8	3·55	0·214	0·470	0·022
5000	6450	121·2	3·88	0·201	0·535	0·022
5100	6430	113·3	4·38	0·189	0·612	0·022
5040	6400	111·2	5·28	0·183	0·633	0·021
5040	6380	103·9	6·57	0·171	0·725	0·021
4975	6360	96·9	7·62	0·162	0·827	0·022
5460	6340	90·0	8·90	0·142	0·960	0·022
20080	6440	230·2	− 0·42	0·515	0·148	0·039
20705	6410	187·2	+ 0·08	0·425	0·224	0·041
19960	6380	167·2	0·97	0·372	0·279	0·039
20185	6350	157·8	1·25	0·353	0·312	0·039
20005	6325	148·1	1·50	0·331	0·353	0·039
20200	6300	131·6	2·98	0·294	0·447	0·038
20195	6278	122·8	3·47	0·274	0·510	0·038
20595	6260	113·2	4·58	0·255	0·596	0·039
20535	6242	107·2	5·77	0·242	0·660	0·039
20080	6232	102·0	6·08	0·226	0·732	0·038
20180	6222	97·8	6·92	0·217	0·795	0·038
20930	6210	92·6	8·50	0·209	0·882	0·039
29665	6380	208·0	− 0·30	0·574	0·179	0·059
29675	6358	185·0	0·12	0·510	0·227	0·059
29835	6326	168·9	0·68	0·466	0·272	0·059
29840	6300	150·8	1·25	0·416	0·339	0·059
30215	6282	134·3	2·45	0·375	0·422	0·060
30190	6270	127·0	3·22	0·345	0·476	0·060
30390	6260	108·0	5·12	0·303	0·658	0·060
30440	6245	113·2	4·10	0·318	0·591	0·060
30130	6230	97·0	7·28	0·269	0·808	0·059
30070	6215	103·6	6·53	0·287	0·709	0·059
30535	6200	95·6	7·58	0·267	0·831	0·060

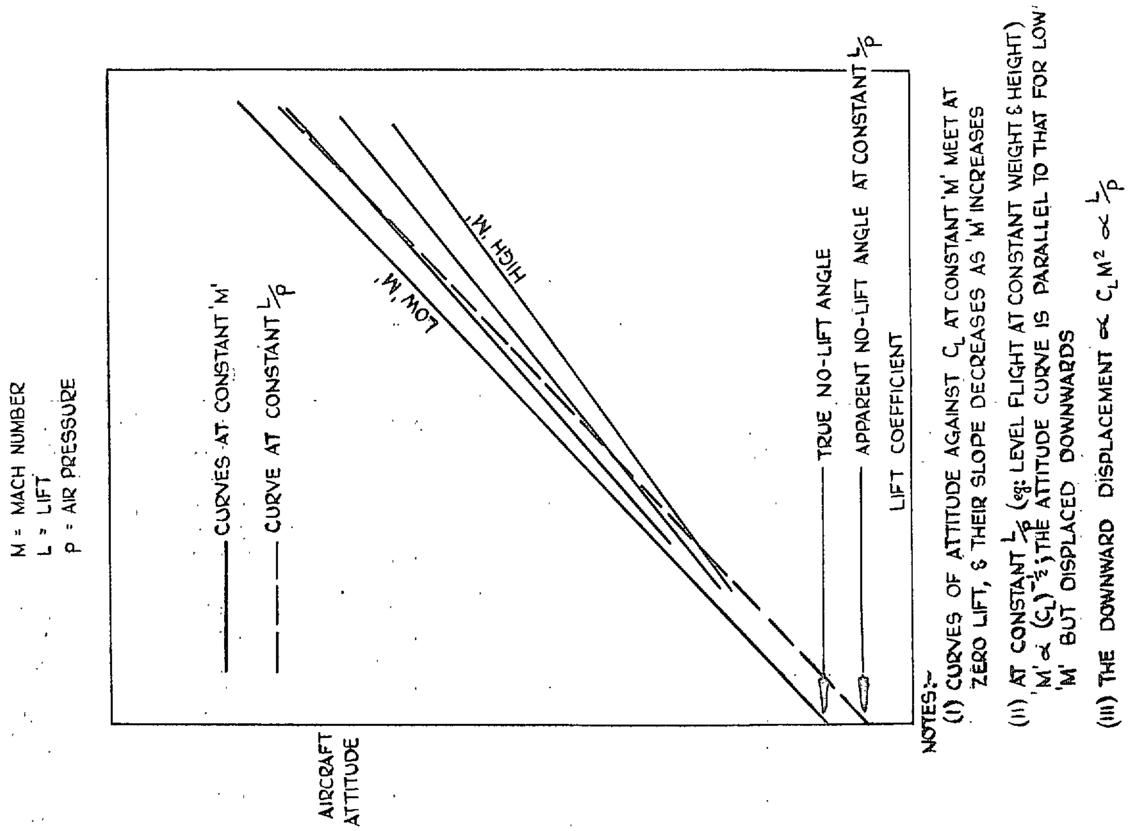

FIG. 1. Sketch illustrating effect of compressibility on the attitude of an aircraft in straight flight.

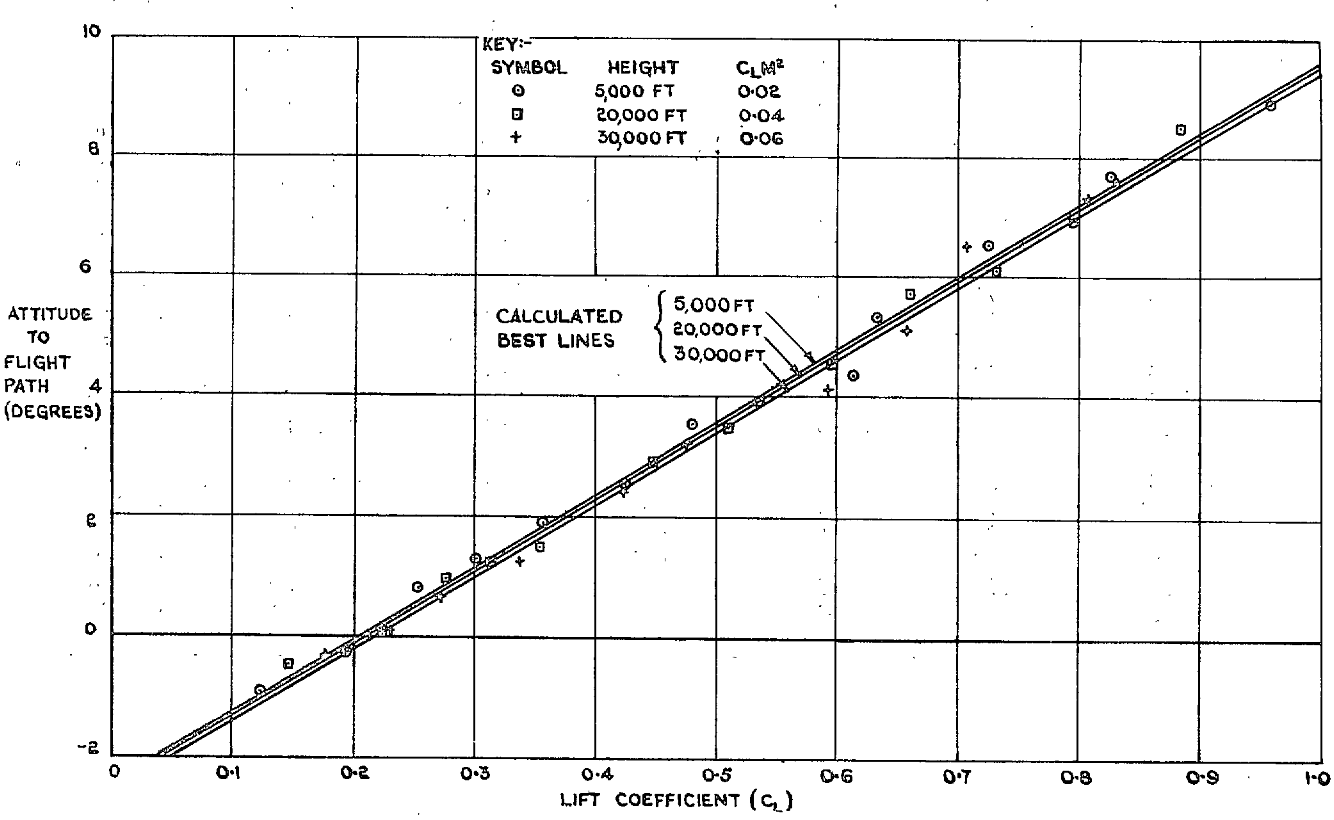

FIG. 2. Variation of attitude with lift coefficient.

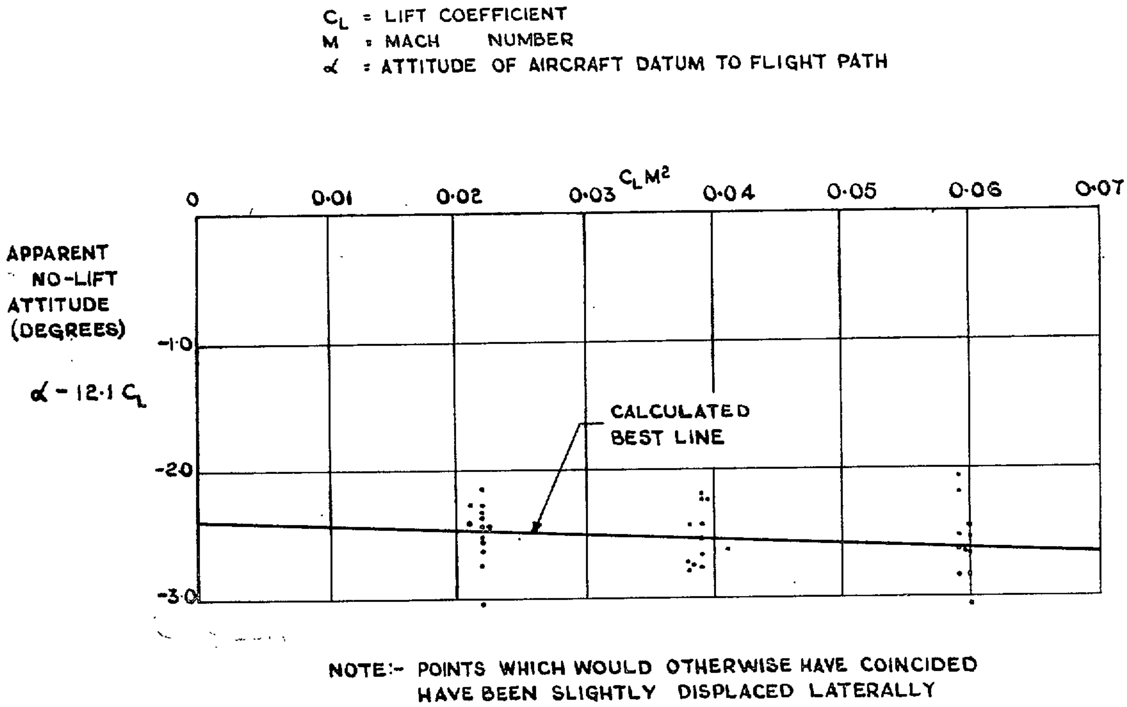

FIG. 3. Apparent no-lift attitude: variation with $C_L M^2$.

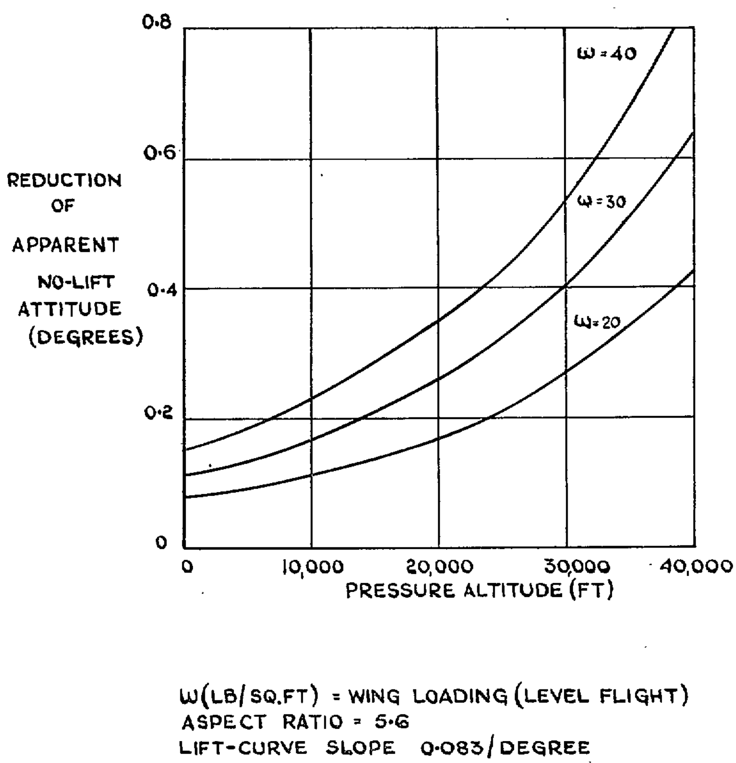

FIG. 4. Reduction of no-lift attitude of *Spitfire* IX due to compressibility: deduced from experiment.

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