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By

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Summary.—In this note, the equations of the flexural-torsional flutter of a swept wing are established, assuming the wing to be semi-rigid and fixed at the root. The general effects of sweep-back, wing stiffness and position of the inertia axis are determined. The critical speeds for flutter and for wing divergence are determined (i) for incompressible flow (ii) for compressible flow, applying the Glauert correction.

The critical flutter speed is in general higher for a swept-back wing having the same wing stiffness as the unswept wing; for a swept-forward wing, divergence will occur before flutter.

NOTATION

Dimensions and Displacements of Wing (see Fig. 1)

c	Chord at distance y from root chord (parallel to the root chord)
c_0	Root chord
c_m	Mean chord
c_t	Tip chord
$d =$	$0 \cdot 9s$
f and F	define the flexural and torsional modes of oscillation
gc	Chordwise distance from leading edge to inertia axis
hc	Chordwise distance from leading edge to flexural axis
jc	Chordwise distance from flexural axis to inertia axis
$l =$	$0 \cdot 7s$. Perpendicular distance from wing root to flexural centre of reference section
s	Perpendicular distance from wing root to tip
s'	Distance from wing root to tip, measured along flexural axis
y	Perpendicular distance from wing root to a given chordwise element
α	Angle of incidence of wing

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$\eta = y/l$

ϕ Normal displacement of flexural centre at a given chordwise element

ψ Slope of flexural axis at a given chordwise element

θ Angle of twist of a given section perpendicular to the flexural axis.

β Angle of sweep-back of flexural axis.

Density

ε Air density/wing density $= \rho/\sigma_\omega$

ρ Air density in slugs per cubic foot

σ_ω Wing density = wing mass per unit area/mean chord, in slugs per cubic foot.

Stiffness coefficients

l_ϕ Elastic moment about perpendicular to flexural axis for unit displacement ϕ_r at the reference section

m_θ Elastic moment about flexural axis for unit displacement θ_r at the reference section

$$B = \frac{V_c\sqrt{\rho}}{(m_\theta/dc_m^2)^{1/2}}$$

$$\gamma = \frac{l_\phi/d^3}{m_\theta/dc_m^2}$$

V Forward speed of aircraft

V_c Critical flutter speed

Introduction.—In this note, the equations of the flexural-torsional flutter of a swept wing are established, assuming the wing to be semi-rigid and fixed at the root. The general effects of sweep-back, wing stiffness and position of the inertia axis are determined. The critical speeds for flutter and for wing divergence are determined (i) for incompressible flow, (ii) for compressible flow, applying the Glauert correction.

Data and Assumptions.—*General.*—A straight tapered swept wing is considered (Fig. 1). The flexural and inertia axes are taken at given constant percentage chord distances behind the leading edge.

Principal Dimensions.

s Span (root to tip), perpendicular to root chord

d Perpendicular distance from root to 'equivalent tip section' $= 0{\cdot}9s$

l Perpendicular distance from root to flexural centre of the 'reference section' $= 0{\cdot}7s$

c_0 Root chord

c_t	Tip chord
c_m	Mean chord
hc	Distance of flexural axis aft of leading edge (measured parallel to root chord)
gc	Distance of inertia axis aft of leading edge (measured parallel to root chord)
$1 - \tau$	Taper ratio $= c_t/c_0$.
β	Angle of sweep back of flexural axis.
	Corresponding distances along the flexural axis are indicated by dashes; thus
s'	Span measured along the flexural axis.

Axes Ox, Oy are taken parallel and perpendicular to the root chord through the point O, where the flexural axis meets the root chord. Axes Ox', Oy' are taken perpendicular to and along the flexural axis.

Modes of Motion and Displacement Co-ordinates.—The wing is assumed to be semi-rigid, the modes of displacement in flexure and in torsion being taken to be independent of the speed; all displacement of either kind are taken to be in phase with one another. The modes of displacement are taken to be linear in torsion and parabolic in flexure; this approximates closely to the natural modes of the system. The displacement co-ordinates are defined as follows:—

The flexural co-ordinate ϕ is the flexural displacement of the flexural centre at a given section divided by y' (positive for downward bending).

The torsional co-ordinate θ is the angle of twist of a given section perpendicular to the flexural axis measured relative to the corresponding root section Ox' (positive when the trailing edge moves down relative to the leading edge). θ_r and ϕ_r are the flexural and torsional co-ordinates of the reference section (the section perpendicular to the flexural axis at 70 per cent of the span, measured along the flexural axis).

The wing is supposed to be placed at a small angle of incidence in a uniform airstream of speed V (Mach number M) and the wing root is supposed to be rigidly fixed.

The displacement θ, ϕ of any point are related to the corresponding displacements at the reference section θ_r, ϕ_r by the equations

$$\frac{\phi}{\phi_r} = \frac{l'\mathrm{f}(\eta)}{y'}; \qquad \frac{\theta}{\theta_r} = \mathrm{F}(\eta)$$

where $\eta = y/l = y'/l'$.

The symbols used in the equation of motion conform with those in R. & M. 1782[1] and 1869[2].

Elastic Stiffness Coefficients.—The flexural and torsional coefficients are denoted by l_θ and m_ϕ respectively. The non-dimensional flutter speed coefficients are plotted against the modified stiffness ratio r defined by

$$r = \frac{l_\phi}{d^3} \Big/ \frac{m_\theta}{d{c_m}^2} = \frac{l_\phi}{m_\theta} \times \frac{{c_m}^2}{d^2}\,.$$

Wing Density.—The wing density σ_ω is defined to be the total wing mass in slugs divided by the product of the wing area in square feet and the mean chord in feet.

Also, $\varepsilon = \rho/\sigma_\omega$ where ρ is the air density in slugs per cubic foot.

Let $\sigma_{\omega\beta}$, $\sigma_{\omega 0}$ be the wing densities for a swept and for an unswept wing of the same area and mean chord. The swept wing will have a larger weight due to its larger span, measured along the flexural axis. It can be shown on theoretical grounds that the weight of a swept wing should vary approximately as $\sec^2 \beta$

Therefore, $\sigma_{\omega\beta} = \sigma_{\omega 0} \sec^2 \beta$

and $\varepsilon = \sec^2 \beta \,.\, \varepsilon_0$.

The Inertial Coefficients.—To find the inertial coefficient, we replace the given wing by the wing ABB′A′, considering the section AA′ to be rigidly fixed to the fuselage.

As in R. & M. 1782[1] and 1869[2] we assume that the mass per unit span (measured along the flexural axis) is $m_\beta \bar{c}^2$ where $\bar{c}$ is the local chord perpendicular to the flexural axis and m_β is constant for a given angle of sweepback.

We have approximately $\bar{c} = c \cos \beta$ where c is the local chord measured parallel to the line of flight.

$$\text{Therefore, total wing mass} = 2m_\beta \int_0^{s'} \bar{c}^2 \, dy'$$

$$= 2m_\beta \cos \beta \,.\, c_0^2 s\left(1 - \tau + \frac{\tau^2}{3}\right)$$

For the unswept wing, total wing mass $= 2m_0 c_0^2 s\left(1 - \tau + \dfrac{\tau^2}{3}\right)$.

Assuming as above that the wing weight varies as $\sec^2 \beta$,

$$m_\beta = m_0 \sec^3 \beta \,.$$

For both swept and unswept wings, total wing area $= 2sc_0 \left(1 - \dfrac{\tau}{2}\right)$

and mean chord $= c_0 \left(1 - \dfrac{\tau}{2}\right)$.

$$\text{Now } \sigma_\omega = \frac{\text{wing mass}}{\text{wing area} \times \text{mean chord}}$$

$$\text{Therefore,} \quad \frac{\sigma_{\omega\beta}}{m_\beta} = \frac{\sigma_{\omega 0}}{m_0} = \frac{4}{3} \, \frac{3 - 3\tau + \tau^2}{4 - 4\tau + \tau^2} \,.$$

We also assume (as in R. & M. 1782[1] and 1869[2]) that the radius of gyration $k\bar{c}$ of a chord wise section about a transverse axis through the inertia centre of the section is a constant percentage of the chord. ($k = 0{\cdot}294$).

Let δm = weight of wing element $\delta x' \, \delta y'$ at point (x', y').

As in R. & M. 1782[1] and 1869[2] the inertia coefficients are given by the following formulae:—

$$A_1 = \Sigma \delta m y'^2 \left(\frac{\phi}{\phi_r}\right)^2$$

$$= \int_0^{s'} m_0 \sec^3 \beta \,.\, c^2 \cos^2 \beta \,.\, l'^2 \mathrm{f}^2 \, dy' = \int_0^{10/7} m_0 c^2 l^3 \mathrm{f}^2 \sec^4 \beta \, d\eta$$

$$= \frac{\rho l^3 c_0^2 a_1}{\varepsilon_0}$$

where $$a_1 = \frac{m_0}{\sigma_{\omega 0}} \int_0^{10/7} \left(\frac{c}{c_0}\right)^2 \mathrm{f}^2 \sec^4 \beta \, d\eta$$

$$\varepsilon_0 = \frac{\rho}{\sigma_{\omega 0}} \text{ and } y' = \eta l'.$$

Similarly $$A_3 = G_1 = \Sigma \delta m \, x'y' \left(\frac{\theta}{\theta_r}\right)\left(\frac{\phi}{\phi_r}\right)$$

$$= \int_0^{s'} m_0 \sec^3 \beta \,.\, c^2 \cos^2 \beta \,.\, l' \,\mathrm{fF} \,.\, jc \cos \beta \, dy'$$

$$= \int_0^{10/7} m_0 c^3 l'^2 j \mathrm{fF} \sec^2 \beta \, d\eta = \frac{\rho l^2 c_0^3 a_3}{\varepsilon_0}$$

where $$a_3 = g_1 = j \frac{m_0}{\sigma_{\omega 0}} \int_0^{10/7} \left(\frac{c}{c_0}\right)^3 \mathrm{fF} \sec^2 \beta \, d\eta$$

and the centre of inertia of any section is distance $j\bar{c}$ behind the flexural axis.

Also $$G_3 = \Sigma \delta m \, x'^2 \left(\frac{\theta}{\theta_r}\right)^2$$

$$= \int_0^{s'} m_0 \sec^3 \beta \,.\, c^2 \cos^2 \beta \,.\, \mathrm{F}^2 \,.\, \lambda_1^2 c^2 \cos^2 \beta \, dy'$$

$$= \int_0^{10/7} m_0 c^4 \lambda_1^2 l \mathrm{F}^2 \, d\eta = \frac{\rho l c_0^4 g_3}{\varepsilon_0}$$

where $$g_3 = \lambda_1^2 \frac{m_0}{\sigma_{\omega 0}} \int_0^{10/7} \left(\frac{c}{c_0}\right)^4 \mathrm{F}^2 \, d\eta$$

and $$\lambda_1^2 = k^2 + j^2.$$

Thus a_1 varies as $\sec^4 \beta$, a_3 $(= g_1)$ as $\sec^2 \beta$ and g_3 is independent of β.

The Aerodynamic Coefficients.—We consider the forces acting on a chordwise strip of the wing (parallel to the line of flight). The geometrical angle of incidence α and the downward displacement of the leading edge of this chordwise strip are given by

$$\alpha = \theta \cos \beta + \psi \sin \beta$$

$$z = \phi \eta l' - \theta h c \cos \beta$$

where ψ is the slope of the flexural axis at the section considered, and any chordwise change of camber has been neglected.

$$\phi = \phi_r \mathrm{f}/\eta$$

$$\psi = \phi_r \, \partial \mathrm{f}/\partial \eta = \phi_r \mathrm{f}'$$

$$\theta = \theta_r \mathrm{F}$$

f and F being functions of $\eta = y/l$.

For the aerofoil element, the lift and moment coefficients referred to the leading edge are given by

$$C_L = \alpha \frac{\partial C_L}{\partial \alpha} + \dot{\alpha} \frac{\partial C_L}{\partial \dot{\alpha}} + \dot{z} \frac{\partial C_L}{\partial \dot{z}}$$

$$Cz = \alpha \frac{\partial C_m}{\partial \alpha} + \dot{\alpha} \frac{\partial C_m}{\partial \dot{\alpha}} + \dot{z} \frac{\partial C_m}{\partial \dot{z}}$$

where α is the geometric angle of incidence and $\dot{z}$ is the downward velocity of the leading edge.

In the standard notation, the downward normal force is given by

$$\begin{aligned} \delta Z &= - \tfrac{1}{2}\rho V^2 c \, \delta C_L l \, d\eta \\ &= - \rho V c l(\alpha V l_a + \dot{z} l_{\dot{z}} + \dot{\alpha} c l_{\dot{a}}) \, d\eta . \end{aligned}$$

Therefore, substituting for α, $\dot{\alpha}$ and $\dot{z}$,

$$\begin{aligned} - \frac{\delta Z}{\rho V l c \, d\eta} &= (\theta_r \mathrm{F} \cos \beta + \phi_r \mathrm{f}' \sin \beta) V l_a + (\dot{\phi}_r l' \mathrm{f} - \dot{\theta}_r h c \mathrm{F} \cos \beta) l_{\dot{z}} \\ &\quad + (\dot{\theta}_r \mathrm{F} \cos \beta + \dot{\phi}_r \mathrm{f}' \sin \beta) l_{\dot{a}} c \\ &= \theta_r \mathrm{F} \cos \beta \, . \, V l_a + \phi_r \mathrm{f}' \sin \beta \, . \, V l_a + \dot{\theta}_r (\mathrm{F} \cos \beta \, . \, l_{\dot{a}} c - h c \mathrm{F} l_{\dot{z}} \cos \beta) \\ &\quad + \dot{\phi}_r (\mathrm{f}' \sin \beta \, . \, l_{\dot{a}} c + l' \mathrm{f} l_{\dot{z}}). \end{aligned}$$

Similarly if δM is the pitching moment on the strip, about the flexural centre,

$$\begin{aligned} \delta M &= \tfrac{1}{2}\rho V^2 l c^2 (\delta C_m + h \delta C_L) \, d\eta' \\ &= \rho V l c^2 (\alpha V m_a + \dot{z} m_{\dot{z}} + \dot{\alpha} c m_{\dot{a}} + h \alpha V l_a + h \dot{z} l_{\dot{z}} + h \dot{\alpha} c l_{\dot{a}}) \, d\eta' . \end{aligned}$$

Substituting for α, $\dot{\alpha}$ and z,

$$\begin{aligned} \frac{\delta M}{\rho V l c^2 \, d\eta'} &= (\theta_r \mathrm{F} \cos \beta + \phi_r \mathrm{f}' \sin \beta)\,(V m_a + h V l_a) \\ &\quad + \dot{\phi}_r l' \mathrm{f} (m_{\dot{z}} + h l_{\dot{z}}) - \dot{\theta}_r h c \mathrm{F} (m_{\dot{z}} + h l_{\dot{z}}) \cos \beta \\ &\quad + (\dot{\theta}_r \mathrm{F} \cos \beta + \dot{\phi}_r \mathrm{f}' \sin \beta)\,(m_{\dot{a}} c + h c l_{\dot{a}}). \end{aligned}$$

Considering the work done in a given displacement,

let δL_a be the increment in the flexural moment

and δM_a the increment in the torsional moment.

Then $\quad \delta L_a = l\mathrm{f} \, \delta Z \sec \beta + \mathrm{f}' \sin \beta \, \delta M'$

$\qquad \delta M_a = \mathrm{F} \, \delta M' \cos \beta .$

Therefore $\dfrac{L_a}{\rho V l^2 c_0} = -\sec\beta \displaystyle\int \mathrm{f}\,\frac{c}{c_0}\Big\{\theta_r \mathrm{F}\cos\beta\,.\,Vl_a + \phi_r \mathrm{f}'\sin\beta\,.\,Vl_a$

$+\,\dot\theta_r\,(\mathrm{F}\cos\beta\,.\,l_{\dot a}c - hc\cos\beta\,.\,\mathrm{F}l_{\dot z})$

$+\,\dot\phi_r\,(\mathrm{f}'\sin\beta\,.\,l_{\dot a}c + l'\mathrm{f}l_{\dot z})\Big\}\,d\eta'$

$+\,\dfrac{c_0}{l}\sin\beta\displaystyle\int\left(\frac{c}{c_0}\right)^2\mathrm{f}'\Big\{\theta_r\mathrm{F}\cos\beta\,(Vm_a + hVl_a) + \phi_r\mathrm{f}'\sin\beta\,(Vm_a + hVl_a)$

$+\,\dot\theta_r\,[\mathrm{F}\cos\beta\,(m_{\dot a}c + hcl_{\dot a}) - hc\mathrm{F}\cos\beta\,(m_{\dot z} + hl_z)]$

$+\,\dot\phi_r\,[\mathrm{f}'\sin\beta\,(m_{\dot a}c + hcl_{\dot a}) + l'\mathrm{f}\,(m_{\dot z} + hl_{\dot z})]\Big\}\,d\eta',$

and $\dfrac{M_a}{\rho V l c_0^2} = \cos\beta\displaystyle\int \mathrm{F}\left(\frac{c}{c_0}\right)^2\Big\{\theta_r\mathrm{F}\cos\beta\,(Vm_a + hVl_a) + \phi_r\mathrm{f}'\sin\beta\,(Vm_a + hVl_a)$

$+\,\dot\theta_r\,[\mathrm{F}\cos\beta\,(m_{\dot a}c + hcl_{\dot a}) - hc\mathrm{F}\cos\beta\,(m_{\dot z} + hl_{\dot z})]$

$+\,\dot\phi_r\,[\mathrm{f}'\sin\beta\,(m_{\dot a}c + hcl_{\dot a}) + l'\mathrm{f}\,(m_{\dot z} + hl_{\dot z})]\Big\}\,d\eta'.$

Now $L_a = \theta_r L_\theta + \phi_r L_\phi + \dot\theta_r L_{\dot\theta} + \dot\phi_r L_{\dot\phi}$
and $M_a = \theta_r M_\theta + \phi_r M_\phi + \dot\theta_r M_{\dot\theta} + \dot\phi_r M_{\dot\phi}$.

Therefore the non-dimensional aerodynamic coefficients are given by

$$b_1 = -\frac{L_{\dot\phi}}{\rho V l^3 c_0} = \int_0^{10/7} \mathrm{f}\,\frac{c}{c_0}\left[\mathrm{f}l_{\dot z}\sec^2\beta + \mathrm{f}'\tan\beta\,.\,l_{\dot a}\,\frac{c}{l}\right]d\eta'$$
$$-\frac{c_0}{l}\int_0^{10/7}\mathrm{f}'\left(\frac{c}{c_0}\right)^2\Big[\mathrm{f}\tan\beta\,(m_{\dot z} + hl_{\dot z})$$
$$+\frac{c}{l}\,\mathrm{f}'\sin^2\beta\,(m_{\dot a} + hl_{\dot a})\Big]\,d\eta'$$

$$c_1 = -\frac{L_\phi}{\rho V^2 l^3} = \frac{c_0}{l}\int_0^{10/7}\mathrm{f}\mathrm{f}'\,\frac{c}{c_0}\tan\beta\,.\,l_a\,d\eta'$$
$$-\left(\frac{c_0}{l}\right)^2\sin^2\beta\int_0^{10/7}\mathrm{f}'^2\left(\frac{c}{c_0}\right)^2(m_a + hl_a)\,d\eta'$$

$$j_1 = -\frac{L_{\dot\theta}}{\rho V l^2 c_0^2} = \int_0^{10/7}\mathrm{f}\left(\frac{c}{c_0}\right)^2(\mathrm{F}l_{\dot a} - h\mathrm{F}l_{\dot z})\,d\eta'$$
$$-\frac{c_0}{l}\sin\beta\cos\beta\int_0^{10/7}\left(\frac{c}{c_0}\right)^3\mathrm{F}\mathrm{f}'\Big[(m_{\dot a} + hl_{\dot a}) - h\,(m_{\dot z} + hl_{\dot z})\Big]\,d\eta'$$

$$k_1 = -\frac{L_\theta}{\rho V^2 l^2 c_0} = \int_0^{10/7} l_a\mathrm{F}\mathrm{f}\,\frac{c}{c_0}\,d\eta' - \frac{c_0}{l}\sin\beta\int_0^{10/7}\mathrm{f}'\left(\frac{c}{c_0}\right)^2\mathrm{F}\cos\beta\,(m_a + hl_a)\,d\eta'$$

$$b_3 = -\frac{M_{\dot\phi}}{\rho V l^2 c_0^2} = -\int_0^{10/7}\mathrm{F}\left(\frac{c}{c_0}\right)^2\left[\mathrm{f}\,(m_{\dot z} + hl_{\dot z}) + \mathrm{f}'\sin\beta\cos\beta\,(m_{\dot a} + hl_{\dot a})\,\frac{c}{l}\right]d\eta'$$

$$c_3 = -\frac{M_\phi}{\rho V^2 l^2 c_0} = -\frac{c_0}{l}\sin\beta\cos\beta\int_0^{10/7} F\left(\frac{c}{c_0}\right)^2 f'\,(m_a + hl_a)\,d\eta'$$

$$j_3 = -\frac{M_{\dot\theta}}{\rho V l c_0^3} = -\cos^2\beta\int_0^{10/7} F^2\left(\frac{c}{c_0}\right)^3\left[m_{\dot a} + hl_a - h\,(m_{\dot z} + hl_{\dot z})\right]d\eta'$$

$$k_3 = -\frac{M_\theta}{\rho V^2 l c_0^2} = -\cos^2\beta\int_0^{10/7} F^2\left(\frac{c}{c_0}\right)^2 (m_a + hl_a)\,d\eta'.$$

It is to be noted that c_1 and c_3 are not in general zero for a swept-back wing.
For the assumed modes of flexure and torsion,

$$f = \eta^2, \qquad f' = 2\eta, \qquad F = \eta.$$

Derivation of Critical Flutter Speed.—As in R. & M. 1782[1] and 1869[2] the equations of motion are

$$A_1\ddot\phi_r + B_1\dot\phi_r + C_1\phi_r + G_1\ddot\theta_r + J_1\dot\theta_r + K_1\theta_r = 0$$
$$A_3\ddot\phi_r + B_3\dot\phi_r + C_3\phi_r + G_3\ddot\theta_r + J_3\dot\theta_r + K_3\theta_r = 0$$

Let $\phi_r = \Phi e^{\lambda t}, \qquad \theta_r = \Theta e^{\lambda t}.$

Substituting and eliminating Θ, Φ we get

$$(a_1\lambda'^2 + b_1\sqrt{\varepsilon_0}\lambda' + X)\,(g_3\lambda'^2 + j_3\sqrt{\varepsilon_0}\lambda' + Y)$$
$$-\,(g_1\lambda'^2 + j_1\sqrt{\varepsilon_0}\lambda' + k_1)\,(a_3\lambda'^2 + b_3\sqrt{\varepsilon_0}\lambda' + c_3) = 0$$

where $\lambda' = \lambda c_0/V\sqrt{\varepsilon_0}$

$$X = \frac{C_1}{\rho V^2 l^3} = \frac{l_\phi}{\rho V^2 l^3} + c_1 = X_c' + c_1$$

$$Y = \frac{K_3}{\rho V^2 l c_0^2} = \frac{m_\theta}{\rho V^2 l c_0^2} + k_3 = Y_c' + k_3$$

i.e., $q_0\lambda'^4 + q_1\lambda'^3 + q_2\lambda'^2 + q^3\lambda' + q_4 = 0$

where
$$q_0 = a_1 g_3 - a_3 g_1$$
$$q_1 = (a_1 j_3 - a_3 j_1 + b_1 g_3 - b_3 g_1)\sqrt{\varepsilon_0}$$
$$q_2 = [a_1 Y - a_3 k_1 + (b_1 j_3 - b_3 j_1)\varepsilon_0 + X g_3 - c_3 g_1]$$
$$q_3 = (b_1 Y - b_3 k_1 + X j_3 - c_3 j_1)\sqrt{\varepsilon_0}$$
$$q_4 = XY - c_3 k_1.$$

The test function is $T_3 = q_1 q_2 q_3 - q_0 q_3^2 - q_1^2 q_4$

$T_3 = 0$ at the critical flutter speed V_c.

$q_4 = 0$ is the condition for wing divergence.

Estimation of the Aerodynamic Coefficients.—Using R. & M. 1782[1] amd 1869[2], the aerodynamic coefficients for incompressible flow over an unswept wing are given by

$$l_{\dot z} = 1\cdot5, \qquad l = 1\cdot4, \qquad l = 1\cdot6$$
$$-m_{\dot z} = 0\cdot375, \qquad -m_{\dot a} = 0\cdot7, \qquad -m_a = 0\cdot4$$

The values of these coefficients have been derived from experimentally determined derivatives for a wing of finite span. The effect of variation of the frequence parameter with sweep-back has been neglected.

The aerodynamic acceleration coefficients have also been neglected in comparison with the structural inertia coefficients.

For the calculation of wing divergence speeds, quasi-static values of the derivatives are used.

There is very little experimental data on the variation of the derivatives with sweep-back and with Mach number. For incompressible flow we assume that the coefficients vary as $\cos \beta$; for the swept wing, applying the Glauert correction as for the quasi-static condition, the derivatives are multiplied by the factor

$$\frac{\cos \beta}{(1 - M^2)^{1/4} (1 - M^2 \cos^2 \beta)^{1/4}}$$

Results.—The calculations were performed for a wing of aspect ratio 5 and taper ratio $\frac{1}{2}$. For the unswept wing, ε_0 was taken as $0 \cdot 10$, giving a wing density of $0 \cdot 765$ lb/cu ft. The flexural axis was taken at $0 \cdot 4$ chord and the inertia axis at (i) $0 \cdot 5$ chord, (ii) $0 \cdot 4$ chord. The sweep-back of the flexural axis was varied from $+$ 60 deg to $-$ 60 deg. Thus the geometric plan forms of the swept wings are obtained by shearing the unswept wing.

The non-dimensional critical speed coefficient

$$B = \frac{V_c \sqrt{\rho}}{\sqrt{(m_\theta / d c_m^2)}}$$

is plotted for various angles of sweep-back and sweep-forward, showing the critical flutter speed and the critical speed for wing divergence.

Curves are drawn for two values of the non-dimensional stiffness ratio

$$r = \frac{l_\phi / d^3}{m_\theta / d c_m^2} .$$

Figs. 2 and 3 are drawn for incompressible flow; Figs. 4 and 5 for compressible flow ($M = 0 \cdot 8$).

Conclusions.—Critical Flutter Speed.—Effect of Sweep-back and Sweep-forward.—From Figs. 2, 3, 4 and 5 we see that the minimum flutter speed occurs for sweep-back angles of 5 to 20 deg. For highly swept-back or swept-forward wings the flutter speed is double that for unswept wings with the same wing stiffness. (*NOTE: In these calculations we have neglected the effect of any rigid body freedoms of the aircraft, e.g.*, pitch and vertical translation. Recent theoretical and experimental work[3] has shown that when these body freedoms are neglected, the calculated flutter speed is liable to be seriously over-estimated. The calculations in this report can be applied to an aircraft for which the fuselage is relatively heavy compared with the wings. For such an aircraft, both the inertia effect of the fuselage and damping due to the tailplane tend to suppress the body freedoms in pitch and vertical translation).

Effect of Change of Flexural Stiffness l_ϕ and Torsional Stiffness m_θ.—The curves have been plotted against the non-dimensional parameter B for two values of the non-dimensional stiffness ratio r. Thus if the ratio of the stiffnesses is kept constant, the critical flutter speed is proportional to $\sqrt{m_\theta}$, and thus increases as the torsional stiffness increases. Over the range of stiffness ratios considered ($r = 1$ to 2) the critical flutter speed is increased slightly when the flexural stiffness is decreased.

Effect of Variation of the Position of the Inertia Axis.—The critical flutter speed increases rapidly as the inertia axis is moved forward. The effect is less beneficial with highly swept-back wings.

Effect of Compressibility.—In general, at a Mach number of $0 \cdot 8$, the critical flutter speed is some 15 per cent lower than in the incompressible case. For wings of lower densities of loading the reduction in critical flutter speed would be smaller.

Wing Divergence.—Effect of Sweep-back and Sweep-forward.—Wing divergence is not important for swept-back wings. The reverse is true for swept-forward wings, where for angles of sweep greater than 5 to 15 deg wing divergence will occur at a lower speed than the critical flutter speed.

Effect of Change of Flexural Stiffness l_ϕ and Torsional Stiffness m_θ.—As in the case of flutter, if the ratio of the stiffnesses is kept constant, the divergence speed is proportional to $\sqrt{m_\theta}$, and thus increases as the stiffness increases. For highly swept-forward wings, the wing divergence speed is almost independent of the torsional stiffness, while for unswept wings the divergence speed is independent of the flexural stiffness.

Effect of Variation of the Position of the Inertia Axis.—The wing divergence speed is unaffected by a change in the position of the inertia axis, the flexural axis remaining fixed.

Effect of Compressibility.—At a Mach number of $0 \cdot 8$, the critical speed for wing divergence is 15 to 20 per cent lower than in the incompressible case.

General Conclusions.—From the above results it is seen that the critical flutter speed is in general higher for a swept-back wing; for a swept-forward wing, divergence will occur before flutter.

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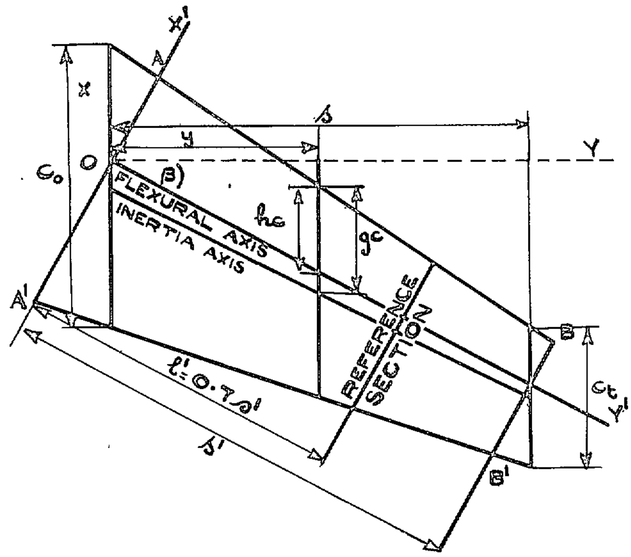

ASPECT RATIO 5

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FIG. 1. Wing plan form.

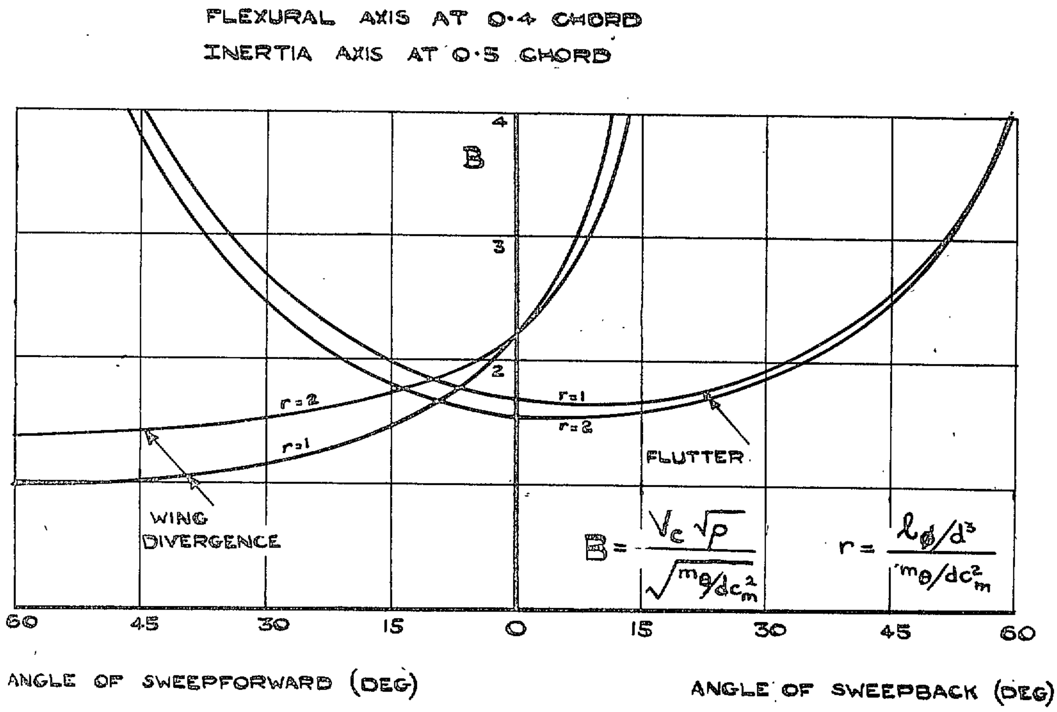

FIG. 2. Incompressible flow.

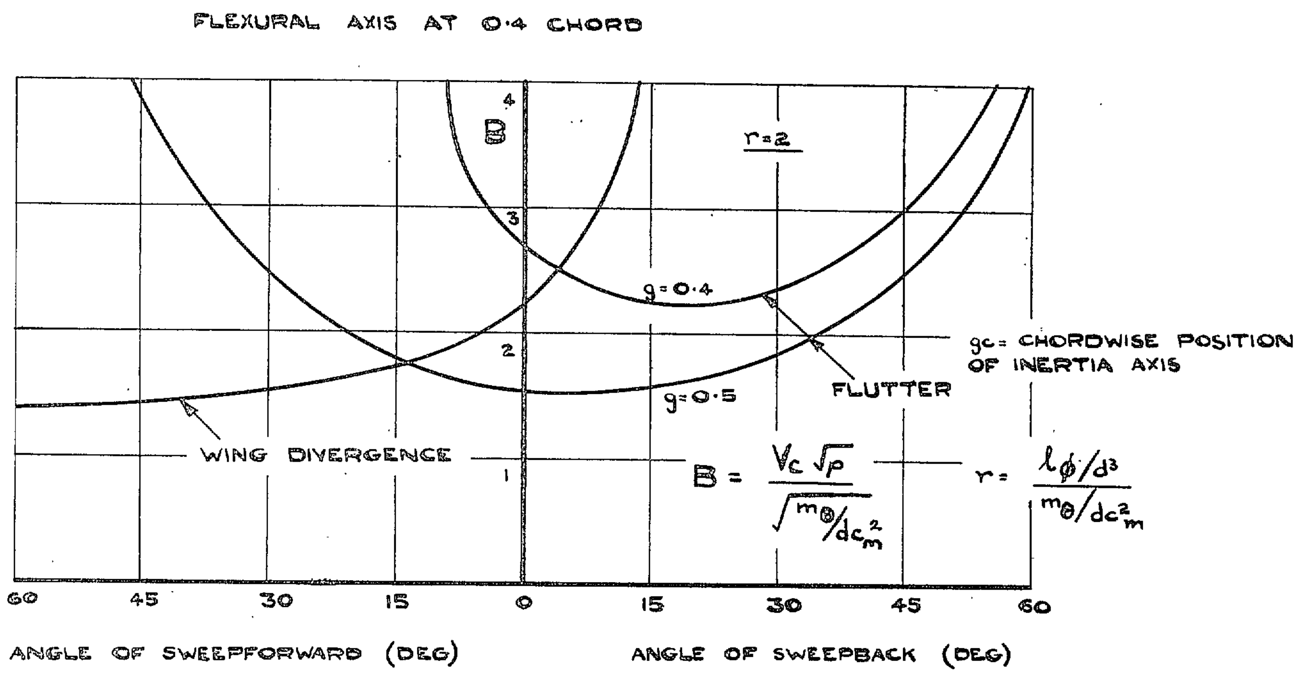

FIG. 3. Incompressible flow.

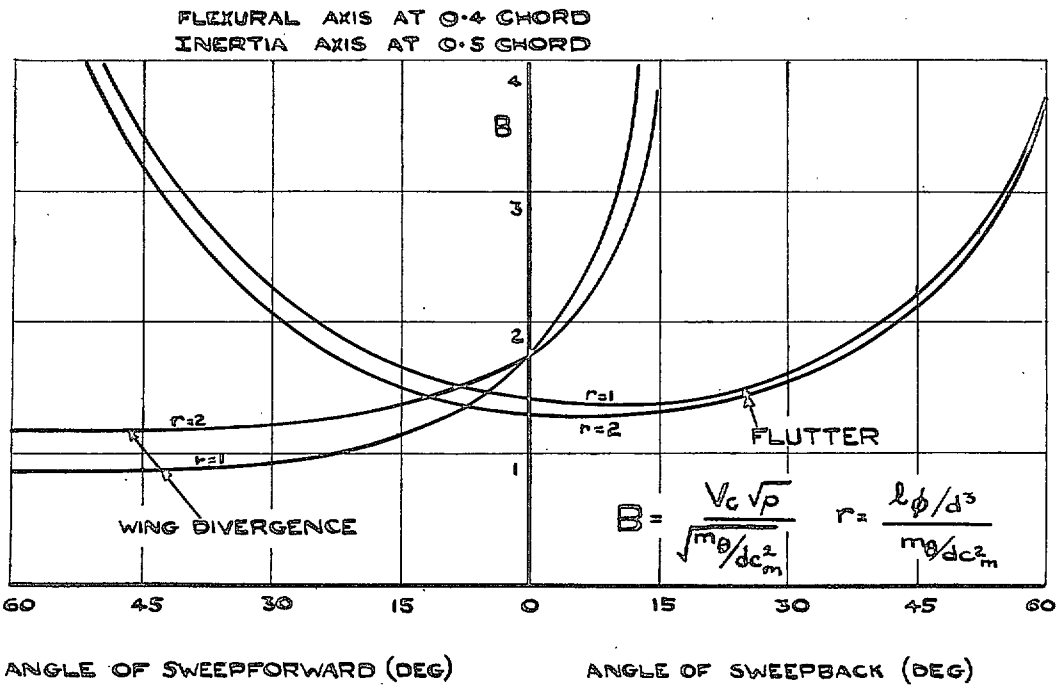

FIG. 4. Compressible flow; $M = 0{\cdot}8$.

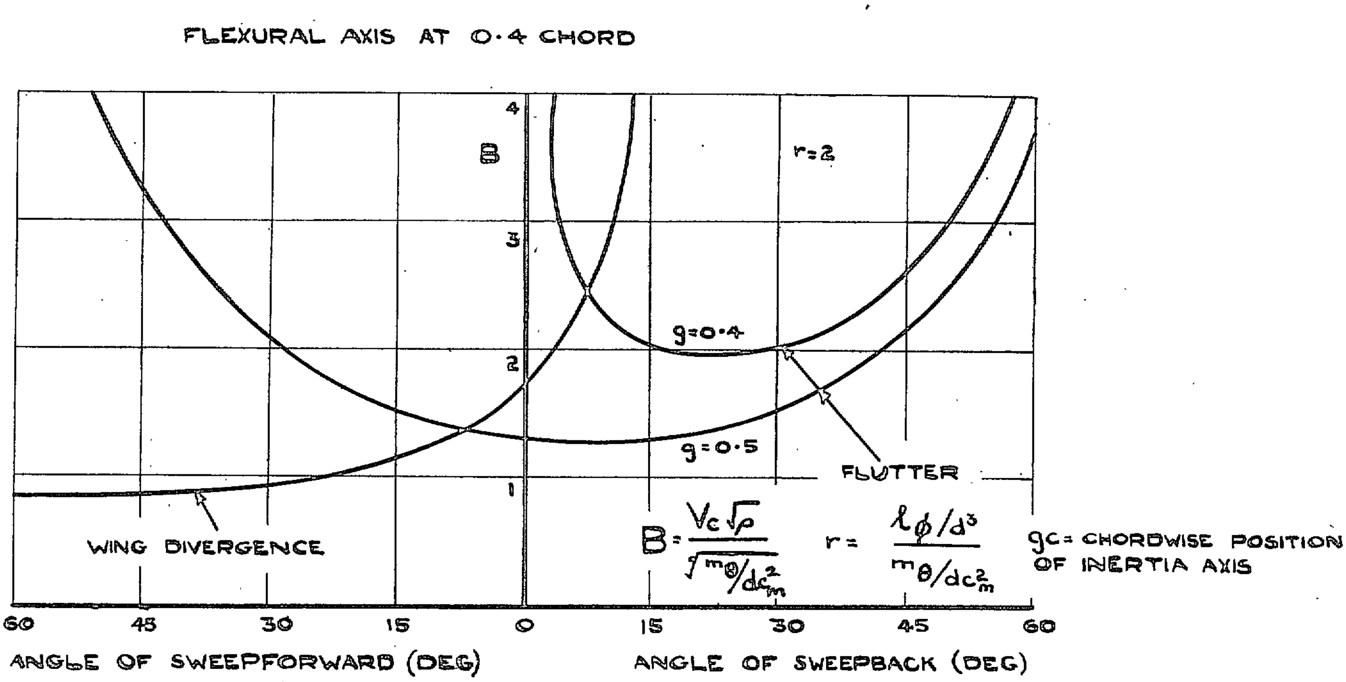

FIG. 5. Compressible flow; $M = 0{\cdot}8$.

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