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Notation for Rotorcraft Work

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NOTATION FOR ROTORCRAFT WORK

compiled by

A. Armitage

SUMMARY

A suggested standard notation for rotorcraft work is presented. The notation is the outcome of study of numerous papers of many sources of origin.

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Introduction

In addition to the References listed, the notations employed in a large number of British, Continental and American papers have been carefully examined. In compiling the notation, the aims have been:

(i) to employ symbols already in accepted use, except where this would lead to confusion,

(ii) where feasible, to adhere as closely as possible to standard aerodynamic and fixed-wing notation,

(iii) throughout, to make a clear distinction, by definition and appropriate symbols, between quantities that have different values according to the axis of reference and "helicopter" or "autogiro" notation used. Failure to distinguish between such quantities leads to complete confusion (Ref.8),

(iv) to keep the list of symbols as short as possible, for general application. The list presented here does not attempt to cover various specialised cases, e.g. convertiplanes.

Part 1. General Symbols and Physical Quantities.

Symbol	Quantities and Notes	Units
a	slope of lift curve, c_ℓ / α_r	
A	disc area of one rotor, πR^2	$ft.^2$
A_p	total projected disc area (multi-rotor aircraft)	$ft.^2$
b	number of blades per rotor	
B	rotor blade tip-loss factor	
c	rotor blade chord (general symbol)	ft.
c_e	equivalent blade chord (thrust basis) $c_e = \int_0^R cr^2 / \int_0^R r^2 dr$	ft.
$\bar{\bar{c}}$	equivalent blade chord (torque basis) $\bar{\bar{c}} = \int_0^R cr^3 / \int_0^R r^3 dr$ This symbol is rarely encountered, but is retained here for the sake of completeness.	ft.
c_o	blade chord at root, projected to rotor axis	ft.
c_r	blade chord at radius r ft.	ft.
c_R	blade chord at tip	ft.
c_x	blade chord at radius ratio x	ft.
c_d	blade element profile drag coefficient c_d is used to denote the profile drag coefficient at a blade element, in distinction from C_D which denotes the drag coefficient for the whole rotor or wing. c_{d_o} is the constant term in the profile drag power series, e.g. $c_d = c_{d_o} + c_{d_1} \alpha_r + c_{d_2} \alpha_r^2$	
c_ℓ	blade element lift coefficient	
$\overline{c_\ell}$	average blade lift coefficient in hovering	
D	drag (general symbol)	lb.
D_f	parasite drag of rotorcraft less main rotors	lb.
D_{100}	D_f at V = 100 $ft.sec.^{-1}$	lb.

Symbol	Quantities and Notes	Units
eR	flapping hinge offset; distance of flapping hinge from centre of rotor hub	ft.
fR	lag hinge offset; distance of lag hinge from centre of rotor hub	ft.
F_c	centrifugal force of one rotor	lb.
g	acceleration due to gravity	$ft.sec.^{-2}$
hR	general symbol to denote height above a reference point	ft.
	It is suggested that this symbol be used with suitable subscripts which should be defined in the text. Such subscripts should indicate the particular height dimension being measured and also the reference point.	
H	longitudinal force at the rotor in the plane of the disc	lb.
HP	general symbol for horsepower	horse power
	See P for appropriate subscripts	
I	moment of inertia (general symbol)	$slugs.ft.^2$
I_b	moment of inertia of a blade about its flapping hinge	$slugs.ft.^2$
ℓR	general symbol to denote distance in a substantially horizontal plane, measured from a reference point	ft.
	See note under hR above.	
L	lift (general symbol)	lb.
L	rolling moment	lb.ft.
m	mass (general symbol)	slugs.
m_b	total mass of a rotor blade to its flapping hinge	slugs.
M	pitching moment	lb.ft.
M	Glauert's Figure of Merit, $C_T^{3/2} / C_Q$	
	Although this quantity has been largely superseded by other merit criteria, it is retained here since it appears in most general basic rotary wing papers.	
M	Mach number	
M_b	weight moment of a rotor blade about its flapping hinge	lb.ft.
M_f	pitching moment of fuselage	lb.ft.
n	number of rotors per aircraft	
	This refers to the number of main rotors.	
N	yawing moment	lb.ft.

Symbol	Quantities and Notes	Units
N	general symbol for revolutions per minute the symbol n is sometimes used to denote rev.sec.$^{-1}$	rev.min.$^{-1}$
P	power (general symbol)	ft.lb.sec.$^{-1}$
P_i	rotor induced power	ft.lb.sec.$^{-1}$
P_p	total rotor profile drag power, inclusive of power due to rotor H force	ft.lb.sec.$^{-1}$
P_f	parasite drag of rotorcraft less main rotor(s)	ft.lb.sec.$^{-1}$
P_E	power available at power plant output	ft.lb.sec.$^{-1}$
P_R	total power required at rotor ($P_R = \eta P_E$)	ft.lb.sec.$^{-1}$
Q	rotor shaft torque	lb.ft.
r	distance of blade element from centre of rotor	ft.
R	radius of rotor	ft.
s	blade solidity ratio, $bc_e / \pi R$	
s_a	blade solidity ratio, $\frac{\text{actual geometrical blade area}}{\text{disc area}}$	
s_p	blade solidity ratio based on total projected disc area	
s_x	local solidity ratio at radius ratio x	
S	area of fixed wing	ft.^{2}
t	time	secs.
T	rotor thrust, normal to rotor disc.	lb.
T	air temperature	deg. C.
V	velocity of flight along flight path, positive for forward flight	ft.sec.$^{-1}$
V_c	rate of climb The symbol RC or R/C is sometimes used to denote rate of climb in ft. min.$^{-1}$	ft.sec.$^{-1}$
V_T	rotor tip speed The use of ΩR is preferred for this quantity	ft.sec.$^{-1}$
w	disc loading, W/nA	lb.ft.$^{-2}$
w_p	disc loading based on total projected area, W/A_p	lb.ft.$^{-2}$
W	weight of complete rotorcraft	lb.
x	blade element radius ratio, r/R	
Y	lateral force (general symbol)	lb.
γ	mass constant of blade, $c_e \rho a R^4 / I_b$	
δ	average blade profile drag coefficient	

Symbol	Quantities and Notes	Units
η	ratio of power available at main rotor(s) to power available at power plant output	
κ	angle between plane of symmetry and rotor shaft, i.e. lateral tilt of rotor shaft	rad.
μ_d	tip speed ratio relative to rotor disc, $V \cos \alpha_d / \Omega R$	
μ_{nf}	tip speed ratio relative to plane of no feathering, $V \cos \alpha_{nf} / \Omega R$; $\mu_{nf} \simeq \mu_d$	
μ_s	tip speed ratio relative to rotor shaft axis, $V \cos \alpha_s / \Omega R$	
μ'	approximate tip speed ratio, $V/\Omega R$	
ρ	air density	slugs.ft.$^{-3}$
ρ_o	standard air density at sea level, ICAO	slugs.ft.$^{-3}$
σ	relative density of air, ρ / ρ_o	
Ψ	blade azimuth angle, measured in the direction of rotation (rotor viewed from above), and measured between the downwind position and a line passing through the centres of the rotor hub and the lag hinge.	rad.
Ω	rotational velocity of rotor	rad.sec.$^{-1}$

Subscripts

d	relative to rotor disc
nf	relative to plane of no feathering
s	relative to rotor shaft axis
t	tail rotor

Coefficients

Many coefficients are used referred to the rotor. In the coefficients given below the reference area is that of the rotor disc, and the reference length is the rotor radius

C_{D_f}	parasite drag coefficient	$D_f / \frac{1}{2}\rho AV^2$
C_L	lift coefficient	$L / \frac{1}{2}\rho AV^2$
C_ℓ	rolling moment coefficient	$L / \frac{1}{2}\rho AV^2 R$
C_{m_f}	fuselage pitching moment coefficient	$M_f / \frac{1}{2}\rho AV^2 R$
C_n	yawing moment coefficient	$N / \frac{1}{2}\rho AV^2 R$
C_p	rotor power coefficient	$P / \rho A(\Omega R)^3$
C_Q	rotor torque coefficient	$Q / \rho A(\Omega R)^2$
C_y	lateral force coefficient	$Y / \frac{1}{2}\rho AV^2$

Other parameters

Certain other parameters, useful for expressing rotorcraft performance, and used fairly widely, are given below for reference.

$\left(\frac{D}{L}\right)_c$ Drag/lift ratio representing angle of climb, positive for climb.

$\left(\frac{D}{L}\right)_f$ parasite drag of rotorcraft less main rotor(s) dividend by rotor lift

$\left(\frac{D}{L}\right)_i$ rotor induced drag/lift ratio

$\left(\frac{D}{L}\right)_p$ rotor profile drag/lift ratio

$\left(\frac{D}{L}\right)_r$ rotor drag/lift ratio, ratio of equivalent drag of rotor to rotor lift, i.e.

$$\left(\frac{D}{L}\right)_i + \left(\frac{D}{L}\right)_p$$

$\left(\frac{D}{L}\right)_t$ equivalent drag contribution of tail rotor divided by main rotor lift

$\left(\frac{D}{L}\right)_u$ component of rotor resultant force along flight path (useful component of rotor resultant force) divided by rotor lift, i.e.

$$\left(\frac{D}{L}\right)_f + \left(\frac{D}{L}\right)_c + \left(\frac{D}{L}\right)_t$$

$\left(\frac{P}{L}\right)$ shaft power parameter, where P is equal to rotor shaft power divided by velocity along flight path, and is therefore equal to the drag force equivalent to the shaft power at the velocity of flight, i.e.

$$\left(\frac{D}{L}\right)_r + \left(\frac{D}{L}\right)_u$$

Part 2. Parameters at the Rotor Disc (Figures 1 and 2)

Considerable confusion can arise if the signs of the various quantities are not clearly defined. Some authors, particularly in earlier works, use the original autogiro notation based on rearward tilt of the disc and on flow upward through the disc. Others employ the so-called "helicopter" notation, based on forward tilt of the disc and on flow downward through it.

Here, the former notation is observed, i.e. velocities are positive for flow upwards through the disc and angles at the disc are positive for rearward tilt of the disc. Not only does this notation fall into line with the bulk of British and U.S.A. rotary wing literature, but it is also consistant with fixed-wing practice.

Symbol	Quantity and Notes	Units
a_1	tilt of rotor disc relative to no feathering axis, positive for rearward tilt	rad.
a_{1_s}	tilt of rotor disc relative to rotor shaft axis, positive for rearward tilt	rad.
b_1	lateral tilt of rotor disc relative to no feathering axis, positive for tilt to starboard	rad.
B_{1_s}	forward tilt of no feathering axis relative to rotor shaft axis, $(a_1 - a_{1_s})$.	rad.
u	total velocity normal to rotor disc, $(V \sin\alpha_d + v)$, negative for flow downward through disc (power-on flight)	$ft.sec.^{-1}$
v	induced velocity at rotor disc, negative for flow downward through disc.	$ft.sec.^{-1}$
v_h	induced velocity at rotor disc in hovering, assuming uniform distribution and referred to the rotor disc; negative for flow downward through disc. $v_h = (T / 2\pi\rho R^2)^{\frac{1}{2}}$	$ft.sec.^{-1}$
V'	resultant velocity at rotor disc, $(V^2 \cos^2\alpha_d + u^2)^{\frac{1}{2}}$	$ft.sec.^{-1}$
α_d	tilt of rotor disc relative to flight path, negative for forward tilt (flow downward through disc)	rad.
α_{nf}	tilt of no feathering axis relative to flight path, negative for forward tilt	rad.
α_s	tilt of rotor shaft axis relative to flight path, negative for forward tilt	rad.
γ	angle between flight path and horizontal, positive for climb	rad.
ϵ	angle between flight path and rotor wake, positive for downwash	rad.
λ	inflow ratio relative to rotor disc, $(u/\Omega R)$, positive for flow upward through disc	

Symbol	Quantity and Notes	Units
λ_{nf}	inflow ratio relative to no feathering plane, positive for flow upward through disc $\lambda_{nf} = (V \sin \alpha_{nf} + v) / \Omega R$ $\lambda_{nf} \simeq \lambda + \mu_d a_1$. (See figure 2)	

Part 3. Airflow Parameters at a Blade Element. (Figure 3)

The planes of reference here may be either the plane of no feathering or the plane of the rotor disc (tip path plane). It is essential that the plane of reference be clearly defined in the text of any paper.

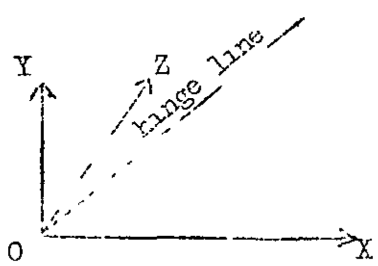
Symbol	Quantities and Notes	Units
U	Resultant velocity at blade element, normal to blade span axis: $U = (U_A{}^2 + U_T{}^2)^{\frac{1}{2}}$	$ft.sec.^{-1}$
U_A	axial component of velocity at blade element, normal to both blade-span axis and U_T, positive for flow upward through disc. The following approximations are given: Relative to plane of the disc: $U_A = u = V \sin \alpha_d + v - a_o \mu_d \cos \psi$. Relative to plane of no feathering: $\frac{U_A}{\Omega R} = \lambda_{nf} - \frac{\dot{\beta}}{\Omega} x - \beta \mu_{nf} \cos \psi$	$ft.sec.^{-1}$
U_T	tangential component of velocity at blade element, normal to both blade-span axis and axis of reference. The following approximations are given: Relative to plane of the disc: $U_T = \Omega r + V \cos \alpha_d . \sin \psi$ Relative to plane of no feathering: $U_T = \Omega r + \mu_{nf} \Omega R \sin \psi$	$ft.sec.^{-1}$
α_r	blade element angle of attack, measured from line of zero lift, $(\theta + \phi)$ It is suggested that the symbol $\alpha_{r(x)(\psi)}$ be used to denote the blade element angle of attack at any radial station and at any blade azimuth angle. Thus, $\alpha_{r(.5)(90)}$ would indicate the blade element angle of attack at 50% radius on	rad.

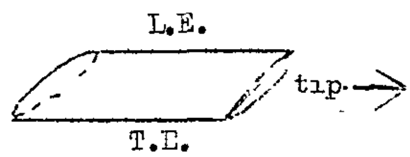
Symbol	Quantities and Notes	Units
θ_o	blade pitch angle at blade root, projected to rotor axis For this quantity, when referred to the tail rotor, the symbol should be θ_{o_t}. For the sake of simplicity, however, this is usually written θ_t.	rad.
θ_R	blade pitch angle at blade tip	rad.
θ_1	difference between tip and (projected) root pitch angles, $(\theta_R - \theta_o)$, positive when tip angle is larger	rad.
ϕ	inflow angle at blade element in plane normal to blade-span axis, $\tan^{-1} \frac{U_A}{U_T}$	rad.

Part 4. Rotor Blade Motion (Figure 4)

Symbol	Quantities and Notes	Units
a_o	constant term in the Fourier series that expresses β (flapping); also, the coning angle in the case of non-flexible blades	rad.
a_n, b_n	coefficient in the Fourier series that expresses β (flapping) (See β below)	
A_o	constant term in the Fourier series that expresses θ (feathering) For untwisted blades, $A_o = \theta_o$	rad.
A_n, B_n	coefficients in the Fourier series that expresses θ (feathering) (See θ below)	
E_o	constant term in the Fourier series that expresses ζ (lagging) (See ζ below)	
E_n, F_n	coefficients in the Fourier series that expresses ζ (lagging)	
$\alpha_{1,2,3}$	angular inclination of lag hinge	rad.
$\delta_{1,2,3}$	angular inclination of flapping hinge	rad.

Notation for these angles is:





angle of projection of hinge line on XY plane to OX = δ_1 or α_1

" " " " " " " YZ " " OY = δ_2 or α_2

" " " " " " " ZX " " OZ = δ_3 or α_3

Angles are positive in the directions X → Y, Y → Z, Z → X.

β	blade flapping angle at particular azimuth position, measured from plane of no feathering; positive for upward flapping	rad.

$$\beta = a_o - a_1 \cos\psi - b_1 \sin\psi - a_2 \cos 2\psi - b_2 \sin 2\psi \ldots$$

For other than simple blades (e.g. for flexible blades), the coefficients a_n, b_n in the above series are variable.

If any other plane of reference is used, this must be clearly indicated and subscripts used for β, a_1, b_1, etc. to distinguish them. If the angle is referred to the rotor shaft axis, the relationship between the equations is.

$$\beta_s = a_o - a_{1_s} \cos \psi - b_{1_s} \sin \psi \ldots$$

where

$$a_{1_s} = a_1 - B_{1_s} \text{, and}$$

$$b_{1_s} = b_1 + A_{1_s} \quad \text{(lateral)}$$

The flapping coefficients with respect to the no feathering axis are given approximately as follows:

$$a_o = \tfrac{1}{2}\gamma \left\{ \frac{\theta}{4} (1 + \mu_{nf}^2) + \frac{\lambda_{nf}}{3} \right\}$$

$$a_1 = \frac{2\mu_{nf} \left(\frac{4}{3} + \lambda_{nf}\right)}{1 - \tfrac{1}{2}\mu_{nf}^2}$$

$$b_1 = \frac{4}{3} \frac{\mu_{nf} a_o}{1 + \tfrac{1}{2}\mu_{nf}^2}$$

When λ is referred to the disc axis, the expressions become:

$$a_o = \tfrac{1}{2}\gamma \left\{ \frac{\theta}{4} (1 + \mu_d^2) - \frac{B_1}{3\mu_d} + \frac{\lambda}{3} \right\}$$

$$a_1 = \frac{2\mu_d \left(\frac{4}{3}\theta + \lambda\right)}{1 + \frac{3}{2}\mu_d^2}$$

$$b_1 = \frac{4}{3} \frac{\mu_d a_o}{1 + \tfrac{1}{2}\mu_d^2}$$

ζ lag angle; measured between span-axis of blade and line passing through centres of rotor hub and lag hinge pin. Positive for lag. rad.

$$\zeta = E_o + E_1 \cos \psi + F_1 \sin \psi + E_2 \cos 2\psi + F_2 \sin 2\psi \ldots$$

θ blade pitch angle, referred to axis of disc rad.

$$\theta = A_o - A_1 \cos \psi - B_1 \sin \psi - A_2 \cos 2\psi - B_2 \sin 2\psi \ldots$$

If the angle θ is referred to the rotor shaft axis, the relation between the equations is:

$$\theta_s = A_o - A_{1_s} \cos \psi - B_{1_s} \sin \psi \ldots$$

Part 5. Stability and Control. (Ref. 7)

The aim here is to adopt a notation and set of stability axes that correspond as closely as possible with fixed-wing aircraft practice and also conform generally with current British and American helicopter notation. Whilst it may yet prove that a different set of axes for the helicopter is more suitable, at present, however, the familiar wind-body axes appear to be quite convenient in helicopter work.

If, as is expected, some of the definitions and notation in fixed-wing aircraft work are changed shortly, it may become necessary to revise helicopter notation to correspond.

Symbol	Quantities	Units
A, B, C	moments of inertia about Gx, Gy, Gz axes respectively	slugs.ft.^{2}
A, B, C, D, E	constants of stability quartic $A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$ (Care should be taken not to confuse these symbols with those for moments of inertia)	
E	product of inertia about Gy axis	slugs.ft.^{2}
L	moment about Gx axis	lb.ft.
M	moment about Gy axis	lb.ft.
N	moment about Gz axis	lb.ft.
p	angular velocity about Gx axis	rad.sec.$^{-1}$
q	angular velocity about Gy axis	rad.sec.$^{-1}$
r	angular velocity about Gz axis	rad.sec.$^{-1}$
t	time	seconds
$\hat{t}$	dimensionless unit of time, $W/g\rho sA\Omega R$	airsec.
u	increment of velocity along flight path	ft.sec.$^{-1}$
v	velocity along Gy axis	ft.sec.$^{-1}$
V	velocity of undisturbed flight	ft.sec.$^{-1}$
w	velocity along Gz axis	ft.sec.$^{-1}$
X	force along Gx axis	lb.
Y	force along Gy axis	lb.
Z	force along Gx axis	lb.
θ	angle of pitch	rad.

Notes on Dynamic Stability of the Helicopter

Assumptions

(1) Disturbances of the helicopter from the equilibrium condition are small enough for only first order derivatives to be considered.

(2) The error in neglecting the coupling between the longitudinal and lateral motions is small, i.e. there is a longitudinal plane of symmetry.

(3) The products of inertia $D \equiv \Sigma myz$ and $F \equiv \Sigma mxy$ are zero.

(4) The extra degree of freedom due to rotor speed variation is neglected.

Definition of Axes

A set of right-handed axes is employed, with the origin at the centre of gravity G. Axes Gx and Gz are always in the plane of symmetry with Gx pointing initially along the direction of steady motion, and Gz pointing approximately downwards. Gy is directed to starboard. After disturbance the axes can be moved to coincide with the original positions by rotations ψ about Gz, θ about the resulting Gy and ϕ about the resulting Gx.

Angular moments and velocities are positive when they rotate the helicopter in the senses $y \rightarrow z$, $z \rightarrow x$, $x \rightarrow y$.

The inertial quantities and instantaneous motion of the helicopter are defined with respect to the axes Gxyz, using the notation of the following table.

TABLE 1

Description	Symbol			Units
Axes	Gx	Gy	Gz	
Moments of inertia	A	B	C	slugs.ft.^{2}
Product of inertia	–	E	–	slugs ft.^{2}
Velocity in steady motion	V	–	–	ft.sec.$^{-1}$
Velocity in disturbed motion	V+u	v	w	ft.sec.$^{-1}$
Disturbed angular velocity	p	q	r	rad.sec.$^{-1}$
Aerodynamic forces	X	Y	Z	lb.
Aerodynamic moments	L	M	N	lb.ft.

Equations of Motion

Longitudinal

$$\frac{W}{g}\dot{u} - X_u u - X_w w - X_q q + W\theta\cos\gamma = X_{B_1} B_1 + X_{\theta_o}\theta_o$$

$$\frac{W}{g}(\dot{w} - Vq) - Z_u u - Z_w w - Z_q q + W\theta\sin\gamma = Z_{B_1} B_1 + Z_{\theta_o}\theta_o$$

$$B\ddot{\theta} - M_u u - M_w w - M_q q = M_{B_1} B_1 + M_{\theta_o}\theta_o$$

$$-q + \frac{d\theta}{dt} = 0$$

Lateral

$$\frac{W}{g}(\dot{v} + V\dot{\psi}) - Y_v v - Y_p p - Y_r r - W\cos\gamma\,(\phi + \psi\sin\gamma) = Y_{A_1} A_1 + Y_{\theta_t}\theta_t$$

$$A\ddot{\phi} - E\ddot{\psi} - L_v v - L_p p - L_r r = L_{A_1} A_1 + L_{\theta_t}\theta_t$$

$$C\ddot{\psi} - E\ddot{\phi} - N_v v - N_p p - N_r r = N_{A_1} A_1 + N_{\theta_t}\theta_t$$

$$-p + \frac{d\phi}{dt} = 0$$

$$-r + \frac{d\psi}{dt} = 0$$

Dynamic stability is most conveniently treated by expressing the quantities concerned in non-dimensional form, by means of the scheme given in Table 2.

TABLE 2

I Units of quantities in II & III	II Quantities	III Divisors to obtain column IV	IV Symbol	V Name
lb.	$X \quad Y \quad Z$	$\rho sA(\Omega R)^2$	$C_x \; C_y \; C_z$	Force coefficients
lb.ft.	$L \quad M \quad N$	$\rho sA(\Omega R)^2 R$	$C_\ell \; C_m \; C_n$	Moment coefficients
$\frac{\text{lb}}{\text{ft.sec.}^{-1}}$	$X_u \; Y_v \; Z_u$ $X_w \quad Z_w$	$\rho sA\Omega R$	$x_u \; y_v \; z_u$ $x_w \quad z_w$	Force velocity derivatives
$\frac{\text{lb}}{\text{rad.sec.}^{-1}}$	Y_p $X_q \quad Z_q$ Y_r	$\rho sA\Omega R.R$	y_p $x_q \quad z_q$ y_r	Force angular velocity derivatives
$\frac{\text{lb.ft.}}{\text{ft.sec.}^{-1}}$	M_u $L_v \quad N_v$ M_w	$\rho sA\Omega R.R.$	m_u $\ell_v \quad n_v$ m_w	Moment velocity derivatives
$\frac{\text{lb.ft.}}{\text{rad.sec.}^{-1}}$	$L_p \; M_q \; N_p$ $L_r \quad N_r$	$\rho sA\Omega R.R^2$	$\ell_p \; m_q \; n_p$ $\ell_r \quad n_r$	Moment angular velocity derivatives
$\frac{\text{lb.ft.}}{\text{ft.sec.}^{-2}}$	$M_{\dot{w}}$	$\rho sA.R^2$	$m_{\dot{w}}$	Moment downwash lag derivative
slugs ft.^{2}	$A \quad B \quad C$ E	$\frac{WR^2}{g}$	$i_A \; i_B \; i_C$ i_E	Inertia coefficients

It is convenient to adopt a dimensionless unit of time $\hat{t}$ (the airsec) defined by

$$\hat{t} = \frac{W}{g\rho sA\Omega R} = \frac{\mu_2}{\Omega},$$

where

$$\mu_2 = \frac{W}{g\rho sAR} = \Omega\hat{t}$$

is the relative density parameter, being the same for both longitudinal and lateral motions. Time measured in this unit is denoted by τ,

$$\tau = t/\hat{t}$$

It is also convenient to write

$$C_T' = \frac{W}{\rho A(\Omega R)^2}$$

and also

$$C_T' = C_T \cos\alpha_d - C_H \sin\alpha_d .$$

In level flight

$$C_T' = C_T$$

Once again, it must be pointed out that the C_T and C_H used here are not the same as the American C_T and C_H, since the latter coefficients are referred to the no feathering plane, whereas in all British work these coefficients are referred to the plane of the disc.

The equations of motion are now,

Longitudinal

$$(D - x_u)\hat{u} - x_w\hat{w} + \left(\frac{C_T'}{s}\cos\gamma - \frac{x_q}{\mu_2}D\right)\theta = x_{B_1}B_1 + x_{\theta_0}\theta_0$$

$$-z_u\hat{u} + (D - z_w)\hat{w} + \left\{\frac{C_T'}{s}\sin\gamma - \left(\mu_d + \frac{z_q}{\mu_2}D\right)\right\}\theta = z_{B_1}B_1 + z_{\theta_0}\theta_0$$

$$-\mu_2\frac{m_u}{i_B}\hat{u} - \left(\frac{m_w}{i_B}D + \frac{\mu_2 m_w}{i_B}\right)\hat{w} + \left(D^2 - \frac{m_q}{i_B}D\right)\theta = \frac{\mu_2}{i_B}\left(m_{B_1}B_1 + m_{\theta_0}\theta_0\right)$$

Lateral

$$(D - y_v)\hat{v} - \left(\frac{y_p}{\mu_2} + \frac{C_T'}{s}\cos\gamma\right)\phi + \left\{\left(\frac{\mu}{d} - \frac{y_r}{\mu_2}\right)D - \frac{C_T'}{s}\sin\gamma\right\}\psi = y_{A_1}A_1 + y_{\theta_t}\theta_t$$

$$-\mu_2\frac{\ell_v}{i_A}\hat{v} + \left(D^2 - \frac{\ell_p}{i_A}D\right)\phi - \left(\frac{i_E}{i_A}D^2 + \frac{\ell_r}{i_A}D\right)\psi = \frac{\mu_2}{i_B}\left(\ell_{A_1}A_1 + \ell_{\theta_t}\theta_t\right)$$

$$-\mu_2\frac{n_v}{i_C}\hat{v} - \left(\frac{i_E}{i_C}D^2 + \frac{n_p}{i_C}D\right)\phi + \left(D^2 - \frac{n_r}{i_C}D\right)\psi = \frac{\mu_2}{i_B}\left(n_{A_1}A_1 + n_{\theta_t}\theta_t\right)$$

where

$$D \equiv \frac{d}{d\tau}, \quad \hat{u} = \frac{u}{\Omega R}, \quad \text{etc.}$$

Coefficients of Stability Quartic

The equations of motion are solved by assuming that

$$u = \hat{u}_0 e^{\lambda\tau} \qquad \text{etc.}$$

If the controls are fixed,

$$x_{B_1} = z_{B_1} = m_{B_1} = y_{A_1} = y_{\theta_t} = \ell_{A_1} = \ell_{\theta_t} = n_{A_1} = n_{\theta_t} = 0$$

and the frequency equation is of the form

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$$

where for the longitudinal case

$$A_1 = 1$$

$$B_1 = -(x_u + z_w) - \frac{m_q}{i_B} - \left(\mu_d + \frac{z_q}{\mu_2}\right) \frac{m_{\dot{w}}}{i_B}$$

$$C_1 = (x_u z_w - x_w z_u) + \frac{m_q}{i_B}(x_u + z_w) + \frac{m_{\dot{w}}}{i_B}\left\{x_u\left(\mu_d + \frac{z_q}{\mu_2}\right) - z_u \frac{x_q}{\mu_2} + \frac{C_T'}{s} \sin \gamma\right\}$$
$$- \frac{\mu_2 m_w}{i_B}\left(\mu_d + \frac{z_q}{\mu_2}\right) - \frac{\mu_2 m_u}{i_B} \frac{x_q}{\mu_2}$$

$$D_1 = -\frac{m_q}{i_B}(x_u z_w - x_w z_u) + \frac{C_T'}{s}(z_u \cos\gamma - x_u \sin\gamma)\frac{m_{\dot{w}}}{i_B} + \frac{\mu_2 m_w}{i_B}\left\{x_u\left(\mu_d + \frac{z_q}{\mu_2}\right) - z_u \frac{x_q}{\mu_2} + \frac{C_T'}{s} \sin\gamma\right\}$$
$$+ \frac{\mu_2 m_u}{i_B}\left\{\frac{C_T'}{s} \cos\gamma - x_w\left(\mu_d + \frac{z_q}{\mu_2}\right) + z_w \frac{x_q}{\mu_2}\right\}$$

$$E_1 = \frac{\mu_2 m_w}{i_B}(z_u \cos\gamma - x_u \sin\gamma)\frac{C_T'}{s} - \frac{\mu_2 m_w}{i_B}(z_w \cos\gamma - x_w \sin\gamma)\frac{C_T'}{s}$$

In level flight, to a good approximation,

$$E_1 = \frac{C_T'}{s}\left(\frac{\mu_2 m_w}{i_B} z_u - \frac{\mu_2 m_u}{i_B} z_w\right)$$

For the lateral case

$$A_2 = 1 - \frac{i_E^2}{i_A i_C}$$

$$B_2 = -y_v\left(1 - \frac{i_E^2}{i_A i_C}\right) - \left(\frac{\ell_p}{i_A} + \frac{n_r}{i_C} + \frac{i_E n_p}{i_A i_C} + \frac{i_E \ell_r}{i_C i_A}\right)$$

$$C_2 = y_v\left(\frac{\ell_p}{i_A} + \frac{n_r}{i_C} + \frac{i_E}{i_A}\frac{n_p}{i_C} + \frac{i_E}{i_C}\frac{\ell_r}{i_A}\right) + \left(\frac{\ell_p}{i_A}\frac{n_r}{i_C} + \frac{\ell_r}{i_A}\frac{n_p}{i_C}\right)$$
$$+ \frac{\mu_2 n_v}{i_C}\left(\mu_d - \frac{y_r}{\mu_2} - \frac{i_E}{i_A}\frac{y_p}{\mu_2}\right) + \frac{\mu_2 \ell_v}{i_A}\left\{\frac{i_E}{i_C}\left(\mu_d - \frac{y_r}{\mu_2}\right) - \frac{y_p}{\mu_2}\right\}$$

$$D_2 = -y_v\left(\frac{\ell_p}{i_A}\frac{n_r}{i_C} - \frac{\ell_r}{i_A}\frac{n_p}{i_C}\right) - \frac{\mu_2 n_v}{i_C}\left\{\frac{\ell_p}{i_A}\left(\mu_d - \frac{y_r}{\mu_2}\right) + \frac{\ell_r y_p}{i_A \mu_2} + \frac{C_T'}{s}\sin\gamma\right.$$
$$\left. + \frac{i_E}{i_A}\frac{C_T'}{s}\cos\gamma\right\} + \frac{\mu_2 \ell_v}{i_A}\left\{\frac{n_p}{i_C}\left(\mu_d - \frac{y_r}{\mu_2}\right) + \frac{n_r y_p}{i_C \mu_2} - \frac{C_T'}{s}\cos\gamma - \frac{i_E}{i_C}\frac{C_T'}{s}\sin\gamma\right\}$$

$$E_2 = -\frac{\mu_2 n_v}{i_C}\left(\frac{\ell_r}{i_A}\frac{C_T'}{s}\cos\gamma - \frac{\ell_p}{i_A}\frac{C_T'}{s}\sin\gamma\right) + \frac{\mu_2 \ell_v}{i_A}\left(\frac{n_r}{i_C}\frac{C_T'}{s}\cos\gamma - \frac{n_p}{i_C}\frac{C_T'}{s}\sin\gamma\right)$$

In level flight, to a good approximation,

$$E_2 = \frac{C_T}{s}\frac{\mu_2}{i_A i_C}(\ell_v n_r - n_v \ell_r)$$

Part 6. Omnibus List of Symbols

Symbol	Quantities	Units	Part
a	slope of lift curve, c_{ℓ}/α_r		1
a_o	constant term in the Fourier series that expresses β (flapping). Also, the coning angle in the case on non-flexible blades	rad.	4
a_n, b_n	coefficients in the Fourier series that expresses β (flapping)		4
a_1	tilt of rotor disc relative to no feathering axis, positive for rearward tilt	rad.	2
a_{1_s}	tilt of rotor disc relative to rotor shaft axis, positive for rearward tilt	rad.	2
A	disc area of one rotor, πR^2	$ft.^2$	1
A	moment of inertia about Gx axis	slugs $ft.^2$	5
A	constant of stability quartic		5
A_o	constant term in the Fourier series that expresses θ (feathering)	rad.	4
A_n, B_n	coefficients in the Fourier series that expresses θ (feathering)		4
A_p	total projected disc area (multi-rotor aircraft)	$ft.^2$	1
b	number of blades per rotor		1
b_1	lateral tilt of rotor disc relative to no feathering axis, positive for tilt to starboard	rad.	2
B	rotor blade tip-loss factor		1
B	constant of stability quartic		5
B	moment of inertia about Gy axis	slugs $ft.^2$	5
B_{1_s}	forward tilt of no feathering axis relative to rotor shaft axis	rad.	2
c	blade chord (general symbol)	ft.	1
c_e	equivalent blade chord (thrust basis)	ft.	1
$\bar{\bar{c}}$	equivalent blade chord (torque basis)	ft.	1
c_o	blade chord at root, projected to rotor axis	ft.	1
c_r	blade chord at radius r ft.	ft.	1
c_R	blade chord at tip	ft.	1
c_x	blade chord at radius ratio x	ft.	1
c_d	blade element profile drag coefficient		1
c_{ℓ}	blade element lift coefficient		1

$\overline{C_\ell}$	average blade lift coefficient in hovering		1
C	moment of inertia about Gz axis	slugs ft.^{2}	5
C	constant of stability quartic		5
C_D	drag coefficient		1
C_{D_f}	parasite drag coefficient		1
C_H	longitudinal force coefficient (see note in part 1)		1
C_L	lift coefficient		1
C_ℓ	rolling moment coefficient		1
C_{m_f}	fuselage pitching moment coefficient		1
C_n	yawing moment coefficient		1
C_P	rotor power coefficient		1
C_Q	rotor torque coefficient		1
C_T	rotor thrust coefficient (see note in part 1)		1
C_y	lateral force coefficient		1
D	drag (general symbol)	lb.	1
D_f	parasite drag of rotorcraft less main rotor(s)	lb.	1
D_{100}	D_f at V = 100 ft.sec.$^{-1}$	lb.	1
eR	flapping hinge offset; distance of flapping hinge from centre of rotor hub	ft.	1
E	product of inertia about Gy axis	slugs ft.^{2}	5
E	constant of stability quartic		5
E_o	constant in the Fourier series that expresses ζ(lagging)		4
E_n, F_n	coefficients in the Fourier series that expresses ζ (lagging)		4
fR	lag hinge offset; distance of lag hinge from centre of rotor hub	ft.	1
F_c	centrifugal force of one rotor	lb.	1
g	acceleration due to gravity	ft.sec.$^{-2}$	1
hR	general symbol to denote height above a reference point (see note in part 1)	ft.	1
H	longitudinal force at the rotor in the plane of the disc	lb.	1
HP	general symbol for horsepower	horsepower	1
I	moment of inertia (general symbol)	slugs ft.^{2}	1

I_b	moment of inertia of a blade about its flapping hinge	slugs ft.^{2}	1
ℓ_R	general symbol to denote distance in a substantially horizontal plane, measured from a reference point	ft.	1
L	lift (general symbol)	lb.	1
L	rolling moment	lb.ft.	1, 5
m	mass (general symbol)	slugs	1
m_b	total mass of a rotor blade to its flapping hinge	slugs	1
M	pitching moment	lb.ft.	1, 5
M	Glauert's Figure of Merit, $C_T^{3/2}/C_Q$		1
M	Mach number		1
M_b	weight moment of a blade about its flapping hinge	lb.ft.	1
M_f	pitching moment of fuselage	lb.ft.	1, 5
n	number of (main)rotors per aircraft		1
N	yawing moment	lb.ft.	1, 5
N	general symbol for revolutions per minute	revs.min.$^{-1}$	1
p	angular velocity about Gx axis	rad.sec.$^{-1}$	5
P	power (general symbol)	ft.lb.sec.$^{-1}$	1
P_i	rotor induced power	ft.lb.sec.$^{-1}$	1
P_p	total rotor profile drag power, inclusive of power due to rotor H force	ft.lb.sec.$^{-1}$	1
P_f	parasite drag power of rotorcraft less main rotor(s)	ft.lb.sec.$^{-1}$	1
P_E	power available at power plant output	ft.lb.sec.$^{-1}$	1
P_R	total power required at rotor	ft.lb.sec.$^{-1}$	1
q	angular velocity about Gy axis	rad.sec.$^{-1}$	1
Q	rotor shaft torque	lb.ft.	1
r	angular velocity about Gz axis	rad.sec.$^{-1}$	5
r	radius of blade element from rotor centre	ft.	1

S	area of fixed wing	$ft.^2$	1
t	time	seconds	1, 5
$\hat{t}$	dimensionless unit of time, $W/g\rho sA\Omega R$	airsec	5
T	rotor thrust normal to rotor disc	lb.	1
T	air temperature	^oC	1
u	total velocity normal to rotor disc, negative for flow downward through disc	$ft.sec.^{-1}$	2
u	increment of velocity along flight path	$ft.sec.^{-1}$	5
U	resultant velocity at blade element, normal to blade-span axis	$ft.sec.^{-1}$	3
U_A	axial component of velocity at blade element normal to both blade-span axis and U_T, negative for flow downward through disc	$ft.sec.^{-1}$	3
U_T	tangential component of velocity at blade element, normal to both blade-span axis and axis of reference	$ft.sec.^{-1}$	3
v	induced velocity at rotor disc, negative for flow downward through disc	$ft.sec.^{-1}$	2
v	velocity along Gy axis	$ft.sec.^{-1}$	5
v_h	induced velocity at rotor disc in hovering assuming uniform distribution, and referred to the rotor disc; negative for flow downward through disc	$ft.sec.^{-1}$	2
V	steady velocity of flight along flight path	$ft.sec.^{-1}$	1, 5
V'	resultant velocity at rotor disc	$ft.sec.^{-1}$	2
V_c	rate of climb	$ft.sec.^{-1}$	1
V_T	rotor tip speed	$ft.sec.^{-1}$	1
w	velocity along Gx axis	$ft.sec.^{-1}$	5
w	disc loading, W/nA	$lb.ft.^{-2}$	1
w_p	disc loading based on total projected disc area, W/A_p	$lb.ft.^{-2}$	1
W	weight of complete rotorcraft	lb.	1
x	blade element radius ratio, r/R		1
X	force along Gx axis	lb.	5
Y	force along Gy axis	lb.	1, 5
Z	force along Gz axis	lb.	5
α_d	tilt of rotor disc relative to flight path, negative for forward tilt	rad.	2
α_{nf}	tilt of no feathering axis relative to flight path, negative for forward tilt	rad.	2

α_s	tilt of rotor shaft axis relative to flight path, negative for forward tilt	rad.	2
α_r	blade element angle of attack, measured from line of zero lift	rad.	3
$\alpha_{1,2,3}$	angular inclination of lag hinge	rad.	4
β	blade flapping angle at particular azimuth position, measured from plane of no feathering; positive for upward flapping	rad.	4
γ	angle between flight path and horizontal, positive for climb	rad.	2
γ	mass constant of blade of blade, $c_e \rho a R^4 / I_b$		1
δ	average blade profile drag coefficient		1
$\delta_{1,2,3}$	angular inclination of flapping hinge	rad.	4
ε	angle between flight path and rotor wake, positive for downwash	rad.	2
ζ	lag angle, measured between span axis of blade and line passing through centres of rotor and lag hinge pin	rad.	4
η	ratio of power available at main rotor(s) to power available at power plant output		1
θ	angle of pitch (of rotorcraft)	rad.	5
θ	blade pitch angle, measured between line of zero lift and plane of reference	rad.	3, 4
θ_o	blade pitch angle at blade root, projected to rotor axis	rad.	3
θ_R	blade pitch angle at blade tip		
θ_1	difference between tip and (projected) root pitch angles, $\theta_R - \theta_o$	rad.	3
κ	angle between plane of symmetry and rotor shaft (lateral tilt of shaft)	rad.	1
λ	root of stability quartic		5
λ	inflow ratio relative to disc, negative for flow downward through disc		2
λ_{nf}	inflow ratio relative to no feathering plane negative for flow downward through disc		2
μ_d	tip speed ratio relative to disc, $V \cos \alpha_d / \Omega R$		1
μ_{nf}	tip speed ratio relative to no feathering plane, $V \cos \alpha_{nf} / \Omega R$		1
μ_s	tip speed ratio relative to rotor shaft axis, $V \cos \alpha_s / \Omega R$		1
μ'	approximate tip speed ratio, $V / \Omega R$		1
μ_2	relative density parameter, $W / g \rho s A R$		5

ρ	air density	slugs $ft.^{-3}$	1
ρ_o	air density at sea level, ICAO	slugs $ft.^{-3}$	1
σ	relative density of air, ρ / ρ_o	1	
τ	non-dimensional measure of time, $t/\hat{t}$		5
ϕ	inflow angle at blade element	rad.	3
ϕ	angle of roll	rad.	5
ψ	angle of yaw	rad.	5
ψ	blade azimuth angle, measured in the direction of rotation, between the downwind position and blade-span axis ($\zeta = 0$)	rad.	1
Ω	rotor angular velocity	rad. $sec.^{-1}$	1

For subscripts see part 1

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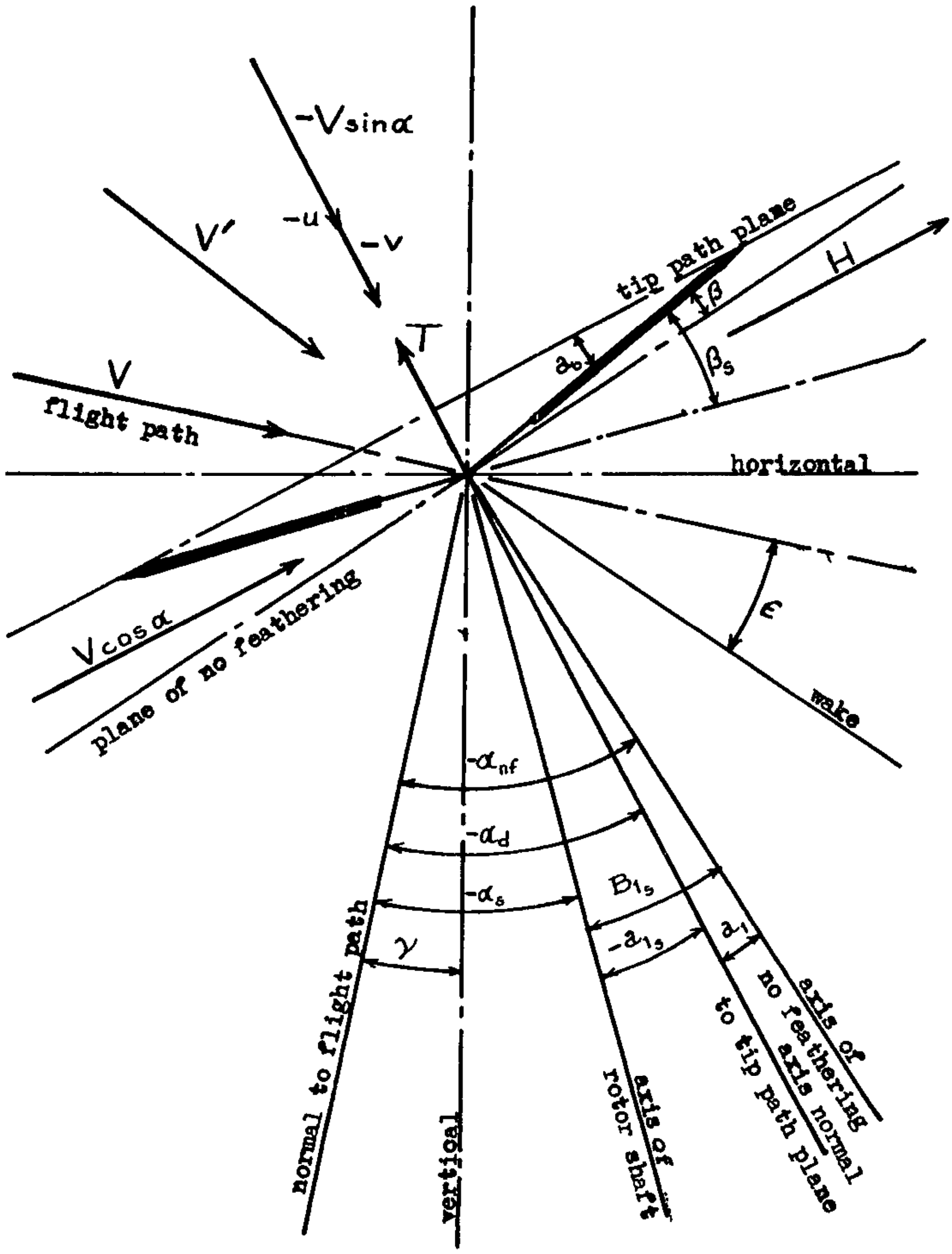


Figure 1

PARAMETERS AT THE ROTOR DISC

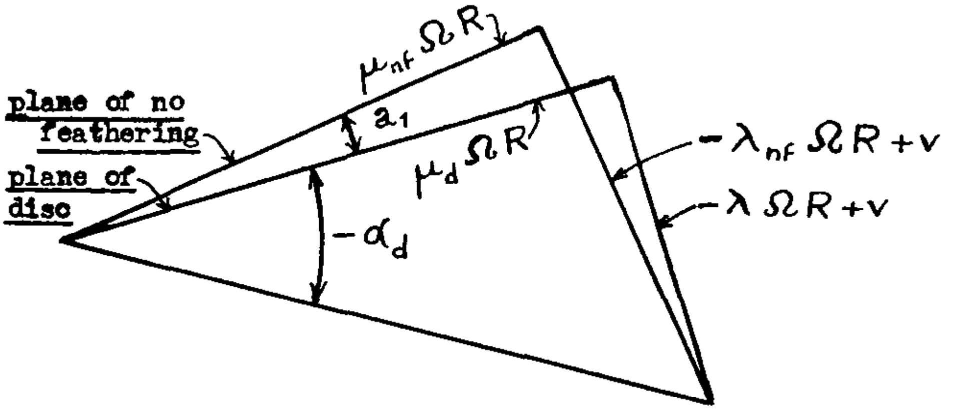


Figure 2

REFERENCE PLANES FOR $\mu_d, \mu_{nf}, \lambda, \lambda_{nf}$

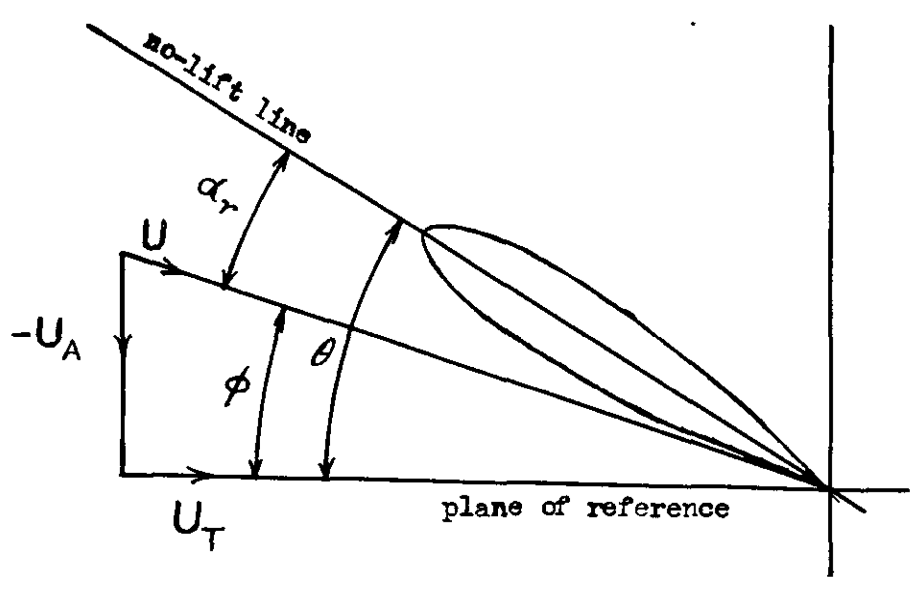


Figure 3

PARAMETERS AT A BLADE ELEMENT

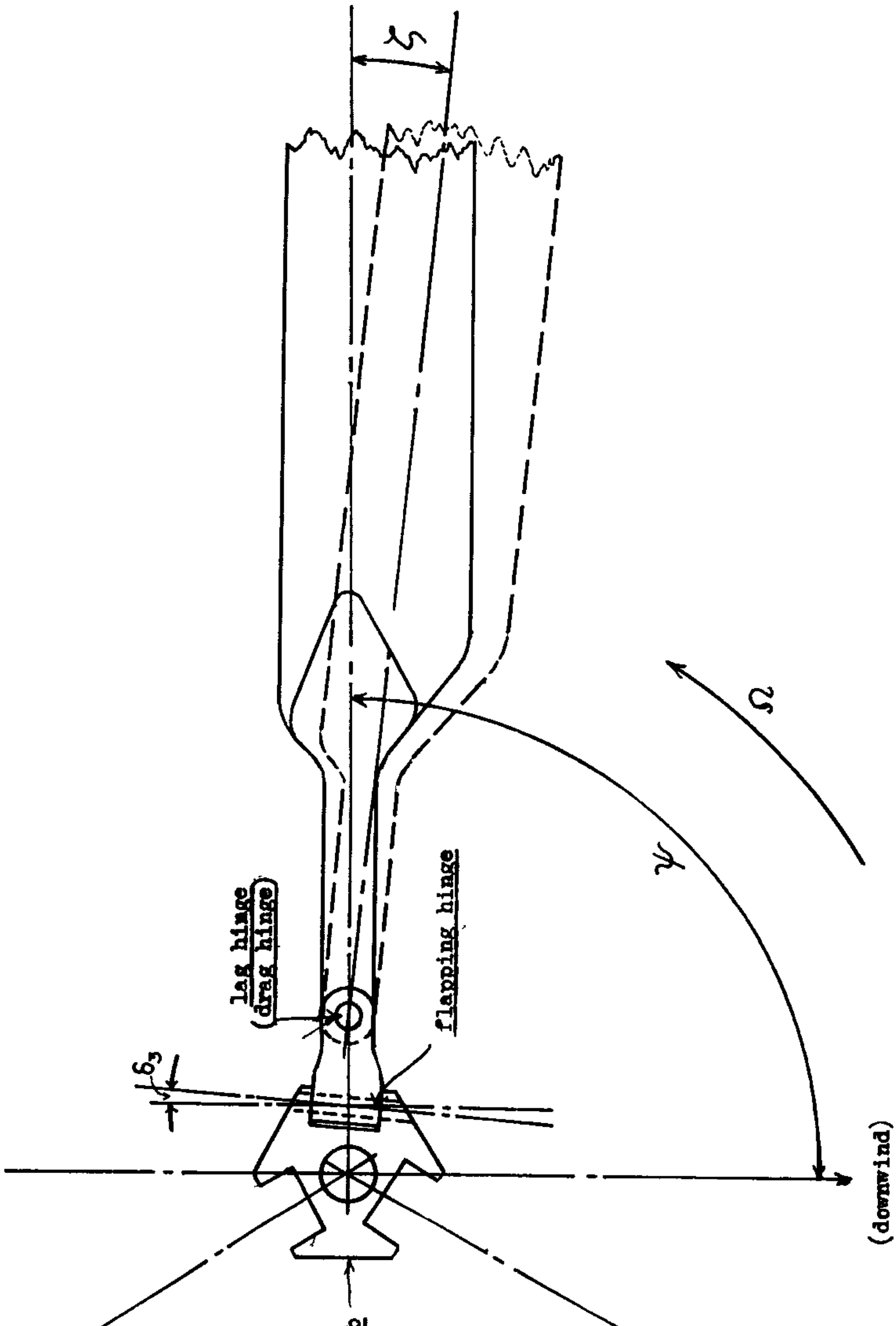
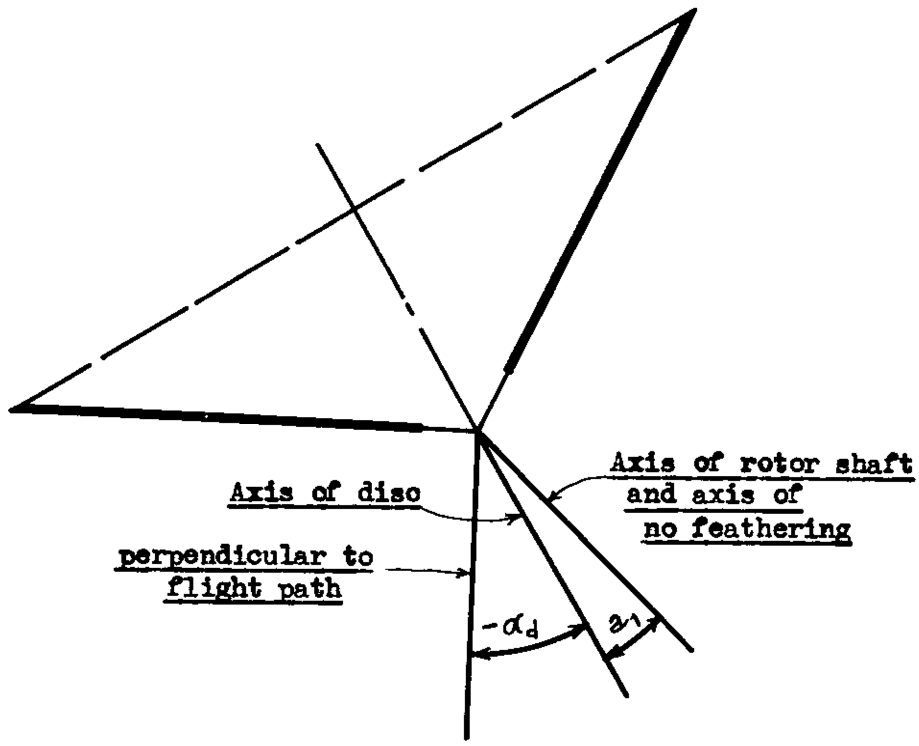
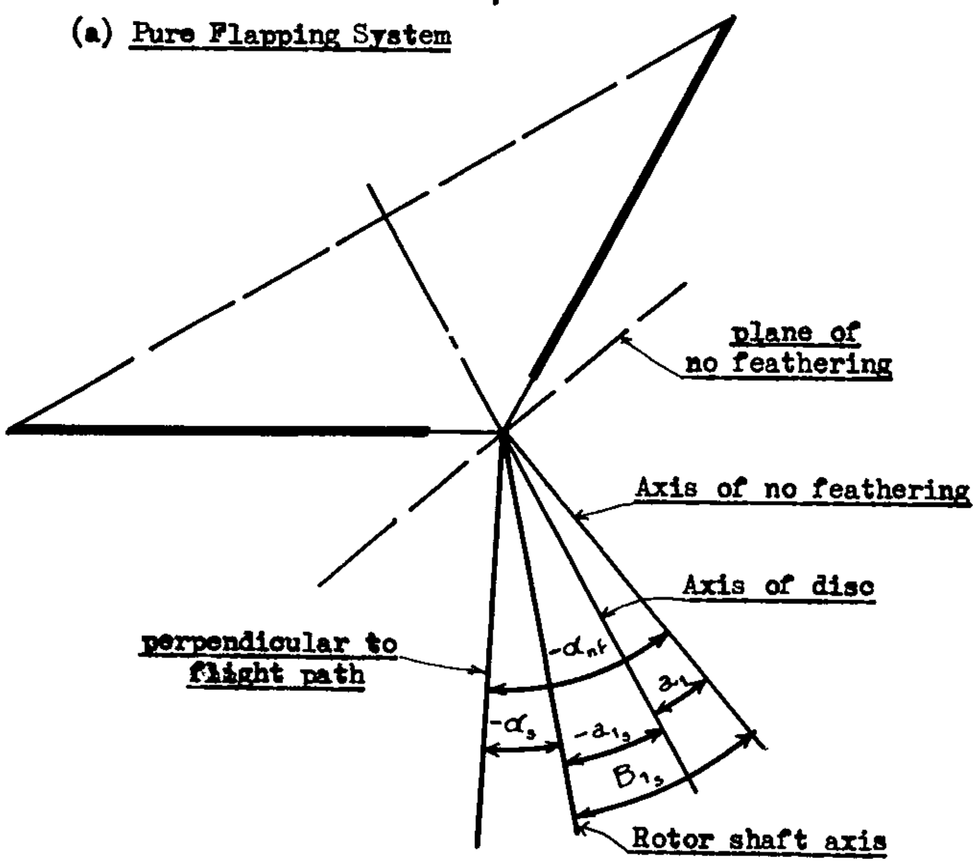


Figure 4

BLADE HINGES & MOTION



(a) Pure Flapping System



(b) System involving both Flapping and Feathering

Longitudinal cross-section of the rotor cone showing

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