N. A. E.

R. & M. No. 2671
(12,043)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

Theoretical Investigations of Ternary Lifting Surface — Control Surface — Trimming Tab Flutter and Derivation of a Flutter Criterion

By

H. WITTMEYER, DR.ING.

LONDON: HER MAJESTY'S STATIONERY OFFICE
1952
ELEVEN SHILLINGS NET

Theoretical Investigations of Ternary Lifting Surface — Control Surface — Trimming Tab Flutter and Derivation of a Flutter Criterion

By

H. WITTMEYER, DR.ING.

COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR), MINISTRY OF SUPPLY

Reports and Memoranda No. 2671*
October, 1948

Summary.—Theoretical investigations have been made of the flutter of an idealised trimming tab system having three degrees of freedom—normal translation of the main lifting surface, rotation of the control surface and rotation of the tab. All the structural parameters of the system have been varied except the out-of-balance moment of the control surface. The cases in which the system is free from flutter have been particularly investigated.

From these investigations criteria for the avoidance of flutter have been derived. If the structural parameters of the system satisfy these criteria, flutter of the system with these three degrees of freedom should be impossible.

The results are applicable to trimming tabs, servo-tabs with zero follow-up ratio, and generally to all systems in which the tab can be regarded as connected elastically only to the control surface.

1. *Introduction.*—In flutter investigations of control systems in which the tab can be regarded as connected elastically to the control surface only† it has been found desirable to consider the following degrees of freedom:

(a) Normal translation of main lifting surface .. .. .. (z)
(b) Rotation of main lifting surface .. .. (α)
(c) Rotation of control surface .. .. (β)
(d) Rotation of tab .. .. (γ)

and essential to consider one of the main lifting surface freedoms (z or α) in addition to both control surface rotations (β and γ)[1 to 5].

Initial German investigations by Leiss[1] (1939) which were restricted to the control degrees of freedom (β and γ) showed that the system was free from flutter if the tab centre of gravity was sufficiently far forward, but subsequent experiments by Voigt and Walter[2] (1941) with three (z, β and γ) and four degrees of freedom and theoretical investigations by the author[3] (1941) with three degrees (z, β and γ) showed that with more than two degrees flutter could occur with a far forward position of the centre of gravity of the tab. These researches followed accident investigations in Germany during the period 1939–41. The above facts were also discovered in this country in theoretical investigations by Buxton and Sharpe[4], to explain an accident to a *Mosquito*

* R.A.E. Report Structures 19, received 7th January, 1949.
† Spring tabs are excluded from this report.

aircraft fitted with an experimental *g*-restrictor device (involving heavy mass over-balance of the elevator tab) and confirmed in experiments by Scruton, Ray and Dunsdon[5] in 1945. The former report[4] brought out new important facts on the effect of the mass-balance of the main control.

The present report is part of a comprehensive consideration of the flutter of tabs* and is an extension of the previous work of the author[3] to cover the effect of the elastic connection between the main lifting surface and the main control and to present the results in the form of a criterion The method used is to compare systematically the results of flutter calculations† and then to find rules of a non-dimensional form which the given results satisfy. From the mathematical point of view, this method is comparable with the well-known method of the approximation to a function given by a set of points by a special kind of analytical function with a reasonable number of parameters available (*e.g.*, a polynomial).

The degrees of freedom covered in the present report are normal translation of the wing and rotation of the control surface and the tab. A wide range of variation of the plan-form, mass, and inertia parameters of the tab and of the frequency ratios of the tab, control and lifting surface have been investigated and criteria developed to cover a large part of the practical variation of these design parameters. General principles for the avoidance of flutter are given and the criteria are presented in the form of frequency ratios of the three components and the mass-balance of the tab (in non-dimensional form). Values for the criteria are suggested based on the flutter calculations with a suitable safety margin. Comparisons are made with the existing Collar-Sharpe criterion for spring tabs as it might be applied to trimming tabs by putting the follow-up ratio equal to zero, and the relation between the new criteria and the current official requirements for trimming tabs is also considered.

2. *Description of the System and the Method of Investigation.*—2.1. *The System and its Degrees of Freedom.*—The system investigated (Fig. 1) is a rectangular lifting surface (referred to in short as a wing) of chord c_w fitted with a rectangular control surface of the same span s and chord $c_c = E_1 c_w$. On the trailing edge of the control surface there is a rectangular cut-out; and this space is filled with a tab of span qs and chord $c_t = E_2 c_w$.

The system incorporates the following elastic constraints:

(i) Constraint in respect of the normal translation of the wing.

(ii) Constraint in respect of the rotation of the control surface relative to the wing.

(iii) Constraint in respect of the rotation of the tab relative to the main control.

The wing is fixed against rotation.

There are three degrees of freedom:

(i) Degree of freedom (z) normal translation of the wing.

(ii) Degree of freedom (β) rotation of the control surface relative to the wing.

(iii) Degree of freedom (γ) rotation of the tab relative to the control surface.

The choice of the degrees of freedom was discussed in the previous report[6].

2.2. *The Structural Parameters Considered.*—In the author's previous report[6], the same system with the same degrees of freedom was investigated in regard to the possibility of flutter when the following structural parameters are varied (the corresponding non-dimensional parameters are shown for each structural parameter in curly brackets for the new notation of the present report and in square brackets for the notation of the first report[6]).

* Spring tabs are excluded from this report.

† The extensive flutter calculations which are turned to account in the present report were made under the direction of the author at the firm of Focke-Wulf by his collaborators Herren Korte, Mewes and Schäfer. The results have not hitherto been published.

(A) (i) Out-of-balance moment of the tab ($\{p_t\}$, $[y\,\mu_2\,\sigma_2]$) .

(ii) Moment of inertia of the tab ($\{i_t\}$, $[y\,\vartheta_2]$) .

(iii) Ratio of the uncoupled natural frequency of tab rotation to that of wing translation ($\{f_\gamma/f_z\}$, $[n_{Q2}/n_B]$) .

(iv) Moment of inertia $[y\vartheta z]$ of a pair of masses kinematically coupled with the tab and the gear ratio $[\alpha]$ of this connection.

(v) Ratio of the tab span to the wing span ($\{q\}$, $[\chi_1]$) .

(vi) Damping of the tab ($\{d_t\}$, $[W_2/\pi\rho l_F^4 b\chi_1\omega]$) .

In the present paper the effect of the following structural parameters is also investigated.

(B) (i) Ratio of the uncoupled natural frequency of control surface rotation to that of wing translation ($\{f_\beta/f_z\}$, $[n_{Q1}/n_B]$) .

(ii) Moment of inertia of the control surface ($\{i_c\}$, $[y\vartheta_1]$) .

(iii) Mass of the wing ($\{\mu\}$, $[\mu]$) .

(iv) Ratios of the control surface chord and of the tab chord, respectively, to the wing chord ($\{E_1, E_2\}$, $[\tau_1\tau_2]$) .

The structural parameters mentioned under A have all been varied again in the present paper except A (iv) and A (vi).

It should be noted that the out-of-balance moment of the control surface has not been varied; the control surface is supposed to be always statically balanced. This corresponds almost exactly to the former German requirements, and for this reason the author has not made calculations for control surfaces which are not mass-balanced. The investigations by Buxton and Sharpe[4], however, provide some information on the effect of control surface mass balance.

2.3. *Choice of Non-dimensional Structural Parameters.*—It is clear that the results must be presented in a non-dimensional form if they are to be as general as possible. In the attempt to find suitable parameters the author at first made use of the parameters introduced by Küssner, but later changed these parameters in such a way that the results were to a large extent independent of the values of the ratios E_1, E_2 of the control surface chord and the tab chord to the wing chord. For this purpose the Küssner parameters $\mu_1\sigma_1$, ϑ_1, $\mu_2\sigma_2$, ϑ_2 were replaced by the parameters

$$p_c = \frac{u_1\sigma_1}{E_1^2}\ ;\quad i_c = \frac{\vartheta_1}{E_1^3}\ ;\quad p_t = \frac{\mu_2\sigma_2}{E_2^2}\ ;\quad i_t = \frac{\vartheta_2}{E_2^3}\,.$$

In the present paper two additional non-dimensional constants have been found to be appropriate and have been introduced into the presentation of the results. The non-dimensional mass coupling between the degrees of freedom control surface rotation (β) and tab rotation (γ) would be, in Küssner's notation, $\vartheta_2 + 2(E_1 - E_2)\mu_2\sigma_2$. It is to be noted that of the above mentioned parameters the two introduced by the author in his report[6], *viz.*,

$$p_t = \frac{\mu_2\sigma_2}{E_2^2} \text{ and } i_t = \frac{\vartheta_2}{E_2^3}$$

have been proved to be expedient. This suggests the introduction of a new non-dimensional parameter

$$\frac{\vartheta_2 + 2(E_1 - E_2)\mu_2\sigma_2}{E_1E_2^2} = \frac{E_2}{E_1}\,i_t + 2\left(\frac{E_1 - E_2}{E_1}\right)p_t = p_{ct}\ .$$

For the same reason the parameter q which has been used exclusively in the previous report has been replaced by $\bar{q} = jq$, where j is a function of E_1, E_2. For particulars of this, *see* section 11 and Fig. 2b.

The non-dimensional quantities which have been introduced by the author, though not very complicated, are nevertheless a little more so than those already existing. In order to demonstrate the advantages of the new parameters, the parameter p_t may be compared with the well known quantity x in the phrase " x per cent over mass balance ".

Let the out-of-balance moment of the part aft of the hinge of a tab be $\tilde{M}_t\tilde{x}_t$. If this tab is x per cent over mass balanced the out-of-balance moment of the tab is $M_t\, x_t = -\tilde{M}_t\tilde{x}_t(x/100)$ and according to section 11,

$$p_t = -\frac{8 \,.\, \tilde{M}_t\tilde{x}_t x}{\pi \,.\, 100 \,.\, \rho c_w \,.\, c_t^2 qs} \,.$$

This means that two tabs with the same geometric form (*i.e.*, with equal values of c_w, c_t, q, s) and with the same degree x of mass balance and the same air density ρ can have different values of p_t if the values of $\tilde{M}_t\tilde{x}_t$ are different. These different values of $\tilde{M}_t\tilde{x}_t$ may arise from the different kind of structure of the tab, *e.g.*, the surface of the first tab may be covered by a metal sheet of uniform thickness while the second tab may have a strong torsion-tube near the hinge line, the surface being fabric covered. Because of these different p_t values one of the tabs may flutter and the other not, for p_t is an essential parameter, as will be shown later.

This means that tabs with the same value of x need not have the same safety against flutter. The value x in the phrase x per cent mass balanced is, therefore, not a suitable parameter for characterising the flutter safety of a tab. Similar considerations apply to the corresponding parameter for an aileron. It will be shown later that the non-dimensional parameters introduced in this paper are generally more appropriate for the purpose of describing the flutter characteristics of the system.

Besides the qualification of the new non-dimensional parameters introduced above to characterise the flutter capacity of the system, there is another advantage to be gained by using them. The author has found by experience that the values for the parameters i_c and i_t do not vary very much if control surfaces or tabs for aeroplanes with similar types of construction and similar load factors are considered. Therefore, the flutter specialist of a firm is able to guess at an early stage of the design the probable values of the parameters i_c and i_t (and thence I_c and I_t) if he has made a statistical survey of the values of i_c and i_t for former aeroplanes designed by this firm. It is clear that these values will be higher for highly loaded aeroplanes than for others. This is one reason why the values of the parameters i_c and i_t are often especially high for modern aeroplanes.

2.4. *Range of Values of the Parameters Considered.*—The range of values of the parameters which have been varied are now given. For comparison, Table 1 gives both the ranges of values investigated in the previous report[6] and the ranges of values occurring in practice, as compiled mainly from Focke-Wulf machines.

TABLE 1

Parameter	E_1	μ	i_c	p_c	$\frac{f_\beta}{f_z}$	$\frac{E_2}{E_1}$
Statistical	0·18 to 0·50	4 to 50	2 to 8	0	0 to 2	0·15
Previous Report[6]	0·3	5·718	7·778	0	0	0·25
Present Report	0·2 to 0·4	5·718 to 50	1 to 7·78	0	0 to ∞	0·13 to 0·25

TABLE 1—*continued*

Parameter	i_t	p_t	$\frac{f_Y}{f_z}$	d_t	q
Statistical	4 to 8	2 to 4	0 to ∞	—	0·3 to 0·5
Previous Report[6]	3·93 to 13·1	−1·2 to +1·2	0 to ∞	0 to 3 × 10⁻²	0·3 to 1·0
Present Report	1·31 to 13·1	−4 to +4	0 to 5	0	0·08 to 1·0

It should be noticed that with regard to the ranges of the values of the structural parameters of British aircraft the ranges of values of E_1, i_c, i_t are greater than could have been investigated in the present report, using only the available flutter calculations made in Germany in 1941.

2.5. *The Hypotheses (including aerodynamic assumptions) used in Calculating the Critical Speed of the System.*—The formulae used for calculating the critical speed of the system investigated have been established in former work by the author[3]. In deriving these formulae the usual assumption was made that the theory of small oscillations would be valid. The airforces were calculated by strip-theory using the values of Dietze[7] for the case of a thin aerofoil with two hinged flaps in an incompressible medium. The simplification sometimes made in British work that the damping and stiffness derivatives may be assumed independent of frequency parameter for the purpose of calculating the air forces has not been made.

2.6. *Presentation of the Results.*—As was shown in the previous report[6] it is convenient to plot the results in terms of $V_c/f_z c_w = v_c$ instead of V_c, and in terms of the ratios f_β/f_z, f_Y/f_z and f_c/f_z instead of the parameters f_β, f_Y and f_c themselves. These ratios are all non-dimensional if the same system of units is used for all parameters. However, to conform to practical requirements V_c will be expressed in m.p.h., c_w in ft and f_z in c.p.s. Then v_c will have the dimension (m.p.h./sec^{-1} ft). To obtain a value of V_c in m.p.h. we have only to multiply the value of v_c given in the diagrams by c_w in feet and f_z in c.p.s. This product can easily be determined from the ground resonance test. Its normal range is 20 to 40 ft/sec.

Dependence upon three structural variables will be shown in the following manner: along each curve only one parameter (*e.g.*, p_t) will be varied, while a second (*e.g.*, f_Y/f_z) will vary from curve to curve and the third (*e.g.*, f_β/f_z) from one family of curves to the next. A typical family of curves will be denoted by

$$v_c = f\left(p_t \,;\, \frac{f_Y}{f_z}\right) f_\beta/f_z = \text{constant}\,.$$

Several such families will be denoted by

$$v_c = f\left(p_t \,;\, \frac{f_Y}{f_z} \,;\, \frac{f_\beta}{f_z}\right).$$

The first argument of the function f will therefore always denote the independent variable which varies along each curve, the second the parameter which varies from one curve to another, and the third a parameter which varies from one family of curves to another.

3. *Flutter Characteristics of the System.*—3.1. *The Dependence of the Variation of Critical Speed with Tab Out-of-balance Moment on the Remaining Parameters (except the chord ratios E_1, E_2).*—3.1.1. *The four branches of the curve $v_c = f(p_t)$.*—Amongst all the functional dependences which it is possible to investigate, priority has been given to the critical speed (more exactly its non-dimensional equivalent v_c) as a function of the out-of-balance moment of the tab (more exactly its non-dimensional equivalent p_t), the remaining non-dimensional parameters varying from

curve to curve, because this relationship is the most interesting in practice. In order to study the variation of these curves $v_c = f(p_t)$ with the variation of the remaining parameters it is convenient to mark the particular branches of the curve suitably. Consideration will, however, be confined to those branches in which "lower critical speeds" are plotted; only these parts of the curves are important in practice, of course, since they represent the critical speeds at which the system begins to flutter if the speed is increasing. (The other critical speeds are called "upper critical speeds": the system stops fluttering if such a speed is exceeded.)

As regards their behaviour, we distinguish four branches of lower critical speeds in the curves under consideration. In Figs. 3 to 8 they are marked by Roman figures. It should first be explained why just four branches are distinguished.

Branch I is the only one which lies principally in the region where the centre of gravity of the tab is forward of its hinge. The Branches II, III and IV, on the other hand, lie principally in the region where the centre of gravity of the tab is aft of its hinge. Differentiation of three branches in the under mass-balanced region would seem to be unnecessary if the natural frequency of the tab is high relative to the natural frequencies of control surface rotation and of wing translation. The curves in Figs. 3d, 5f and 6 (for $f_\gamma/f_z = 5$) are, for example, marked with II, III and IV respectively, only because they have developed from different types of curves marked in that way. Otherwise we should not have been able to distinguish between them as regards their shape. But it is quite another thing when the natural frequency of the tab is low. Then Branch II exhibits a resonance phenomenon if the natural frequencies of tab rotation and control surface rotation coincide (Fig. 4b) while Branches III and IV exhibit a resonance phenomenon if the natural frequencies of tab rotation and wing translation (Figs. 3c, 4d and 5c) coincide. A distinction between the Branches III and IV is convenient because of their behaviour when the natural frequency of the control surface is increased: on increasing the value of f_β Branch III disappears upwards in the direction of the v_c-axis (Figs. 7c, d) while Branch IV approaches a finite curve (Fig. 7f). (Incidentally, Branch III appears only in curves for which $f_\beta/f_z \leqslant 0 \cdot 5$ and Branch IV only in curves for which $f_\beta/f_z \geqslant 1$). Under certain conditions (*e.g.*, $f_\gamma/f_\beta \leqslant 1$; $0 \cdot 5 \leqslant f_\beta/f_z \leqslant 10$; $q = 1$; *see* Figs. 4a, b; 5a, b, c; 7c, d, e) Branch I is connected with Branch II and then a boundary between these two branches will not be defined. In Fig. 1 of the previous report[6] it is shown that Branch I is connected with Branch III if the moment of inertia of the tab is high enough. The Branches II and III are sometimes indirectly connected via a branch of upper critical speeds, as for example in Fig. 3a or Fig. 8c of the present report. A summary of the features of all four branches is given in Table 2.

3.1.2. *Flutter frequencies.*—(a) *Branch I.* The ratio f_c/f_z for all the flutter cases belonging to Branch I increases with increase of f_β/f_z (Fig. 7); of f_γ/f_z (Figs. 3 to 5) or of μ (Fig. 10) and with decrease of i_c (Fig. 9), i_t (R. & M. 2418[5], Fig. 1, where there is only a small influence) and q (Figs. 7a, 8a). The smallest value that has appeared in the calculations for Branch I is $f_c/f_z = 0 \cdot 33$ (Fig. 3a). If $f_\gamma/f_z \geqslant 1$ and $f_\beta/f_z \leqslant 1$ then $f_c/f_z \simeq 1$ (Figs. 3d, 4d, 5d, e, f). For large values of f_β/f_z the value of f_c/f_z can become far greater than unity. If, for example, $f_\beta/f_z = 10$ and $f_\gamma/f_z = 0$, f_c/f_z lies on Branch I between $6 \cdot 8$ and $8 \cdot 6$ (Fig. 7c). With increasing values of f_β/f_z and probably small values of f_γ/f_z, the flutter frequency f_c obviously tends to the frequency of control surface rotation. With appropriate values of the tab frequency, the flutter frequency, therefore, behaves in nearly the same way as a system with the two degrees of freedom wing translation (z) and control surface rotation (β) for a sufficiently large out-of-balance moment of the control surface.

(b) *Branch II.* It may be deduced from Figs. 3, 4, 5 and 7 that for

$$\frac{f_\gamma}{f_\beta} \leqslant 1, \quad f_c \simeq f_\beta$$

and for

$$\frac{f_\gamma}{f_\beta} > 1, \quad f_c \simeq f_\gamma \, .$$

This means that the flutter frequencies behave in nearly the same way as those of a system with the two degrees of freedom control surface rotation (β) and tab rotation (γ) which has a sufficiently large mass-coupling. Though it may not be important, it should be mentioned that in the curves from which the above conclusions for $f_\gamma/f_\beta > 1$ were derived, $f_\beta/f_z \leqslant 1$ always.

For convenience of reference in a future report, it is opportune to state here that if the critical speeds on Branch II are low, the flutter frequency f_c is higher than the natural frequency of the tab, but sometimes approaches very closely to it. The same statement is true for Branch III but not for Branches I and IV.

(*c*) *Branch III.* It may be deduced from the Figs. 3, 4 and 7 that for

$$\frac{f_\gamma}{f_z} \leqslant 1, \quad f_c \simeq f_z$$

and for

$$\frac{f_\gamma}{f_z} > 1, \quad f_c \simeq f_\gamma \; .$$

This means that the flutter frequencies behave in nearly the same way as those of a system with the two degrees of freedom wing translation (z) tab rotation (γ) which has a sufficiently large mass coupling.

(*d*) *Branch IV.* It may be deduced from Figs. 5 and 6 that for

$$\frac{f_\gamma}{f_z} \leqslant 2, \quad f_c \simeq f_z$$

and for

$$\frac{f_\gamma}{f_z} > > 2, \quad f_c \simeq f_\gamma \; .$$

This means that the flutter frequencies behave just as for Branch III in nearly the same way as those of a system with the two degrees of freedom wing translation (z), tab rotation (γ) which has a sufficiently large mass coupling.

3.1.3. *Resonance phenomena.*—There are exceptionally low values of the non-dimensional critical speed on Branch I if $f_\beta/f_z = 1$ and if simultaneously $f_\gamma/f_z \geqslant 1 \cdot 2$; $f_\gamma/f_\beta \geqslant 1$. The minimum moves to the left if f_γ/f_z increases (Figs. 5c, d, e). It is noteworthy that for Branch I the coincidence of the two frequencies f_β and f_z alone is not sufficient. The reason is that the flutter motion, though consisting predominantly of wing translation (z) and control surface rotation (β), has to be steered by a (perhaps small) tab rotation. The above-mentioned auxiliary conditions provide the right phase for the tab rotation.

Branch III lies very low if $f_\beta = f_\gamma$ (Figs. 3, 4, 5).

Branches III and IV lie low if $f_\gamma = f_z$ (Figs. 3c, 4d, for Branch III and Fig. 5c for Branch IV).

3.1.4. *Flutter modes.*—Flutter modes have not been calculated for this report, but we can in several cases refer to calculations of the previous report[6]. In addition, reference can be made to the behaviour of the flutter frequencies as mentioned in section 3.1.2 and the resonance phenomena as mentioned in section 3.1.3.

(a) *Branch I.* One example of flutter modes for Branch I in which the degrees of freedom wing translation (z) and control surface rotation (β) are predominantly engaged is shown in the previous report[6] (Fig. 2, "Flutter modes for II"). Considering also the behaviour of the flutter frequencies on this branch as well as the resonance phenomenon, this type of flutter mode seems to be typical for this branch. It should, however, be emphasised that the tab rotation, small as it may be, is essential for steering this flutter motion, for flutter is not possible if the tab is not sufficiently over mass-balanced, or if the tab is fixed. It may therefore be said that on Branch I the flutter modes are similar to the modes of a wing with an under mass-balanced control surface. The behaviour similar to an under mass-balanced control surface is produced by over mass-balancing the tab. (In this connection it is well to remember the fact found by Buxton and Sharpe[4] that the effect of over mass-balancing the tab can be eliminated by over mass-balancing the control surface.)

(b) *Branch II.* The flutter modes given in the previous report[6] [Fig. 7, $n_{fl}/n_B = 0{\cdot}22$] show that, especially near resonance ($f_\beta = f_\gamma$) the modes for Branch II will consist predominantly of control surface rotation (β) and tab rotation (γ). The behaviour of the flutter frequencies and the nature of the resonance phenomenon on this branch support this statement.

(c) *Branch III.* In the same way as under (b), it may be concluded from the previous report[6] (flutter modes for I), that especially near resonance ($f_z = f_\gamma$) the modes for Branch III will consist predominantly of wing translation (z) and tab rotation (γ).

(d) *Branch IV.* The behaviour of the flutter frequencies, the nature of the resonance phenomena and the behaviour when f_β/f_z tends to infinity make it probable that the flutter modes corresponding to the Branch IV consist predominantly of wing translation (z) and tab rotation (γ), particularly in the case of resonance. There are, however, no calculations of flutter modes to support this statement.

3.1.5. *Existence of a region free from flutter.*—It was shown[6] that for the case $f_\beta = 0$ there is a region free from flutter in the neighbourhood of $p_t = 0$ if the values of i_t and q are small enough. Figs. 3 to 8 show that these ranges become smaller and smaller with increasing f_β, sometimes disappearing entirely. However, if the value of f_γ is increased a gap appears between the branches on the right side and the left side when f_γ/f_β exceeds the figure $1{\cdot}2$ (Figs. 4c, 5d). For $f_\gamma/f_\beta \geqslant 2$ and $f_\gamma/f_z \geqslant 2$ nearly the whole region between the asymptotes is free from flutter (Figs. 3d, 4d, 5e, f). It would appear that the condition $f_\gamma/f_\beta \geqslant 2$ ensures that the Branches II or III are shifted near or to the right of the right-hand asymptote whilst the condition $f_\gamma/f_z \geqslant 2$ has a corresponding effect on Branch I on the left-hand side of the p_t range (Figs. 4d, 5c). In all curves which had been considered for getting these conditions for a region free from flutter the condition $f_\beta/f_z \leqslant 1$ was operative. Because of the lack of further curves with $f_\beta/f_z > 1$ and $f_\gamma/f_z \geqslant 2$, it is not certain whether the condition $f_\beta/f_z \leqslant 1$ is really necessary or not. For reasons of caution this condition must, however, be kept in mind.

For sufficiently high values of f_β/f_z another statement will probably be true, viz. that the region between the asymptotes for which the tab c.g. is forward of the hinge will be free from flutter. This may be conjectured from Figs. 8e, f, but a sufficient number of calculations does not exist to make any quantitative statement possible.

In any case it can be stated that for sufficiently high values of f_β/f_z there are only high critical speeds, if any, for over mass-balanced tabs, because for $f_\beta \to \infty$ this range of p_t must become flutter-free.

The above mentioned conditions $f_\gamma/f_\beta \geqslant 2$; $f_\gamma/f_z \geqslant 2$; $f_\beta/f_z \leqslant 1$ are, however, sufficient to ensure that the region between the asymptotes is free from flutter. (Incidentally, the first of the three inequalities can be omitted if the third is added because the first follows from the second and the third).

The conditions for the existence of these asymptotes, and where they lie, will be investigated in section 3.1.7.

If in addition to the condition $f_\beta/f_z \leqslant 1$ only the condition $f_\gamma/f_\beta \geqslant 2$ is satisfied, Branch I always lies, for the examples investigated in the previous report[6] and the present report, almost entirely in the region in which the tab c.g. is before its hinge. In this case it can therefore be stated that the range between $p_t = 0{\cdot}1$ and the right hand asymptote is free from flutter if for this asymptote we have $p_t \geqslant 0{\cdot}1$.

3.1.6. *Zero critical speeds.*—It has been shown[6] that the critical speed may become zero* in a certain p_t-range if $f_\beta = f_\gamma = 0$ (*see* Fig. 7a, Branch II). If f_β or f_γ or both are increased from the value zero these zero-critical speeds will disappear, but for small values of f_β and f_γ Branch II will lie near the above mentioned range of p_t. Therefore, a knowledge of the position of the zero critical speeds is essential for those cases for which $f_\beta \neq 0$ and $f_\gamma \neq 0$. We have investigated the dependence of the left-hand boundary of the range of zero critical speeds on the structural parameters. The p_t values of these 'zero limits' are plotted in the curves in Figs. 12 and 13 as abscissae whilst the ordinates of the curves are the non-dimensional tab span q. It can be seen from these figures that the zero limits move into a range of greater p_t if q or i_t decreases or i_c increases. The influence was already known for i_t and q from the previous report[6]. The influence of these parameters is also apparent in the curves $v_c = f(p_t)$: the influence of i_c may be seen in Fig. 9 and of q in Figs. 7 and 8. The (theoretically exact) independence of the position of the zero limit of the value of μ may be seen in Fig. 10.

3.1.7. *Asymptotes defining the flutter-free region.*—It was found in section 3.1.5 that the asymptotes are the boundaries of the region free from flutter if $f_\gamma/f_z \geqslant 2$ and $f_\beta/f_z \leqslant 1$. For this reason it is important to consider how the position of these asymptotes depends on all parameters under consideration except p_c, that is, on i_c, i_t, q and μ. The other parameters have no influence because the position of the asymptotes is independent of the stiffness parameters.

In the interests of accuracy it is emphasized at this point that there are generally more than two asymptotes to the curves $v_c = f(p_t)$. In this report, however, we are concerned only with the two asymptotes which lie nearest to the v_c axis and which approach each other if q increases. In Figs. 10c, 14 and 15 the values of q have been plotted against those values of p_t which are the abscissae of the asymptotes of the curve $v_c = f(p_t)$ for the above mentioned values of q. The results are bell-shaped curves (*see* for example, Fig. 15a for the parameter value $i_t = 13 \cdot 1$). The region inside the bell is stable because no flutter can be excited if the parameters (q and p_t) for the system lie in this region and the system has suitable values of the natural frequencies f_z, f_β and f_γ, as mentioned above.

The branch to the left of a bell-shaped curve (apart from its right-hand boundary) hardly depends at all on the parameter i_t (Fig. 14). With increasing i_c or increasing μ it moves to the left (Figs. 15 and 10). From the work of Buxton[4], it is known that this branch also moves to the left if p_c decreases (by an over mass-balance of the control surface).

The branch on the right of a bell-shaped curve moves a little to the right if μ or i_c increases. With increasing i_t it moves, however, to the left. By plotting the right-hand branch against the parameter p_{ct} of the mass-coupling between control surface rotation and tab rotation (Fig. 14e) it is seen that the dependence of these curves on i_t is much less than in the corresponding curves in Fig. 14a. The variable more appropriate to the right-hand branch is therefore the parameter p_{ct} whilst for the left-hand branch the variable p_t is on all counts the more suitable one.

The movement of the two branches of the bell-shaped curve as described above determines the position of the top of the bell. The effect is that these tops have a very low value of q if i_t is large. That means that for such values of i_t there may be no asymptotes of the kind considered here, if q is moderately large. For such values of i_t and q it is not possible to obtain a range of p_t free from flutter by choosing the values of the natural frequencies appropriately, although the critical speed can, of course, theoretically be increased as much as desired by increasing the natural frequencies.

3.2. *Influence of the Chord Ratios E_1 and E_2.*—Although the parameters i_c, i_t and p_t have been introduced instead of Küssner's parameters ϑ_1, ϑ_2 and $\mu_2\sigma_2$, it will be seen that the results still depend on E_1 and E_2 (Figs. 13a, 16a), even if the more suitable variable p_{ct} has been introduced

* The branch of upper critical speeds which meets the p_t-axis at the point where the range of zero critical speeds begins (Fig. 3a) should have a vertical tangent at that point. This follows from the fact that the curves for V_c^2 (*see* Fig. 13 in R. & M. 2418[5]) do not touch the p_t-axis. The author is indebted for this remark to Mr. G. H. L. Buxton, who noticed this when translating the report.

for the right-hand branch (Fig. 13b). To minimise this effect a new parameter $\bar{q} = jq$ is introduced in place of q on the following principle. Let qs be the tab span, E_2c_w the tab chord, s the span and E_1c_w the chord of the control surface of the system considered. We now choose an equivalent system with the same control surface span as above but the chord ratios $E_1' = 0\cdot3$ and $E_2' = 0\cdot075$ irrespective of the values of E_1 and E_2. The tab span $\bar{q}s$ will be determined in such a way that for both systems the following ratio is equal; aerodynamic moment about the control surface hinge per unit angle between the tab and the control surface (b_3 in aerodynamics notation) over the corresponding moment caused by an angle of 1 radian between the control surface and the wing (b_2 in aerodynamics notation). The value of $\bar{q} = jq$ found in this way will be used in the following investigations instead of q.

With the help of Dietze's expression for $\omega = 0$ the expression for j is easily found to be

$$j = 0\cdot714 \; \frac{2f_8(E_1)\,f_1(E_2) + f_{10}(E_1, E_2)}{2f_8(E_1)\,f_1(E_1) + f_{10}(E_1)} = F\left(\frac{E_2}{E_1}\,;\; E_1\right) ,$$

where the functions f_i have the same meaning as in Dietze's report[7]. For an easier treatment of this formula the function $j = F(E_2/E_1\,;\,E_1)$ with E_2/E_1 as an independent variable and E_1 as a parameter has been plotted in Fig. 2b.

If the variable q is transformed into $\bar{q}$, Figs. 13a and 16a change into 13c and 16b respectively, and the curves for different values of E_1 and E_2 lie closer together than before the transformation. A still better coincidence of the right-hand branches of Figs. 16b or 13c is obtained if the variable p_t is transformed into the variable p_{ct} (Figs. 16c and 13d). The value of the introduction of the parameters i_t, p_t, $\bar{q}$ instead of ϑ_2, μ_2, $\sigma_2 q$ and of the parameter p_{ct} for the right-hand branch can also be demonstrated for a curve representing the function $v_c = f(p_t)$. In each of the Figs. 11a to e are plotted two curves $v_c = f(p_t)$. For the full lined $E_1 = 0\cdot3$ and $E_2 = 0\cdot075$ and for the dotted ones $E_1 = E_1' = 0\cdot3$ and $E_2 = E_2' = 0\cdot04$. (The parameters for the dotted curves will be distinguished by a dash.) In Fig. 11a, the systems compared have the same values of ϑ_2 and $\mu_2\sigma_2$. The two curves are quite different. No improvement is obtained by transforming only the parameter $\mu_2\sigma_2$ into p_t (Fig. 11b), but if curves with the same values of p_t and i_t are compared (Fig. 11c) they become more alike. The coincidence becomes still better if the curves compared have not the same value for q as in Fig. 11c but the same value for $\bar{q}$, as in Fig. 11d. In view of the benefit derived with the parameter p_{ct}, the right-hand branch of Fig. 11d has finally been plotted against p_{ct} in Fig. 11e. It is then seen that the coincidence of the two curves near the asymptote and near the zero critical speed is better in Fig. 11e than in Fig. 11d. The two minima of p_{ct} for the full lined and the dotted curve in Fig. 11e are, however, more diverse than the corresponding values of p_t in the curves of Fig. 11d. The reason for this is probably that to the branches concerned belong flutter modes which consist predominantly of wing translation (z) and tab rotation (γ) (Branch III, see section 3.1.4). These flutter cases, therefore, will depend more on the coupling p_t between these two degrees of freedom (z), (γ) than on the coupling p_{ct} between the two degrees of freedom control surface rotation (β) and tab rotation (γ). It should be emphasized, however, that this statement applies only to the arc of a curve for which $f_\beta = f_\gamma = 0$. This statement does not contradict previous statements that the best variable for the zero critical speeds and the right-hand asymptotes is the parameter p_{ct}.

Therefore, it can be said that by introducing the non-dimensional parameters i_c, i_t, p_t, $\bar{q}$ the influence of the geometry of the system on curves becomes very small. Further, for characterising the situation of the right-hand asymptote and of the zero critical speeds of the curves $v_c = f(p_t)$ a still greater improvement is obtained by introducing the parameter p_{ct} instead of p_t.

4. *The Trimming Tab Criterion I.*—4.1. *Derivation.*—In order to establish a simple criterion for freedom from flutter of a tab of the type considered we use the fact stated in section 3.1.5 that the region between the asymptotes of the curves is free from flutter if the following conditions are satisfied:

$$\frac{f_\gamma}{f_\beta} \geqslant 2k_1 \;;\; \frac{f_\gamma}{f_z} \geqslant 2k_2 \;;\; \frac{f_\beta}{f_z} \leqslant k_3 \quad \ldots \quad (1)$$

where $k_i \simeq 1$ $(i = 1 \ldots 3)$ . .. . (2)

If the conditions (1) and (2) are satisfied, the required criterion has only to have the additional conditions for the structural parameters that the points corresponding to them lie in the interior of the bell-shaped curves in Figs. 14 and 15. In order to represent the bell-shaped curves in Figs. 14 and 15 analytically it has been assumed that they are of the form:

$$\frac{q\,[p_t - a_3 + a_4\,(a_2 - q)]^2\,a_1}{a_2 - q} = 1\,. \qquad (3)$$

The dependence of the free constants a_1, a_2, a_3, a_4 on i_c and i_t has been determined graphically in such a way that the curves in Figs. 14 and 15 are represented as well as possible by equation(3). Sufficient curves do not exist to determine the dependence of the a_i on μ and p_c. (No curves exist for determining the dependence on p_c). It should, therefore, be stated that the following relations are derived under some restrictions for the parameters, *i.e.*, for cases in which

$$\mu = 5\cdot718\,;\quad i_c = 1 - 7\cdot78\,;\quad p_c = 0\,;\quad i_t = 1\cdot31 - 13\cdot1$$
$$E_1 = 0\cdot3\,;\quad E_1 = 0\cdot075\,. \qquad (4)$$

The relations found are:

$$\begin{aligned} a_1 &= 0\cdot222 + 0\cdot013i_t + i_c(-0\cdot0145 + 0\cdot00149i_t) \\ a_2 &= 1\cdot12 - 0\cdot0267i_t + i_c(0\cdot0365 - 0\cdot001i_t) \\ a_3 &= -\,0\cdot164 - 0\cdot0965i_t + i_c(0\cdot0778 + 0\cdot00489i_t) \\ a_4 &= \frac{i_t}{0\cdot448 + 0\cdot277i_t} - 0\cdot238i_c\,. \end{aligned} \qquad (5)$$

The graphical significance of the constants a_2, a_3, a_4 and $d = 2\sqrt{[(2a_2 - 1)/a_1]}$ is shown in Fig. 2a. From the expression (3) we derive the following inequality for p_t.

$$a_3 - a_4(a_2 - q) - k_5\sqrt{\left(\frac{a_2 - q}{a_1 q}\right)} \leqslant p_t \leqslant a_3 - a_4(a_2 - q) + k_4\sqrt{\left(\frac{a_2 - q}{a_1 q}\right)} \qquad (6)$$

with

$$q \leqslant a_2 \qquad (7)$$

and

$$k_4 \simeq 1, \quad k_5 \simeq 1\,. \qquad (8)$$

In order now, to include the dependence of this approximate formula on E_1 and E_2 the parameter $\bar{q}$ is introduced in place of q and, in the right-hand part of the inequality, the parameter $p_{ct} = (E_2/E_1)i_t + 2p_t(E_1 - E_2)/E_1$ instead of p_t, remembering from sections 3.1.7 and 3.2 that the bell-shaped curve is nearly independent of the values of E_1, E_2 if they are plotted in terms of these new variables. In this way we get for the right-hand branch of the bell-shaped curves the approximate formula

$$\frac{2}{3}p_{ct} = \frac{2}{3}\frac{E_2}{E_1}i_t + \frac{4}{3}\left(\frac{E_1 - E_2}{E_1}\right)p_t \leqslant 0\cdot167i_t + a_3 - a_4(a_2 - \bar{q})$$
$$+ k_4\sqrt{\left(\frac{a_2 - \bar{q}}{a_1\bar{q}}\right)} \qquad (9)$$

and for the left-hand branch

$$p_t \geqslant a_3 - a_4(a_2 - \bar{q}) - k_5\sqrt{\left(\frac{a_2 - \bar{q}}{a_1\bar{q}}\right)} \quad . \quad .. \quad .. \quad .. \quad .. \quad .. \qquad (10)$$

In order that inequalities (9) and (10) may have a real sense, a condition corresponding to inequality (7) must be added, viz.

$$\bar{q} \leqslant a_2 . \quad .. \quad .. \quad .. \quad .. \quad .. \qquad (11)$$

The coefficients on the left-hand side and that of the first element on the right-hand side of inequality (9) have been chosen in such a way that for $E_1 = 0\cdot 3$ and $E_2 = 0\cdot 075$ the inequality (9) becomes identical with the right-hand part of inequality (6).

The inequalities (9) and (10) indicate that for safety against flutter the mass coupling p_t between wing translation and tab rotation must not fall below a certain value and that the dynamical mass coupling p_{ct} between control surface rotation and tab rotation must not exceed a certain value.

4.2. *Range of Validity*.—The formulae (1), (2), (5), (8), (9), (10), (11) together form the new criterion with the help of which the structural parameters can be chosen in such a way that flutter is eliminated for the three degrees of freedom: wing translation, control surface rotation and tab rotation. In order to determine the range of validity of these formulae the hypotheses employed in deriving them will be stated here.

The criterion was exactly satisfied by the curves corresponding to the parameters given in Fig. 3. It was then assumed that the asymptotes were also the boundaries of the region free from flutter for the range of parameters given in equation (4) provided that the frequencies satisfied the formulae (1), (2). While this was a kind of extrapolation, the dependence of the position of the asymptotes themselves on the values of these parameters was found from Fig. 14 by an interpolation. The introduction of the parameters i_c, i_t, p_t, p_{ct} and $\bar{q}$ enables us to enlarge the range of validity of the formulae for the asymptote positions to the following range of chord ratios

$$E_1 = 0\cdot 2 \text{ to } 0\cdot 4 \; ; \quad \frac{E_2}{E_1} = 0\cdot 13 \text{ to } 0\cdot 25 \quad .. \quad .. \quad .. \quad .. \quad .. \qquad (12)$$

This has actually been shown to be true only for the values of μ, p_c, i_c and i_t given in Fig. 16. Because this extension was largely based on physical considerations it was assumed that it would be permissible not only for these special parameter values but also for the whole range of the parameters given by equations (4) and (12) together, this being of course an extrapolation.

Regarding the influence of μ it could be shown by some examples that if the value of μ is greater than $5\cdot 718$—as is usually the case—the range between the asymptotes of the curve $v_c = f(p_t)$ is in any case larger than it would be if the coefficients k_i ($i = 1, 2, \ldots 5$) were put equal to unity. The range is enlarged particularly on the left-hand side, *i.e.*, the side for which the tab c.g. lies forward of the hinge.

The effect of μ could, therefore, be taken into account by making the constants k_4 and k_5—especially k_5—greater than unity. For the moment however we cautiously refrain from doing this because no calculations have been made which prove that the whole region between the asymptotes is also free from flutter for great values of μ if the frequencies satisfy the expressions (1), (2). From the example in Figs. 10a and 10b we notice only that the range of p_t has not become smaller if μ has increased: but $f_\beta = f_\gamma = 0$ in this example. It seems probable that for values of f_z, f_β, f_γ satisfying the conditions (1), (2) an increase in μ will have a more favourable effect upon the flutter free range than it is seen to have in Figs. 10a and 10b, because the curves will then be pressed against their asymptotes and the greater the value of μ, the greater the range between the asymptotes.

Altogether, it can be said that the criterion has been found valid for the range of values given in Table 1, partly by interpolation and partly by extrapolation, starting from the values given in Fig. 3. Therefore, the more the given parameters differ from those in Fig. 3, the more the constants $k_1, \ldots k_5$ will differ from 1. An estimate of this difference must be kept for further experience to decide, perhaps in the form of wind tunnel tests.

For this reason it seems expedient to keep the constants $k_1, \ldots k_5$ in the criterion, so as to retain the possibility of modifying it in the light of further experience.

By giving those constants appropriate values we shall be able to allow for the inaccuracy of the formulae arising from the simplifications made in its derivation, and at the same time allow for the influence of structural damping and friction as well as the fact that the theoretical air forces are different from those occurring in practice. The last mentioned facts will probably mean that the values of k_3, k_4 and k_5 are in practice greater than unity or that the constants k_1, k_2 are less than unity.

4.3. *Introduction of a Safety Margin.*—Up to the moment no safety margin has been introduced. In practice a certain safety margin would be provided by the friction and structural damping which have not been taken into account in the calculation; but it is possible that this margin is not large enough. It seems reasonable to introduce this margin by the requirement that for an increase or decrease of p_t by certain amounts δ_1, δ_2 the system shall still be free from flutter. That is, we replace in equation (9) p_t by $p_t + \delta_1$ and in inequality (10) p_t by $p_t - \delta_2$. The values of δ_1, δ_2 are proposed arbitrarily as $0 \cdot 5$.

With the safety margins included the criterion is given in full in the summary (section 8) as "Trimming Tab Criterion I".

5. *Simplified Forms of the Trimming Tab Criterion I.*—5.1. *Derivation of Trimming Tab Criterion II.*—The second of conditions (1) of section 4.1 is sometimes inconvenient to satisfy. But it may be avoided if we remember the last section in section 3.1.5 from which it may be deduced that if $f_y/f_z \geqslant 2$ is not satisfied sufficient conditions for preventing flutter are still obtained if the condition (10) is replaced by

$$p_t \geqslant k_6 \quad \ldots \quad (13)$$

where

$$k_6 \simeq 0 \cdot 1 \,. \quad \ldots \quad (14)$$

This simplified criterion is given in full in the summary (section 8) as Trimming Tab Criterion II. The range of validity of this criterion is the same as that of the Trimming Tab Criterion I. However, it must be emphasised that the condition (13) has not been proved by very many examples.

5.2. *Derivation of Trimming Tab Criterion III.*—To find a still more simplified criterion for use we start from the Trimming Tab Criterion I and first simplify in it the condition (9). We write condition (9) in the abbreviated form

$$p_{ct} \leqslant \zeta\,(\mu, i_c, i_t, p_c, \bar{q}) \,. \quad \ldots \quad (15)$$

Then the function ζ is determined anew from the right-hand branches in Fig. 14. Though trying to get a rather simpler approximation of the function ζ care is taken, however, that the differences between approximation and exact representation do not become too great and are in any case on the safe side with regard to flutter prevention. In this way the requirement (9) becomes simplified to

$$p_{ct} \leqslant -0 \cdot 14 + 0 \cdot 25 i_c - \frac{i_c}{i_t}\left(0 \cdot 634 + \frac{1 \cdot 27}{i_c}\right) + \frac{1}{\bar{q}}\left(0 \cdot 751 + 0 \cdot 69\frac{i_c}{i_t} - 0 \cdot 0435 i_c\right) \quad \ldots \quad (16)$$

in which the safety margin, added later to condition (9), is already present. The condition (7) which was necessary in order that the expression (9) had a real meaning is for that reason no longer necessary for the expression (16) derived from (9). The additional restriction (7) would, however, be necessary also for inequality (16) if our criterion should be right within just the range of the parameters given in section 4.2; but normally the non-dimensional mass μ of the wing is greater than the value $\mu = 5 \cdot 718$ for which the formula (9) has been derived. For such values of μ the condition (7) will be too restrictive. To understand this it will be remembered that a_2 is the top value of $\bar{q}$ in the bell-shaped curves (*see* Fig. 2), and according to Fig. 1 a_2 increases very much if μ increases. Therefore, the value of a_2 will be for the normal practical values of μ, *i.e.*, for values of μ greater than $5 \cdot 718$, greater than the value of a_2 given by relations (5). For this reason it seems intelligible to omit the additional condition (7) when deriving here a criterion for practical use.

Condition (16) can now be transposed into a form similar to that of the Collar-Sharpe Criterion. Remembering that

$$\bar{q} = jq \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

and (*see* the list of symbols, section 11)

$$\frac{p_{ct} \cdot q}{i} = \frac{I_t + M_t x_t (E_1 - E_2) c_w}{I_c p^2}$$

with

$$p = \frac{E_2}{E_1} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

we finally derive from condition (16)

$$\frac{I_t + M_t x_t (E_1 - E_2) c_w}{I_c} \leqslant C p^{3/2} = C_1 \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

with

$$C = \frac{\sqrt{p}}{j} \left\{ -0 \cdot 0435 + \frac{0 \cdot 751}{i_c} + \frac{0 \cdot 69}{i_t} + jq \left[0 \cdot 25 - \frac{0 \cdot 14}{i_c} - \frac{1}{i_t} \left(0 \cdot 634 + \frac{1 \cdot 27}{i_c} \right) \right] \right\} \quad \ldots \quad (20)$$

and

$$j = \sqrt{p} \, [0 \cdot 93 + 1 \cdot 28 \, (1 \cdot 97 - E_1) \, (0 \cdot 745 - p)] \, . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

The set of formulae (19), (20) and (21) represent the simplified form of the requirement (9) we wished to derive. For a complete criterion like the Trimming Tab Criterion II we should add the requirements

$$\frac{f_\gamma}{f_\beta} \geqslant 2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

$$\frac{f_\beta}{f_z} \leqslant 1 \; ; \quad p_t \geqslant k_6 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

but it is still possible to make some further simplifications. The first of conditions (23) was only to be added in the criteria I and II because the computed material was only available within this range. However, the necessity for this condition not having been proved it is considered that it should be omitted in a criterion proposed for practical use. Further, the condition $p_t \geqslant k_6$ may be written in the dimensional form

$$M_t x_t \geqslant 0 \cdot 4 k_6 \rho c_w c_t^2 q s \; . \; \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

The formulae (19), (20), (21), (22), (24) constitute a recommendation for practical use to prevent flutter which are summarised in a slightly generalised form in the Summary (section 8) as Trimming Tab Criterion III.

The range of validity of the Criterion III just derived is with respect to all structural parameters except the non-dimensional mass μ the same as that of Criterion II. With regard to μ we have made the additional assumption that μ is normally greater than $5 \cdot 718$ in order that the formula (19) would be valid without the restriction (7). This is an extrapolation in respect to μ that is only justified by some examples in Fig. 10. Because of this simplification and because we have omitted the first of conditions (23) the Criterion III is not so exact as the Criteria I and II. In this respect Criterion III can therefore be regarded only as a " recommendation to prevent flutter ". Nevertheless, it has the advantage over Criteria I and II of greater simplicity. Furthermore it is to be expected that by attaching correction factors to C in condition (19) and to the numerical value 2 in (22) (*see* Summary) it will be possible to adapt the Criterion III easily to practical experience even in a wider range of the values of the structural parameters than assumed until now.

6. *Comparison Between Criterion III and the Collar-Sharpe Criterion.*—If the existing Collar-Sharpe criterion for spring tabs [10, 11] is applied to the case of a trimming tab by putting the follow-up ratio N equal to zero the criterion takes exactly the form (19) of Criterion III with K' replacing C and K replacing C_1. The important difference is that whereas C and C_1 in Criterion III are functions of i_c, i_t, p, q, and E_1 the Collar-Sharpe criterion in its first form [10] has K constant and in its second form[11] has K' constant. These relationships obtained by Collar and Sharpe were, of course, admittedly approximate and were in any case obtained on a different basis. No attempt was made, by means of frequency ratio requirements, to ensure that the curve of flutter speed against tab out-of-balance moment lay wholly on the under-balance side of the asymptote, and the Collar-Sharpe relationships in fact define the position of the nose of the curve, not of the asymptote. It is, nevertheless, instructive to compare the criteria for the case of the trimming tab, and in Table 3 values of C and C_1 as given by conditions (19), (20) and 21) for a range of values of the relevant parameters are compared with the Collar-Sharpe values for K' and K. For all cases except Case 1 the Sharpe (K') Criterion is more restrictive than that of (19), (20) and (21). In the extreme Case 1 the small value of C comes from high values of i_t, i_c and small values of p, q and E_1. Following Cases 1 to 5 in Table 3 step by step we see that C increases by decreasing i_c and i_t and increasing p and E_1. Comparing 1 and 1a and 5 and 5a respectively it is seen that for small values of i_c, i_t the value of C decreases if q increases, but for large values of i_c and i_t the value of C increases if q increases. Inspection of the formulae (20), (21) confirms these conclusions drawn from Table 3 and shows in addition that for small values of i_c and great values of q the value of C might increase if i_t increases. Further, it should be noted that the influence of i_c upon C is much greater than that of i_t. This influence of i_c is in such a direction that a large value of I_c is not so beneficial as it would be if C in formula (19) were a constant, as K' is in the Sharpe criterion. Comparing finally the values of C and C_1 in Table 3 it appears that the constant K of the Collar-Sharpe Criterion varies much more than the constant K' of the Sharpe Criterion, thus confirming that the latter is at any rate an improvement on the former.

The further requirements (22) and (24) of Criterion III have no counterpart in the criteria of Collar and Sharpe. This means that while conditions (19), (20) and (21) is normally less restrictive than the Sharpe (K') Criterion the requirements (22) and (24) provide additional restrictions. However, on the whole, Criterion III will in practice give normally some relief compared with the Sharpe (K') Criterion with a zero follow-up ratio N and only prove more restrictive in exceptional cases of high (non-dimensional) moments of inertia of the control surface or unusually flexible tab connections.

It is not proposed to pursue this comparison any further at the present stage as it will be considered again in the later report on spring tabs ($N \neq 0$) for which the Collar-Sharpe criterion is more directly applicable and comparison therefore more relevant.

7. *Practical Deductions and Applications.*—7.1. *Theoretical Deductions from the Criteria.*—To draw deductions of a qualitative kind from the criteria we note that the formulae (9) and (10) represent a synopsis of the curves of Figs. 14 and 15, considering also Fig. 16 for the influence of

the parameters E_1 and E_2. From these curves we derive the following guiding principles for obtaining a system as advantageous as possible. In order to obtain a range of p_t which is as wide as possible and lies as far as possible in the region where the tab c.g. is aft of its hinge, we would have, besides satisfying the conditions (1), (2) for the natural frequencies, to make the value of i_c as high as possible and the values of $\bar{q}$, i_t and E_2/E_1 as low as possible.

7.2. *Practical Interpretation of the Deductions.*—The designer cannot simply comply with the recommendations just given in section 7.1 because he has to consider other points of view as well.

He will not make the value of i_c (non-dimensional moment of inertia of the control surface) exceptionally high because this would promote flutter with the two degrees of freedom wing rotation, control surface rotation, and besides this would involve additional weight.

The value E_1 of the ratio of the control surface chord to the lifting surface chord will be fixed by aerodynamic considerations for the aeroplane as a whole.

With regard to the tab, the designer must primarily meet the desired requirement for aerodynamic performance of the tab system. For our purposes it will be sufficient to define the performance of a trimming tab system in the following way. Two trimming tab systems will have by definition the same performance if their maximum tab deflections give the same lift to the system. If we compare only systems the planforms of which are identical except in regard to the tabs—as is done here—this definition can be simplified for our purposes if the tab chord has not exceptionally high or low values. Two trimming tab systems of this kind will have nearly the same aerodynamic performance if the ratio of the aerodynamic surface hinge moment due to the maximum tab angle to the corresponding moment due to a certain required control surface angle is the same for both systems. Remembering the definition of $\bar{q}$ in section 3.2 we see that for such equivalent trimming tab systems the following equation holds

$$\gamma_{\max 1} \cdot \bar{q}_1 = \gamma_{\max 2} \cdot \bar{q}_2 .$$

The value of $\bar{q}$ is thus determined by the standard of aerodynamic performance required of the tab system and by the value of $\gamma_{\max}$ adopted.

Since from the flutter point of view the smaller $\bar{q}$ is the better, $\gamma_{\max}$ should be chosen as high as possible compatible with aerodynamic considerations.

It is possible to make the value of i_t (non-dimensional moment of inertia of the tab) small by placing the load-bearing parts of the tab as far as possible in the neighbourhood of the hinge, and by giving the balance arm of the tab a suitable length*. According to the author's experience i_t depends only slightly on E_2 if different tabs with similar kinds of construction are compared for the same aircraft, *viz.*, i_t increases slightly if E_2 decreases.

We can now regard the values of E_1, i_c, $\bar{q}$ and i_t as being fixed in accordance with the above mentioned considerations. It remains only to choose the value of E_2.

For the following reasons it appears that this value should be as small as possible:

(a.1) The range of values of p_t increases a little if E_2/E_1 decreases (*see* Figs. 16b, c).

(a.2) The tab frequency f_γ increases if the stiffness of the tab control circuit is regarded as constant because the moment of inertia of the tab I_t decreases if E_2 decreases.

This is true even if the aerodynamic efficiency is kept constant.

---

* It would be possible to get a particularly small moment of inertia of the tab by placing additionally the tab hinge a reasonable distance aft of the leading edge of the tab, so that the hinge and the c.g. of the tab without mass balance would be near to each other. But then the tab would have an aerodynamic balance. According to the experience of Voigt, Walter and Heger[8, 9] on ailerons this might have the same effect as reducing the stiffness of the control circuit of the tab, *i.e.*, a bad effect.

A limit to the possible decrease of the tab chord is given by the following considerations:

(b.1) In order to keep the same aerodynamic efficiency a tab with a smaller chord must have a greater span. But the tab span cannot be longer than the span of the main control.

Further a lower limit to the tab chord itself is given by the fact that the efficiency of tabs with very small chords is greatly reduced by boundary layer effects.

(b.2) When E_2 becomes very small the value of i_t will increase by a certain amount even though only tabs for the same aircraft with similar designs are considered.

(b.3) A tab with a small chord will have a large span and such a tab will not be very stiff against torsion, especially if more than two hinges should be necessary, which often decreases the torsional stiffness in itself.

The advantage cited in (a.1) may be nullified by the disadvantage in (b.2) or eventually even outweighed.

But there remains still one advantage in decreasing the tab chord, *viz.*, that the tab frequency f_y is increased. This is valid in so far as with very small tab chords the torsional stiffness has not become so small that the tab frequency is not reduced by actual torsion of the tab itself, as in (b.3). If it should not be possible to get a region free from flutter according to the formulae (9) and (10) which is wide enough by reducing the non-dimensional moment of inertia of the tab i_t we have to reduce the parameter $\bar{q}$ and with it the efficiency of the tab. This is possible by reducing the tab surface area, preferably by means of a reduction in the tab chord.

7.3. *Application of the Criteria to Actual Lifting Systems.*—That tabs of the kind treated here are sometimes liable to flutter in practice is shown by German experience[3] and by the example of the experimental *g*-restrictor tab of the Mosquito[4]. Although British experience has indicated that the stiffness of trimming tab connections is normally high enough for flutter to be unlikely, at least below a certain speed, current official requirements (about which more is said later in section 8) do not entirely exclude the possibility. Furthermore, if backlash should develop on a tab in service it is evident that the tab would not be so susceptible to oscillation trouble if the out-of-balance moment lies within the limits of the criteria.

In order that our criteria can be used for real aircraft it remains to explain how to replace a real wing, rudder or tailplane by our simple system.

A strict investigation has not been made into this question, and the following recommendations, therefore, should be regarded as provisional.

The spans of our idealised wing and control surface will normally be taken equal to the span of the actual control surface. Some reduction of this span will be made only at that end where the control surface ends together with the lifting surface. The tab will be given the same span as it has in reality (perhaps with a similar reduction as mentioned above).

The wing chord will be taken as the mean chord of that part of the surface which is supplied with a control surface. The idealised control surface chord and tab chord will be found by calculating the root-mean-squares of these chords.

The non-dimensional quantities q, E_1, E_2, μ, i_c, i_t, p_c, p_t can then be calculated and our criteria applied.

If the movement of the lifting surface to be taken into account is more rolling than vertical translation, or contains pitching, the calculation of the p_t value in condition (10) should be modified. $M_t x_t$ in the expression for p_t should then be replaced by

$$\frac{\Sigma T m_t \,.\, xy}{y_m}$$

where

ΣT the summation over the tab,

m_t a mass element of the tab,

x the distance of the element aft of tab hinge,

y the distance of an element of the lifting surface (including control surface and tab) from the axis of rolling of the lifting surface (*i.e.*, in case of a rudder, distance of the element above the axis of twist of the fuselage).

y_m the mean y-value of all elements of the part of the lifting surface (including control surface and tab) under consideration.

(For the manner of calculating mass couplings of parts of actual aeroplanes, *see* Duncan, Ellis and Gadd[12].)

Similar considerations would have to be taken into account if p_c had to be calculated. However, the criterion should not be applied to cases where the mode of the lifting surface contains a considerable amount of pitching.

The reason why the treatment of p_t in condition (10) is different from that in condition (9) is that condition (10) implies a restriction of the coupling between wing movement (translation) and tab rotation whilst condition (9) implies restriction of the coupling between control surface rotation and tab rotation.

Finally it should be mentioned that in the case where the actual lifting surface (*e.g.*, wing) has a greater span than the control surface (*e.g.*, aileron) the part of the wing without control surface will damp the tab flutter motion in those modes in which the wing motion plays an essential part. (It is assumed here that the system is flutter free without the tab). In consequence of this, the constant k_5 in condition (10) may then be allowed to have higher values.

The above suggested modifications to inequality (10) will also apply to the inequalities (13), (23) and (24), and to the corresponding inequalities listed in the Summary (section 8) as (B), (E) and (H).

8. *Summary and Discussion of Results.*—We summarise the results of this paper, chiefly from the point of view of their application in practice.

If the system investigated with the degrees of freedom wing translation, control surface rotation and tab rotation flutters at all there will be a low critical speed

(1) if the tab c.g. is aft of the hinge and there is resonance

  (a) between wing translation and tab rotation, or

  (b) between control surface rotation and tab rotation;

(2) if the tab c.g. is forward of the hinge and there is resonance between wing translation and control surface rotation, and if in addition the natural frequency of tab rotation is higher than the natural frequency of wing translation.

The flutter modes appertaining to these cases consist predominantly of those degrees of freedom which are just in resonance.

If the natural frequencies of the tab and the control surface are zero, there is a range of zero critical speeds which lies predominantly in the region where the values of p_t are positive (tab c.g. aft of its hinge), but goes further into the region with negative values of p_t (*i.e.*, tab c.g. before its hinge) the greater the moment of inertia of the tab, the greater the aerodynamic efficiency (b_3/b_2), and the smaller the moment of inertia of the control surface.

With regard to the flutter frequencies reference should be made to Table 2.

The fact that for appropriate values of the natural frequencies of the system investigated the range of values of p_t between two asymptotes of the curve $v_c = f(p_t)$ is free from flutter has been used to set up a flutter criterion which is given below as Criterion I. Successive developments of this criterion, with appropriate frequency conditions, give rise to Criteria II and III, also given below. In the order I, II, III the criteria become simpler though more approximate, but for most practical purposes Criterion III should be adequate. Factors k_1 to k_7 are introduced into the criteria to provide scope for further refinements in the light of experience.

Trimming Tab Criterion I.—A system with a trimming tab is free from flutter with the degrees of freedom wing translation, control surface rotation, tab rotation if the structural parameters satisfy the following inequalities, where all quantities k_i ($i = 1$ to 5) are provisionally taken equal to unity and $\delta_1 = \delta_2 = 0 \cdot 5$:

$$\frac{f_\gamma}{f_\beta} \geqslant 2k_1 \; ; \quad \frac{f_\gamma}{f_z} \geqslant 2k_2 \; ; \quad \frac{f_\beta}{f_z} \leqslant k_3 \quad \ldots \quad \text{(A)}$$

$$p_t \geqslant \delta_2 + a_3 - a_4(a_2 - \bar{q}) - k_5 \sqrt{\left(\frac{a_2 - \bar{q}}{a_1 \bar{q}}\right)} \quad \ldots \quad \text{(B)}$$

$$p_t \leqslant \frac{3}{4}\left(\frac{E_1}{E_1 - E_2}\right) \left[\frac{1}{6}\left(1 - 4\frac{E_2}{E_1}\right) i_t + a_3 - a_4(a_2 - \bar{q}) + k_4 \sqrt{\left(\frac{a_2 - \bar{q}}{a_1 \bar{q}}\right)}\right] - \delta_1 \quad \ldots \quad \text{(C)}$$

and

$$\bar{q} = jq \leqslant a_2 \, .$$

Here a_1, a_2, a_3, a_4 and j are given by the formulae (5) and (21).

It is to be expected that further practical experience will show that the coefficients k_1 and k_2 may be reduced whilst the coefficients k_3, k_4 and especially k_5 may be increased. We remember that δ_1, δ_2 are safety margins for p_t, arbitrarily chosen. If at the moment all k_i ($i = 1 \ldots 5$) are taken equal to unity, the first of conditions A is, of course, superfluous.

The range of validity of the tab flutter criterion is:

$$\mu = 5 \cdot 718 \; ; \quad i_c = 1 \text{ to } 7 \cdot 78 \; ; \quad p_c = 0 \; ; \quad i_t = 1 \cdot 31 \text{ to } 13 \cdot 1 \; ;$$
$$E_1 = 0 \cdot 2 \text{ to } 0 \cdot 4 ; \quad E_2/E_1 = 0 \cdot 13 \text{ to } 0 \cdot 25 \, .$$

The criterion will probably also be valid for values of μ up to $\mu = 50$. From calculations by Buxton and Sharpe[9], it seems probable that the above mentioned criterion is still valid if the elevator is over mass-balanced, in fact for an even greater value of k_5.

If the second of conditions (A) in the tab Criterion I cannot be satisfied one of the following criteria should be used.

Trimming Tab Criterion II.—A system with a trimming tab is free from flutter with the degrees of freedom wing translation, control surface rotation, tab rotation if the structural parameters satisfy the following inequalities, where all quantities k_1 ($i = 1$ to 5) are provisionally taken equal to unity, further $k_6 \simeq 0 \cdot 1$, and $\delta_1 = 0 \cdot 5$

$$\frac{f_\gamma}{f_\beta} \geqslant 2k_1 \; ; \quad \frac{f_\beta}{f_z} \leqslant k_3 \quad \ldots \quad \text{(D)}$$

$$p_t \geqslant k_6 \quad \ldots \quad \text{(E)}$$

$$p_t \leqslant \frac{3}{4}\left(\frac{E_1}{E_1 - E_2}\right)\left[\frac{1}{6}\left(1 - 4\frac{E_2}{E_1}\right)i_t + a_3 - a_4(a_2 - \bar{q})\right.$$

$$\left. + k_4\sqrt{\left(\frac{a_2 - \bar{q}}{a_1\bar{q}}\right)}\right] - \delta_1 \qquad \text{(F)}$$

with

$$\bar{q} \leqslant a_2$$

Here the values of a_1, a_2, a_3, a_4, $\bar{q}$ are to be computed in the same way as in the Trimming Tab Criterion I. The range of validity is the same as that of the Criterion I. It should however be emphasised that the exact value of the constant k_6 could not be determined very closely.

The criterion recommended for practical use at the present stage is the following:

Trimming Tab Criterion III.—To prevent a trimming tab system from flutter with the degrees of freedom wing translation, control surface rotation, tab rotation, the structural parameters of the system should satisfy the following equations, where we take provisionally $k_1 = 1$; $k_6 = 0 \cdot 1$; $k_7 = 1$

$$\frac{f_\gamma}{f_\beta} \geqslant 2k_1 \qquad \text{(G)}$$

$$M_t x_t \geqslant 0 \cdot 4 k_6 \rho c_w c_t^2 q s \qquad \text{(H)}$$

$$\frac{I_t + M_t x_t (E_1 - E_2) c_w}{I_c} \leqslant k_7 C . p^{3/2} \qquad \text{(K)}$$

Here C is given by the formulae (20) and (21).

The range of validity of this criterion is about the same as that of Criterion II. It is only assumed that the non-dimensional mass μ of the lifting surface is greater than $5 \cdot 7$. It is to be expected that by a slight modification of the constants k_1, k_6, k_7 the criterion can easily be adapted to practical experience and made valid for the whole normal range of the structural parameters.

It appears that the non-dimensional quantities i_c, i_t, p_t, p_{ct}, $\bar{q}$ introduced by the author are useful in a wider field than in the above criteria. They could be used, for example, when compiling statistics of structural parameters and in the formulation of flutter requirements.

In the case of a lifting surface with a more complicated planform and a more complicated degree of freedom than investigated above, certain mean values of the structural parameters have to be put into the criteria as explained in section 7.3.

The best possibility for the designer to satisfy any of the above mentioned criteria is to make the maximum tab angle as great as possible and the chord of the tab as small as possible (by choosing a suitable length for the tab span), and to give the tab with this chosen chord a moment of inertia as small as possible by placing the load-bearing parts of the tab as near as possible to the hinge line and by giving the balance arm a suitable length. In carrying out these measures the designer must not, however, forget the demand of a sufficiently high stiffness of the tab and of the tab control circuit to meet the conditions for the tab frequency. If it is impossible to meet the conditions for tab out-of-balance moment by the measures just mentioned, the criterion can still be satisfied by reducing the value of $\bar{q}$, that is, by reducing the aerodynamic efficiency of the tab. Another possibility to prevent flutter in this case, which is not yet covered by the criterion, would be to over mass-balance both the control surface and tab.

It should be mentioned here that the above criteria give sufficient conditions for eliminating flutter entirely with the three degrees of freedom considered. For some practical purposes (*e.g.*, in relation to unsymmetrical elevator flutter) it is useful to know in addition that only high critical speeds exist, if any, if the natural frequency f_β of the mass-balanced control surface is very high and the tab is slightly over mass-balanced.

Finally, a word should be said about the relation between the criteria evolved in this report and the current official requirement relating to the flutter of trimming tabs[13], which states in effect that trimming tabs shall be statically or slightly over mass-balanced for speeds greater than 350 knots E.A.S. Superficially this requirement appears to have little in common with the criteria of the present report, but in reality the A.P. 970 requirement is based on the fact that the majority of trimming tab systems have connections which are so stiff that there is little danger of flutter below 350 knots. Below this speed limit the A.P. 970 requirement, therefore, relies on the stiffness of the tab connection and makes no stipulation about tab mass-balance: in other words, though flutter may occur it will be at a speed well above 350 knots. Above the speed limit tab mass-balance is defined to ensure that, on the same principle as the new criteria, flutter will not occur at any speed. The main criticism of the A.P. 970 requirement is that above the speed limit it appears unnecessarily restrictive. It is to be noted, incidentally, that A.P. 970 makes special reservations with regard to trimming tab systems having unusually flexible connections (Chapter 500, section 7.44).

The Collar-Sharpe criterion for spring tabs[10, 11] can of course as it stands be applied to trimming tabs by putting N, the follow-up ratio, equal to zero, though in the derivation of the criterion such an extension was not contemplated and in fact the application is not generally made. Nevertheless, comparisons made between Criterion III of the present report and the Collar-Sharpe criterion with $N = 0$ show that for the alternative forms K and $K'p$ of the Collar-Sharpe criterion neither K' nor K is constant by Criterion III, though K' is much more so than K. Also, for all but extreme cases the Criterion III is less restrictive than the Collar-Sharpe criterion with $K' = 0 \cdot 1$: at the same time the Criterion III involves an additional requirement in respect of the tab natural frequency, which should however in most cases be easily met.

9. *Conclusions.*—For the avoidance of flutter of trimming tabs under the conditions considered certain qualitative conclusions can be drawn from the results obtained, *viz.*, that the following effects will be favourable:

(1) High stiffness of the tab connection.

(2) Low moment of inertia of the tab about its hinge.

(3) High moment of inertia of the control surface about its hinge, compatible with the avoidance of additional weight and with freedom from binary flutter involving main surface rotation and control surface rotation.

(4) Low tab span parameter $\bar{q}$, which is, however, largely determined by the desired aerodynamic performance of the tab but will be reduced by adopting as high a maximum tab angle as possible.

(5) Reasonably low tab chord (assuming the control surface chord fixed by aerodynamic considerations).

Quantitative requirements for the avoidance of flutter are obtained in the form of alternative Criteria I, II and III, given in detail in section 8. For practical purposes Criterion III is recommended.

10. *Further Developments.*—It is considered desirable that the investigation should be extended as follows:

(a) Research, possibly by wind-tunnel tests, to obtain more accurate values for the factors k_1 to k_7 in the criteria.

(b) Effect of over mass-balance of the main control surface and extension of the present range of values of tab and control surface inertias and of wing mass.

(c) Effect of aerodynamic balance of the tab.

11. *List of Symbols*.—The term 'wing' is used generally to denote a lifting surface with control surface and tab, *i.e.*, a main-plane unit, a tail-plane unit or a fin and rudder unit.

$a_1, \ldots, a_4$	Constants defined in formulae (5) in section 4.1	
c_w	Complete wing chord	
c_c	Control surface chord	
c_t	Tab chord	
d	Constant, defined in Fig. 2a	
D_t	Artificial damping moment about the hinge line of the tab per unit angular velocity of rotation of the tab about its hinge line	
E_1	Control surface chord ratio c_c/c_w	
E_2	Tab/chord ratio c_t/c_w	
f	Symbol (if without index) for an arbitrary function	
f_1, f_8, f_{10}	Functions of E_1, E_2 defined by Dietze[7]	
f_c	Flutter frequency (in c.p.s.)	
f_z	Natural frequency of wing vertical translation (assumed to be different from zero throughout the report)	taking into account the virtual inertia of the air
f	Natural frequency of control surface rotation	taking into account the virtual inertia of the air
f_γ	Natural frequency of tab rotation	taking into account the virtual inertia of the air
$F(E_2/E_1;\ E_1) = j$	Constant defined in formula (21) in section 5.2 and shown in Fig. 2b	
I_c	Moment of inertia of the control surface (including tab) about its hinge line	
I_t	Moment of inertia of the tab about its hinge line	
j	Constant defined in formula (21) in section 5.2	
$k_1, \ldots, k_5, k_7$	Constants nearly equal to unity which are used in the Trimming Tab Criteria in section 8	
k_6	Constant approximately equal to $0 \cdot 1$ which is used in the Trimming Tab Criteria in section 8	
M_w	Mass of the wing (including control surface and tab)	
M_c	Mass of the control surface (including tab)	
M_t	Mass of the tab	
p	Tab-control surface/chord ratio E_2/E_1	
q	Tab span/control surface span	
$\bar{q}$	Tab span parameter defined in section 3.2	
s	Control surface span	
V_c	Critical speed	
v_c	Reduced critical speed $= V_c$ (m.p.h.)$/c_w$ (ft) $.\ f_z$(sec^{-1})	
$x_c = \sigma_1 c_w/2$	Distance of control surface (including tab) c.g. behind control surface hinge	
$x_t = \sigma_2 c_w/2$	Distance of tab c.g. behind tab hinge	
(z)	Abbreviation for the degree of freedom 'vertical wing translation'	

(α)	Abbreviation for the degree of freedom 'wing rotation'.
(β)	Abbreviation for the degree of freedom 'control surface rotation'.
(γ)	Abbreviation for the degree of freedom 'tab rotation'.
δ_1, δ_2	Safety margins introduced into the criteria (*see* sections 4.3 and 8).
ρ	Air density.
ρ_0	Air density at sea level.
$\sigma_w = M_w/sc^2_w$	Wing density.

Derived quantities (*all non-dimensional*)

$$\mu = \frac{4M_w}{\pi\rho c_w^{\,2} \,.\, s} = \frac{4}{\pi\rho}\,\sigma_w \;;\quad \mu_1 = \frac{4M_c}{\pi\rho c_w^{\,2} s} \;;\quad \mu_2 = \frac{4M_t}{\pi\rho c_w^{\,2} q s} \;;$$

$$\vartheta_1 = \frac{16I_c}{\pi\rho c_w^{\,4} s} \;;\quad \vartheta_2 = \frac{16I_t}{\pi\rho c_w^{\,4} q s} \;;\quad d_t = \frac{8D_t}{\pi^2\rho c_w^{\,4} q s f_z} \;;$$

$$i_c = \frac{\vartheta_1}{E_1^{\,3}} = \frac{16I_c}{\pi\rho c_w s c_c^{\,3}} \;;\quad i = \frac{\vartheta_2}{E_2^{\,3}} = \frac{16I_t}{\pi\rho c_w s c_t^{\,3} q} \;;$$

$$p_c = \frac{\mu_1\sigma_1}{E_1^{\,2}} = \frac{8M_c x_c}{\pi\rho c_w c_c^{\,2} s} \;;\quad p_t = \frac{\mu_2\sigma_2}{E_2^{\,2}} = \frac{8M_t x_t}{\pi\rho c_w c_t^{\,2} q s} \;;$$

$$p_{ct} = \frac{E_2}{E_1}\, i_t + 2\left(\frac{E_1 - E_2}{E_1}\right) p_t \,.$$

REFERENCES

No.	Author	Title, etc.
1	K. Leiss	The Influence of Tabs on Flutter. (Zum Einfluss von Hilfsrudern auf das Flattern.) *Deutsche Luftfahrtforschung.* F.B. Nr. 1109. 1939.
2	H. Voigt and F. Walter	Wind Tunnel Experiments on the Flutter Behaviour of a Wing with a Servo Tab. (Windkanalversuche über das Schwingungsverhalten eines Flügels mit Flettner-Hilfsruder.) *Deutsche Luftfahrtforschung.* F.B. Nr. 1204. 1941.
3	H. Wittmeyer	On the Influence of a Tab on the Critical Speed of an Aeroplane I. (Über den Einfluss eines Hilfsruders auf die kritische Geschwindigkeit von Flugzeugen I.) *Technische Berichte* 3/1939, and *Jahrbuch der Deutschen Luftfahrtforschung.* 1940.
4	G. H. L. Buxton and G. D. Sharpe	Ternary Tailplane Elevator-tab Flutter. R. & M. 2418. November, 1946.
5	C. Scruton, P. M. Ray and D. V. Dunsdon.	Experiments on the Influence of Tab Mass-Balance on Flutter. R. & M. 2418. November, 1946.
6	H. Wittmeyer	Theoretical Investigations on the Flutter Characteristics of Wings and Tail Units with Control Surfaces and Tabs. R.A.E. Library Translation No. 196 (Theoretische Untersuchungen der Flattereigenschaften von Flügeln und Leitwerken mit Rudern und Hilfsrudern). ZWB (FB) 1468. 1941. Report of the Focke-Wulf Aircraft Co., Bremen. A.R.C. Report No. 11,058. 1947.
7	F. Dietze	The Air Forces on a Harmonically Oscillating, Deformable Plate (two-dimensional problem), *e.g.*, for the Calculation of a Wing with Control Surface and Tab. (Die Luftkräfte der harmonisch schwingenden, in sich verformbaren Platte (Ebenes Problem), u.a. zur Berechnung des Flügels mit Ruder und Hilfsruder.) *Luftfahrtforschung,* Bd. 16. 1939. p. 84/96.
8	H. Voigt, F. Walter and W. Heger..	Flutter Experiments on the Influence of the Aerodynamic Balance Given by a Geared Tab. (Flatterversuche über den Einfluss des aerodynamischen Ausgleichs dürch Weggesteuertes Hilfsruder.) *Jahrbuch der Deutschen Luftfahrtforschung.* 1941. Vol. I, pp. 338/350
9	F. Walter	Flutter Experiments on the Influence of a Control Surface Balanced by Setting Back the Hinge. (Flatterversuche über den Einfluss des aerodynamischen Ruderinnenausgleischs.) *Jahrbuch der Deutschen Luftfahrtforschung.* 1942. Vol. I, p. 549.
10	A. R. Collar and G. D. Sharpe ..	Mass-balance for Spring Tab Flutter Prevention. R. & M. 2637. June, 1946.
11	G. D. Sharpe	Consideration of the Effect of Tab Dimensions on Spring Tab Flutter. R. & M. 2637. June, 1946.
12	W. J. Duncan, D. L. Ellis and A. G. Gadd	Experiments on Servo-rudder Flutter. R. & M. 1652. 1934.
13	—	A.P. 970. Design Requirements for Aeroplanes. Chapter 500, Paragraph 7.4. (A.L. 50).

TABLE 2

Summary of the Characteristics of the Branches of 'Lower Critical Speeds' of the Curves $v_c = f(p_t)$. (See Figs. 3 to 10.)

Branch	Necessary condition for existence of the branch	Branch lies principally in the range where the tab c.g. is :—	Approximate value of the Flutter Frequency f_c under the conditions in the first row‖								Resonance phenomenon occurs when :—	Flutter modes near resonance consist predominantly of :—	Results are derived from figures †
			$\frac{f_\beta}{f_z} \leqslant 1$ $\frac{f_\gamma}{f_z} \geqslant 1 \cdot 2$ $\frac{f_\gamma}{f_\beta} \geqslant 1$	$\frac{f_\beta}{f_z} >> 1$	$\frac{f_\gamma}{f_\beta} \leqslant 1$	$\frac{f_\gamma}{f_\beta} > 1$ *	$\frac{f_\gamma}{f_z} \leqslant 1$	$\frac{f_\gamma}{f_z} > 1$	$\frac{f_\gamma}{f_z} \leqslant 2$	$\frac{f_\gamma}{f_z} >> 2$			
I	—	forward of the hinge of the tab.	f_z	$(<) f_\beta$	—	—	—	—	—	—	$f = f_\beta$ $\frac{f_\gamma}{f_z} \geqslant 1$	(z), (β)	3d, 4d, 5d, e, f, 7e {1}, {2}
II	—	aft of the hinge of the tab	—	—	f_β	f_γ	—	—	—	—	$f_\beta = f_\gamma$	(β), (γ)	3, 4, 5, 7 {7}
III	$\frac{f_\beta}{f_z} \leqslant 0 \cdot 5$		—	—	—	—	f_z	f_γ	—	—	$f_z = f_\gamma$	(z), (γ)	3, 4, 7 {2}
IV	$\frac{f_\beta}{f_z} \geqslant 1$		—	—	—	—	—	—	f	$(<) f_\gamma$	$f_z = f_\gamma$	(z), (γ)‡	5, 6.

* Only derived from curves in which $f_\gamma/f_z \leqslant 1$.

‡ No calculations were made.

† The figures in curly brackets are those of previous report[6].

‖ Except near the asymptotes, where $f_c \to \infty$.

TABLE 3

Comparison between Criterion III and Collar-Sharpe Criterion with $N = 0$

(1) Values of the constants C, C_1 in the requirement

$$\frac{I_t + M_t x_t (E_1 - E_2) c_w}{I_c} \leqslant C p^{3/2} = C_1 \qquad (22)$$

with

$$C = \frac{\sqrt{p}}{j}\left\{-0{\cdot}0435 + \frac{0{\cdot}751}{i_c} + \frac{0{\cdot}69}{i_t} + jq\left[0{\cdot}25 - \frac{0{\cdot}14}{i} - \frac{1}{i}\left(0{\cdot}634 + \frac{1{\cdot}27}{i_c}\right)\right]\right\} \qquad (23)$$

and

$$j = \sqrt{p}\,[0{\cdot}93 + 1{\cdot}28\,(1{\cdot}97 - E_1)\,(0{\cdot}745 - p)] \qquad (24)$$

for different sets of values of the relevant parameters.

No.	i	i_t	p	q	E_1	C	$C_1 = C.p^{3/2}$
1	7	10	0·15	0·25	0·2	0·0727	0·00422
2	3	10	0·15	0·25	0·2	0·140	0·0081
3	3	3	0·15	0·25	0·2	0·177	0·0103
4	3	3	0·3	0·25	0·2	0·205	0·0338
5	3	3	0·3	0·25	0·5	0·228	0·0375
1a	7	10	0·15	1	0·2	0·116	0·00673
5a	3	3	0·3	1	0·5	0·165	0·0273

(2) Values of the constants K and K' in condition (19) according to Collar and Sharpe (independent of the value of the structural parameters), if used for $N = 0$.

Reference	K' (compare with C)	K (compare with C_1)
Collar and Sharpe[10] ..	—	0·02
Sharpe[11]	0·1	0·015 if greater than $K'p^{3/2}$

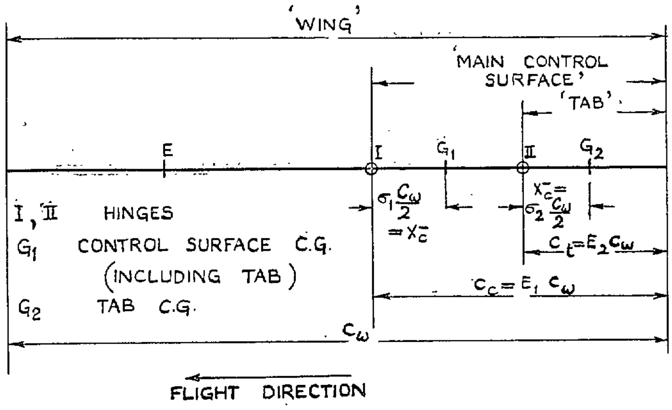


FIG.1. a. CROSS-SECTION OF THE WING WITH CONTROL SURFACE AND TAB.

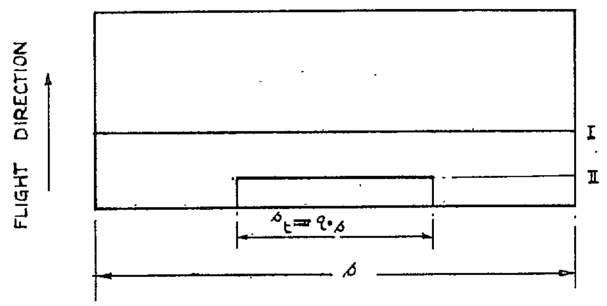


FIG.1. b. PLAN FORM OF THE WING WITH CONTROL SURFACE AND TAB.

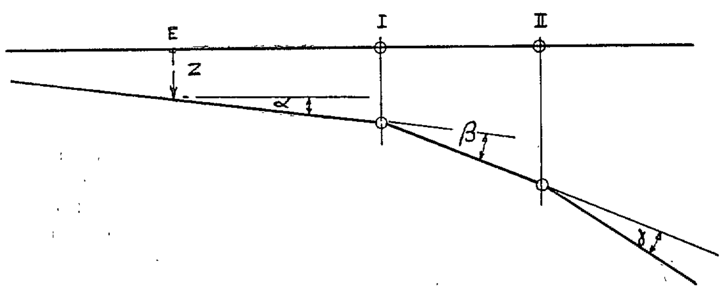


FIG.1 c. DEGREES OF FREEDOM OF THE SYSTEM. (IN THIS REPORT IT HAS BEEN ASSUMED THAT $\alpha = 0$.)

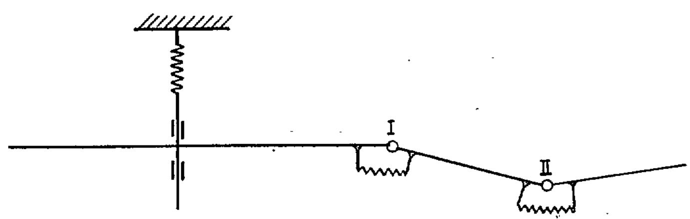


FIG. 1. d. THE ELASTIC SYSTEM.

FIG. 1. The system considered.

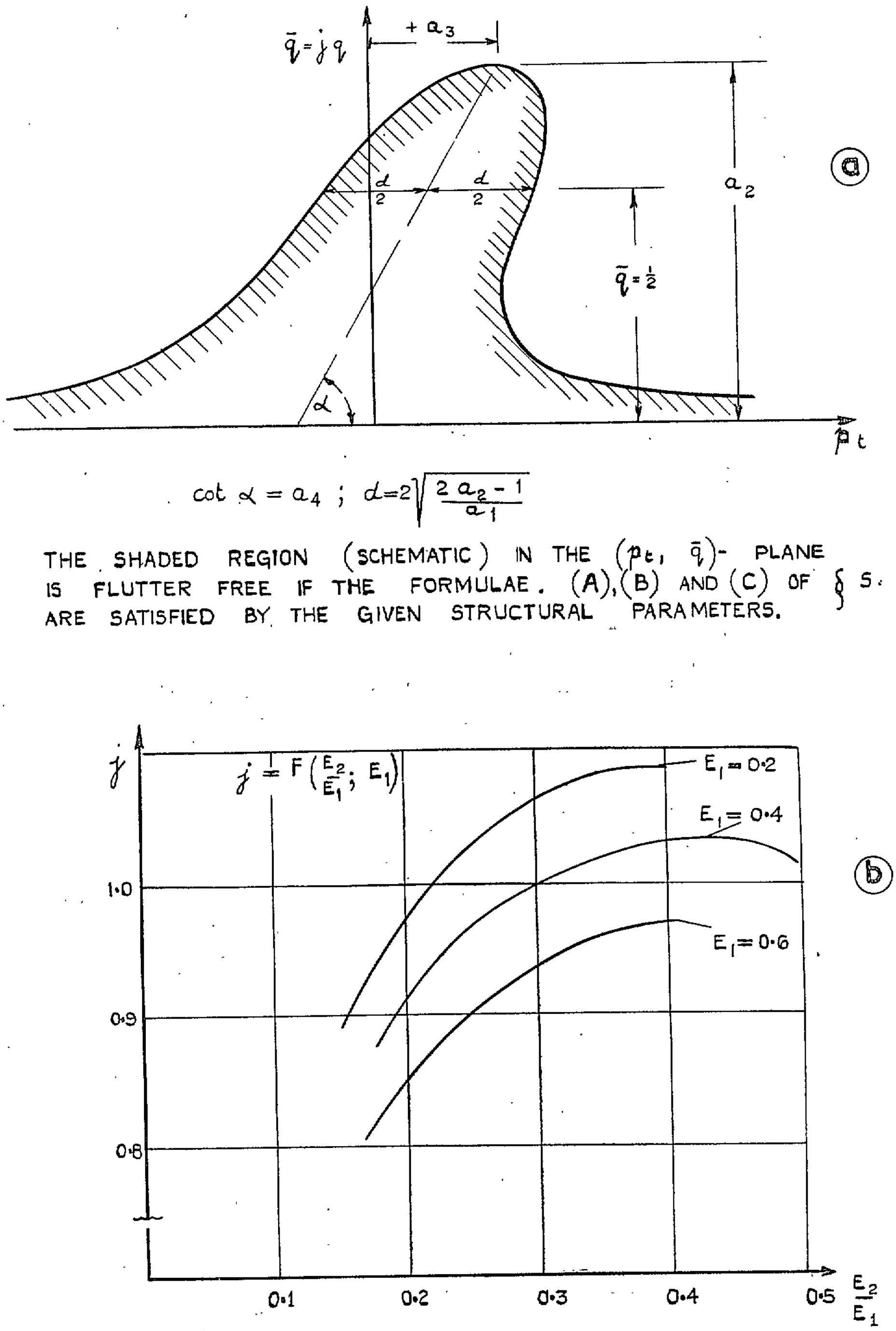
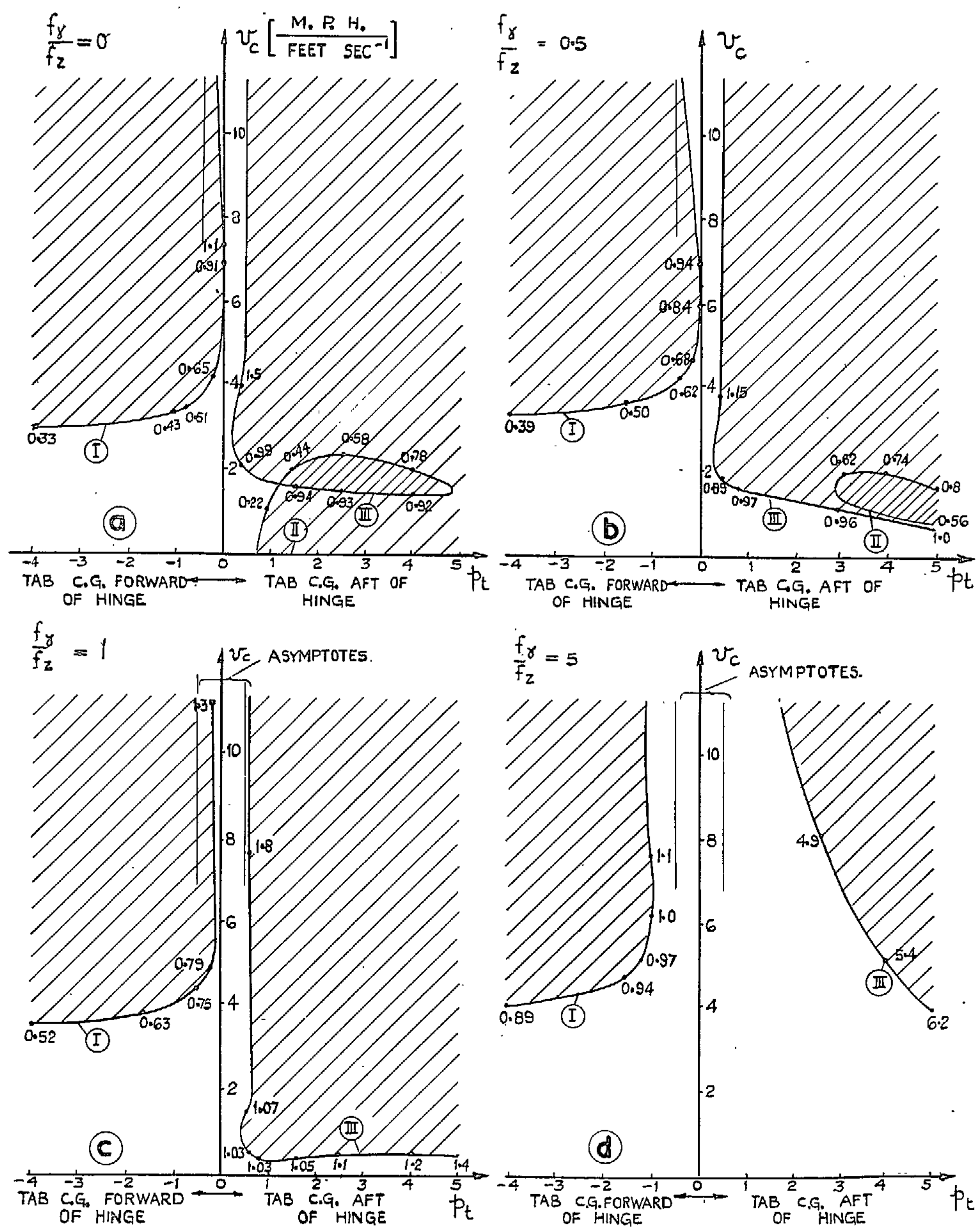
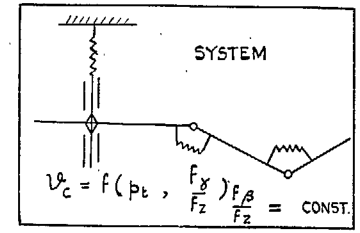


FIG. 2. Graphical illustration of the trimming tab criterion.



THE REDUCED CRITICAL SPEED v_c AS A FUNCTION OF THE POSITION OF THE TAB CENTRE OF GRAVITY FOR SEVERAL VALUES OF THE RATIO OF THE TAB NATURAL FREQUENCY f_γ TO THE WING NATURAL FREQUENCY f_z :

SCHEME FOR FIGURES 3 TO 5



DEGREES OF FREEDOM (z), (β), (γ)

VALUES OF PARAMETERS

$\mu = 5.718$, $p_c = 0$, $i_c = 7.78$,

$i_t = 6.55$, $q = 1$, $E_1 = 0.3$, $E_2 = 0.075$

NO FRICTION , NO DAMPING

THE ROMAN FIGURES DENOTE THE BRANCHES OF 'LOWER CRITICAL SPEEDS' AS DEFINED IN PARAGRAPH 3.1.1

THE NUMBERS ON THE CURVES DENOTE THE VALUES OF $\frac{f_c}{f_z}$

SPECIAL PARAMETER VALUE FOR THIS FIGURE

$$\frac{f_\beta}{f_z} = 0$$

FIG. 3. Critical speed as a function of tab centre of gravity position.

FOR THE TITLE AND THE SCHEME SEE FIG. 3.

TYPICAL PARAMETER VALUE FOR THIS FIGURE:

$$\frac{f_\beta}{f_z} = 0{\cdot}5$$

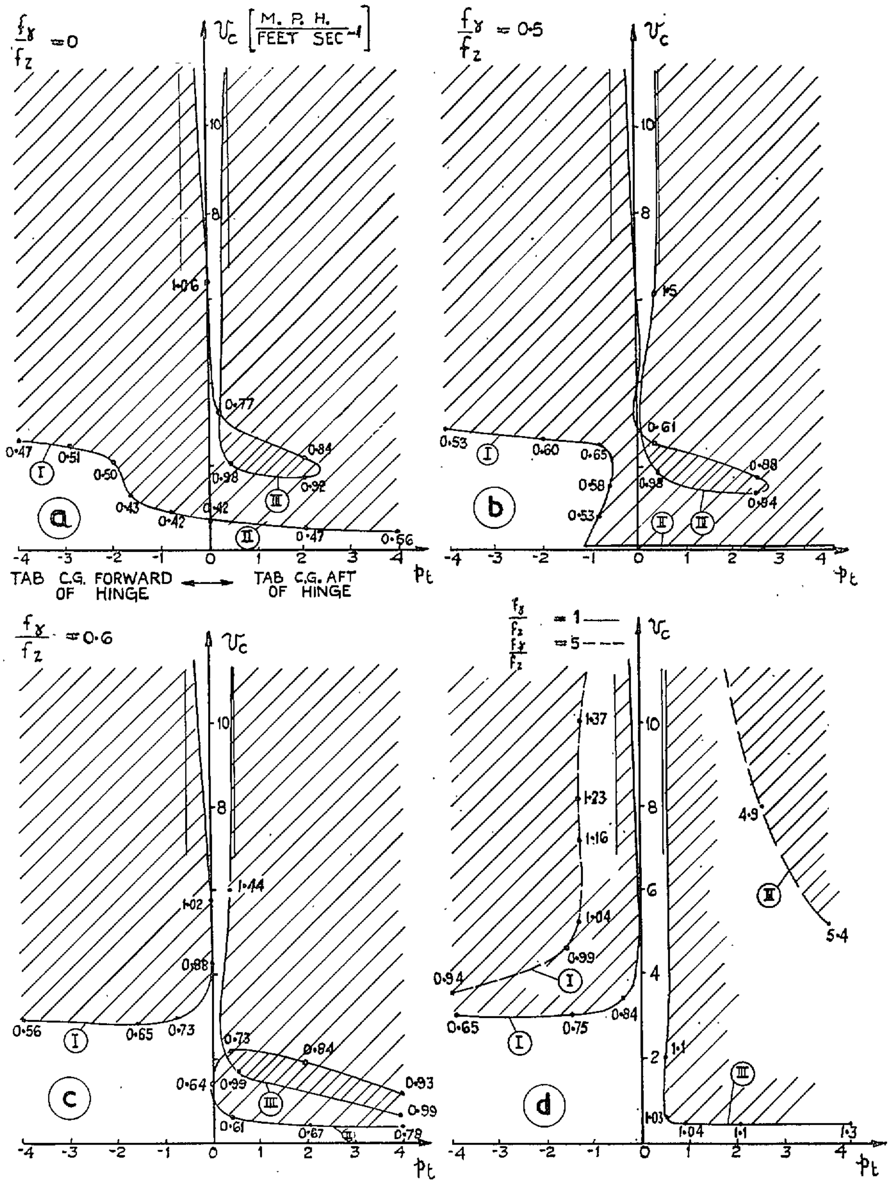


FIG. 4. Critical speed as a function of tab centre of gravity position.

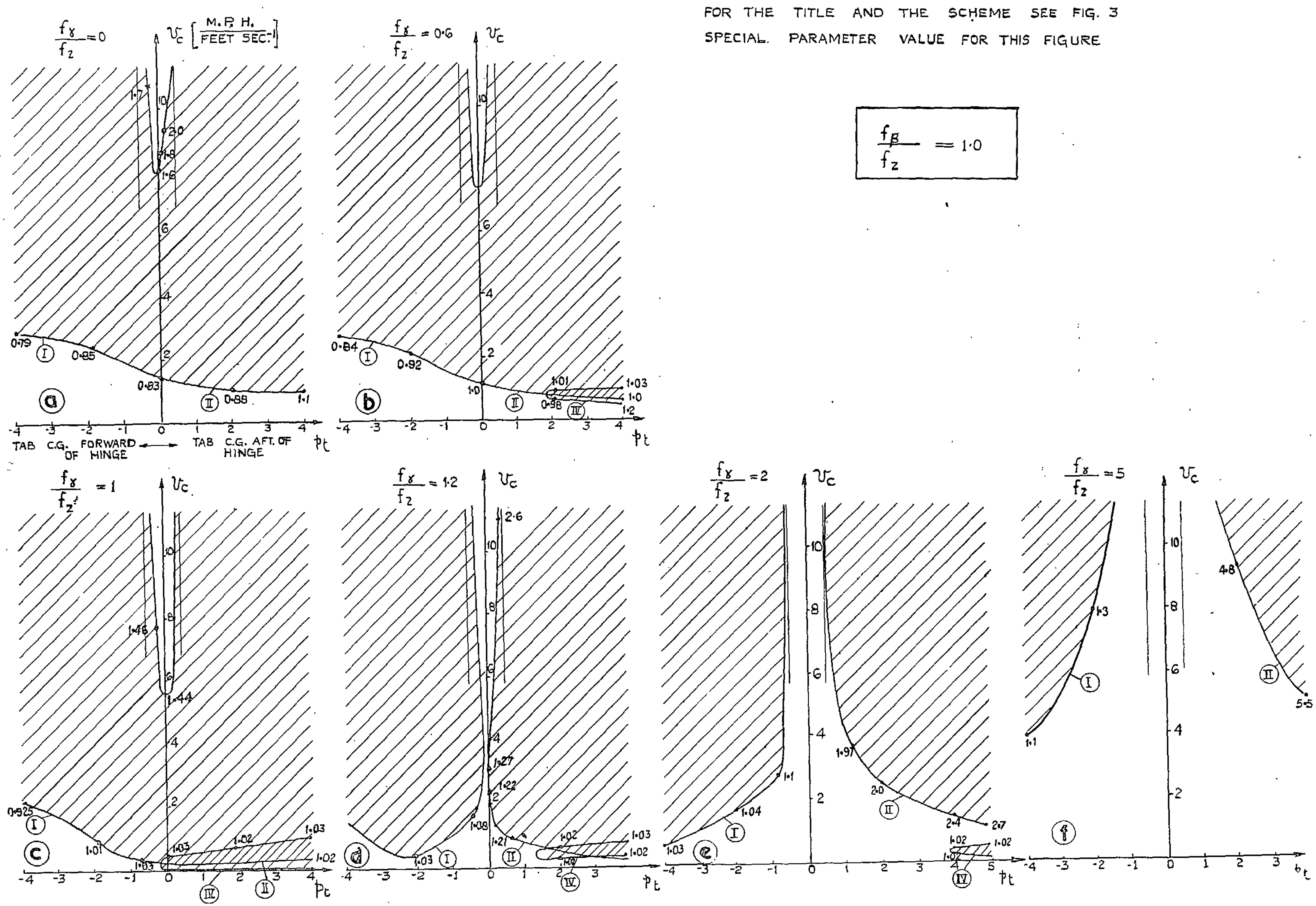


FIG. 5. Critical speed as a function of tab centre of gravity position.

FOR THE TITLE AND THE SCHEME SEE FIG. 3.

SPECIAL PARAMETER VALUE FOR THIS FIGURE:

$$\frac{f_\beta}{f_z} = \infty ,$$

i.e. THERE ARE ONLY TWO DEGREES OF FREEDOM (z), (γ)

THERE ARE NO CRITICAL SPEEDS IF THE TAB C.G. IS FORWARD OF HINGE.

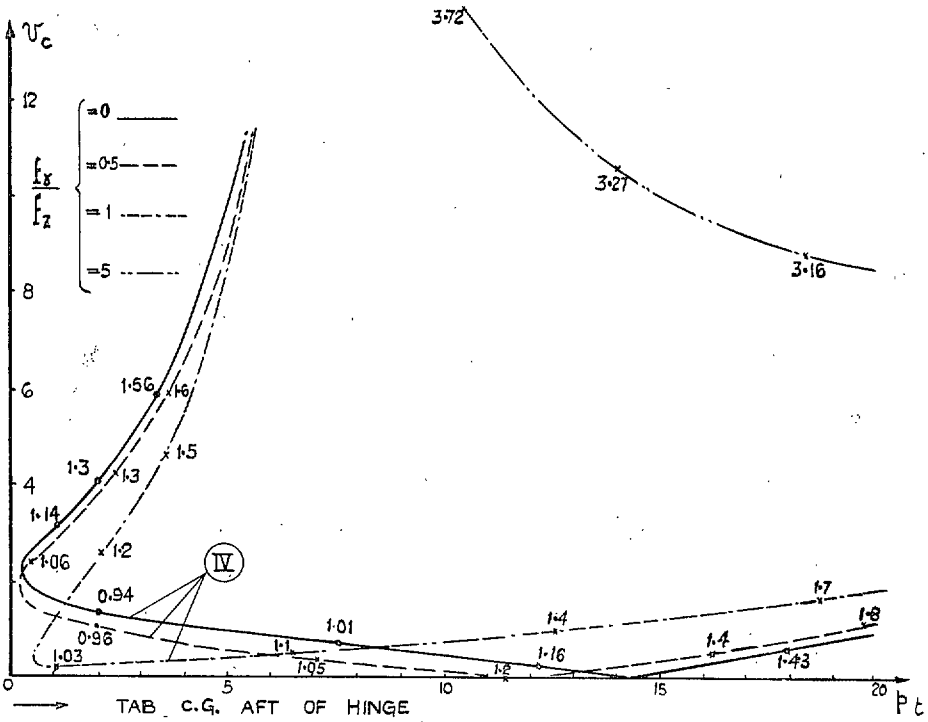


FIG. 6. Critical speed as a function of tab centre of gravity position.

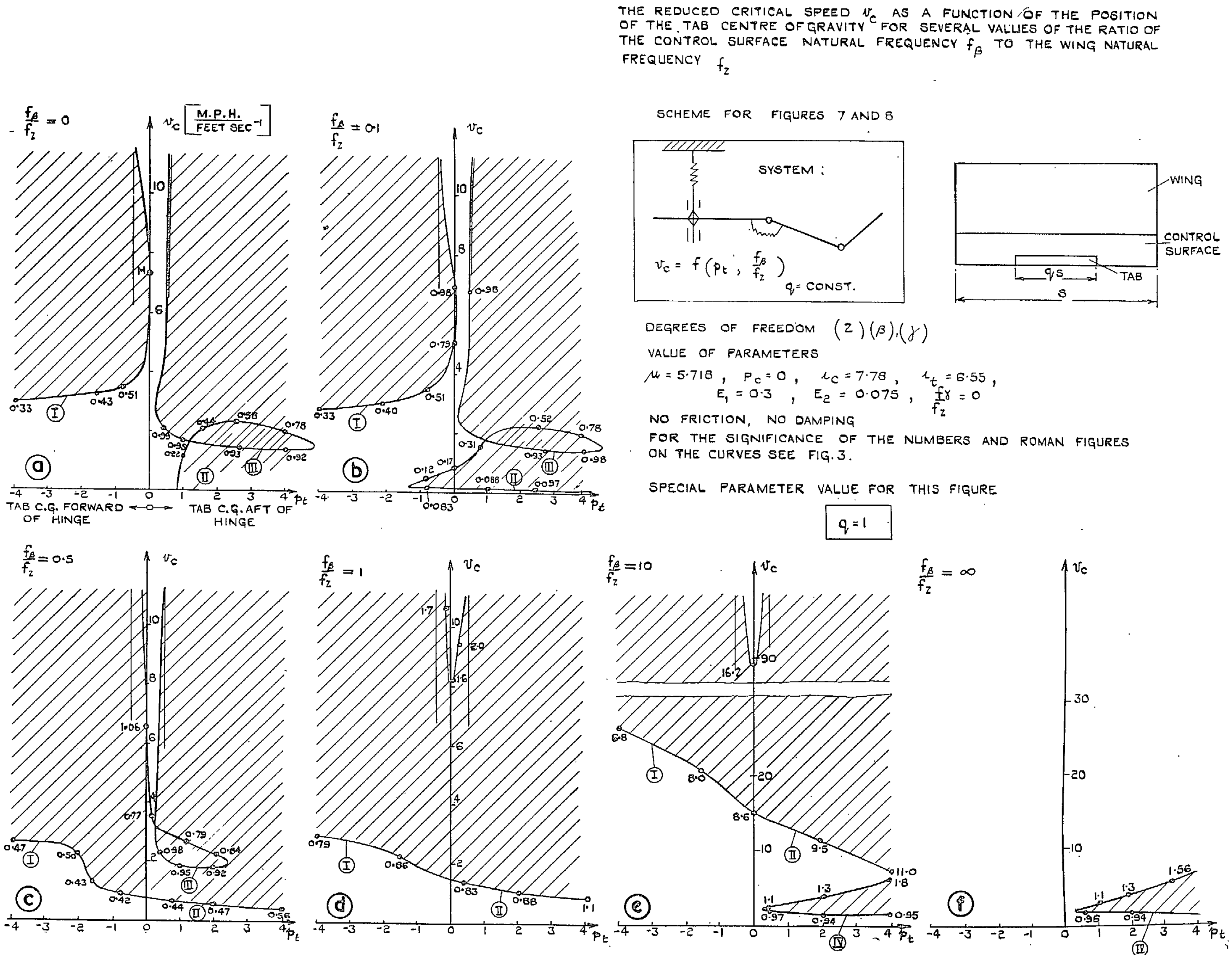


FIG. 7. Critical speed as a function of tab centre of gravity position.

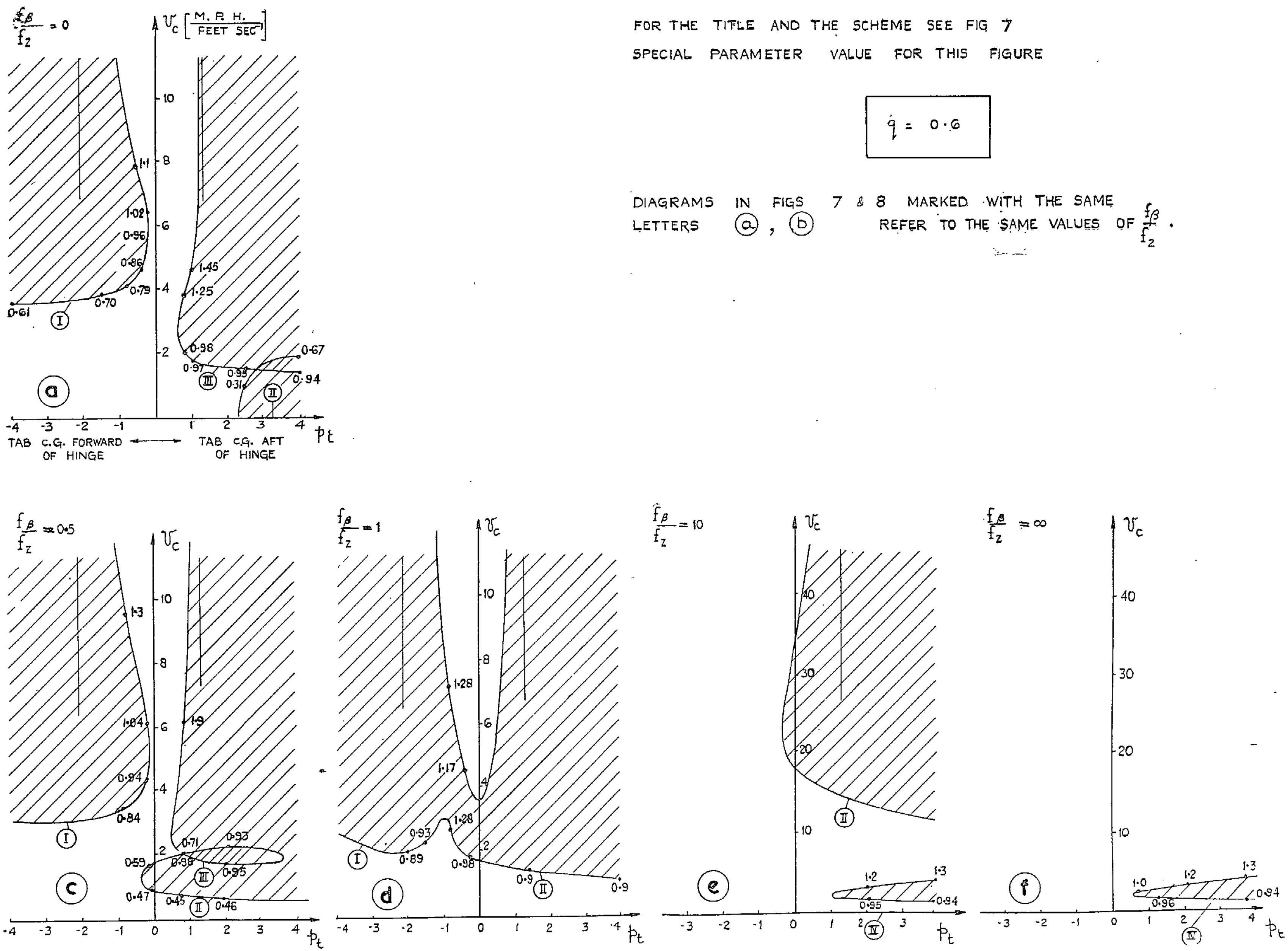


FIG. 8. Critical speed as a function of tab centre of gravity position.

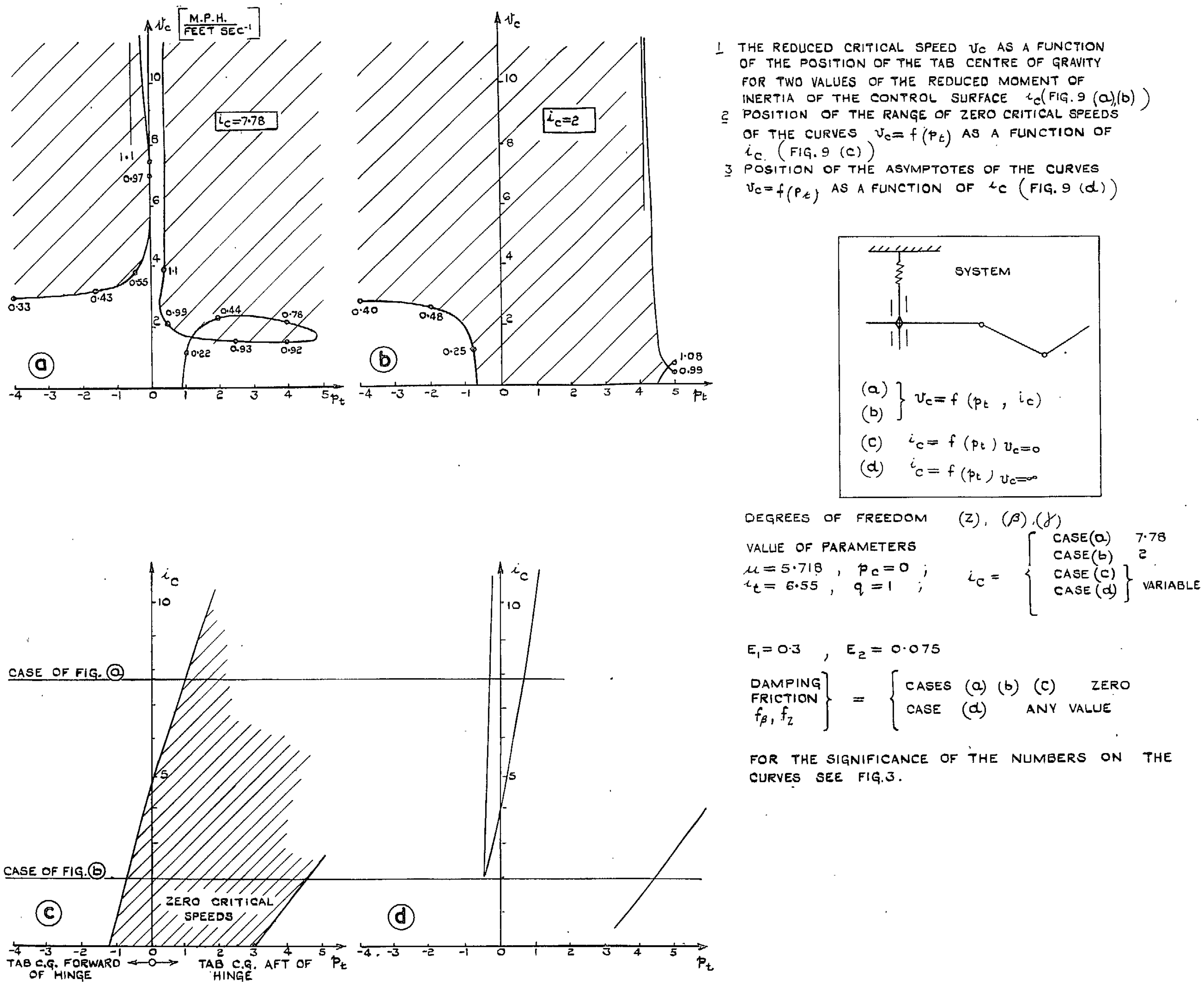


FIG. 9. Critical speed as a function of tab centre of gravity position.

(51320) C*

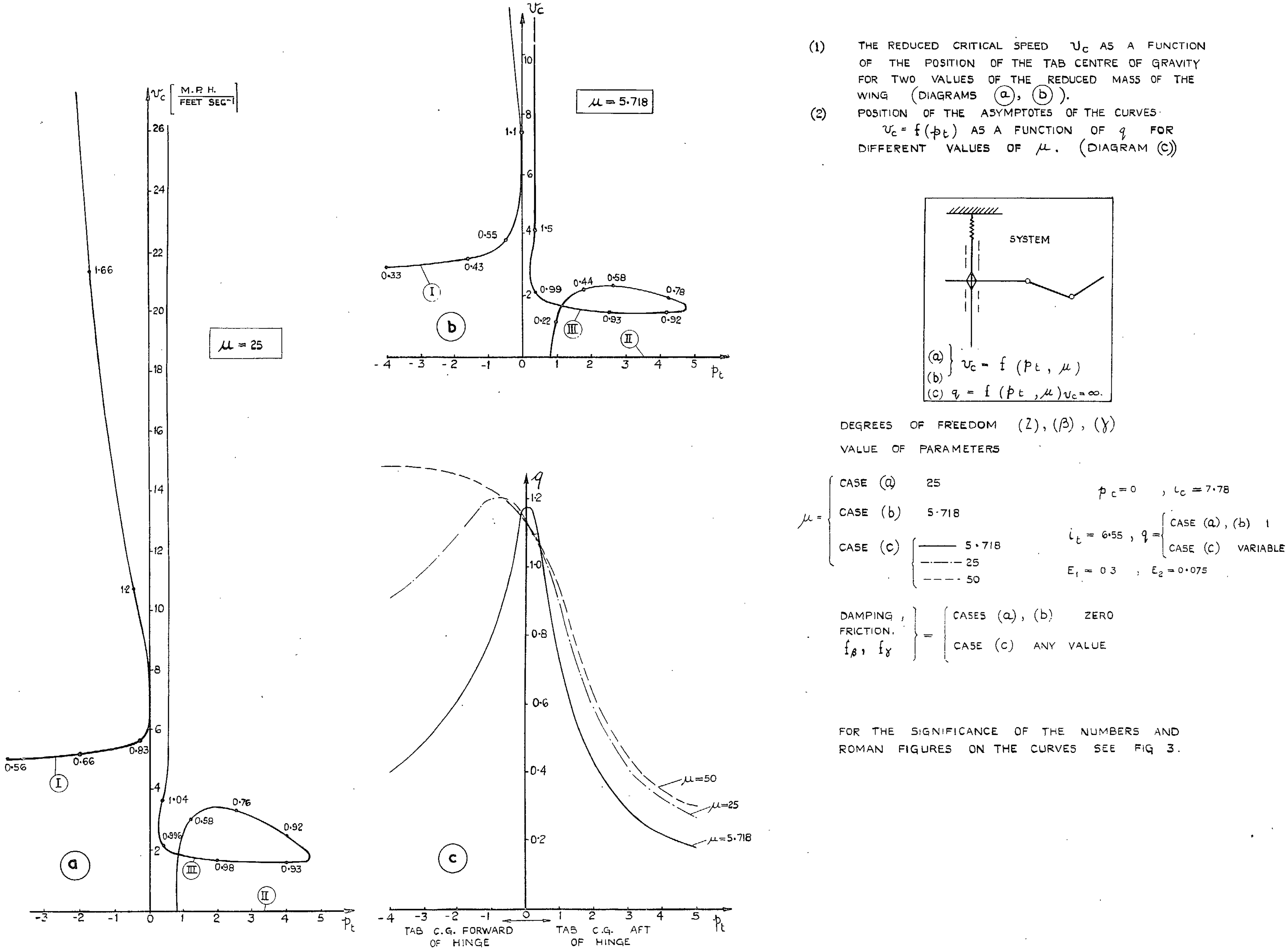


FIG. 10. Critical speed as a function of tab centre of gravity position.

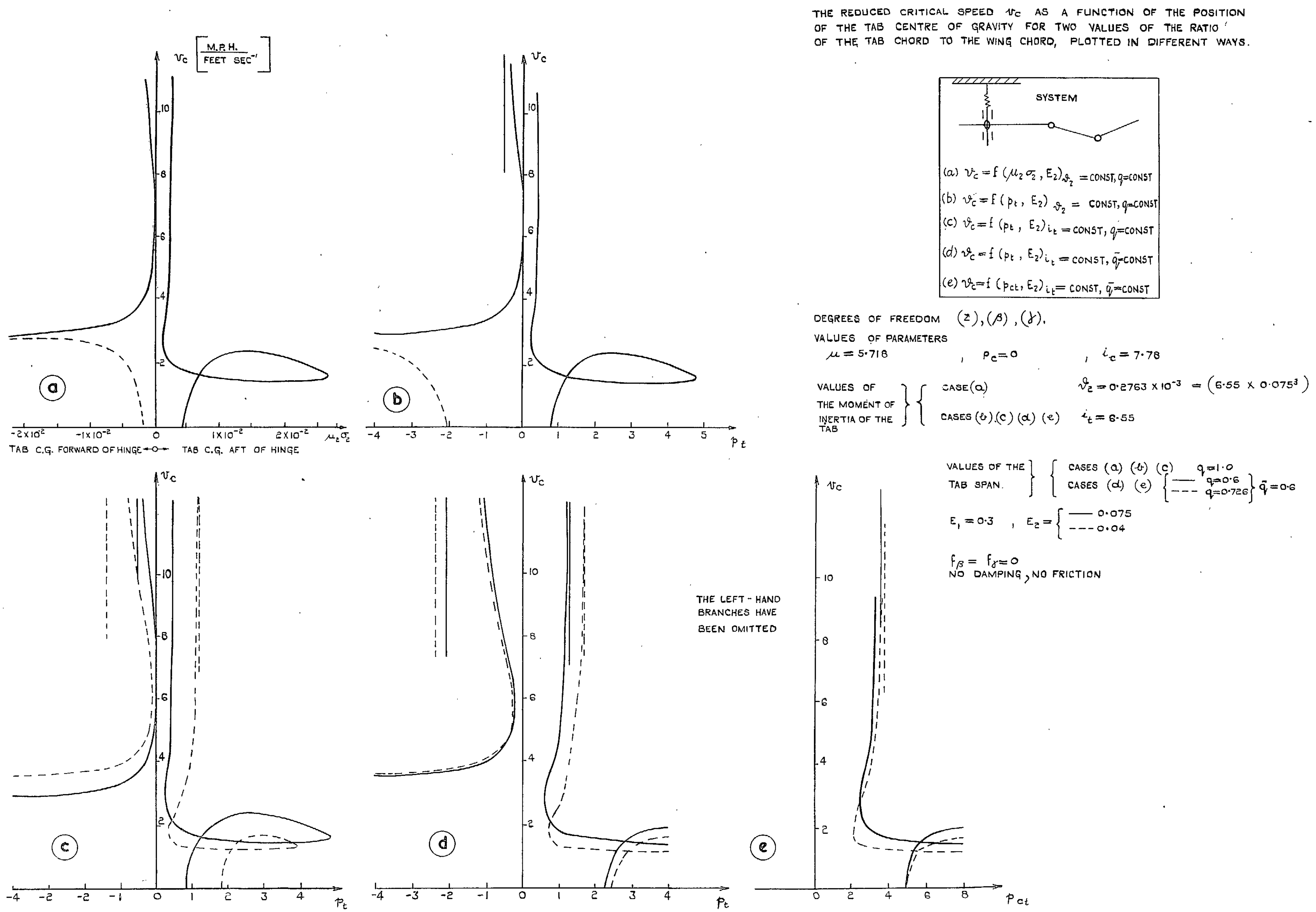


FIG. 11. Critical speed as a function of tab centre of gravity position.

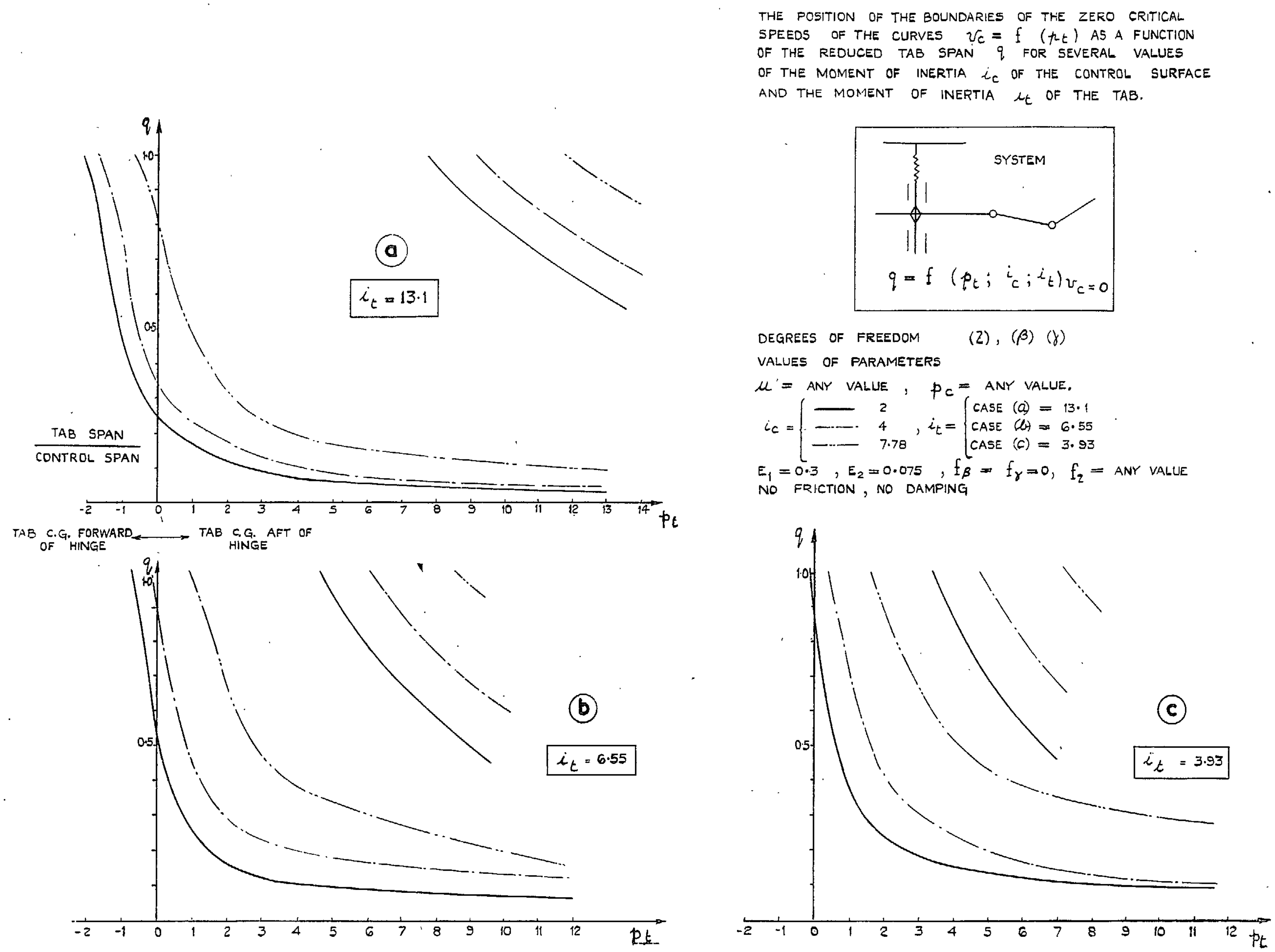


FIG. 12. Position of the boundaries of the range of zero critical speeds as a function of tab span.

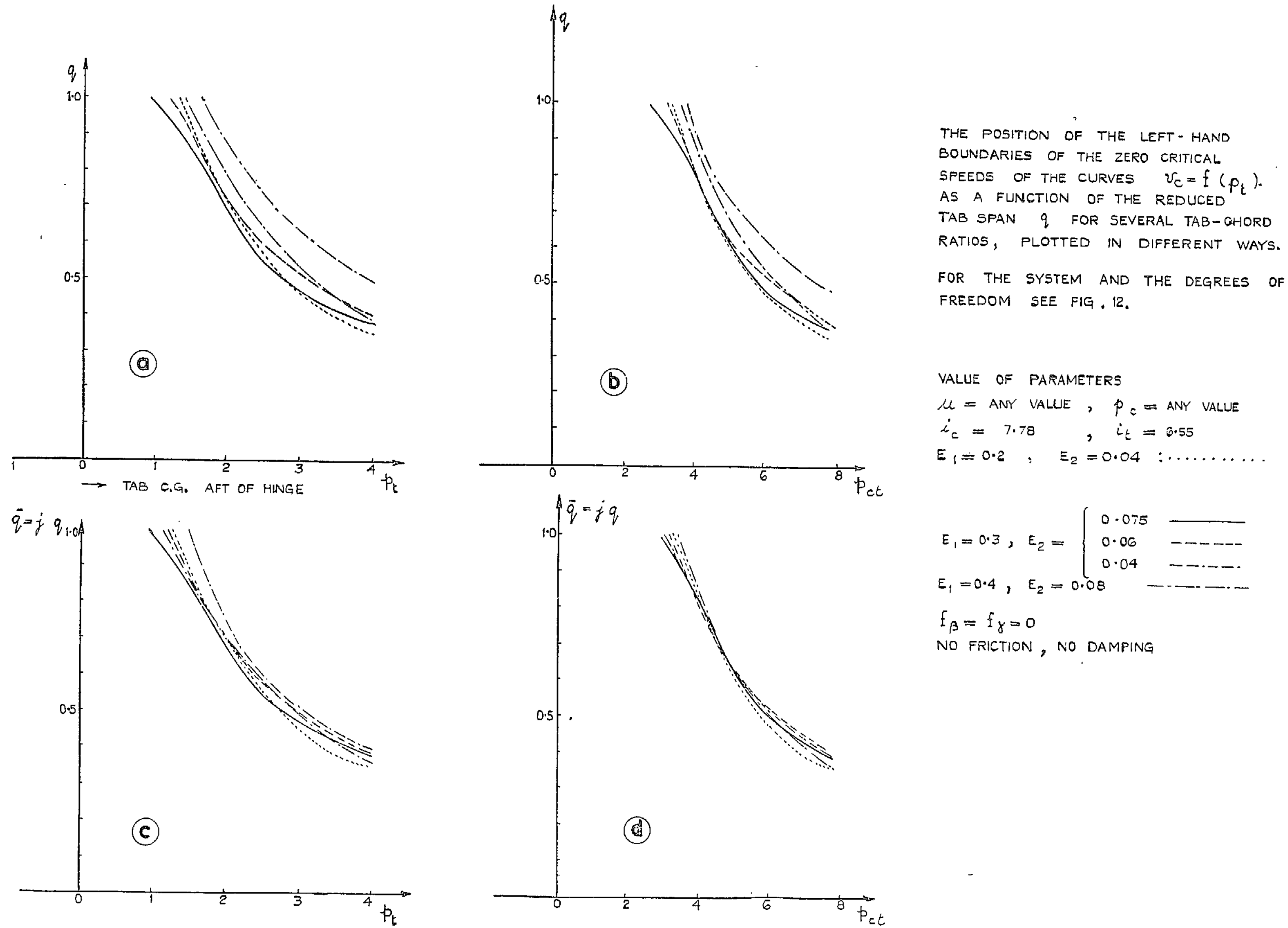


FIG. 13. Position of the boundaries of the range of zero critical speeds as a function of tab span.

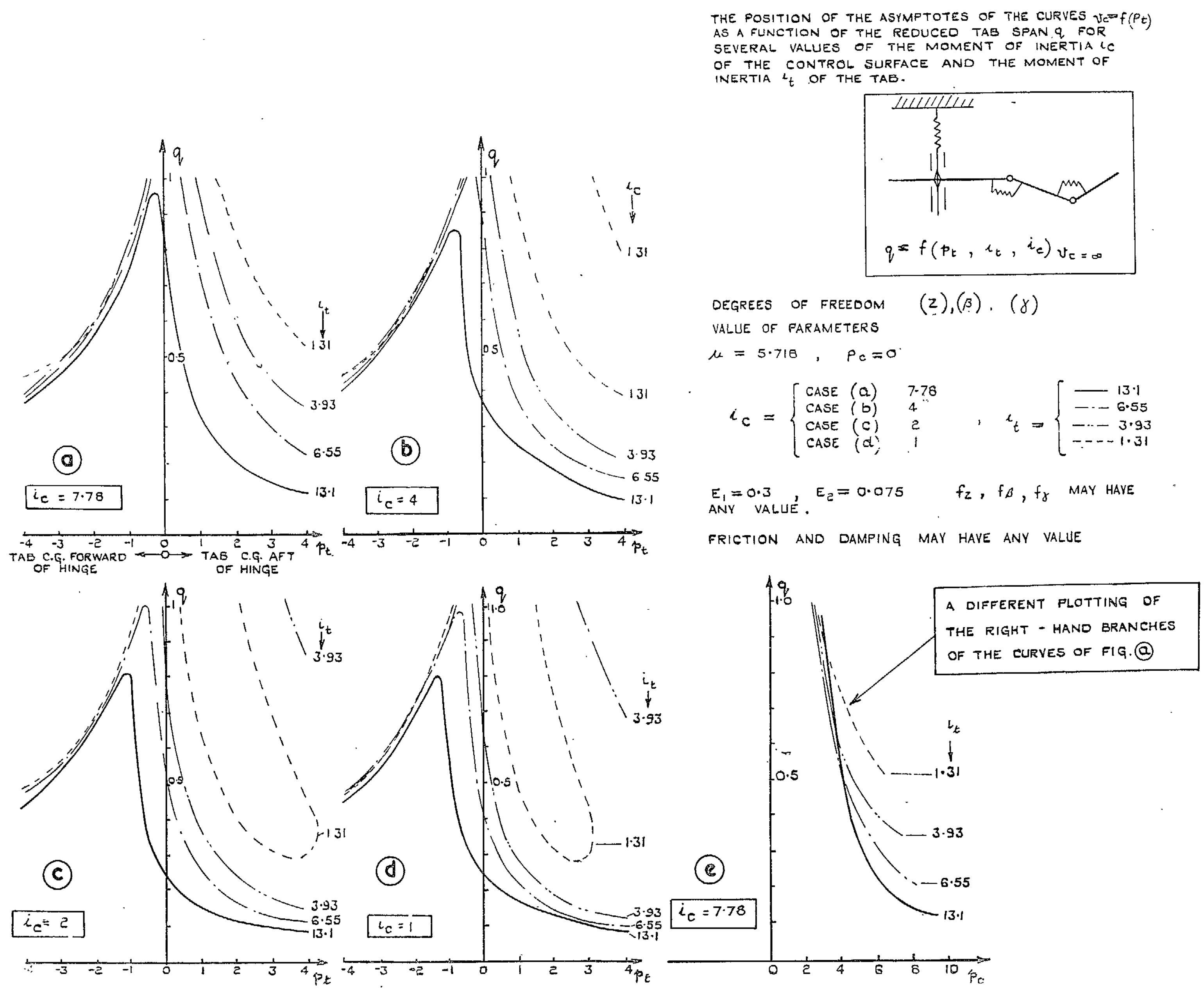


FIG. 14. Position of the asymptotes of the curves $v_c = f(p_t)$ as a function of tab span.

(51820)

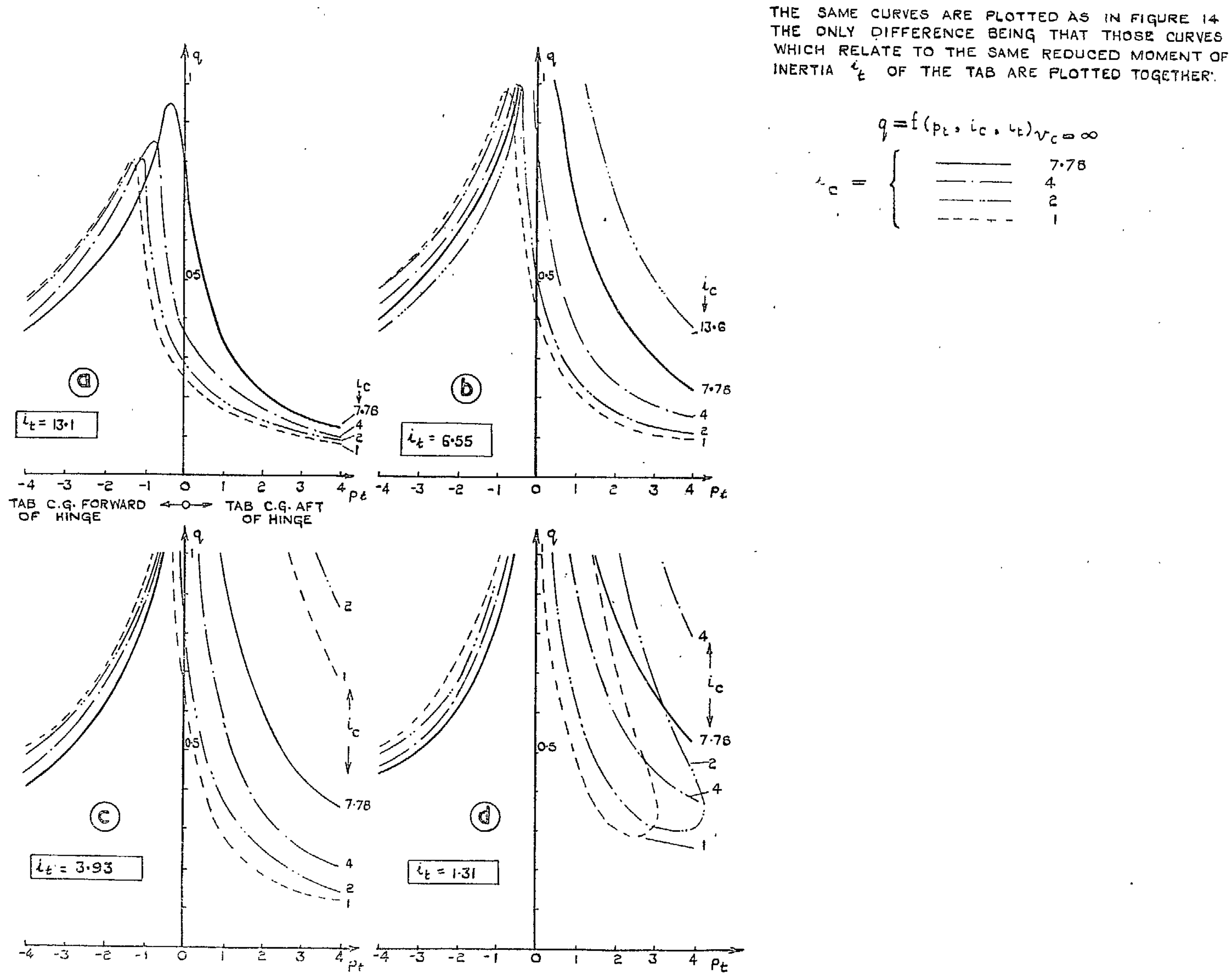


FIG. 15 Position of the asymptotes of the curves $v_c = f(p_t)$ as a function of tab span.

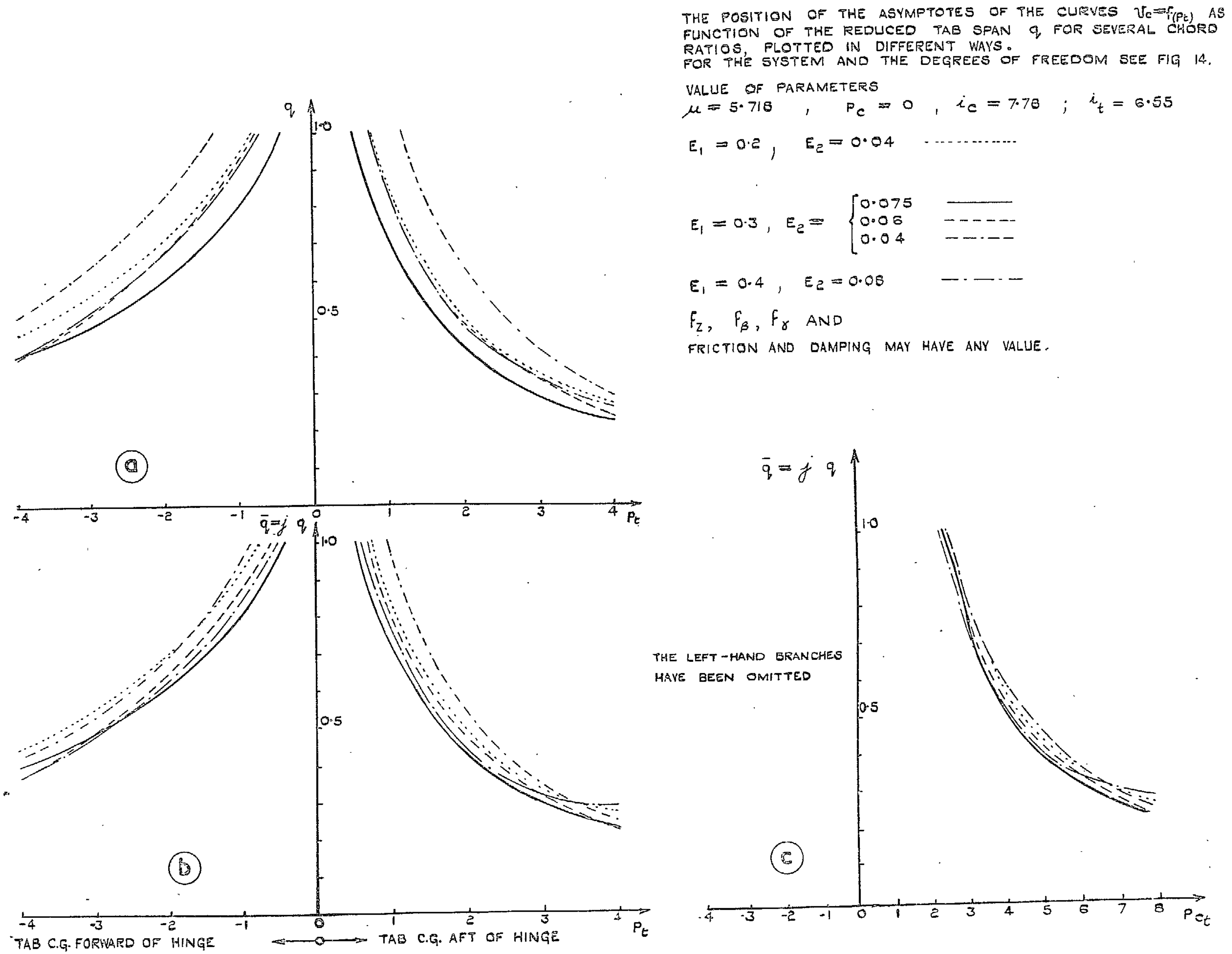


FIG. 16. Position of the asymptotes of the curves $v = f(p_t)$ as a function of tab span.

(51320) Wt. 15/680 K.9 10/52 Hw.
PRINTED IN GREAT BRITAIN

R. & M. No. 2671
(12,043)
A.R.C. Technical Report

Publications of the Aeronautical Research Council

ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)—

1934–35 Vol. I. Aerodynamics. *Out of print.*
Vol. II. Seaplanes, Structures, Engines, Materials, etc. 40*s.* (40*s.* 8*d.*)

1935–36 Vol. I. Aerodynamics. 30*s.* (30*s.* 7*d.*)
Vol. II. Structures, Flutter, Engines, Seaplanes, etc. 30*s.* (30*s.* 7*d.*)

1936 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40*s.* (40*s.* 9*d.*)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 50*s.* (50*s.* 10*d.*)

1937 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40*s.* (40*s.* 10*d.*)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 60*s.* (61*s.*)

1938 Vol. I. Aerodynamics General, Performance, Airscrews. 50*s.* (51*s.*)
Vol. II. Stability and Control, Flutter, Structures, Seaplanes, Wind Tunnels, Materials. 30*s.* (30*s.* 9*d.*)

1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50*s.* (50*s.* 11*d.*)
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63*s.* (64*s.* 2*d.*)

1940 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Icing, Stability and Control, Structures, and a miscellaneous section. 50*s.* (51*s.*)

Certain other reports proper to the 1940 *volume will subsequently be included in a separate volume.*

ANNUAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

1933–34	1*s.* 6*d.* (1*s.* 8*d.*)
1934–35	1*s.* 6*d.* (1*s.* 8*d.*)
April 1, 1935 to December 31, 1936.	4*s.* (4*s.* 4*d.*)
1937	2*s.* (2*s.* 2*d.*)
1938	1*s.* 6*d.* (1*s.* 8*d.*)
1939–48	3*s.* (3*s.* 2*d.*)

INDEX TO ALL REPORTS AND MEMORANDA PUBLISHED IN THE ANNUAL TECHNICAL REPORTS, AND SEPARATELY—

April, 1950 R. & M. No. 2600. 2*s.* 6*d.* (2*s.* 7½*d.*)

INDEXES TO THE TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

December 1, 1936 — June 30, 1939.	R. & M. No. 1850.	1*s.* 3*d.* (1*s.* 4½*d.*)
July 1, 1939 — June 30, 1945.	R. & M. No. 1950.	1*s.* (1*s.* 1½*d.*)
July 1, 1945 — June 30, 1946.	R. & M. No. 2050.	1*s.* (1*s.* 1½*d.*)
July 1, 1946 — December 31, 1946.	R. & M. No. 2150.	1*s.* 3*d.* (1*s.* 4½*d.*)
January 1, 1947 — June 30, 1947.	R. & M. No. 2250.	1*s.* 3*d.* (1*s.* 4½*d.*)

Prices in brackets include postage.

Obtainable from

HER MAJESTY'S STATIONERY OFFICE

York House, Kingsway, LONDON, W.C.2 423 Oxford Street, LONDON, W.1
P.O. Box 569, LONDON, S.E.1
13a Castle Street, EDINBURGH, 2 1 St. Andrew's Crescent, CARDIFF
39 King Street, MANCHESTER, 2 Tower Lane, BRISTOL, 1
2 Edmund Street, BIRMINGHAM, 3 80 Chichester Street, BELFAST

or through any bookseller.

S.O. Code No. 23-2671