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A Comparison Between Plain and Stringer-reinforced Sheet from the Shear Lag Standpoint

By
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Summary.—In R. & Ms. 2098[1], 2099[2], 2100[3] the stringer-sheet method of solving shear lag problems in stringer reinforced sheet was developed. The present report compares for two simple cases the solution for the plain sheet with that for the stringer-reinforced sheet. The solutions are practically identical by the two methods provided that the sheet is considered fully effective in taking end load. This leads to the conclusion that, in regions of tensile stress, at all events, all the skin area is to be included in the stringer area when applying this method.

1. *Introduction.*—The stringer-sheet method of solving stress distribution problems in stringer-reinforced sheet is founded upon the following assumptions:—(a) skin between the stringers is capable of taking shear stress only, any capacity for taking direct stress being allowed for by adding a suitable fraction of the skin area to the stringer area; (b) sheet stresses normal to the direction of the stringers are neglected as unimportant. It was shown in R. & M. 2098[1] that the stringers may be replaced by a ' stringer-sheet ' obtained by spreading the total stringer area uniformly across the sheet, thus giving a very large number of very small stringers.

If it was desired to include the longitudinal stress carrying capacity of the skin then an appropriate fraction of the skin cross-sectional area was added to that of the stringers. This fraction would depend upon the degree of buckling present in the sheet and would approach unity for a plane unbuckled sheet. Thus the only stresses considered were the direct longitudinal stresses and the shear stress. The cross-tensional stresses in the sheet (*i.e.* direct stresses normal to the longitudinal stress) were assumed to be unimportant. This was subsequently justified by introducing, after the solution had been obtained, such cross-tensile stresses as were necessary to equilibrate fully the shear stress. This demonstration, however, was not fully conclusive in that the condition of compatibility of displacements was not satisfied.

The purpose of this report is to show for two simple cases that the assumption made in regard to the unimportance of the cross-tensile stresses was justified. We take a sheet unreinforced by stringers and, by the method of R. & M. 2098[1], we consider it as made up of a sheet of equal thickness capable of carrying shear only and another sheet of the same thickness to carry end stresses alone. The solution thus obtained is compared with the rigorously correct one (easily obtained in the simple cases chosen) in which the conditions for equilibrium and compatibility are both satisfied. If it is shown that the cross tensile stresses may be neglected in these cases where they are likely to be most serious then ' a fortiori ' they must be negligible when the sheet is reinforced by stringers.

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2. *Problems Treated.*—The first problem treated was that of the metal sheet bounded at its longer sides by heavy flanges and loaded along these flanges, as shown in Fig. 1. (This is representative of the under surface of a two-spar wing.) The entire root end was fixed, *i.e.* prevented from moving longitudinally.

The second problem was that of the same sheet subjected to a transverse load at its free end in the plane of the sheet as shown in Fig. 3. The root was again prevented from moving longitudinally, the flanges being pin-jointed at this end. The flanges were treated as perfectly flexible in the plane of the surface and their distance apart was taken as unconstrained by any ribs.

In the problems treated there are no stringers. Therefore, the stringer-sheet was composed wholly of the contribution from the skin cover. In applying the method of R. & M. 2098[1] the entire sheet was assumed to be effective in taking tensile load. The mathematical solutions are given in an appendix.

3. *Discussion of Results.*—As may be seen from Figs. 1 to 3, the stress distributions obtained by the two methods are practically the same. It follows that the shear stresses in the sheet, being in equilibrium with the tensile stresses, must also show the same degree of agreement by the two methods and therefore, it was considered unnecessary to work out and plot the former.

The effect of the presence of ribs and stiff flanges would only be to increase somewhat the cross stiffness of the sheet and therefore, only to accentuate an action that has already been shown to be of no importance.

We infer from the agreement between the two methods of solution for the loading cases of Figs. 1 and 3, there being no lateral movement of the section as a whole in the former while such movement does take place in the latter, that there will, in general, be good agreement also in structures such as lightly cambered box beams. It is also highly probable that the agreement will also be satisfactory for oval-section shells in view of the smallness of the cross-tensile stresses.

Conclusions.—In regions of tensile stress, when applying the stringer-sheet method of determining stress distribution, the skin is to be considered fully effective both in shear and in tension. In regions of compressive stress the contribution of the skin in taking end stresses may be found from an equivalent width formula, while any reduction of shear stiffness can be allowed for by a reduced shear modulus. The best basis for such allowances must of course be experimental.

REFERENCES

No.	*Author*	*Title*
1	D. Williams, R. D. Starkey and R. H. Taylor.	Distribution of Stress Between Spar Flanges and Stringers for a Wing under Distributed Loading. R. & M. 2098. June, 1939.
2	D. Williams and M. Fine	Stress Distribution in Reinforced Flat Sheet, Cylindrical Shells and Cambered Box Beams under Bending Actions. R. & M. 2099. September, 1940.
3	M. Fine and D. Williams	Stress Concentration Due to Four Point Fixing at Front End of Monocoque Fuselage. R. & M. 2100. April, 1941.

APPENDIX

Definition of symbols

Let l length of beam

t thickness of skin

$2d$ width of beam

A area of section of flange

G, E elastic moduli

$W(1 - x/l)$ shear applied to flanges in Fig. 1

W transverse end load applied in Fig. 3

Co-ordinates x, y are taken as shown in Fig. 2.

Solutions of problems

It may be shown that the stress function for the problem of Fig. 1 is

$$\phi = \sum_q D_q \left\{ y \sinh \frac{q\pi y}{2l} - d \tanh \frac{q\pi d}{2l} \cosh \frac{q\pi y}{2l} \right\} \cos \frac{q\pi x}{2l} \quad \ldots \quad (1)$$

where q takes odd positive values and

$$D_q = \frac{16Wl^2}{q^2\pi^2} \left\{ \frac{1 - (2/q\pi)\,(-)\,q - \frac{1}{2}}{q\pi\,(2A \cosh q\pi d/2l + td \operatorname{sech} q\pi d/2l) + 2lt \sinh q\pi d/2l} \right\}. \quad \ldots \quad (2)$$

The longitudinal displacement u for the stringer-sheet solution is

$$u = \sum_q A_q \cosh q\,k\pi y/2l \sin q\pi x/2l \quad \ldots \quad (3)$$

Where $\quad k^2 = E/G \quad \ldots \quad (4)$

and

$$A_q = \frac{16kWl^2}{Eq^2\pi^2} \left\{ \frac{1 - (2/q\pi)\,(-)\,q - \frac{1}{2}}{2lt \sinh q\,k\pi d/2l + kA\pi q \cosh qk\pi d/2l} \right\}. \quad \ldots \quad (5)$$

The corresponding solutions for the problem of Fig. 3 are

$$\phi = \sum_q C_q\,(y \cosh q\pi y/2l - d \coth q\pi d/2l \sinh q\pi y/2l) \cos q\pi x/2l + Wxy/2td \quad (6)$$

where

$$C_q = \frac{8l^3W}{q^2\pi^2\,d(2q\pi A \sinh q\pi d/2l - tdq\pi \operatorname{cosech} q\pi d/2l + 2lt \cosh q\pi d/2l)} \quad \ldots \quad (7)$$

and

$$u = \sum_q B_q \sinh qk\pi y/2l \sin q\pi x/2l \quad \ldots \quad (8)$$

where

$$B_q = \frac{8Wl^2}{q\pi\,\{(q^2\pi^2 dAE - 4l^2\,Gt) \sinh qk\pi d/2l + 2lkq\pi d\,Gt \cosh qk\pi d/2l\}} \quad \ldots \quad (9)$$

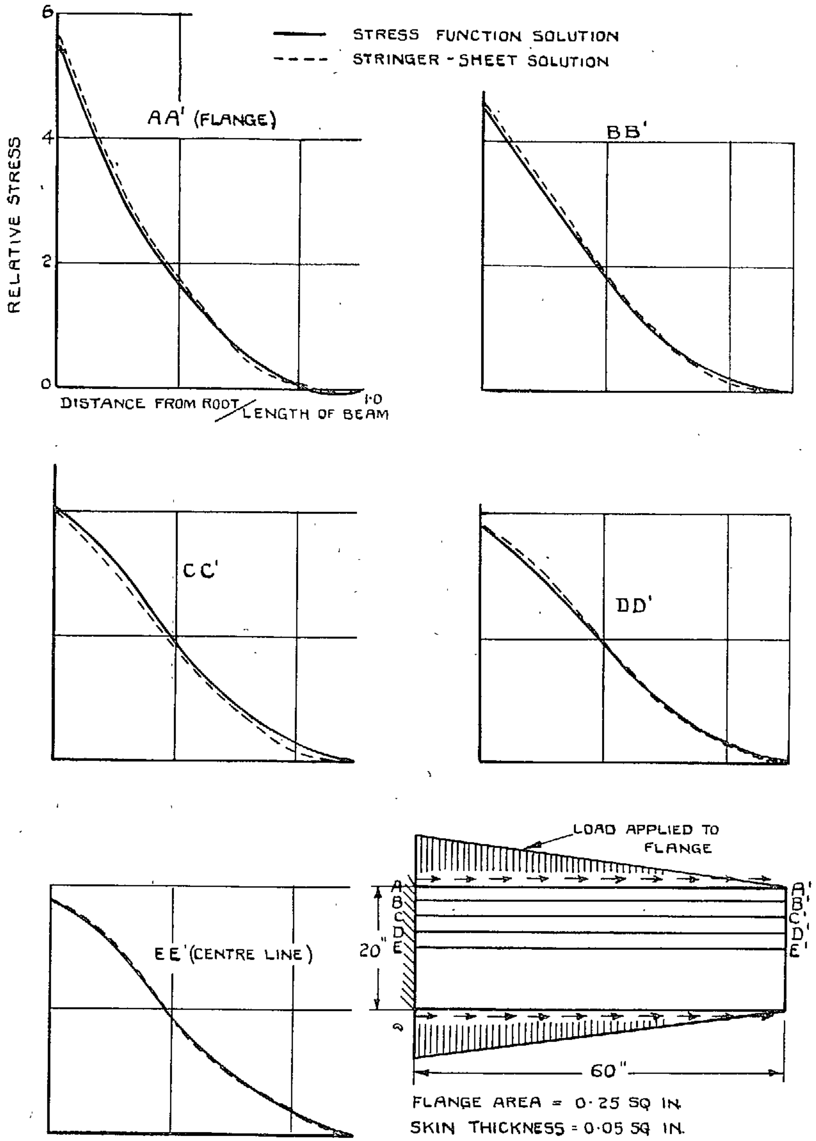

FIG. 1. Spanwise Stress in Flat Surface of Box Beam under Uniformly Distributed Transverse Load (Spanwise Plot of Stress)

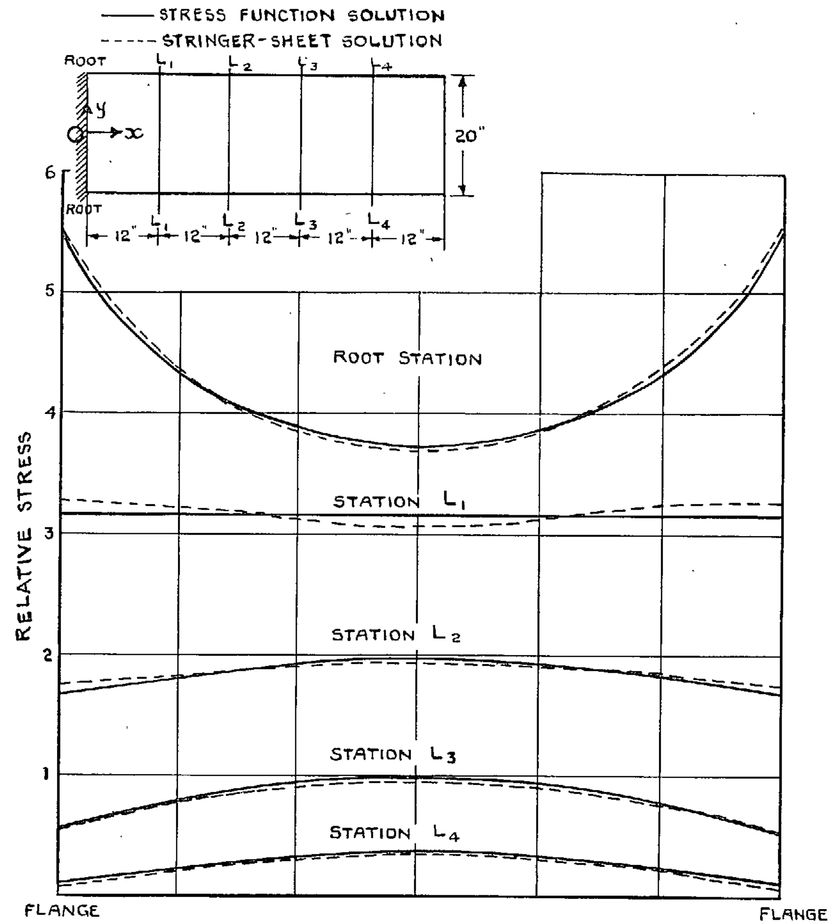

FIG. 2. Spanwise Stress in Flat Surface of Box Beam under Uniformly Distributed Transverse Load (Chordwise Plot of Stress)

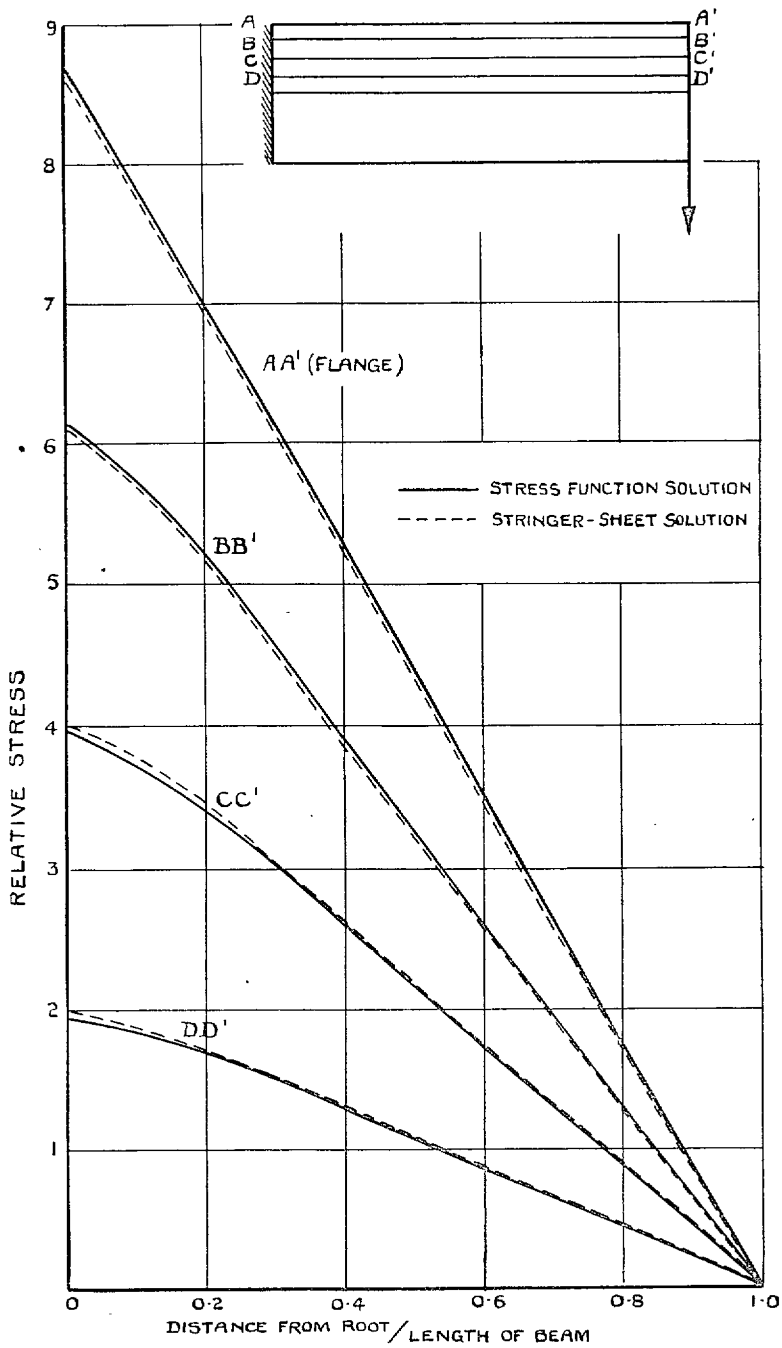

FIG. 3. Longitudinal Stress in Beam of Fig. 1 Subject to Single Transverse Load at Tip

(96735) Wt. 14/655 K.5 4/51 Hw.

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