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By

B. THWAITES, M.A., PH.D., A.F.R.AE.S.
of the Aerodynamics Division, N.P.L.

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Summary.—It has been shown in R. & M. 2611[1] how lift may be obtained on aerofoils independently of the incidence. In this paper mathematical processes are set out of designing such aerofoils to have specified velocity distributions at certain incidences and lift-coefficients. Approximate and exact methods are given, corresponding to the methods employed in the design of ordinary aerofoils. Several shapes are worked out, some of them being the product of ideas not given in R. & M. 2611[1]. A full discussion of the characteristics of such aerofoils is given.

Introduction.—In R. & M. 2611[1], a method was explained whereby lift could be obtained on aerofoils independently of incidence. An essential part of the method is the prevention of separation of flow by continuous suction over the parts of the surface where large adverse velocity gradients occur. Such aerofoils can certainly be called low-drag aerofoils, expecially if the amount of suction is sufficiently great not only to prevent separation but also to keep the boundary layer everywhere very thin. This amount of suction depends on whether the boundary layer has remained laminar or become turbulent over the non-porous parts of the surface.

Much economy, therefore, can be gained by designing such aerofoils to have suitable velocity distributions so that the flow remains laminar on the non-porous parts and also so that there is a large range of C_L for which low drag can be obtained with a small amount of suction. The now well-known ideas for achieving low drag on ordinary aerofoils are just as applicable to aerofoils which, for example, fly at zero incidence always. It is necessary, therefore, to extend the usual methods of aerofoil design to the case in which incidence and circulation are not related.

1. *Exact Theory.*—1.1. The general method given by Lighthill[2] is followed and generalised.

Let the region outside an aerofoil be transformed conformally into the region outside a circle. Then, if z and ζ are complex variables in the planes of the aerofoil and circle respectively, there is a unique analytic function $z(\zeta)$ of transformation such that $dz/d\zeta \to 1$ as $|\zeta| \to \infty$. If the radius of the circle is unity and the velocity of fluid at infinity is also unity, the most general flow about the circle is given by the complex potential $w(\zeta)$ for which

$$w(\zeta) = w_{\alpha,K} = \zeta e^{-i\alpha} + \frac{1}{\zeta e^{-i\alpha}} + iK \log \zeta \qquad \ldots \quad (1)$$

α is called the incidence of the stream, and $2\pi K$ the circulation.

$$\frac{dw_{\alpha,K}}{d\zeta} = e^{-i\alpha} - \frac{1}{\zeta^2 e^{-i\alpha}} + \frac{iK}{\zeta} \qquad \ldots \quad (2)$$

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The velocity of fluid on the circle at the point $\zeta = e^{i\theta}$ is easily found from equation (2) to be

$$\left|\frac{dw_{\alpha,K}}{d\zeta}\right|_{\zeta=e^{i\theta}} = |2 \sin (\theta - \alpha) + K| = 2\,|\sin (\theta - \alpha) + \sin \beta| \quad \ldots \quad \ldots \quad (3)$$

in which $K = 2 \sin \beta$ for convenience.

If $q_{\alpha,K}$ and q_0 are the velocities at the surface of the aerofoil for general values of incidence and circulation, and for zero values respectively, the following relations hold:—

$$q_{\alpha,K} = \left|\frac{dw_{\alpha,K}}{d\zeta} \cdot \frac{d\zeta}{dz}\right|_{\zeta=e^{i\theta}} \qquad \text{and } q_0 = \left|\frac{dw_0}{d\zeta}\,\frac{d\zeta}{dz}\right|_{\zeta=e^{i\theta}}.$$

Hence

$$q_0 = 2 \sin \theta \left|\frac{d\zeta}{dz}\right|_{\zeta=e^{i\theta}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

Further,

$$\frac{q_{\alpha,K}}{q_0} = \left|\frac{dw_{\alpha,K}}{d\zeta}\right|_{\zeta=e^{i\theta}} \div \left|\frac{dw_0}{d\zeta}\right|_{\zeta=e^{i\theta}}.$$

or, using equation (3),

$$\frac{q_{\alpha,K}}{q_0} = \left|\frac{\sin (\theta - \alpha) + \sin \beta}{\sin \theta}\right| = |\cos \alpha - \cot \theta \sin \alpha + \operatorname{cosec} \theta \sin \beta|. \quad \ldots \quad (5)$$

By means of this formula, the value of q_0 in terms of $q_{\alpha,K}$, α and β is obtained.

Certain conditions must be imposed upon q_0 which are now found.

Since the aerofoil is a totally enclosed region, the integrals $\int dz$ or $\int (dz/d\zeta)\, d\zeta$ taken round contours enclosing the aerofoil and circle must be zero.

From equation (2),

$$\frac{dw_0}{d\zeta} = 1 - \frac{1}{\zeta^2} \qquad \text{and so the closing-up condition becomes}$$

$$\int \frac{dz}{dw_0}\left(1 - \frac{1}{\zeta^2}\right) d\zeta = 0. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

Now $dw_0/dz \to 1$ as $|z| \to \infty$, hence dz/dw_0 may be written as

$$\frac{dz}{dw_0} = 1 + \frac{a_1}{\zeta} + \frac{a_2}{\zeta^2} + O\left(\frac{1}{\zeta^2}\right) \text{ as } \zeta \to \infty$$

and equation (6) then gives the result $a_1 = 0$.

Hence $$\log\left(\frac{dz}{dw_0}\right) = \frac{a_2}{\zeta^2} + O\left(\frac{1}{\zeta^3}\right) = -\log\left(\frac{dw_0}{dz}\right) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

And so, on the aerofoil, if $\dfrac{dw_0}{dz} = q_0\, e^{-i\chi}$,

$$\log\left(\frac{d\zeta}{dw_0}\right) = -\log q_0 + i\chi = \left[\frac{a_2}{\zeta^2} + \ldots\ldots\right]_{\zeta=e^{i\theta}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

Thus $\log q_0$ may be expanded in a Fourier series containing terms in $\cos n\theta$ and $\sin n\theta$ only for $n \geqslant 2$. Therefore,

$$\left.\begin{aligned} \int_{-\pi}^{\pi} \log q_0 \, d\theta &= 0 \\ \int_{-\pi}^{\pi} \sin\theta \log q_0 \, d\theta &= 0 \\ \int_{-\pi}^{\pi} \cos\theta \log q_0 \, d\theta &= 0. \end{aligned}\right\} \quad \ldots \quad (9)$$

These equations must always be satisfied.

From equation (7), $\log dw_0/dz = \log q_0 - i\chi$, and since w_0 is an analytic function in the region outside the circle, $\log q_0$ and χ on the circle are expansible as conjugate Fourier series in θ. Therefore, χ is determinable from q_0 by Poisson's integral

$$\chi(\theta) = -\frac{1}{2\pi} P\int_{-\pi}^{\pi} \log q_0(t) \cot \tfrac{1}{2}(t-\theta)dt. \quad \ldots \quad (10)$$

P denoting that the Cauchy principal value of the integral is taken at $t = \theta$.

The ordinates and abscissae of the aerofoil are given by the two relations

$$x = \int ds \cos\chi, \quad y = \int ds \sin\chi$$

which, using equation (4) can be written

$$x = 2\int \frac{\sin\theta}{q_0} \cos\chi \, d\theta, \quad y = 2\int \frac{\sin\theta}{q_0} \sin\chi \, d\theta \quad . \quad \ldots \quad (11)$$

The theory having been set out, methods of design are now described.

1.2. If q_0 is given, χ is obtainable from equation (10), and the shape is deriveable from equation (11). q_0 in its turn can be obtained in terms of q_α, from equation (5), which gives

$$\log q_0 = \log q_{\alpha K} - \log|\cos\alpha - \sin\alpha \cot\theta + \sin\beta \operatorname{cosec}\theta| \quad . \quad \ldots \quad (12)$$

Thus in the general case it is necessary to derive the conjugate and first three Fourier coefficients of the last term in equation (12), taken over any range of θ in which $q_{\alpha K}$ is specified. This, however, is complicated, and since it seems that the practical use of aerofoils for which lift is independent of incidence will be confined to zero incidence, only the case $\alpha = 0$ will be considered.

1.3. For $\alpha = 0$, equation (12) becomes

$$\log q_0 = \log q_K - \log|1 + \sin\beta \operatorname{cosec}\theta| \quad \ldots \quad (13)$$

and equation (5) becomes, $q_{0,K} = q_K$

$$q_0 = \frac{q_K \sin\theta}{\sin\beta + \sin\theta} \, . \quad \ldots \quad (14)$$

The conjugate and first three Fourier coefficients of the last term in equation (13) are therefore required. Here again the work in the general case is complicated, and the following simplifications are made which are justified at any rate by practical requirements. Only the case of the specification of q_K over the whole range of θ for one K is considered. There are then two cases: symmetrical and cambered aerofoils, with which the paper deals separately.

1.4. *Symmetrical Aerofoils:*—in which $\theta = 0$ and $\theta = \pi$ correspond respectively to the trailing and leading-edges. $\log q_0$ is then an even function of θ, and for this case, equation (10) becomes

$$\chi(\theta) = -\frac{\sin\theta}{\pi} P \int_0^\pi \frac{\log q_0(t)}{\cos\theta - \cos t}\, dt.$$

The conjugate of the even function which takes the value $\log |1 + \sin\beta \operatorname{cosec}\theta|$ for $0 \leqslant \theta \leqslant \pi$, shown in Appendix I to be

$$G(\beta, \theta) = -\frac{\pi}{2} + F\left(\tan\frac{\beta}{2}\cot\frac{\theta}{2}\right) + F\left(\cot\frac{\beta}{2}\cot\frac{\theta}{2}\right),$$

in which $F(p) = \dfrac{2}{\pi}\displaystyle\int_0^p \frac{\log x}{x^2 - 1}\, dx.$

Thus

$$\chi(\theta) = -G(\beta, \theta) - \frac{\sin\theta}{2\pi} P \int_0^\pi \frac{\log q_K(t)}{\cos\theta - \cos t}\, dt. \qquad \ldots \quad (15)$$

Also in Appendix I are calculated the first three Fourier coefficients of the function, and using these results, equation (9) becomes

$$\left.\begin{aligned} \int_0^\pi \log q_K\, d\theta &= 2\int_0^\beta \log\cot\frac{p}{2}\, dp \\ \int_0^\pi \log q_K \cos\theta\, d\theta &= 0. \end{aligned}\right\} \qquad \ldots \quad (16)$$

In the case of symmetrical aerofoils the condition $\int_{-\pi}^{\pi} \sin\theta \log q_0\, d\theta = 0$ is automatically satisfied.

It is worthwhile to point out an interesting result from the second of equations (16). Integration by parts gives

$$\left(\sin\theta \log q_K\right)_0^\pi - \int_{qK(0)}^{qK(\pi)} \frac{\sin\theta}{q_K}\, dq_K = 0$$

or $$\int_0^\pi \frac{\sin\theta}{q_K} \frac{dq_K}{d\theta}\, d\theta = 0.$$

Since $\sin\theta/q_K \geqslant 0$ for $0 \leqslant \theta \leqslant \pi$, $dq_K/d\theta$ must take both positive and negative values in this range, unless it is everywhere zero. Thus the interesting result is obtained that the velocity cannot be monotonically increasing on the whole of the upper surface.

1.5. *Cambered Aerofoils.*—If q_K is specified over the whole surface for a certain K, the conjugate is required of $\log |1 + \sin\beta \operatorname{cosec}\theta|$. In Appendix II, it is shown that this is identically zero.

Thus, simply,

$$\chi(\theta) = -\frac{1}{2\pi} P\int_{-\pi}^{\pi} \log q_K(t) \cot \tfrac{1}{2}(t-\theta)dt. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

Also in Appendix II are found the necessary Fourier coefficients of $\log | 1 + \sin\beta \operatorname{cosec}\theta |$, and therefore, equation (9) becomes

$$\left.\begin{aligned} &\int_{-\pi}^{\pi} \log q_K \, d\theta = 0 \\ &\int_{-\pi}^{\pi} \sin\theta \log q_K \, d\theta = 2\pi\sin\beta \\ &\int_{-\pi}^{\pi} \cos\theta \log q_K \, d\theta = 0. \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

The design of some aerofoils according to these methods will be explained in sections 3 and 4.

2. *Approximate Theory*.—2.1. In this paragraph the general process of Goldstein's approximations based on Theodorsen's method of transformation of an aerofoil into a circle is followed.

The aerofoil shape is given by

$$\left.\begin{aligned} x &= 2a\,(\cosh\psi_L - \cosh\psi\cos\theta) \\ y &= 2a\sinh\psi\cos\theta \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

in which $\theta = 0$ is the leading edge,* and the chord, c, lies along $y = 0$.

The circle $|\zeta| = e^{[\psi]}$ is transformed into this aerofoil by the relation $\zeta = \zeta(z)$ for which $d\zeta/dz \to 1$ as $z \to \infty$, and the modulus of this transformation at the point θ on the aerofoil is

$$\left|\frac{d\zeta}{dz}\right| = \frac{e^{[\psi]}[1 + \varepsilon'(\theta)]}{2\,[1 + \{\psi'(\theta)\}^2]^{1/2}\,[\sinh^2\psi + \sin^2\theta]^{1/2}} = \tfrac{1}{2}F(\theta) \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

in which $$[\psi] = \frac{1}{2\pi}\int_{-\pi}^{\pi} [\psi(t) - \varepsilon(t)\,\psi'(t)\,]\,dt \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

and

$$\varepsilon(\theta) = -\frac{1}{2\pi}\int_{-\pi}^{\pi} \psi(t)\,\{1 + \varepsilon'(t)\}\cot\tfrac{1}{2}\,[t + \varepsilon(t) - \theta - \varepsilon(\theta)]\,dt. \quad \ldots \quad (22)$$

Equation (22) is an integral to determine $\varepsilon(\theta)$ from $\psi(\theta)$.

Thus from equation (20), we get

$$\begin{aligned} q_{\alpha,K} &= \tfrac{1}{2}F(\theta)\,\{2\sin(\theta + \varepsilon + \alpha) + K/e^{[\psi]}\} \\ &= F(\theta)\left\{\sin(\theta + \varepsilon + \alpha) + \frac{cC_L}{8\pi e^{[\psi]}}\right\} \end{aligned} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

since the point $e^{i\phi + [\psi]}$ on the circle corresponding to the point θ on the aerofoil is related by $\phi = \theta + \varepsilon$.

2.2. Approximations are carried out, for thin aerofoils. a and ψ_L take their asymptotic values given by $4a = c$ and $\cosh\psi_L = 1$. If second and higher powers of the thickness are

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* In contrast to the convention in section 1, the stream is in the direction $-e^{-i\alpha}$. x is measured positive from the leading to the trailing edge.

neglected, we can neglect occasionally second and higher powers and products of ψ, ψ', ε, ε', so that equations (19) to (23) become, taking the chord to be of unit length,

$$\left.\begin{aligned} x &= \tfrac{1}{2}(1-\cos\theta),\quad y = \tfrac{1}{2}\psi\sin\theta \\ [\psi] &= C_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\psi\,(t)\,dt \\ \varepsilon\,(\theta) &= -\frac{1}{2\pi}\int_{-\pi}^{\pi}\psi\,(t)\cot\tfrac{1}{2}(t-\theta)\,dt \\ q_{\alpha,K} &= (q_{\alpha K})_3 = \frac{e^{C_0}\,(1+\varepsilon)}{(\psi^2+\sin^2\theta)^{1/2}}\left\{\sin\,(\theta+\varepsilon+\alpha)+\frac{C_L}{2\pi\,e^{C_0}}\right\}. \end{aligned}\right\} \qquad \ldots \quad \ldots \quad (24)$$

The last equation is the formula corresponding to Goldstein's third approximation and is the formula which is used to obtain the velocity distribution about an aerofoil of given shape.

If it is desired to find the aerofoil shape to have a specified velocity distribution, it is necessary to make use of inferior approximations. These approximations will now be obtained for the case of symmetrical aerofoils at zero incidence which is the case most applicable in practice to aerofoils such as we are considering. It is perfectly simple to derive the approximations for cambered aerofoils, and it is left to the reader to develop them if he requires them.

2.3. For a symmetrical aerofoil, $y(\theta)$ and $\varepsilon(\theta)$ are odd functions and $\psi(\theta)$ and $\varepsilon\,(\theta)$ are even functions of θ. Suffix s denotes values corresponding to a symmetrical aerofoil.

To continue the approximations ψ^2 is ignored in comparison with $\sin^2\theta$, and products and squares of ε, ε', C_0, $C_L/2\pi$ in comparison with unity. The third approximation for $q_{\alpha,K}$ given in equation (24) then reduces to

$$(q_K)_1 = 1 + \varepsilon_s' + C_0 + \varepsilon_s\cot\theta + \frac{C_L}{2\pi}\,\mathrm{cosec}\,\theta$$

which is called the first approximation. This is clearly inaccurate near the leading edge where θ is small, and this is due primarily to the neglect of ψ^2 in comparison with $\sin^2\theta$.

This can be remedied. With

$$g_s\,(\theta) = \varepsilon_s' + C_0 + \varepsilon_s\cot\theta$$

so that
$$(q_K)_1 = 1 + g_s(\theta) + \frac{C_L}{2\pi}\,\mathrm{cosec}\,\theta \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

and by approximating to $F(\theta)$ in equation (20) by $(1+\varepsilon'(\theta)+C_0+\tfrac{1}{2}C_0^2)/(\psi^2+\sin^2\theta)^{1/2}$, it is easy to obtain the formula

$$(q_K)_2 = \frac{(1+\tfrac{1}{2}C_0^2)}{(\psi^2+\sin^2\theta)^{1/2}}\left[(1+g_s)\sin\theta+\frac{C_L}{2\pi}\right] \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

which is called the second approximation. The difficulty near $\theta = 0$ has now been avoided.

In practical work, $g_s(\theta)$ is never used explicitly in finding the velocity distribution about an aerofoil. It is introduced because of its use in the design of aerofoils.

In Ref. 3, it is shown that $g_s(\theta)\sin\theta$ is expansible in a Fourier sine series conjugate to that of $2y^{s\prime}(\theta)$. Hence from a knowledge of $g_s(\theta)$ the shape $y^s(\theta)$ can be derived. The procedure in design to be adopted, therefore, follows closely that given in R. & M. 2166[4]. Briefly, it consists of using the relation (26) to obtain $g_s(\theta)$ when $(q_K)_2$, C_L have been given, and C_0 and ψ guessed. The shape is then immediately derivable from $g_s(\theta)$ by the methods given in R. & M. 2166[4].

2.4. Comparison between approximate and exact calculations is made on one of the aerofoils designed in section 3. Fig. 2 demonstrates graphically the comparison between the exact velocity distribution at certain C_L's with the third approximation. This approximation gives a slight deviation from the flat distribution at $C_L = 1{\cdot}033$ and gives $C_L = 0{\cdot}96$ as the top of the C_L range. However, the third approximation is clearly very good except very near the leading and trailing edges, for values of C_L of about unity.

Also on Fig. 2 is shown Approximation II for the upper-surface velocity at $C_L = 1{\cdot}033$. This also lies close to the exact velocity distributions and therefore, we may have confidence in the reverse process of designing aerofoils to have specified velocity distributions as given in R. & M. 2166[4], since this method uses Approximation II as a basis.

3. *Three Symmetrical Aerofoils.*—3.1. In this section, details will be given of the design of three aerofoils, designed to operate at zero incidence with a large low-drag range of C_L. Some remarks on the low-drag range are necessary first.

For conventional aerofoils whose lift is derived by incidence, the term low-drag is applied when there are positive velocity gradients over the front part of the surface of the aerofoil within a range of lift-coefficients. This definition implies first, adverse velocity gradients over this part of the surface outside this range of C_L and second, adverse gradients at all C_L's over at least some part of the rest of the aerofoil's surface.

Such a simple definition of low-drag range (there are, of course, many others possible) with its implications cannot be applied to aerofoils whose incidence is independent of lift, and we do not attempt to make any definition for the general case. However, we may justifiably restrict ourselves to the case of zero incidence since this appears to be the case most important practically. A perfectly good definition of low-drag range on such an aerofoil then is the range of C_L's within which there are positive velocity gradients over the part of the surface over which porous suction is not necessarily applied. This implies that within the C_L-range there will be no danger of separation from non-porous parts of the surface, that laminar flow will be possible over these parts, and finally that the amount of suction required elsewhere will be kept as small as possible. It does not follow that the velocity gradients over the porous parts will necessarily be always adverse, although this is true for C_L's within the low-drag range as defined above. We may expect in fact that outside the C_L-range, there will be positive velocity gradients over some parts of the porous surface.

3.2. We shall now consider the simplest possible case of a low-drag aerofoil, that for which the velocity is constant over the upper surface of a symmetrical aerofoil at a certain C_L and zero incidence. It has already been remarked in section 1 that it is impossible for the velocity to increase over the whole of the upper surface, and the case of constant velocity is, therefore, the next most desirable distribution.

Suppose $q_K = e^l$, $0 \leqslant \theta \leqslant \pi$, for a circulation corresponding to β, as given in section 1.

The conjugate of log q_K is zero, and equation (15) gives in this case

$$\chi(\theta) = \frac{\pi}{2} - F\left(\tan\frac{\beta}{2}\cot\frac{\theta}{2}\right) - F\left(\cot\frac{\beta}{2}\cot\frac{\theta}{2}\right) = -\,G(\beta, \theta).$$

Equation (16) immediately gives l in terms of β:

$$l = \frac{2}{\pi} \int_0^\beta \log \cot \frac{p}{2}\, dp$$

and $\sin\theta/q_0$ is, from equation (14), given by $\sin\theta/\,q_0 = e^{-l}\,(\sin\beta + \sin\theta)$.

The ordinates of the aerofoil are then easily computed by the formulae (11). If c is the length of the chord of the aerofoil, the lift coefficient corresponding to β is

$$C_L = \frac{8\pi \sin\beta}{c}.$$

Such aerofoils are doubly-symmetrical. Within the C_L-range the velocity increases over the front half, and decreases over the rear half. Outside the C_L-range, there will be a velocity maximum near the leading-edge and also near the trailing-edge, and the velocity will be increasing over some of the rear part. Porous suction will be required over the rear half of these aerofoils for C_L's within the C_L-range.

Figs. 1, 2, and 3 demonstrate three such aerofoils corresponding to $\beta = 5$, 8 and 20 deg. Comments and comparisons will be given in section 5.

4. *Two Cambered Aerofoils.*—In this paragraph, aerofoils are described which do not use porous suction, but suction at a slot. It has already been pointed out that, for aerofoils whose lift does not depend only on the incidence, camber is hardly necessary. However, the analysis is section 1 enables a particular type of cambered aerofoil to be designed, although it does not come strictly in the class of lift-independent-of-incidence aerofoils. The type has considerable interest.

Suction at a slot has been applied to aerofoils which, by having a very sudden fall in velocity at some point or points on the surface at which the boundary layer is removed, can be designed so that elsewhere on the surface there are only small negative velocity gradients. An obvious extension to this idea is an aerofoil on whose surface the velocity takes two (different) constant values over two parts of the surface. In this way, there are no adverse pressure gradients except at one single point.

Hughes[5] has worked out the potential flow for a particular case of such an aerofoil, and found that the thickness of the aerofoil turned out to be negative. In his analysis, he assumed that the two stagnation points on the circle into which the aerofoil is transformed correspond to the two points separating the two regions of constant velocity on the aerofoil.

If, however, the two separating points are taken to correspond to points other than the stagnation points on the circle a reasonable aerofoil shape can be derived.

The analysis is very simple by the use of section 1.5.

$$\text{Suppose } \left.\begin{aligned} q_K &= e^{l},\ 0 < \theta < \pi \\ &= e^{-l},\ -\pi < \theta < 0 \end{aligned}\right\}$$

The necessary conditions that $\int_{-\pi}^{\pi} \log q_K\, d\theta = 0$ and $\int_{-\pi}^{\pi} \cos\theta \log q_K\, d\theta = 0$ are hereby automatically satisfied. The remaining condition of equation (18) gives

$$l = \frac{\pi}{2} \sin\beta$$

and equation (17) gives immediately that

$$\chi(\theta) = \sin\beta \log\cot\frac{\theta}{2}$$

and equation (14) that $\dfrac{\sin\theta}{q_0} = (\sin\beta + \sin\theta)\, e^{-t},\ 0 < \theta < \pi$

$$= (\sin\beta + \sin\theta)\, e^{t},\ -\pi < \theta < 0$$

Hence there will be cusps at the points $\theta = -\beta$ and $\theta = -\pi + \beta$, and spirals at the points $\theta = 0$ and π, where the discontinuities in velocity occur.

Figs. 4, 5 demonstrate two such aerofoils corresponding to $\beta = 20$ and 38 deg. Remarks on these aerofoils will be given in section 5.

5. *General Properties of Such Aerofoils and Comments on the Aerofoils Designed.*—5.1. For an aerofoil for which the relation $C_L = a_0 \sin(\alpha - \alpha_0)$ is approximately true, the centre of pressure is well-known to be near the quarter-chord point. Further, if the moment on the aerofoil is zero at zero C_L, then the position of the centre of pressure remains constant.

For aerofoils whose circulation is independent of incidence, the line of action of the resultant force may pass through any point of the chord, and since this is an undesirable state of affairs it is as well to be restricted to aerofoils with certain properties of symmetry of shape and also constant incidence. It seems that such aerofoils will have little need to operate incidences other than zero, although in exceptional cases is might be desirable to delay stalling by decreasing the incidence as the lift increases (as was suggested in R. & M. 2611[1]). The moment coefficient on such an aerofoil, following the method of R. & M. 2112[2], may be easily evaluated.

Choosing the axes in the aerofoil plane so that $dw_0/dz \to 1$ and $z - \zeta \to 0$ as $z \to \infty$, equation (7) gives

$$\log\frac{dw_0}{dz} = \frac{a_2}{\zeta^2} + \frac{a_3}{\zeta^3} + \ldots \text{ as } \zeta \to \infty$$

But $z - \zeta \to 0$ as $\zeta \to \infty$, hence

$$\log\frac{dw_0}{dz} = \frac{a_2}{z^2} + \ldots \text{ as } z \to \infty \quad \ldots \quad (27)$$

and
$$\frac{dw_0}{dz} = 1 + \frac{a_2}{z^2} + \ldots\ldots .$$

Also
$$\frac{dw_K}{d\zeta} = 1 - \frac{1}{\zeta^2} + \frac{iK}{\zeta}$$

$$= \frac{dw_0}{d\zeta}\left(1 + \frac{iK}{\zeta} + \ldots\right) .$$

Hence,
$$\frac{dw_K}{dz} = \frac{dw_0}{dz}\left(1 + \frac{iK}{z} + \ldots\right), \text{ as } z \to \infty .$$

Blasius' theorem says that the moment about the origin is the real part of $\frac{1}{2}\rho\int(dw_K/dz)^2\, z\, dz$, the integral being taken round a contour enclosing the aerofoil.

This integral equals

$$\tfrac{1}{2}\rho \int \left(1 + \frac{a_2}{z^2} + \ldots\right)^2 \left(1 + \frac{iK}{z} + \ldots\right)^2 z\, dz = \tfrac{1}{2}\rho 2\pi i\,(2a_2 - K^2).$$

Hence, the moment is equal to $-2\pi\rho c_2$, where $a_2 = b_2 + ic_2$.

But from equation (27), $\log q_0 - i\chi = a_2 (\cos 2\theta \rho\ i \sin 2\theta) + \ldots$

hence the moment coefficient is

$$C_m = \frac{-4}{(\text{Chord})^2} \int_{-\pi}^{\pi} \sin 2\theta \log q_0\, d\theta\,.$$

The forces on the aerofoil having been resolved into a force through and a moment about the origin in the z-plane, we can find the position of this point as follows.

$$\lim_{z \to \infty} (z - \zeta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} z\, d\theta$$

hence, since $z - \zeta \to 0$ as $z \to \infty$,

$$\int_{-\pi}^{\pi} z\, d\theta = 0\,. \qquad \ldots \qquad (28)$$

It is simple therefore to find the origin of co-ordinates to satisfy equation (28), after the aerofoils shape has been found.

The above analysis shows that:

(i) the moment coefficient does not vary with lift coefficient, and

(ii) the line of action of the lift force remains constant.

These two results would appear to have the greatest practical importance, since the problem of control for different lift-coefficients is greatly simplified.

For a symmetrical aerofoil, $\log q_0$ is an even function of θ, and hence $C_m = 0$, and equation (28) gives

$$x_0 = \int_{-\pi}^{\pi} x\, d\theta$$

where x_0 is the position of the origin 0 with respect to the aerofoil co-ordinates (x, y). This origin will clearly lie near the half chord point. Since the centre of pressure of a supersonic aerofoil lies near the half chord point the problems of control in supersonic aircraft may thus be considerably simplified by the use of aerofoils which in the subsonic range of speeds also have the centre of pressure near this point.

5.2. A few brief comments will now be made on the aerofoils designed already.

The three symmetrical aerofoils all have a constant velocity at some C_L over their upper surfaces. The principal advantages of such aerofoils, used at zero incidence, are:

(i) A very large C_L-range, which is approximately double that of ordinary aerofoils of similar thickness.

(ii) The lowest possible magnitude of velocity at the top of the C_L-range which implies a high critical Mach number.

(iii) Constant centre of pressure at the mid-chord point.
(iv) Zero moment about mid-chord point for all C_L's.

Apart from those aerodynamic advantages, there is also a simplicity of construction.

Comment upon the two aerofoils of Figs. 4, 5 is rather more difficult especially since they are not aerofoils for which the lift is independent of the incidence. Since they have two cusps, the physical flow will be close to the potential flow only when the latter gives a finite velocity at each of those cusps. This can only occur for one value of the circulation and zero incidence. At other incidences, potential flow must give an infinite velocity at one of the cusps, and will, therefore, no longer represent at all closely the physical flow. However, at zero incidence, the C_L's obtained are very large indeed for the thickness/chord ratio. Boundary-layer suction must be applied at the single place at which the velocity decreases. Application of this type of aerofoil to aircraft could only be considered after the stability characteristics had been found—it may be that the aerofoil is unstable at zero incidence and it is certainly difficult to visualise the real flow at other incidences. Also the fact that the aerofoils operate at one and only one lift coefficient would demand a special flying technique and a particular type of aircraft. It must be emphasised that these aerofoils have been included in this paper only for their particular interest: they are not at all typical of the aerofoils which would use the Thwaites Flap and whose design primarily is considered in this paper.

Conclusion.—Methods of designing aerofoils whose lift is independent of incidence have been displayed. Some such aerofoils have been designed and the advantages to be gained are pointed out. In particular, aerofoils can be flown at zero incidence, under which condition the moment coefficient is independent of lift coefficient, and the centre of pressure remains fixed.

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APPENDIX I

The even function which takes the value $\log \mid 1 + \sin\beta \operatorname{cosec}\theta \mid$ in the range $0 \leqslant \theta \leqslant \pi$.

1. The conjugate $G(\beta, \theta)$ of this function is given by

$$G\,(\beta, \theta) = -\frac{\sin\theta}{\pi}\int_{-0}^{\pi} \frac{\log \mid 1 + \sin\beta \operatorname{cosec} t \mid}{\cos\theta - \cos t}\, dt.$$

If $q = \tan\theta/2$, $r = \tan\beta/2$, $p = \cot\theta/2 \tan t/2$, this reduces tc

$$G\,(\beta, \theta) = -\frac{2}{\pi}\left\{h\left(\frac{r}{q}\right) + h\left(\frac{1}{rq}\right) - h\,(0)\right\}$$

in which $\qquad h(s) = \displaystyle\int_0^{\infty} \frac{\log\,(p + s)}{p^2 - 1}\, dp.$

Now $\dfrac{dh}{ds} = \displaystyle\int_0^{\infty} \frac{1}{(p^2 - 1)}\,\frac{1}{p + s}\, dp = -\frac{\log s}{s^2 - 1}.$

But $h(0) = \pi^2/4$, hence $h(s) = \pi^2/4 - \displaystyle\int_0^{s} (\log s)/(s^2 - 1)\, ds.$

Therefore $G(\beta, \theta) = -\dfrac{\pi}{2} + F\left(\tan\dfrac{\beta}{2}\cot\dfrac{\theta}{2}\right) + F\left(\cot\dfrac{\beta}{2}\cot\dfrac{\theta}{2}\right)$

where $\qquad F(p) = \dfrac{2}{\pi}\displaystyle\int_0^{p} \frac{\log x}{x^2 - 1}\, dx,$

a function which has already been tabulated[2].

2. With the same substitutions

$$I_0 = \int_0^{\pi} \log \mid 1 + \sin\beta \operatorname{cosec}\theta \mid d\theta = 2\left[\,j(r) + j\left(\frac{1}{r}\right)\right] - \pi \log\left(\frac{1 + r^2}{r}\right)$$

and in which $j(s) = \displaystyle\int_0^{\infty} \frac{\log\,(p + s)}{p^2 + 1}\, dp.$

Now $\dfrac{dj(s)}{ds} = \displaystyle\int_0^{\infty} \frac{1}{(p^2 + 1)\,(p + s)}\, dp = -\frac{1}{1 + s^2}\log s + \frac{s}{1 + s^2}\,\frac{\pi}{2}.$

But $j(0) = 0$, therefore

$$j(s) = \frac{\pi}{4}\log\,(1 + s^2) - \int_0^{s} \frac{\log s}{1 + s^2}\, ds\,.$$

Therefore $I_0 = -4\displaystyle\int_0^{\tan\frac{\beta}{2}} \frac{\log s}{s^2 + 1}\, ds = 4\int_0^{\frac{\beta}{2}} \log \cot t\, dt.$

3. It is obvious that $\displaystyle\int_0^{\pi} \cos\theta \log\,(1 + \sin\alpha \operatorname{cosec}\theta)\, d\theta = 0.$

APPENDIX II

The function $\log | 1 + \sin \beta \operatorname{cosec} \theta |$

1. The conjugate of this function is given by

$$J(\theta, \beta) = -\frac{1}{2\pi} \int_{-\pi}^{\pi} \log | 1 + \sin \beta \operatorname{cosec} t | \cot \tfrac{1}{2}(t - \theta)\, dt.$$

Hence $$\frac{\partial J}{\partial \beta} = -\frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\cos \beta}{\sin \beta + \sin t} \cot \tfrac{1}{2}(t - \theta)\, dt$$

$$= -\frac{(1 - r^2)}{2\pi} \int_{-\infty}^{\infty} \frac{(1 + pq)}{(p + r)(1 + pr)(q - p)}\, dp = 0$$

all the terms being logarithmic and the substitutions of Appendix I having been used.

Also, through integration by parts,

$$J(\theta, \beta) = -\frac{\sin \beta}{\pi} \int_{-\pi}^{\pi} \frac{\cot t}{\sin \alpha + \sin t} \log \sin \tfrac{1}{2}(t - \theta)\, dt$$

and so $$\frac{\partial J}{\partial \theta} = +\frac{\sin \beta}{2\pi} \int_{-\pi}^{\pi} \frac{\cot t}{(\sin \alpha + \sin t)} \cot \tfrac{1}{2}(t - \theta)\, dt$$

$$= \frac{\sin \beta}{2\pi} \int_{-\infty}^{\infty} \frac{(1 - p^2)}{2p} \frac{(1 + p^2)(1 + r^2)}{(p + r)(1 + pr)} \frac{(1 - qp)}{(p - q)} \frac{2}{(1 + p^2)}\, dp$$
$$= 0$$

since, again, all the terms are logarithmic.

Hence $J(\theta, \beta)$ is constant and zero, since its average value is zero. Thus

$$J(\theta, b) \equiv 0.$$

2. $$J_0(\beta) = \int_{-\pi}^{\pi} \log | 1 + \sin \beta \operatorname{cosec} \theta |\, d\theta$$

$$\frac{dJ_0}{d\beta} = \int_{-\pi}^{\pi} \frac{d\theta}{\sin \theta + \sin \beta}$$
$$= 0.$$

But $J_0(0) = 0$, and since J_0 is regular, $J_0 \equiv 0$.

3. $$J_1(\beta) = \int_{-\pi}^{\pi} \sin \theta \log | 1 + \sin \beta \operatorname{cosec} \theta |\, d\theta$$

$$= \Big[-\cos \theta \log | 1 + \sin \beta \operatorname{cosec} \theta | \Big]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{\cos^2 \theta \sin \beta}{(\sin \beta + \sin \theta) \sin \theta}\, d\theta$$

$$= \int_{-\pi}^{\pi} \left[\frac{1}{\sin\beta + \sin\theta} - \frac{1}{\sin\theta}\right] d\theta + \sin\beta \int_{-\pi}^{\pi} d\theta - \sin^2\beta \int_{-\pi}^{\pi} \frac{d\theta}{\sin\beta + \sin\theta}$$

$$= 2\pi \sin\beta.$$

4. $$J_2(\beta) = \int_{-\pi}^{\pi} \cos\theta \log \mid 1 + \sin\beta \operatorname{cosec}\theta \mid d\theta$$

$$= 0$$

which is obvious.

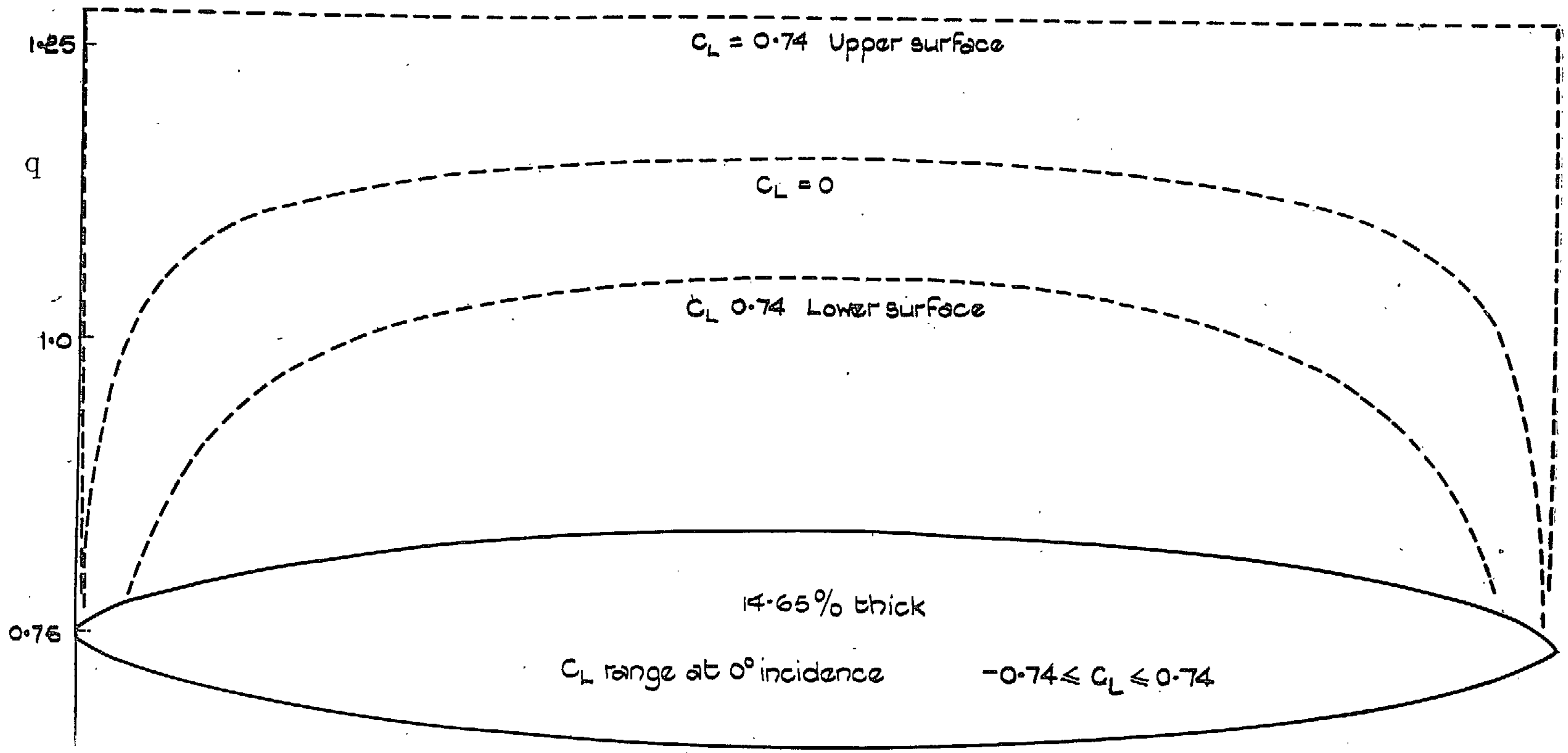
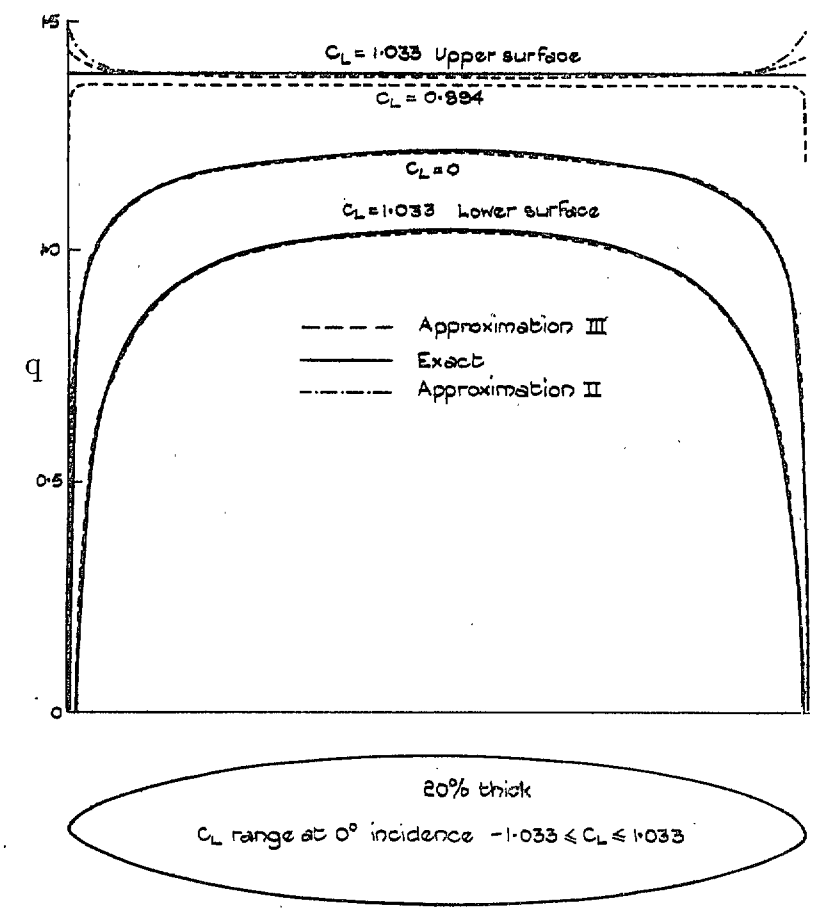

1·25
q
1·0
0·76
C_L = 0·74 Upper surface
C_L = 0
C_L 0·74 Lower surface
14·65% thick
C_L range at 0° incidence
$-0{\cdot}74 \leq C_L \leq 0{\cdot}74$

FIG. 1. T.F.A. V.



1·5
C_L = 1·033 Upper surface
C_L = 0·884
C_L = 0
C_L = 1·033 Lower surface
1·0
Approximation III
Exact
Approximation II
q
0·5
0
20% thick
C_L range at 0° incidence −1·033 ≤ C_L ≤ 1·033

FIG. 2.

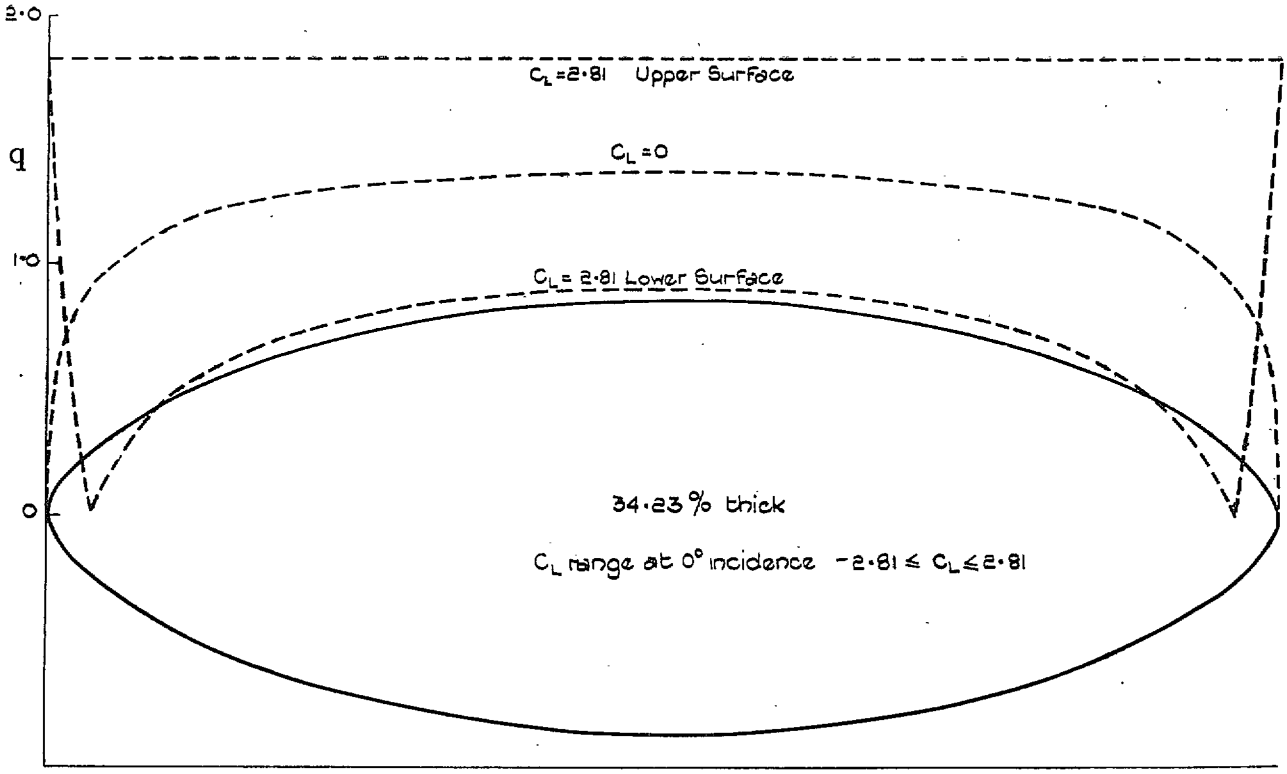


FIG. 3. T.F.A. III.

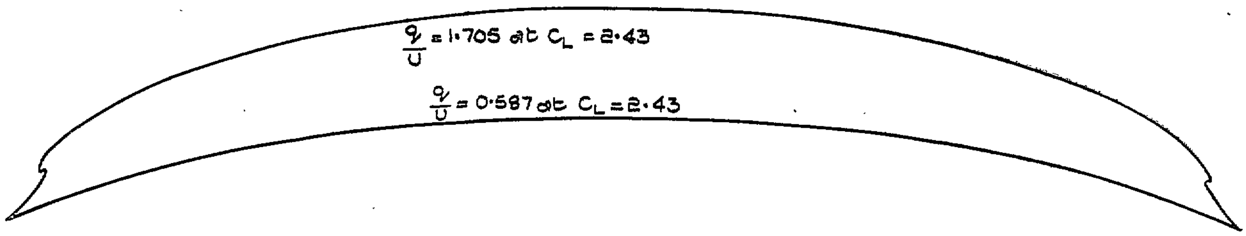


FIG. 4. C.V.A. I, 9·2 per cent thick, $C_L = 2{\cdot}43$, $C_M = 0$.

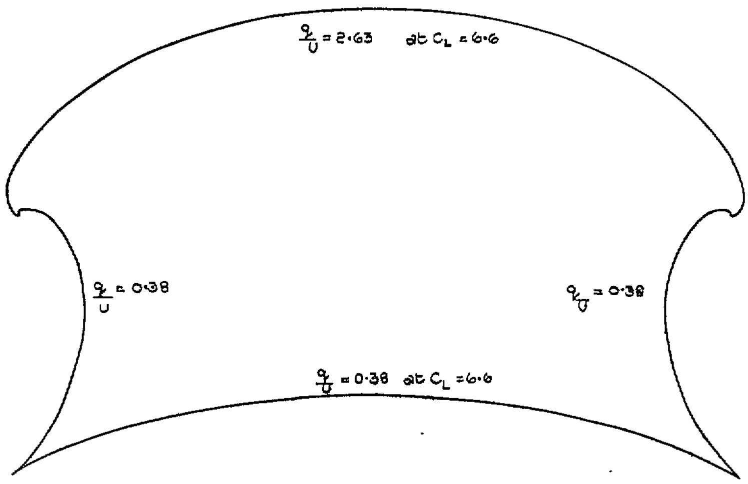


FIG. 5. C.V.A. II, 53·1 per cent thick, $C_L = 6{\cdot}6$, $C_M = 0$.

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