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Estimation of Wing Flutter Speeds from the Curves of R. & M. 1869

By

MARY E. K. GRAHAM, B.Sc.

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COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR),
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Summary.—Normal classical theory[1] is somewhat complicated for design use, and this report describes a simple method for the use of designers: briefly, this is to estimate a straight-tapered wing equivalent to the wing under consideration, and to apply the results of R. & M. 1869[2] to determine its flutter characteristics. A graph is given, in which flutter speeds calculated by the classical theory for a number of typical aeroplane wings, are plotted against the speeds found by the use of the curves given in R. & M. 1869. The method strictly applies only to plain cantilever wings, but would probably give conservative results for wings with wing engines.

1. *Introduction.*—Since the classical theory of wing flutter was first established, attempts, of which Pugsley's theory[3] is typical, have been made to simplify its highly mathematical form and render it more suitable for design use. There is still need, however, for an easy process which will enable designers to find, without much calculation, the flexural-torsional flutter speeds of aircraft wings, and in this report it is shown that such speeds can be estimated with good accuracy, compared with the results given by classical theory, by the direct use of the curves of R. & M. 1869—Influence of Wing Taper on the Flutter of Cantilever Wings—by Duncan and Griffith.

While the method suggested is suitable for plain cantilever wings of normal proportions, recourse to the classical theory (on which R. & M. 1869 is based) may be advisable when the wing has any abnormality, or as a final check when data such as displacement modes are more accurately known. In such cases, the method suggested in R. & M. 2605[4], which is a simplification of the classical theory, is recommended as being quite adequate for all practical purposes.

2. *Data and Assumptions*—

Notation

S	span (root to tip).
d	distance from root to " equivalent tip " $= 0\cdot9s$.
l	distance from root to " reference section " $= 0\cdot7s$.
c_0	root chord.
c_t	tip chord.
c_m	mean chord.
k	taper ratio $= c_t/c_0$
ρ_0	standard air density $= 0\cdot002378$ slugs/cu. ft.
ρ	air density in slugs/cu. ft.
σ_w	wing density which is total wing mass in slugs divided by the product of the wing area in square feet and the mean chord in feet.
ε	air density divided by wing density $= \rho/\sigma_w$.
l_ϕ	flexural-stiffness coefficient at the reference section.
m_θ	torsional stiffness coefficient at the reference section.
h	distance of flexural axis aft of leading edge as a fraction of the chord.
j	distance of inertia axis aft of flexural axis as a fraction of the chord.

* R.A.E. Report S.M.E. 3216—received 1st June, 1942.

Duncan and Griffith have considered straight tapered wings with square tips. Usually, aeroplane wings have rounded tips or a double taper. It is shown below that good agreement, between the critical flutter speed found by classical theory for the actual wing, and a critical speed derived from R. & M. 1869, can be obtained by choosing an equivalent straight tapered wing with the same root chord, span and area as the given wing.

3. *Method of Calculation.*—In R. & M. 1869 are given curves for particular values of ε, with the corresponding values of the wing density in pounds per cubic foot for the standard air density ($\rho_0 = 0 \cdot 002378$) alongside. If flutter speeds at altitude are required, it is necessary to calculate the value of ε corresponding to the appropriate air density and wing density. To obtain the results of the present report, interpolation has been made for wing densities, but in the case of other variables, namely, wing taper ratio and flexural axis position, the curve having the values nearest to those required has been taken and no attempt at interpolation made. It would obviously be more accurate to interpolate for all variables but the results show that sufficient accuracy is obtained without such interpolation.

4. *Application.*—To illustrate this method seven typical cantilever wings, for which the flutter speeds had previously been calculated by the classical theory, have been considered. These seven include wings with rounded tips and double tapered wings.

Duncan and Griffith have plotted $B \equiv V_c C_m \sqrt{(\rho\ d/m_\theta)}$ against $r \equiv (l_\phi/d^3)\ /\ (m_\theta/dc^2_m)$ for a range of taper ratios, flexural axis positions, and wing densities. To apply the curves it is therefore necessary to know the flexural axis position, the torsional stiffness and flexural stiffness coefficients as well as the principal dimensions and inertia characteristics.

On substitution for r the value of B can be read off from the appropriate curve and hence, V_c can be found. In the attached table are shown the data required and the results obtained for the aircraft considered. The values of k, h, and j from the corresponding curves in R. & M. 1869 are also given for comparison with the required values. The flutter speeds obtained have been plotted in Fig. 1 against the flutter speeds found by classical theory. It will be seen that the maximum error is less than 10 per cent.

No allowance is made for compressibility effects in the calculations.

5. *Conclusions.*—For all practical purposes the flexural-torsional flutter speeds for plain cantilever wings can be obtained directly from R. & M. 1869, and will approximate closely to the flutter speeds given by classical theory. R. & M. 1869 did not treat cantilever wings with wing engines, but the above process probably leads to conservative results for such wings.

REFERENCES

No.	*Authors.*	*Title.*
1.	R. A. Frazer and W. J. Duncan.	The Flutter of Aeroplane Wings. R. & M. 1155. August, 1928.
2.	W. J. Duncan and C. L. T. Griffith.	The Influence of Wing Taper on the Flutter of Cantilever Wings. R. & M. 1869. 1939.
3.	A. G. Pugsley.	A simplified Theory of Wing Flutter. R. & M. 1839. April, 1937.
4.	G. A. Naylor.	An Approximation Simplifying Wing-flutter Calculations. R. & M. 2605. April, 1942.

TABLE OF REQUIRED DATA AND COMPARISON WITH CLASSICAL THEORY OF RESULTS OBTAINED USING R. & M. 1869.

Aircraft Number	d ft.	C_m ft.	σ_w lb. per cu. ft.	l_φ	m_θ	$r = \frac{l_\varphi}{d^3} \Big/ \frac{m_\theta}{d c_m^2}$	k		h		j		R.&M. 1869 Fig. No.	$B = \frac{V_c C_m \sqrt{\rho d}}{\sqrt{m_\theta}}$	V_c from Curve) m.p.h.	V_c (Classical Theory) m.p.h.
							Value Reqd.	Value from Curve	Value Reqd.	Value from Curve	Value Reqd.	Value from Curve				
1 ..	20·7	7·226	0·942	$9 \cdot 52 \times 10^6$	$1 \cdot 39 \times 10^5$	0·834	0·447	0·4	0·342	0·35	0·06	0·05	8	2·7	1,346	1,390
2 ..	11·053	4·7	0·87	$2 \cdot 25 \times 10^6$	$2 \cdot 78 \times 10^5$	1·4625	0·371	0·4	0·31	0·3	0·13	0·1	9	2·15	1,000	971
3 ..	17·27	8·7	0·95	$3 \cdot 33 \times 10^6$	$1 \cdot 33 \times 10^6$	0·635	0·385	0·4	0·28	0·25	0·17	0·15	10	2·6	1,167	1,249
4 ..	15·2	6·47	0·664	$2 \cdot 47 \times 10^6$	$4 \cdot 2 \times 10^5$	1·065	0·52	0·524	0·22	0·25	0·18	0·15	18	2·5	900	968
5 ..	16·4	6·25	0·857	$2 \cdot 85 \times 10^6$	$2 \cdot 39 \times 10^5$	1·79	0·48	0·5	0·3	0·3	0·13	0·1	13	2·0	541	532
6 ..	12·55	4·92	1·258	$1 \cdot 61 \times 10^6$	$2 \cdot 8 \times 10^5$	0·883	0·42	0·4	0·287	0·25	0·21	0·15	10	2·15	900	827
7 ..	12·4	5·25	1·14	$1 \cdot 018 \times 10^6$	$1 \cdot 684 \times 10^5$	1·083	0·47	0·5	0·25	0·25	0·17	0·15	14	2·1	637	592

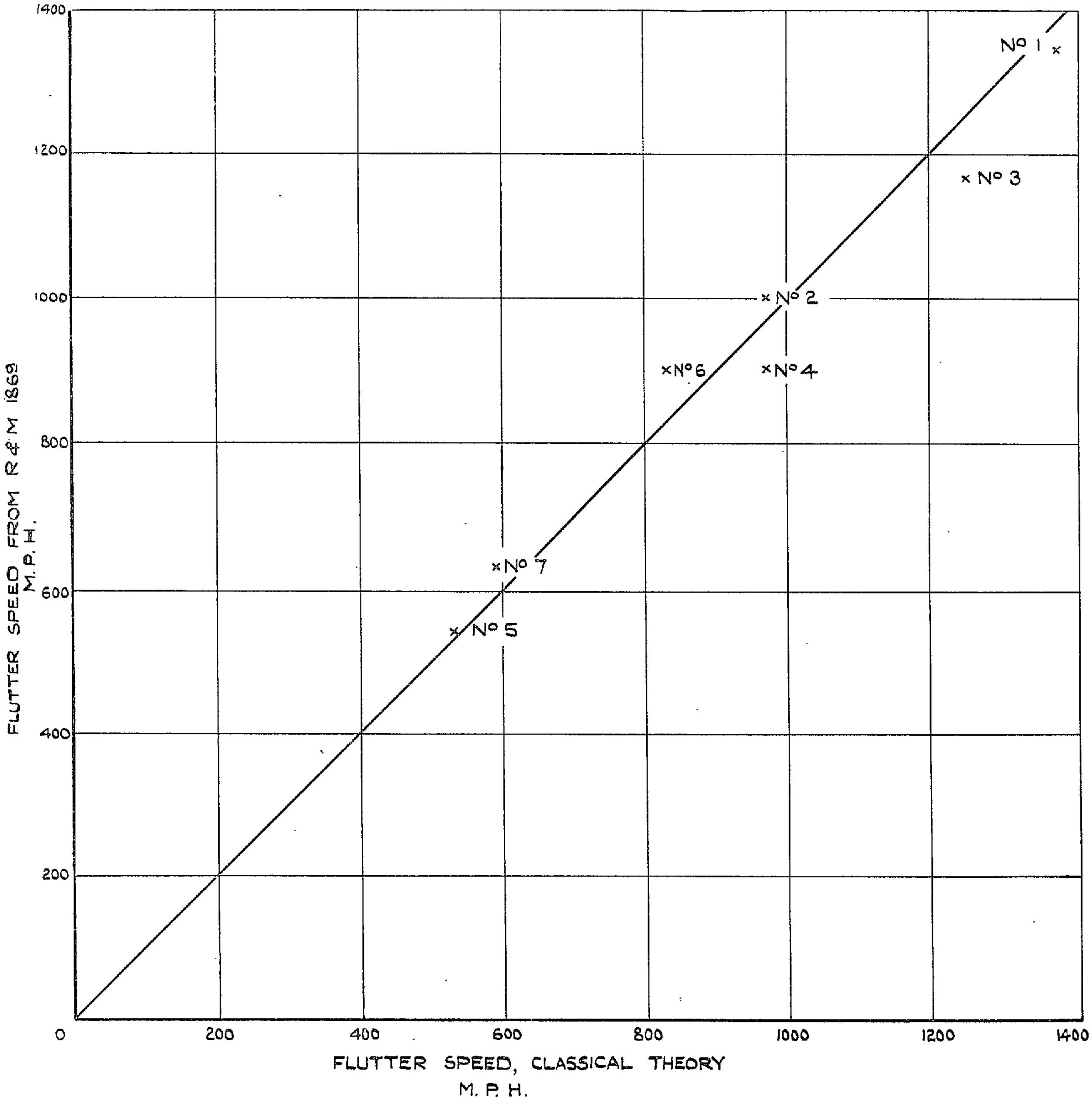

FIG. 1. Flutter Speeds Obtained using R. & M. 1869 compared with Results using Classical Theory.

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