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ON THE EVALUATION OF WALL INTERFERENCE IN TWO-DIMENSIONAL VENTILATED WIND TUNNELS BY SUBSONIC LINEAR THEORY

by

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SUMMARY

The interference potentials for lift, solid blockage and wake blockage have previously been presented in the form of integral expressions. In this Report the expression for wake blockage, which has been in error, is corrected, and the integral expressions are converted into power series. The series for wake blockage do not appear to have been given before, and the series for slotted walls also appear to be new. The motivation was to provide far-field boundary conditions, especially at large distances downstream, for computations of transonic flows within wind tunnels. Additionally these power series facilitate the calculation of the interference in the neighbourhood of the model. As an aid to the user the effects of parametric variations of wall porosity and slot geometry are demonstrated. Lift and blockage interference velocities, their streamwise gradients, and wall conditions for zero solid blockage are presented in both tabular and graphical form.

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CONTENTS

1 INTRODUCTION

When a model is placed in a wind tunnel, and measurements are made of the pressures and forces on it, these measured quantities differ from those which would be obtained if the model were tested under similar conditions, for example the same angle of incidence and Mach number, in free air, because the tunnel walls interfere with the flow. It is usual to make 'corrections' to the measured quantities and to the incidence and Mach number, so that the 'corrected' results might correspond to what would be obtained if the model were tested in free air at the 'corrected' Mach number and incidence. For subsonic flow, linear theory is usually considered adequate for the calculation of these corrections (see, for example, Ref.1).

The need for the present work arose during the development of an extension of a transonic small-perturbation method for the computation of the flow around an aerofoil to include the effects of the wall constraint in a tunnel[2,3]. This method requires a knowledge of the flow far upstream and downstream of the model, and, since the non-linear term in the transonic small-perturbation equation becomes increasingly small compared to the other terms at large distances from the model, the linear equation as used in subsonic linear theory becomes applicable.

In subsonic linear theory the interference is due to the reaction of the boundaries of the test section to the effect of three characteristics of the test model. These characteristics are its lift, its displacement and the displacement due to the wake. The interference reaction is usually estimated by representing these by respectively a vortex, a doublet and a source. The corresponding modes of interference are referred to respectively as lift interference, solid blockage and wake blockage. If the velocity potential is obtained for these three singularities within the tunnel then the values of this potential far upstream and downstream will provide the relevant far-field conditions for the computational method. Interference is represented by an interference potential, which is the difference between the potentials in the tunnel and in free air. The interference potential in the neighbourhood of the model may be used to estimate the interference corrections to be made to measured quantities.

Integral expressions for the interference potential due to a doublet have been obtained, using a Fourier transform method, by Baldwin *et al.*[4] who introduce the widely used homogeneous wall boundary condition. Wright[5] uses the same method to obtain integral expressions for a vortex and a source, although in Ref.5 there is a misprint in the former and an error in the latter affecting the results when the tunnel wall is slotted. These errors have been repeated in later literature (see for example, Ref.6) and are indicated in section 2.2 below. The integral expressions cannot be used directly to provide results either far from the model or close to the model, although in the latter instance they have been evaluated for some special cases such as solid or perforated walls. Murman[7] has obtained asymptotic expansions of the integral expressions for lift interference and solid blockage far from the model, applicable only when the tunnel walls are not slotted. He did not treat wake blockage.

In this Report the integral expressions for lift interference, solid blockage and wake blockage are converted into power series convergent at large streamwise distances from the model, including the slotted wall case as well as the simpler special cases. These power series provide directly the required far-field boundary conditions for the computational method referred to above, and can be converted into power series convergent at the model position for the calculation of interference corrections.

The Report begins with a statement of the basic equations and boundary conditions (section 2.1) and a list of the integral expressions for the interference potential (section 2.2). These are converted into power series convergent at large streamwise distances from the model in section 3. To obtain interference corrections these power series are evaluated at the model position (sections 4 and 5). In section 6 a new procedure is given for deriving conditions for zero solid blockage. These are slightly different from the approximate ones given by Baldwin *et al.*[4] and reproduced in other literature[1]. A brief resumé on the application of the interference corrections is given in section 7 and some tables and graphs of the interference parameters are supplied as practical aids.

2 THE INTERFERENCE POTENTIAL

2.1 Subsonic linear theory

The linearised equation for two-dimensional subsonic compressible flow is

$$\beta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad , \tag{1}$$

where ϕ is the perturbation velocity potential of the flow. A single homogeneous boundary condition for ventilated walls has been derived by Baldwin *et al.*[4] in the form

$$\left(\frac{\partial\phi}{\partial x} \pm \tfrac{1}{2}Fh\,\frac{\partial^2\phi}{\partial x\partial z} \pm \frac{1}{P}\,\frac{\partial\phi}{\partial z}\right)_{z=\pm\frac{1}{2}h} = 0 \quad , \tag{2}$$

where P is a porosity parameter and F is a slot parameter given by

$$F = \frac{2s}{\pi h}\,\ln\,\mathrm{cosec}\left(\frac{\pi a}{2s}\right)$$

with a = slot width and s = distance between slot centres.

Equation (2) contains the special cases

(i) solid wall: $P = 0$
(ii) perforated wall: $F = 0$
(iii) ideal slotted wall: $1/P = 0$
(iv) free jet: $F = 0,\ 1/P = 0$.

It should be noted here that equation (2) is not universally accepted as a valid wall boundary condition (see, for example, Ref 8) although it is still widely used in the absence of an agreed alternative, especially since alternative forms are usually non-linear and therefore more difficult to apply.

As explained in the Introduction, the model is represented by a vortex, a doublet and a source, and thus boundary conditions on the model itself are not included in the analysis. We write $\phi = \phi_1 + \phi^*$ where ϕ_1 is the perturbation velocity potential of the flow about each singularity in turn in free air, and ϕ^* is the interference potential. In the next section we use the Fourier transform method of Baldwin *et al.*[4] to obtain integral expressions for ϕ^* .

2.2 The interference potential in integral form

2.2.1 Lift interference

In free air the potential due to a two-dimensional vortex at the origin is

$$\phi_1 = -\frac{\Gamma}{2\pi}\tan^{-1}\left(\frac{\beta z}{x}\right) ,$$

and the interference potential may be written

$$\phi^*(\xi,\eta) = -\frac{\Gamma}{2\pi}\left\{\frac{\beta}{P}\int_0^\infty \frac{1}{I_F}\sinh\,(q\eta)\cos\,(q\xi)\,\frac{dq}{q} + \int_0^\infty \frac{I_G}{I_F}\sinh\,(q\eta)\sin\,(q\xi)\,\frac{dq}{q} + \frac{\pi}{2}\,\frac{J\eta}{1+F}\right\} , \tag{3}$$

where $I_G = [(1 - Fq)(\sinh q + Fq \cosh q) - (\beta/P)^2 \cosh q]\, e^{-q}$,

$I_F = (\sinh q + Fq \cosh q)^2 + (\beta/P)^2 \cosh^2 q$,

with $\xi = 2x/\beta h$,

$\eta = 2z/h$

and $J = 1$ when $1/P = 0$ (ideal slotted wall or free jet)

and $J = 0$ otherwise.

The last term in equation (3) is in itself a solution of the differential equation (1) and the tunnel wall boundary condition (2), when $1/P = 0$, and is added merely to make the interference potential vanish far upstream, as may be seen in section 3. In Ref.5 the first integrand was given as

$$[\sinh q + Fq \cos q + (1 - Fq) \cosh q]\, e^{-q} \frac{\sinh (q\eta) \cos (q\xi)}{I_F q}$$

whereas the cos q should have read cosh q . With this correction the expression

$$[\sinh q + Fq \cosh q + (1 - Fq) \cosh q]\, e^{-q}$$

reduces to unity.

2.2.2 Solid blockage

In free air the potential due to a doublet of strength d is

$$\phi_1 = \frac{d}{2\pi}\left(\frac{x}{x^2 + \beta^2 z^2}\right) .$$

The interference potential may be written

$$\phi^* = -\frac{d}{\pi\beta h}\left\{\frac{\beta}{P}\int_0^\infty \frac{1}{I_A} \cosh (q\eta) \cos (q\xi) dq + \int_0^\infty \frac{I_C}{I_A} \cosh (q\eta) \sin (q\xi) dq - \frac{\pi}{2} B\right\} , \qquad (4)$$

where $I_C = [(1 - Fq)(\cosh q + Fq \sinh q) - (\beta/P)^2 \sinh q]\, e^{-q}$

$I_A = (\cosh q + Fq \sinh q)^2 + (\beta/P)^2 \sinh^2 q$,

and, to make ϕ^* vanish far upstream, $B = 1$ when $P = 0$ (solid wall) and $B = 0$ otherwise.

2.2.3 Wake blockage

For a two-dimensional source of strength m

$$\phi_1 = \frac{m}{2\pi\beta} \ln (x^2 + \beta^2 z^2)^{\frac{1}{2}} \quad ;$$

$$\phi^* = -\frac{m}{2\pi\beta}\left\{\frac{\beta}{P}\int_0^\infty \frac{1}{I_A}\left[\cosh (q\eta) \sin (q\xi) - \sin q\right]\frac{dq}{q} - \int_0^\infty \frac{I_C}{I_A}\left[\cosh (q\eta) \cos (q\xi) - \cos q\right]\frac{dq}{q} - \frac{\pi}{2} B(\xi + 1) + \pi(1 - B)\frac{\beta}{P}\right\} \quad . \qquad (5)$$

The corresponding expression given in Ref.5 is

$$\phi^* = -\frac{m}{2\pi\beta}\left\{\frac{\beta}{P}\int_0^\infty \frac{I_D}{I_A} \cosh (q\eta) \sin (q\xi)\frac{dq}{q} - \int_0^\infty \frac{I_C}{I_A}\cosh (q\eta) \cos (q\xi)\frac{dq}{q}\right\} ,$$

with

$$I_D = (\sinh q + \cosh q + Fq \sinh q)\, e^{-q} \quad .$$

The expression for I_D is incorrect, and, when corrected, reduces to unity. This error has been repeated in later literature[1,6].

The expression for ϕ^* given in equation (5) above has had two extra terms included in the integrands. The second of these makes the second integrand exist at $q = 0$; the first one merely simplifies the analysis, and the effect of these additions is merely to add a constant to ϕ^* .

3 CONVERSION OF INTEGRALS INTO POWER SERIES

We follow here the technique used by Murman[7] when he evaluated the integrals in (3) and (4) for the case $F = 0$. A fuller account is given in

Appendix A. Consider first equation (3). This may be written as

$$\phi^* = -\frac{\Gamma}{2\pi}\left\{\frac{\beta}{P}\,\mathrm{Re}(L_1) + \mathscr{I}m(L_2) + \frac{\pi}{2}\frac{J\eta}{1+F}\right\}$$

where $L_1 = \int_0^\infty \frac{1}{I_F}\sinh(q\eta)e^{iq\xi}\frac{dq}{q}$; $L_2 = \int_0^\infty \frac{I_G}{I_F}\sinh(q\eta)e^{iq\xi}\frac{dq}{q}$.

I_F has zeros where $\sinh q + Fq\cosh q = ik\cosh q$ $(k = \pm\beta/P)$ or

$$e^{2q} = \frac{1 - Fq + ik}{1 + Fq - ik} .$$

Now if we write $q = a + ib$ where a and b are both real we find

$$e^{2a} = \left\{\frac{(1 - Fa)^2 + (k - Fb)^2}{(1 + Fa)^2 + (k - Fb)^2}\right\}^{\frac{1}{2}} .$$

Since, for $a \gtrless 0$ (and $F > 0$), the right hand side of this expression is $\lesssim 1$ while the left hand side is $\gtrsim 1$ it follows that $a = 0$ and so all the zeros of I_F lie on the imaginary axis. The integrals may thus be evaluated for $\xi > 0$ by replacing q by $q + ip$ and integrating round a contour (see Fig.1) which consists of the real axis from 0 to R, the quarter circle $Re^{i\theta}$ $(0 \leqslant \theta \leqslant \pi/2)$ and the imaginary axis from iR to 0 indented at the points $i\alpha_r$ (the roots of $I_F = 0$). We then let $R \to \infty$. For $\xi < 0$ the quarter circle $Re^{i\theta}$ $(3\pi/2 \leqslant \theta \leqslant 2\pi)$ and the imaginary axis from $-iR$ to 0 (again indented) are used. The α_r satisfy

$$\tan\alpha_r = \beta/P - F\alpha_r . \tag{6}$$

I_F is also zero at the points $i\bar{\alpha}_r$ where $\tan\bar{\alpha}_r = -\beta/P - F\bar{\alpha}_r$; and it is shown in Appendix A that the sum of all the residues at these points is zero. When $1/P = 0$ (ideal slotted wall or free jet) I_F also has a zero at the origin. For the solid wall case $(P = 0)$ all the α_r are odd multiples of $\pi/2$.

For the evaluation of the integrals (4) and (5) similar contours are used, except that the indentations are at the points $i\gamma_r$ where the γ_r are the roots of

$$\cot \gamma_r = F\gamma_r - \beta/P \quad . \tag{7}$$

For the solid wall case there is a pole at the origin and the other roots are multiples of π .

When the integrals have been evaluated in this way the resulting power series expansions for equations (3), (4) and (5) are as follows:

For lift interference

$$\left.\begin{aligned} \phi^* &= \frac{\Gamma}{2}\left\{\frac{1}{\pi}\tan^{-1}\left(\frac{\eta}{\xi}\right) + \sum_{\alpha_r<0} \frac{\sin(\alpha_r\eta)e^{-\alpha_r\xi}}{\alpha_r(1 + F\cos^2\alpha_r)}\right\} && (\xi < 0) \\ \phi^* &= \frac{\Gamma}{2}\left\{\frac{1}{\pi}\tan^{-1}\left(\frac{\eta}{\xi}\right) - \frac{J\eta}{1 + F} - \sum_{\alpha_r>0} \frac{\sin(\alpha_r\eta)e^{-\alpha_r\xi}}{\alpha_r(1 + F\cos^2\alpha_r)}\right\} && (\xi > 0) \quad , \end{aligned}\right\} \tag{8}$$

where $J = 1$ when $1/P = 0$ (ideal slotted wall or free jet), and $J = 0$ otherwise.

For solid blockage

$$\left.\begin{aligned} \phi^* &= \frac{d}{\beta h}\left\{-\frac{1}{\pi}\frac{\xi}{\xi^2 + \eta^2} - \sum_{\gamma_r<0} \frac{\cos(\gamma_r\eta)e^{-\gamma_r\xi}}{1 + F\sin^2\gamma_r}\right\} && (\xi < 0) \\ \phi^* &= \frac{d}{\beta h}\left\{-\frac{1}{\pi}\frac{\xi}{\xi^2 + \eta^2} + B + \sum_{\gamma_r>0} \frac{\cos(\gamma_r\eta)e^{-\gamma_r\xi}}{1 + F\sin^2\gamma_r}\right\} && (\xi > 0) \quad , \end{aligned}\right\} \tag{9}$$

where $B = 1$ when $P = 0$ (solid wall), and $B = 0$ otherwise.

For wake blockage

$$\left.\begin{aligned}
\phi^* &= \frac{m}{2\beta}\left\{-\frac{1}{\pi}\ln(\xi^2+\eta^2)^{\frac{1}{2}} + \sum_{\gamma_r<0}\frac{\cos(\gamma_r\eta)e^{-\gamma_r\xi}}{\gamma_r(1+F\sin^2\gamma_r)}\right\} & (\xi<0)\\
\phi^* &= \frac{m}{2\beta}\left\{-\frac{1}{\pi}\ln(\xi^2+\eta^2)^{\frac{1}{2}} + B\xi - (1-B)\frac{\beta}{P} - \sum_{\gamma_r>0}\frac{\cos(\gamma_r\eta)e^{-\gamma_r\xi}}{\gamma_r(1+F\sin^2\gamma_r)}\right\} & (\xi>0)
\end{aligned}\right\}$$

...... (10)

where $B = 1$ when $P = 0$ (solid wall), and $B = 0$ otherwise. A constant term

$$\frac{m}{2\beta}\sum_{\gamma_r>0}\frac{e^{-\gamma_r}}{\gamma_r(1+F\sin^2\gamma_r)}$$

has been omitted from both expressions for ϕ^* in (10) above (see Appendix A).

Expressions for the perturbation potential ϕ may be obtained by adding the appropriate free-air potential ϕ_1 to the above expressions for the interference potential ϕ^*. These provide far-field boundary conditions for use with computational methods[2,3,7]. Murman[7] performs a computation in the neighbourhood of the model and matches this with an asymptotic expansion for large (ξ), whereas in Catherall's[2,3] method values of the perturbation potential are required only at $|\xi| = \infty$. In either case equations (8) to (10), with the appropriate expressions for ϕ_1 added, provide the necessary conditions.

Interference velocity components are obtained by differentiating equations (8) to (10) and are given in Appendix B.

4 INTERFERENCE FOR SOLID OR PERFORATED WALLS, OR FREE JET, $F = 0$

The evaluation of interference corrections requires the knowledge of the interference velocity components u^* and w^* and their streamwise derivatives at the origin. When $F = 0$ the power series given in section 3 may be summed quite easily, and then ξ and η set to zero.

4.1 Lift interference

The general expressions for u^* and w^* are given in Appendix B. With the substitution $F = 0$ and

$$\alpha_r = \tan^{-1}(\beta/P) + r\pi \qquad (r = 0,1,\ldots)$$

equations (B-5) and (B-6) simplify to give

$$\left.\begin{aligned} u^* &= \frac{\partial\phi^*}{\partial x} = \frac{\Gamma}{\beta h}\,\mathcal{I}m(I_L) \\ w^* &= \frac{\partial\phi^*}{\partial z} = \frac{\Gamma}{h}\,\mathrm{Re}(I_L) \;, \end{aligned}\right\} \tag{11}$$

with

$$I_L = \frac{1}{\zeta\pi} - \frac{e^{-\theta\zeta}}{1 - e^{-\pi\zeta}} \;, \tag{12}$$

where $\zeta = \xi + i\eta = 2(x/\beta + iz)/h$ and $\theta \doteq \tan^{-1}(\beta/P)$.

At the origin both u_L^* and $\partial u_L^*/\partial x$ are zero while

$$\left.\begin{aligned} w_L^* &= \frac{\Gamma}{h}\left[\frac{\theta}{\pi} - \frac{1}{2}\right] \;, \\ \frac{\partial w_L^*}{\partial x} &= -\frac{\pi\Gamma}{\beta h^2}\left[\frac{1}{6} - \frac{\theta}{\pi} + \left(\frac{\theta}{\pi}\right)^2\right] \;. \end{aligned}\right\} \tag{13}$$

Equations (13) are the general results for a perforated wall and include the free jet ($\theta = 0$) and solid wall ($\theta = \pi/2$) as special cases.

4.2 Solid blockage

Similarly from equations (B-7) and (B-8) there results

$$\left.\begin{aligned} u^* &= \frac{2d}{\beta^2 h^2}\,\mathrm{Re}(I_S) \\ w^* &= -\frac{2d}{\beta h^2}\,\mathcal{I}m(I_S) \;, \end{aligned}\right\} \tag{14}$$

with

$$I_s = \frac{1}{\pi\zeta^2} - \frac{e^{\left(\frac{\pi}{2}-\theta\right)\zeta}}{e^{\pi\zeta} - 1} \left[\theta + \frac{\pi}{2} + \frac{\pi}{e^{\pi\zeta} - 1}\right] , \tag{15}$$

and again $\theta = \tan^{-1}(\beta/P)$. At the origin w_s^* and $\partial w_s^*/\partial x$ are zero while

$$\left.\begin{aligned} u_s^* &= \frac{\pi d}{\beta^2 h^2} \left[\left(\frac{\theta}{\pi}\right)^2 - \frac{1}{12}\right] , \\ \frac{\partial u_s^*}{\partial x} &= \frac{4\pi d}{3\beta^3 h^3}\,\theta\left[\frac{1}{4} - \left(\frac{\theta}{\pi}\right)^2\right] . \end{aligned}\right\} \tag{16}$$

The second of equations (16) appears to be a new result.

4.3 Wake blockage

Similarly from equations (B-10) and (B-11)

$$\left.\begin{aligned} u^* &= \frac{m}{\beta^2 h} \mathrm{Re}(I_w) \\ w^* &= -\frac{m}{\beta h} \mathcal{I}m(I_w) , \end{aligned}\right\} \tag{17}$$

with

$$I_w = -\frac{1}{\pi\zeta} + \frac{e^{\left(\frac{\pi}{2}-\theta\right)\zeta}}{e^{\pi\zeta} - 1} + B . \tag{18}$$

At the origin both w^* and $\partial w^*/\partial x$ are zero while

$$\left.\begin{aligned} u_w^* &= -\frac{m}{\beta^2 h}\left(\frac{\theta}{\pi} - B\right) , \\ \frac{\partial u_w^*}{\partial x} &= \frac{\pi m}{\beta^3 h^2}\left[\left(\frac{\theta}{\pi}\right)^2 - \frac{1}{12}\right] . \end{aligned}\right\} \tag{19}$$

Equations (19) with $B = 0$ are the general results for a perforated wall and include the free jet ($\theta = 0$) as a special case; for the solid wall, however, it is necessary to substitute $B = 1$ and $\theta = \pi/2$.

5 INTERFERENCE FOR SLOTTED TUNNELS, $F \neq 0$

When $F \neq 0$ the evaluation at the origin of the power series in section 3 is not so simple as when $F = 0$. The series are convergent only for large ξ and must be modified to make them convergent at the origin. Details of this procedure are given in Appendix C. We write α_r and γ_r from equations (6) and (7) in the form

$$\begin{aligned} \alpha_r &= (r - \tfrac{1}{2})\pi + \mu_r \\ \gamma_r &= r\pi + \lambda_r \end{aligned} \tag{20}$$

where, for any integer r, μ_r and λ_r lie between 0 and π. By means of the analysis of Appendix C the following expressions are obtained for the interference velocities and their streamwise derivatives at the origin.

For lift interference

$$w_L^* = \frac{\Gamma}{h}\left\{-1 + \sum_0^\infty \frac{F \sin^2\mu_r}{1 + F \sin^2\mu_r}\right\} \tag{21}$$

and

$$\frac{\partial w_L^*}{\partial x} = \frac{2\Gamma}{\beta h^2}\left\{-\frac{11\pi}{24} - \frac{1}{F\pi} - \sum_0^\infty \frac{F\pi(r - \frac{1}{2}) \sin^2\mu_r - \mu_r}{1 + F \sin^2\mu_r}\right\} . \tag{22}$$

For solid blockage

$$u_s^* = \frac{2d}{\beta^2 h^2}\left\{\frac{\pi}{12} + \frac{1}{F\pi} + \sum_0^\infty \frac{F\pi r \sin^2\lambda_r - \lambda_r}{1 + F \sin^2\lambda_r}\right\} \tag{23}$$

and

$$\frac{\partial u_s^*}{\partial x} = \frac{4d}{\beta^3 h^3}\left\{\frac{1}{2F} - \frac{\beta/P}{F^2\pi} + \sum_0^\infty \frac{2\pi r\lambda_r - F\pi^2 r^2 \sin^2\lambda_r - 1/F + \lambda_r^2 - \sin^2\lambda_r}{1 + F \sin^2\lambda_r}\right\} . \tag{24}$$

For wake blockage

$$u_w^* = \frac{m}{\beta^2 h}\left\{\frac{1}{2} - \sum_0^\infty \frac{F\sin^2\lambda_r}{1 + F\sin^2\lambda_r}\right\} \tag{25}$$

and

$$\frac{\partial u_w^*}{\partial x} = \frac{2m}{\beta^3 h^2}\left\{\frac{\pi}{12} + \frac{1}{F\pi} + \sum_0^\infty \frac{F\pi r\sin^2\lambda_r - \lambda_r}{1 + F\sin^2\lambda_r}\right\} . \tag{26}$$

The above power series converge rapidly (except when F is close to zero) and in general only a few terms need be taken, as may be seen in the next section. Note the similar expressions in equations (23) and (26) for the solid blockage velocity and the wake blockage gradient; there is a similar correspondence between the first of equations (16) and the second of equations (19) when $F = 0$.

6 CONDITIONS FOR ZERO SOLID BLOCKAGE

In Fig.2 of Ref.4 a graph is given which relates the slot and porosity parameters for zero solid blockage. This graph is reproduced in Ref.1 also. The major part of this graph consists of interpolated points of doubtful accuracy. In this section we use the results of equation (23) to correct this graph. If we write

$$U(R) = \frac{\pi}{12} + \frac{1}{F\pi} + \sum_0^R \frac{F\pi r\sin^2\lambda_r - \lambda_r}{1 + F\sin^2\lambda_r} \tag{27}$$

where from equations (7) and (20) λ_r is given by

$$\cot\lambda_r = F\pi r - \beta/P + F\lambda_r ,$$

then, from equation (23), the solid blockage is zero when $U(\infty)$ is zero.

For each pair of values of F and β/P the λ_r are calculated iteratively. If $\bar{\lambda}$ is an approximation to λ_r then a better approximation is

$$\bar{\lambda} + (\cot\bar{\lambda} - F\pi r + \beta/P - F\bar{\lambda})/(\mathrm{cosec}^2\bar{\lambda} + F) .$$

A good initial guess is $1/(F\pi r)$ for $r > 0$ and 1 for $r = 0$.

For each value of β/P , $U(\infty)$ must be evaluated for various values of F until that value of F which makes $U(\infty)$ zero is found. It is easy to show that the truncation error $U(\infty) - U(R)$ can be expanded as a power series in $1/R$ and that a better approximation to $U(\infty)$ may be obtained by calculating

$$V(R) = \tfrac{1}{2}R^2U(R) - (R-1)^2U(R-1) + \tfrac{1}{2}(R-2)^2U(R-2) \quad .$$

$V(R)$ tends to $U(\infty)$ much faster than does $U(R)$, in fact the truncation error is of order $(1/R^3)$. It has been found that usually only about 10 terms of the series (27) need be used to obtain $U(\infty)$ to within an accuracy of 10^{-5}.

The relationship between F and β/P for zero solid blockage as obtained by the above procedure is tabulated in Table 1, and Fig.2 shows the comparison with Baldwin's[4] approximate curve.

7 APPLICATION OF INTERFERENCE CORRECTIONS

The strengths of the various singularities introduced in section 2.2 are determined from physical quantities via the relations

$$\Gamma = \tfrac{1}{2}UcC_L \quad , \qquad d = AU/\beta \qquad \text{and} \qquad m = \tfrac{1}{2}UcC_D$$

where A is the 'equivalent' cross-sectional area of the model (see Ref.1, section 6.3.2).

The following formulae for the interference corrections are taken from Ref.1 and are included here for convenience.

7.1 Blockage corrections

'Corrections' need to be made to the Mach number and drag coefficient:

$$\Delta M = (1 + 0.2M^2)M(\varepsilon_s + \varepsilon_w) \tag{28}$$

$$\begin{aligned} \Delta C_D &= \frac{2A}{\rho U^2 c}\left(\frac{\partial p_s}{\partial x} + \frac{\partial p_w}{\partial x}\right) \\ &= -\frac{2A}{c}\left(\frac{\partial \varepsilon_s}{\partial x} + \frac{\partial \varepsilon_w}{\partial x}\right) = -\frac{2A}{c}\frac{\partial \varepsilon_s}{\partial x} - C_D\varepsilon_s \end{aligned} \tag{29}$$

where p_s and p_w are the interference pressures due to solid and wake blockage.

In terms of their ratios Ω_s and Ω_w for the solid wall case

$$\varepsilon_s = \frac{u_s^*}{U} = \frac{\pi A}{6\beta^3 h^2}\Omega_s \tag{30}$$

$$\varepsilon_w = \frac{u_w^*}{U} = \frac{cC_D}{4\beta^2 h}\Omega_w \ . \tag{31}$$

A 'non-dimensional' pressure gradient parameter π_s is introduced, where

$$\pi_s = -\frac{\beta^4 h^3}{\rho U^2 A}\frac{\partial p_s}{\partial x} = \frac{\beta^4 h^3}{A}\frac{\partial \varepsilon_s}{\partial x} \ . \tag{32}$$

Ω_s, Ω_w and π_s are tabulated in Tables 2, 3 and 4 and are plotted in Figs.3, 4 and 5 for various values of the porosity. Fig.3 may be compared with Fig.6.2 of Ref.1 where the impression is given that the porosity has little effect on the solid blockage. It may be seen from Fig.3 that this is far from true unless F is very large. Concerning Fig.4 it should be noted that, as regards wake blockage, the solid wall cannot be taken as the limit of a perforated wall as β/P tends to infinity, since in this case $\Omega_w \to -1$, whereas, by definition, $\Omega_w = 1$ for the solid wall.

In Ref.1, Ω_w is plotted only for $F = 0$ (Fig.6.26 of Ref.1); although this is correct, the numerator in the integrand in equations (6-23) and (6-24) of Ref.1 should be unity, and any calculations based on these equations (derived from Ref.5) would be incorrect if $F \neq 0$. Fig.6.7 of Ref.1 gives the blockage gradient parameter, here defined as π_s , only for $\beta/P = 0.6$. The results in Fig.5 show a large dependence on β/P . Incidently the largest value of $\pi_s = \pi^2/9\sqrt{3} = 0.633$ occurs when $F = 0$ and $\beta/P = \tan(\pi/\sqrt{12}) = 1.278$, the extreme condition for zero solid blockage (Table 1).

7.2 Lift corrections

The lift coefficient, quarter-chord moment coefficient and incidence have the following corrections to be applied to them:

$$\Delta C_L = -\frac{\pi}{2}\left(\frac{c}{\beta h}\right)^2 C_L \delta_1 \tag{33}$$

$$\Delta C_m = -\tfrac{1}{4}\Delta C_L \tag{34}$$

$$\Delta\alpha = \frac{cC_L}{h}\delta_0 + \frac{c^2}{\beta h^2}\delta_1(\tfrac{1}{4}C_L + C_m) \qquad (35)$$

where

$$\delta_0 = \frac{h}{UcC_L} w_L^* = \frac{h}{2\Gamma} w_L^* \qquad (36)$$

$$\delta_1 = \frac{\beta h^2}{UcC_L}\frac{\delta w_L^*}{\delta x} = \frac{\beta h^2}{2\Gamma}\frac{\partial w_L^*}{\partial x} \ . \qquad (37)$$

δ_0 and δ_1 are tabulated in Tables 5 and 6 and are plotted in Figs.6 and 7 for various values of the porosity. Fig.6 for δ_0 is comparable to Fig.6.14 of Ref.1, but for δ_1 (Fig.7) Ref.1 gives curves only for $\beta/P = 0$, the ideal slotted wall case (Fig.6.15 of Ref.1) and for $F = 0$, the perforated wall case (Fig.6.29 of Ref.1). It may be noted that δ_0 only vanishes for the closed tunnel ($\beta/P = \infty$), but that, provided $\beta/P < 0.7824$, F can be chosen to eliminate δ_1 .

8 CONCLUDING REMARKS

The motivation for the present work was to obtain the far-field (large $|\xi|$) solution for the interference potential for application in numerical flow-field calculations within two-dimensional wind tunnels. However the general solutions given in equations (8), (9) and (10) may be used to calculate the interference potential close to the model also. In particular, when $F = 0$ (solid or perforated walls or free jet) explicit expressions are obtainable for the interference velocities and their streamwise gradients at the model position (section 4).

More general expressions for the interference at the model position for slotted tunnels are given in section 5. Most of the tabulated numerical results supplement existing data; for example, those of Ref.1 are, on the whole, for special cases. However, the formulation of wake blockage in Ref.5 is incorrect, and the present values for Ω_w are based on a corrected expression for the interference potential.

Section 7, with the aid of the tables and graphs, should enable interference corrections to be readily calculated for general slotted wall cases as well as for the special cases. Alternatively corrections may be calculated using the power series given in section 5 and the computational method of section 6.

Appendix A

CONVERSION OF INTEGRALS INTO POWER SERIES

A.1 Equation (3)

As described in section 3, equation (3) is equivalent to

$$\phi^* = -\frac{\Gamma}{2\pi}\left\{\frac{\beta}{P}\,\mathrm{Re}(L_1) + \mathcal{I}m(L_2) + \frac{\pi}{2}\,\frac{J\eta}{1+F}\right\} \tag{A-1}$$

where $L_1 = \int_0^\infty \frac{1}{I_F}\sinh(q\eta)e^{iq\xi}\,\frac{dq}{q}$;

$$L_2 = \int_0^\infty \frac{I_G}{I_F}\sinh(q\eta)e^{iq\xi}\,\frac{dq}{q}$$

and $I_G = \left[(1 - Fq)(\sinh q + Fq\cosh q) - (\beta/P)^2\cosh q\right]e^{-q}$,

$I_F = (\sinh q + Fq\cosh q)^2 + (\beta/P)^2\cosh^2 q$.

The integrals may be evaluated for $\xi > 0$ by replacing q by $q + ip$ and integrating round the contour shown in Fig.1a. This has indentations at the points $i\alpha_r$ and $i\bar{\alpha}_r$ where α_r and $\bar{\alpha}_r$ are the positive roots of

$$\tan\alpha_r = \beta/P - F\alpha_r \tag{A-2}$$

and

$$\tan\bar{\alpha}_r = -\beta/P - F\alpha_r \tag{A-3}$$

respectively. For $1/P = 0$ (for which we write $J = 1$; otherwise $J = 0$) there is also an indentation at the origin. Hence, letting R tend to infinity and the radii of the indentations tend to zero, we have

$$0 = L_1 - i\int_0^\infty \frac{\sin(p\eta)e^{-p\xi}dp}{p\left[-(\sin p + Fp\cos p)^2 + (\beta/P)^2\cos^2 p\right]}$$

$$-\pi i\sum_{\alpha_r>0}\frac{i\sin(\alpha_r\eta)e^{-\alpha_r\xi}}{i\alpha_r I_F'(i\alpha_r)} - \pi i\sum_{\bar{\alpha}_r>0}\frac{i\sin(\bar{\alpha}_r\eta)e^{-\bar{\alpha}_r\xi}}{i\bar{\alpha}_r I_F'(i\bar{\alpha}_r)} \quad . \tag{A-4}$$

There will also be a contribution to L_1 from the pole at the origin when $1/P = 0$; however, the contribution to ϕ^* in equation (A-1) is zero when $1/P = 0$ and so this term may be ignored.

From equations (A-2) and (A-3) it is easy to show that

$$I_F'(i\alpha_r) = 2i\beta/P(1 + F\cos^2\alpha_r)$$

and

$$I_F'(i\bar{\alpha}_r) = -2i\beta/P(1 + F\cos^2\bar{\alpha}_r) \quad .$$

Hence from equation (A-4) we have

$$\mathrm{Re}(L_1) = \frac{\pi}{2}\frac{P}{\beta}\sum_{\alpha_r>0}\frac{\sin(\alpha_r\eta)e^{-\alpha_r\xi}}{\alpha_r(1 + F\cos^2\alpha_r)} - \frac{\pi}{2}\frac{P}{\beta}\sum_{\bar{\alpha}_r>0}\frac{\sin(\bar{\alpha}_r\eta)e^{-\bar{\alpha}_r\xi}}{\bar{\alpha}_r(1 + F\cos^2\bar{\alpha}_r)} \quad . \tag{A-5}$$

For L_2 we have

$$0 = L_2 - \int_0^\infty \frac{I_G(ip)i\sin(p\eta)e^{-p\xi}dp}{pI_F(ip)} - \frac{J\pi i}{2}\frac{\eta}{1 + F}$$

$$- \pi i\sum_{\alpha_r>0}\frac{iI_G(i\alpha_r)\sin(\alpha_r\eta)e^{-\alpha_r\xi}}{i\alpha_r I_F'(i\alpha_r)} - \pi i\sum_{\bar{\alpha}_r>0}\frac{iI_G(i\bar{\alpha}_r)\sin(\bar{\alpha}_r\eta)e^{-\bar{\alpha}_r\xi}}{i\bar{\alpha}_r I_F'(i\bar{\alpha}_r)} \quad ,$$

the third term arising from the pole at the origin when $1/P = 0$, and so

$$\mathcal{I}m(L_2) = -\int_0^\infty \frac{\sin(p\eta)e^{-p\xi}dp}{p} + J\frac{\pi}{2}\frac{\eta}{1 + F} + \frac{\pi}{2}\sum_{\alpha_r>0}\frac{\sin(\alpha_r\eta)e^{-\alpha_r\xi}}{\alpha_r(1 + F\cos^2\alpha_r)}$$

$$+ \frac{\pi}{2}\sum_{\bar{\alpha}_r>0}\frac{\sin(\bar{\alpha}_r\eta)e^{-\bar{\alpha}_r\xi}}{\bar{\alpha}_r(1 + F\cos^2\bar{\alpha}_r)} \quad . \tag{A-6}$$

Since

$$\int_0^\infty \sin(p\eta)e^{-p\xi}/p\,dp = \tan^{-1}(\eta/\xi)$$

the second of equations (8), for $\xi > 0$, results from substituting equations (A-5) and (A-6) into equation (A-1).

For $\xi < 0$ the contour shown in Fig.1b is used. The expressions for $\mathrm{Re}(L_1)$ and $\mathcal{I}m(L_2)$ given in equations (A-5) and (A-6) above apply, except that each term has the opposite sign, the negative roots of equations (A-2) and (A-3) are taken, and $e^{-p\xi}$ is replaced by $e^{p\xi}$ in the first term in equation (A-6). Thus the first of equations (8), for $\xi < 0$, results.

A.2 Equation (4)

Equation (4) is equivalent to

$$\phi^* = -\frac{d}{\pi\beta h}\left\{\frac{\beta}{P}\mathrm{Re}(L_3) + \mathcal{I}m(L_4) - B\frac{\pi}{2}\right\} \tag{A-7}$$

where $L_3 = \int_0^\infty \frac{1}{I_A}\cosh(q\eta)e^{iq\xi}dq$; $L_4 = \int_0^\infty \frac{I_C}{I_A}\cosh(q\eta)e^{iq\xi}dq$

and

$$I_C = \left[(1 - Fq)(\cosh q + Fq\sinh q) - (\beta/P)^2\sinh q\right]e^{-q} , \tag{A-8}$$

$$I_A = (\cosh q + Fq\sinh q)^2 + (\beta/P)^2\sinh^2 q . \tag{A-9}$$

The same contours are used as in section A.1 except that the indentations are now at $i\gamma_r$ and $i\bar{\gamma}_r$ where γ_r and $\bar{\gamma}_r$ are the roots of

$$\cot\gamma_r = F\gamma_r - \beta/P \tag{A-10}$$

and

$$\cot\bar{\gamma}_r = F\bar{\gamma}_r + \beta/P \tag{A-11}$$

respectively. For $P = 0$ (for which we write $B = 1$; otherwise $B = 0$) there is also an indentation at the origin. Thus for $\xi > 0$

$$0 = L_3 - i \int_0^\infty \frac{\cos (p\ )e^{-p\xi} dp}{(\cos p - Fp \sin p)^2 - (\beta/P)^2 \sin^2 p}$$

$$- \pi i \sum_{\gamma_r > 0} \frac{\cos (\gamma_r \eta) e^{-\gamma_r \xi}}{I_A'(i\gamma_r)} - \pi i \sum_{\bar{\gamma}_r > 0} \frac{\cos (\gamma_r \eta) e^{-\bar{\gamma}_r \xi}}{I_A'(i\bar{\gamma}_r)}$$

where, from (A-10) and (A-11)

$$I_A'(i\gamma_r) = - 2i\beta/P(1 + F \sin^2 \gamma_r)$$

and

$$I_A'(i\bar{\gamma}_r) = 2i\beta/P(1 + F \sin^2 \bar{\gamma}_r) \quad .$$

Hence

$$Re(L_3) = - \frac{\pi}{2} \frac{P}{\beta} \sum_{\gamma_r > 0} \frac{\cos (\gamma_r \eta) e^{-\gamma_r \xi}}{1 + F \sin^2 \gamma_r} + \frac{\pi}{2} \frac{P}{\beta} \sum_{\bar{\gamma}_r > 0} \frac{\cos (\bar{\gamma}_r \eta) e^{-\bar{\gamma}_r \xi}}{1 + F \sin^2 \bar{\gamma}_r} \quad . \qquad \text{(A-12)}$$

For L_4 we have

$$0 = L_4 - \int_0^\infty \frac{I_C(ip)}{I_A(ip)} \cos (p\eta) e^{-p\xi}\ idp + B \frac{\pi i}{2}$$

$$- \pi i \sum_{\gamma_r > 0} \frac{I_C(i\gamma_r) \cos (\gamma_r \eta) e^{-\gamma_r \xi}}{I_A'(i\gamma_r)} - \pi i \sum_{\bar{\gamma}_r > 0} \frac{I_C(i\bar{\gamma}_r) \cos (\bar{\gamma}_r \eta) e^{-\bar{\gamma}_r \xi}}{I_A'(i\bar{\gamma}_r)}$$

and so

$$\mathcal{I}m(L_4) = \int_0^\infty \cos (p\eta) e^{-p\xi} dp - B\pi/2 - \frac{\pi}{2} \sum_{\gamma_r > 0} \frac{\cos (\gamma_r \eta) e^{-\gamma_r \xi}}{1 + F \sin^2 \gamma_r}$$

$$- \frac{\pi}{2} \sum_{\bar{\gamma}_r > 0} \frac{\cos (\bar{\gamma}_r \eta) e^{-\bar{\gamma}_r \xi}}{1 + F \sin^2 \bar{\gamma}_r} \quad . \qquad \text{(A-13)}$$

Since

$$\int_0^\infty \cos\,(p\eta)e^{-p\xi}dp \;=\; \xi/(\xi^2 + \eta^2)$$

the second of equations (9), for $\xi > 0$, results from substituting equations (A-12) and (A-13) into equation (A-7).

For $\xi < 0$ the contour shown in Fig.1b is used with indentation at $i\gamma_r$ and $i\bar{\gamma}_r$. The expressions in equations (A-12) and (A-13) apply, except that each term has the opposite sign, the negative roots of equations (A-10) and (A-11) are taken, and $e^{-p\xi}$ is replaced by $e^{p\xi}$ in the first term in equation (A-13). Thus the first of equations (9), for $\xi < 0$, results.

A.3 Equation (5)

Equation (5) is equivalent to

$$\phi^* \;=\; -\frac{m}{2\pi\beta}\left\{\frac{\beta}{P}\,\mathcal{I}m(L_5) - Re(L_6) - \frac{\pi}{2}\,B(\xi + 1) + \pi(1 - B)\,\frac{\beta}{P}\right\} \tag{A-14}$$

where, for $\xi > 0$

$$L_5 \;=\; \int_0^\infty \frac{1}{I_A}\left(\cosh\,(q\eta)e^{iq\xi} - e^{iq}\right)\frac{dq}{q} \quad ;$$

$$L_6 \;=\; \int_0^\infty \frac{I_C}{I_A}\left(\cosh\,(q\eta)e^{iq\xi} - e^{iq}\right)\frac{dq}{q}$$

and I_C and I_A are given above in equations (A-8) and (A-9). The same contours as in section A.2 are used and, for $\xi > 0$,

$$0 \;=\; L_5 - \int_0^\infty \frac{(\cos\,(p\eta)e^{-p\xi} - e^{-p})dp}{p\left[(\cos p - Fp\sin p)^2 - (\beta/P)^2\sin^2 p\right]}$$

$$- \pi i \sum_{\gamma_r > 0} \frac{\cos\,(\gamma_r\eta)e^{-\gamma_r\xi} - e^{-\gamma_r}}{i\gamma_r I_A'(i\gamma_r)} - \pi i \sum_{\bar{\gamma}_r > 0} \frac{\cos\,(\bar{\gamma}_r\eta)e^{-\bar{\gamma}_r\xi} - e^{-\bar{\gamma}_r}}{i\bar{\gamma}_r I_A'(i\bar{\gamma}_r)}$$

Hence

$$m(L_5) = \frac{\pi}{2}\frac{P}{\beta}\sum_{\gamma_r>0}\frac{\cos(\gamma_r\eta)e^{-\gamma_r\xi} - e^{-\gamma_r}}{\gamma_r(1 + F\sin^2\gamma_r)} - \frac{\pi}{2}\frac{P}{\beta}\sum_{\bar{\gamma}_r>0}\frac{\cos(\bar{\gamma}_r\eta)e^{-\bar{\gamma}_r\xi} - e^{-\bar{\gamma}_r}}{\bar{\gamma}_r(1 + F\sin^2\bar{\gamma}_r)} .$$

...... (A-15)

For L_6 we have

$$0 = L_6 - \int_0^\infty \frac{I_C(ip)}{I_A(ip)}(\cos(p\eta)e^{-p\xi} - e^{-p})\frac{dp}{p} - \frac{\pi i}{2}(i - i\xi)B$$

$$- \pi i\sum_{\gamma_r>0}\frac{I_C(i\gamma_r)\left(\cos(\gamma_r\eta)e^{-\gamma_r\xi} - e^{-\gamma_r}\right)}{i\gamma_r I_A'(i\gamma_r)}$$

$$- \pi i\sum_{\bar{\gamma}_r>0}\frac{I_C(i\bar{\gamma}_r)\left(\cos(\bar{\gamma}_r\eta)e^{-\bar{\gamma}_r\xi} - e^{-\bar{\gamma}_r}\right)}{i\bar{\gamma}_r I_A'(i\bar{\gamma}_r)}$$

and so

$$\mathrm{Re}(L_6) = \int_0^\infty (\cos(p\eta)e^{-p\xi} - e^{-p})\frac{dp}{p} + \frac{\pi}{2}(\xi - 1)B$$

$$- \frac{\pi}{2}\sum_{\gamma_r>0}\frac{\cos(\gamma_r\eta)e^{-\gamma_r\xi} - e^{-\gamma_r}}{\gamma_r(1 + F\sin^2\gamma_r)}$$

$$- \frac{\pi}{2}\sum_{\bar{\gamma}_r>0}\frac{\cos(\bar{\gamma}_r\eta)e^{-\bar{\gamma}_r\xi} - e^{-\bar{\gamma}_r}}{\bar{\gamma}_r(1 + F\sin^2\bar{\gamma}_r)} . \qquad \text{(A-16)}$$

Since

$$\int_0^\infty (\cos(p\eta)e^{-p\xi} - e^{-p})/p\,dp = -\ln(\xi^2 + \eta^2)^{\frac{1}{2}}$$

we have

$$\phi^* = -\frac{m}{2\beta}\left\{\frac{1}{\pi}\ln(\xi^2+\eta^2)^{\frac{1}{2}} - B\xi + (1-B)\frac{\beta}{P} + \sum_{\gamma_r>0}\frac{\cos(\gamma_r\eta)e^{-\gamma_r\xi} - e^{-\gamma_r}}{\gamma_r(1+F\sin^2\gamma_r)}\right\}$$

which reduces to the form given in the second of equations (10) for $\xi > 0$ when the constant term

$$\frac{m}{2\beta}\sum_{\gamma_r>0}\frac{e^{-\gamma_r}}{\gamma_r(1+F\sin^2\gamma_r)}$$

has been subtracted. This term will also be subtracted from the expression for ϕ^* for $\xi < 0$ to be derived in equation (A-19).

When $\xi < 0$ we replace L_5 and L_6 in equation (A-14) by

$$L_7 = \int_0^\infty \frac{1}{I_A}\left(\cosh(q\eta)e^{iq\xi} + e^{-iq}\right)\frac{dq}{q} \quad ;$$

$$L_8 = \int_0^\infty \frac{I_C}{I_A}\left(\cosh(q\eta)e^{iq\xi} - e^{-iq}\right)\frac{dq}{q}$$

and, using the usual contour for $\xi < 0$, and noting that for L_7 there is now a pole at the origin provided $P \neq 0$, we find that

$$0 = L_7 - \int_0^\infty \frac{(\cos(p\eta)e^{p\xi} + e^{-p})dp}{p\left[(\cos p - Fp\sin p)^2 - (\beta/P)^2\sin^2 p\right]} + \frac{\pi i}{2}2(1-B)$$

$$+ \pi i\sum_{\gamma_r<0}\frac{(\cos(\gamma_r\eta)e^{-\gamma_r\xi} + e^{\gamma_r})}{i\gamma_r I_A'(i\gamma_r)} + \pi i\sum_{\bar{\gamma}_r<0}\frac{\cos(\bar{\gamma}_r\eta)e^{-\bar{\gamma}_r\xi} + e^{\bar{\gamma}_r}}{i\bar{\gamma}_r I_A'(i\bar{\gamma}_r)} \quad .$$

Hence

$$\mathrm{m}(L_7) = -\pi(1-B) - \frac{\pi}{2}\frac{P}{\beta}\sum_{\gamma_r<0}\frac{\cos(\gamma_r\eta)e^{-\gamma_r\xi} + e^{\gamma_r}}{\gamma_r(1+F\sin^2\gamma_r)}$$

$$+ \frac{\pi}{2}\frac{P}{\beta}\sum_{\bar{\gamma}_r<0}\frac{\cos(\bar{\gamma}_r\eta)e^{-\bar{\gamma}_r\xi} + e^{\bar{\gamma}_r}}{\bar{\gamma}_r(1+F\sin^2\bar{\gamma}_r)} \quad . \qquad \text{(A-17)}$$

For L_8 we have

$$0 = L_8 - \int_0^\infty \frac{I_C(-ip)}{I_A(-ip)}\left(\cos(p\eta)e^{p\xi} - e^{-p}\right)\frac{dp}{p} + \frac{\pi i}{2}(-i\xi - i)B$$

$$+ \pi i \sum_{\gamma_r<0}\frac{I_C(i\gamma_r)\left(\cos(\gamma_r\eta)e^{-\gamma_r\xi} - e^{\gamma_r}\right)}{i\gamma_r I_A'(i\gamma_r)}$$

$$+ \pi i \sum_{\bar{\gamma}_r<0}\frac{I_C(i\bar{\gamma}_r)\left(\cos(\bar{\gamma}_r\eta)e^{-\bar{\gamma}_r\xi} - e^{\bar{\gamma}_r}\right)}{i\bar{\gamma}_r I_A'(i\bar{\gamma}_r)}$$

and so

$$\mathrm{Re}(L_8) = \int_0^\infty \left(\cos(p\eta)e^{p\xi} - e^{-p}\right)\frac{dp}{p} - \frac{\pi}{2}(\xi+1)B$$

$$+ \frac{\pi}{2}\sum_{\gamma_r<0}\frac{\cos(\gamma_r\eta)e^{-\gamma_r\xi} - e^{\gamma_r}}{\gamma_r(1+F\sin^2\gamma_r)} + \frac{\pi}{2}\sum_{\bar{\gamma}_r<0}\frac{\cos(\bar{\gamma}_r\eta)e^{-\bar{\gamma}_r\xi} - e^{\bar{\gamma}_r}}{\bar{\gamma}_r(1+F\sin^2\bar{\gamma}_r)} \quad . \qquad \text{(A-18)}$$

Thus

$$\phi^* = -\frac{m}{2\beta}\left\{\frac{1}{\pi}\ln(\xi^2+\eta^2)^{\frac{1}{2}} - \sum_{\gamma_r<0}\frac{\cos(\gamma_r\eta)e^{-\gamma_r\xi}}{\gamma_r(1+F\sin^2\gamma_r)} + \sum_{\bar{\gamma}_r<0}\frac{e^{\bar{\gamma}_r}}{\bar{\gamma}_r(1+F\sin^2\bar{\gamma}_r)}\right\} \quad .$$

...... (A-19)

Since $\bar{\gamma}_{-r} = -\gamma_r$ (from equations (A-10) and (A-11)) the last term in equation (A-19) is equal to

$$\frac{m}{2\beta} \sum_{\gamma_r > 0} \frac{e^{-\gamma_r}}{\gamma_r(1 + F \sin^2\gamma_r)} \quad ,$$

and when it has been subtracted from ϕ^*, as was done for $\xi > 0$, the first of equations (10) for $\xi < 0$ results.

Appendix B

INTERFERENCE VELOCITY COMPONENTS

Interference velocity components u^* and w^* are related to ϕ^* by

$$u^* = \frac{\partial \phi^*}{\partial x} = \frac{2}{\beta h}\frac{\partial \phi^*}{\partial \xi} \; ; \qquad w^* = \frac{\partial \phi^*}{\partial z} = \frac{2}{h}\frac{\partial \phi^*}{\partial \eta} \; . \tag{B-1}$$

Expressions for u^* and w^* derived from equations (8), (9) and (10) are obtained below.

B.1 Lift interference

$$u^* = \frac{\Gamma}{\beta h}\left\{ -\frac{\eta}{\pi(\xi^2+\eta^2)} + \sum_{\alpha_r>0} \frac{\sin(\alpha_r\eta)e^{-\alpha_r\xi}}{1+F\cos^2\alpha_r} \right\} \quad \text{for } \xi > 0 \, , \tag{B-2}$$

$$w^* = \frac{\Gamma}{h}\left\{ \frac{\xi}{\pi(\xi^2+\eta^2)} - \frac{J}{1+F} - \sum_{\alpha_r>0} \frac{\cos(\alpha_r\eta)e^{-\alpha_r\xi}}{1+F\cos^2\alpha_r} \right\} \quad \text{for } \xi > 0 \, , \tag{B-3}$$

with corresponding expressions for $\xi < 0$.

An examination of the roots of equation (6) reveals that

$$\left.\begin{aligned} \alpha_r &= (r+\tfrac{1}{2})\pi && \text{for } P = 0 \\ \text{and} \quad (r-\tfrac{1}{2})\pi \leqslant \alpha_r &< (r+\tfrac{1}{2})\pi && \text{for } P \neq 0 \end{aligned}\right\} \tag{B-4}$$

and so equations (B-2) and (B-3) may be written

$$u^* = \frac{\Gamma}{\beta h}\left\{ -\frac{\eta}{\pi(\xi^2+\eta^2)} + \sum_{r=0}^{\infty} \frac{\sin(\alpha_r\eta)e^{-\alpha_r\xi}}{1+F\cos^2\alpha_r} \right\} \quad \text{for } \xi > 0 \, , \tag{B-5}$$

$$w^* = \frac{\Gamma}{h}\left\{ \frac{\xi}{\pi(\xi^2+\eta^2)} - \sum_{r=0}^{\infty} \frac{\cos(\alpha_r\eta)e^{-\alpha_r\xi}}{1+F\cos^2\alpha_r} \right\} \quad \text{for } \xi > 0 \, . \tag{B-6}$$

For $\xi < 0$ the summations run from $r = -1$ to $-\infty$. It may be noted that for $1/P = 0$ $\alpha_0 = 0$ and so the constant term in equation (B-3) may be absorbed into the summation as in (B-6).

B.2 Solid blockage

$$u^* = \frac{2d}{\beta^2 h^2}\left\{\frac{\xi^2 - \eta^2}{\pi(\xi^2 + \eta^2)^2} - \sum_{r=0}^{\infty} \frac{\gamma_r \cos(\gamma_r \eta) e^{-\gamma_r \xi}}{1 + F \sin^2\gamma_r}\right\} \quad \text{for } \xi > 0 \text{ ,} \tag{B-7}$$

$$w^* = \frac{2d}{\beta h^2}\left\{\frac{2\xi\eta}{\pi(\xi^2 + \eta^2)^2} - \sum_{r=0}^{\infty} \frac{\gamma_r \sin(\gamma_r \eta) e^{-\gamma_r \xi}}{1 + F \sin^2\gamma_r}\right\} \quad \text{for } \xi > 0 \text{ .} \tag{B-8}$$

For $\xi < 0$ the summations run from $r = -1$ to $-\infty$. From equation (7) we find that

$$\left.\begin{aligned} \gamma_r &= r\pi && \text{for } P = 0 \\ r\pi < \gamma_r &< (r + 1)\pi && \text{for } P \neq 0 \text{ .} \end{aligned}\right\} \tag{B-9}$$

B.3 Wake blockage

$$u^* = \frac{m}{\beta^2 h}\left\{- \frac{\xi}{\pi(\xi^2 + \eta^2)} + \sum_{r=0}^{\infty} \frac{\cos(\gamma_r \eta) e^{-\gamma_r \xi}}{1 + F \sin^2\gamma_r}\right\} \quad \text{for } \xi > 0 \text{ ,} \tag{B-10}$$

$$w^* = \frac{m}{\beta h}\left\{- \frac{\eta}{\pi(\xi^2 + \eta^2)} + \sum_{r=0}^{\infty} \frac{\sin(\gamma_r \eta) e^{-\gamma_r \xi}}{1 + F \sin^2\gamma_r}\right\} \quad \text{for } \xi > 0 \text{ .} \tag{B-11}$$

For $\xi < 0$ the summations run from $r = -1$ to $-\infty$. γ_r is defined as in equation (B-9) and the constant term is u^* for $P = 0$ has been absorbed into the summation.

Appendix C

INTERFERENCE AT THE ORIGIN WHEN $F \neq 0$

The power series expansions given in section 3 and in Appendix B are only convergent for large ξ and must be modified before they can be made convergent at the origin. We note first that α_r and γ_r may be written

$$\alpha_r = (r - \tfrac{1}{2})\pi + \mu_r \tag{C-1}$$

$$\gamma_r = r\pi + \lambda_r \tag{C-2}$$

where μ_r and λ_r lie between 0 and π for finite β/P. For large r and $F \neq 0$ it is easy to show that

$$\mu_r = \frac{1}{F\pi r} + 0\left(\frac{1}{r^2}\right) ; \qquad \lambda_r = \frac{1}{F\pi r}\left(1 + \frac{\beta/P}{F\pi r} + 0\left(\frac{1}{r^2}\right)\right) . \tag{C-3}$$

These properties of α_r and γ_r will be utilised in the following analysis.

C.1 Lift interference

The components u^* and $\partial u^*/\partial x$ are both zero at the origin, while w^* and $\partial w^*/\partial x$ at the origin are to be obtained. From equation (B-6) we may write

$$\begin{aligned}\frac{h}{\Gamma} w_L^* &= \lim_{\xi \to 0} \left\{ \frac{1}{\pi\xi} - \sum_{r=0}^{\infty} \frac{e^{-\alpha_r \xi}}{1 + F\cos^2\alpha_r} \right\} \\ &= \lim_{\xi \to 0} \left\{ \frac{1}{\pi\xi} - \sum_{r=0}^{\infty} e^{-\pi(r-\frac{1}{2})\xi} + \sum_{r=0}^{\infty} e^{-\pi(r-\frac{1}{2})\xi} \frac{(1 + F\sin^2\mu_r - e^{-\mu_r\xi})}{1 + F\sin^2\mu_r} \right\}\end{aligned}$$

from equation (C-1). The second term is equal to $-e^{\frac{1}{2}\pi\xi}/(1 - e^{-\pi\xi})$ and when expanded near $\xi = 0$ provides a term to cancel the term $1/\pi\xi$. Thus

$$\frac{h}{\Gamma} w_L^* = -1 + \lim_{\xi \to 0} \sum_{r=0}^{\infty} e^{-\pi(r-\frac{1}{2})\xi} \frac{F\sin^2\mu_r}{1 + F\sin^2\mu_r} + \lim_{\xi \to 0} \sum_{r=0}^{\infty} e^{-\pi(r-\frac{1}{2})\xi} \frac{(1 - e^{-\mu_r\xi})}{1 + F\sin^2\mu_r} .$$

The first summation tends to $\sum_{r=0}^{\infty} \frac{F \sin^2\mu_r}{1 + F \sin^2\mu_r}$ as $\xi \to 0$ and, from equation (C-3), the r + 1th term in the second summation is: $\frac{e^{-\pi(r-\frac{1}{2})\xi}}{1 + F \sin^2\mu_r} \, O\left(\frac{\xi}{r}\right)$ for large r. Thus the series is convergent, since $\sum \frac{e^{-\pi r\xi}}{r}$ is convergent for large r, and tends to zero as $\xi \to 0$.

Hence

$$\frac{h}{\Gamma} w_L^* = -1 + \sum_{r=0}^{\infty} \frac{F \sin^2\mu_r}{1 + F \sin^2\mu_r} .$$

The other quantity that is required at the origin is the streamline curvature term $\partial w_L^*/\partial x$. From equation (B-6) we obtain

$$\frac{\partial w^*}{\partial x} = \frac{2\Gamma}{\beta h^2} \left\{ \frac{\eta^2 - \xi^2}{\pi(\xi^2 + \eta^2)^2} + \sum_{r=0}^{\infty} \frac{\alpha_r \cos(\alpha_r\eta) e^{-\alpha_r\xi}}{1 + F \cos^2\alpha_r} \right\} .$$

Thus, by the same procedure as above

$$\frac{\beta h^2}{2\Gamma} \frac{\partial w_L^*}{\partial x} = \lim_{\xi\to 0} \left\{ -\frac{1}{\pi\xi^2} + \sum_{r=0}^{\infty} \frac{\alpha_r e^{-\alpha_r\xi}}{1 + F \cos^2\alpha_r} \right\}$$

$$= \lim_{\xi\to 0} \left\{ -\frac{1}{\pi\xi^2} + \sum_{r=0}^{\infty} e^{-\pi(r-\frac{1}{2})\xi} \left(\pi(r - \tfrac{1}{2}) - \frac{\xi}{F}\right) \right.$$

$$- \sum_{r=0}^{\infty} e^{-\pi(r-\frac{1}{2})\xi} \, \frac{(F\pi(r - \frac{1}{2}) \sin^2\mu_r - \mu_r e^{-\mu_r\xi})}{1 + F \sin^2\mu_r}$$

$$\left. + \sum_{r=0}^{\infty} \frac{e^{-\pi(r-\frac{1}{2})\xi}}{1 + F \sin^2\mu_r} \left(\pi(r - \tfrac{1}{2})(e^{-\mu_r\xi} - 1) + \frac{\xi}{F}(1 + F \sin^2\mu_r)\right) \right\} .$$

As above the third summation tends to zero as $\xi \to 0$. The first summation is equal to

$$e^{\frac{1}{2}\pi\xi}\left\{\frac{\pi e^{-\pi\xi}}{(1 - e^{-\pi\xi})^2} - \frac{(\pi/2 + \xi/F)}{1 - e^{-\pi\xi}}\right\}$$

and, when expanded near $\xi = 0$, gives a contribution

$$\frac{1}{\pi\xi^2} - \frac{11}{24}\pi - \frac{1}{F\pi} + 0(\xi) \quad ,$$

which cancels the singularity and provides the first two terms of equation (22). The second summation tends to

$$\sum_{r=0}^{\infty} \frac{F\pi(r - \frac{1}{2}) \sin^2\mu_r - \mu_r}{1 + F \sin^2\mu_r}$$

as $\xi \to 0$ and provides the remaining terms of equation (22).

C.2 Solid blockage

The components w^* and $\partial w^*/\partial x$ are both zero at the origin while u^* and $\partial u^*/\partial x$ at the origin may be obtained from equation (B-7) using the techniques of section C.1.

$$\frac{\beta^2 h^2}{2d} u^*_s = \lim_{\xi\to 0}\left\{\frac{1}{\pi\xi^2} - \sum_{r=0}^{\infty} \frac{\gamma_r e^{-\gamma_r\xi}}{1 + F \sin^2\gamma_r}\right\}$$

$$= \lim_{\xi\to 0}\left\{\frac{1}{\pi\xi^2} - \sum_{r=0}^{\infty} e^{-\pi r\xi}\left(\pi r - \frac{\xi}{F}\right) + \sum_{r=0}^{\infty} e^{-\pi r\xi} \frac{(F\pi r \sin^2\lambda_r - \lambda_r e^{-\lambda_r\xi})}{1 + F \sin^2\lambda_r}\right.$$

$$\left. + \sum_{r=0}^{\infty} \frac{e^{-\pi r\xi}}{1 + F \sin^2\lambda_r}\left(\pi r - \frac{\xi}{F} - \xi \sin^2\lambda_r - \pi r e^{-\lambda_r\xi}\right)\right\}$$

which reduces to equation (23). From equation (B-7)

$$\frac{\partial u^*}{\partial x} = \frac{4d}{\beta^3 h^3}\left\{\frac{2\xi(3\eta^2 - \xi^2)}{\pi(\xi^2 + \eta^2)^3} + \sum_{r=0}^{\infty} \frac{\gamma_r^2 \cos(\gamma_r\eta)e^{-\gamma_r\xi}}{1 + F\sin^2\gamma_r}\right\}$$

for $\xi > 0$. Thus

$$\frac{\beta^3 h^3}{4d}\frac{\partial u_s^*}{\partial x} = \lim_{\xi\to 0}\left\{-\frac{2}{\pi\xi^3} + \sum_{r=0}^{\infty}\frac{\gamma_r^2 e^{-\gamma_r\xi}}{1 + F\sin^2\gamma_r}\right\}$$

$$= \lim_{\xi\to 0}\left\{-\frac{2}{\pi\xi^3} + \sum_{r=0}^{\infty} e^{-r\pi\xi}\left[r^2\pi^2 + \frac{1}{F} - \frac{r\pi\xi}{F} - \frac{\xi\beta/P}{F^2}\right]\right.$$

$$+ \sum_{r=0}^{\infty}\frac{e^{-r\pi\xi}}{1 + F\sin^2\lambda_r}\left(2r\pi\lambda_r e^{-\lambda_r\xi} - F\pi^2 r^2\sin^2\lambda_r - \frac{1}{F} - \sin^2\lambda_r + \lambda_r^2 e^{-\lambda_r\xi}\right)$$

$$\left. + \sum_{r=0}^{\infty}\frac{e^{-r\pi\xi}}{1 + F\sin^2\lambda_r}\left[r^2\pi^2(e^{-\lambda_r\xi} - 1) + \frac{\xi}{F}\left(r\pi + \frac{\beta/P}{F}\right)(1 + F\sin^2\lambda_r)\right]\right\}$$

The first summation is equal to

$$\pi^2 e^{-\pi\xi}\frac{(1 + e^{-\pi\xi})}{(1 - e^{-\pi\xi})^3} + \frac{1}{F(1 - e^{-\pi\xi})} - \frac{\pi\xi}{F}\frac{e^{-\pi\xi}}{(1 - e^{-\pi\xi})^2} - \frac{\xi\beta/P}{F^2(1 - e^{-\pi\xi})}$$

which, when expanded near $\xi = 0$ is equal to $\frac{2}{\pi\xi^3} + \frac{1}{2F} - \frac{\beta/P}{F^2\pi} + O(\xi)$. The third summation tends to zero as $\xi \to 0$ and equation (24) results.

C.3 Wake blockage

The components w^* and $\partial w^*/\partial x$ are both zero at the origin while, from equation (B-10)

$$\frac{\beta^2 h}{m} u_w^* = \lim_{\xi\to 0}\left\{-\frac{1}{\pi\xi} + \sum_{r=0}^{\infty}\frac{e^{-\gamma_r\xi}}{1 + F\sin^2\gamma_r}\right\}$$

$$= \lim_{\xi\to 0}\left\{-\frac{1}{\pi\xi} + \sum_{r=0}^{\infty} e^{-\pi r\xi} - \sum_{r=0}^{\infty} e^{-\pi r\xi}\frac{F\sin^2\lambda_r}{1 + F\sin^2\lambda_r} + \sum_{r=0}^{\infty}\frac{e^{-\pi r\xi}(e^{-\lambda_r\xi} - 1)}{1 + F\sin^2\lambda_r}\right\}$$

which reduces to equation (25). From (B-10)

$$\frac{\partial u^*}{\partial x} = \frac{2m}{\beta^3 h^2}\left\{\frac{\xi^2 - \eta^2}{\pi(\xi^2 + \eta^2)^2} - \sum_{r=0}^{\infty} \frac{\gamma_r \cos(\gamma_r \eta) e^{-\gamma_r \xi}}{1 + F \sin^2 \gamma_r}\right\}$$

for $\xi > 0$. Thus

$$\frac{\beta^3 h^2}{2m} \frac{\partial u_w^*}{\partial x} = \frac{\beta^2 h^2}{2d} u_s^*$$

and equation (26) results from section C.2 above.

Table 1

CONDITIONS FOR ZERO SOLID BLOCKAGE

β/P	F
0	1.1844
0.1	1.1799
0.2	1.1662
0.3	1.1432
0.4	1.1106
0.5	1.0681
0.6	1.0149
0.7	0.9501
0.8	0.8724
0.9	0.7795
1.0	0.6676
1.1	0.5287
1.2	0.3409
1.278 (= $\tan \pi/\sqrt{12}$)	0

Note: Blanks in the following tables indicate very slow convergence of the power series of section 5 for these values of F .

Table 2

SOLID BLOCKAGE (Ω_s)

F \ β/P	0	0.3	0.6	1	2	5
0	-0.5	-0.4484	-0.3225	-0.125	0.2452	0.6467
0.1	-0.4126	-0.3738	-0.2761	-0.1096		
0.2	-0.3432	-0.3129	-0.2335	-0.0911	0.2314	
0.3	-0.2858	-0.2611	-0.1948	-0.0713	0.2284	0.6326
0.4	-0.2369	-0.2162	-0.1596	-0.0512	0.2273	0.6285
0.5	-0.1945	-0.1767	-0.1277	-0.0315	0.2277	0.6245
0.75	-0.1082	-0.0954	-0.0592	0.0148	0.2333	0.6158
1.0	-0.0412	-0.0313	-0.0030	0.0561	0.2429	0.6085
1.5	0.0589	0.0654	0.0844	0.1256	0.2673	0.5976
2	0.1317	0.1365	0.1505	0.1812	0.2933	0.5907
3	0.2340	0.2370	0.2457	0.2653	0.3416	0.5849
4	0.3046	0.3066	0.3127	0.3266	0.3829	0.5854
5	0.3573	0.3589	0.3635	0.3740	0.4176	0.5893
10	0.5066	0.5072	0.5090	0.5132	0.5316	0.6226
20	0.6299	0.6301	0.6308	0.6324	0.6396	0.6809
50	0.7533	0.7533	0.7535	0.7539	0.7559	0.7683
150	0.8520	0.8520	0 8520	0.8521	0.8525	0.8551
∞	1.0	1.0	1.0	1.0	1.0	1.0

Table 3

WAKE BLOCKAGE (Ω_W)

β/P F	0	0.3	0.6	1	2	5
0	0	-0.1855	-0.3440	-0.5	-0.7048	-0.8743
0.1	0	-0.1709	-0.3214	-0.4765	-0.6908	
0.2	0	-0.1595	-0.3029	-0.4559	-0.6771	-0.8674
0.3	0	-0.1502	-0.2874	-0.4376	-0.6639	-0.8639
0.4	0	-0.1426	-0.2742	-0.4213	-0.6511	-0.8603
0.5	0	-0.1360	-0.2627	-0.4067	-0.6388	-0.8567
0.75	0	-0.1231	-0.2395	-0.3759	-0.6104	-0.8476
1.0	0	-0.1134	-0.2217	-0.3513	-0.5849	-0.8383
1.5	0	-0.0995	-0.1957	-0.3139	-0.5416	-0.8197
2	0	-0.0898	-0.1772	-0.2864	-0.5063	-0.8015
3	0	-0.0768	-0.1522	-0.2480	-0.4520	-0.7669
4	0	-0.0683	-0.1355	-0.2219	-0.4120	-0.7355
5	0	-0.0621	-0.1234	-0.2027	-0.3809	-0.7070
10	0	-0.0455	-0.0907	-0.1500	-0.2899	-0.5996
20	0	-0.0328	-0.0655	-0.1087	-0.2136	-0.4785
50	0	-0.0210	-0.0420	-0.0699	-0.1388	-0.3304
150	0	-0.0122	-0.0244	-0.0407	-0.0811	-0.1994
∞	0	0.0	0	0	0	0

Table 4

PRESSURE GRADIENT (π_s)

β/P F	0	0.3	0.6	1	2	5
0	0	0.2947	0.4989	0.6169	0.5834	0.3388
0.1	0					
0.2	0	0.1899	0.3429	0.4672	0.5145	
0.3	0	0.1607	0.2952	0.4137	0.4822	
0.4	0	0.1389	0.2584	0.3700	0.4523	
0.5	0	0.1221	0.2291	0.3337	0.4248	0.3148
0.75	0	0.0930	0.1771	0.2658	0.3659	0.3006
1.0	0	0.0744	0.1431	0.2190	0.3188	0.2864
1.5	0	0.0523	0.1017	0.1592	0.2497	0.2589
2	0	0.0396	0.0776	0.1232	0.2024	0.2341
3	0	0.0260	0.0512	0.0825	0.1433	0.1932
4	0	0.0188	0.0373	0.0605	0.1086	0.1623
5	0	0.0145	0.0288	0.0471	0.0861	0.1386
10	0	0.0062	0.0123	0.0203	0.0389	0.0753
20	0	0.0025	0.0050	0.0082	0.0161	0.0351
50	0	0.0007	0.0014	0.0023	0.0046	0.0110
150	0	0.0001	0.0003	0.0005	0.0010	0.0024
∞	0	0	0	0	0	0

Table 5

LIFT INTERFERENCE (δ_0)

F \ β/P	0	0.3	0.6	1	2	5
0	-0.25	-0.2036	-0.1640	-0.125	-0.0738	-0.0314
0.1	-0.2273	-0.1867	-0.1529	-0.1190	-0.0723	
0.2	-0.2083	-0.1727	-0.1433	-0.1135	-0.0707	-0.0312
0.3	-0.1923	-0.1609	-0.1350	-0.1084	-0.0691	-0.0310
0.4	-0.1786	-0.1506	-0.1276	-0.1038	-0.0676	-0.0308
0.5	-0.1667	-0.1417	-0.1211	-0.0995	-0.0660	-0.0306
0.75	-0.1429	-0.1235	-0.1075	-0.0903	-0.0623	-0.0301
1	-0.125	-0.1096	-0.0967	-0.0827	-0.0589	-0.0296
1.5	-0.1000	-0.0896	-0.0807	-0.0708	-0.0529	-0.0284
2	-0.0833	-0.0758	-0.0693	-0.0619	-0.0480	-0.0272
3	-0.0625	-0.0581	-0.0541	-0.0495	-0.0404	-0.0249
4	-0.0500	-0.0471	-0.0445	-0.0413	-0.0348	-0.0229
5	-0.0417	-0.0396	-0.0377	-0.0354	-0.0306	-0.0211
10	-0.0227	-0.0221	-0.0215	-0.0207	-0.0190	-0.0150
20	-0.0119	-0.0117	-0.0115	-0.0113	-0.0108	-0.0094
50	-0.0049	-0.0049	-0.0048	-0.0048	-0.0047	-0.0044
150	-0.0017	-0.0017	-0.0016	-0.0016	-0.0016	-0.0016
∞	0	0	0	0	0	0

Table 6

STREAMWISE CURVATURE CORRECTION (δ_1)

F \ β/P	0	0.3	0.6	1	2	5
0	-0.2618	-0.1296	-0.0381	0.0327	0.0967	0.1247
0.1	-0.2161	-0.1080	-0.0319	0.0306		
0.2	-0.1806	-0.0900	-0.0253	0.0302	0.0904	
0.3	-0.1519	-0.0748	-0.0188	0.0309	0.0882	
0.4	-0.1283	-0.0616	-0.0125	0.0321	0.0865	0.1212
0.5	-0.1083	-0.0501	-0.0067	0.0337	0.0851	0.1204
0.75	-0.0700	-0.0268	0.0063	0.0385	0.0832	0.1187
1	-0.0423	-0.0090	0.0172	0.0436	0.0824	0.1173
1.5	-0.0051	0.0166	0.0343	0.0530	0.0830	0.1151
2	0.0189	0.0342	0.0470	0.0609	0.0848	0.1137
3	0.0481	0.0569	0.0645	0.0731	0.0892	0.1122
4	0.0652	0.0709	0.0759	0.0819	0.0934	0.1119
5	0.0765	0.0804	0.0840	0.0883	0.0970	0.1122
10	0.1016	0.1028	0.1039	0.1053	0.1085	0.1153
20	0.1156	0.1160	0.1163	0.1167	0.1177	0.1201
50	0.1246	0.1247	0.1248	0.1248	0.1250	0.1255
150	0.1288	0.1288	0.1288	0.1288	0.1288	0.1289
∞	0.1309	0.1309	0.1309	0.1309	0.1309	0.1309

SYMBOLS

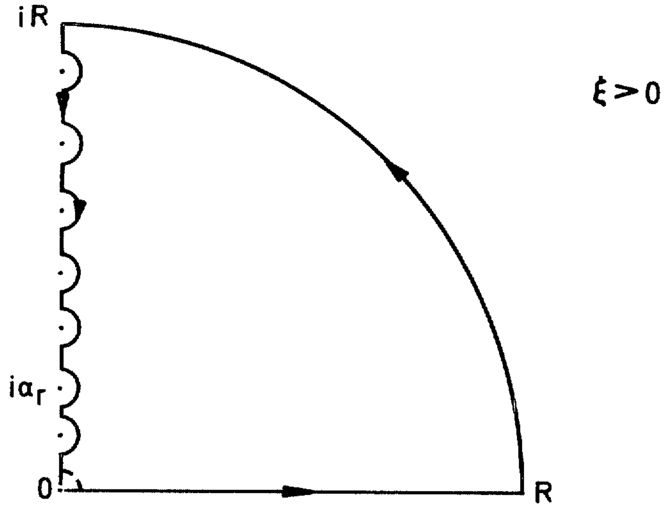
a	width of slots
A	'equivalent' cross-sectional area of model
B	1 for a solid wall, zero otherwise
c	aerofoil chord
C_D	drag coefficient
C_L	lift coefficient
C_m	quarter chord moment coefficient
d	strength of doublet ($= AU/\beta$)
F	slot parameter $= (2s/\pi h)\ \ln \operatorname{cosec}\ (\pi a/2s)$
h	tunnel height
I_A, I_C, I_F, I_G	expressions in integrals, defined in sections 2.1 and 2.2
J	1 for an ideal slotted wall or free jet, zero otherwise
m	strength of source ($= \frac{1}{2}UcC_D$)
M	Mach number far upstream
p	pressure
P	porosity parameter
s	distance between slot centres
U	velocity far upstream
u^*, w^*	interference velocity components along and perpendicular to free-stream direction
u^*_L, w^*_L	interference velocity components at model position due to lift interference
u^*_s, w^*_s	interference velocity components at model position due to solid blockage
u^*_w, w^*_w	interference velocity components at model position due to wake blockage
x, z	coordinates along and perpendicular to free-stream direction
α	incidence
α_r	roots of the equation $\tan \alpha_r = \beta/P - F\alpha_r$
β	$\sqrt{1 - M^2}$
γ_r	roots of the equation $\cot \gamma_r = F\gamma_r - \beta/P$
Γ	strength of vortex ($= \frac{1}{2}UcC_L$)
δ_0	$(h/UcC_L)w^*_L$
δ_1	$(\beta h^2/UcC_L)\partial w^*_L/\partial x$
ΔC_L etc.	interference correction to be made to C_L etc.
ε_s	solid blockage factor ($= u^*_s/U$)

SYMBOLS (concluded)

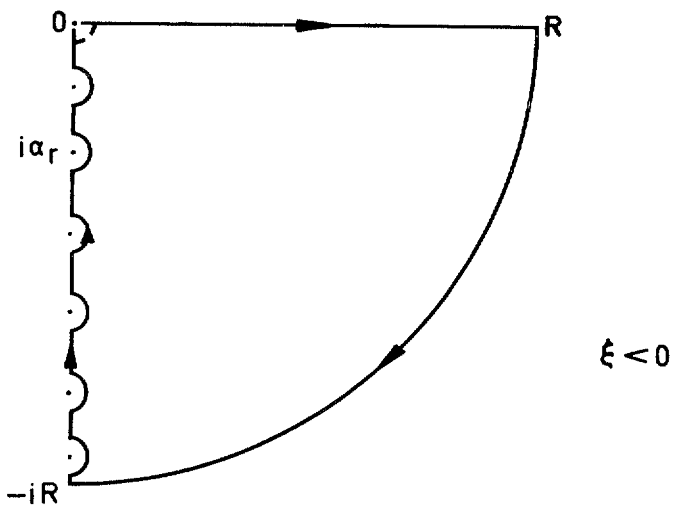
ε_w	wake blockage factor (= u_w^*/U)
ζ	$\xi + i\eta$
η	$2z/h$
θ	$\tan^{-1} (\beta/P)$
λ_r	$\gamma_r - r\pi$
μ_r	$\alpha_r - (r - \frac{1}{2})\pi$
ξ	$2x/\beta h$
π_s	pressure-gradient parameter = $-(\beta^4 h^3/\rho U^2 A)\partial p_s/\partial x$
ρ	density
ϕ	perturbation velocity potential
ϕ_1	perturbation velocity potential due to model in free air
ϕ^*	interference potential
Ω_s	ratio of ε_s to solid blockage factor for closed tunnel
Ω_w	ratio of ε_w to wake blockage factor for closed tunnel

REFERENCES

No.	Author	Title, etc.
1	H.C. Garner E.W.E. Rogers W.E.A. Acum E.C. Maskell	Subsonic wind tunnel wall corrections. AGARDograph 109 (1966)
2	D. Catherall	The computation of transonic flows past aerofoils in solid, porous or slotted wind tunnels. AGARD conference proceedings 174 (1975)
3	D. Catherall	The computation of two-dimensional flows in transonic wind tunnels. RAE Technical Report (to be issued)
4	B.S. Baldwin J.B. Turner E.D. Knechtel	Wall interference in wind tunnels with slotted and porous boundaries at subsonic speeds. NACA TN 3176 (1954)
5	R.H. Wright	The effectiveness of the transonic wind-tunnel as a device for minimising tunnel-boundary interference for model tests at transonic speeds. AGARD Report 294 (1959)
6	M. Pindzola C.F. Lo	Boundary interference at subsonic speeds in wind tunnels with ventilated walls. AEDC-TR-69-47 (1969)
7	E.M. Murman	Computation of wall effects in ventilated transonic wind tunnels. AIAA paper 72-1007 (1972)
8	S.B. Berndt H. Sörensén	Flow properties of slotted walls for transonic test sections. AGARD Conference Proceedings 174 (1975)



a



b

Fig 1a&b Contours used for evaluation of integrals

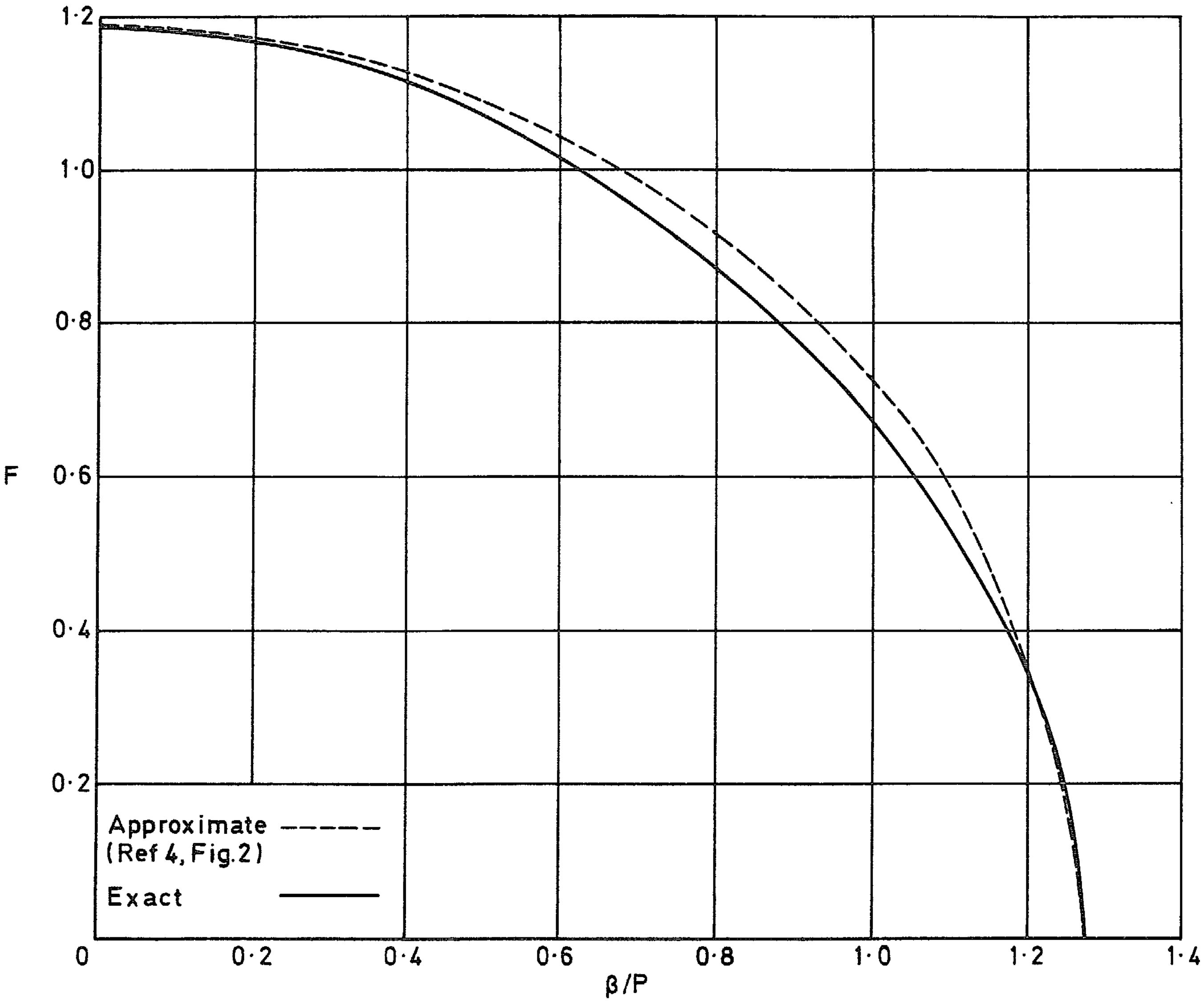


Fig 2 Comparison of exact and approximate conditions for zero solid blockage

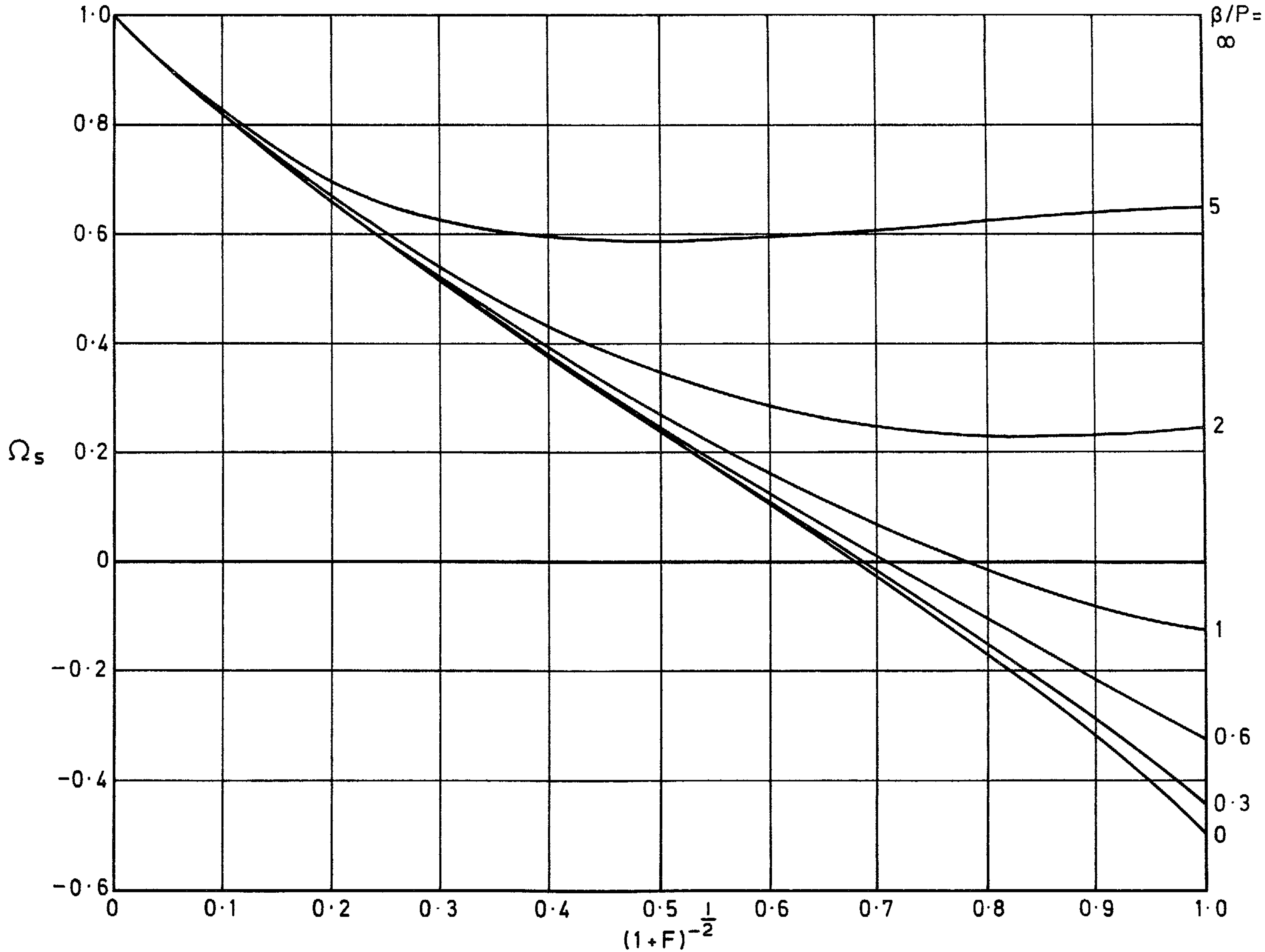


Fig 3 Solid blockage at model position

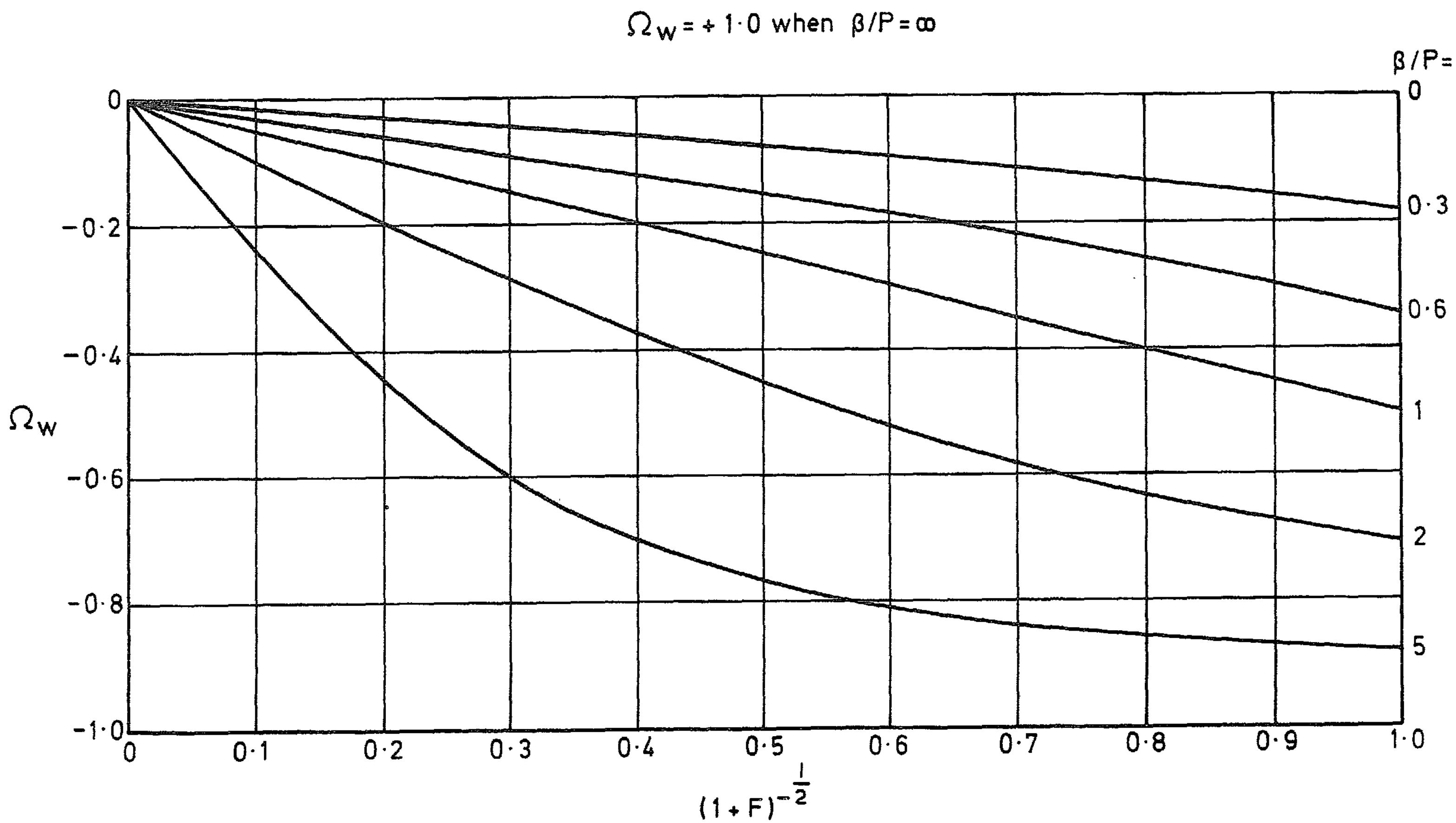


Fig 4 Wake blockage at model position

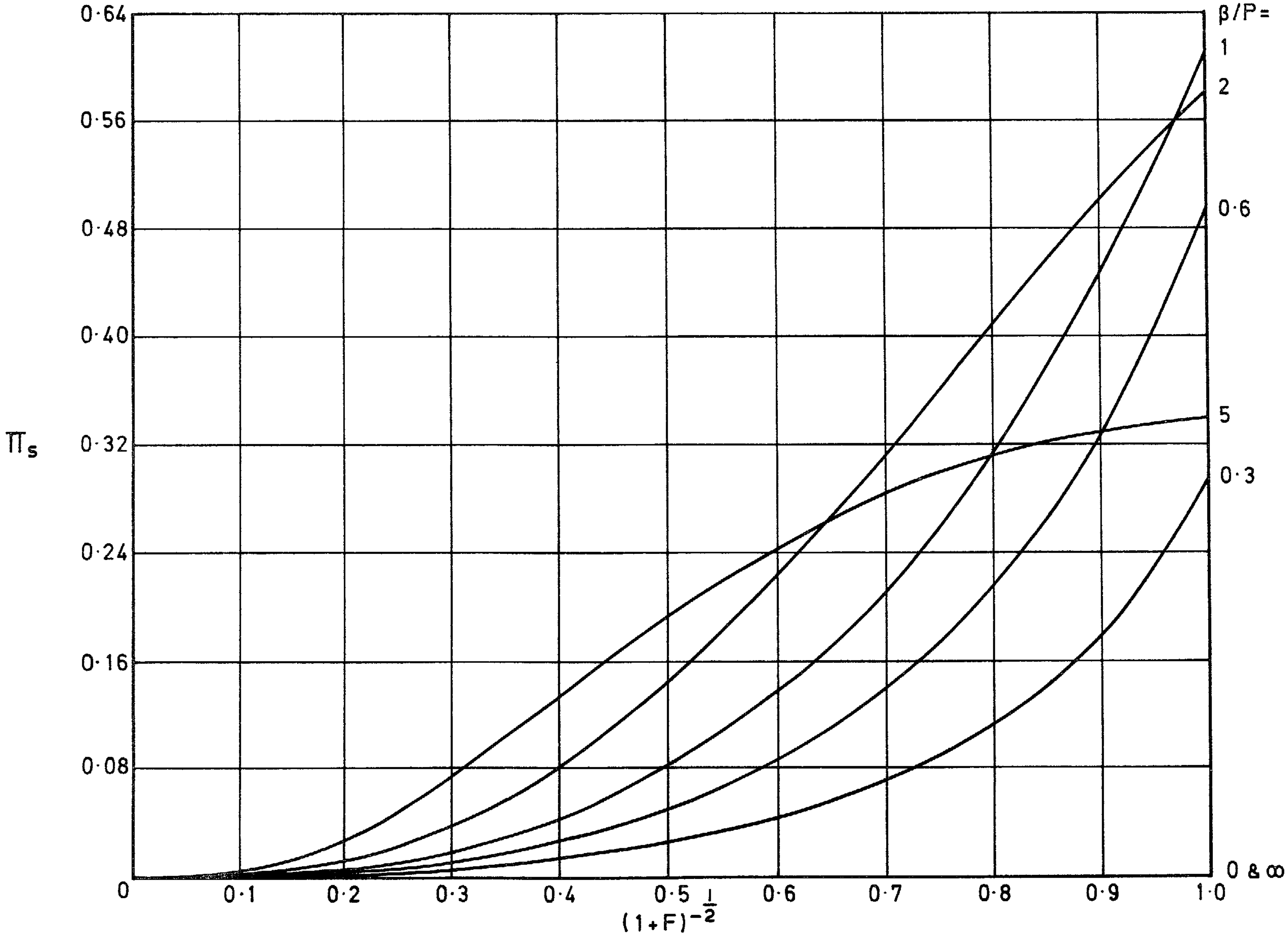


Fig 5 Longitudinal pressure gradient at model position associated with solid blockage

Fig 5

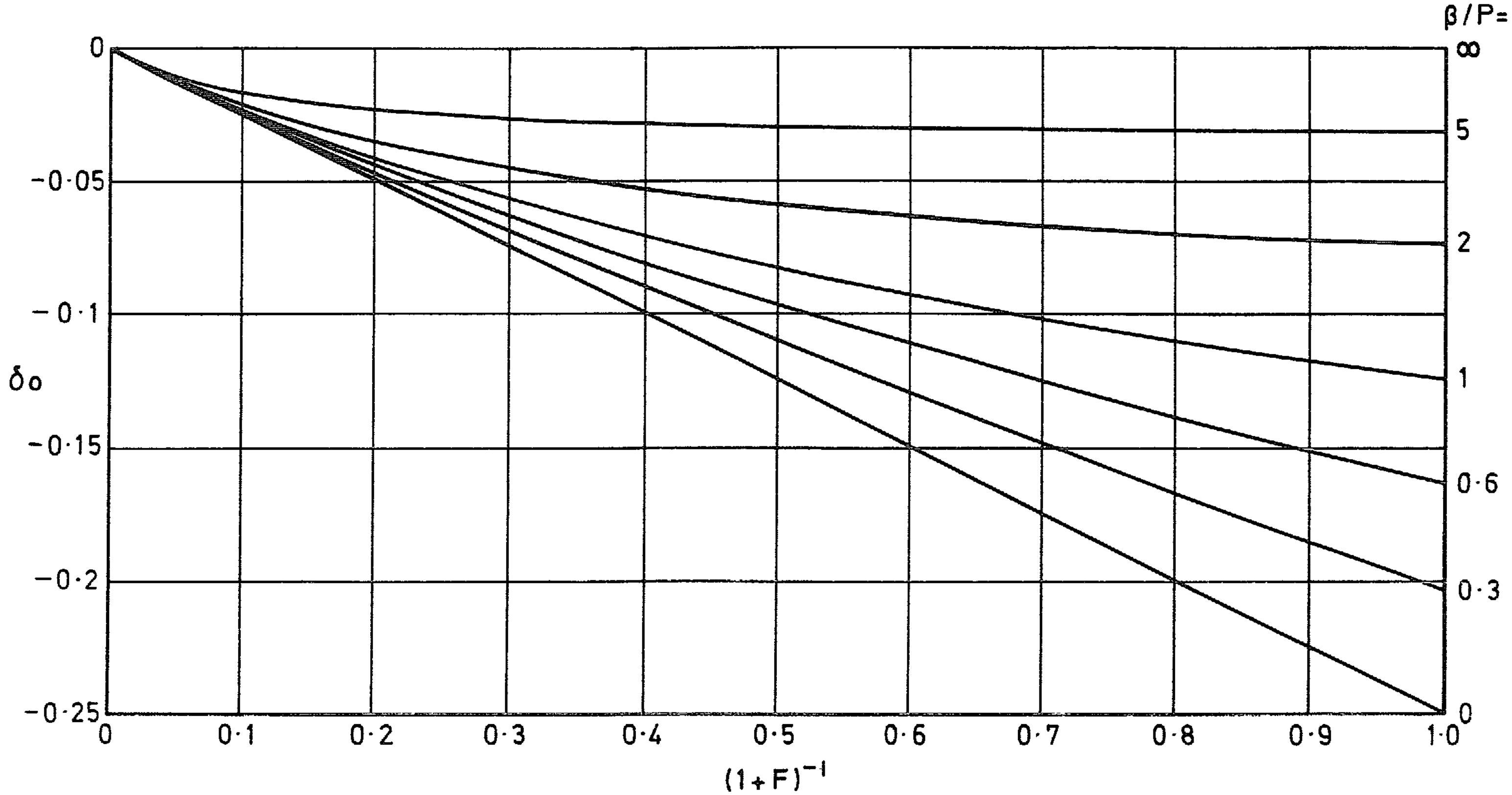


Fig 6 Lift interference at model position

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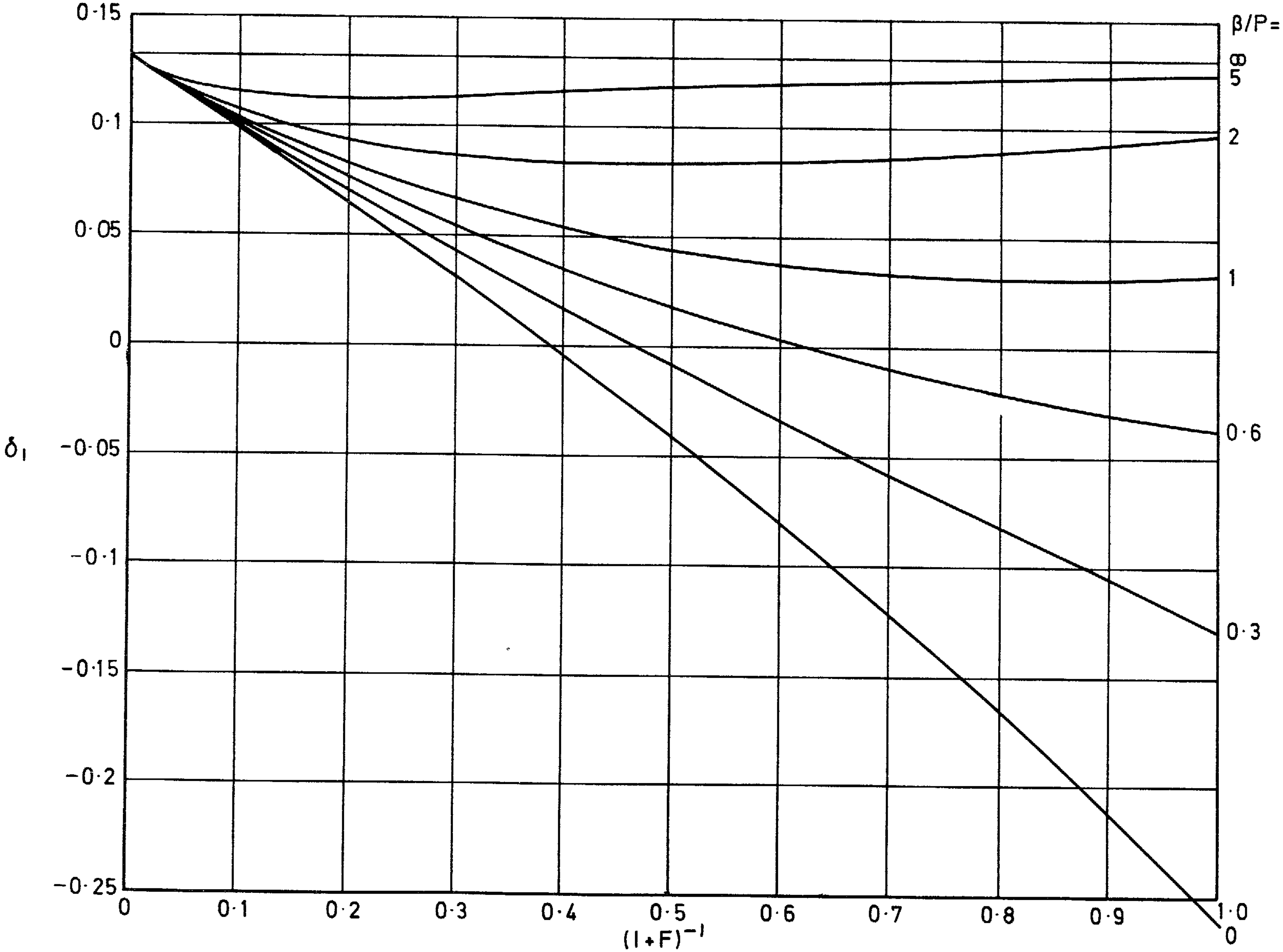

Fig 7 Streamwise curvature correction at model position

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