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Some Unseparated Base Flows with Heat Addition

By E. G. Broadbent

Aerodynamics Dept., R.A.E., Farnborough

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Summary

Analytical solutions are derived for a class of axisymmetric base flows with heat addition. The assumed upstream conditions are non-uniform in velocity and temperature, which vary with r in spherical polar co-ordinates (r, θ, ϕ) in a prescribed manner such that pressure and Mach number are independent of r. The turning flow expands about the base axisymmetrically and without change in r-dependence, so that the flow is self-similar with respect to conical surfaces of constant θ. The magnitude and distribution of heat addition is then calculated and results are given for a few examples.

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* Replaces R.A.E. Technical Report 73094—A.R.C. 34 954.

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1. Introduction

Isentropic expansion flows of a supersonic stream are accompanied by a rapid fall in pressure, which can, however, be alleviated, or even turned into a pressure rise, by the addition of heat. Some two-dimensional flows of this type have been discussed by Steffen,[1] Foster and Clarke,[2] Broadbent and Townend.[3] For axisymmetric flows Marsh and Horlock[4] obtained some solutions for duct flow with a specified mode of heat addition, and Bartlmä[5] considered base flow through an expansion corner followed by heat addition.

The present report was preceded by a theoretical consideration of axisymmetric flow in spherical polar coordinates (r, θ, ϕ) with variation only in the θ direction,[6] such that the flow was self similar over conical surfaces of constant θ. Solutions were derived both for internal duct flow and external base flow, but the latter were incomplete because of a singularity on the axis which meant terminating the solution for some value of $\theta > 0$. It was noticed that the singularity disappeared for flows with a certain type of r-dependence, and a class of such flows is analysed in the present report. The r-dependence is such that although the velocity and density must vary with r, it is possible for the pressure and Mach number to be independent of r, and these conditions are in fact imposed. It follows that the upstream flow is cylindrically stratified in temperature and velocity, but uniform in pressure and Mach number.

Again the flow fields are self similar in the variable θ but they can now be completed as far as the axis $\theta = 0$. The streamlines are still converging at this point, but provided the flow remains supersonic the self-similar region could be terminated by a shock, in principle, without upstream influence. The shock would then define a surface marking the downstream boundary of the self-similar flow.

Some examples are given for a few body shapes and upstream Mach numbers, but it is concluded that heated base flows of this type are probably less practical than those involving a flow separation.

2. Analysis

The equations of motion in spherical polar coordinates (r, θ, ϕ) may be written as follows for axisymmetric flow i.e. no variation in the ϕ direction—*see* Fig. 1.

Continuity:

$$r\frac{\partial \rho u}{\partial r} + 2\rho u \sin\theta + \rho v \cos\theta + \frac{\partial \rho v}{\partial \theta}\sin\theta = 0, \tag{1}$$

r component of momentum:

$$\frac{1}{\rho}\frac{\partial p}{\partial r} + u\frac{\partial u}{\partial r} + \frac{v}{r}\left(\frac{\partial u}{\partial \theta} - v\right) = 0, \tag{2}$$

θ component of momentum:

$$ru\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial \theta} + uv + \frac{1}{\rho}\frac{\partial p}{\partial \theta} = 0, \tag{3}$$

energy:

$$\left(u\frac{\partial}{\partial r} + \frac{v}{r}\frac{\partial}{\partial \theta}\right)\{h + \tfrac{1}{2}(u^2 + v^2)\} = q, \tag{4}$$

where

- ρ is density,
- u, v are velocity components in the r and θ directions respectively (Fig. 1),
- p is pressure,
- h is specific enthalpy

and

- q is rate of heat addition per unit mass.

In a previous report,[6] the above equations were restricted by the condition that the physical parameters should be invariant with r, so that those terms involving derivatives with respect to r dropped out of the equations. Some solutions were found in which the streamlines either converged (appropriate to base flow) or diverged (appropriate to internal duct flow, e.g. for a ramjet), but the base-flow solutions were incomplete on account of a singularity on the axis $\theta \to 0$. This meant that although they might apply to the outer part of a base the solution could not be carried through to the axis.

What had been hoped for were solutions that could be completed down to $\theta = 0$, and although the flow would be converging there, it could in principle be terminated by a shock provided the Mach number at the axis was supersonic. The present analysis continues the search for flows of this type, with some degree of success, albeit for non-uniform conditions upstream of the base. If the r-derivatives are assumed to vanish in equations (1) to (4), the singularity arises from the explicit appearance of θ in equation (1), together with the fact that u and v are linked by the relation $v = du/d\theta$ from equation (2). By introducing the variation with r, however, it is possible to find solutions belonging to a certain class in which the singularity is no longer present.

We consider a type of flow in which the physical parameters u, v, ρ and p vary with r in a specified way:

$$\left.\begin{aligned} u(r,\theta) &= \hat{u}(\theta)\left(\frac{r}{r_1}\right)^n, \\ v(r,\theta) &= \hat{v}(\theta)\left(\frac{r}{r_1}\right)^n, \\ \rho(r,\theta) &= \hat{\rho}(\theta)\left(\frac{r}{r_1}\right)^m \\ \text{and} \quad p(r,\theta) &= p(\theta)\left(\frac{r}{r_1}\right)^{m+2n} \end{aligned}\right\} \tag{5}$$

where m and n are constants, r_1 is some reference length, and the relations (5) are chosen to satisfy equations (1) to (4). The solutions given later will be such that n is negative and m positive, so that we are considering a flow in which the velocity is decreasing and the density increasing as one moves away from the axis, and those conditions must also apply upstream. However, the Mach number, M, is constant along a ray, since

$$M^2(r) = \frac{u^2(r) + v^2(r)}{\gamma p(r)/\rho(r)} = \frac{\hat{u}^2 + \hat{v}^2}{\gamma \hat{p}/\hat{\rho}}. \tag{6}$$

In fact the solutions given will be restricted to the condition

$$\left.\begin{aligned} & p(r) = \text{constant} \\ \text{i.e.} \quad & m + 2n = 0. \end{aligned}\right\} \tag{7}$$

It is clearly unlikely that upstream flow, satisfying equations (5), (6) and (7) exactly, would be found in aeronautical engineering applications, but the conditions may be satisfied approximately in some circumstances, e.g. where hot exhaust gases flow over a base or centre-body. Moreover, in some examples the radial gradients are quite small. A type of flow in which the postulated conditions do arise has been discussed by Horlock[7] following Vazsonyi[8] who attributed the suggestion to Emmons.

If the relations (5) are substituted in equations (1) to (4), a new set of equations in $\hat{u}$, $\hat{v}$, $\hat{\rho}$ and $\hat{p}$ is obtained, and in which r only appears in the heat addition term. For convenience the accents are dropped, and the equations become,

$$(m+n)\rho u + 2\rho u \sin\theta + \rho v \cos\theta + \frac{d\rho v}{d\theta}\sin\theta = 0, \tag{8}$$

$$(m+2n)\frac{p}{\rho} + nu^2 + v\left(\frac{du}{d\theta} - v\right) = 0, \tag{9}$$

$$nuv + v\frac{dv}{d\theta} + uv + \frac{1}{\rho}\frac{dp}{d\theta} = 0 \tag{10}$$

and

$$\left(2nu + v\frac{d}{d\theta}\right)\left(\frac{\gamma}{\gamma-1}\frac{p}{\rho} + \tfrac{1}{2}(u^2+v^2)\right) = \frac{qr}{(r/r_1)^{3n}}, \tag{11}$$

where the perfect gas relation has been assumed for the enthalpy, and γ is the ratio of specific heats.
In order that there shall be no singularity as $\theta \to 0$, it follows from equation (8) that

$$(m + n)u(0) + v(0) = 0$$

or

$$\frac{u(0)}{v(0)} = -\frac{1}{m + n}. \tag{12}$$

A fairly general possibility would be

$$\left.\begin{aligned} \frac{u}{v} &= -c_1 + c_2\theta^s, \\ c_1 &= \frac{1}{m+n}, \qquad c_2 = \text{const}, \end{aligned}\right\} \tag{13}$$

but the solutions given here will be restricted to $c_2 = 0$. The upstream flow is assumed to be parallel for $\theta \geqslant \theta_1$, so that if the subscript 1 attached to a variable refers to the value at $\theta = \theta_1$, there follows

$$-\frac{u}{v} = -\frac{u_1}{v_1} = \cot\theta_1 = c_1 = \frac{1}{m + n} = -\frac{1}{n}, \tag{14}$$

where the final relation in equation (14) follows under the condition (7). These results can be used in (8) to give a differential equation for ρv,

$$\frac{d\rho v}{d\theta} = 2\rho v c_1 + \rho v\frac{(1 - \cos\theta)}{\sin\theta}, \tag{15}$$

which has the solution

$$\rho v = \rho_0 v_0\, e^{2c_1\theta} \sec^2\tfrac{1}{2}\theta, \tag{16}$$

where the subscript 0 refers to values at $\theta = 0$. A solution for v follows from equation (9) with $m + 2n = 0$ and $u = -c_1 v$, which gives

$$v = v_0 \exp\left\{-\frac{(1 - nc_1^2)\theta}{c_1}\right\} \tag{17}$$

or, by (14)

$$v = v_0\, e^{(n-1)\theta}. \tag{18}$$

The form (17) applies more generally than (18) and does not depend on the assumption $m + 2n = 0$, as may be seen by eliminating p from (9) and (10) and substituting for ρ, u and v by means of (16) and (17) with $u = -c_1 v$.
From (16) and (18)

$$\rho = \frac{2\rho_0 \exp\{\theta(1 - n - 2/n)\}}{1 + \cos\theta}. \tag{19}$$

The corresponding expression for the pressure gradient from equation (10) becomes

$$\frac{dp}{d\theta} = -\frac{2(1 + n^2)}{n}\frac{\rho_0 v_0^2 \exp\{\theta(n + 1)(n - 2)/n\}}{1 + \cos\theta}, \tag{20}$$

from which $p(\theta)$ is obtained by numerical integration. Since it is convenient to integrate in the streamwise direction, a change of variables was made by the substitution

$$\theta = \theta_1(1 - \alpha) \tag{21}$$

so that α increases from 0 to 1 as θ falls from θ_1 to zero. The pressure is then given by

$$\frac{p(\alpha)}{p_1} = 1 + \gamma M_\infty^2\left(\frac{1 + n^2}{n}\right)\theta_1(1 + \cos\theta_1)\sin^2\theta_1 \int_0^\alpha \frac{\exp\{-\theta_1\alpha(n + 1)(n - 2)/n\}}{1 + \cos\{\theta_1(1 - \alpha)\}}\,d\alpha. \tag{22}$$

The flow parameters are now completely specified by equations (14), (18), (19) and (22). It remains to derive an expression from equation (11) for the mode of heat addition that satisfies these relations, which is found by substitution to be

$$\begin{aligned} r\left(\frac{r_1}{r}\right)^{3n}\frac{q(r,\alpha)}{u_\infty^3} = 2n\frac{u}{u_\infty}\left(\frac{\gamma}{\gamma - 1}\frac{p}{p_1}\frac{\rho_1}{\rho}\frac{1}{\gamma M_\infty^2} + \tfrac{1}{2}\exp\{2(1 - n)\alpha\theta_1\}\right) - \\ -\frac{v}{u_\infty}\left[\frac{\gamma}{\gamma - 1}\frac{1}{\gamma M_\infty^2}\frac{p}{p_1}\left\{\sin(\theta_1(1 - \alpha)) + \left(1 - n - \frac{2}{n}\right)\left(1 + \cos(\theta_1(1 - \alpha))\right)\right\} \times \right. \\ \times \frac{\exp\{\alpha\theta_1[1 - n - (2/n)]\}}{1 + \cos\theta_1} + \frac{\gamma}{\gamma - 1}\frac{\rho_1}{\rho}\left(\frac{1 + n^2}{n}\right)\frac{(1 + \cos\theta_1)\sin^2\theta_1}{1 + \cos(\theta_1(1 - \alpha))} \times \\ \left. \times \exp\{-\theta_1\alpha(n + 1)(n - 2)/n\} + (1 - n)\exp\{2(1 - n)\alpha\theta_1\}\right] = B(\alpha),\ \text{say}, \end{aligned} \tag{23}$$

where u_∞ is the upstream flow speed given by $u_\infty^2 = u_1^2 + v_1^2$ and M_∞ is the corresponding Mach number $M_\infty = u_\infty\sqrt{\rho_1/\gamma p_1}$, and it should be remembered that on returning to the original form (5), u_∞ depends on r whereas M_∞ does not. A non-dimensional parameter, Q, defining the heat addition per unit volume is introduced in the form

$$Q = \left(\frac{qr}{u_\infty^3}\right) \times \left(\frac{\rho}{\rho_1}\right) \times \left(\frac{r_1}{r}\right)$$

so that

$$Q(r,\alpha) = \frac{\rho}{\rho_1}\left(\frac{r}{r_1}\right)^{3n-1} B(\alpha) \tag{24}$$

where ρ/ρ_1 is a function of α defined by (19) and (21).

The shape of the streamlines is given by

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{u}{v} = \frac{1}{n} \tag{25}$$

whence

$$r = r_0\,e^{\theta/n}. \tag{26}$$

If we suppose the flow to be that past a uniform cylinder for $\theta > \theta_1$, then the shape of the base for $\theta \leqslant \theta_1$ is given by

$$\frac{r}{r_a} = \frac{e^{-(\theta_1 - \theta)/n}}{\sin\theta_1} \tag{27}$$

where $r_a = r_1\sin\theta_1$ is the radius of the cylinder.

The shape of the base may also be given in rectangular coordinates (x, y) where y is measured radially from the axis, and x is measured axially from the downstream end of the cylindrical part, i.e. $x = 0$ corresponds with $\theta = \theta_1$. If x and y are made non-dimensional by r_a, the expressions may be written, by (14),

$$\left.\begin{aligned} x &= \frac{1}{n} + e^{-\alpha\theta_1/n}\left(\sin\alpha\theta_1 - \frac{1}{n}\cos\alpha\theta_1\right) \\ \text{and} \qquad y &= e^{-\alpha\theta_1/n}\left(\cos\alpha\theta_1 + \frac{1}{n}\sin\alpha\theta_1\right). \end{aligned}\right\} \qquad (28)$$

It follows that for small $-n$, i.e. small θ_1, that the length to diameter of the base increases like $-1/n$ or $1/\theta_1$, since $-\alpha\theta_1/n \to 1$ with $\alpha = 1$ and $\theta_1 \to 0$.

It may be remarked that by use of the more general form (13) for u/v it is possible to find solutions that avoid the necessity of terminating the flow in a shock by deriving body shapes that end in a cusp. This happens, for example, with $s = -1$ when the equations can be integrated, but the solutions are unrealistic in that the spike is infinitely long. Solutions with other values of s are unsatisfactory in other respects.

3. Results

Results are presented in Figs. 2 to 4 for different values of θ_1 (i.e. different values of n) and for a range of upstream Mach number. The shape of the body is illustrated at the top of each figure, and it can be seen that the bodies become shorter and blunter with increasing θ_1 as expected. Two other features restrict the practical range of parameters. For a given θ_1 the heat addition is positive for flows with a low upstream Mach number, but becomes negative for a sufficiently high upstream Mach number. The Mach number range for positive heat addition becomes smaller as θ_1 is increased, which is why the curve for $M_1 = 4$ is omitted from Figs. 3 and 4 since the corresponding heat addition would be negative. The other restrictive condition is that for the flow to be free of upstream influence the final Mach number at $\theta = 0$ should be supersonic, such that the subsequent turning of the flow can be through a shock. The results show that the Mach number falls in the downstream direction, so that to meet this condition the upstream Mach number must be substantially greater than one.

Figure 2 shows results for $\theta_1 = 15°$, $n = -0{\cdot}268$, and for four upstream values of Mach number, $M_1 = 0{\cdot}8$, 1·4, 2·0 and 4·0. For simplicity the results are presented as functions of x along the streamline illustrated at the top of the figure, which may be regarded either as the streamline along the body surface, or as one out in the free stream. It should, of course, be remembered that the self-similar behaviour is with respect to θ whose origin is on the x axis not at $x = 0$ but at $x = -\cot\theta_1$.

As the flow proceeds downstream the pressure falls like it would in an unheated expansion but less rapidly, and meanwhile the temperature increases relatively quickly, which is why the Mach number falls despite an increase in velocity. The actual velocity increase is given by the same form as equation (18) since the two components vary in proportion to each other; with n negative this shows an exponential fall with increasing θ, i.e. in the upstream direction.

With regard to the fall in Mach number, even the flow starting from $M_1 = 2$ becomes just subsonic ($M = 0{\cdot}98$) at the axis, so for wholly supersonic flow the upstream Mach number must exceed 2. However, it is of interest to compare the results for all given values of M_1 since they establish the trend. The intensity of heat addition falls rapidly with increasing M_1, although in absolute terms this is largely accounted for by the non-dimensional form of Q which has u_∞^3 in the denominator. The temperature rise is large for the smaller values of M_1, and in some sense this must mean that the corresponding efficiency is low since a lot of energy is being wasted in heat with comparatively little return in terms of pressure. This point will be discussed further in the next section.

The results shown in Figs. 3 and 4 for $\theta_1 = 20$ and 25 degrees respectively are qualitatively similar to those of Fig. 2. The trends are, that as the body becomes shorter and blunter the fall in Mach number is reduced, but the pressure drop is increased. The distribution of heat addition remains much the same, but there is some reduction in the magnitude, although the temperature increase remains about the same, partly because less heat per unit volume is needed to produce a given temperature rise as the pressure is reduced. In addition to these trends, there is the effect mentioned earlier that the upper limit of M_1 for positive heat addition is reduced as the body becomes blunter.

The physical reason why the heat addition goes negative at sufficiently high M_1 follows from the forms of the density and pressure gradients given by (19) and (20). It can be seen that whereas the density gradient $(1/\rho_0)(d\rho/d\theta)$ is independent of Mach number, the corresponding pressure gradient $(1/p_0)(dp/d\theta)$ is proportional to M_0^2, i.e. for given θ_1 to M_1^2. Moreover $dp/d\theta$ is positive so that the pressure falls in the flow direction at a rate that increases rapidly with M_1, and eventually exceeds the rate appropriate to isentropic flow, thus leading to negative heat addition.

4. Discussion and Conclusions

Some examples have been given of exact solutions for shock-free flow with heat addition past axisymmetric base shapes. Such solutions are always of interest for comparative purposes, but those of the present report cannot be considered very suitable for practical applications in aeronautics where it is desired to reduce the base drag as cheaply as possible.

One difficulty is that of obtaining satisfactory combustion. A stable flame is usually produced by forcing a flow separation and using the recirculating flow to carry some of the hot burning gases back to the separation point where they can maintain ignition. No such mechanism exists in the continuous flows of the present report.

In addition, the efficiency in terms of reduction in drag for unit heat addition cannot be expected to be high. This tends to be true of all non-ducted flows since there is no wall present to sustain a high pressure on one side of the flow (*see* Ref. 9 for example) and the flows of the present report are effectively non-ducted in that the pressure is constant with respect to r and falling with respect to α. In thermodynamic terms the cycle has a very poor compression ratio. The same is generally true of base-burning involving separated flow, but there it may be possible to obtain some compensation in other respects. In separated base flow without heat addition there is an instantaneous pressure drop around the base corner for a supersonic free stream as the flow turns through a centred expansion, and the reduced pressure is roughly constant across the recirculating flow in the base region. Further downstream, there is a pressure rise as the converging flow is turned parallel to the axis to close the base region, so that the recirculating flow is subject to strong pressure gradients. Base-burning works by filling out the stream-tubes through heat addition, in which the recirculating flow is also hot, and the sudden expansion is reduced or prevented. It seems possible that if the heat can be added to relatively low-velocity air but at free-stream pressure, then the quantity of heat required (which depends on the mass flow of heated air) may be relatively small.[10] This may account for the experimental result that small quantities of heat added have a relatively large effect, but that further improvements in base pressure become increasingly expensive in fuel consumption. A similar result for the unseparated flows of the present report may also hold if the boundary layer is sufficiently thick, but this is not easy to demonstrate analytically since there is no combination of m and n in equation (5) that closely resembles a boundary-layer flow. Base-burning with separated flow may also be efficient at sufficiently high Mach numbers if it takes place downstream of a strong shock, so that there is a large pressure ratio relative to free stream.[11,12] Alternatively a high pressure ratio can be achieved through a shock without separation,[2,5] but again the efficiency may not be good since the flow field is not contained.

In view of these arguments, it seems that heat addition is best used either in a duct or a separated flow region, where the flow is contained in some way. However, the results of the present report may possibly have some practical application to afterbodies which are intended for use without heat addition, except for occasional bursts of heat to reduce the drag for a short time.

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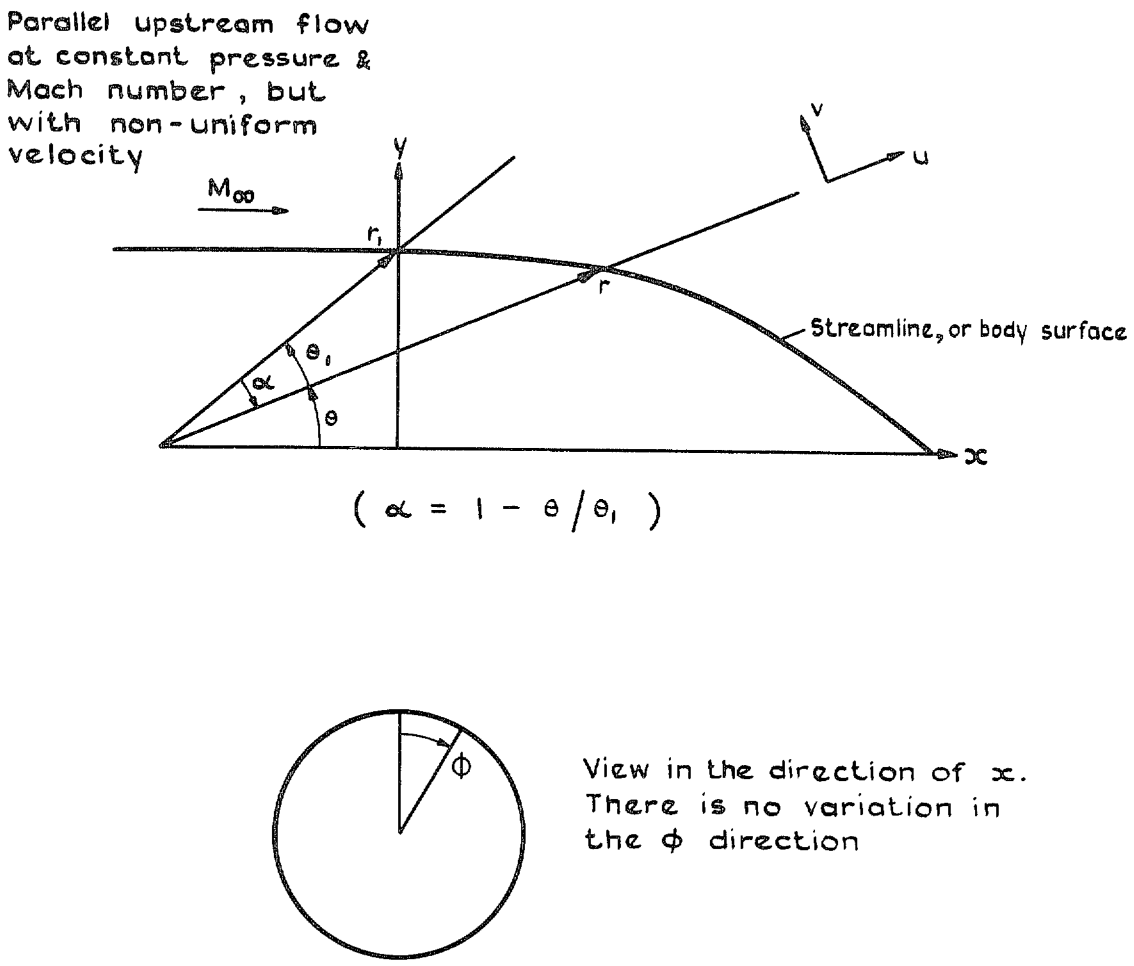

FIG. 1. Co-ordinates and streamline.

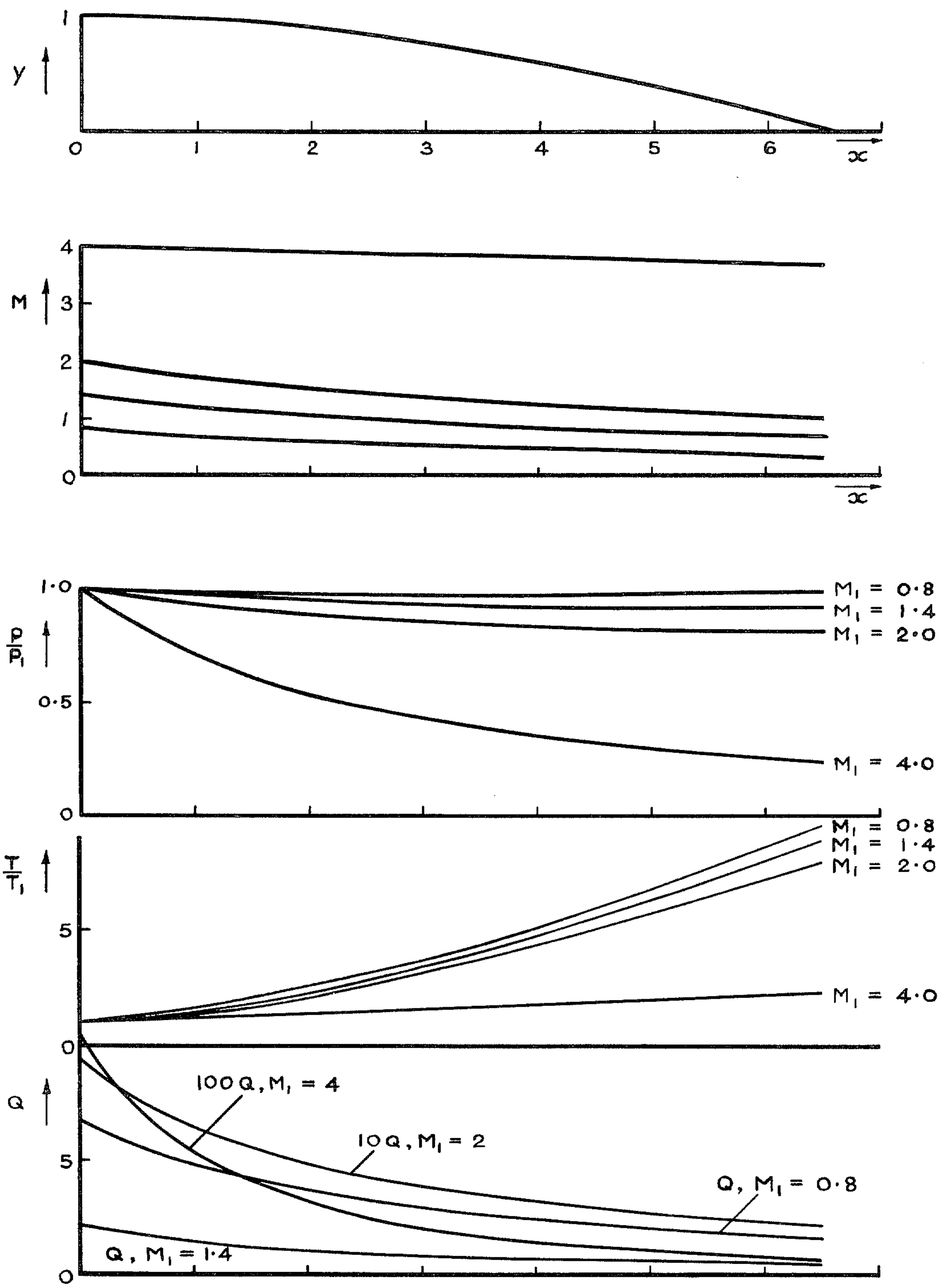

FIG. 2. Results for $\theta_1 = 15°$; $n = -0{\cdot}268$.

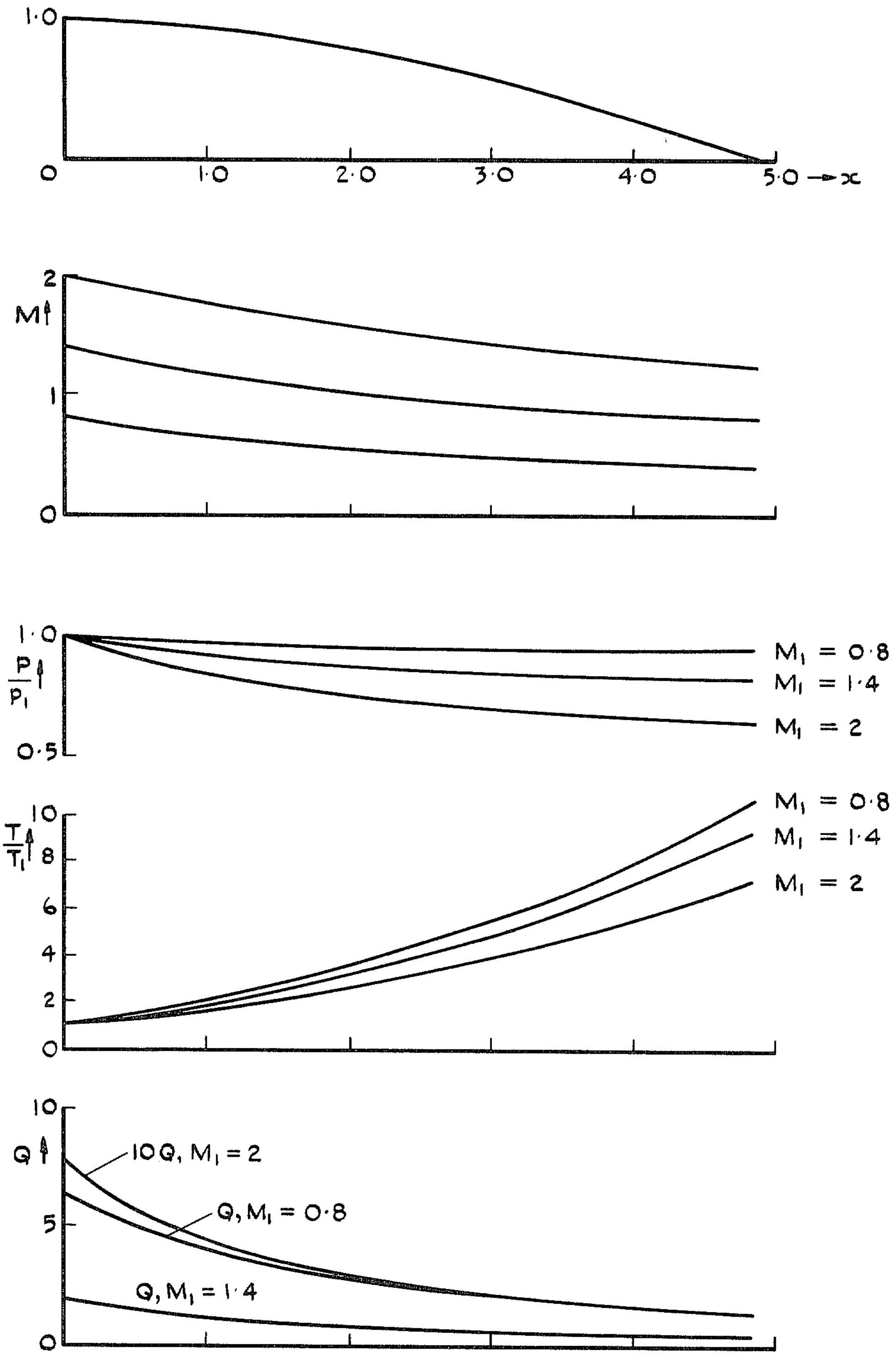

FIG. 3. Results for $\theta_1 = 20° : n = -0{\cdot}364$.

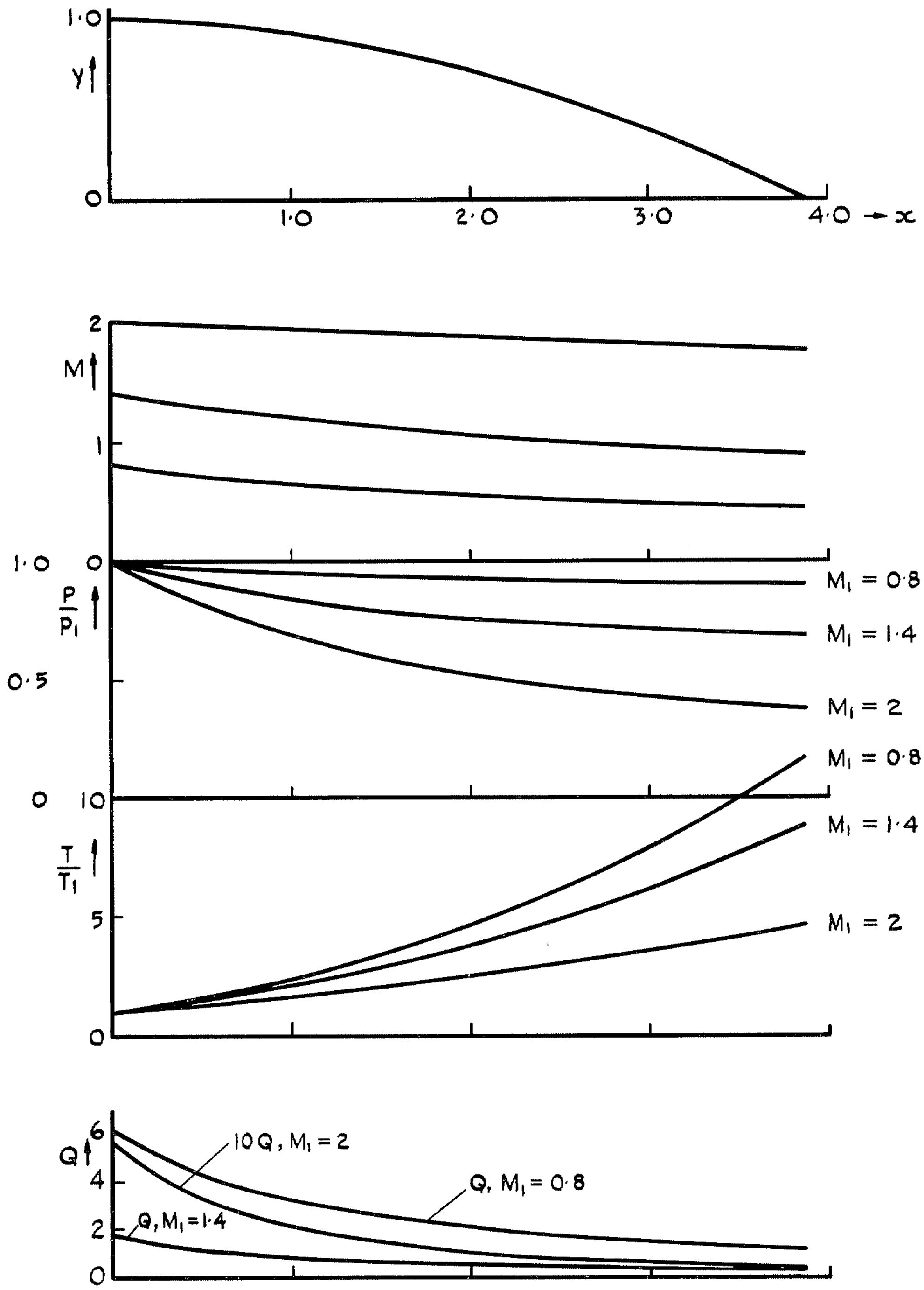

FIG. 4. Results for $\theta_1 = 25°: n = -0{\cdot}466$.

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