R. & M. No. 3754

R. & M. No. 3754

PROCUREMENT EXECUTIVE, MINISTRY OF DEFENCE

AERONAUTICAL RESEARCH COUNCIL

REPORTS AND MEMORANDA

The Effect of Compressibility on Unstalled Torsional Flutter

By D. S. WHITEHEAD

University Engineering Dept., Cambridge

LONDON: HER MAJESTY'S STATIONERY OFFICE

1974

PRICE £1.20 NET

The Effect of Compressibility on Unstalled Torsional Flutter

By D. S. WHITEHEAD

University Engineering Dept., Cambridge

Reports and Memoranda No. 3754*

August, 1973

Summary

The report presents the results of calculations of torsional flutter of unstalled cascade blades at zero mean deflection and subsonic Mach numbers. Three kinds of flutter are found. Subcritical flutter occurs when no acoustic waves can propagate in the duct. The effect of increasing Mach number is highly favourable, and tends to suppress the flutter predicted by incompressible theory. Acoustic resonance flutter occurs when the blade frequency is very slightly less than the acoustic resonant frequency of the upstream and downstream ducts. It is critically dependent on this coincidence, and would give very small areas of flutter on a compressor performance map. Supercritical flutter occurs when some acoustic waves can propagate in the duct. It only occurs at Mach numbers close to unity, and is probably not of much practical importance.

* Replaces A.R.C. 35 018

LIST OF CONTENTS

1. Introduction

When attempts are made to calculate the aerodynamic forces and moments on compressor and turbine blades due to their vibration, it is usual to use a theory which assumes that the flow is incompressible and two-dimensional. A theoretical and experimental investigation of torsional flutter has been reported by Whitehead.[1] Whitehead[2] has also reported some calculations which allow for the compressibility of the fluid, but the very few results did no more than indicate that compressibility is important. More recently, Smith[3] has reported a very much better method of calculation, and it is this method which is used in the present study. The assumptions are that the blades can be represented by flat plates at zero mean incidence, that the flow is subsonic, isentropic, and two-dimensional, and that the Kutta–Joukouski condition at the trailing edge is satisfied.

It seems to be more appropriate to look at torsional flutter than at bending flutter under these assumptions, since incompressible theory has shown that bending flutter depends critically on the steady mean loading on the blades, whereas torsional flutter is less dependent on this effect.

2. Method of Calculation

If a cascade of blades is vibrating in torsion, about an axis position distant ηc from the leading edge, then the moment coefficient is given by

$$(C_{M\alpha})_\eta = C_{M\alpha} - \eta C_{F\alpha} - i\lambda\eta C_{Mq} + i\lambda\eta^2 C_{Fq}. \tag{1}$$

The four complex coefficients $C_{M\alpha}$, $C_{F\alpha}$, C_{Mq} and C_{Fq} are defined for an axis position at the leading edge. They are functions of the space to chord ratio (s/c), stagger angle (ξ), frequency parameter (λ), Mach number (M), and phase angle between adjacent blades (ϕ). They have been calculated by the method given by Smith,[3] and throughout this investigation the number of points at which the upwash velocities are matched has been 4.

It is assumed that all blades are identical, so that flutter will occur with a constant phase angle between each blade and the next. Since differences between blades are known to have a stabilizing effect,[1] this is the most pessimistic assumption. If it is also assumed that the mechanical damping is zero, then the condition for flutter is that the imaginary part of $(C_{M\alpha})_\eta$ is zero. Equation (1) can then be regarded as a quadratic equation for η, which may or may not have real roots. If there are real roots, then this gives a range of axis positions for which flutter will occur.

Flutter may occur at phase angles given by

$$\phi = 2\pi m/N, \tag{2}$$

where N is the number of blades in the row, and m is any integer. Normally, N is large enough so that virtually any value of ϕ can be obtained, but if the flutter is very critically dependent on ϕ, then equation (2) may provide some effective constraint.

It is therefore necessary to hunt for the most critical phase angles for flutter to occur. In order to assist in this hunting process, a computer program was written, using graphical input and output techniques. Results are presented on a screen which shows a plot of axis position (η) against phase angle (ϕ). In order to input a starting value of ϕ for the next hunt, an adjustable cursor is set up over that part of the diagram where, on the basis of previous results, the flutter is to be expected. If no real roots of equation (1) are obtained, then another try can be made.

3. Results

It has been found that three different regimes of flutter can be distinguished, and it will be convenient to discuss them separately. They will be called 'sub-critical flutter,' which occurs in a regime where any acoustic waves generated cannot propagate upstream and downstream, 'acoustic resonance flutter,' which occurs when a pair of acoustic waves is just on the verge of being able to propagate, and 'super-critical flutter' which occurs in a regime where at least one pair of acoustic waves can propagate. These three regimes can merge into each other, so that the distinction between them is not always clear cut.

3.1. Sub-Critical Flutter

This is an extension of the flutter found in incompressible flow.

Fig. 1 shows results for a space-to-chord ratio of 1·0, a stagger angle of 45 degrees, and a position of the torsional axis at 58·8 per cent chord. This corresponds to one of the experimental arrangements used in Ref. 1. The interblade phase angle is 60 degrees, which is the most critical angle possible in a 12 bladed cascade. The imaginary part of the moment coefficient is plotted against frequency parameter. This is the part of the moment coefficient which can do work on the vibration, and if it is positive then flutter can occur when there is no mechanical damping. In fact in the experiments reported in Ref. 1 there was significant mechanical damping, and allowing for an average value of this gives the line shown as a flutter limit in Fig. 1. If the value of the imaginary part of the moment coefficient is above this line, then flutter is predicted.

The results for the $M = 0$ line correspond to the incompressible theory. It is seen that as the Mach number is increased the points at which flutter is just possible move to progressively lower values of the frequency parameter. This corresponds to higher fluid velocities or to lower blade stiffness. The effect of Mach number is therefore highly favourable.

Figs. 2, 3 and 4 show the results of a more general flutter investigation for three cascades. In obtaining these results, the 'most critical' phase angle has been found, and the mechanical damping has been neglected. The frequency parameter below which torsional flutter is just possible has been plotted against axis position.

It will be seen in all three cascades that as the Mach number increases, the frequency parameter at which flutter appears is markedly reduced, so that the effect of increasing Mach number is highly favourable.

It is also seen that the worst position for the torsional axis is around 50 to 70 per cent chord.

This flutter is not sensitive to interblade phase angle, so that the effect of using only integral values of m in equation (2), rather than the most critical value of ϕ, is negligible.

The optimum phase angles for sub-critical flutter have been plotted in Fig. 11, for the cascade illustrated in Fig. 4. In each case the lower value of $\phi/2\pi$ plotted in Fig. 11 corresponds to the higher value of η plotted in Fig. 4. The optimum phase angles for the other two cascades are similar, and are therefore not presented.

3.2 Acoustic Resonance Flutter

As the Mach number of the flow through the cascade is increased, for given geometry, frequency parameter, and phase angle, then it becomes possible for successive pairs of waves to propagate in the regions upstream and downstream of the cascade. This is the 'cut-off' phenomenon reported by Tyler and Soffrin.[4] Of the two waves in each pair, one carries energy upstream and the other carries energy downstream. The cut-off condition is

$$\frac{\lambda(s/c)}{(\phi - 2\pi n)} = \pm\sqrt{\frac{1}{M^2} - \cos^2\xi} - \sin\xi, \qquad (3)$$

where n is an integer, chosen to put ϕ in the range $o \leqslant \phi < 2\pi$. This condition can also be considered as an acoustic resonance of the annular duct, without blades, since at the cut-off condition the waves carry energy in a purely tangential direction, and no energy is dissipated down the duct.

It has been found that very close to this acoustic resonance condition, torsional flutter can be predicted theoretically, and this has been investigated in detail in one cascade which corresponds roughly to the tip sections in a modern fan engine. The results are shown in Figs. 5 to 9. These figures show loops plotted in the phase-angle axis-position plane, and inside these loops flutter is predicted. It will be seen that the range of phase angle for flutter is very narrow. The constraint provided by the finite number of blades in an actual row is therefore likely to be substantial, since the allowable phase angles given by equation (2) may all be outside the region of flutter.

In Figs. 5 to 9 the arrows pointing vertically downwards indicate the phase angles for the acoustic resonance condition, given by equation (3). It is seen that some of the loops show cusps at this phase angle. These cusps caused some difficulty in the development of the computer program, since simple procedures for finding maxima and minima fail at the cusps. The axis position for torsional flutter are largely forward of the blade mid-chord point ($\eta = 0{\cdot}5$) and in some cases extend to axis positions far forward of the leading edge. However, no case has been found in which the axis position for flutter extended to minus infinity, which would correspond to pure bending flutter.

Fig. 10, which has been obtained from Figs. 5 to 9, shows frequency parameter plotted against the most rearward axis position for which this type of acoustic resonance flutter is possible. As the Mach number increases, the critical axis position moves forward.

The range of phase angle for which this type of flutter occurs has been plotted on Fig. 11 (for $M = 0{\cdot}6$, 0·7 and 0·8), although the range is so narrow that it is difficult to show on this figure. The straight lines on the figure indicate the acoustic resonance condition, given by equation (3). The regions of acoustic resonance flutter are bounded by these lines, and lie at slightly lower frequency parameters. The flutter therefore occurs in a regime very close to the 'cut-off' condition, but in which the acoustic waves just do not propagate.

For a Mach number of 0·8 there are two lines for the acoustic resonance condition, which intersect at a frequency parameter of approximately 1·93. These correspond to the alternative signs in equation (3). At this point the blades are in resonance with two different acoustic modes. Figs. 7 and 8 show that the flutter regions are enlarged close to this point, and this also accounts for the kink in the $M = 0{\cdot}8$ line on Fig. 10.

Fig. 12 shows a similar region of acoustic resonance flutter, for a Mach number of 0·9.

3.3 Supercritical Flutter

As the flow Mach number approaches unity, the lines for acoustic resonance move to comparatively low frequency parameters, as can be seen in Fig. 12. It is then possible for torsional flutter to occur in a regime where one or more pairs of acoustic waves can propagate. This is referred to as super-critical flutter. It has only been investigated for one cascade at a Mach number of 0·9. At higher Mach numbers than this the linearization assumed in the theory becomes of dubious validity.

Like sub-critical flutter, this kind of flutter is not sensitive to the exact value of the phase angle. The values of the optimum phase angle for flutter are shown on Fig. 12. The corresponding limiting axis positions are shown on Fig. 13, where it can be seen that the worst axis position is near the leading edge.

4. Conclusions

Armstrong and Stevenson[5] have stated that in order to avoid torsional flutter of stalled compressor blades, the frequency parameter should not be less than 1·6. This figure is commonly used as a design rule. Therefore practical blades will usually have frequency parameters greater than 1·6, at least at a part-speed condition where some stalling is to be expected, but at the design point frequency parameters somewhat less than this are sometimes used.

For sub-critical flutter, the effect of Mach number has been found to be highly favourable, in that flutter is less likely as the Mach number increases. The effect is so strong that the flutter predicted by incompressible theory is likely to be completely supressed on practical blades if the Mach number is greater than 0·5. For this type of flutter, the critical axis position is near to or just aft of the mid-chord point, and the phase angle is not critical.

Acoustic resonance flutter is predicted in a very narrow regime, where a pair of acoustic waves just fail to propagate in an annular duct. It occurs right in the practical range of frequency parameter, but is very sensitive to phase angle. It might be considered to be similar to the coupled flutter which occurs on aircraft wings, in that two or more degrees of freedom are involved, but here the degrees of freedom are one of blade motion and one (or more) acoustic resonances in the duct. Kaji and Okazaki[6] have shown that acoustic waves very close to the cut-off condition are almost totally reflected by a blade row. This suggests that the acoustic resonance flutter may be markedly affected by the presence of adjacent blade rows. Also, on practical blades, three-dimensional effects are likely to be important. There is also a substantial constraint on the flutter, due to the finite number of blades in a row. For these reasons the actual quantitative results presented here are not thought to be of great practical significance. But the qualitative result, that flutter may occur if the blade natural frequency is just below an acoustic resonant frequency of the duct, is thought to be a valid and important conclusion. The critical axis position for this type of flutter is near the leading edge of the blade.

Supercritical flutter only occurs at Mach numbers approaching unity. Here the linearization used in the theory becomes of doubtful validity, and three-dimensional effects would be expected to be important. In the one case calculated, the flutter was only found at frequency parameters rather less than those for practical blades. For these reasons the calculation of supercritical flutter is not thought to be of great practical significance.

Since the effect of compressibility on unstalled torsional flutter is substantial, it is unsatisfactory to use incompressible theory for design calculations. However, although theories now exist which can allow for compressibility and steady lift on the blades separately, there is no theory which can allow for both of these effects at the same time. The development of such a theory is highly desirable.

Acknowledgment

Grateful acknowledgment is made to Dr. R. B. Morris, of the Computer Aided Design group at the Cambridge University Engineering Laboratory, who gave much help with the graphical input and output techniques, and to the Procurement Executive, Ministry of Defence, who supported this work.

LIST OF SYMBOLS

c	Blade chord
m	Integer
n	Integer
q	Translational velocity of leading edge due to bending
s	Blade spacing
$C_{M\alpha}$	Moment coefficient due to torsion $= T/\pi\rho U^2c^2\alpha$
$C_{F\alpha}$	Force coefficient due to torsion $= F/\pi\rho U^2c\alpha$
C_{Mq}	Moment coefficient due to translation $= T/\pi\rho Uc^2q$
C_{Fq}	Force coefficient due to translation $= F/\pi\rho Ucq$
$(C_{M\alpha})_\eta$	Moment coefficient for axis position η
F	Aerodynamic force
M	Mach number
N	Number of blades in row
T	Aerodynamic moment
U	Mainstream fluid velocity
α	Angular displacement of blade
ηc	Distance of torsional axis from leading edge
λ	Frequency parameter $= \omega c/U$
ξ	Stagger angle
ρ	Density
ϕ	Phase angle between each blade and the next (radians)
ω	Angular frequency

REFERENCES

No.	*Author(s)*	*Title, etc*
1	D. S. Whitehead	Torsional flutter of unstalled cascade blades at zero deflection. A.R.C. R. & M. 3429 (1964).
2	D. S. Whitehead	Vibration and sound generation in a cascade of flat plates in subsonic flow. A.R.C. R. & M. 3685 (1970).
3	S. N. Smith	Discrete frequency sound generation in axial flow turbomachines. A.R.C. R. & M. 3709 (1972).
4	J. M. Tyler and T. G. Sofrin ..	Axial flow compressor noise studies. *Trans. S.A.E.*, Vol. 70, p. 309 (1962).
5	E. K. Armstrong and R. G. Stevenson	Some practical aspects of compressor blade vibration. *J. Roy. Aero. Soc.*, Vol. 64, p. 117 (1960).
6	S. Kaji and T. Okazaki	Propagation of sound waves through a blade row. II Analysis based on the acceleration potential method. *J. Sound Vib.*, Vol. 11, p. 355 (1970).

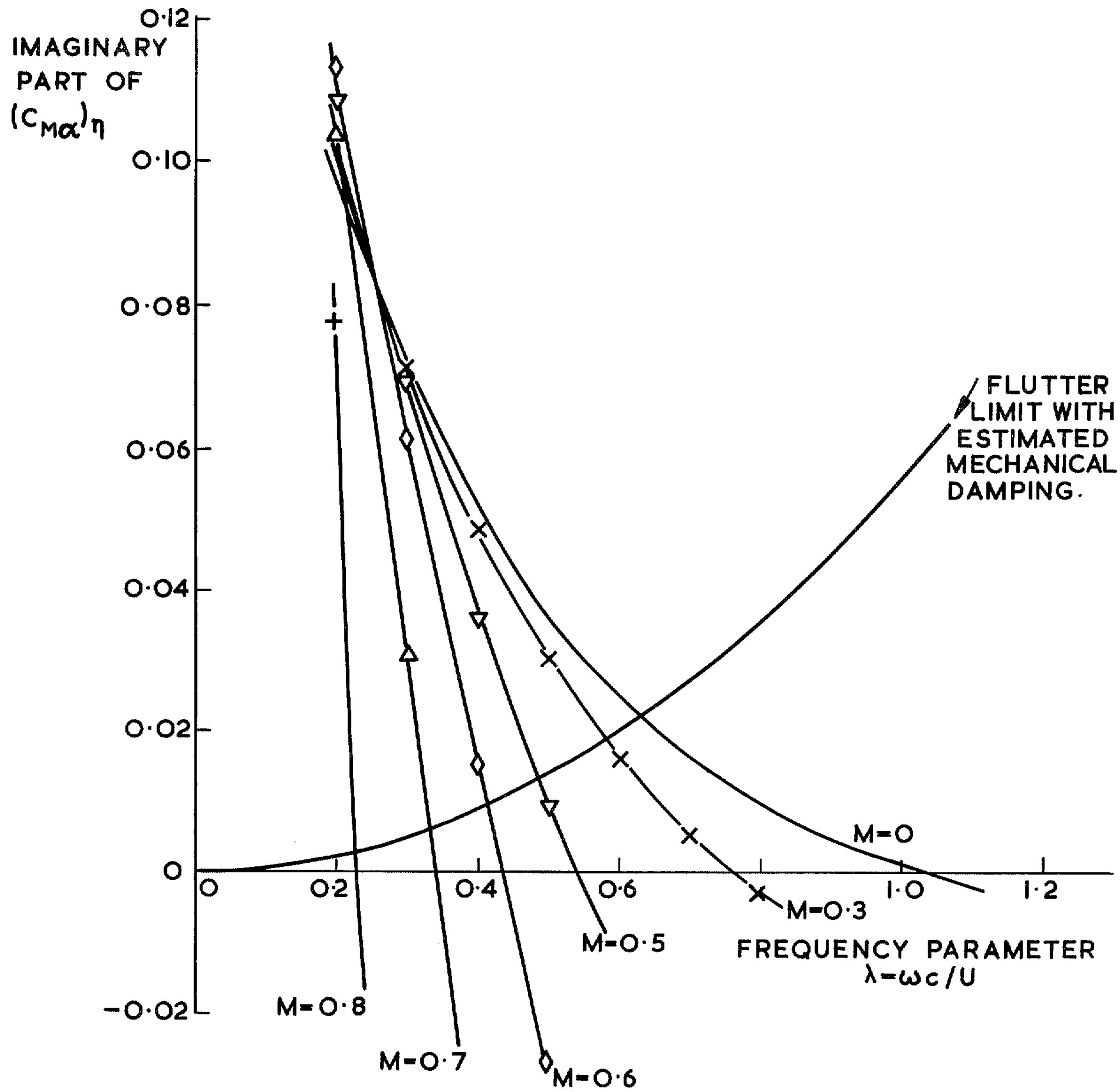

FIG. 1. Torsional flutter prediction s/c = 1·0, $\xi = 45°$, $\eta = 0{\cdot}588$, $\phi = 60°$.

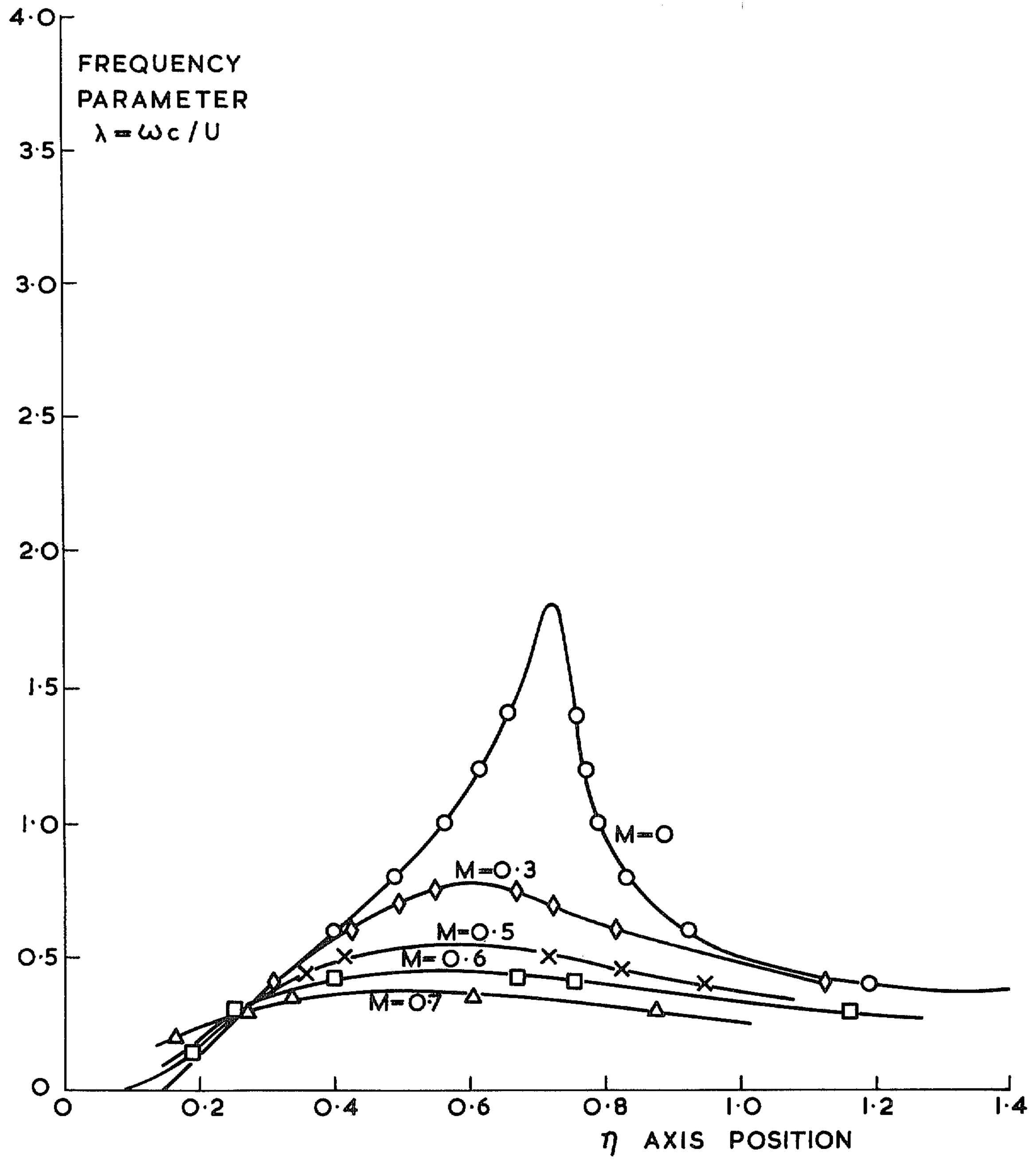

FIG. 2. Sub-critical flutter s/c = 1·0, ξ = 45°.

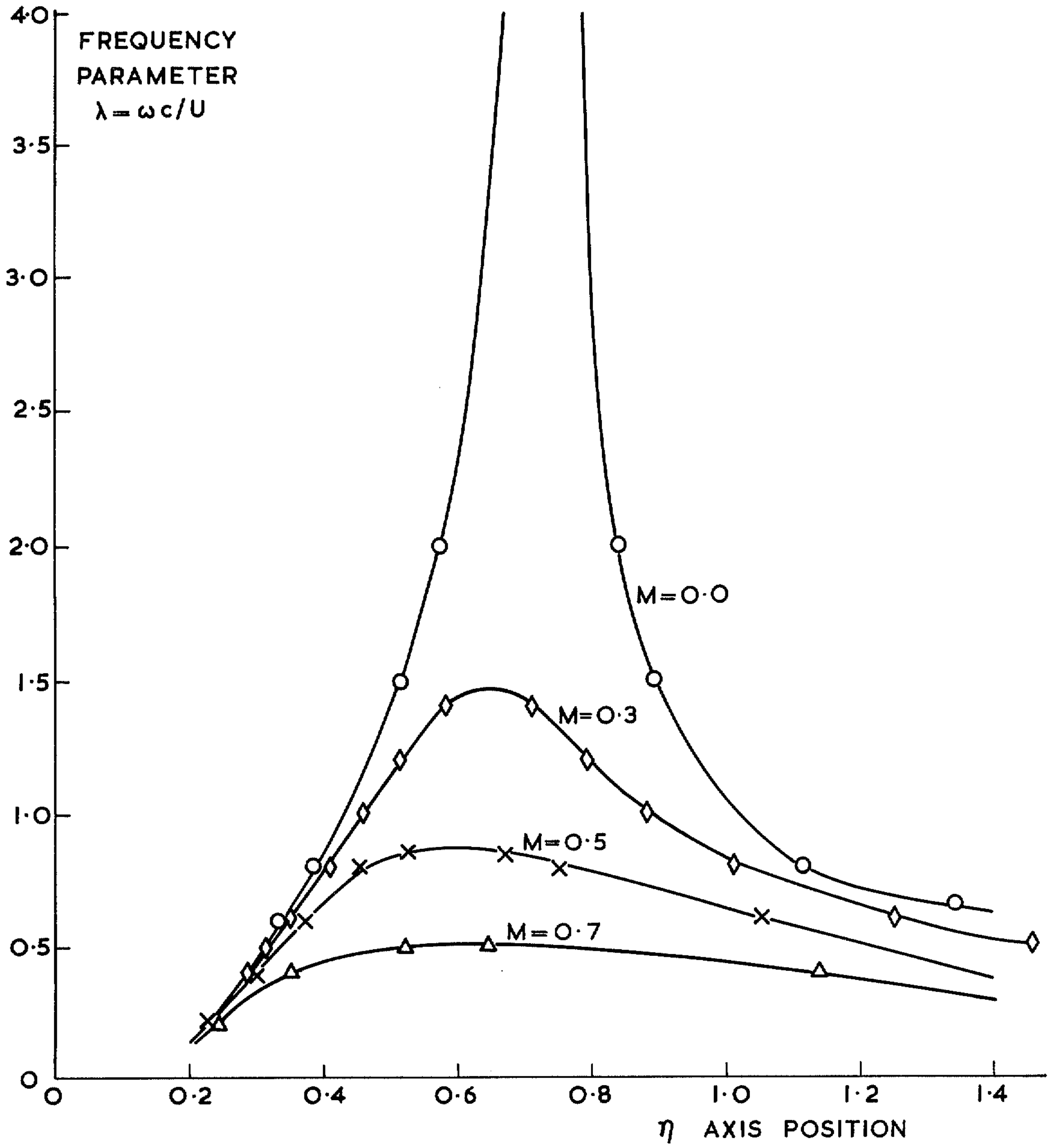

FIG. 3. Sub-critical flutter s/c = 1, $\xi = 60°$.

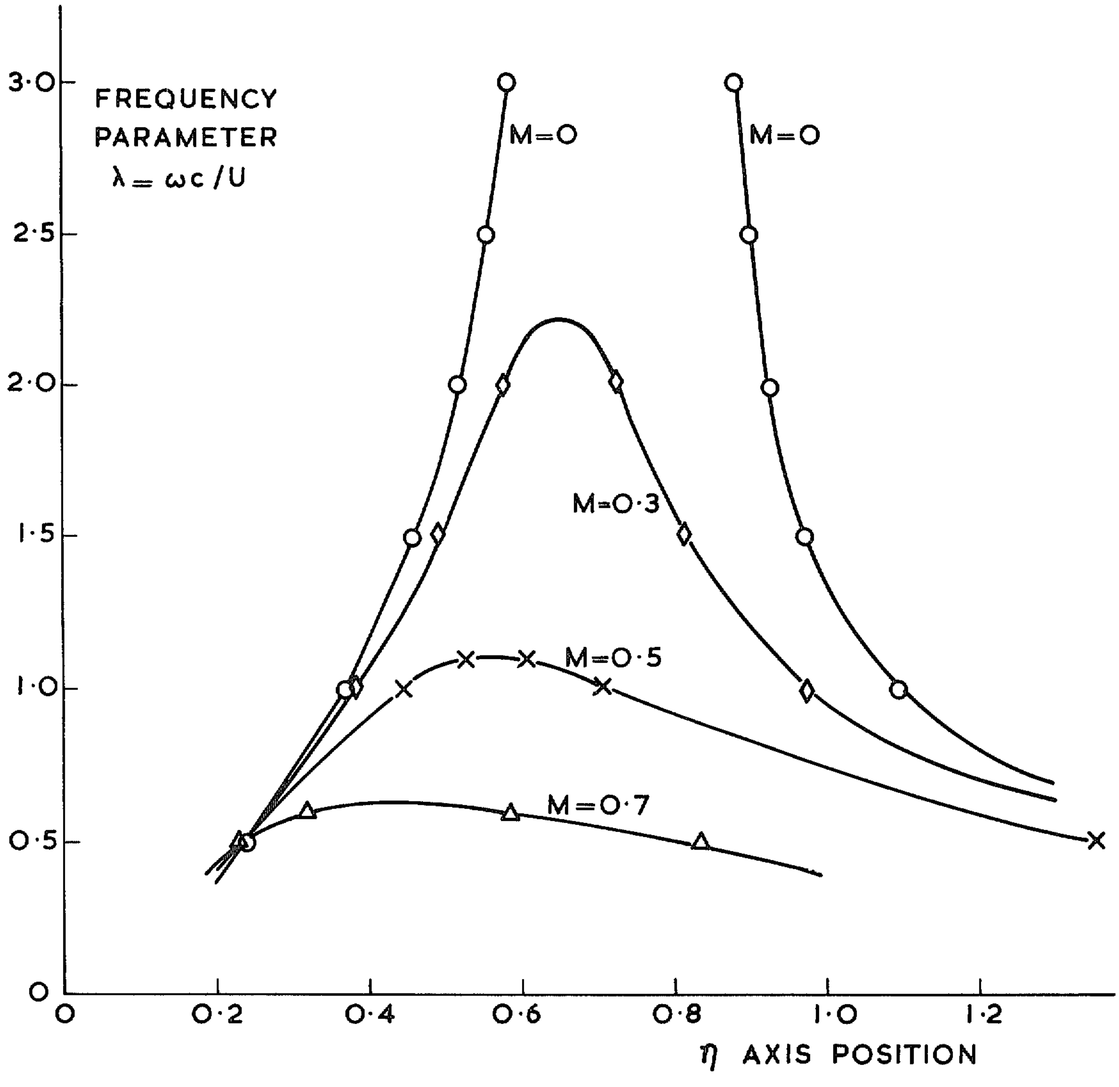

FIG. 4. Sub-critical flutter s/c = 0·8, $\xi = 60°$.

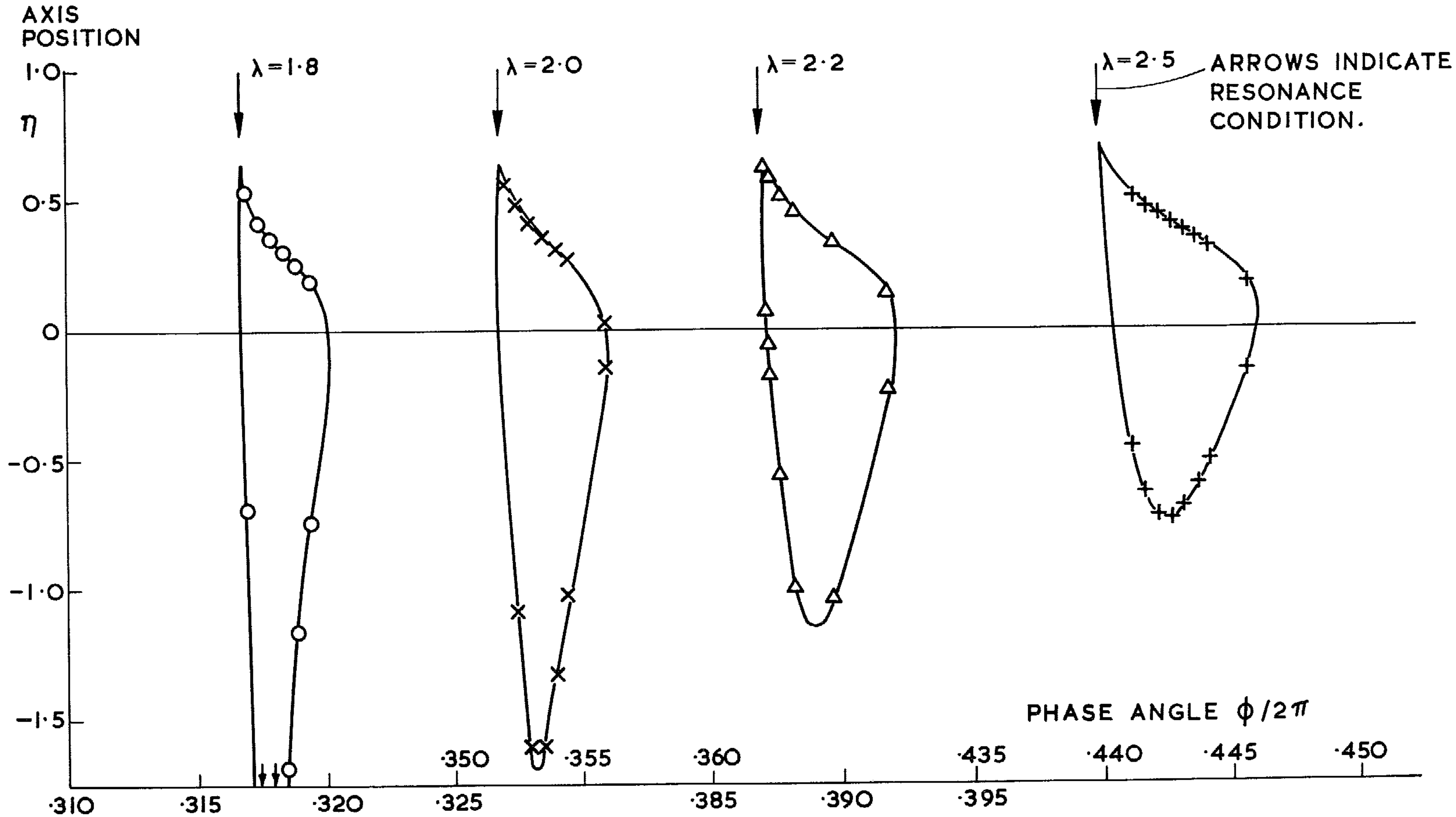

FIG. 5. Acoustic resonance flutter s/c = 0·8, $\xi = 60°$, $M = 0{\cdot}6$.

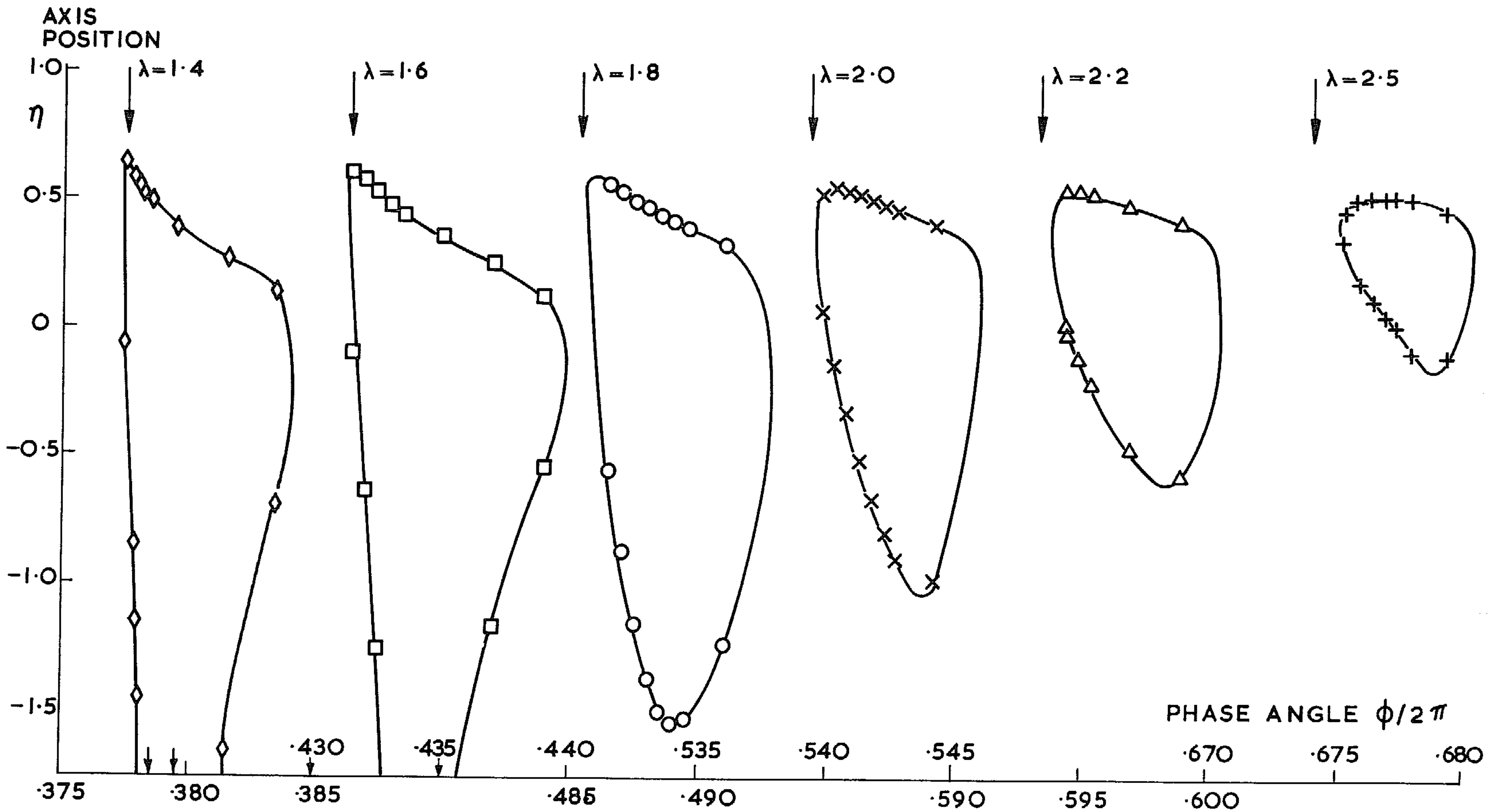

FIG. 6. Acoustic resonance flutter s/c = 0·8, $\xi = 60°$, $M = 0·7$.

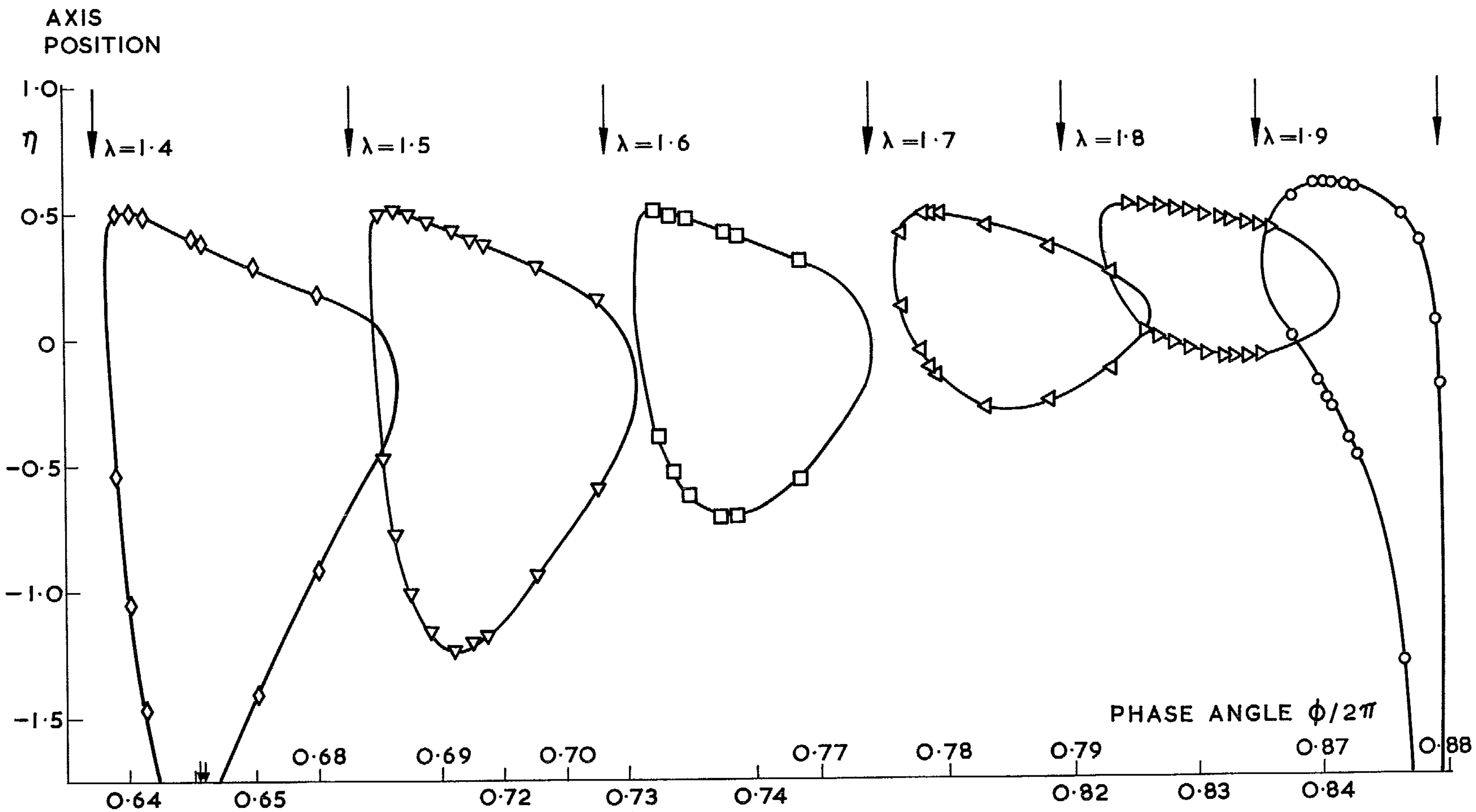

FIG. 7. Acoustic resonance flutter s/c = 0·8, ξ = 60°, M = 0·8.

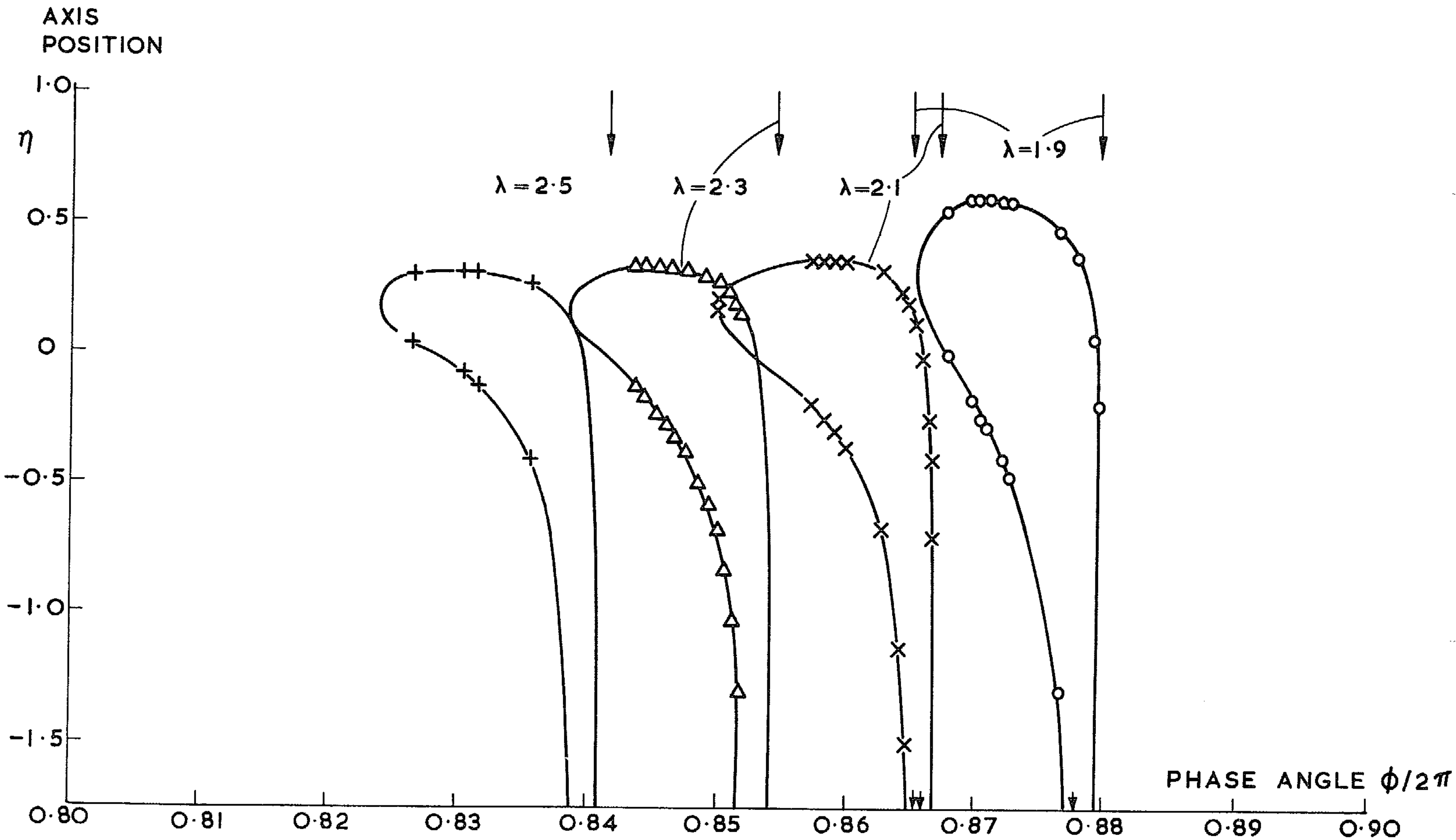

FIG. 8. Acoustic resonance flutter s/c = 0·8, ξ = 60°, M = 0·8 (continued).

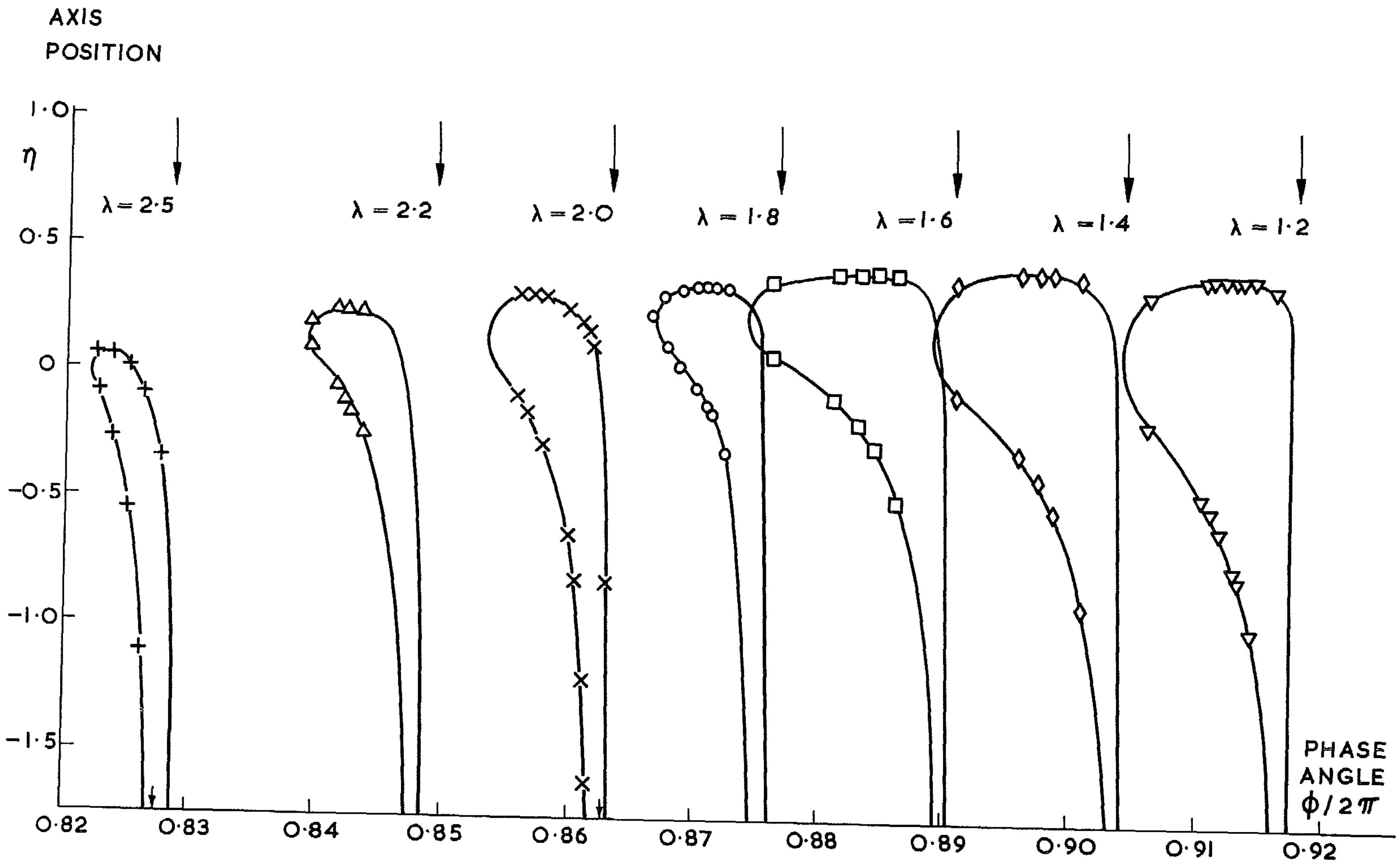

FIG. 9. Acoustic resonance flutter s/c = 0·8, $\xi = 60°$, $M = 0{\cdot}9$.

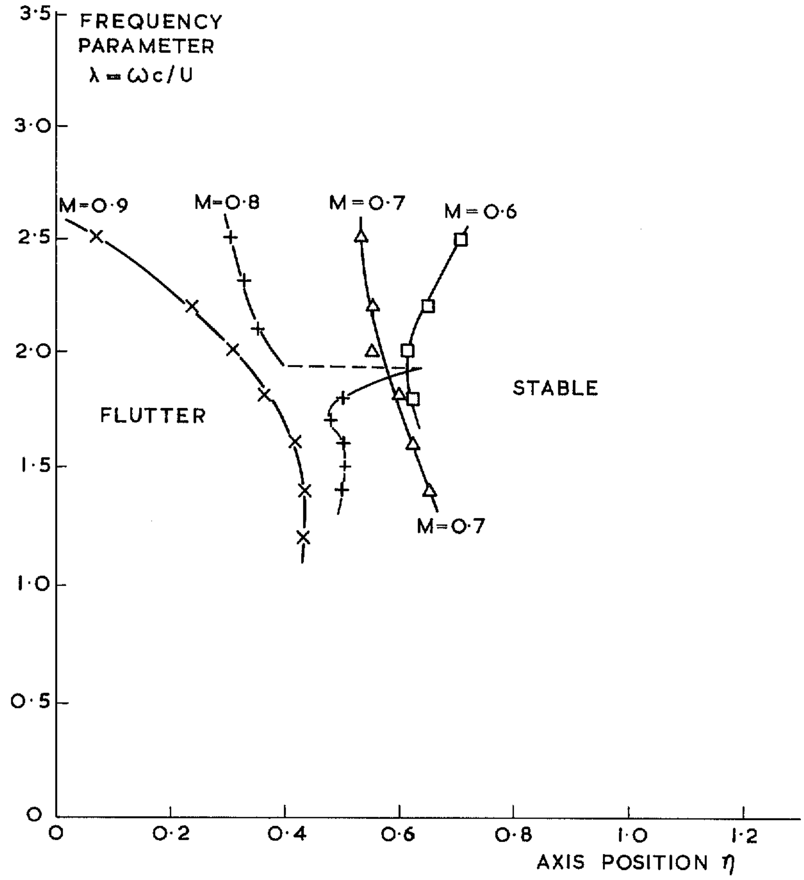

FIG. 10. Acoustic resonance flutter s/c = 0·8, ξ = 60°.

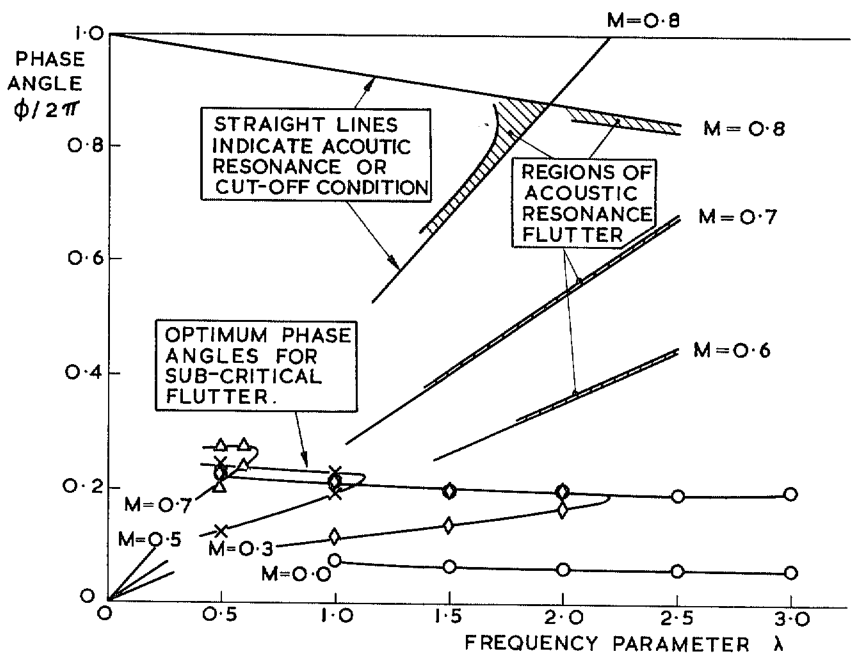

FIG. 11. Phase angles for flutter s/c = 0·8, ξ = 60°.

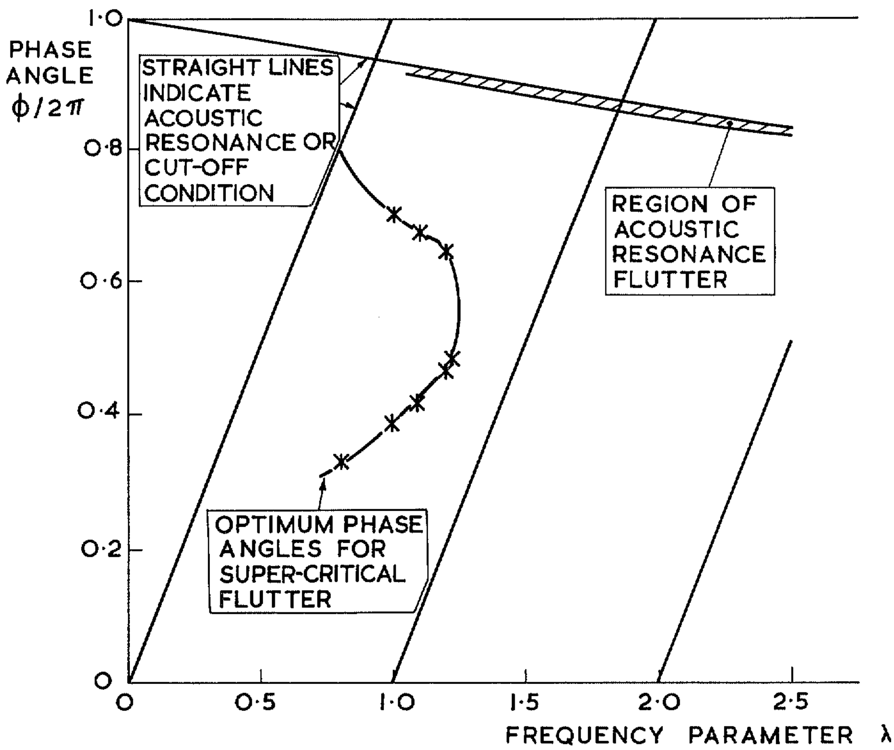

FIG. 12. Phase angles for flutter s/c = 0·8, $\xi = 60°$, $M = 0{\cdot}9$.

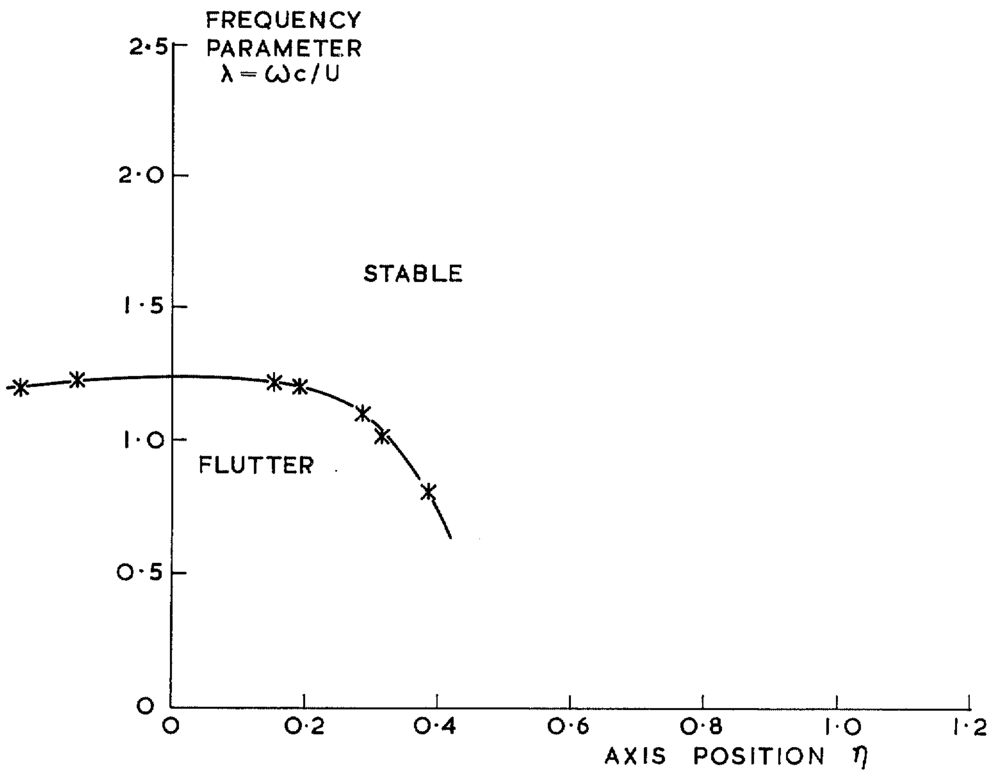

FIG. 13. Super-critical flutter s/c = 0·8, $\xi = 60°$, $M = 0{\cdot}9$.

Printed in England for Her Majesty's Stationery Office by J. W. Arrowsmith Ltd., Bristol BS3 2NT.
Dd. 505374 K5 10/74

R. & M. No. 3754

HER MAJESTY'S STATIONERY OFFICE

Government Bookshops

49 High Holborn, London WC1V 6HB
13a Castle Street, Edinburgh EH2 3AR
41 The Hayes, Cardiff CF1 1JW
Brazennose Street, Manchester M60 8AS
Southey House, Wine Street, Bristol BS1 2BQ
258 Broad Street, Birmingham B1 2HE
80 Chichester Street, Belfast BT1 4JY

Government publications are also available through booksellers

R. & M. No. 3754
ISBN 0 11 470847 9