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# Relative Frequency of Occurrence of Different Normal Accelerations at the Centre of Gravity of Aircraft in Turbulence 

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## SUMMARY

An examination is made of the relative frequency of occurrence of different normal accelerations, at points near the aircraft centre of gravity of 5 different aircraft, from about 200 hours of research flying sub-divided into about 12000 time periods mainly of about 1 minute. It is shown that the commonly used assumption of a Rayleigh distribution for vertical gust velocity maxima for each period gives poor estimates of the cumulative totals of all the periods for each aircraft. If, however, it is assumed that
(i) the frequency of occurrence of different magnitudes of the maxima of the gust velocity vector is a Rayleigh distribution,
(ii) the vector changes direction sufficiently slowly for the maxima of the components to occur at the same time as the maxima of the vector, hold for each period, the estimates of the cumulative totals of all the periods for each aircraft do not differ significantly from the measurements. Nor is there any significant difference between the measurements and the cumulative totals of gust velocity maxima in 36 traverses of severe turbulence encountered in 23000 flying hours of civil aircraft operations.

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## 1. Introduction

The data on turbulence loads that are most extensive are in the form of normal accelerations at a point near the aircraft centre of gravity, together with height, speed and weight of the aircraft at the time of their occurrence. These accelerations are almost invariably tabulated as the frequency with which various increments from 1 g are exceeded. For any particular aircraft on which measurements are taken, these accelerations give a good indication of the overall load on that aircraft. For the purpose of design calculation, it is usually necessary to make some assumption of the functional relationship of the number of exceedances at the different acceleration levels. Much theoretical work has been done on the assumption that the vertical gust velocity (and also the centre of gravity acceleration) has a normal distribution on a time scale in any traverse of homogeneous turbulence and that any particular turbulence encounter is made up of a series of traverses of homogeneous turbulence of different intensities. On the usual assumption that the rate of change of gust velocity is independent of the magnitude of the velocity, the normal distribution of gust velocity is equivalent to a Rayleigh distribution of gust velocities exceeding specified values (this is usually referred to as a distribution of crossings).

Practically all workers in the field have noticed that on this assumption, no matter to what extent the areas are sub-divided, there is still a tendency for the higher gusts to occur more frequently than the Rayleigh distribution would predict. King ${ }^{1}$ has found that for 36 severe turbulence areas the highest peak gust velocity was invariably greater than would have been expected from a Rayleigh distribution with the same root mean square of peak velocities and also that the average flying time in each was only half a minute. The purpose of the present investigation is to examine experimental data and sub-divide it into short periods of time which are mostly about 1 minute and check a distribution based on a Rayleigh distribution of the magnitude of the gust velocity vector rather than a Rayleigh distribution of the magnitude of its component.

The investigation also gives an opportunity of examining the total number of gusts in each interval (i.e. zero crossings), and compare these with theory. A recent report by Kaynes ${ }^{2}$ gives an estimate of zero crossings that forms the basis of this comparison.

## 2. Theoretical Model of Turbulence

The relative frequency of occurrence of vertical gust components of different magnitude is calculated on the following assumptions:
(i) the frequency of occurrence of different magnitudes of the maxima of the gust velocity vector is a Rayleigh distribution,
(ii) the vector changes direction sufficiently slowly for the maxima of the components to occur at the same time as the maxima of the vector.
The proportion of maxima of gust velocity vectors greater than $U$ are equal to $\exp \left(-U^{2} / 2 \sigma^{2}\right)$ where $\sigma^{2}=$ the mean square of the turbulence velocity. It is shown in the Appendix that on the above assumptions the corresponding proportions of maxima of vertical component of gust velocity greater than $U$ will be equal to

$$
\begin{equation*}
\int_{0}^{1} \exp \left(-U^{2} / 2 \sigma^{2} x^{2}\right) d x \tag{1}
\end{equation*}
$$

These two distributions are shown in Fig. 1 and expression (1) is tabulated for a range of values of $U / \sigma$ in Table 17.

## 3. Experimental Normal Acceleration Records

A large amount of records was available in unpublished documents for the following aircraft:

## Anson

Canberra
Meteor
Viking
York

In the earlier documents the basic data have been added together for presentation. In the present analysis the records are sub-divided into short periods of time, usually about 1 minute with the exception of the Meteor which is in periods of a quarter of a minute. The records are all as stepped traces at prescribed levels of normal acceleration, the Anson, Viking and York being at multiples of 0.1 g increment the Meteor at multiples of 0.2 g and the Canberra at increments of $0.2 \mathrm{~g}, 0.3 \mathrm{~g}, 0.4 \mathrm{~g}, 0.6 \mathrm{~g}$, and thereafter in multiples of 0.2 g . The analysis is done in two parts, firstly the records are reduced to a series of maxima and minima and secondly the maxima above 1 g and the minima below 1 g determined for various thresholds $T g$ (i.e. a maximum $P g$ above 1 g is only counted when a subsequent, but not necessarily the consecutive minimum of less than ( $P-T$ )g occurs). It was found that the size of threshold did not matter much provided it was not greater than necessary to return to 1 g and for convenience of analysis only, the one largest increment between 1 g crossings was chosen. By being able to neglect differences due to the size of threshold, it is possible to calculate the number of crossings of any level as the number of maxima greater than that level. The number of crossings are determined for each short period of the records. These individual periods have been examined separately and the root mean square acceleration determined on two assumptions. Firstly that the distribution of maxima is a Rayleigh arising from a normal distribution of the vertical gust component; and secondly that the distribution of maxima is that given by expression (1) which arises from a Rayleigh distribution of the maxima of the gust vector. For any individual period the number of gusts is so small that the observed values can rarely be said to be significantly different from either of the assumptions. A typical period is shown in Fig. 2. When the periods are added together it is immediately apparent that the higher gust velocities occur more frequently than would have been expected from the summation of the Rayleigh distributions for the components. The totals for the Anson, Canberra, Meteor, Viking and York are given respectively in Tables 1 to 5 for a range of height bands; the calculated values are tabulated to the nearest 0.1 of a digit. In preparing the tables the root mean square of the measured peaks has been calculated and that value used to determine the appropriate Rayleigh distributions. Throughout the tables calculated values are given as follows:

Calculation A: A Rayleigh distribution of the maxima of the gust velocity vector.
Calculation B: A Rayleigh distribution of the maxima of the vertical component of the gust velocity.
The totals of the accelerations for all heights grouped together are plotted in Figs. 3 to 7 respectively for the Anson, Canberra, Meteor, Viking and York; the calculated values are not rounded off before plotting. It will be seen that the distribution based on a Rayleigh distribution of the maxima of the velocity vector, that is now proposed, gives a much better representation of the observed data than the distribution based on Rayleigh distribution of the vertical component. Also the Rayleigh distribution of the vertical component is invariably appreciably below the experimental data at the higher gust velocities. The Canberra and the Meteor were both recording data below 500 feet. It will be seen that at the higher increments the observed data on the Canberra is higher than the distribution that is now proposed, and that for the Meteor is lower. The amounts that the Meteor are lower, admittedly on a much shorter distance, are approximately equal to the amounts that the Canberra data are too high. The Viking, Anson and York are all recording data at higher altitudes, for the most part between 1500 and 9500 feet. York and Viking agree closely with the formula now being proposed, but the Anson does have rather higher experimental records than the theoretical which is discussed in Section 4.

## 4. Regions of Continuous Turbulence

### 4.1. Moderate Intensity Turbulence

By examining the records in large numbers of periods of time it is possible to make an assessment of when the turbulence is reasonably continuous. This has been done for all the aircraft examined. An assessment has been made of the intensity and total number of gusts (i.e. zero crossings) in each period. The intensity has been based on the mean square of the peak accelerations and converted to gust velocities by means of the gust response factor given by Kaynes. ${ }^{2}$ The total number of gusts given in each case is the percentage of the total number that would be expected in continuous turbulence based on Kaynes' estimate. For the purpose of determining turbulence that is reasonably continuous and also of moderate intensity it was decided that traverses should be selected using as a basis a parameter that was equal to
the product of intensity and percentage of zero crossings. Traverses of about 50 kilometres at each height band were selected where this parameter was high. In the case of the Anson, the total recorded time is 620 minutes mainly in the height band $1500-5500$ feet and the two 50 kilometre traverses which give this parameter the highest value are given in Table 6, together with the numbers of gusts encountered.

Figure 8 gives a plot of the distributions of normal acceleration crossings for each of these two traverses of turbulence and compares the data with the theoretical predictions, of the two distributions being considered, on the assumption that the turbulence was homogeneous over the whole area. To give an indication of the intensity of the turbulence the scale used is in fact not normal acceleration but gust velocity. To make this conversion the turbulence is represented by a two-dimensional Von Karman spectrum with a scale of turbulence chosen empirically ${ }^{3}$ to be equal to the altitude up to 1000 feet and thereafter equal to 1000 feet times the ratio of the air density at ground level to that at the altitude of the turbulence. Kaynes' gust response factors ${ }^{2}$ for this spectrum are used and an additional dynamic factor ${ }^{3}$ of 1.2 is made to allow empirically for the effect of structural flexibility on the acceleration records. It will be seen that the distribution based on gust vector gives a good representation in both cases, but the distribution based on velocity component gives appreciably too low values for the higher gusts in both cases.

The Meteor and the Canberra flights were all made below 500 feet. The Canberra data were sub-divided into flights at different intensities of solar radiation. The same sub-division is used in the present Report as was used by Bullen. ${ }^{4}$ A 50 kilometre flight for each of the ranges of solar radiation is selected on the same basis as was used for the Anson. These are given in Table 7 and Fig. 9. The Meteor results are given in Table 8 and Fig. 10. The same procedure was repeated for the Viking and one traverse of turbulence for each of the height bands $1500-5500$ feet, $5500-9500$ feet was selected on the same basis. These are shown in Table 9 and Fig. 11.

The number of records taken for the York was small but it was possible to select one area of turbulence at 1000 feet, which is given in Table 10 and Fig. 12, but none at higher altitudes.

### 4.2. Moderately Severe Turbulence

An examination of the data from the Viking and Anson showed that there were a few individual minutes that had a fairly severe intensity of turbulence, being about double the intensity of the adjacent periods. In each of these cases a more detailed examination has been made. Each record from a period containing a high intensity turbulence was taken together with the period immediately preceding and immediately following it, and the total examined as a single period. This record was then sub-divided into three consecutive periods, the middle one being determined as that containing the high intensity turbulence. It was noticed that these high intensity traverses were about 2 to 5 kilometres. In the case of the Anson there was one such encounter in the height band 1500-5500 feet and for the Viking there were such encounters at each of the three height bands above 1500 feet. The most intense areas in each height band are shown in Table 11 and Fig. 13. Because of the short duration of these encounters of moderately severe turbulence the number of gusts is relatively small and appreciable scatter is to be expected. Nevertheless the agreement is again good for the Rayleigh distribution of the gust velocity vector and poor for the Rayleigh distribution of gust component at the higher gust velocities. In all these cases of moderately severe turbulence it is not possible to make an accurate estimate of the time duration as the time scale on the original records was only accurately made at discrete minute intervals.

The moderately severe turbulence traverse by the Anson occurred during the 50 kilometre traverse given in Fig. 8b and it is evident that it was significantly more intense than the rest of the area. On the particular day of the flying there were widespread thunderstorms and it is reasonable to suppose that there were a number of moderately severe areas of turbulence. As this day's flying accounted for much of the 620 minutes recorded, the total number of gusts encountered was nearly as many as those recorded on the Viking 4360 minutes. It would be expected therefore that any influence of the records of moderately severe turbulence would be greater for the Anson than for the Viking; in particular it is apparent from the above discussion that moderately severe turbulence will often have a duration of less than 1 minute and in such cases individual 1 minute periods will contain a combination of moderate and moderately
severe turbulence. This could account for the higher number of large gusts than even the theory based on the gust vector would predict.

### 4.3. Severe Turbulence in Civil Operational Flying

An examination by $\mathrm{King}^{1}$ of records from 23000 flying hours by 3 four-engined pure jet civil transports gave 36 traverses of severe turbulence. The actual measured values of the peak centre of gravity normal accelerations in each of these traverses have been made available for analysis as used in Section 4.2. In order to make a cumulative comparison of a Rayleigh distribution of the magnitude of the gust vector and of the gust component, the individual traverses were converted from centre of gravity normal accelerations to gust velocity with Kaynes' response factors. The sum of the 36 traverses is given in Table 12 and Fig. 14. It can be seen that the agreement for the Rayleigh distribution based on gust vector is good and that based on gust component poor, especially at the high gust velocities.

### 4.4. Probabilities of High Gusts in Particular Traverses of Turbulence

The probability that the largest gust would have been greater than that measured has been calculated for each area of turbulence, where the number of gusts were sufficiently small for the calculation to be practicable ; the method is the same as that used by King ${ }^{2}$. Tables 13, 14, 15 give the probabilities respectively for the 50 kilometre traverses, the 2 to 5 kilometre traverses, and the traverses of severe turbulence met in civil operations.

## 5. Total Number of Gusts in Continuous Turbulence

On general reasoning it is thought that the traverse of 50 kilometres given in Section 4.1 are in turbulence that is practically continuous and are therefore good samples to compare theoretical estimates of total numbers of gusts (i.e. zero crossings) with measured values. The estimates of the measured values are obtained by taking a Rayleigh distribution with the same root mean square velocity as that for the measured peaks and extrapolating it to zero velocity. This is done for each traverse and given in Table 16 on the assumption that the magnitude of the gust vector is a Rayleigh distribution; the corresponding values for the gust component having a Rayleigh distribution are much smaller but have approximately the same consistency. Whilst the theoretical values ${ }^{2}$ of the zero crossings vary considerably between the different aircraft, the proportion of gusts is near 100 per cent for each of them.

## 6. Conclusions

An examination of the relative frequency of occurrence of different normal accelerations at points near the aircraft centre of gravity of about 200 hours of research flying, sub-divided into about 12000 time periods mainly of about 1 minute, has shown that whilst for the individual time periods of about 1 minute there is not a significant difference between observed values of normal acceleration crossings and the commonly used Rayleigh distribution, the cumulative effect makes the difference definitely significant when a large number of periods are combined. Similarly there is no significant difference for individual time periods of about 1 minute if it is assumed that
(i) the frequency of occurrence of different magnitudes of the maxima of the gust velocity vector is a Rayleigh distribution,
(ii) the vector changes direction sufficiently slowly for the maxima of the components to occur at the same time as the maxima of the vector.
For this condition it was found that the cumulative effect did not produce significant differences when all the periods for each aircraft were grouped together. Nor was there any significant difference when the measurements from 36 traverses of severe turbulence encountered in 23000 flying hours of civil operations were grouped together.

On the above assumptions the proportion of maxima of gust velocity vectors greater than $U$ is equal to $\exp \left(-U^{2} / 2 \sigma^{2}\right)$ and the proportion of maxima of vertical component of gust velocity greater than $U$ is
equal to

$$
\int_{0}^{1} \exp \left(-U^{2} / 2 \sigma^{2} x^{2}\right) d x
$$

It is also shown in this detailed analysis that Kaynes' aircraft responses based on the two-dimensional form of Von Karman's spectrum of turbulence give estimates of total numbers of gusts (zero crossings) that are close to the observed values for several aircraft with widely differing numbers of gusts per kilometre.

## REFERENCES

| No. |  | Author(s) | Title, etc. |
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| 2 | I. W. Kaynes |  | Aircraft centre of gravity response to two-dimensional spectra of turbulence. <br> A.R.C. R. \& M. 3665 (1969). |
| 3 | J. Taylor | . $\quad$ - | Manual on aircraft loads. <br> AGARDograph 83, Pergamon Press (1965). |
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## APPENDIX

## Proportion of Gust Components in One Direction for Gust Vectors having a Proportion $\exp \left(-U^{2} / 2 \sigma^{2}\right)$ Greater than $U$

It is assumed that the proportion of the maxima of gust velocity vectors that are greater than $U$ is $\exp \left(-U^{2} / 2 \sigma^{2}\right)$.

The proportion that lies between $U$ and $U+\delta U$ is

$$
\frac{U}{\sigma^{2}} \exp \left(-U^{2} / 2 \sigma^{2}\right) \delta U
$$

The volume of the spherical shell bounded by $U$ and $U+\delta U$ is $4 \pi U^{2} \delta U$. Hence the proportion per unit volume in the shell is

$$
\frac{U}{\sigma^{2}} \exp \left(-U^{2} / 2 \sigma^{2}\right) \delta U /\left(4 \pi U^{2} \delta U\right)=\exp \left(-U^{2} / 2 \sigma^{2}\right) /\left(4 \pi U \sigma^{2}\right)
$$

The volume of the ring bounded by $U$ and $U+\delta U$ and by $\theta$ and $\theta+\delta \theta$ is

$$
2 \pi U^{2} \cos \theta \delta U \delta \theta
$$

where $\theta$ is the angle between the direction of the gust vector and the plane perpendicular to the axis $y$.
Hence the proportion of gusts in this ring is

$$
\frac{U}{2 \sigma^{2}} \cos \theta \exp \left(-U^{2} / 2 \sigma^{2}\right) \delta U \delta \theta
$$

Let the magnitude of the gust vector component in the direction $y$ be $U_{y}$ so that $U=U_{y} \operatorname{cosec} \theta$. Then the proportion of gust vector components in the direction $y$ that are greater than $U_{y}$ is

$$
\begin{aligned}
& \int_{0}^{\pi / 2} \int_{U_{y} \operatorname{cosec} \theta}^{\infty} \frac{U}{2 \sigma^{2}} \cos \theta \exp \left(-U^{2} / 2 \sigma^{2}\right) d U d \theta \\
& =\frac{1}{2} \int_{0}^{\pi / 2} \cos \theta \exp \left(-U_{y}^{2} \operatorname{cosec}^{2} \theta / 2 \sigma^{2}\right) d \theta \\
& =\frac{1}{2} \int_{0}^{1} \exp \left(-U_{y}^{2} / 2 \sigma^{2} x^{2}\right) d x
\end{aligned}
$$

where $x=\sin \theta$.
The corresponding proportion of components less than $-U_{y}$ is the same and hence the total proportion of gust components in the direction $y$ of magnitude greater than $U_{y}$ is

$$
\int_{0}^{1} \exp \left(-U_{y}^{2} / 2 \sigma^{2} x^{2}\right) d x
$$

TABLE 1
Comparison on an Anson of Measured Numbers of Exceedances of Different Normal Acceleration Increments and Calculated Numbers with the Same Number at the Lowest Measured Increment

A measured increment is the highest value between crossing of the 1 g datum. The calculations are made for each of 620 time intervals of 1 minute.

| Height ft | Distance km | Accel. g-incr. | Number of times exceeded |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Calc. A | Calc. B | Measured |
| 1500-5500 | 1992.1 | 0.00 | 46207.6 | 22064.9 |  |
|  |  | 0.10 | 11220.0 | 11220.0 | 11220 |
|  |  | 0.20 | 2050.5 | 2121.5 | 2072 |
|  |  | 0.30 | 338.3 | 251.4 | 388 |
|  |  | 0.40 | 55.8 | 27.9 | 87 |
|  |  | 0.50 | 10.9 | 4.4 | 17 |
|  |  | 0.60 | 2.8 | 0.9 | 5 |
|  |  | 0.70 | 0.9 | 0.2 | 1 |
| $5500-9500$ | 319.7 | 0.00 | 1069.6 | 441.2 |  |
|  |  | 0.10 | 162.0 | 162.0 | 162 |
|  |  | 0.20 | 15.7 | 15.2 | 18 |
|  |  | 0.30 | 1.6 | $1 \cdot 1$ | 1 |
| All | 2311.8 | 0.00 | 47277.2 | 22506.0 |  |
|  |  | 0.10 | 11382.0 | 11382.0 | 11382 |
|  |  | 0.20 | 2066.2 | 2136.7 | 2090 |
|  |  | 0.30 | 339.9 | 252.5 | 389 |
|  |  | 0.40 | 56.0 | 28.0 | 87 |
|  |  | 0.50 | 10.9 | 4.5 | 17 |
|  |  | 0.60 | 2.8 | 0.9 | 5 |
|  |  | 0.70 | 0.9 | 0.2 | 1 |

## TABLE 2

## Comparison on a Canberra of Measured Numbers of Exceedances of Different Normal Acceleration Increments and Calculated Numbers with the Same Number at the Lowest Measured Increment

A measured increment is the highest value* between crossing of the 1 g datum. The calculations are made for each of 6133 time intervals of an average of 0.86 minute.

| Height ft | $\begin{aligned} & \text { Distance } \\ & \mathrm{km} \end{aligned}$ | Accel. g -incr. | Number of times exceeded |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Calc. A | Calc. B | Measured |
| 200 | 46685.5 | 0.00 | 877238.8 | 443664.7 |  |
|  |  | 0.20 | 252549.0 | 252549.0 | 252549 |
|  |  | 0.30 | 122333.5 | 133533.8 | 129860 |
|  |  | 0.40 | 55814.2 | 58698.8 | 59871 |
|  |  | 0.60 | 9940.0 | 7304.3 | 8980 |
|  |  | 0.80 | 1498.3 | 592.5 | 1153 |
|  |  | 1.00 | 202.8 | 40.7 | 184 |
|  |  | $1-20$ | 27.6 | 4.9 | 28 |
|  |  | 1.40 | 5.0 | 1.6 | 12 |
|  |  | 1.60 | 1.6 | 0.7 | 4 |

*The actual measurements were not precisely the highest value between crossings but the numerical values should not be significantly different.

TABLE 3
Comparison on a Meteor of Measured Numbers of Exceedances of Different Normal Acceleration Increments and Calculated Numbers with the Same Number at the Lowest Measured Increment

A measured increment is the highest value between crossing of the 1 g datum. The calculations are made for each of 240 time intervals of a quarter of a minute.

| Height ft | $\begin{aligned} & \text { Distance } \\ & \mathrm{km} \end{aligned}$ | Accel. <br> g-incr. | Number of times exceeded |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Calc. A | Calc. B | Measured |
| 300 | 494.2 | 0.00 | 10331.2 | 6213.8 |  |
|  |  | 0.20 | 4576.0 | 4576.0 | 4576 |
|  |  | 0.40 | 1877.6 | 2135.3 | 1941 |
|  |  | 0.60 | $733 \cdot 1$ | 751.5 | 831 |
|  |  | 0.80 | $272 \cdot 1$ | 213.7 | 334 |
|  |  | 1.00 | $96.2$ | $51.7$ | $94$ |
|  |  | $1.20$ | $32.7$ | $11 \cdot 1$ | 19 |
|  |  | 1.40 | 10.8 | 2.2 | 4 |
|  |  | 1.60 | 3.5 | 0.4 | 1 |

TABLE 4

## Comparison on a Viking of Measured Numbers of Exceedances of Different Normal Acceleration Increments and Calculated Numbers with the Same Number at the Lowest Measured Increment

A measured increment is the highest value between crossing of the 1 g datum. The calculations are made for each 4360 time intervals of 1 minute.

| Height <br> ft | $\begin{gathered} \text { Distance } \\ \mathrm{km} \end{gathered}$ | Accel. g-incr. | Number of times exceeded |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Calc. A | Calc. B | Measured |
| 0-1500 | 40.7 | $\begin{aligned} & 0.00 \\ & 0.10 \\ & 0.20 \\ & 0.30 \end{aligned}$ | $\begin{array}{r} 389.3 \\ 56.0 \\ 4.7 \\ 0.3 \end{array}$ | $\begin{array}{r} 158.8 \\ 56.0 \\ 4.4 \\ 0.2 \end{array}$ | $\begin{array}{r} 56 \\ 3 \\ 1 \end{array}$ |
| 1500-5500 | 6318.6 | $\begin{aligned} & 0.00 \\ & 0.10 \\ & 0.20 \\ & 0.30 \\ & 0.40 \\ & 0.50 \\ & 0.60 \\ & 0.70 \end{aligned}$ | $\begin{array}{r} 43112.4 \\ 9450.0 \\ 1794.6 \\ 371.6 \\ 82.6 \\ 20.4 \\ 5.8 \\ 1.9 \end{array}$ | $\begin{array}{r} 19802 \cdot 4 \\ 9450.0 \\ 1892.4 \\ 308.6 \\ 49.1 \\ 9.0 \\ 2.1 \\ 0.6 \end{array}$ | $\begin{array}{r} 9450 \\ 1849 \\ 414 \\ 86 \\ 30 \\ 8 \\ 3 \end{array}$ |
| 5500-9500 | $10086 \cdot 9$ | $\begin{aligned} & 0.00 \\ & 0.10 \\ & 0.20 \\ & 0.30 \\ & 0.40 \\ & 0.50 \\ & 0.60 \end{aligned}$ | $\begin{array}{r} 18375.1 \\ 2675.0 \\ 328.0 \\ 59.0 \\ 10.8 \\ 1.8 \\ 0.3 \end{array}$ | $\begin{array}{r} 7466.3 \\ 2675 \cdot 0 \\ 337.8 \\ 46.5 \\ 4.9 \\ 0.4 \\ 0.0 \end{array}$ | $\begin{array}{r} 2675 \\ 319 \\ 62 \\ 13 \\ 3 \\ 1 \end{array}$ |
| 9 500-13500 | $6163 \cdot 2$ | $\begin{aligned} & 0.00 \\ & 0.10 \\ & 0.20 \\ & 0.30 \\ & 0.40 \\ & 0.50 \\ & 0.60 \\ & 0.70 \end{aligned}$ | $\begin{array}{r} 12807.8 \\ 1794.0 \\ 204.7 \\ 50.2 \\ 18.8 \\ 7.7 \\ 3.1 \\ 1.2 \end{array}$ | $\begin{array}{r} 5164.4 \\ 1794.0 \\ 211.3 \\ 50.2 \\ 16.4 \\ 4.9 \\ 1.3 \\ 0.3 \end{array}$ | $\begin{array}{r} 1794 \\ 213 \\ 48 \\ 17 \\ 7 \\ 3 \\ 1 \end{array}$ |
| All | 22609.4 | $\begin{aligned} & 0.00 \\ & 0.10 \\ & 0.20 \\ & 0.30 \\ & 0.40 \\ & 0.50 \\ & 0.60 \\ & 0.70 \end{aligned}$ | $\begin{array}{r} 74684.6 \\ 13975 \cdot 0 \\ 2331.9 \\ 481 \cdot 1 \\ 112.3 \\ 29.9 \\ 9.1 \\ 3.1 \end{array}$ | $\begin{array}{r} 32591.9 \\ 13975 \cdot 0 \\ 2445.9 \\ 405 \cdot 3 \\ 70.4 \\ 14.3 \\ 3.3 \\ 0.8 \end{array}$ | $\begin{array}{r} 13975 \\ 2384 \\ 525 \\ 116 \\ 40 \\ 12 \\ 4 \end{array}$ |

## TABLE 5

Comparison on a York of Measured Numbers of Exceedances of Different Normal Acceleration Increments and Calculated Numbers with the Same Number at the Lowest Measured Increment

A measured increment is the highest value between crossing of the 1 g datum. The calculations are made for each of 360 time intervals of 1 minute.

| Height ft | $\begin{aligned} & \text { Distance } \\ & \text { km } \end{aligned}$ | Accel. g -incr. | Number of times exceeded |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Calc. A | Calc. B | Measured |
| 0-1500 | 108.1 | 0.00 | 1313.2 | 633.2 |  |
|  |  | 0.10 | 330.0 | $330 \cdot 0$ | 330 |
|  |  | 0.20 | 65.9 | 69.0 | 72 |
|  |  | 0.30 | 11.9 | 9.0 | 12 |
|  |  | 0.40 | 2.0 | 0.8 | 2 |
| 5500-9500 | 1902.5 | 0.00 | 4578.5 | 1880.2 |  |
|  |  | 0.10 | 681.0 | 681.0 | 681 |
|  |  | 0.20 | 70.9 | 70.8 | 67 |
|  |  | 0.30 | $10 \cdot 1$ | 7.5 | 10 |
|  |  | 0.40 | 1.6 | 0.6 | 2 |
| All | 2010.6 |  | $5891.7$ | $2513.4$ |  |
|  |  | $0.10$ | $1011.0$ | $1011 \cdot 0$ |  |
|  |  | $0.20$ | $136 \cdot 8$ | $139.8$ | $139$ |
|  |  | $0.30$ | $22 \cdot 1$ | $16 \cdot 6$ | $22$ |
|  |  | 0.40 | 3.6 | 1.5 | 4 |

## TABLE 6

Comparison on an Anson of Measured Numbers of Exceedances of Different Normal Acceleration Increments and Calculated Numbers with the Same Number at the Lowest Measured Increment for Approximately 50 km Traverses of Turbulence of Moderate Intensity

A measured increment is the highest value between crossing of the 1 g datum. The calculations are made for each traverse taken as one entity.

| Height ft | $\begin{aligned} & \text { Distance } \\ & \text { km } \end{aligned}$ | Accel. g -incr. | Number of times exceeded |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Calc. A | Calc. B | Measured |  |  |
|  |  |  |  |  | Total | -ve | +ve |
| 3000 | 50.4 | 0.00 | 1386.04 | 820.35 |  |  |  |
|  |  | 0.100 | 589.00 | 589.00 | 589 | 289 | 300 |
|  |  | 0.200 | 197.43 | 218.00 | 174 | 92 | 82 |
|  |  | 0.300 | 50.78 | 41.60 | 53 | 30 | 23 |
|  |  | 0.400 | 9.81 | 4.09 | 19 | 10 | 9 |
|  |  | 0.500 | 1.40 | 0.21 | 4 | 2 | 2 |
|  |  | 0.600 | 0.15 | 0.01 | 1 | 0 | 1 |
| 3000 | 50.4 | 0.00 | 1357.30 | 852.22 |  |  |  |
|  |  | 0.100 | 660.00 | 660.00 | 660 | 310 | 350 |
|  |  | 0.200 | $270 \cdot 30$ | 306.56 | 234 | 116 | 118 |
|  |  | 0.300 | 91.54 | 85.41 | 85 | 43 | 42 |
|  |  | 0.400 | 25.26 | 14.27 | 32 | 17 | 15 |
|  |  | 0.500 | 5.62 | 1.43 | 14 | 9 | 5 |
|  |  | 0.600 | 1.00 | 0.09 | 4 | 3 | 1 |
|  |  | 0.700 | 0.14 | 0.00 | 2 | 1 | 1 |
|  |  | 0.800 | 0.02 | 0.00 |  | 1 | 0 |

TABLE 7
Comparison on a Canberra of Measured Numbers of Exceedances of Different Normal Acceleration Increments and Calculated Number with the Same Numbers at the Lowest Measured Increment for Approximately 50 km Traverses of Turbulence at Midday over Flat Desert with a Range of Solar Radiations

A measured increment is the highest value* between crossing of the 1 g datum. The calculations are made for each traverse taken as an entity.

| Height ft | Solar radiation $\mathrm{mW} / \mathrm{cm}^{2}$ | $\begin{gathered} \text { Distance } \\ \text { km } \end{gathered}$ | Accel. g-incr. | Number of times exceeded |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Calc. A | Calc. B | Measured |  |  |
|  |  |  |  |  |  | Total | -ve | + ve |
| 200 | 30-34 | 50.0 | 0.00 | 866.09 | 531.36 |  |  |  |
|  |  |  | 0.200 | 400.00 | 400.00 | 400 | 176 | 224 |
|  |  |  | 0.300 | 252.98 | 280.48 | 248 | 100 | 148 |
|  |  |  | 0.400 | 151.91 | 170.64 | 149 | 54 | 95 |
|  |  |  | 0.600 | 46.42 | 41.25 | 47 | 15 | 32 |
|  |  |  | 0.800 | 11.22 | 5.65 | 14 | 3 | 11 |
|  |  |  | 1.000 | 2.12 | 0.44 | 3 | 0 | 3 |
| 200 | 35-39 | 61.3 | 0.00 | 1416.48 | 734.33 |  |  |  |
|  |  |  | 0.200 | 427.00 | 427.00 | 427 | 152 | 275 |
|  |  |  | 0.300 | 199.35 | 216.82 | 210 | 53 | 157 |
|  |  |  | 0.400 | 82.62 | 83.95 | 85 | 13 | 72 |
|  |  |  | 0.600 | 9.68 | 5.58 | 9 | 0 | 9 |
| 200 | 40-44 | 61.1 | 0.00 | $1240 \cdot 11$ | 649.05 |  |  |  |
|  |  |  | 0.200 | 384.00 | 384.00 | 384 | 166 | 218 |
|  |  |  | 0.300 | 182.89 | 199.25 | 202 | 83 | 119 |
|  |  |  | 0.400 | 77.71 | 79.52 | 83 | 35 | 48 |
|  |  |  | 0.600 | 9.72 | 5.76 | 7 | 4 | 3 |
| 200 | 45-49 | 49.8 | 0.00 | 991.24 | 582.43 |  |  |  |
|  |  |  | 0.200 | 414.00 | 414.00 | 414 | 206 | 208 |
|  |  |  | 0.300 | 244.43 | 270.21 | 245 | 111 | 134 |
|  |  |  | 0.400 | 135.15 | 148.69 | 151 | 69 | 82 |
|  |  |  | 0.600 | 33.51 | 26.98 | 32 | 12 | 20 |
|  |  |  | 0.800 | 6.17 | 2.47 | 6 | 4 | 2 |
| 200 | 50-54 | 49.5 | 0.00 | 1094.72 | 622.55 |  |  |  |
|  |  |  | 0.200 | 422.00 | 422.00 |  |  |  |
|  |  |  | 0.300 | 235.56 | 259.56 | 237 | 89 | 148 |
|  |  |  | 0.400 | 121.69 | 131.44 | 129 | 48 | 81 |
|  |  |  | 0.600 | 25.34 | 18.81 | 25 | 7 | 18 |
|  |  |  | 0.800 | 3.71 | 1.24 | 3 | 1 | 2 |
|  |  |  | 1.000 | 0.38 | 0.04 | 1 | 0 | 1 |

[^1]TABLE 7 (Contd)

| $\begin{gathered} \text { Height } \\ \mathrm{ft} \end{gathered}$ | Solar radiation $\mathrm{mW} / \mathrm{cm}^{2}$ | $\begin{array}{\|c} \text { Distance } \\ \mathrm{km} \end{array}$ | Accel. g -incr. | Number of times exceeded |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Calc. A | Calc. B | Measured |  |  |
|  |  |  |  |  |  | Total | -ve | +ve |
| 200 | 55-59 | 50.3 | 0.00 | 897.57 | 516.33 |  |  |  |
|  |  |  | 0.200 | 356.00 | 356.00 | 356 | 161 | 191 |
|  |  |  | 0.300 | 202.74 | 223.67 | 206 | 90 | 116 |
|  |  |  | 0.400 | 107.32 | 116.68 | 105 | 46 | 59 |
|  |  |  | 0.600 | 23.80 | 18.18 | 26 | 11 | 15 |
|  |  |  | 0.800 | 3.79 | 1.35 | 3 | 0 | 3 |
|  |  |  | 1.000 | 0.42 | 0.05 | 1 | 0 | 1 |
| 200 | 60-64 | 43.5 | 0.00 | 885.01 | 506.72 |  |  |  |
|  |  |  | 0.200 | 347.00 | 347.00 | 347 | 149 | 198 |
|  |  |  | 0.300 | 196.03 | 216.16 | 202 | 82 | 120 |
|  |  |  | 0.400 | 102.75 | 111.43 | 109 | 41 | 68 |
|  |  |  | 0.600 | 22.21 | 16.78 | 19 | 8 | 11 |
|  |  |  | $0.800$ |  | 1.18 | 5 | 2 | 3 |
| 200 | 65-69 | 57.5 | 0.00 | 1320.91 | 783.97 |  |  |  |
|  |  |  | 0.200 | 565.00 | 565.00 | 565 | 269 | 296 |
|  |  |  | 0.300 | 339.12 | 375.17 | 348 | 157 | 191 |
|  |  |  | 0.400 | 191.27 | 211.49 | 226 | 100 | 126 |
|  |  |  | 0.600 | 49.88 | 41.12 | 42 | 14 | 28 |
|  |  |  | 0.800 | 9.81 | 4.15 | 8 | 4 | 4 |
|  |  |  | 1.000 | 1.43 | 0.22 | 1 | 1 | 0 |
| 200 | 70-74 | 54.8 | 0.00 | 1166.72 | 685.95 |  |  |  |
|  |  |  | 0.200 | 488.00 | 488.00 | 488 | 223 | 265 |
|  |  |  | 0.300 | 288.41 | 318.85 | 290 | 115 | 175 |
|  |  |  | 0.400 | 159.67 | 175.71 | 151 | 58 | 93 |
|  |  |  | 0.600 | 39.71 | 32.02 | 37 | 15 | 22 |
|  |  |  | 0.800 | 7.34 | 2.95 | 10 | 4 | 6 |
|  |  |  | 1.000 | 0.99 | 0.14 | 3 | 1 | 2 |
|  |  |  | 1.200 | $0 \cdot 10$ | 0.00 | 1 | 0 | 1 |
| 200 | 75-79 | 54.9 |  |  |  |  |  |  |
|  |  |  | 0.200 | 474.00 | 474.00 | 474 |  |  |
|  |  |  | 0.300 | 285.91 | 316.39 | 291 | 111 | 180 |
|  |  |  | 0.400 | 162.23 | 179.65 | 165 | 45 | 120 |
|  |  |  | 0.600 | 42.95 | 35.66 | 41 | 10 | 31 |
|  |  |  | 0.800 | 8.62 | 3.71 | 10 | 2 | 8 |
|  |  |  | $1-000$ | 1.29 | 0.20 | 2 | 0 | 2 |
| 200 | 80-84 | 61.8 | 0.00 | 1395.48 | 821.81 |  |  |  |
|  |  |  | 0.200 | 586.00 | 586.00 |  | 275 | 311 |
|  |  |  | 0.300 | 347.27 | 383.98 | 362 | 150 | 212 |
|  |  |  | 0.400 | 192.89 | 212.46 | 206 | 72 | 134 |
|  |  |  | 0.600 | 48.38 | 39.17 | 51 | 14 | 37 |
|  |  |  | 0.800 | 9.05 | 3.67 | 5 | 1 | 4 |
|  |  |  | 1.000 | 1.24 | 0.17 | 1 | 0 | 1 |

## TABLE 8

Comparison on a Meteor of Measured Numbers of Exceedances of $\operatorname{Different}$ Normal Acceleration Increments and Calculated Numbers with the Same Number at the Lowest Measured Increment for Approximately 50 km Traverses of Turbulence of Moderate Intensity

A measured increment is the highest value between crossing of the 1 g datum. The calculations are made for the traverse taken as one entity.

| $\begin{gathered} \text { Height } \\ \mathrm{ft} \end{gathered}$ | $\begin{aligned} & \text { Distance } \\ & \text { km } \end{aligned}$ | Accel. g-incr. | Number of times exceeded |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Calc. A | Calc. B | Measured |  |  |
|  |  |  |  |  | Total | -ve | $+\mathrm{ve}$ |
| 300 | 49.4 | 0.00 | 1127.82 | 742.31 |  |  |  |
|  |  | 0.200 | 606.00 | 606.00 | 606 | 304 | 302 |
|  |  | 0.400 | 286.14 | 329.71 | 283 | 131 | 152 |
|  |  | 0.600 | 117.22 | 119.55 | 128 | 52 | 76 |
|  |  | 0.800 | 41.23 | 28.89 | 48 | 15 | 33 |
|  |  | 1.000 | 12.34 | 4.65 | 11 | 1 | 10 |
|  |  | 1.200 | 3.12 | 0.50 | 1 | 0 | 1 |

TABLE 9
Comparison on a Viking of Measured Numbers of Exceedances of Different Normal Acceleration Increments and Calculated Numbers with the Same Number at the Lowest Measured Increment for Approximately $50 \mathbf{k m}$ Traverses of Turbulence of Moderate Intensity

A measured increment is the highest value between crossing of the 1 g datum. The calculations are made for each traverse taken as one entity.

| $\begin{gathered} \text { Height } \\ \mathrm{ft} \end{gathered}$ | $\begin{gathered} \text { Distance } \\ \text { km } \end{gathered}$ | Accel. g -incr. | Number of times exceeded |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Calc. A | Calc. B | Measured |  |  |
|  |  |  |  |  | Total | -ve | + ve |
| 3000 | 48.4 | 0.00 | 916.13 | 526.80 |  |  |  |
|  |  | 0.100 | 363.00 | 363.00 | 363 | 183 | 180 |
|  |  | 0.200 | 109.26 | 118.77 | 108 | 61 | 47 |
|  |  | 0.300 | 24.17 | 18.45 | 28 | 18 | 10 |
|  |  | 0.400 | 3.84 | 1.36 | 5 | 3 | 2 |
| 8000 | 48.8 | 0.00 | 807.65 | 444.29 |  |  |  |
|  |  | 0.100 | 286.00 | 286.00 | 286 | 145 | 141 |
|  |  | 0.200 | 72.08 | 76.29 | 76 | 47 | 29 |
|  |  | 0.300 | 12.39 | 8.43 | 13 | 10 | 3 |
|  |  | 0.400 | 1.41 | 0.39 | 2 | 1 | 1 |

TABLE 10
Comparison on a York of Measured Numbers of Exceedances of Different Normal Acceleration Increments and Calculated Numbers with the Same Number at the Lowest Measured Increment for Approximately 50 km Traverses of Turbulence of Moderate Intensity

A measured increment is the highest value between crossing of the 1 g datum. The calculations are made for each traverse taken as one entity.

| Height <br> ft | $\begin{gathered} \text { Distance } \\ \text { km } \end{gathered}$ | Accel. g-incr. | Number of times exceeded |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Calc. A | Calc. B | Measured |  |  |
|  |  |  |  |  | Total | -ve | + ve |
| 1000 | 44.4 | 0.00 | 540.24 | 286.21 |  |  |  |
|  |  | 0.100 | 173.00 | 173.00 | 173 | 74 | 99 |
|  |  | 0.200 | 37.01 | 38.21 | 37 | 14 | 23 |
|  |  | 0.300 | 5.02 | 3.08 | 6 | 2 | 4 |
|  |  | 0.400 | 0.42 | 0.09 | 1 | 0 | 1 |

TABLE 11
Comparison on an Anson and Viking of Measured Numbers of Exceedances of Different Normal Acceleration Increments and Calculated Numbers with the Same Number at the Lowest Measured Increment for Traverses of Moderately Severe Turbulence

A measured increment is the highest value between crossing of the 1 g datum. The calculations are made for each traverse taken as an entity.

| Aircraft | Height ft | $\begin{gathered} \text { Distance } \\ \mathrm{km} \end{gathered}$ | Accel. g -incr. | Number of times exceeded |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Calc. A | Calc. B | Measured |  |  |
|  |  |  |  |  |  | Total | -ve | +ve |
| Anson | 3000 | 4.1 | 0.00 | 159.49 | 112.48 |  |  |  |
|  |  |  | 0.100 | 98.00 | 98.00 | 98 | 50 | 48 |
|  |  |  | 0.200 | 55.53 | 64.82 | 58 | 29 | 29 |
|  |  |  | 0.300 | 28.82 | 32.55 | 26 | 13 | 13 |
|  |  |  | 0.400 | 13.62 | 12.41 | 15 | 7 | 8 |
|  |  |  | 0.500 | 5.83 | 3.59 | 6 | 4 | 2 |
|  |  |  | 0.600 | 2.25 | 0.79 | 3 | 2 | 1 |
|  |  |  | 0.700 | 0.78 |  | 1 | 0 | 1 |
| Viking | 2000 | 1.5 | 0.00 | 37.99 | 26.58 |  |  |  |
|  |  |  | 0.100 | 23.00 | 23.00 | 23 | 10 | 13 |
|  |  |  | 0.200 | 12.78 | 14.90 | 15 | 6 | 9 |
|  |  |  | 0.300 | 6.47 | 7.23 | 6 | 2 | 4 |
|  |  |  | 0.400 | 2.96 | 2.63 | 3 | 1 | 2 |
|  |  |  | 0.500 | 1.22 | 0.72 | 2 | 1 | 1 |
|  | 8000 | 1.7 |  |  |  |  |  |  |
|  |  |  | 0.100 | 20.00 | 20.00 | 20 | 13 | 7 |
|  |  |  | 0.200 | $10 \cdot 10$ | 11.70 | 8 | 5 | 3 |
|  |  |  | 0.300 | 4.51 | 4.79 | 5 | 3 | 2 |
|  |  |  | 0.400 | 1.77 | 1.37 | 3 | 1 | 2 |
|  |  |  | 0.500 | 0.61 | 0.27 | 1 | 1 | 0 |
|  | 11000 | 5.8 | 0.00 | 89.39 | 64.97 |  |  |  |
|  |  |  | 0.100 | 58.00 | 58.00 | 58 | 30 | 28 |
|  |  |  | 0.200 | 35.29 | 41.26 | 39 | 22 | 17 |
|  |  |  | 0.300 | 20.03 | 23.39 | 21 | 12 | 9 |
|  |  |  | 0.400 | 10.56 | 10.57 | 11 | 5 | 6 |
|  |  |  | 0.500 | 5.15 | 3.80 | 5 | 3 | 2 |
|  |  |  | 0.600 | 2.32 | 1.09 | 2 | 1 | 1 |
|  |  |  | 0.700 | 0.96 | 0.25 | 1 | 1 | 0 |

TABLE 12
Comparison on Three 4-Engined Pure Jet Transports of Measured Numbers of Exceedances of Different Gust Increments and Calculated Numbers with the Same Number at the Lowest Measured Increment During 36 Traverses of Severe Turbulence Encountered in 23000 Hours of Civil Flying

A measured increment is the highest value between crossing of the 1 g datum. The calculations are made for each traverse taken as one entity.


TABLE 12 (Contd.)

| Height | Distance km | Gust incr. $\mathrm{m} / \mathrm{s}$ eas | Number of times exceeded |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Calc. A | Calc. B | Measured |
| $9500-13500$ | 22.4 | 9.00 | 4.9 | 4.6 | 5 |
|  |  | 10.00 | 3.2 | 2.8 | 2 |
|  |  | 11.00 | 2.1 | 1.7 | 2 |
|  |  | 12.00 | 1.4 | 1.1 | 1 |
|  |  | 13.00 | 1.0 | 0.8 | 1 |
|  |  | 14.00 | 0.7 | 0.5 | 1 |
|  |  | 15.00 | 0.5 | 0.4 | 1 |
| 13500-17500 |  | 0.00 | 128.7 | 68.6 |  |
|  |  | 1.00 | 93.7 | 65.3 |  |
|  |  | 2.00 | 65.9 | 56.3 |  |
|  |  | 3.00 | 44.7 | 44.0 |  |
|  |  | 4.00 | 29.2 | 31.2 | 29 |
|  |  | 5.00 | 18.3 | 20.1 | 17 |
|  |  | 6.00 | 11.1 | 11.8 | 8 |
|  |  | 7.00 | 6.5 | 6.4 | 7 |
|  |  | 8.00 | 3.6 | 3.2 | 4 |
|  |  | 9.00 | 2.0 | 1.5 | 3 |
|  |  | 10.00 | 1.1 | 0.7 | 1 |
|  |  | 11.00 | 0.6 | 0.3 | 1 |
| 17500-21500 | 4.8 | 0.00 | 24.5 | 15.4 |  |
|  |  | 1.00 | 20.1 | 15.1 |  |
|  |  | 2.00 | 16.2 | 14.1 |  |
|  |  | 3.00 | 12.9 | 12.5 |  |
|  |  | 4.00 | 10.1 | 10.7 | 8 |
|  |  | 5.00 | 7.8 | 8.7 | 4 |
|  |  | 6.00 | 5.9 | 6.7 | 4 |
|  |  | 7.00 | 4.4 | 5.0 | 4 |
|  |  | 8.00 | 3.3 | 3.6 | 4 |
|  |  | 9.00 | 2.4 | 2.4 | 4 |
|  |  | 10.00 | 1.7 | 1.6 | 3 |
|  |  | 11.00 | 1.2 | 1.0 | 3 |
|  |  | 12.00 | 0.8 | 0.6 | 2 |
| $21500-29500$ | 47.1 | 0.00 | 310.7 | 166.5 |  |
|  |  | 1.00 | 206.1 | 153.7 |  |
|  |  | 2.00 | 129.8 | $121 \cdot 1$ |  |
|  |  | 3.00 | 77.9 | 82.3 |  |
|  |  | 4.00 | 44.7 | 49.1 | 40 |
|  |  | 5.00 | 24.9 | 26.5 | 21 |
|  |  | 6.00 | 13.6 | 13.5 | 13 |
|  |  | 7.00 | 7.4 | 6.7 | 9 |
|  |  | 8.00 | 4.1 | 3.2 | 6 |
|  |  | 9.00 | $2 \cdot 3$ | 1.5 | 2 |
|  |  | 10.00 | 1.3 | 0.7 | 1 |
|  |  | 11.00 | 0.7 | 0.3 | 1 |

TABLE 12 (Contd.)

| Height ft | $\begin{aligned} & \text { Distance } \\ & \text { km } \end{aligned}$ | Gust incr. $\mathrm{m} / \mathrm{s}$ eas | Number of times exceeded |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Calc. A | Calc. B | Measured |
| $29500-37500$ | 161.8 | 0.00 | 585.7 | 322.1 |  |
|  |  | 1.00 | 421.1 | 305.1 |  |
|  |  | 2.00 | 292.8 | 260.0 |  |
|  |  | 3.00 | 197.2 | 200.2 |  |
|  |  | 4.00 | 129.0 | 140.6 | 130 |
|  |  | 5.00 | 82.3 | 91.1 | 87 |
|  |  | 6.00 | 51.5 | 55.3 | 51 |
|  |  | 7.00 | 31.8 | 32.0 | 30 |
|  |  | 8.00 | 19.5 | 17.8 | 20 |
|  |  | 9.00 | 11.8 | 9.6 | 14 |
|  |  | 10.00 | 7.1 | 5.0 | 7 |
|  |  | 11.00 | 4.3 | 2.5 | 4 |
|  |  | 12.00 | 2.5 | 1.2 | 3 |
| All | 331.6 |  | 1531.7 | 844.5 |  |
|  |  | 1.00 | 1095.5 | 797.9 |  |
|  |  | 2.00 | 760.3 | 675.7 |  |
|  |  | 3.00 | 514.2 | 518.9 |  |
|  |  | 4.00 | 340.6 | 367.4 | 333 |
|  |  | 5.00 | 222.2 | 244.1 | 210 |
|  |  | 6.00 | 143.4 | 154.8 | 138 |
|  |  | 7.00 | 92.0 | 94.8 | 88 |
|  |  | 8.00 | 58.8 | 56.5 | 62 |
|  |  | 9.00 | 37.4 | 32.9 | 46 |
|  |  | 10.00 | 23.8 | 18.9 | 27 |
|  |  | 11.00 | 15.1 | 10.7 | 19 |
|  |  | 12.00 | 9.6 | 6.0 | 11 |
|  |  | 13.00 | 6.1 | 3.4 | 5 |
|  |  | 14.00 | 3.9 | 1.9 | 5 |
|  |  | 15.00 | 2.5 | $1 \cdot 1$ | 3 |

TABLE 13
Probabilities of Larger Gusts Being Encountered than those Recorded in Traverses of Approximately $50 \mathbf{k m}$ of Turbulence of Moderate Intensity

| Aircraft | Height <br> ft | Solar radiation <br> $\mathrm{mW} / \mathrm{cm}^{2}$ | Probability based on a Rayleigh distribution of |  |
| :--- | :---: | :---: | :---: | :---: |
|  |  | A-the gust vector | B-the vertical component |  |
| Anson | 3000 |  | 0.069 | 0.002 |
| Canberra | 3000 |  | 0.008 | 0.000 |
|  | 200 | $30-34$ | 0.504 | 0.071 |
|  | 200 | $35-39$ | 0.730 | 0.287 |
|  | 200 | $40-44$ | 0.855 | 0.454 |
|  | 200 | $45-49$ | 0.827 | 0.307 |
|  | 200 | $50-54$ | 0.164 | 0.012 |
|  | 200 | $55-59$ | 0.189 | 0.015 |
|  | 200 | $60-64$ | 0.616 | 0.159 |
|  | 200 | $65-69$ | 0.538 | 0.078 |
|  | 200 | $70-74$ | 0.045 | 0.001 |
|  | 200 | $75-79$ | 0.360 | 0.036 |
| Meteor | 200 | $80-84$ | 0.480 | 0.061 |
| Viking | 300 |  | 0.868 | 0.214 |
|  | 3000 |  | 0.666 | 0.185 |
| York | 8000 |  | 0.497 | 0.124 |

TABLE 14
Probabilities of Larger Gusts Being Encountered than Those Recorded in Traverses of Turbulence of Moderately Severe Intensity

|  |  | Length of <br> Height <br> Araverse <br> ft | km | Probability based on a Rayleigh distribution of |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  |  |  | B-the vertical component |  |  |
| Anson | 3000 | 4.1 | 0.430 | 0.071 |  |
| Viking | 2000 | 1.5 | 0.610 | 0.373 |  |
|  | 8000 | 1.7 | 0.354 | 0.149 |  |
|  | 11000 | 5.8 | 0.520 | 0.144 |  |

TABLE 15
Probabilities of Larger Gusts Being Encountered than Those Recorded in Traverses of Turbulence of Severe Intensity During 23000 Hours of Civil Flying

| Aircraft | $\begin{gathered} \text { Height } \\ \mathrm{ft} \end{gathered}$ | Length of traverse km | Probability based on a Rayleigh distribution of |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | A-the gust vector | B-the vertical component |
| Type I | 33800 | 3.2 | 0.323 | 0.288 |
|  | 33000 | 3.3 | 0.214 | 0.118 |
|  | 6000 | $2 \cdot 1$ | 0.362 | 0.249 |
|  | 18800 | 2.2 | 0.302 | 0.189 |
|  | 19100 | 2.6 | 0.245 | 0.193 |
|  | 9500 | 4.6 | 0.128 | 0.050 |
|  | 15500 | 19.7 | 0.233 | 0.089 |
|  | 8300 | 6.7 | 0.392 | 0.269 |
|  | 11000 | 2.3 | 0.282 | 0.231 |
|  | 31000 | 3.2 | 0.430 | 0.369 |
|  | 32300 | 29.3 | 0.020 | 0.004 |
|  | 6000 | 3.7 | 0.182 | 0.086 |
|  | 29000 | 3.1 | 0.396 | 0.284 |
| Type III (later version of Type I) | 22200 | 8.5 | 0.377 | 0.182 |
|  | 12000 | 5.5 | 0.060 | 0.012 |
|  | 13500 | 9.8 | 0.561 | 0.347 |
|  | 9000 | 2.4 | 0.068 | 0.014 |
|  | 27200 | 35.5 | 0.109 | 0.023 |
|  | 9000 | 9.1 | 0.612 | 0.434 |
|  | 10300 | 2.5 | 0.317 | 0.368 |
|  | 8600 | 2.5 | 0.116 | 0.058 |
|  | 32000 | 12.7 | 0.404 | 0.158 |
| Type II | 7500 | 7.1 | 0.301 | 0.221 |
|  | 1800 | 4.1 | 0.161 | 0.095 |
|  | 34900 | 7.4 | 0.308 | 0.163 |
|  | 32600 | 10.3 | 0.223 | 0.127 |
|  | 8000 | 14.1 | 0.649 | 0.553 |
|  | 30800 | 3.5 | 0.236 | 0.163 |
|  | 36000 | 3.7 | 0.536 | 0.455 |
|  | 9100 | 5.2 | 0.275 | 0.185 |
|  | 35000 | 3.9 | 0.341 | 0.268 |
|  | 35700 | 11.2 | 0.153 | 0.104 |
|  | 16100 | 2.7 | 0.348 | 0.302 |
|  | 33800 | 3.5 | 0.259 | 0.142 |
|  | 34000 | $66 \cdot 5$ | 0.425 | 0.186 |
|  | 5600 | 13.8 | 0.015 | 0.005 |

TABLE 16
Comparison of Calculated Number of Gusts in Continuous Turbulence with Estimates from Approximately 50 km Traverses of Turbulence of Moderate Intensity, Based on the Assumption that there is a Rayleigh Distribution of the Magnitudes of the Maxima of the Gust Velocity Vector

| Aircraft | Height$\mathrm{ft}$ | Solar radiation $\mathrm{mW} / \mathrm{cm}^{2}$ | Number of sub-divisions | Calculated number of gusts per km in cont. turb. | Estimates from the measurements of the number of gusts encountered in continuous turbulence |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Individual sub-divisions percentage of calculated number |  |  |  |  |  |  |  | Mean of values in sub-divisions percentage of calculated number | Whole traverse taken together percentage of calculated number |
| Anson | 3000 |  | 12 | 31.07 | 56.1 | $83.0$ | $109.7$ | $96.5$ | 154.4 | 119.3 | 97.0 | 117.4 | 93.5 | 88.6 |
|  | 3000 |  | 12 | 31.07 | 44.1 | 107.1 | 73.3 | 86.6 | 133.5 | 86.1 | 78.4 | 115.5 | 94.2 | 86.7 |
|  |  | , |  |  | 93.6 | 128.8 | 93.4 | 90.5 |  |  |  |  |  |  |
| Canberra | 200 | 30-34 | 7 | 24.67 | 73.3 | 59.8 | 75.0 | 92.5 | 53.2 | 85.0 | 70.6 |  | 72.8 | 70.3 |
|  |  | 35-39 | 7 | 24.17 | 97.3 | 104.9 | 111.4 | 99.1 | 107.3 | 79.6 | 80.5 |  | 97.3 | 95.6 |
|  |  | 40-44 | 7 | 24.15 | 92.3 | 74.7 | 89.2 | 74.8 | 113.4 | 78.2 | 66.3 |  | 84.1 | 84.1 |
|  |  | 45-49 | 7 | 24.62 | 102.2 | 70.0 | 92.4 | 69.5 | 76.0 | 77.1 | 90.7 |  | 82.6 | 80.9 |
|  |  | 50-54 | 7 | 24.13 | 91.4 | 119.0 | 111.5 | 81.1 | 97.8 | 62.7 | 84.2 |  | 92.5 | 89.8 |
|  |  | 55-59 | 7 | 24.13 | 72.2 | 81.1 | 59.8 | 70.0 | 74.7 | 74.0 | 109.5 |  | 77.3 | 74.0 |
|  |  | 60-64 | 7 | 24.41 | 81.9 | 92.7 | 93.8 | 85.0 | 78.1 | $60 \cdot 6$ | 108.7 |  | 85.8 | 83.4 |
|  |  | 65-69 | 7 | 24.02 | 168.2 | 68.1 | 101.0 | $62 \cdot 1$ | 103.7 | 69.3 | 124.8 |  | 99.6 | 95.6 |
|  |  | 70-74 | 7 | 25.03 | 78.4 | 74.4 | 96.9 | 90.6 | 90.6 | 110.9 | 101.6 |  | 91.9 | 85.1 |
|  |  | 75-79 | 7 | 22.69 | 86.8 | 116.6 | 96.3 | 84.0 | 92.2 | 74.9 | 83.7 |  | 90.7 | 88.3 |
|  |  | 80-84 | 7 | 24.72 | 123.8 | 97.3 | 96.8 | 77.1 | 96.3 | 84.8 | 75.1 |  | 93.0 | 91.0 |
| Meteor | 300 |  | 24 | 26.66 | 74.0 | 84.2 | 48.1 | 117.0 | 111.2 | 65.2 | 81.7 | 100.8 | 88.7 | $85 \cdot 6$ |
|  |  |  |  |  | 62.3 | 88.5 | 136.2 | 70.9 | 31.7 | 75.4 | 57.4 | 47.9 |  |  |
|  |  |  |  |  | 78.5 | 123.7 | 115.8 | 95.8 | 86.7 | 77.3 | 151.3 | 147.1 |  |  |
| Viking | 3000 |  | 10 | 20.80 | 90.1 | 100.5 | 98.5 | 86.7 | 98.8 | 96.6 | 98.6 | 86.9 | 96.7 | 92.2 |
|  |  |  |  |  | 119.6 | 91.6 |  |  |  |  |  |  |  |  |
|  | 8000 |  | 10 | 19.21 | $83.3$ | $59.2$ | 115.5 | 60.8 | 93.7 | 88.5 | 79.4 | 81.8 | 90.8 | 86.2 |
|  |  |  |  |  | $110.9$ | $135.4$ |  |  |  |  |  |  |  |  |
| York | 1000 |  | 9 | 17.43 | 98.7 | 54.5 | 77.0 | 83.0 | 92.8 | 126.6 | 81.2 | 63.5 | 83.8 | 69.7 |
|  |  |  |  |  | 77.0 |  |  |  |  |  |  |  |  |  |

TABLE 17
Calculated Values of $\int_{0}^{1} \exp \left(-U^{2} / 2 \sigma^{2} x^{2}\right) d x$

| $U / \sigma$ | $\int_{0}^{1} \exp \left(-U^{2} / 2 \sigma^{2} x^{2}\right) d x$ | $U / \sigma$ | $\int_{0}^{1} \exp \left(-U^{2} / 2 \sigma^{2} x^{2}\right) d x$ |
| :---: | :---: | :---: | :---: |
| 0.1 | $8.79664,-1$ | 2.6 | $3.66940,-3$ |
| 0.2 | $7.69271,-1$ | 2.7 | $2.65729,-3$ |
| 0.3 | $6.68671,-1$ | 2.8 | $1.90776,-3$ |
| 0.4 | $5.77625,-1$ | 2.9 | $1.35771,-3$ |
| 0.5 | $4.95802,-1$ | 3.0 | $9.57919,-4$ |
| 0.6 | $4.22800,-1$ | 3.1 | $6.69894,-4$ |
| 0.7 | $3.58145,-1$ | 3.2 | $4.64345,-4$ |
| 0.8 | $3.01315,-1$ | 3.3 | $3.19017,-4$ |
| 0.9 | $2.51744,-1$ | 3.4 | $2.17224,-4$ |
| 1.0 | $2.08841,-1$ | 3.5 | $1.46590,-4$ |
| 1.1 | $1.72004,-1$ | 3.6 | $9.80367,-5$ |
| 1.2 | $1.40628,-1$ | 3.7 | $6.49749,-5$ |
| 1.3 | $1.14122,-1$ | 3.8 | $4.26737,-5$ |
| 1.4 | $9.19134,-2$ | 3.9 | $2.77728,-5$ |
| 1.5 | $7.34612,-2$ | 4.0 | $1.79105,-5$ |
| 1.6 | $5.82590,-2$ | 4.1 | $1.14450,-5$ |
| 1.7 | $4.58407,-2$ | 4.2 | $7.24647,-6$ |
| 1.8 | $3.57836,-2$ | 4.3 | $4.54603,-6$ |
| 1.9 | $2.77091,-2$ | 4.4 | $2.82567,-6$ |
| 2.0 | $2.12830,-2$ | 4.5 | $1.74013,-6$ |
| 2.1 | $1.62137,-2$ | 4.6 | $1.06171,-6$ |
| 2.2 | $1.22499,-2$ | 4.7 | $6.41772,-7$ |
| 2.3 | $9.17823,-3$ | 4.8 | $3.84326,-7$ |
| 2.4 | $6.81914,-3$ | 4.9 | $2.28010,-7$ |
| 2.5 | $5.02363,-3$ | 5.0 | $1.34008,-7$ |



Fig 1. Relative frequency of occurrence of gusts with Rayleigh distribution of maxima of gust velocity vector.

Calculated values on following assumptions of a Rayleigh distribution
--- Maxima of vertical component of gust velocity
-_ Maxima of gust velocity vector - Observed values


Fig 2. Number of gusts encountered in one minute of typical flying turbulence.


Fig. 3. Anson, number of gusts encountered in 620 minutes flying.


Fig. 4. Canberra, number of gusts encountered in 6133 periods of flying, each period being about 0.86 minutes.


Fig. 5. Meteor, number of gusts encountered in 240 periods of flying, each period being $\frac{1}{4}$ minute.


Fig. 6. Viking, number of gusts encountered in 4360 minutes flying.


Fig. 7. York, number of gusts encountered in 360 minutes flying.


Fig. 8a. Anson, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 8b. Anson, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 9a. Canberra, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 9b. Canberra, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 9c. Canberra, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig 9d. Canberra, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 9e. Canberra, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 9f. Canberra, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 9g. Canberra, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 9h. Canberra, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 9i. Canberra, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 9j. Canberra, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 9k. Canberra, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 10. Meteor, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 11a. Viking, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 11b. Viking, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.

Fig. 12. York, number of gusts encountered in approximately 50 km flying through individual areas of turbulence.


Fig. 13a. Anson, number of gusts encountered in individual areas of moderately severe turbulence.


Fig. 13b. Viking, number of gusts encountered in individual areas of moderately severe turbulence.


FIG. 13c. Viking, number of gusts encountered in individual areas of moderately severe turbulence.


Fig. 13d. Viking, number of gusts encountered in individual areas of moderately severe turbulence.


Fig. 14. 36 traverses of the most severe turbulence in 23000 hours of civil flying by 34 -engined pure jet transports.

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[^0]:    * Replaces RAE Technical Report 71169—A.R.C. 33503.

[^1]:    * The actual measurements were not precisely the highest value between crossings but the numerical values given should not be significantly different.

