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By

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A Flight Investigation of the Wake Behind a Meteor Aircraft, with some Theoretical Analysis

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SUMMARY

An experimental investigation of the wake behind a Meteor 4 aircraft has been carried out, and theory used wherever possible to confirm and extend the results obtained.

Measurements show that for the conditions covered in these tests, the jet velocity has fallen to a negligible value by about 200-300 ft behind the jet exit. The jet would be expected to persist Longest under take-off conditions although no measurements were made of this specific case.

The major disturbances behind on aircraft are due to the trailing vortices and these decay only slowly. Tests with a Vampire flying in the wake of the Meteor show that the strength of these vortices has only fallen to about half its initial value by 8000 ft behind the aircraft.

Theory and flight test experience show that the rolling moment imposed on a tracking aircraft constitutes the most severe disturbance from those vortices, and that in some circumstances this rolling moment can be sufficiently large to overpower the aileron control of the tracking aircraft,

Theory indicates that the disturbances from the trailing vortices will be very severe for a small slow aircraft flying in the make of a large heavily loaded aircraft at low speed. The disturbances encountered by a missile attacking an aircraft will depend largely upon the relative sizes of the missile and aircraft, but in many cases may be small, as the closing speeds are usually high and the missile wing span small.

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1 Introduction

Some brief teats were recently carried out by Armament *Department* to determine the position of the wake of a target aircraft and its general effect upon a tracking aircraft. These teats showed that the disturbances in the wake were severe even at extreme range and that a stern attack would only be practicable if the wake was avoided. To enable the guns of an attacking aircraft to be pre-set to avoid the wake of the target, measurements of the angle of the wake to the flight path of the target aircraft were made. Although the information obtained in these tests was thought sufficient from the point of view of an air-to-air combat, it was not considered sufficient for guided weapon purposes, since in many instances it would not be possible to arrange the flight path of a guided missile to avoid the wake of the target aircraft. Some quantitative idea of the magnitude of the disturbances likely to be encountered by such a missile was therefore required.

At the opposite end of the speed range difficulty was being experienced, and even accidents caused, by light aircraft flying into the wakes of larger aircraft under approach conditions at busy civil airports. Information on the nature of the wake and the factors affecting its decay were required to enoble existing airfield control procedure to be modified to ensure the safety of the small aircraft.

The series of teats and the theoretical work described in this note were carried out with the above objects in view.

It is convenient to consider the wake behind a jet aircraft as having the following components:

1. The jet (or jets). This consists of a small diameter region having a large longitudinal velocity combined with perhaps a relatively small rotation due to swirl. Considerable small scale turbulence would be expected to be present.

2. The body and other wakes. These are characterised by regions of turbulence and reduced total head behind drag producing components. In most cases, the largest region would be behind the fuselage.

3. The vortex sheet and trailing vortices. The vortex sheet shed from the wing rapidly rolls up into a pair of trailing vortices. These trailing vortices may be *represented* by a double rotational motion of the classical form. Small scale turbulence is confined to the cores of these vortices.

The teats were planned in such a way as to investigate each of the above components separately, an attempt being made to measure the magnitude of these components as well as their rate of decay with distance behind the aircraft. Since the difficulties involved in teats of this nature were considerable, both the successful as well as the unsuccessful techniques have been detailed rather more fully then is perhaps usual so as to guide any further investigations of this nature.

A theoretical approach has been used to estimate the disturbances to a tracking aircraft or missile flying in the wake and to predict the way in which these disturbances would very with speed, altitude, and size of the target and tracking aircraft, The estimates have been checked, wherever possible, against the observed flight teat results.

2 Visualisation of the Wake

Early in the tests it was found that consistent results could only be obtained when the pilot of the tracking aircraft could "see" the wake. Extensive use was therefore made of the Meteor 4 aircraft used in the tests of Reference 1. This aircraft was equipped with means for injecting a low viscosity oil into the jet pipe to produce a dense white smoke. The oil was carried in underwing tanks which were pressurised with nitrogen to provide the necessary pressure feeding. Each tank fed only to the engine on its own side, but the pilot could select either port, starboard, or both engines. The smoke, as well as defining the boundaries of the jet, also showed the positions of the trailing vortices, since when the vortex sheet from the wing rolled up, the smoke was wrapped round these vortices. This system was not entirely satisfactory as care had to be taken that the tracking aircraft did not become completely immersed in the smoke, otherwise its windscreen became covered in unburnt oil. a fair mount of practice was necessary to perfect a technique of switching the smoke on and off at the right times so that the tracking aircraft never became completely immersed in the smoke. The total duration of the smoke was only about 30-40 seconds, even though a considerable amount of oil was carried, and this short duration proved a severe limitation.

At high altitudes use was made of condensation trails to visualise the trailing vortices. This was found preferable to using the oil smoke as it eliminated the difficulties mentioned above.

It is felt that if further tests of this nature are to be carried out, a smoke trail obtained by injecting cressylic acid into the Jet pipe would probably give a considerable improvement. The smoke trail produced by this system is not unduly dense, persists far a considerable time and can, of course, be switched on and off at will. Further, sufficient acid can be carried to produce a long duration of smoke.

Smoke canisters positioned at suitable points on the aircraft were found useful for investigating the rolling up of the vortex sheet behind the wing. However, as the type of canisters used would not ignite at altitude their use was limited to flights at altitudes less than about 5,000 ft. At these low altitudes, care had to be taken to choose calm conditions so that the tendency for atmospheric turbulence to break up the smoke pattern was reduced.

There is little doubt that the success of a series of tests to investigate the wake behind an aircraft depends almost fundamentally upon an adequate means of visualising the wake.

3 Characteristics of the Jet

3.1 Determination of the Rate of Decay of Jet Velocity

3.1.1 "Pitot Traverse" Method

The first technique used to measure the velocity in the jet was surprisingly successful. A two-seat Meteor 7 was flown in formation with the target Meteor and to its starboard side. The snoke(para.2) was then switched on and the tracking Meteor 7 edged in until its wing tip pitot-static head was immersed in the centre of the smoke. The observer in the Meteor 7 then read the increase in ASI reading. The distance between the two aircraft was obtained with a fixed ring gun sight. The closest distance that the tracking aircraft was flown to the target aircraft was dictated by the strength of the tip vortex effects and the controllability of the Meteor at the particular flight speed. Thus, although at 250 knots a traverse could be made only a few feet behind the fin, it was not deemed safe to fly closer than about 40 ft from the tail of the target at other speeds.

The results obtained in tests at 10,000 ft altitude and at speeds of 178, 250 and 346 knots are shown in figures 1 (a), (b) and (c), together with estimated values for the mean velocity at the jet exit as derived from Derwent 5 brochure data.

The results are plotted as the difference, ΔV_i, between the EAS in the jet stream and the flight EAS (V_i). It is seen that the jet velocity falls to a low value very rapidly and that for the particular flight conditions of these tests, the jet velocity can be considered negligible at distances greater than about 200-300 ft behind the aircraft. The jet velocity persists longest at high R.P.M. and low speed (Fig.1(a)). It may be noted that in deriving jet velocity from the ASI reading it is assumed that the static pressure in the jet is atmospheric. This is almost certainly true in all cases considered here as the jet exit is not choked.

When the engine is at low R.P.M. or idling there is a drop in ASI reading in the jet signifying a loss of total head.

This "pitot traverse" method demanded appreciable flying skill, since the disturbances due to the trailing vortices were so severe that the pilot could only hold the pitot-static head in the jet stream for a short space of time. This was particularly true at the very close ranges, where often the jet velocity has to be obtained from a momentary jump in ASI reading as the pitot-static head traversed through the jet stream. Where this jump in ASI reading was large, it was felt that errors due to lag in the ASI system could be appreciable. Further there was no guarantee that the velocity at the centre of the jet (i.e. the peak velocity) was being measured every time. These uncertainties mainly applied to the closer ranges and it is felt that the values of jet velocity obtained at the larger ranges are reasonably accurate.

3.1.2 Photographic Method

In an attempt to confirm the results obtained above, another technique was used.

A Meteor 7 was flown in formation with the target aircraft but a few thousand feet directly below it. The observer in the rear seat then took a ciné record of the smoke as it emerged from the jet pipes of the target aircraft. A specimen record obtained in this way is shown in Figure 2. The camera used was a caméflex Eclair camera set for its maximum speed of about 50 pictures per sec. and equipped with a 35 mm focal length lens. This camera was found convenient in that the sight could be rotated through 90° to enable the camera to be sighted with the lens pointing vertically upwards. To assist the operator in keeping the camera level, a spirit level bubble was attached to the side of the camera. The camera speed was obtained by photographing a high speed watch (3 secs per rev.) in flight immediately after the test.

From the ciné record a plot of the distance of the end of the smoke from the jet exit against time can be obtained, the linear scale being obtained by measuring the size of the aircraft on the film and the time scale from the measured camera speed. Figure 3 shows the plot obtained from the photographs of Figure 2, which were taken at 30,000 ft and at 255 knots EAS. The slope of the curve at any point clearly represents the true velocity of the jet at that point plus the true airspeed of the aircraft. For the example of Figure 3, it appears that this total velocity has fallen to a stabilised value of 418 knots by a distance of about 150 ft behind the jet exit. The true air speed of the aircraft, as derived from the IAS and ambient temperature is 426 knots. Attributing the 2 per cent difference between these two speeds to experimental errors it follows that the velocity of the Jet has fallen to zero by about 150 ft behind the Jet exit. At 10,000 ft altitude the corresponding distance at the same r.p.m. and speed is about 250 ft (traverse method Fig.1(b)). This difference between the results obtained at the two altitudes might be expected, for, at a given r.p.m. and speed, the E.A.S. at the Jet exit decreases with increasing altitude. Hence the Jet would be expected to persist longest at low altitude.

The record shown at Figure 3 is one of two taken at 30,000 ft to illustrate that the rapid decay of jet velocity shown by the results of the traverse method (para.3.1.1.) was also true at high altitude. A series of records, of which Figures 4(a) and 4(b) are typical, were also taken at about 10,000 ft altitude to give a direct comparison with the curves of Figure 1.

Figure 4(a) suggests that at 178 knots at 13,000 ft altitude and at 12,200 r.p.m., the Jet velocity has fallen to zero by about 500 ft behind the jet exit. The results obtained from the traverse method (Fig.1(a)) give a value of about 300-350 ft. Thus the two different techniques produce an answer of the same order, if not exactly of the same magnitude. However, exact agreement could hardly be expected, as the almost asymptotic decay of jet velocity makes it most difficult to determine the exact point where it has fallen to zero.

Figure 4(b), is a record taken with the starboard engine idling and gives a good idea of the possible errors in the method. The steady flight speed was recorded as 221 knots, while the stabilised speed determined from the port engine smoke trail is 213 knots and from the starboard engine 207 knots. Although these three figures agree to only about 7% accuracy, there is still little doubt of the order of the distance downstream for the Jet velocity to decay to a small value.

As the above particular examples show, the results of this method confirm conclusively the results of the traverse method in showing that the Jet velocity decays to a small value within several hundred feet of the target aircraft. This photographic technique is, however, not accurate enough to be used for a general investigation of the effects of speed, altitude and engine r.p.m. on the decay of the Jet.

3.2 Turbulence in the Jet

The turbulence in the jet could be felt as a high frequency buffet on the tracking aircraft. This turbulence seemed to die away in much the same manner as the jet velocity and was not apparent at large range.

A vibrograph was installed in a Vampire aircraft to measure the magnitude and rate of decay of this turbulence (at least, as far as its effect on the Vampire was concerned). Some trial records were taken to prove the method, but no systematic tests were made owing to lack of time.

3.3 Summary of conclusions from the Jet Investigation

As two widely different techniques of measuring the velocity in the Jet had given practically the same result, and as the real interest lay in the characteristics of the wake at further distances downstream than where the jet had been shown to exist in any serious form, no further tests were carried out.

Before proceeding further, it is convenient to summarise the main results, together with the Inferences that can be drawn from them:

1. For the speed range 178-350 knots EAS at 10,000 ft and for engine speeds up to 13,500 r.p.m., the jet velocity has fallen to a negligibly low value by about 200-250 ft behind the jet exit.

2. It seem probable from the results that the distance for which the jet persists is mainly dependent upon the difference between the speed of the jet at the jet exit and the speed of the aircraft, the larger this difference the longer the jet persists.

3. For a given r.p.m. and flight EAS, this excess speed will be greatest at low altitude. Hence the conditions favouring a long persistence of the jet velocity might be expected to be low altitude, high engine power, and low aircraft speed: e.g. take-off conditions.

4 Extent of Rake behind Fuselage and Cther Drag Producing Bodies

The wake behind the fuselage and other drag producing bodies is defined by a region of reduced total head and of small scale turbulence.

Some brief tests using the traverse technique (para.3.1.1.) indicated that the loss in total head in the wake of the fuselage was relatively small. For example, at 250 knots at 10,000 ft altitude, the drop in ASI reading at about 40 ft behind the fuselage was only about 5-10 knots. No systematic tests were made.

5 The Vortex Sheet and Trailing Vortioes

The investigation of paras.3 and 4 nave shown that the disturbances at large distances behind a turbo-Jet aircraft can only be due to the trailing vortex system. Indeed, it is probable that at all distances behind the target, the trailing vortices predominate in their effect on a tracking aircraft and this might be expected to be equally true for a propeller driven aircraft.

In the following paragraphs, the theoretical characteristics of the trailing vortex sheet and vortices will be discussed and their effects on a tracking aircraft estimated. Flight measurements of the disturbances in the wake and of the decay of these disturbances with distance behind the target aircraft will then be presented and compared with theory.

The characteristics and basic theory of the vortex sheet and trailing vortices are given in many text-books, and reference should be made to these standard works for more detailed information than is given in the following outline.

5.1 "Rolling-up" of the Vortex Sheet

The vortex sheet shed from the wing, being unstable, rapidly rolls up into two finite vortices commonly referred to as the trailing vortices. Theoretical and experimental investigations of this rolling-up process are detailed in Reference 2.

The point where the vortices are fully rolled-up is herd to specify precisely as the vortices approach the fully rolled-up condition asymptotically as the distance from the wing approaches infinity. However it can be shown[2] that the distance, e, for the vortex sheet to become essentially rolled-up is of the form

$$e = K.\ \frac{A}{C_L}\ b \qquad (1)$$

where A = aspect ratio of wing
b = wing span
K = some constant

To illustrate the order of magnitude of this quantity for a Meteor aircraft, it will be assumed that the Meteor has elliptic span loading and hence that a value for K of 0.28 can be used. (Reference 2). Substituting in equation (1) we obtain Figure 5, which shows that the vortex sheet becomes essentially rolled up in quite a short distance behind the aircraft. Hence as far as a tracking aircraft is concerned, little error will be involved by assuming that the disturbances at all practicable ranges are due entirely to the fully rolled-up vortices.

Experimental checks of the distance for the vortex sheet to roll up and of the distance apart of the fully rolled-up vortices can be obtained by using suitably positioned smoke canisters. No tests were carried out with this specifically in mind but Figure 7(b) is typical of the results that can be obtained. The canisters were in this case positioned one at the wing tip to mark the centre of the vortex and the other midway between the nacelle and fuselage (Fig.7(a)). The photo at 160 knots shows clearly how the centre of the vortex moves inboard to its stabilised position, the smoke from the inner canister meanwhile wrapping itself around this centre. The diffusion of the smoke from the inboard canister is very rapid. This is perhaps due to the smoke becoming mixed with the jet or with the turbulent air in the core of the vortex. In any event, the smoke seems to become eventually wrapped in a cylinder about the centre of the vortex instead of remaining as a single filament. Unless particularly calm conditions are chosen, atmospheric turbulence rapidly breaks up the smoke pattern (para.2). This can be seen happening towards the bottom of the photograph at 260 knots, although conditions at the time when this photo was taken were admittedly very turbulent.

5.2 Inclination to Flight Path of Vortex Sheet and Trailing Vortices

To show the position of the wake behind the target aircraft and how this varies with flight speed, the inclination to the flight path of the vortex sheet and of the fully rolled-up vortices will be calculated.

Assuming elliptic span loading, the downward velocity of the vortex sheet immediately behind the wing is[3]

$$\omega_0 = \frac{\Gamma_0}{b}$$

where Γ_0 = circulation at mid span.

b = span of aircraft

and ω_0 = downward velocity

For level flight

$$W = \rho . \frac{\pi b}{4} . \Gamma_0 U \qquad (2)$$

where W = weight of aircraft

U = true air speed of aircraft

Hence the downward inclination of the vortex sheet immediately behind the wing is

$$\frac{\omega_o}{U} = \frac{188\ W}{b^2\ V_i^2} \quad \text{radians} \tag{3}$$

where V_i = EAS of aircraft (Knots)

The vortex sheet rapidly rolls up into two finite vortices (para. 5.1) and once these are formed they impart to each other a downward velocity of

$$\omega = \frac{\Gamma_o}{2\pi b'} \tag{4}$$

where b' = distance apart of fully rolled-up vortices.

If it is assumed that $b' = \pi/4\ b$ (elliptic loading) then (4) becomes after rearrangement

$$\frac{\omega}{U} = \frac{38\ W}{b^2\ V_i^2} \quad \text{radians} \tag{5}$$

The inclinations given by (3) and (5) for a Meteor aircraft are shown in Figure 6. As the vortex sheet rolls up its inclination to the flight path decreases rapidly until the value for the fully rolled-up vortices is reached. Thereafter the inclination below the flight path decreases only slowly as the vortices decay.

The curves of Figure 6 together with the calculated distances for the vortex sheet to roll up (Figure 5) enable the position of the wake to be derived. For example, if it is assumed that the inclination of the wake decreases linearly from its value close behind the wing to its value for the fully rolled-up vortices, then at 250 knots the distance of the wake below the flight path is about 4 ft at 200 ft behind the aircraft, this distance thereafter increasing by about 0.6 ft for every 100 ft increase in range. Thus, except at very low speeds, the centre of wake will lie only slightly below the flight path.

Some measurements of the inclination of the centre of the wake to the flight path are reported in Reference 1. For speeds of 250-450 kts the inclination at about 400 ft behind the aircraft was about $\frac{1}{2}^{\circ}$. This is of the same order as the theoretical values for the fully rolled-up vortices (Figure 6). Hence we may conclude that a theoretical treatment such as given above will give a good guide to the position of the wake.

5.3 Strength and Theoretical Effects of Fully Rolled-up Vortices

In this section the theoretical effects of the vortices upon a tracking aircraft will be discussed. It will be assumed that the vortices have just become rolled-up and have not had time to decay appreciably.

The velocity pattern due to the fully rolled-up vortex pair can be readily derived from the standard theory which, as already mentioned, is presented in most text books. Thus if the origin is taken midway between the two vortices and if y is measured horizontally and perpendicular to the flight path and z vertically downwards, then the downward velocity at any point on the y axis is

$$\omega = \frac{\Gamma_0}{2\pi} \frac{b'}{\left(\frac{b'^2}{4} - y^2\right)} \qquad (6)$$

Where Γ_0 is given by equation (2) and $b' = \pi/4 \, . \, b$ (assuming again elliptic span loading). To avoid the infinite velocity at the centre of each vortex, it is generally assumed that each vortex has a "core" rotating with constant angular velocity. The diameter of this core is about 0.2 b' for elliptic loading[2].

The variation of ω, derived from equation (6) for a Meteor aircraft, is shown in Figure 8 for a flight speed of 248 knots EAS at 15,000 ft altitude. The distance downstream where this velocity distribution would be expected is at the point where the vortex sheet is just fully rolled-up, that is, at about 200 ft range (Figure 5). As the distance downstream increases beyond this range, so the cores spread and the vortex system decays.

Note that the velocity ω in equation (6) is relative to axes moving with the vortices. The velocity relative to axes fixed in space must include the velocity due to the downward movement of the vortices themselves (equation 4). However, for a tracking aircraft flying continuously in the wake, it is the velocity relative to the moving axes that is of importance.

The vortices will cause both longitudinal and lateral disturbances to a tracking aircraft and we will now proceed to examine each of these effects separately.

5.3.1 Longitudinal Effects on a Tracking Aircraft

The longitudinal trim change as an aircraft traverses slowly through the wake from top to bottom is difficult to define and calculate, but the following will indicate the general order of magnitude of the disturbance.

The longitudinal change of trim will clearly be a maximum when the aircraft is at the centre of the vortex pattern. As the velocity distribution here will be of the type shown in Figure 8, it is apparent that the trim change will be influenced by three main factors:

1. The downward velocity between the two vortices will reduce the lift on the tailplane and hence produce a nose-up moment on the aircraft,
2. The downward velocity will also reduce the loading over the root of the wing and hence reduce the downwash at the tail. This will cause a nose-down moment.

3. In order to maintain equilibrium of lift = weight, the incidence of the whole aircraft will have to be increased. The effect of this in *reducing* the change of incidence at the tail caused by (1) will become of greater importance as the span of the tracking aircraft is decreased, so that more of the wing is immersed in the downward velocity field between the two vortices.

Consider now a Meteor aircraft being tracked by a Vampire, both aircraft flying at the same speed of 248 knots and at 15,000 ft altitude. Figure 8 shows that the mean downward velocity over the tailplane of the Vampire, when it is at the centre of the wake, will be about 14 ft/sec. The down elevator required to balance out the increased lift on the tailplane will be $\frac{a_{1t}}{a_{2t}} \cdot \frac{14}{U}$, which gives a value of about $2\frac{1}{2}^{\circ}$. The stick force corresponding to this will be about 5 lb. This elevator angle to trim will however be reduced by probably about half by the second of the factors mentioned above. Further, as a large proportion of the wing of the Vampire will be immersed in the downward velocity field between the two vortices, the change in attitude required to maintain lift = weight will be large and will again considerably reduce the elevator angle to trim.

The longitudinal change of trim would thus be expected to be small. Pilots' opinions from flight tests using Meteor and Vampire aircraft confirmed that this was the case. The lateral disturbances were in all cases stated to be the most severe and difficult to control.

5.3.2 Lateral Effects on a Tracking Aircraft

The way in which the lateral disturbance varies as the tracking aircraft moves across the wake from one side to the other can best be illustrated by reference to Figure 8. As the tracking aircraft moves in from the right so the port wing will become affected by the upward velocity. This will impose a rolling moment on the aircraft tending to roll it to the right. As the aircraft moves further into the centre of the wake, so the rolling moment will increase to a maximum and then decrease as the port wing becomes affected by the downward velocity between the two vortices. As more and more of the port wing becomes immersed in this downward velocity and the starboard wing becomes influenced by the upward velocity, the rolling moment will change to one producing a roll to the left. This will probably reach a maximum value when the aircraft centre-line is at or near the centre of the right hand vortex. Movement of the aircraft further into the centre of the wake will cause this rolling moment to decrease until it becomes zero when the aircraft centre-line is at the centre of the vortex pattern. The reverse sequence will apply as the aircraft then moves out of the wake to the left,

Let us now try to estimate the magnitude of these rolling moments and the aileron angles required to trim.

If L is the rolling moment imposed by the vortex pattern at any particular instant, then the aileron angle, ξ, required to trim is given by

$$L = \ell_{\xi}\, \xi\, \tfrac{1}{2}\rho U^2 S b \,.$$

Now
$$\ell_{\xi} = \ell_p \cdot \frac{pb}{2U\xi}$$

Hence $$L = \ell_p \cdot \frac{pb}{2U\xi} \cdot \xi \cdot \tfrac{1}{2} \rho U^2 S b . \qquad (7)$$

If we assume that the rolling moment, L, and the damping in roll derivative, ℓ_p, obey the same law of variation with Mach number then at any Mach number, M, we can write (7) as

$$(L)_{M=0} = (\ell_p)_{M=0} \left(\frac{pb}{2U\xi}\right)_M \cdot \xi \cdot \tfrac{1}{2} \rho U^2 S b \cdot \qquad (8)$$

Hence we can obtain the aileron angle to trim by estimating the low speed values of L and ℓ_p and by substituting them in equation (8), together with the value of $\left(pb/2U\xi\right)_M$ obtained from flight tests at the appropriate Mach number and EAS.

Both L and ℓ_p have been estimated here by simple strip theory assuming that the increase in local CL at any point along the span is equal to (the change in incidence at that position) x (the section lift curve slope). A check on the accuracy of this strip method was obtained using the more exact theory of Reference 4. The difference was, however, found to be insufficient to warrant the use of this more complicated method.

The particular case of a Vampire 5 aircraft tracking a Meteor will now be considered. Both aircraft will be assumed to be flying at 24.8 knots and at 15,000ft altitude. The velocity distribution in the wake of the Meteor at fairly close range has been estimated in para. 5.3, and is given in Figure 8. Using this velocity distribution, the rolling moment imposed on the Vampire when it is at any specified distance from the centre of the wake has been calculated, and the aileron angle to trim derived in the manner outlined above. Figure 9 shows the variation in this aileron angle to trim as the Vampire traverses across the wake. It is seen that the initial roll to the right as the aircraft traverses in from the right is relatively small compared with the ensuing roll to the left caused by the port wing becoming immersed in the downward velocity field between the two vortices. The maximum aileron angle to trim amounts to 8° (or about 0.8 of the total aileron travel) and occurs when the centre-line of the Vampire is 11 ft from the centre of the wake. This rapidly changes to 8° in the other direction as the aircraft traverses further across the wake, the distance between the two peak values being only 22 ft. The large magnitude of this trim change coupled with the rapid change-over from maximum one way to maximum the other, accounts to a large extent for the difficulty of controlling a tracking aircraft in the wake of another aircraft.

To indicate how the aileron angle to trim in the wake varies with the flight speed let us assume that the core diameter and distance apart of the vortices are unaffected by speed, altitude or Mach number. The vertical velocity ω, at any point in the wake will thus be proportional to Γ_o (equation (6)) and hence to $\frac{1}{\rho U}$ (equation (2)). It therefore follows that the rolling moment L induced on the tracking aircraft will be independent of speed and altitude, since L is proportional to $\frac{\omega}{U} \cdot \frac{1}{2} \rho U^2$.

Hence the aileron angle to trim will vary as $\frac{1}{\frac{1}{2}\rho U^2} \cdot \frac{1}{\frac{pb}{2U\xi}}$ (Eqn.7) or as

$\frac{1}{V_i^2} \cdot \frac{1}{\frac{pb}{2U\xi}}$ where V_i is the flight EAS. The lateral control difficulties would therefore be expected to become more serious as speed is reduced. For the particular example considered above, the aileron control of the Vampire will be insufficient to trim out the lateral disturbances for speeds less than about 226 knots EAS. We see then that the rolling moments imposed on a tracking aircraft flying in the wake can be large and that at low speed it is possible to have insufficient aileron control available to counter the severe lateral disturbances. It should be borne in mind that the above calculations apply to ranges of the order of 200 ft. As the distance behind the target aircraft increases and the trailing vortices decay so the disturbances will decrease and the amount of aileron control required be reduced accordingly.

5.4 Rate of Decay of Trailing Vortices

From what has been said above it is clear that the important characteristic governing the severity of the disturbances in the wake at moderate distances from the target is the rate of decay of the trailing vortices. We will now proceed to describe attempts to measure the rate of decay in flight, as there appears to be little information available on the subject.

As the rotational velocity in the wake is small compared with the flight speed, it was found difficult to devise a technique which would enable the quantity measured to be large in comparison with possible experimental errors. In the end two methods were tried.

The first of these was an attempt to measure the actual rotational velocity in the wake by using smoke canisters suitably positioned on the target aircraft (e.g. Figure 7). Unfortunately the smoke was found to diffuse too rapidly making this method unsuitable for exploring the wake at large distances downstream. It was therefore not pursued further, but as the technique could be used to give information of value on the manner of the rolling-up of the vortex sheet behind the wing it has been briefly described in para.5.1 rather than in this section.

In the second method the difficulty of measuring directly the velocity pattern due to the vortices was avoided, an attempt being made instead to measure the maximum aileron angle to trim as an aircraft traversed the wake at various distances downstream. Using an argument roughly the converse of that developed in para.5.3.2, it was then possible to work back from the measured maximum aileron angles and obtain the rate of spread of the vortex core. This second method proved quite successful and it is therefore discussed in more detail.

5.4.1 Variation in Maximum Aileron Angle to Trim with Distance behind Target Aircraft

A Vampire 5 aircraft was instrumented to measure port aileron angle, aileron stick force, and normal acceleration on a desynn type continuous trace recorder. A 16 mm G.S.A.P. camera was fitted in the nose of the aircraft to measure range. A typical record obtained as the Vampire traversed from right to left across the centre of the wake is shown in Figure 10. Comparing this with the theoretical curve of Figure 9, it is seen that the general shapes of the aileron angle to trim curves are similar except that the small peak predicted at about 35 ft from the centre of the wake does not occur so markedly on the flight record. The flight tests showed that although the pilot could control the aircraft to just beyond the first maximum aileron angle reached, the changeover from maximum aileron angle one way to maximum the other was so rapid that he had difficulty in keeping the aircraft under control. The record shown in Figure 10 is perhaps one of the better ones, as in general the flight record of the variation in aileron angle beyond the first peak does not agree well with the theoretical variation. It was felt however that some reliance could be placed in the first peak value of aileron angle obtained in this way. A series of tests were accordingly carried out to measure the variation of this peak aileron angle with range, speed and altitude. To enable the pilot to "see" the wake, smoke was used at low altitude and vapour trails at high altitude (pera.2).

Only a proportion of the flight records obtained could be used as the pilot found it difficult to traverse always through the centre of the wake. Also, as pilots estimates of range were found to be considerably in error (no gun sight was fitted), only those records where the range could be derived from the G.S.A.P. camera film were used, The difficulty of tracking through the centre of the wake and cf estimating range from the G.S.A.P. film increased as range increased. This accounts to some extent for the few points obtained at large range. Those records where the pilot reported that he had allowed a wing to drop before the maximum aileron angle was reached were also ignored.

The results obtained are shown in Figure 11. In these the maximum port aileron angle to trim as measured from the records has been corrected to a mean angle using ground measurements of the aileron differential movement. No correction has been made for variation in weight of the target aircraft. It is seen from Figure 11 that the vortices persist for a considerable time and have decayed only to about half their initial strength by 8000 ft behind the target aircraft. For the two speeds at 30,000-35,000 ft altitude, the disturbances close behind the aircraft were so strong that although maximum stick travel was used, the wings could still not be kept level. Theoretical values for the maximum aileron angle to trim as derived by the method of psra.5.3.2 are also shown plotted. It has been assumed that these theoretical values apply at the point where the vortices are just fully rolled-up. The agreement between theory and experiment is quite encouraging.

It should be noted that with the limited number of points obtained at the larger ranges there is no real guarantee that the maximum values of aileron angle to trim have been reached. A considerable number of points at eaoh range would be necessary to enable curves to be drawn with confidence. however, it is felt that sufficient has been done here to establish beyond doubt the general order of the rate of decay of the strength of the trailing vortices.

Probably the major difficulty encountered in these tests was in determining range with a camera. Thus at ranges greater than say 5000 ft it was difficult to see the target aircraft distinctly or the film because of haze and the general lack of definition of a 16 mm film. Further when vapour trails were used it was necessary for the tracking aircraft to climb several hundred feet above the wake to photograph the target. Radar ranging equipment would be a satisfactory solution to these difficulties. The smoke producing system used for visualization of the wake at low altitudes was also not entirely satisfactory, the difficulties encountered having been mentioned already in para.2. A smoke system using cressylic acid would probably give much improved results.

It was because of the above difficulties that the programme was limited to those tests necessary to prove that the technique was a workable one and to give some general guide to the rate of decay of the trailing vortices.

5.5 General Discussion

It has been demonstrated here, both by theory and by flight tests, that the rolling moments imposed by the trailing vortices are the most serious of the disturbances felt by an aircraft flying in the wake of another aircraft. It has also been shown that at low speeds these rolling moments become of such magnitude that in general insufficient aileron power is available to counteract them. Further, as the rolling moment changes sign from its maximum in one direction to its maximum in the other in a very short distance laterally across the wake, control of the tracking aircraft will be very difficult. In fact, unless the wake is traversed slowly and by a pilot who knows what disturbances to expect, the aircraft will in many instances be thrown out of control. This is no doubt the reason underlying many of the accidents that have occurred to aircraft flying inadvertently into the wakes of other aircraft.

An estimate of the maximum disturbance likely to be encountered close behind the target aircraft can be obtained by using the method of para. 5.3.2 in conjunction with the core diameter of 0.2b' assumed there. As the actual magnitude of the disturbances depend fundamentally upon the type of aircraft in question, let us consider, briefly by the general methods of dimensional analysis the effects of the various parameters, speed, altitude, size of aircraft, etc.

It has already been shown in para. 5.3.2 that for both target and tracking aircraft flying at the same speed, the aileron angle to trim in the wake is proportional to $\dfrac{1}{V_i^2 \dfrac{pb}{2U\xi}}$. Hence the control difficulties will increase rapidly as the speed is reduced, or as the rolling power of the ailerons decreases. The most severe disturbances would be expected

(1) at low equivalent airspeeds

or (2) at transonic speeds where $\frac{pb}{2U\xi}$ may become small

or (3) at speeds near the aileron reversal speed.

To show the effect of the size of the tracking aircraft on the maximum aileron angle to trim Figure 12 has been prepared by scaling the Vampire planform used for the previous calculations. The speed and altitude are again 24.8 knots EAS and 15,000 ft respectively. The effect on the -tracking aircraft is seen to be most severe when its span is about ¼ that of the target. Thus a fighter of 40 ft span attacking a bomber of 160 ft span would, other things being equal, expect to encounter more severe disturbances than would a large aircraft flying in the wake of a smaller aircraft. On the other hand, a guided missile attacking a large aircraft would probably encounter relatively small disturbances as its span is usually very small relative to that of the target. It should be noted that the curve of Figure 12 applies only to the particular core diameter assumed for Figure 8. As the vortices decay and the core diameter increases so the value of the ratio span of tracking aircraft/span of target aircraft for maximum disturbance will increase, although of course the overall level of the disturbances will be less.

The disturbance in the wake will also clearly be proportional to the span loading ($^{W}/b$) of the target aircraft.

It has been assumed in the above discussion that the speed of the tracking aircraft is the same as that of the target aircraft. If, however, the tracking aircraft flies in the wake at a speed which is different from that of the target, then the aileron angle to trim can easily be shown to be proportional to

$$\frac{1}{V_i \, V_i' \left(\frac{pb}{2U\xi}\right)'}$$

whore the dashed quantities refer to the tracking aircraft. Thus, for a given target aircraft speed, V_i, the lateral disturbance to a tracking aircraft will be greatest when its speed is low.

To summarise, it appears that. a small slow aircraft flying in the wake of a large heavily loaded aircraft flying at low speed will encounter exceptionally severe disturbances. The disturbances encountered by a missile attacking an aircraft will depend to a large extent upon the relative sizes of the aircraft and missile, and in many circumstances may be small espeoially as the closing speeds are usually high.

6 Recommendations for Further Work

It is felt that sufficient work has been carried out here to show that the major disturbances in the wake at all ranges are due to the trailing vortices. The main problem now outstanding is the determination of the laws governing the rate of decay of these vortices. With these laws established it would then be possible to predict the disturbances likely to be encountered by an aircraft or missile flying in the wake of any other aircraft.

It is considered that the first step should be the establishment of an adequate theory for the decay of a pair of trailing vortices, backed in the first instance by laboratory experiments. Further flight experiments would then be required to check the validity of this theory under flight conditions and to determine the values of the empirical constants.

The experimental technique using the values of maximum aileron angle to trim can, it is thought, be used successfully for any further flight experiments. To enable these measurements to be related to the velocity distributions due to the vortices, accurate measurement of the aileron Rower of the tracking aircraft and of the distance apart of the vortices will be required. This latter can be accomplished by the smoke technique mentioned in para5.1.

7 Conclusions

1. Measurements show that, for a Meteor flying within the speed range 175-350 knots at 10,000 ft altitude, the jet velocity has fallen to a negligibly low value by about 200-300 ft behind the jet exit.

2. The jet velocity would be expected to persist longest at low altitude, at low forward speeds, and at high engine r.p.m. The conditions favouring a long persistence of the jet might thus be expected at take-off.

3. The major disturbances at all distances behind an aircraft are due to the trailing vortices and these decay only slowly. Tests using a Vampire aircraft flying in the wake of a Meteor show that the strength of these

vortices has fallen to only about half its initial value by 8000 ft behind the aircraft. Although insufficient experimental work has been carried out here to draw any general conclusions on the decay of the trailing vortices, it is felt that a satisfactory flight technique has been established for further tests.

4. Theory and flight experience show that the rolling moments imposed on a tracking aircraft by the vortices constitute the most severe disturbance due to the wake, and that under some conditions the rolling moments so imposed. cannot be controlled oven with full aileron applied.

5. Theory indicates that the effect of the trailing vortices on an aircraft or missile flying in the wake at a given range increases as

(a) the speed of the target decreases

(b) the span loading of the target increases

(c) the speed of the tracking aircraft relative to the target aircraft decreases

(d) the span of the tracking aircraft relative to the target decreases. The ratio of span of tracking aircraft to span of target to give maximum disturbance will depend largely upon the vortex core diameter at the particular range in question.

(e) the aileron effectiveness of the tracking aircraft decreases.

Thus a small slow aircraft flying in the wake of a large heavily loaded aircraft flying at low speed would be expected to encounter exceptionally severe disturbances. The disturbances encountered by a missile attacking an aircraft will depend largely upon the relative sizes of the aircraft and missile, but in many circumstances may be small especially as the closing speeds are usually high and the missile wing span small.

NOTATION

W = weight of aircraft (lbs)

b = aircraft span (ft)

A = aspect ratio of ring

s = wing area (sq ft)

b' = distance apart of trailing vortices (ft)

e = distance for vortex sheet to roll up fully (ft)

V_i = EAS of aircraft (knots)

U = TAS of aircraft (ft/sec)

ρ = air density (slugs)

ρ_o = air density at sea level (slugs)

C_L = lift coefficient = $W/\frac{1}{2}\rho U^2 S$

Γ_o = circulation around centre section of wing

ω_o = downward velocity of vortex sheet immediately behind wing (ft/sec)

ω = downward velocity at any point due to the fully rolled up vortices (ft/sec)

ΔV_i = difference between jet velocity (EAS) and aircraft EAS (knots)

ξ = mean aileron angle (degrees)

p = rate of roll (degrees/sec)

ℓ_ξ = rate of change of rolling moment coefficient with aileron angle = $L/\frac{1}{2}\rho U^2 s\, b\, \xi$

ℓ_p = rate of change of rolling moment coefficient with rolling velocity = $L/\frac{1}{2}\rho U^2 S\, b\, \frac{pb}{2U}$

M = Mach number

a_{1t} = $dC_L/d\alpha$ for tailplane

a_{2t} = $dC_L/d\eta$ for tailplane

η = elevator angle (degrees)

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3	W.F. Durand	Aerodynamic Theory Vol.11 page 324.
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5	N.P. Durand	Aerodynamic Theory Vol.III page 68-69.

TABLE 1

Aircraft Data

Meteor 4 EE.522

Wing span	37' 4"
Wing area	350 sq ft
Aspect ratio	3.98
Taper ratio	2.2 :1
Wing section	EC 1240/0640 root EC 0940/0640 tip
Wing setting to fuselage datum	1.0°
Aerodynamic mean chard	116.6 ins.
Fuel capacity	325 gals
Fog oil capacity (for smoke)	120 gals
All-up weight	16320 lb
C.G. position with full fuel, pilot but no fog oil	2.4 ins aft of datum

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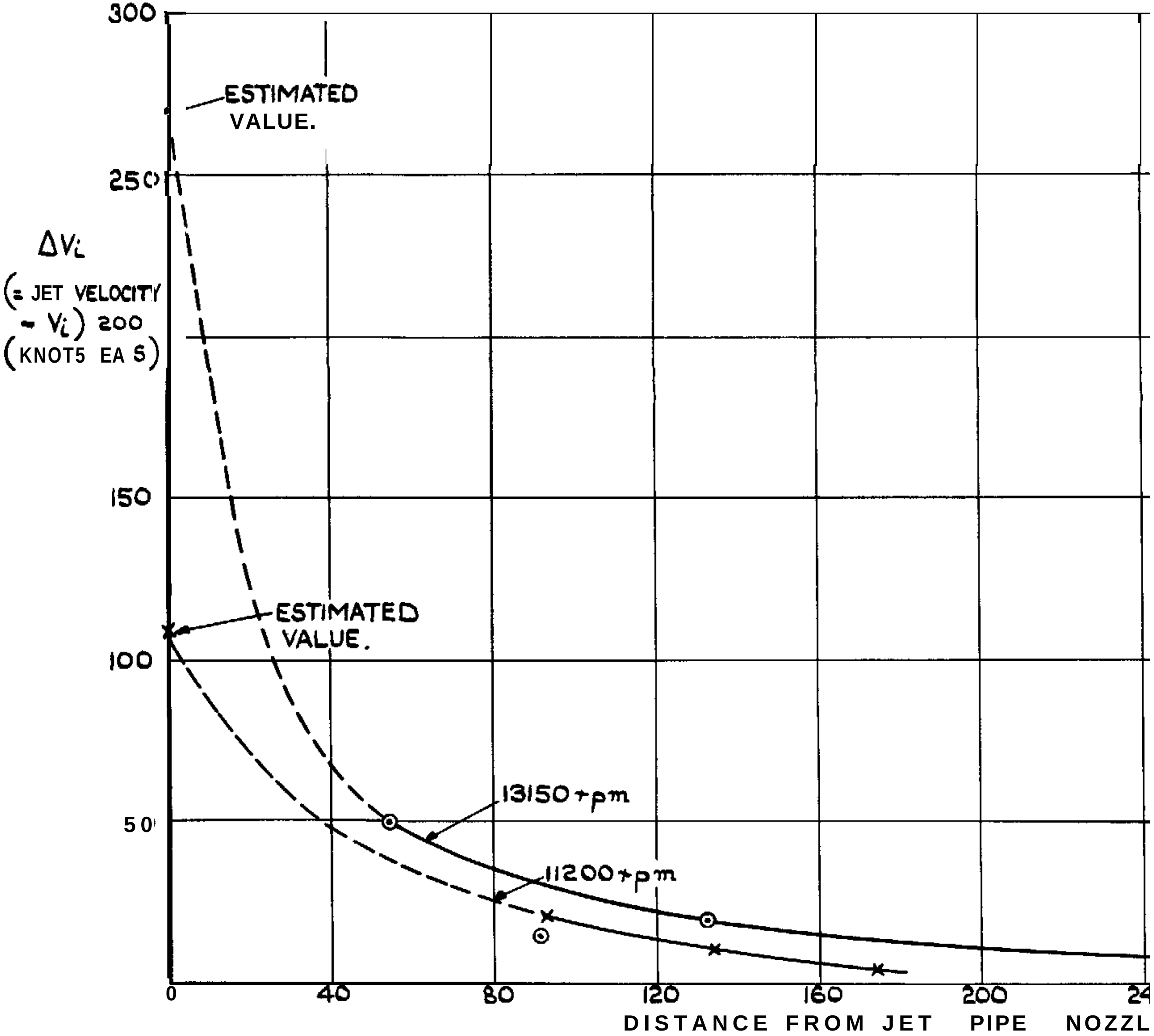


FIG.1 (a) DECAY OF JET VELOCITY WITH
AIRCRAFT – PITOT TRAVERSE”

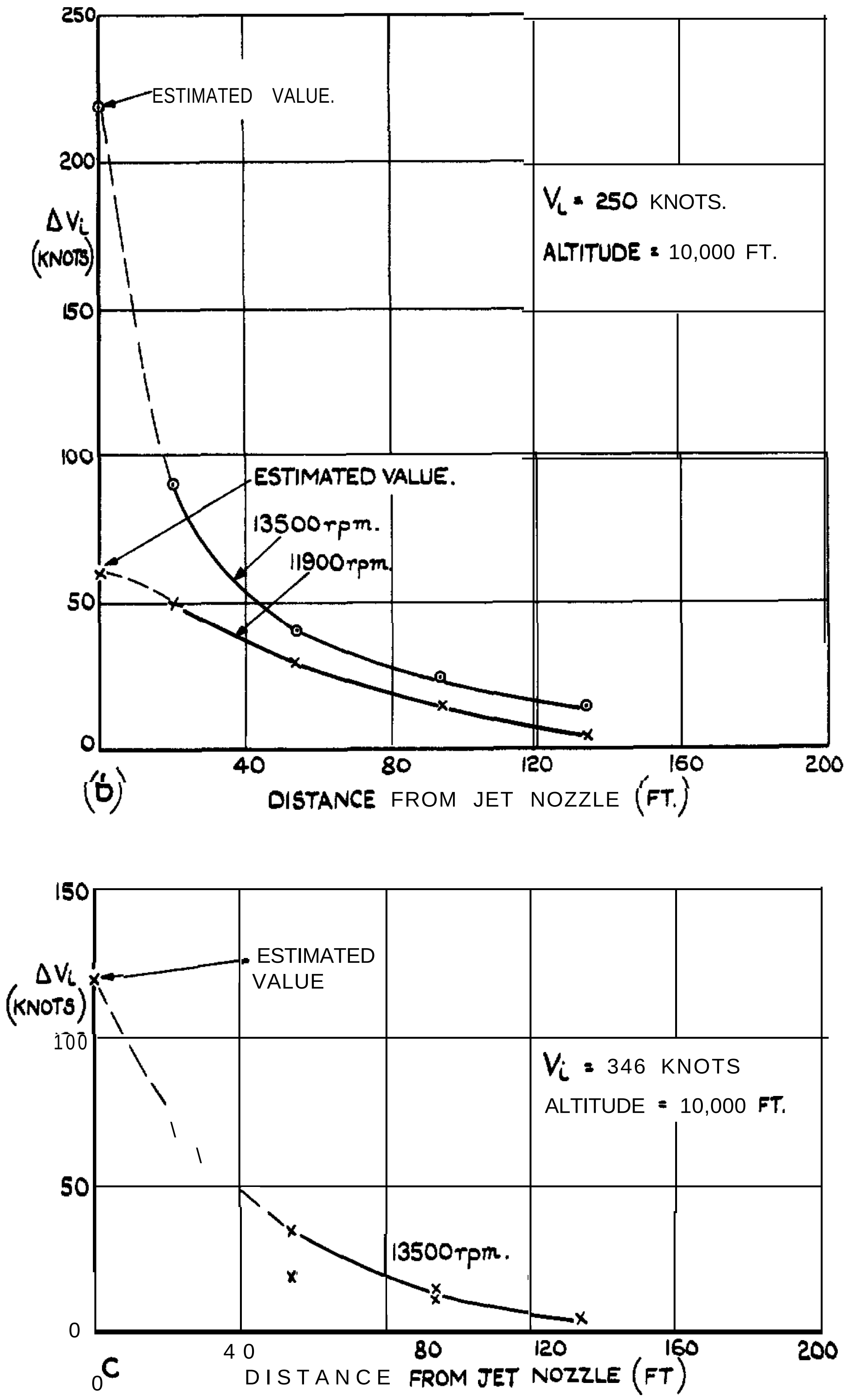


FIG. 1.(b & c) DECAY OF JET VELOCITY WITH DISTANCE BEHIND AIRCRAFT —"PITOT TRAVERSE" TECHNIQUE.

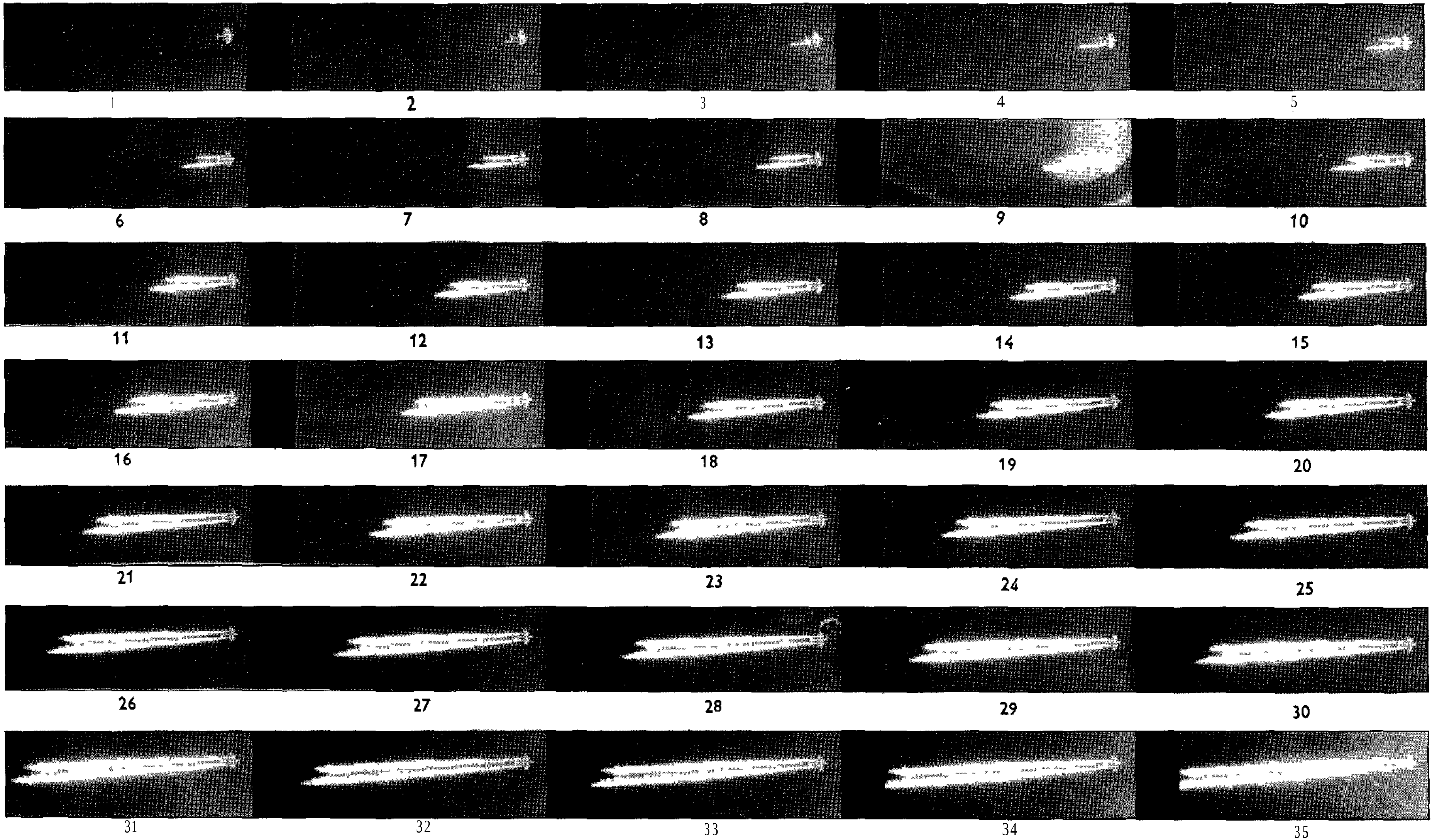


FIG 2. ENLARGEMENT OF A TYPICAL CINE FILM USED FOR DETERMINATION OF JET VELOCITY

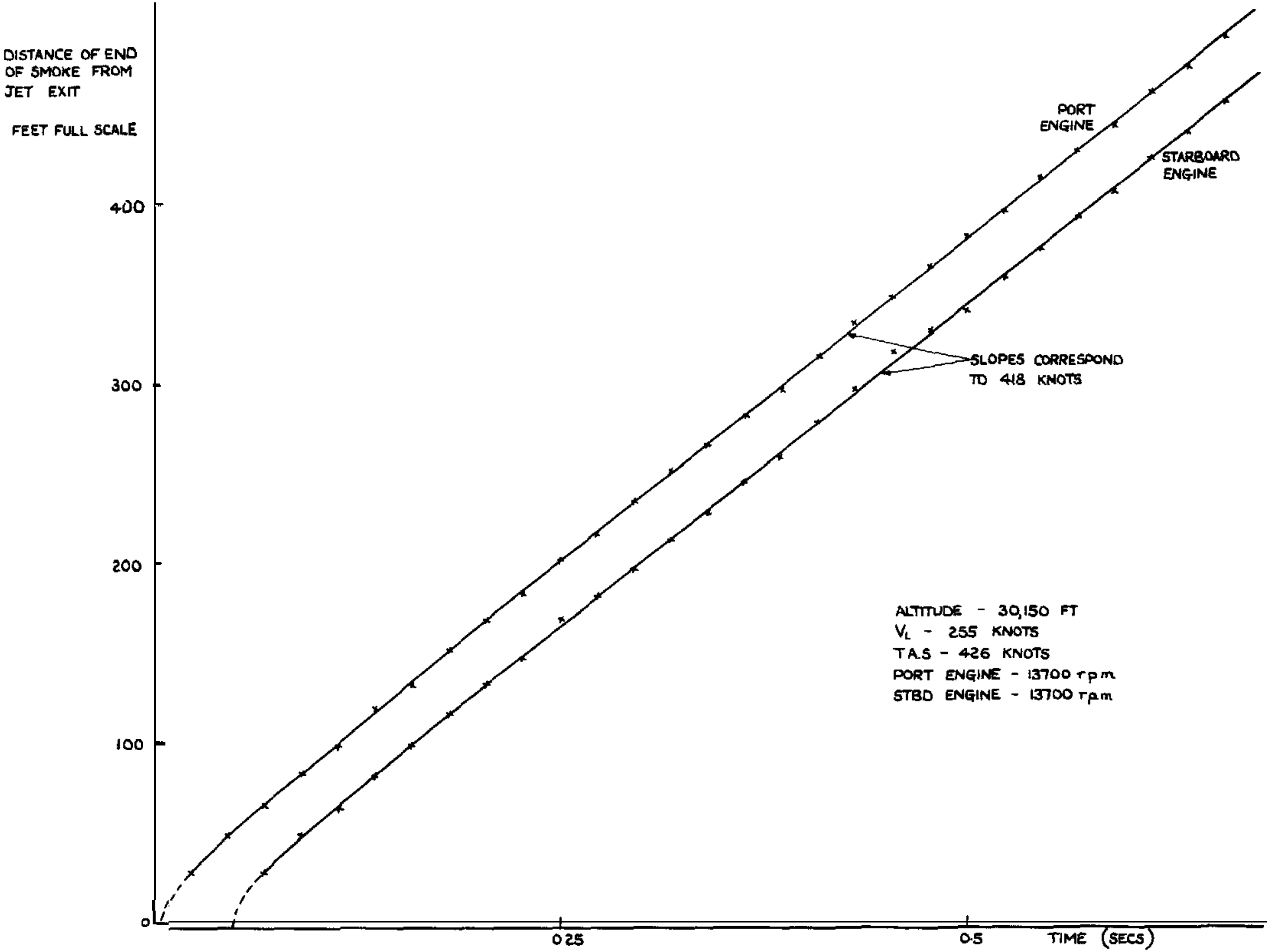


fIG.3. PLOT SHOWING DECAY OF JET VELOCITY OBTAINED BY PHOTOGRAPHIC TECHNIQUE -RESULTS OBTAINED FROM PRINTS OF FIG.2.

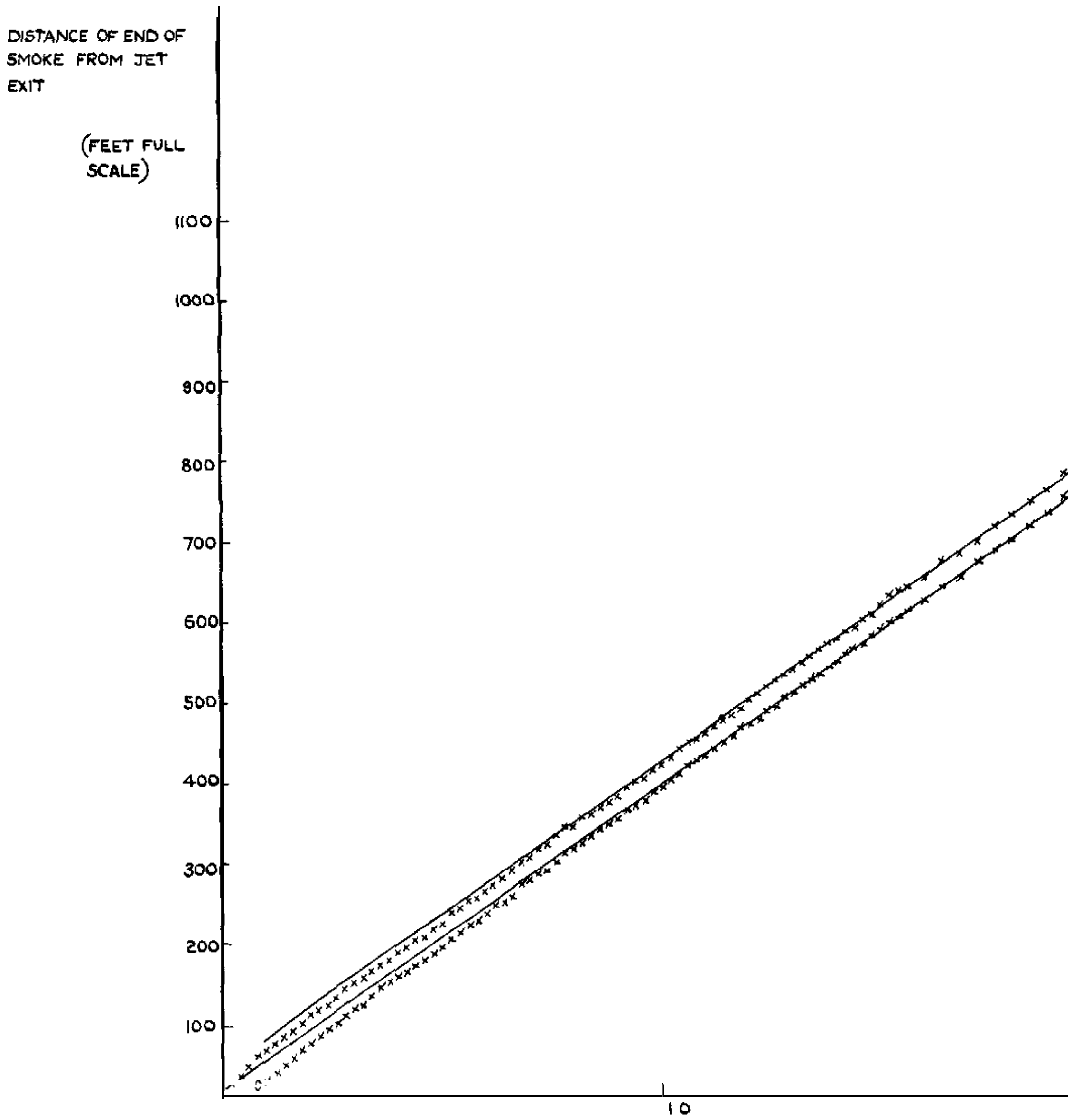


FIG. 4 (a). PLOT SHOWING DECAY OF JET VEL
PHOTOGRAPHIC TECHNIQU

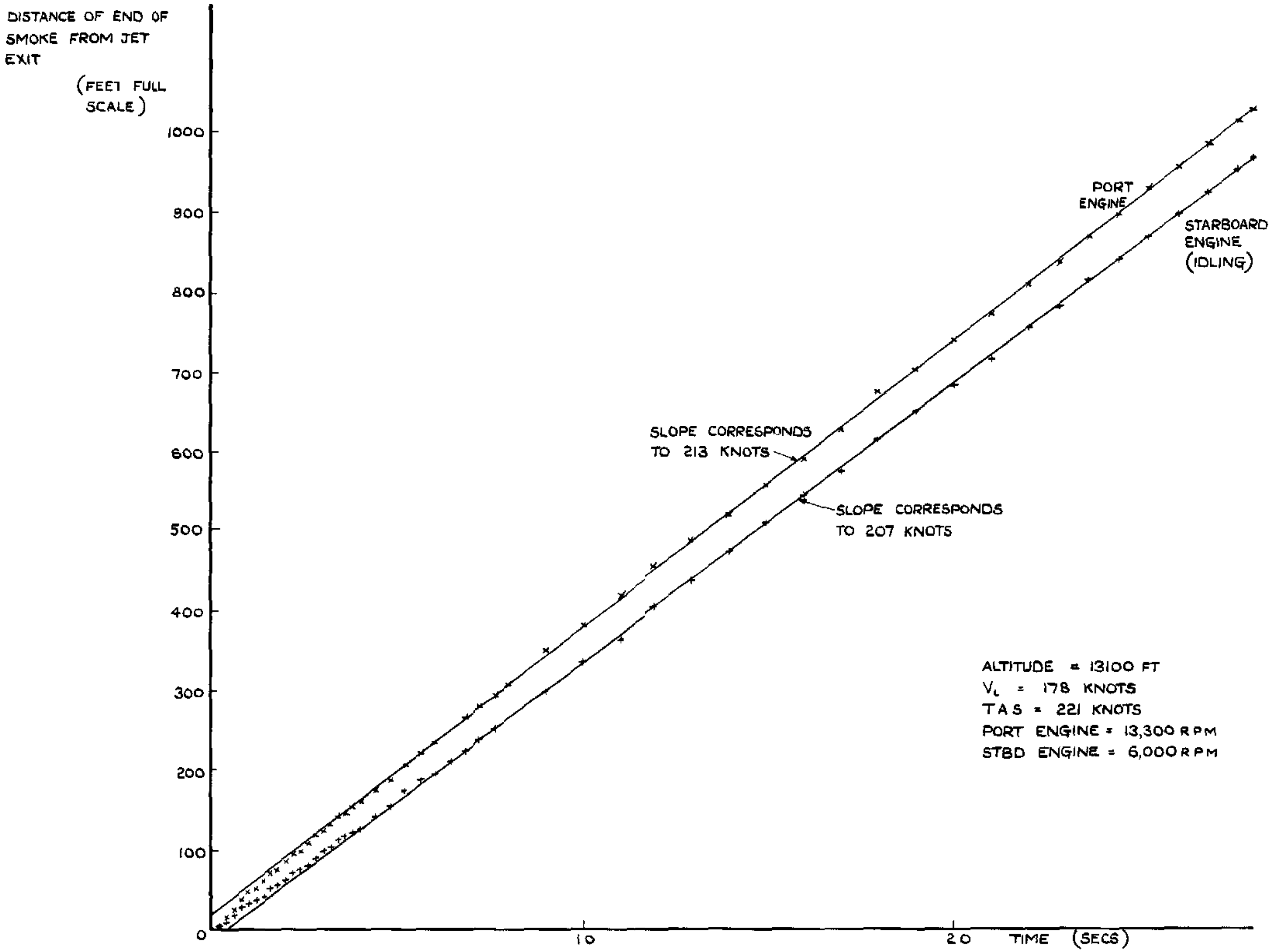


FIG 4 (b). PLOT SHOWING DECAY OF JET VELOCITY AS OBTAINED BY PHOTOGRAPHIC TECHNIQUE.

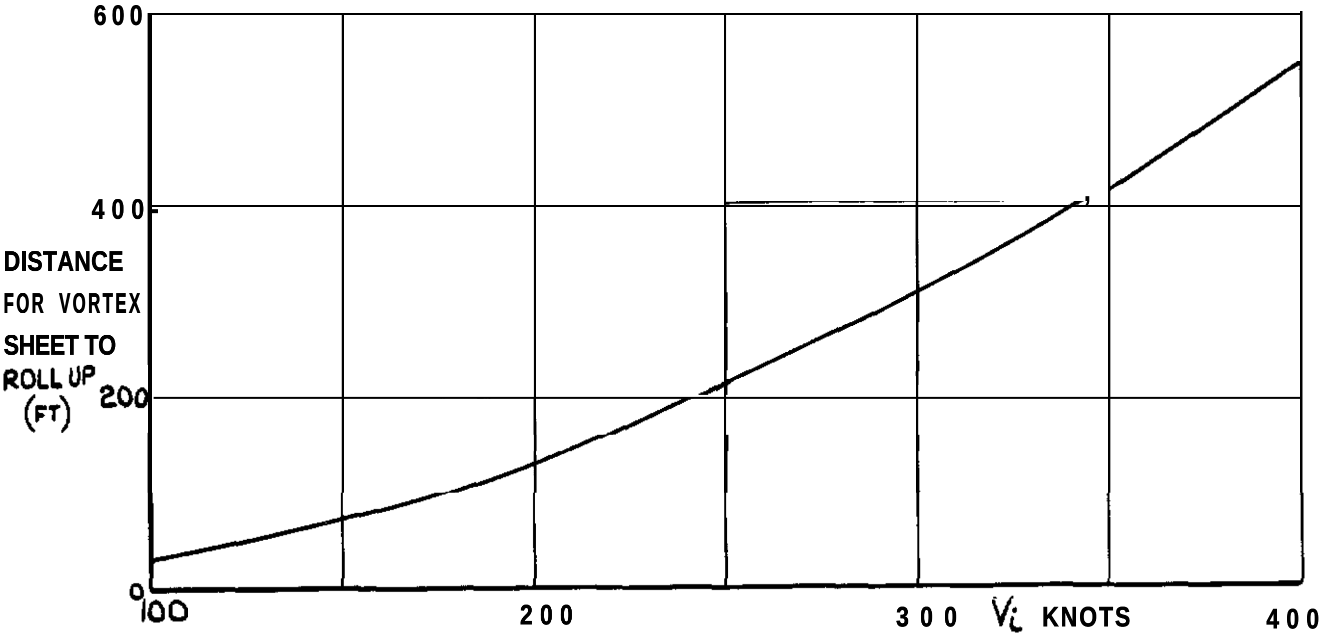


FIG. 5. THEORETICAL DISTANCE FOR VORTEX SHEET TO ROLL UF?

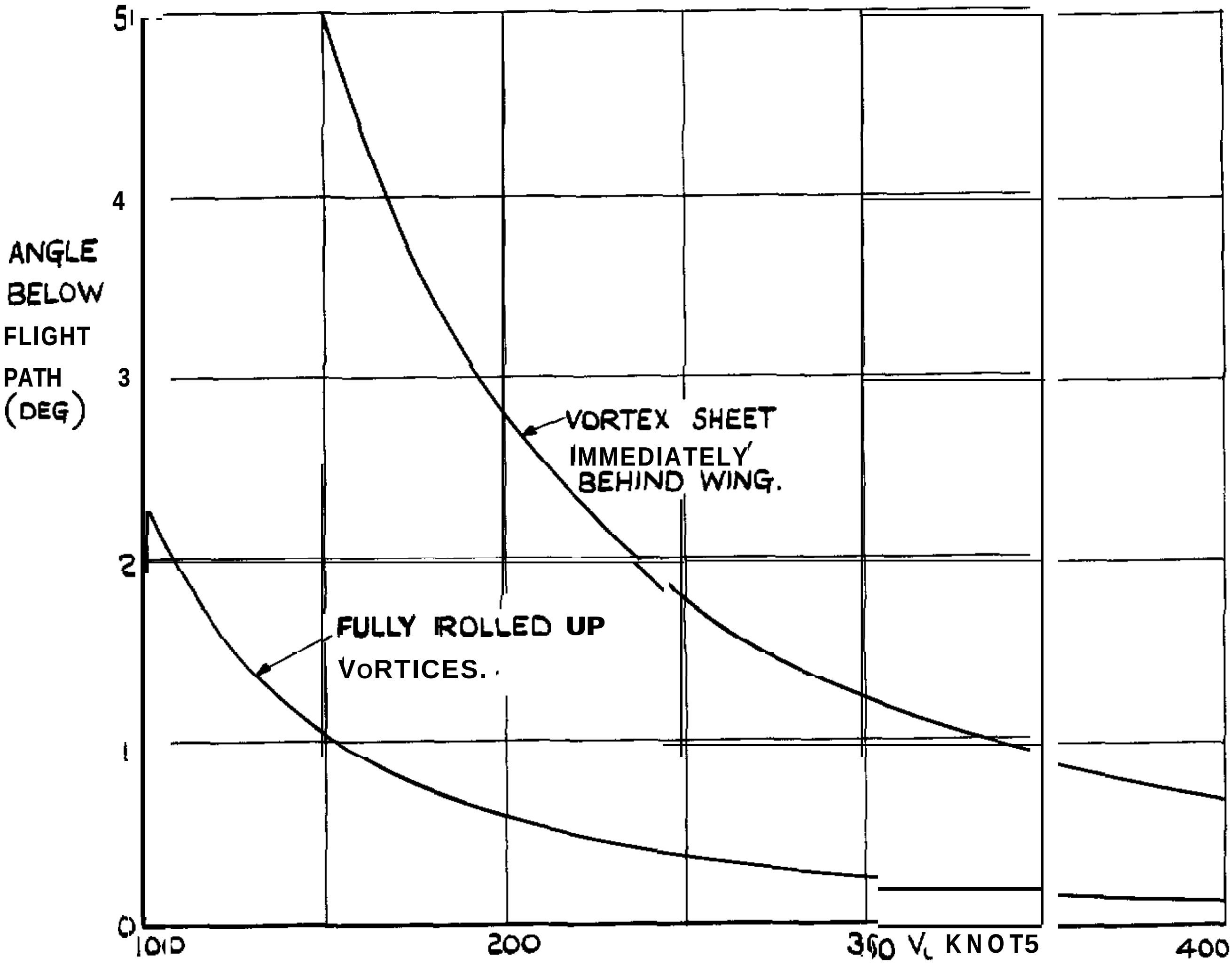
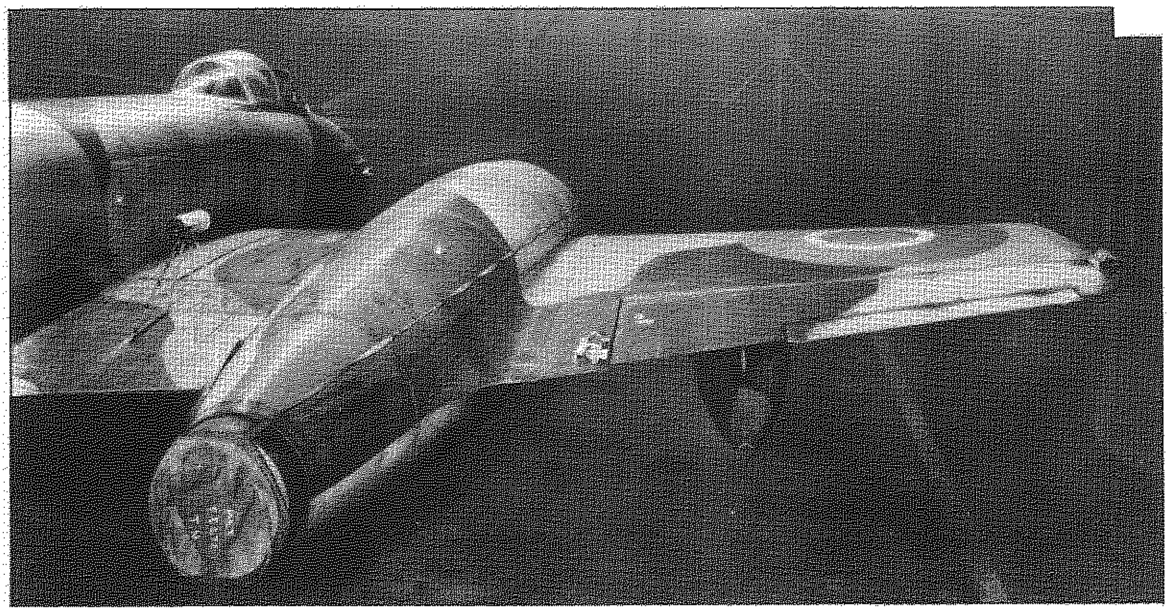
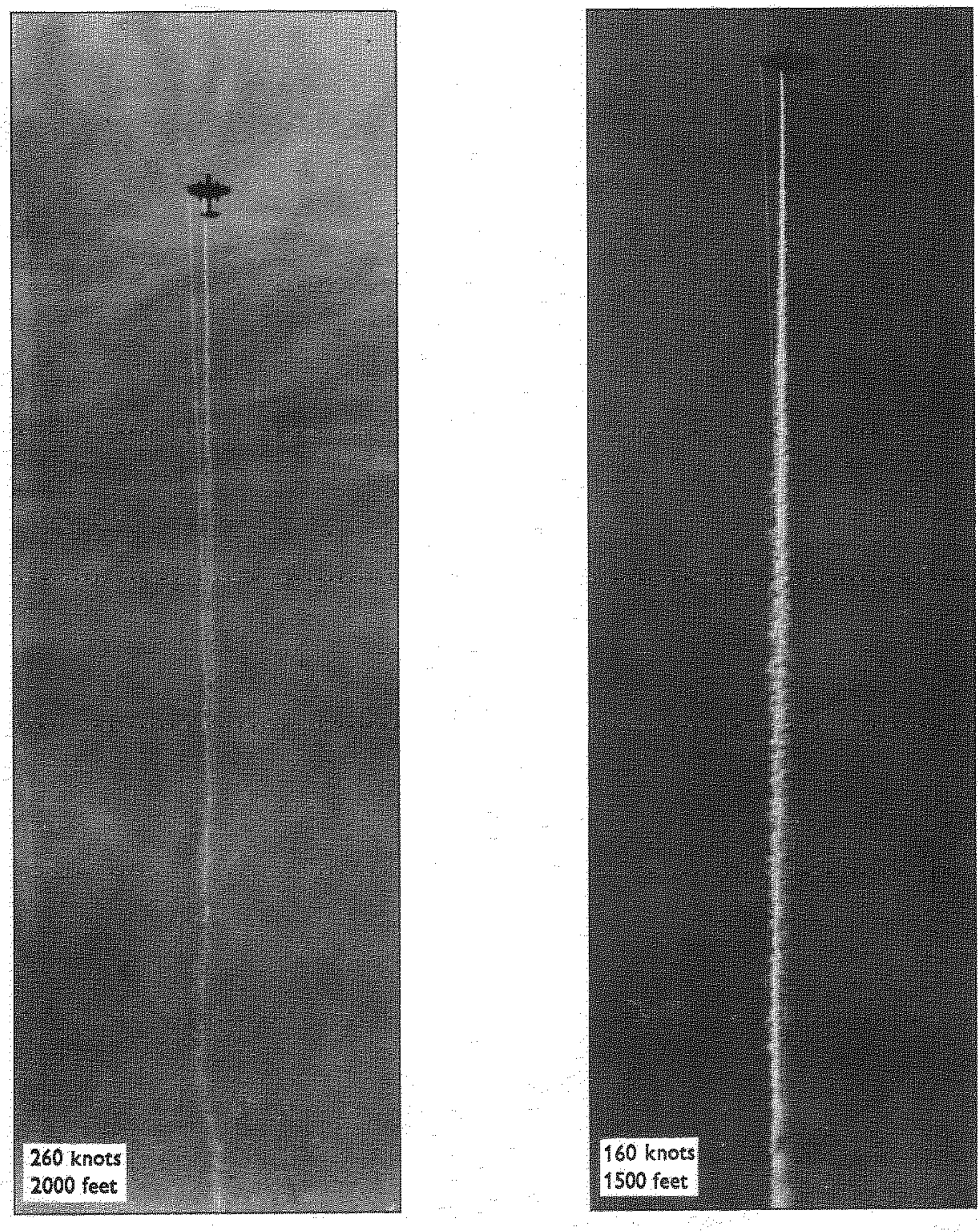


FIG. 6. THEORETICAL INCLINATION TO FLIGHT PATH OF VORTEX SHEET & VORTICES.



a. SMOKE CANISTER INSTALLATION ON METEOR



b. PHOTOS OBTAINED WITH SMOKE CANISTERS IN OPERATION

FIG.7a & b

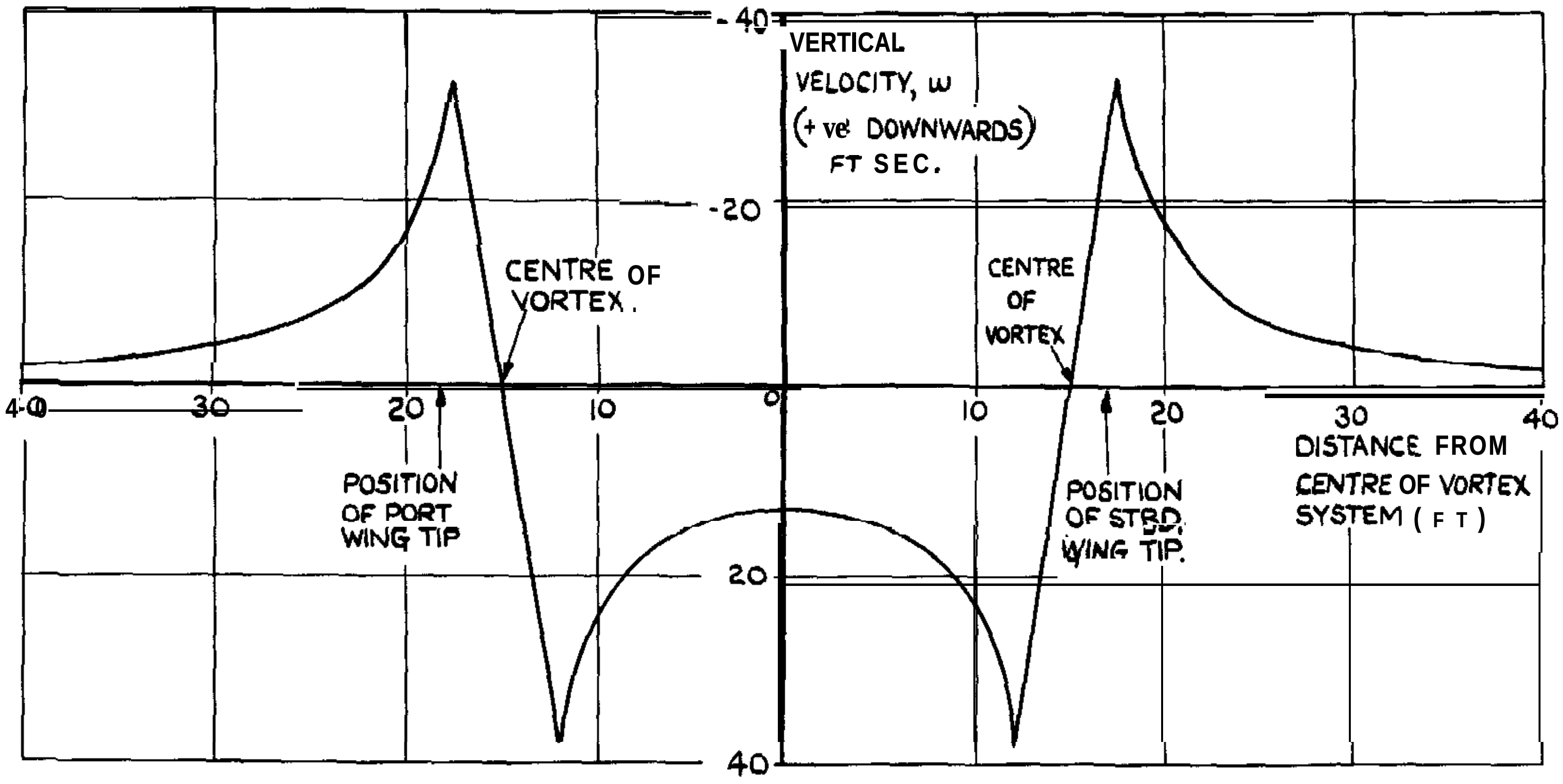


FIG.8. THEORETICAL VELOCITY DISTRIBUTION DUE TO FULLY ROLLED -UP VORTICES BEHIND A METEOR AT 248 KNOTS (EAS) AND 15,000 FT. ALTITUDE,

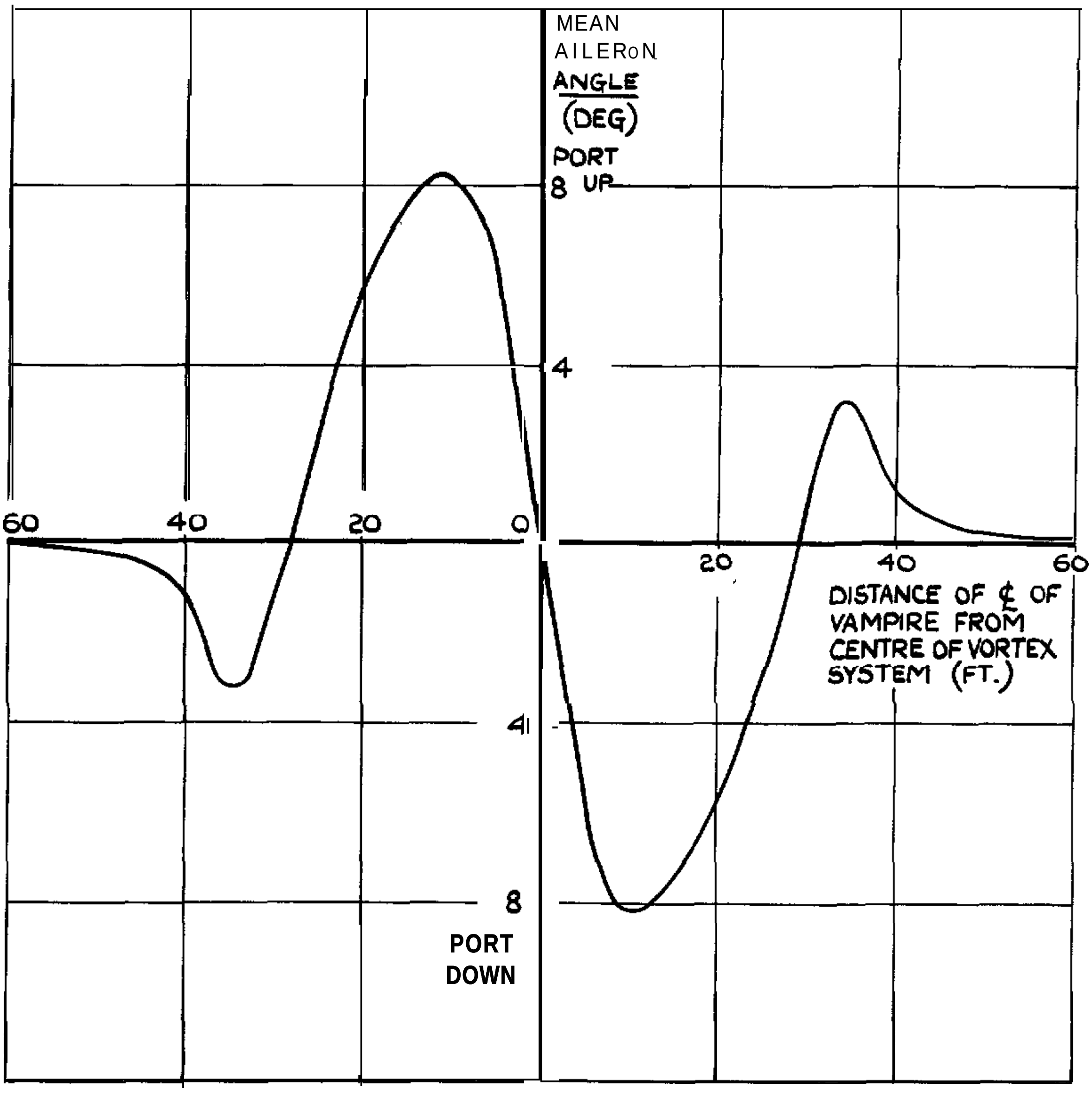
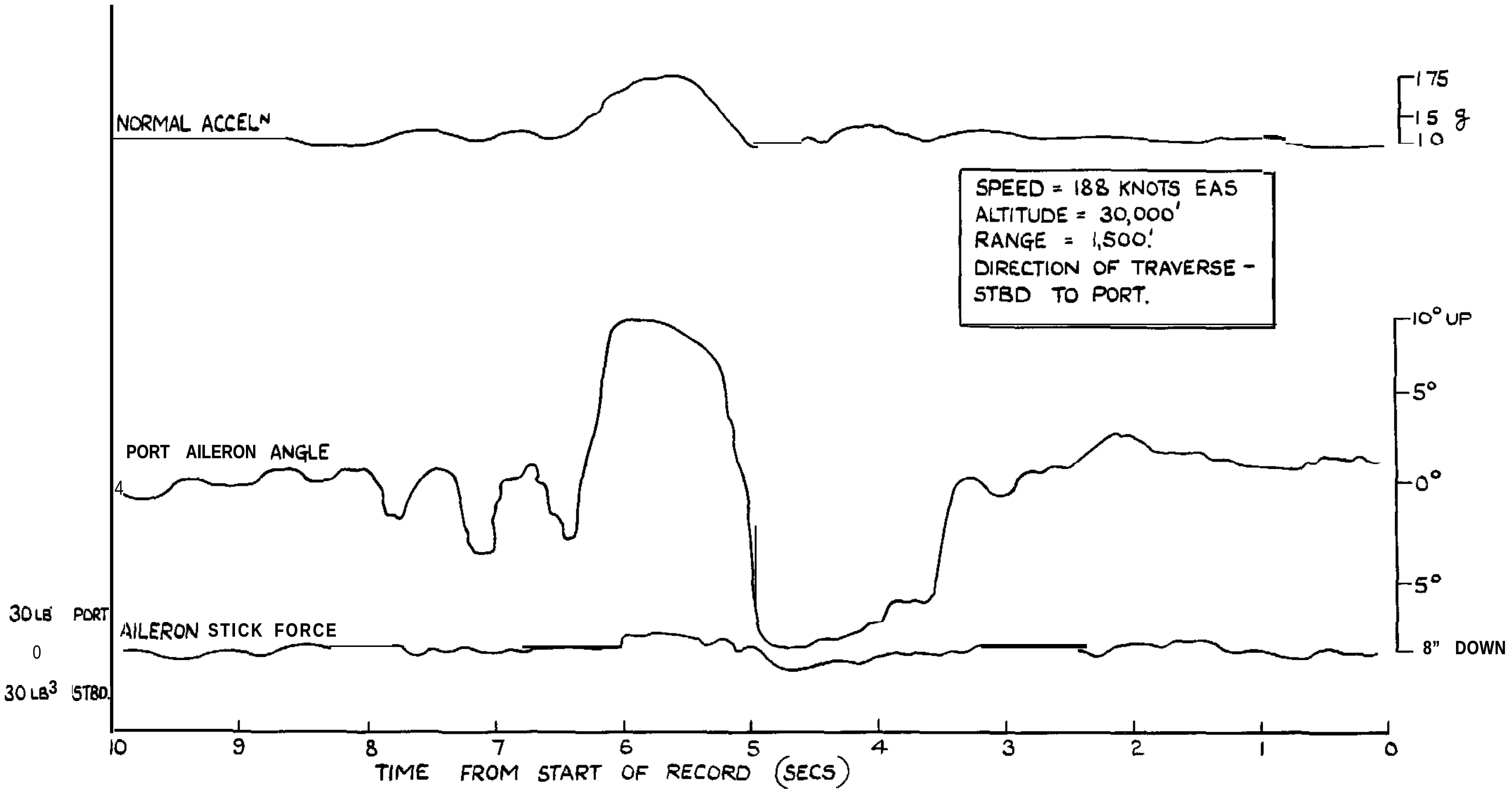


FIG.9. CALCULATED VARIATION IN AILERON ANGLE TO TRIM FOR A VAMPIRE AIRCRAFT MOVING ACROSS THE VELOCITY FIELD OF FIG. 8.



SPEED = 188 KNOTS EAS
ALTITUDE = 30,000'
RANGE = 1,500.'
DIRECTION OF TRAVERSE - STBD TO PORT.

FIG. 10. FLIGHT RECORD OF LATERAL CHANGE OF TRIM ON A VAMPIRE AIRCRAFT TRAVERSING THE WAKE OF A METEOR.

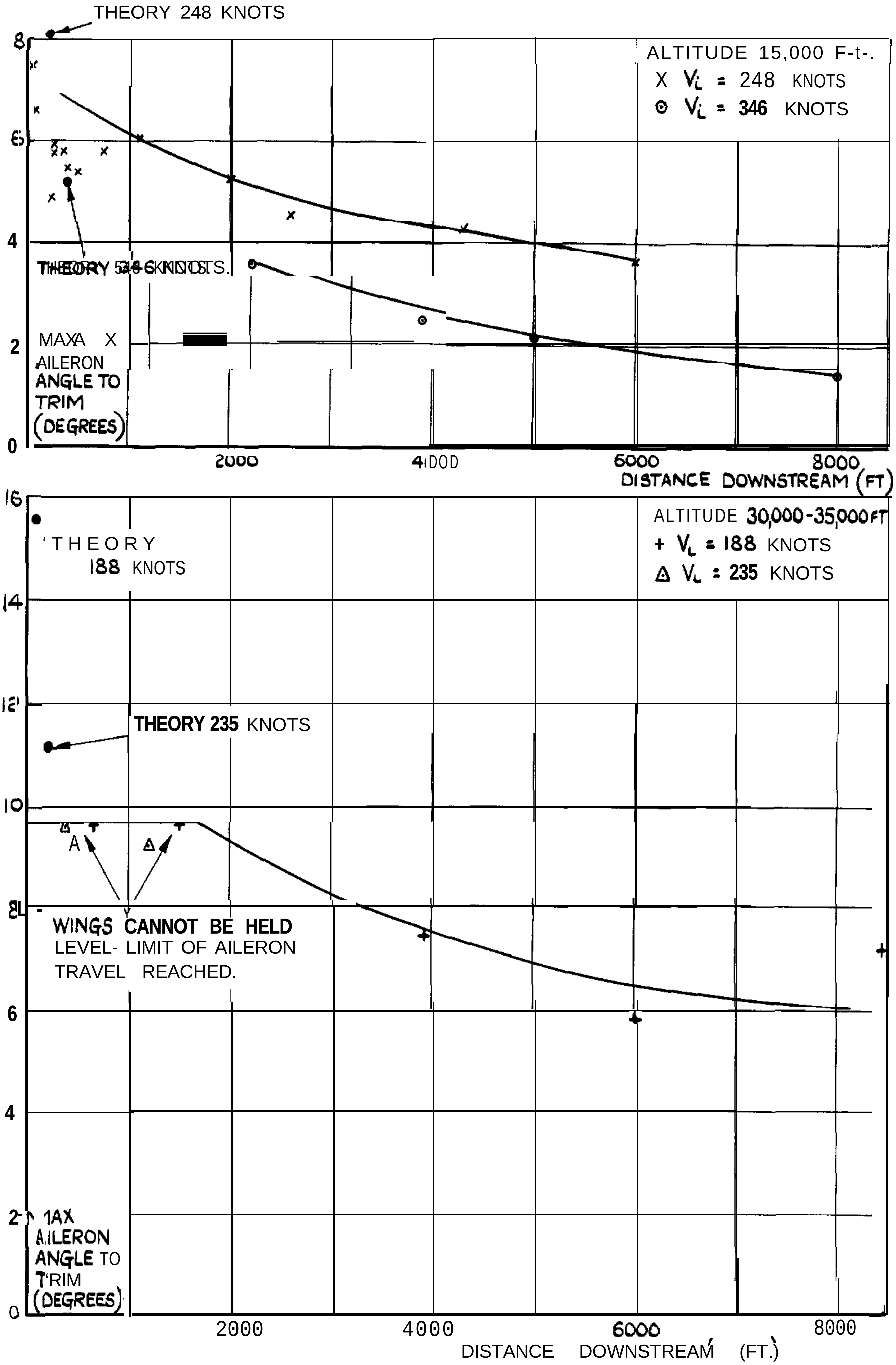


FIG. II. EFFECT OF DISTANCE DOWNSTREAM FROM TARGET AIRCRAFT ON MAXIMUM AILERON ANGLE TO TRIM IN WAKE.

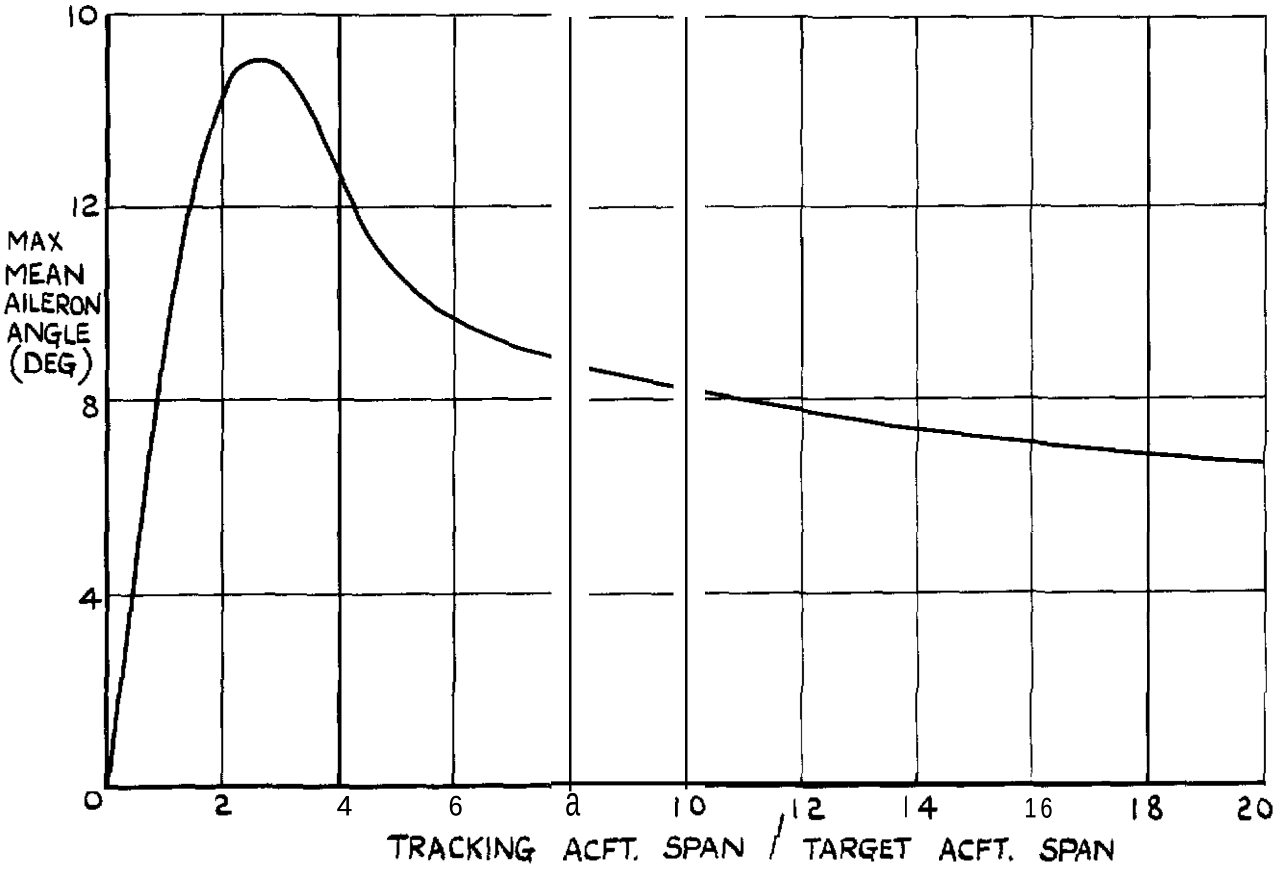


FIG. 12. EFFECT OF SIZE OF TRACKING AIRCRAFTON MAXIMUM AILERON TO TRIM IN WAKE AS CALCULATED FROM VELOCITY DISTRIBUTION OF FIG. 8.

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