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The Measurement of Reynolds Stresses in Low Intensity Turbulent Flow

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CORRECTION

Page 9

(1) *1st line* Equation No. is (14)

(2) *3rd line* *Read:*

$$\text{where } \alpha = 2\alpha_a\alpha_b/(\alpha_a+\alpha_b) \text{ and } D = (\alpha_b-\alpha_a)/(\alpha_a+\alpha_b).$$

(3) *6th line* *Read:*

$$E_{c_a}/(E_{c_a}^2-E_{0_a}^2) = E_{c_b}/(E_{c_b}^2-E_{0_b}^2),$$

Page 14

(4) *1st line of last paragraph of Section 2.4*

Read: In situations where $(\overline{u_2^2}-\overline{u_3^2}) \gg \overline{u_2u_3}$,

Third equation from bottom of page

Subscript of first "u" is 1, i.e.,

$$\alpha_a\alpha_b e_{a-b} = (\alpha_b-\alpha_a)\frac{u_1}{U_0}+(\beta'_a\alpha_b+\beta'_b\alpha_a)\frac{u_4}{U_0},$$

Page 16

(6) *2nd line* Delete one "=" sign.

(7) *5th line from bottom of page*

Delete "/" in $\frac{\overline{u_3^2}}{U_0^2}$ term.

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The Measurement of Reynolds Stresses in Low Intensity Turbulent Flow

By H. J. PERKINS

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Summary.

Some of the fundamentals of hot wire anemometry are reviewed and given a new interpretation. Methods, employing either the inclined single sensor or the X-probe, are described for measuring all six components of the symmetric Reynolds stress tensor in flows where both the mean secondary velocities and the r.m.s. turbulence fluctuations are less than about 5 per cent of the primary velocity. Various methods are compared and general rules established for obtaining high accuracy.

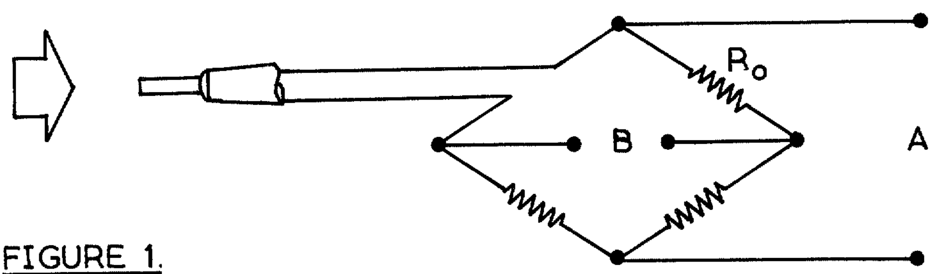

FIGURE 1.

1. *Introduction.*

Several full accounts of hot wire anemometry are listed in the bibliography as Refs. 1 to 8. Only a summary of the salient points will be given here.

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* Replaces A.R.C. 31 748.

The observed convective heat loss from a thin heated wire or cylindrical film exposed to a fluid stream can be used to deduce the local instantaneous velocity field. The wire, which constitutes one arm of a Wheatstone bridge circuit, Fig. 1, can be operated either by

(a) applying a 'constant current' to the wire ($R_o \approx 0$) and observing the instantaneous voltage at B, relating this to the instantaneous wire temperature, or by

(b) using the out of balance voltage at B to modulate the applied bridge voltage at A in such a way as to maintain the wire at 'constant temperature'. The instantaneous bridge voltage at A will then be related to the wire heating current. In this case the thermal inertia of the wire is eliminated from consideration but at the expense of an elaborate electronic control circuit. The operating temperature of the wire is set by adjusting the value of the resistance R_o.

Advances in electronic circuitry during the last decade have so improved the latter mode of operation that its use is now almost exclusive, and so it is to this case that further discussion is restricted.

1.1. *The Response Equation for a Hot Wire.*

The heat balance for a hot wire having resistance R_w at its operating temperature, and excited by an instantaneous bridge voltage E, can be written

$$\text{Heat input} = E^2/R_w = L_R + L_C + L_{FC} + L_{NC}$$

L_R the radiation loss, will in general be very small for tungsten or platinum wires operated below 300°C.

L_C represents the heat lost by conduction to the wire support needles and usually constitutes about 10 per cent of the heat input.

L_{FC} the forced convection loss, is intuitively a function of the flow velocity, wire and fluid properties and temperatures, and the wire geometry, particularly its size and inclination to the flow. This can be written conveniently as

$$L_{CF} = C_1 + C_2\, U^n_{eff}$$

where U_{eff} is the effective flow velocity, and $n \simeq \frac{1}{2}$

L_{NC} represents the natural convection loss $(E_0^2/R_w - L_C)$, where E_o is the bridge voltage at zero flow, and is unimportant above wire Reynolds number of 0·1. It is coincidental that for most wires E_o^2/R_w is approximately equal to C_1, the limiting form of the above expression for L_{FC} as the wire Reynolds number approaches zero.

Hence the response equation for the heated sensor can be written,

$$(E^2 - E_o^2) = KU^n f(\gamma) \tag{1}$$

where $(E^2 - E_o^2)$ is determined from the measured bridge voltages,

U is the instantaneous fluid velocity,

n is a power law index, $n \simeq \frac{1}{2}$,

K embodies the wire and fluid properties and temperatures,

and $f(\gamma)$ is a function describing the yaw response of the heat loss, γ being the included angle between the instantaneous velocity vector and the plane normal to the wire.

The right hand side of equation (1) traditionally represents the product of three separable functions of wire temperature, flow velocity, and yaw angle for a given wire operating in a given fluid. In general, K, n and $f(\gamma)$ are all functions of the flow velocity, as demonstrated by Brunn[9], although over the limited velocity range encountered in the typical experiment the U dependence can legitimately be dropped. The simplest yaw response equation, used by early experimenters, is $f(\gamma) = \cos^n\gamma$, the cosine law, implying that only the velocity component perpendicular to the wire, $U \cos \gamma$, is responsible for the cooling. This would indeed be so, up to quite large yaw angles, if the wire was infinite in extent and maintained at a constant temperature throughout its entire length. Unfortunately the real hot wire must be finite, having a non-uniform temperature distribution due to heat conduction to the supports.

When such a sensor is inclined to the fluid stream the heat loss is greater than that suggested by cosine-law cooling. The classical explanation for this effect suggests that the velocity component parallel to the wire skews the temperature distribution thus increasing the conduction loss to the downstream support. That this is not the case has been demonstrated recently by Champagne *et al*[10], by actually measuring the temperature distribution along hot wires 'normal' and yawed to the fluid stream. The slightly increased loss to the downstream support is almost entirely balanced by a reduction in the loss to the forward support.

A more plausible explanation is attempted here involving an interaction between aerodynamic 'end effects' and the 'transition' in heat transfer mode which takes place as the yaw angle is increased. As the wire is rotated to lie parallel to the fluid stream the heat transfer, which takes place through a shear layer developing along the wire, settles to about 40 per cent of the 'normal' heat loss, retaining its $U^{\frac{1}{2}}$ dependence but becoming erratic due to upstream support interference. At low yaw angles the boundary layer on the wire remote from the supports, although skewed, is essentially two-dimensional so that the cosine law of cooling ought to be adequate. During transition the flow must become three-dimensional over the entire length of the wire. The yaw angle at which transition begins will therefore depend primarily on the three-dimensionality of the flow over the wire, starting earlier on shorter wires. Hence, the increase in heat transfer above that suggested by the cosine law must depend strongly upon the length to diameter ratio of the sensor l/d, and the ratio of support diameter to wire diameter ds/d. Fig. 2(a) shows the postulated yaw response, which is described by an expression of the form,

$$f(\gamma) = \cos^n\gamma + f_1\,(l/d, ds/d, \gamma)$$

such as

$$f(\gamma) = [\cos^2\gamma + C_3 \sin C_4\,\gamma]^{n/2}$$

where C_4 is a function primarily of l/d and C_3 a function of ds/d.

Many other improvements to the cosine law have emerged over the years, each introducing a new yaw parameter. Notable among these are,

(1) Hinze[8], Fig. 2(b)

$$f(\gamma) = (\cos^2\gamma + k^2 \sin^2\gamma)^{n/2}\,,\ \text{with } n = \tfrac{1}{2}$$

(2) Newman and Leary[11], Fig. 2(c),

$$f(\gamma) = \cos^m\gamma,\ \text{with } m < n,$$

and most recently,

(3) Friehe and Schwartz[12], Fig. 2(d),

$$f(\gamma) = (1 - b\,(1 - \cos^{\frac{1}{2}}\gamma)\,)^{2n}$$

the quantities k, m and b being intuitively dependent upon the l/d and ds/d ratios, and to a lesser extent, the yaw angle and flow velocity. The difference between the three expressions, as indicated in Fig. 2, is demonstrative of the difficulties involved in establishing the yaw response experimentally. Each of k, m and b are claimed to be independent of yaw angle for $-60° < \gamma < +60°$ by the various proponents, but clearly they cannot all be right. Dependence on the flow velocity is in general only slight although the choice of n, the power law exponent, crucially affects all three parameters. The l/d dependence, and in the case of k the ds/d dependence, is summarized in Fig. 3. The advantages and disadvantages of each expression are discussed more fully in Refs. 9–13, the hot-wire theoreticians logically favouring Hinze's formulation whilst the hot-wire users prefer the second expression for its inherent simplicity in low turbulence flow.

Concentrating now on the latter; it has been demonstrated by Brunn[9] that for a given sensor operated at a given temperature, m is sensibly constant for $20° < \gamma < 70°$ and has a U dependence which is so much like that of n that the ratio m/n emerges as a very weak function of U. In summary then, a very useful and simple form of the response equation for use in flows where γ is less than 70° will be

$$(E^2 - E_o^2) = KU^n \cos^m\gamma \tag{2}$$

where n is determined from a constant-temperature velocity calibration with $\gamma = 0°$, n being either the local slope of the curve log $(E^2 - E_o^2)$ *versus* log U, or an average slope over the experimental velocity range,

K is given by $(E_c^2 - E_o^2)/U_o^n$, E_c and U_o being calibration values of E and U;

and m, or m/n, is deduced from a yaw calibration. The latter can be performed in two different ways.

(1) The steady flow yaw response in a uniform stream is obtained by comparing the response at γ = constant to that at $\gamma = 0$ (normal). From the expression,

$$(E_\gamma^2 - E_o^2) \quad (E_{\gamma=0}^2 - E_o^2) = \cos^m\gamma$$

the quantity m can be deduced if γ is known accurately; the estimated error in m produced by a 1° degree error in γ is tabulated below.

γ DEGREES	10	20	30	40	50
PERCENTAGE ERROR IN m/1° ERROR IN γ	22·5	10·2	7·0	5·6	4·8

(2) As will be demonstrated in Section 6, it is possible to obtain an indirect calibration for m from actual Reynolds stress† measurements in a known shear flow, fully developed flow along a straight circular pipe.

2. *The Measurement of Reynolds Stresses in the Presence of Small Secondary Flows.*

Consider a three-dimensional flow in which the three mean velocity components, U_i, and the six elements of the symmetric Reynolds stress tensor $\overline{u_i u_j}$ are all non-zero. Referring to the co-ordinate system of Fig. 4, the instantaneous velocity, U, will be

†Double velocity correlations, at a point in the flow will be referred to as Reynolds 'stresses'. The omission of the density multiplier is a convenience in incompressible flow.

$$\left[(U_1+u_1)^2+(U_2+u_2)^2+(U_3+u_3)^2 \right]^{\frac{1}{2}},$$

or equivalently

$$\left[(U_1+u_1)^2+(U_4+u_4)^2+(U_5-u_5)^2\right]^{\frac{1}{2}}$$

where x_5 is the co-ordinate perpendicular to the x_1-x_4 plane. The angle γ will be the included angle between the above vector and the plane normal to the wire, that is,

$\sin\gamma \simeq \cos(\theta+\psi)\cos\psi'$, referring to wire 'a' of Fig. 5,

where

θ is the inclination of the wire to the x_1 axis,

ψ is the time mean inclination of the velocity vector in the x_1-x_4 plane,

and ψ' is the time mean inclination in the x_1-x_5 plane.

Suppose initially that the steady secondary flow velocities are comparable in magnitude to the r.m.s. turbulence fluctuations and that both are always less than 5 per cent of the primary velocity, U_1. Removing secondary quantities from the velocity statement above†, then,

$$U \simeq U_1+u_1$$

and the angle between the velocity vector and the wire direction becomes $(\theta+\psi+d\psi)$. Inclining this projected vector out of the x_1-x_4 plane by up to 10° produces only a 1° change in the effective yaw angle, that is, the effect of a small U_5 or u_5 velocity component on the yaw angle is negligible. Both the velocity and yaw response of the sensor are thus implicitly linearized.

A method is required for measuring the nine dependent variables under these circumstances whilst maintaining the probe axis parallel to the primary direction for experimental convenience. Unfortunately there exist two conflicting requirements here. In order to measure the secondary flows, the yawed sensor must be arranged to have maximum sensitivity to the mean transverse velocity component, that is $dE/d\gamma$ must be maximized; this occurs when γ is approximately 75°. For the Reynolds stress measurements, on the other hand, γ must be about 45°, giving approximately equal sensitivity to longitudinal and transverse velocity fluctuations. It is therefore clear that a single probe of either inclination is unsuitable for measuring all nine variables, two separate experiments being required. However, it will be necessary, in determining the Reynolds stresses to include the influence of the secondary flows, implying added complexity. A discussion of the interpretation of hot wire signals in the presence of mean transverse convections will be postponed until Section 3, the case of small secondary flows, that is, secondary flow velocities considerably less than the r.m.s. turbulence fluctuations, being considered first.

2.1. *A Single Sensor Method* (*METHOD* 1).

The inclined hot wire sensor shown in Fig. 4 and again as wire 'a' in Fig. 5 is to be used to deduce the Reynolds stresses. From the previous section the response of the sensor may be written

$$E_a^2-E_{o_a}^2 = K_a U_1^n \sin^m(\theta+\psi) \tag{3}$$

which under the above conditions, $\sin\psi \simeq \psi \simeq U_4/U_1$ and $\cos\psi \simeq 1$ gives for the steady response,

$$\left\{\frac{(E_a^2-E_{o_a}^2)}{(E_{c_a}^2-E_{o_a}^2)}\right\}^{1/n} = \frac{U_1}{U_0}\left[1+\frac{U_4}{U_1}\cot\theta\right]^{m/n} = A$$

†It should be noted that even in low turbulence flow many of the secondary terms in the series expansion for U, are not negligible under certain conditions.

eliminating K_a *via* calibration at a point where $E = E_c$, $U_1 = U_0$ and $U_2 = U_3 = U_4 = 0$. Expanding the bracketed term as a binomial series, and taking only the first two terms of that series,

$$A = \frac{U_1}{U_0} + \beta \frac{U_4}{U_0} \tag{4}$$

or $A = U_1/U_0$ for small secondary flows.

The quantity $\beta = m/n \cot \theta$ is a constant for a given wire and must be determined by a yaw calibration, as outlined in Sections 1.1 and 6.

The instantaneous wire response can be examined, in this simple case, by differentiating equation (3) to give

$$2E_a\, dE_a = K_a\, nU_1^{n-1}\, dU_1 \sin^m(\theta+\psi) + K_a\, U_1^n\, m \sin^{m-1}(\theta+\psi)\cos(\theta+\psi)\, d\psi,$$

which on substituting e_a for dE_a, u_1 for dU_1, and u_4/U_1 for $d\psi$, becomes

$$e_a = \frac{n(E_a^2 - E_{0_a}^2)}{2E_a}\left[\frac{u_1}{U_1} + \frac{m}{n}\cot(\theta+\psi)\frac{u_4}{U_1}\right] \tag{5}$$

or, writing $m/n \cot(\theta+\psi) = \beta$ in the absence of appreciable secondary flows,

$$\alpha_a e_a = \frac{u_1}{U_0} + \beta \frac{u_4}{U_0} \tag{6}$$

where

$$\alpha_a = \frac{2E_a A}{n(E_a^2 - E_{0_a}^2)}$$

Taking he time averaged square of equation (6)

$$\boxed{\alpha_a^2 \overline{e_a^2} = \frac{\overline{u_1^2}}{U_0^2} + 2\beta \frac{\overline{u_1 u_4}}{U_0^2} + \beta^2 \frac{\overline{u_4^2}}{U_0^2}} \tag{7}$$

an equation for the Reynolds stresses.

The interpretation of the quantities U_4 and u_4 will depend upon the choice of the angle ϕ in Fig. 4. For example, when $\phi = 0°$, orientation (i), $U_4 = U_2$ and $u_4 = u_2$, or when the probe is rotated to $\phi = 45°$, orientation (ii), $U_4 = (U_2 + U_3)/\sqrt{2}$ etc. In general, then, equation (7) will have the form

$$\alpha^2 \overline{e_r^2} = C_{1\,r}\frac{\overline{u_1^2}}{U_0^2} + C_{2\,r}\frac{\overline{u_2^2}}{U_0^2} + C_{3\,r}\frac{\overline{u_3^2}}{U_0^2} + C_{4\,r}\frac{\overline{u_1 u_2}}{U_0^2} + C_{5\,r}\frac{\overline{u_1 u_3}}{U_0^2} + C_{6\,r}\frac{\overline{u_2 u_3}}{U_0^2},$$

using r to denote the various orientations and dropping the subscript a temporarily from α and e. By rotating the sensor about the x_1 axis in increments of 45° from $\phi = 0°$, eight equations for the six Reynolds stresses are given, having the following coefficients.

METHOD 1

ORIENTATION	$\phi°$	C_1	C_2	C_3	C_4	C_5	C_6
i	0	1	β^2	0	2β	0	0
ii	45	1	$\beta^2/2$	$\beta^2/2$	$\sqrt{2}\beta$	$\sqrt{2}\beta$	β^2
iii	90	1	0	β^2	0	2β	0
iv	135	1	$\beta^2/2$	$\beta^2/2$	$-\sqrt{2}\beta$	$\sqrt{2}\beta$	$-\beta^2$
v	180	1	β^2	0	-2β	0	0
vi	225	1	$\beta^2/2$	$\beta^2/2$	$-\sqrt{2}\beta$	$-\sqrt{2}\beta$	β^2
vii	270	1	0	β^2	0	-2β	0
viii	315	1	$\beta^2/2$	$\beta^2/2$	$\sqrt{2}\beta$	$-\sqrt{2}\beta$	$-\beta^2$

The quantities α and β, which are orientation dependent in the general case, can be considered constant when the secondary flows are small. Hence, from the steady response

$$\frac{U_1}{U_0} = A_i \text{ or } A_{ii} \text{ etc.}$$

and from the time-averaged response

$$\frac{\overline{u_1 u_2}}{U_0^2} = \frac{\alpha^2}{4\beta}(\overline{e_i^2} - \overline{e_v^2}) .$$

$$\frac{\overline{u_1 u_3}}{U_0^2} = \frac{\alpha^2}{4\beta}\left[(\overline{e_{iii}^2} - \overline{e_{vii}^2}) , \right]$$

$$\frac{\overline{u_2 u_3}}{U_0^2} = \frac{\alpha^2}{4\beta^2}\left[(\overline{e_{ii}^2} + \overline{e_{vi}^2}) - (\overline{e_{iv}^2} + \overline{e_{viii}^2}) \right]$$

and $$\frac{(\overline{u_2^2}-\overline{u_3^2})}{U_0^2} = \frac{\alpha^2}{2\beta^2}\left[(\overline{e_i^2}+\overline{e_v^2})-(\overline{e_{iii}^2}+\overline{e_{vii}^2})\right].$$

The two stresses $\overline{u_2^2}$ and $\overline{u_3^2}$ cannot be separated unless hn independent measurement of $\overline{u_1^2}$ is made with a second sensor placed normal to the flow. Calling this orientation (*ix*), we have finally,

$$\frac{\overline{u_2^2}}{U_0^2} = \frac{\alpha^2}{2\beta^2}(\overline{e_i^2}+\overline{e_v^2})-\frac{1}{\beta^2}\left(\frac{\overline{u_1^2}}{U_0^2}\right)_{ix}$$

and $$\frac{\overline{u_3^2}}{U_0^2} = \frac{\alpha^2}{2\beta^2}(\overline{e_{iii}^2}+\overline{e_{vii}^2})-\ \frac{1}{\beta^2}\left(\frac{\overline{u_1^2}}{U_0^2}\right)_{ix}.$$

All six Reynolds stresses are thus determined from a total of 9 r.m.s. voltage readings.

2.2. *X-Probe Method.*

Multi-sensor probes, usually in an X-array for Reynolds stress measurements, have largely superseded the single sensor by offering a reduction in labour and an improved accuracy. If it is assumed that the two wires of the X-probe are sufficiently close together that they see the same approaching flow, or put another way, the instantaneous signals from the two wires are highly correlated, then in addition to the signals e_a and e_b it is possible to extract meaningful composite signals such as e_{a+b}, e_{a-b} or more complex combinations like $e_{a+b}\, e_{a-b}$ or $e_{a+b}\, e_{a-b}^{\,2}$ *etc.* In determining the Reynolds stresses, by the present method, only addition and subtraction facilities are required.

Assuming wire 'b' in Fig. 5 is equally inclined to the x_1-axis and has the same index *n* hnd ratio *m*/*n* as wire 'a', then equations (4) and (5) can be rewritten,

$$B = \frac{U_1}{U_0}-\beta\frac{U_4}{U_0} \simeq \frac{U_1}{U_0}, \tag{8}$$

$$\alpha_b\, e_b = \frac{u_1}{U_0}-\beta\frac{u_4}{U_0} \tag{9}$$

and for instantaneous sum and difference composite signals

$$\alpha_a\alpha_b\, e_{a+b} = (\alpha_a+\alpha_b)\frac{u_1}{U_0}+(\alpha_b-\alpha_a)\frac{u_4}{U_0}, \tag{10}$$

$$\alpha_a\alpha_b\, e_{a-b} = (\alpha_b-\alpha_a)\frac{u_1}{U_0}+(\alpha_a+\alpha_b)\frac{u_4}{U_0}, \tag{11}$$

Taking the mean square of equations (9), (10) and (11),

$$\alpha_b^2\,\overline{e_b^2} = \frac{\overline{u_1^2}}{U_0^2}-2\beta\frac{\overline{u_1\, u_4}}{U_0^2}+\beta^2\frac{\overline{u_4^2}}{U_0^2}. \tag{12}$$

$$\alpha^2\,\overline{e_{a+b}^2} = 4\frac{\overline{u_1^2}}{U_0^2}+8\beta\, D\frac{u_1\, u_4}{U_0^2}, \tag{13}$$

$$\alpha^2 \overline{e^2_{a-b}} = 8\beta D \frac{\overline{u_1 u_4}}{U_0^2} + 4\beta^2 \frac{\overline{u_4^2}}{U_0^2} \tag{1}$$

neglecting terms in D^2

where $\alpha = 2\,\alpha_a\alpha_b \quad (\alpha_a+\alpha_b)$ and $D = (\alpha_b-\alpha_a) \quad (\alpha_a+\alpha_b)$.

The ratio 'D' expresses the non-dimensional difference between the two wires of the X-array and can be made equal to zero during calibration by adjusting the temperature of one wire to give,

$$E_{c_a} \quad (E^2_{c_a} - E^2_{0_a}) = E_{c_b} \quad (E^2_{c_b} - E^2_{0_b}),$$

a relatively simple but not essential operation.

If 'zeroed' in this way then subsequent differences arising away from calibration will be caused by a combination of,

(a) secondary flows,

(b) deviations in the two response characteristics,

and (c) velocity gradient errors, the two wires seeing slightly different velocity vectors.

Again the interpretation of the quantities U_4 and u_4 depends on the rotation ϕ of the plane of the X-probe. In each orientation six possible signals E_a, E_b, e_a, e_b, e_{a+b} and e_{a-b} can be extracted but the choice of which signals to observe and in which orientations will determine both the economy and the accuracy of the method. It is possible to deduce a method having maximum economy, *see* table below, in which the six Reynolds stresses are determined from only six r.m.s. voltage readings. (METHOD 2).

PROBE ORIENTATION	ϕ	PLANE OF X-PROBE	OBSERVED QUANTITIES		CALIBRATION	
			STEADY (DC)	INTEGRATED (rms)	ZERO FLOW	$U_1 = U_o$
(i)	0°	$x_1 - x_2$	E_a, E_b	$\overline{e^2_{a_i}}, \overline{e^2_{b_i}}$	E_{0_a} E_{0_b}	E_{c_a} E_{c_b}
(ii)	45°			$\overline{e^2_{a+b_{ii}}}, \overline{e^2_{a-b_{ii}}}$		
(iii)	90°	$x_1 - x_3$		$\overline{e^2_{a_{iii}}}, \overline{e^2_{b_{iii}}}$		

However, as will be shown later, such a method has very poor accuracy for certain of the stresses and is not recommended. Maximum accuracy can be achieved by taking measurements in four planes and this method will be developed in more detail. (METHOD 3).

METHOD 3

ORIENTATION (i)

$$\phi = 0 \qquad \text{so that } u_4 = u_2,$$

$$\alpha_a^2 \overline{e^2_{a_i}} = \frac{\overline{u_1^2}}{U_0^2} + 2\beta \frac{\overline{u_1 u_2}}{U_0^2} + \beta^2 \frac{\overline{u_2^2}}{U_0^2}, \tag{15}$$

$$\alpha_b^2\,\overline{e_{b_i}^2} = \frac{\overline{u_1^2}}{U_0^2} - 2\beta\frac{\overline{u_1 u_2}}{U_0^2} + \beta^2\frac{\overline{u_2^2}}{U_0^2}, \tag{16}$$

$$\alpha^2\,\overline{e_{a-b_i}^2} = 8\beta D\frac{\overline{u_1 u_2}}{U_0^2} + 4\beta^2\frac{\overline{u_2^2}}{U_0^2}. \tag{17}$$

ORIENTATION (ii)

$$\phi = 45^\circ,\, u_4 = \frac{(u_2+u_3)}{\sqrt{2}},$$

$$\alpha^2\,\overline{e_{a+b_{ii}}^2} = 4\frac{\overline{u_1^2}}{U_0^2} + 4\sqrt{2}\,\beta\,D\left(\frac{\overline{u_1 u_2}}{U_0^2} + \frac{\overline{u_1 u_3}}{U_0^2}\right), \tag{18}$$

$$\alpha^2\,\overline{e_{a-b_{ii}}^2} = 4\sqrt{2}\,\beta\,D\left(\frac{\overline{u_1 u_2}}{U_0^2} + \frac{\overline{u_1 u_3}}{U_0^2}\right) + 2\beta^2\left(\frac{\overline{u_2^2}}{U_0^2} + \frac{\overline{u_3^2}}{U_0^2} + 2\frac{\overline{u_2 u_3}}{U_0^2}\right). \tag{19}$$

ORIENTATION (iii)

$$\phi = 90^\circ,\, u_4 = u_3$$

$$\alpha_a^2\,\overline{e_{a_{iii}}^2} = \frac{\overline{u_1^2}}{U_0^2} + 2\beta\frac{\overline{u_1 u_3}}{U_0^2} + \beta^2\frac{\overline{u_3^2}}{U_0^2}. \tag{20}$$

$$\alpha_b^2\,\overline{e_{b_{iii}}^2} = \frac{\overline{u_1^2}}{U_0^2} - 2\beta\frac{\overline{u_1 u_3}}{U_0^2} + \beta_2\frac{\overline{u_3^2}}{U_0^2}, \tag{21}$$

$$\alpha^2\,\overline{e_{a-b_{iii}}^2} = 8\beta D\frac{\overline{u_1 u_3}}{U_0^2} + 4\beta^2\frac{\overline{u_3^2}}{U_0^2}. \tag{22}$$

ORIENTATION (iv)

$$\phi = 135^\circ,\, u_4 = \frac{(u_3-u_2)}{\sqrt{2}},$$

$$\alpha^2\,\overline{e_{a-b_{iv}}^2} = 4\sqrt{2}\,\beta\,D\left(\frac{\overline{u_1 u_3}}{U_0^2} - \frac{\overline{u_1 u_2}}{U_0^2}\right) + 2\beta^2\left(\frac{\overline{u_2^2}}{U_0^2} + \frac{\overline{u_3^2}}{U_0^2} - 2\frac{\overline{u_2 u_3}}{U_0^2}\right). \tag{23}$$

From equations (15) to (23) it follows that

$$\frac{U_1}{U_0} = A_i \text{ or } B_i\ etc.,$$

or more exactly

$$\frac{U_1}{U_0} = \frac{(A_i+B_i)}{2},$$

if secondary flows are present, and for the Reynolds stresses,

$$\frac{\overline{u_1 u_2}}{U_0^2} = \left[\alpha_a^2 \overline{e_{a_i}^2} - \alpha_b^2 \overline{e_{b_i}^2}\right] \Big/ 4\beta,$$

$$\frac{\overline{u_1 u_3}}{U_0^2} = \left[\alpha_a^2 \overline{e_{a_{iii}}^2} - \alpha_b^2 \overline{e_{b_{iii}}^2}\right] \Big/ 4\beta,$$

$$\frac{\overline{u_1^2}}{U_0^2} = \alpha^2 \overline{e_{a+b_{ii}}^2}/4 - \sqrt{2}\,\beta D \left(\frac{\overline{u_1 u_2}}{U_0^2} + \frac{\overline{u_1 u_3}}{U_0^2}\right),$$

$$\frac{\overline{u_2^2}}{U_0^2} = \alpha^2 \overline{e_{a-b_i}^2}/4\beta^2 - \frac{2}{\beta} D \frac{\overline{u_1 u_2}}{U_0^2},$$

$$\frac{\overline{u_3^2}}{U_0^2} = \alpha^2 \overline{e_{a-b_{iii}}^2}/4\beta^2 - \frac{2}{\beta} D \frac{\overline{u_1 u_3}}{U_0^2},$$

or

$$\frac{(\overline{u_2^2} - \overline{u_3^2})}{U_0^2} = \left[\alpha^2 \overline{e_{a-b_i}^2} - \alpha^2 \overline{e_{a-b_{iii}}^2}\right] \Big/ 4\beta^2 + \frac{2}{\beta} D \left[\frac{\overline{u_1 u_3}}{U_0^2} - \frac{\overline{u_1 u_2}}{U_0^2}\right]$$

and

$$\frac{\overline{u_2 u_3}}{U_0^2} = \left[\alpha^2 \overline{e_{a-b_{ii}}^2} - \alpha^2 \overline{e_{a-b_{iv}}^2}\right] \Big/ 8\beta^2 - \frac{\sqrt{2}}{\beta} D \frac{\overline{u_1 u_2}}{U_0^2}.$$

The 'D' terms will in general be small and negligible for a well constructed X-probe, when the cross flows are small, thus simplifying the analysis. Since the longitudinal stress $\overline{u_1^2}$ can be measured in any orientation in terms of $\overline{e_{a+b}^2}$ and the local D-term, the significance of these extra difference terms can be estimated by comparing the values of $\overline{e_{a+b}^2}$ in different orientations; a very quick and useful check. A constant-current method of the above form has been presented by Gessner[15], but unfortunately it was assumed that the two wires of the X-probe were identical and cooled only by the normal velocity component.

2.3. *Comparison of the Methods.*

In order to determine the Reynolds stresses with maximum accuracy each must depend on a minimum number of measured quantities. A table can be drawn up to compare the three methods discussed, on this basis.

REYNOLDS STRESS	NO. OF MEASUREMENTS REQUIRED		
	SINGLE INCLINED SENSOR METHOD 1	X-PROBE (MAX. ECONOMY) METHOD 2	X-PROBE (MAX. ACCURACY) METHOD 3
$\overline{u_1^2}$	1 (NORMAL WIRE)	1	1
$\overline{u_2^2}, \overline{u_3^2}$	3	3	1
$(\overline{u_2^2}-\overline{u_3^2})$	4	4	2
$\overline{u_1 u_2}, \overline{u_1 u_3}$	2	2	2
$\overline{u_2 u_3}$	4	6	2

The X-probe, used in conjunction with an addition/subtraction device is clearly a superior instrument for the measurement of the transverse direct stresses, $\overline{u_2^2}$ and $\overline{u_3^2}$, their difference, $(\overline{u_2^2}-\overline{u_3^2})$ and the secondary shear stress, $\overline{u_2 u_3}$. The reliability of the X-probe estimates of $\overline{u_1^2}$, $\overline{u_1 u_2}$ and $\overline{u_1 u_3}$ will depend upon the lengths to which one is prepared to go in allowing for small differences which might occur in the behaviour of the two wires. Differences caused by a local velocity gradient cannot however be simply eliminated, making the X-probe a secondary device for such measurements. These three stresses must therefore be measured using a single sensor method, a 'normal wire' for $\overline{u_1^2}$ and a 45° inclined wire for the shear stresses, $\overline{u_1 u_2}$ and $\overline{u_1 u_3}$. When all six Reynolds stresses are required it is often necessary to bypass the latter rule both to avoid unnecessary experimental complications and to improve the local consistency between the stresses. Installing a slightly different shaped probe into a slightly different flow some hours or days later to obtain the remaining components of the stress tensor will probably introduce more errors than using the X-probe to measure the entire tensor in the first place. Such a test might however constitute a check on the X-probe performance.

2.4. *Sources of Error.*

Two additional sources of error in the Reynolds stress measurements, not discussed in the previous sections, will be covered here. The first arises due to instrument or reading errors and the second from a mechanical misalignment of the probe plane.

It is required to estimate the likely error produced in each stress by a given percentage error in the r.m.s. voltage measurements. In general the r.m.s. readings arising from the integration of e_a, e_b, $\frac{1}{2}e_{a+b}$ and e_{a-b} do not differ greatly at a point in the flow, so that the stresses very often emerge from the subtraction of two very nearly equal quantities. If e is defined to represent the average of these r.m.s. voltages, in which a reading error of η can occur, then a quantity such as $(\overline{e_a^2}-\overline{e_b^2})$ can be assigned the tolerance $\pm 4\eta e$. The tolerance on each stress can now be estimated in this way (*see* table below) and further, by assigning typical values to the variables, the percentage error in the stress can be assessed.

Example Suppose we are situated near to the wall, $x_2 = 0$, in a zero pressure gradient two-dimensional turbulent boundary layer, and the following readings are observed. $E = 2v$, $E_o = 1{\cdot}4v$, $n = {\cdot}5$, $U_1/U_0 = {\cdot}5$, so that $\alpha = 2$, $e = 25mV$, $\beta = 1$ and η, the reading error, is, say, 1 per cent of e.

REYNOLDS STRESSES			$\dfrac{\overline{u_1^2}}{U_0^2}$	$\dfrac{\overline{u_2^2}}{U_0^2}, \dfrac{\overline{u_3^2}}{U_0^2}$	$\dfrac{\overline{u_1 u_2}}{U_0^2}, \dfrac{\overline{u_1 u_3}}{U_0^2}$	$\dfrac{(\overline{u_2^2}-\overline{u_3^2})}{U_0^2}$	$\dfrac{\overline{u_2 u_3}}{U_0^2}$
SINGLE SENSOR		TOLERANCE $\pm$	$2\alpha^2 \eta e$	$4\alpha^2 \eta e/\beta^2$	$\alpha^2 \eta e/\beta$	$4\alpha^2 \eta e/\beta^2$	$2\alpha^2 \eta e/\beta^2$
SINGLE SENSOR	EXAMPLE—1% READING ERROR	MAGNITUDE OF TOLERANCE $\pm$	$1{\cdot}0 \times 10^{-4}$	$2{\cdot}0 \times 10^{-4}$	$\cdot 5 \times 10^{-4}$	$2{\cdot}0 \times 10^{-4}$	$1{\cdot}0 \times 10^{-4}$
SINGLE SENSOR	EXAMPLE—1% READING ERROR	PERCENTAGE OF STRESS $\pm$	2·0	10·0	3·5	20·0	
X-PROBE (MAX. ACCURACY)		TOLERANCE $\pm$	$2\alpha^2 \eta e$	$\alpha^2 \eta e/2\beta^2$	$\alpha^2 \eta e/\beta$	$\alpha^2 \eta e/\beta^2$	$\alpha^2 \eta e/2\beta^2$
X-PROBE (MAX. ACCURACY)	EXAMPLE—1% READING ERROR	MAGNITUDE OF TOLERANCE $\pm$	$1{\cdot}0 \times 10^{-4}$	$\cdot 25 \times 10^{-4}$	$\cdot 5 \times 10^{-4}$	$\cdot 5 \times 10^{-4}$	$\cdot 25 \times 10^{-4}$
X-PROBE (MAX. ACCURACY)	EXAMPLE—1% READING ERROR	PERCENTAGE OF STRESS $\pm$	2·0	1·5	3·5	5·0	

Under these circumstances, which are fairly realistic, the tolerances on the stress readings are as given in the table and where an appropriate value for the stress can be postulated this tolerance is written also as a percentage error /1 per cent reading error. Estimates for the error in $\overline{u_2 u_3}$ can be made using a different example, the square duct flow. Errors of as much as ± 100 per cent for the single sensor method or ± 25 per cent using the X-probe are suggested. It should be mentioned in concluding the argument that in the author's experience reading errors of ± 3 per cent are frequently unavoidable in r.m.s. turbulence measurements even with extremely long integration times. In general the errors are distributed randomly through the results, within the tolerances given, so that the percentage errors above probably represent overestimates.

The second source of error worthy of mention arises when a small misalignment of $\Delta°$ occurs in the angle ϕ. If it is assumed that the 45° indexing device operates correctly but has a constant misalignment, with respect to the $x_2 - x_3$ frame, of $\Delta°$, then the interpretation of the quantity u_4 must be revised slightly. In orientation (i), for example,

$$\begin{aligned} u_4 &= u_2 \cos \Delta + u_3 \sin \Delta \\ &= u_2 + u_3 \Delta \text{ when } \Delta \text{ is small.} \end{aligned}$$

Hence for the X-probe

$$(\alpha_a^2 \overline{e_a^2} - \alpha_b^2 \overline{e_b^2})_i = 4\beta \left(\frac{\overline{u_1 u_2}}{U_0^2} + \Delta \frac{\overline{u_1 u_3}}{U_0^2} \right).$$

Similarly, in orientation (iii),

$$(\alpha_a^2 \overline{e_a^2} - \alpha_b^2 \overline{e_b^2})_{iii} = 4\beta \left(\frac{\overline{u_1 u_3}}{U_0^2} - \Delta \frac{\overline{u_1 u_2}}{U_0^2} \right).$$

From these equations it is possible to solve for the two stresses $\overline{u_1 u_2}$ and $\overline{u_1 u_3}$ despite the misalignment. Returning to the example used above, where $\overline{u_1 u_2}$ is very much greater than $\overline{u_1 u_3}$ it is clear that the estimate of $\overline{u_1 u_3}$ will depend very heavily on the value of Δ.

Taking the more complex case,

$$(\alpha^2 \overline{e^2_{a-b_i}} - \alpha^2 \overline{e^2_{a-b_{iii}}}) = 4\beta^2 \left[\frac{(\overline{u_2^2}-\overline{u_3^2})}{U_o^2} + 4\Delta \frac{\overline{u_2 u_3}}{U_o^2} \right]$$

and

$$(\alpha^2 \overline{e^2_{a-b_{ii}}} - \alpha^2 \overline{e^2_{a-b_{iv}}}) = 4\beta^2 \left[2\frac{\overline{u_2 u_3}}{U_o^2} - 2\Delta \frac{(\overline{u_2^2}-\overline{u_3^2})}{U_o^2} \right]$$

neglecting the 'D' terms.

Again these two equations can be solved for $\overline{u_2 u_3}$ and $(\overline{u_2^2}-\overline{u_3^2})$. In situations where $(\overline{u_2^2}-\overline{u_3^2}) > > \overline{u_2 u_3}$, large errors will be induced in $\overline{u_2 u_3}$ if the Δ terms are neglected, and *vice versa*. This represents a significant source of error since it is, in general, difficult to align the probe correctly, even though its final position can be checked very accurately.

3. *Reynolds Stress Measurements in the Presence of Large Secondary Flows.*

At the beginning of Section 2 attention was concentrated on the case where the secondary flow velocities are very much less than the r.m.s. turbulence fluctuations. It is instructive to see how this analysis can be extended to deal with the transverse flows originally postulated in the range ± 5 per cent of the local primary velocity. Rewriting equations (4), (6), (8), (9), (10) and (11) for the X-probe, for example,

$$A = \frac{U_1}{U_0} + \beta \frac{U_4}{U_0},$$

$$B = \frac{U_1}{U_0} - \beta \frac{U_4}{U_0},$$

$$\alpha_a e_a = \frac{u_1}{U_0} + \beta'_a \frac{u_4}{U_0},$$

$$\alpha_b e_b = \frac{u_1}{U_0} - \beta'_b \frac{u_4}{U_0},$$

$$\alpha_a \alpha_b e_{a+b} = (\alpha_a + \alpha_b) \frac{u_1}{U_0} + (\beta'_a \alpha_b - \beta'_b \alpha_a) \frac{u_4}{U_0},$$

$$\alpha_a \alpha_b e_{a-b} = (\alpha_b - \alpha_a) \frac{u_4}{U_0} + (\beta'_a \alpha_b + \beta'_b \alpha_a) \frac{u_4}{U_0},$$

where

$$\left.\begin{aligned} \beta &= m/n \cot\theta, \\ \alpha_a &= 2E_a \left\{ \frac{\dfrac{U_1}{U_0}}{n(E_a^2 - E_{0_a}^2)} \right\} \end{aligned}\right\} \quad \frac{U_1}{U_0} = \frac{(A+B)}{2}$$

$$\left.\begin{aligned}\beta'_a &= m/n \cot(\theta+\psi) \\ \text{and} \qquad \beta'_b &= m/n \cot(\theta-\psi)\end{aligned}\right\} \quad \psi = \frac{U_4}{U_1} = \frac{(A-B)}{\beta(A+B)}$$

Retaining the previous definitions for α and D, two further quantities are introduced,

$$D' = (\beta'_a \alpha_b - \beta'_b \alpha_a) \Big/ (\alpha_a + \alpha_b) \text{ and } S' = (\beta'_a \alpha_b + \beta'_b \alpha_a) \Big/ (\alpha_a + \alpha_b)$$

So that the mean square equations can be written,

$$\alpha_a^2 \overline{e_a^2} = \frac{\overline{u_1^2}}{U_0^2} + 2\beta'_a \frac{\overline{u_1 u_4}}{U_0^2} + \beta'^2_a \frac{\overline{u_4^2}}{U_0^2},$$

$$\alpha_b^2 \overline{e_b^2} = \frac{\overline{u_1^2}}{U_0^2} - 2\beta'_b \frac{\overline{u_1 u_4}}{U_0^2} + \beta'^2_b \frac{\overline{u_4^2}}{U_0^2},$$

$$\alpha^2 \overline{e_{a-b}^2} = 4D^2 \frac{\overline{u_1^2}}{U_0^2} + 8DS' \frac{\overline{u_1 u_4}}{U_0^2} + 4S'^2 \frac{\overline{u_4^2}}{U_0^2},$$

$$\alpha^2 \overline{e_{a+b}^2} = 4 \frac{\overline{u_1^2}}{U_0^2} + 8D' \frac{\overline{u_1 u_4}}{U_0^2} + 4D'^2 \frac{\overline{u_4^2}}{U_0^2},$$

The quantities α_a, α_b, β'_a and β'_b hnd hence α, D, D', and S' are now functions of both position in the flow and orientation of the probe, so that no simple expressions for the Reynolds stresses arise. Using the maximum accuracy method, six equations involving the six Reynolds stresses can be derived for simultaneous solution. At each point and in each orientation ψ, the mean inclination of the velocity vector must be calculated from readings of E_a and E_b. As mentioned in the introduction to Section 2, the 45° X-probe is not suitable for yaw measurements so that the accuracy of the resulting Reynolds stresses is likely to be low, the errors arising in estimating coefficients such as $(\beta'_a - \beta'_b) = m/n\,[\cot(\theta+\psi) - \cot(\theta-\psi)]$.

For the mean velocity components,

$$\left(\frac{U_1}{U_0}\right) = (A_i + A_{ii} + A_{iii} + A_{iv} + B_i + B_{ii} + B_{iii} + B_{iv})/8$$

$$\left(\frac{U_2}{U_0}\right) = \left[\left(\frac{U_2}{U_0}\right)_1 + \left(\frac{U_2}{U_0}\right)_2\right] \Big/ 2$$

where

$$\left(\frac{U_2}{U_0}\right)_1 = (A_i - B_i)/2\beta,$$

$$\left(\frac{U_2}{U_0}\right)_2 = [(A_{ii} - B_{ii}) - (A_{iv} - B_{iv})] \Big/ 2\sqrt{2}\beta$$

and

$$\left(\frac{U_3}{U_0}\right) \equiv \left[\left(\frac{U_3}{U_0}\right)_1 + \left(\frac{U_3}{U_0}\right)_2\right] \Big/ 2$$

where

$$\left(\frac{U_3}{U_0}\right)_1 = (A_{iii} - B_{iii})/2\beta\,,$$

$$\left(\frac{U_3}{U_0}\right)_2 = [(A_{ii} - B_{ii}) + (A_{iv} - B_{iv})]\Big/2\sqrt{2}\beta$$

and, for the turbulence correlations,

$$\alpha_{ii}^2 \overline{e_{a+b_{ii}}^2} = 4\frac{\overline{u_1^2}}{U_0^2} + 4\sqrt{2}D'_{ii}\left(\frac{\overline{u_1 u_2}}{U_0^2} + \frac{\overline{u_1 u_3}}{U_0^2}\right) + 2D'^2_{ii}\left(\frac{\overline{u_2^2}}{U_0^2} + \frac{\overline{u_3^2}}{U_0^2} + 2\frac{\overline{u_2 u_3}}{U_0^2}\right),$$

$$\alpha_i{}^2 \overline{e_{a-b_i}^2} = 4D_i^2 \frac{\overline{u_1^2}}{U_0^2} + 8D_i S'_i \frac{\overline{u_1 u_2}}{U_0^2} + 4S'^2_i \frac{\overline{u_2^2}}{U_0^2},$$

$$\alpha_{iii}^2 \overline{e_{a-b_{iii}}^2} = 4D_{iii}^2 \frac{\overline{u_1^2}}{U_0^2} + 8D_{iii}S'_{iii} \frac{\overline{u_1 u_3}}{U_0^2} + 4S'^2_{iii} \frac{\overline{u_3^2}}{U_0^2},$$

$$(\alpha_{a_i}^2 \overline{e_{a_i}^2} - \alpha_{b_i}^2 \overline{e_{b_i}^2}) = 2(\beta'_{a_i} + \beta'_{b_i})\frac{\overline{u_1 u_2}}{U_0^2} + (\beta'^2_{a_i} - \beta'^2_{b_i})\frac{\overline{u_2^2}}{U_0^2},$$

$$(\alpha_{a_{iii}}^2 \overline{e_{a_{iii}}^2} - \alpha_{b_{iii}}^2 \overline{e_{b_{iii}}^2}) = 2(\beta'_{a_{iii}} + \beta'_{b_{iii}})\frac{\overline{u_1 u_3}}{U_0^2} + (\beta'^2_{a_{iii}} - \beta'^2_{b_{iii}})\frac{\overline{u_3^2}}{U_0^2},$$

$$\begin{aligned}(\alpha_{ii}^2 \overline{e_{a-b_{ii}}^2} - \alpha_{iv}^2 \overline{e_{a-b_{iv}}^2}) = {} & 4(D_{ii}^2 - D_{iv}^2)\frac{\overline{u_1^2}}{U_0^2} + 4\sqrt{2}\,(D_{ii}S'_{ii} + D_{iv}S'_{iv}) \\ & \left(\frac{\overline{u_1 u_2}}{U_0^2} + \frac{\overline{u_1 u_3}}{U_0^2}\right) + 2\,(S'^2_{ii} - S'^2_{iv})\left(\frac{\overline{u_2^2}}{U_0^2} + \frac{/\overline{u_3^2}}{U_0^2}\right) \\ & + 4\,(S'^2_{ii} + S'^2_{iv})\frac{\overline{u_2 u_3}}{U_0^2}.\end{aligned}$$

4. *The Use of a Linearizer.*

Consider the general response equation developed at the beginning of Section 1.

$$(E^2 - E_0^2) = KU_{eff}^n = KU^n f(\gamma)$$

$$E+e = K'|U_1+u_1|$$

This has very unfortunate consequences in flows where $|u_1|$ can exceed U_1, that is, in low velocity, highly turbulent flows where reversal of the instantaneous velocity vector can occur. Such reversals are found in the bluff body wake and in the turbulent boundary layer approaching separation, where the velocity scale of the eddy motion exceeds the local mean velocity.

Fig. 6a shows the normal response of the hot wire when $|u_1| \ll U_1$ and Figs. 6b and 6c show the abnormal effect of recification. It is clear that simply measuring the quantities E and $\overline{e^2}$ is not sufficient to separate the variables U_1 and $\overline{u_1^2}$. The distorted signal $e(t)$ is unfortunately meaningless so that its integration yields an equally meaningless quantity. A governing equation exists, however, to relate the four quantities, *viz*:

$$e^2+E^2 = K'^2(U_1^2+\overline{u_1^2}) \tag{24}$$

Hot wire measurements in a separated boundary layer are shown in Fig. 7. The flow reversal suggested by the total pressure-tube traverse is not present in the hot-wire results, but by adjusting U_1 and $\overline{u_1^2}$, according to equation (24), it is possible to recreate the correct form for U_1 and a plausible distribution of $\overline{u_1^2}$. Such a one-dimensional view of the turbulence, although grossly inaccurate, suffices to illustrate the point.

In the highly turbulent flow an alternative measuring device is clearly needed. Such a device has been designed and is presently being developed by Bradbury[20]. The probe consists of two parallel fine filaments which detect a heat pulse released from a central wire. The time taken for the pulse to reach either of the filaments, the 'time of flight', is recorded and *via* calibration can be used to estimate the instantaneous local velocity. By repeated sampling at a point in the flow, the time mean velocity and its r.m.s. fluctuating component can be deduced. The principle can be extended to the measurement of Reynolds stresses other than the longitudinal intensity.

6. *Yaw Calibration in Fully Developed Turbulent Pipe Flow.*

The axial momentum equation in fully developed turbulent flow along a straight smooth circular pipe, has the form

$$\frac{d\tau}{dr}+\frac{\tau}{r}+\frac{1}{\rho}\frac{dp}{dx_1} = 0$$

for which the solution is

$$\tau = \nu\frac{dU_1}{dr} - \overline{u_1 u_r} = -\frac{r}{2\rho}\frac{dp}{dx_1},$$

the total shear stress varying linearly across a diameter from zero on the axis to a value $D/4\rho\, dp/dx_1$ at the wall. In the region away from the pipe wall the viscous stress will be negligible, $\overline{u_1 u_r}$ representing the local total shear stress. Using either the single inclined sensor or the X-probe it is possible to measure this stress and thence deduce the quantity m, m/n or β for the probe, as demonstrated by Newman and Leary[11]. Consider the X-probe situated at a radius r with its axis parallel to the axis of the pipe and the X-array lying in the x_1-r plane, then

$$\overline{u_1 u_r} = \frac{r}{2\rho}\frac{dp}{dx_1} = \frac{U_0^2}{4\beta}(\alpha_a^2\,\overline{e_a^2}-\alpha_b^2\,\overline{e_b^2})$$

or rewriting

$$\beta = (\alpha_a^2\,\overline{e_a^2} - \alpha_b^2\,\overline{e_b^2}) \Bigg/ \frac{r}{p_d}\,\frac{dp}{dx_1}$$

where p is the wall static pressure,

p_d the centreline dynamic pressure when α is written in terms of the calibration voltage at the centreline, and all other quantities are as defined in Section 2.2.

The calibration procedure for an inclined sensor is as follows:

(1) measure θ, preferably on a magnified image in order to obtain an accuracy of better than 0·1°,

(2) calibrate for n in a uniform variable velocity stream with the sensor placed normal to the mean velocity vector

and (3) measure the shear stress distribution across a pipe in the fully developed region.

Step (2) is carried out over a velocity range exceeding that encountered in the subsequent experiments, whereas (3) is conducted at a representative velocity, noting that β is largely insensitive to changes in flow velocity. Fig. 8 shows details of the calibration of a DISA 55 A 38 miniature X-probe used in a boundary-layer experiment where the free stream velocity was 60 ft/second. The wires, which were of platinum plated tungsten, had an l/d ratio of 240, suggesting a value for m/n of approximately 0·9, from the work of Brunn[9].

The pipe flow calibration facility and the rack and pinion turnover probe holder are shown schematically in Fig. 9.

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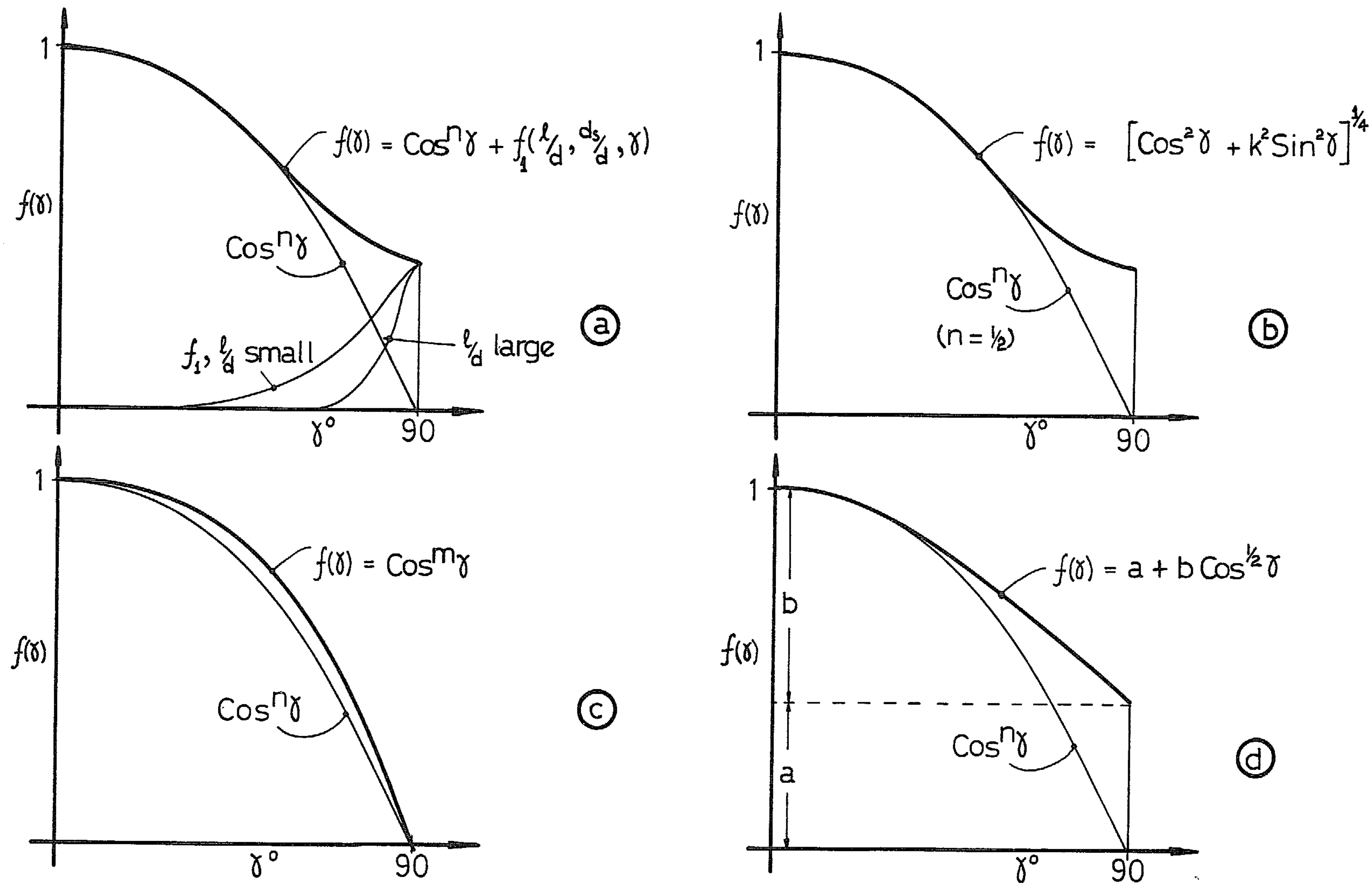

FIG. 2. Hot-wire yaw response.

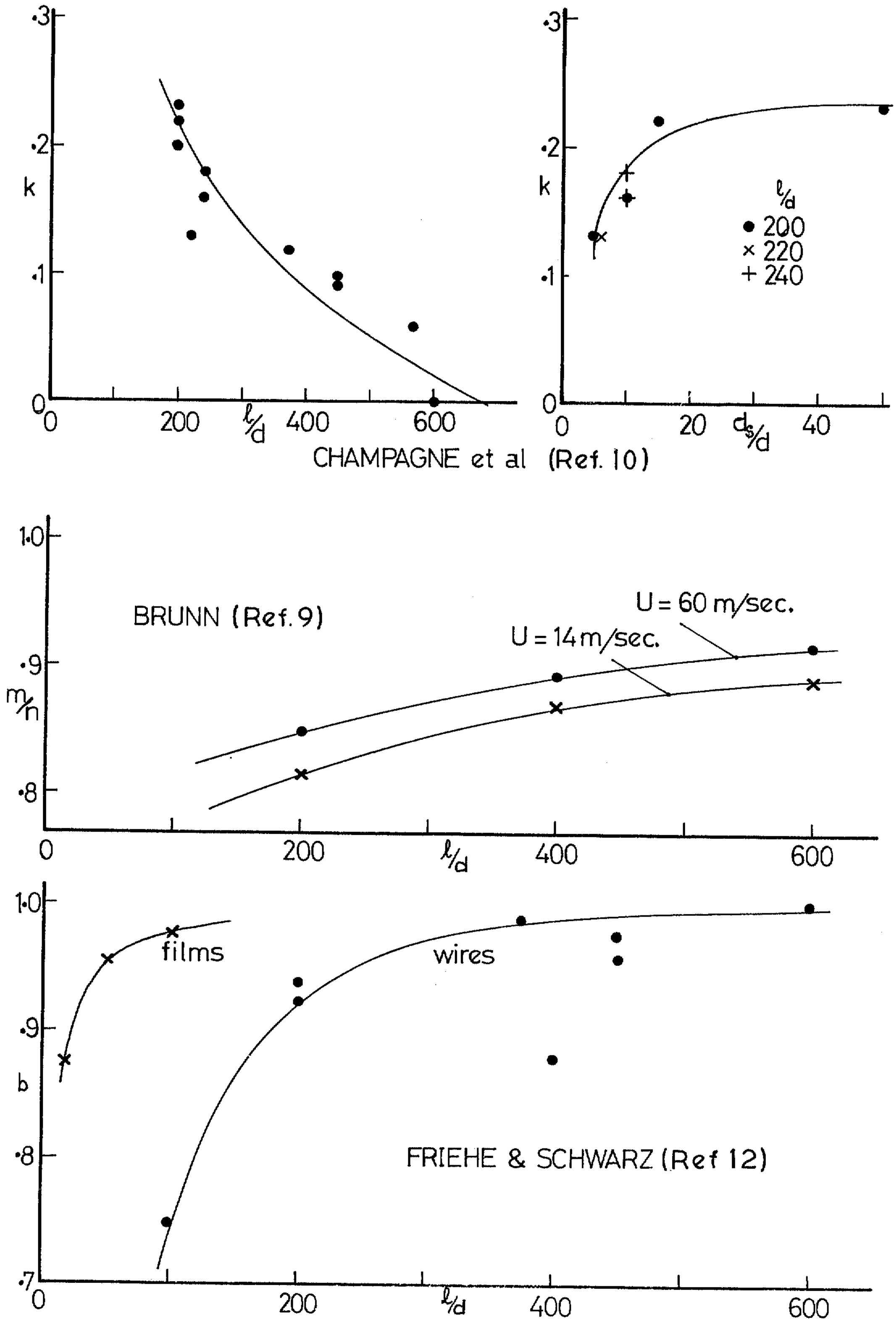

FIG. 3. Yaw parameters.

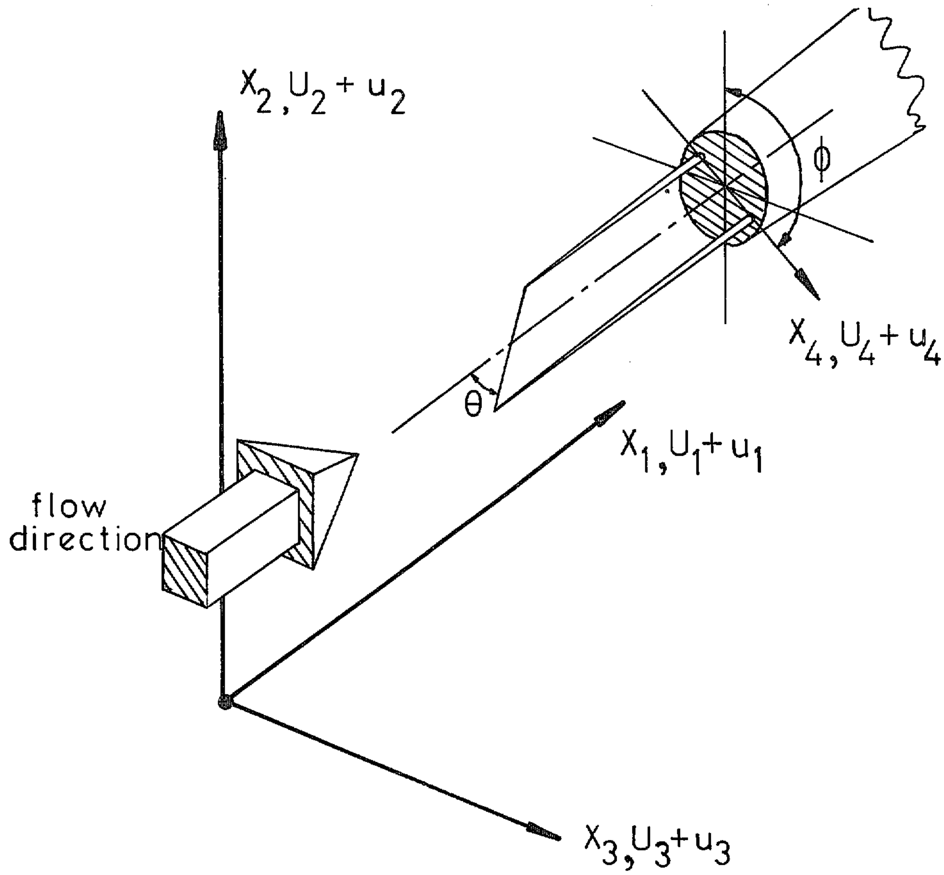
$X_2, U_2 + u_2$
ϕ
$X_4, U_4 + u_4$
θ
$X_1, U_1 + u_1$
flow
direction
$X_3, U_3 + u_3$

FIG. 4.

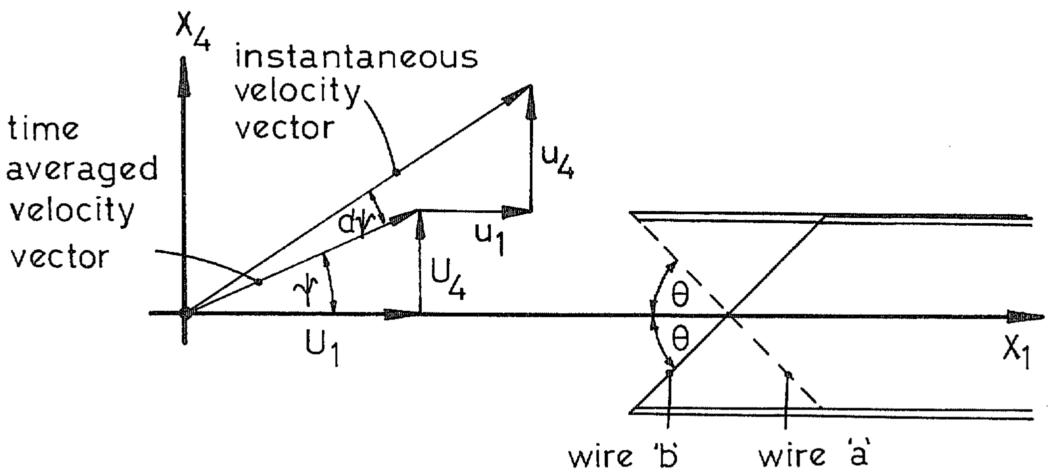
X_4
instantaneous
velocity
vector
time
averaged
velocity
vector
u_4
$d\psi$
u_1
U_4
ψ
U_1
θ
θ
X_1
wire 'b'
wire 'a'

FIG. 5.

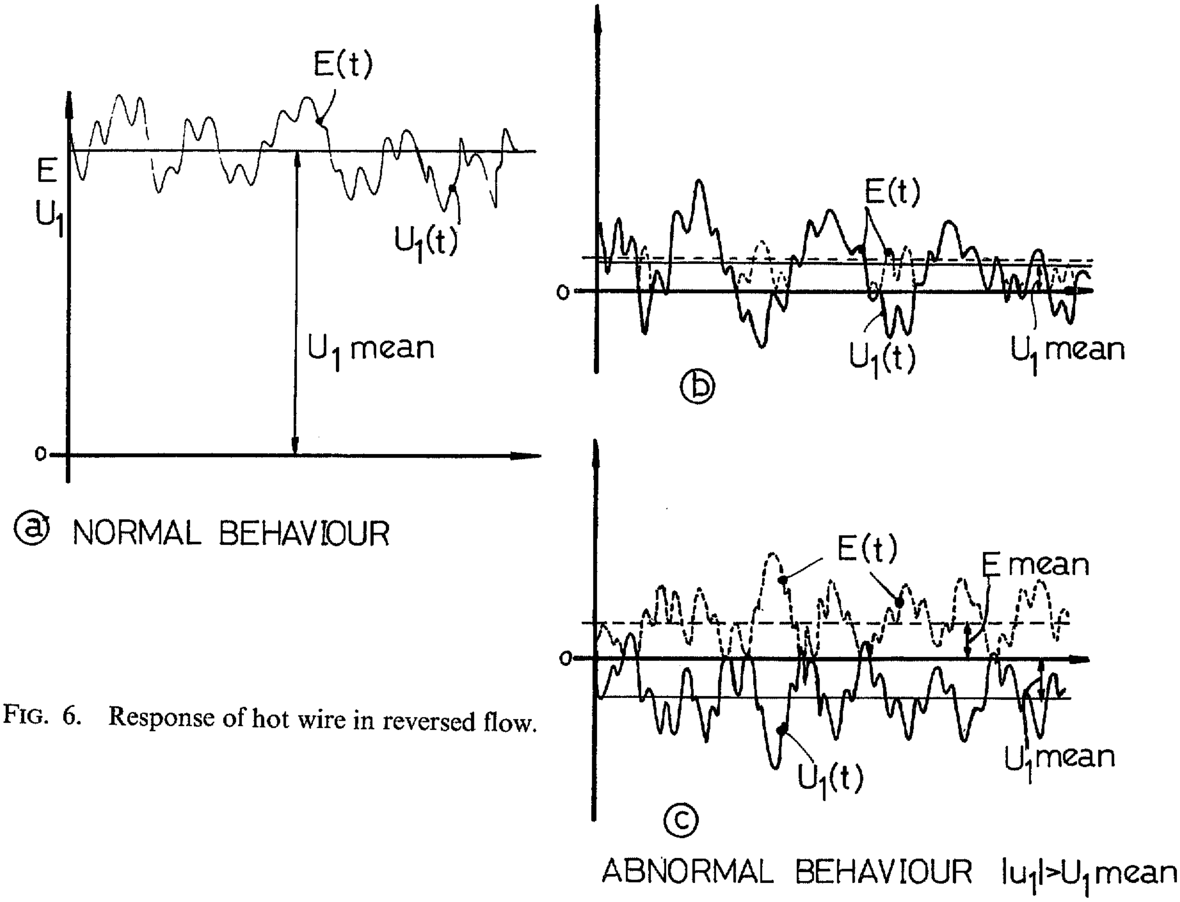

FIG. 6. Response of hot wire in reversed flow.

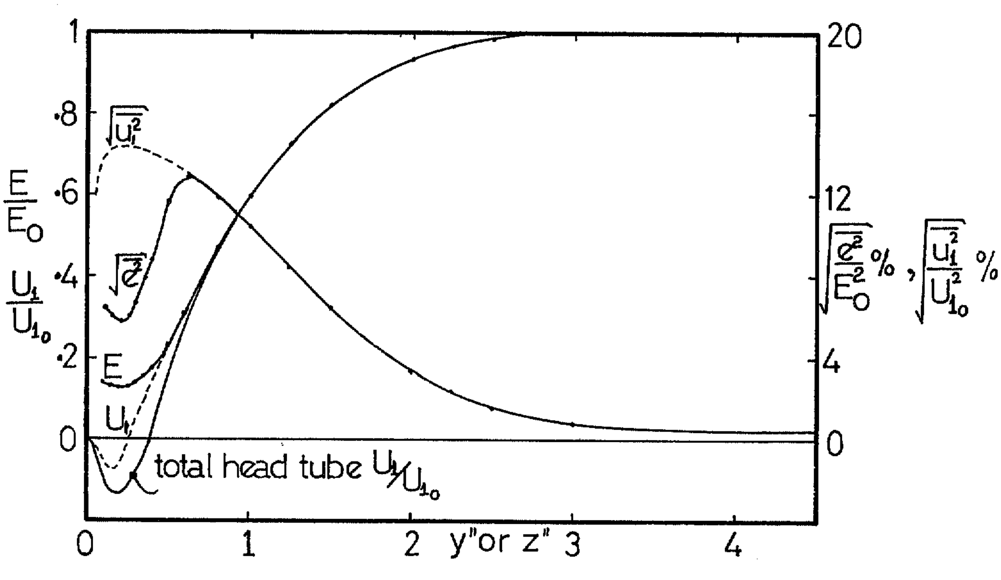

FIG. 7. Response of hot wire in separated flow.

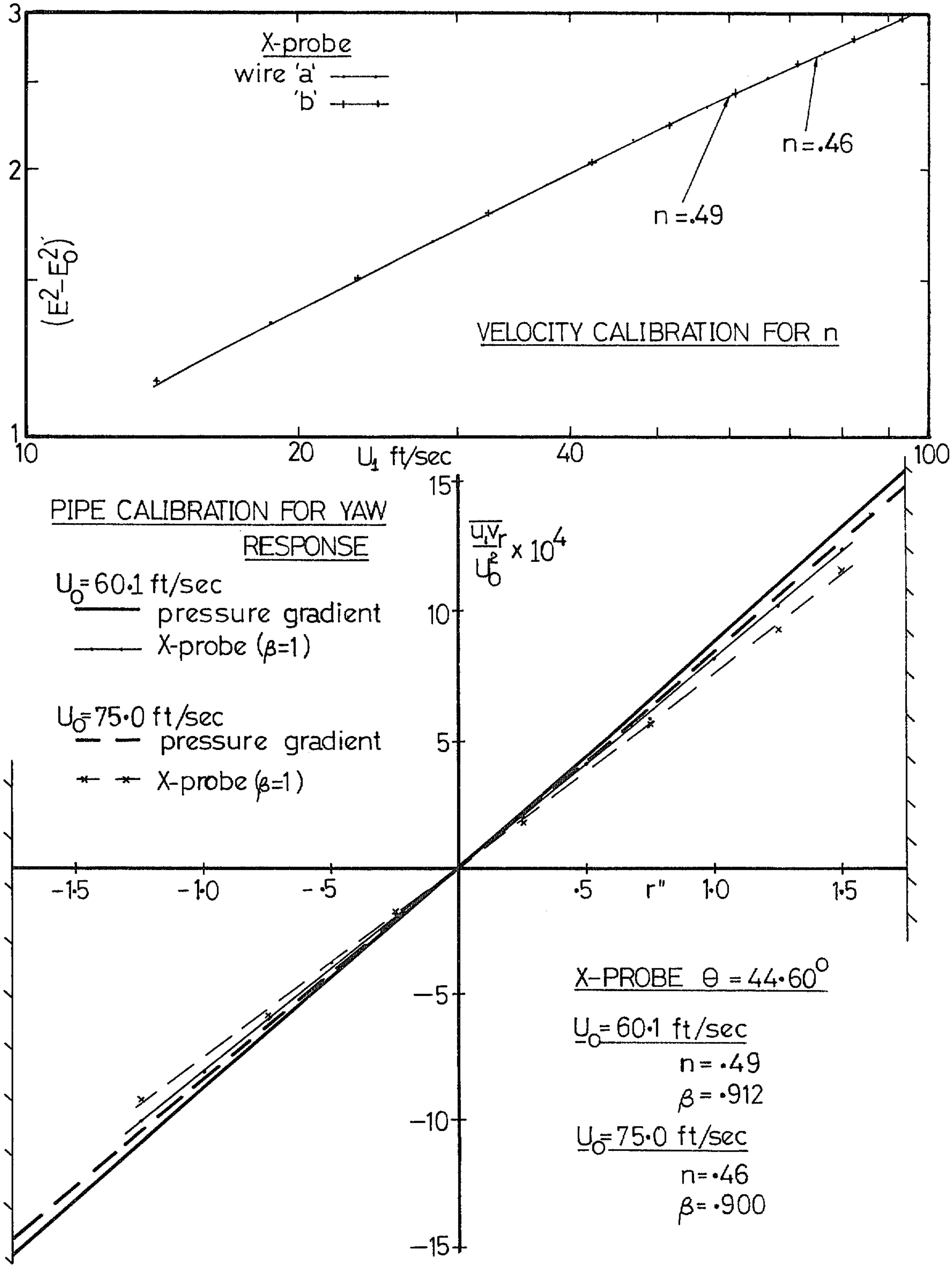

Fig. 8. X-probe calibration.

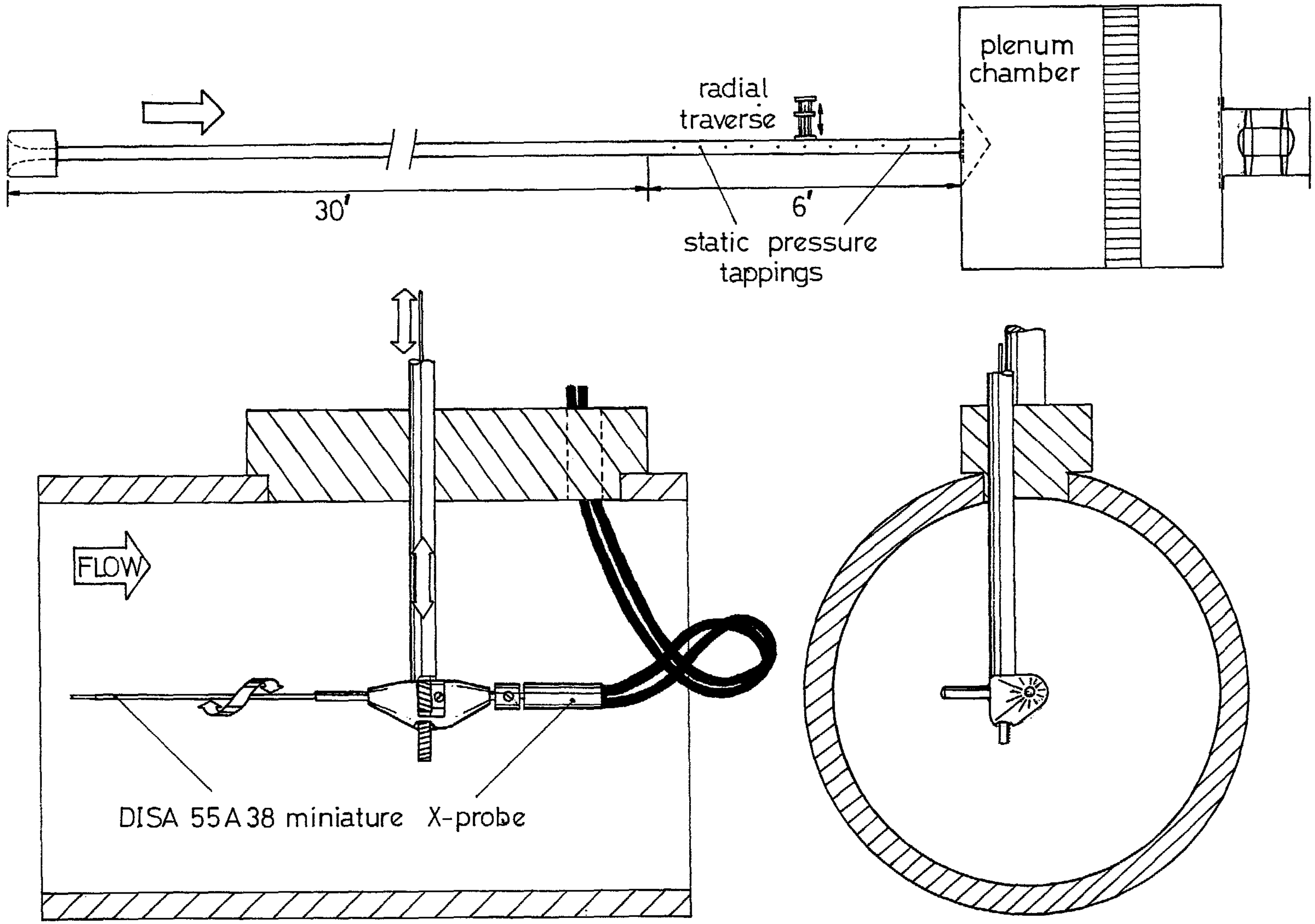

FIG. 9. Hot wire calibration facility.

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