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A Comparative Study of Extrapolation Methods for Creep Data at Small Strains

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A Comparative Study of Extrapolation Methods for Creep Data at Small Strains

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Summary.

Methods of extrapolating creep and rupture data, including the direct use of stress-time charts, the Graham-Walles method and the time-temperature parameters known as Dorn, Larson-Miller and Manson-Haferd, are reviewed with special reference to small creep strains.

Creep data for aluminium alloy DTD 5070A are presented between temperatures of 100 deg. C and 255 deg. C, stresses between 5 and 21 ton/in^2 and times of up to 5000 hours; the main interest centred on strains of up to 0·2 per cent.

Based on these results there was little to choose between the various parametric methods for interpolation and for limited extrapolation. For longer extrapolation, up to 30 000 hours, the Graham-Walles method provided a convincing guide to the expected upper limit of performance. Of the parametric methods, the Dorn expression $t.e^{-\Delta H/RT}$ appeared to be the least unsatisfactory.

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1. *Introduction.*

There has been, during the past ten years, a considerable volume of work published on the question of the interpolation and extrapolation of creep data but the main concern has been with the rupture condition. Less attention has been paid to small creep strains and, in particular, to the question of whether methods accepted for rupture are equally valid much earlier in the creep life.

The present investigation set out to examine this question for one material, an aluminium alloy, concentrating on the region up to about 0·2 per cent creep strain—though wherever practicable the tests were continued to greater strains and in many cases to rupture.

Obviously it would be advantageous to arrive at a single method of data extrapolation for all values of creep strain and for rupture. In any engineering design dual criteria of creep failure exist. Over large areas of fairly constant temperature and stress the attainment of a specified distortion may be the limiting factor while for zones of high stress gradient (i.e. near holes etc.), or high temperature gradient the failure mode would be rupture.

The six possible ways of plotting pairs of the four variables, stress, strain, time and temperature are shown diagrammatically in Fig. 1. In each of the six diagrams the quantities in square brackets are held constant, though of course a series of curves could be plotted on each to show the effect of variation of one of these 'constant' quantities.

Only diagram (i), the familiar strain-time curve at constant stress and temperature is open to direct experimental observation. All the others must be obtained by cross-plotting a series of strain-time curves over a range of temperature and stress.

Since the design problem can pose itself in different ways, all these charts may have their uses under appropriate circumstances. For example where a temperature gradient exists, the stress-temperature chart would be useful if drawn for the desired lifetime and permissible strain (or rupture).

2. *Some Methods of Extrapolation—Summary.*

2.1. *Direct Extrapolation of Basic Curves.*

If the precise stress-strain-time-temperature law were known for a material, any of the six basic diagrams of Fig. 1 could be used for extrapolation purposes as indicated by the broken lines. In practice the law is not known with sufficient degree of precision so that a significant personal element enters into this procedure. Logarithmic scales are sometimes used to linearise the data but this carries the penalty of incurring large errors in the real quantities for small errors in the extrapolated log quantities.

For direct extrapolation, the stress-time plot (Fig. 1 (iv)) is probably the most widely employed, usually in the form of stress-log time or log stress-log time. Extrapolation by eye, long ago employed judiciously by Bailey[1], can be supplemented by more scientific approaches, for example that due to Brozzo[2].

2.2. *Extrapolation of the Constituent Parts of Creep Curves.*

This method was originated and developed by Graham and Walles[3]. The creep strain of a metal is described by a summation of terms of the form

$$C.\,\sigma^{\beta}\,\phi^{K}$$

i.e.

$$\varepsilon = \Sigma\, C.\,\sigma^{\beta}\,\phi^{K} \tag{1}$$

in which C, β and K are constants for any one term and ϕ is a function of time and temperature. By a graphical analysis of the data, with the aid of certain observed regularities in creep behaviour, the values of C, β and K are found for each term. These constants are then used to extrapolate the terms individually to the lower stresses, the extrapolated values then being recombined into the creep equations for those stresses, (*see* Appendix).

2.3. *Use of Time-Temperature Parameters.*

The resistance of a material to deformation cannot be expressed through time alone or temperature alone but only through a combined time-temperature law and this fact has led to the analysis of creep behaviour through such parameters. It is postulated that for any one value of the parameter the same stress would produce the same strain. In effect the four variables have been lumped into two pairs so that the creep law becomes:

$$f(\sigma, \varepsilon) = \Phi(t, T) = P \tag{2}$$

or for constant strain

$$f(\sigma) = \Phi(t, T) = P \tag{3}$$

Now a single numerical value of P can be produced by any number of combinations of time and temperature. Hence if the parameter is a correct representation for all times and temperatures, creep tests can be performed for short times at high temperatures to give a curve of σ against P. This same curve (the 'Master Curve') can then be used to predict the allowable stress for the same values of P given by long times and lower temperatures. The problem of extrapolation is then neatly converted into one of interpolation of a master curve.

The three best known parameters are

$$f(\sigma) = P = t\,e^{-\Delta H}/RT \qquad \text{Dorn}^{4} \tag{4}$$

$$f(\sigma) = P = T\,(C_{LM} + \log t) \qquad \text{Larson-Miller}^{5} \tag{5}$$

$$f(\sigma) = P = (T - To)/(\log t - \log t_0) \qquad \text{Manson-Haferd}^{6} \tag{6}$$

where t is time, T is absolute temperature, R is the gas constant and ΔH, C_{LM}, To, t_o are constants for the material and the strain level, determined by experiment.

A derivation of the parameters with special reference to small strains is given in the Appendix.

3. *Experiments.*

3.1. *Material and Specimens.*

The material used in the investigation was 16 S.W.G. Alclad sheet to specification DTD 5070A. Tensile data are given in Table 1 and the tensile stress-strain curve at 20 deg. C in Fig. 2.

The creep specimens were manufactured to the form and dimensions given in Fig. 3 and were cut from the sheet with their longitudinal axes parallel to the direction of rolling.

3.2. *Test Machines and Procedure.*

All creep tests were of the constant-load type performed in axial tension using free-standing, dead-weight, single-lever machines of two tons capacity. Temperature control was by resistance thermometer and saturable reactor working on a 3 zone furnace 15 inches long, the gauge length being 5 inches. Thermocouple exploration showed that a negligible temperature gradient could be achieved over the gauge length and the long term stability of control was found to be excellent. Axiality of load was checked by the strain replica method, the use of which has been reported elsewhere[7,8].

The creep strains were measured by extensometers of the rod and tube type fitted with micrometer heads. The readings on the two sides of the specimen were averaged to give the nominal strain. Good reproducibility was achieved and the strains compared well with those given by the replica method.

Three thermocouples were fitted to each specimen before testing. The heating up period was 16–20 hours during which time any necessary minor temperature adjustments were made. Dead weights were then applied to the load arm, the loading time usually being about 30 seconds. There was no interruption of test until termination either by rupture or by the attainment of a specified creep strain.

3.3. *Results.*

A sample set of strain-time curves is shown in Fig. 4 while Table 2 gives the times for 0·1 per cent, 0·15 per cent, 0·2 per cent creep and rupture for all the test temperatures. These times were used in the subsequent graphical presentation of data and for the extrapolation exercise involving time-temperature parameters.

The creep curves themselves require little comment. At the lower stresses there was often a well defined primary stage, (slope of about 0·3 on the log—log plot) lasting up to about 1000 hours. At higher stresses and temperatures secondary creep began early, sometimes after only a few minutes. In those tests which continued to rupture a tertiary stage could always be identified and there was a rupture ductility of between 6 and 10 per cent.

4. *Analysis and Discussion of Results.*

4.1. *General.*

This Section of the Report treats the experimental creep results by the methods mentioned in the Introduction and examined in the Appendix.

The aim is to test the ability of these methods to correlate results for DTD 5070A over the experimental range of time and temperature, and, in the following Section, to assess their effectiveness for purposes of interpolation and extrapolation mainly at 125 deg. C* both for the rupture condition and for creep strains 0·1 per cent, 0·15 per cent and 0·2 per cent**.

Table 3 summarises the 30 000* hour prediction by each of the methods. Where a blank appears in the table it was not possible to arrive at a prediction because of a limitation of the particular method, or because there were insufficient data for its use.

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*125 deg. C and 30 000 hours were chosen because they are in the region of interest for supersonic aircraft of aluminium alloy.

**Hence forward called the standard strains.

4.2. *Direct Extrapolation of Strain—Time Cross-Plots.*

Fig. 5a–d shows stress *versus* log-time for the standard strains, and rupture. Contrary to expectation these curves were all concave downwards so that, in the event, it was not possible to apply Brozzo's construction[2] which requires the opposite curvature. The extrapolations to 30 000 hours were, therefore, performed by eye with corresponding uncertainty in the values which are reported in Table 3.

All that can be said in favour of this method of extrapolation is its simplicity. There is no pooling of data; interpolation or extrapolation at any particular temperature can only be undertaken if results are available at that precise temperature. Cross-plots would be necessary if interpolation at other temperatures were required.

The present results lend weight to the argument that unambiguous extrapolation by this method is difficult unless the data extend practically to the required time. As an example consider the curve for 125 deg. C in Fig. 5d. Extrapolation by eye could lead to rupture values anywhere between 13 and 15 tons per sq. inch at 30 000 hours. The value entered in Table 3 is merely the average of these extremes.

Since the experimental curves are concave downwards it can however be surmised that the extrapolated values are on the low (safe) side because sooner or later a point of inflexion must occur. But it could also be urgued that this simply adds to the hazards of extrapolation rather than being a redeeming feature.

Perhaps the two main points against the method of direct extrapolation are that (a) it offers little possibility of using high-temperature, short-time results to forecast lower-temperature, long-time behaviour and (b) the personal factor enters significantly into the extrapolation procedure.

On the other hand for interpolation at the test temperatures these stress-time curves appear perfectly adequate since smooth isothermals can be drawn through the data.

4.3. *The Graham-Walles Method.*

Using the method outlined in Fig. 12 the log stress-log time graphs for the $t^{1/3}$, t^1, t^3, components of the creep curves using a reference strain $\varepsilon = 0{\cdot}1$ per cent were plotted and the best values of T^1 were derived.

For all three components the standard slopes of $-\frac{1}{16}$, $-\frac{1}{8}$ and $-\frac{1}{4}$ could be identified. In each case the $-\frac{1}{4}$ slope was extended to give the optimistic extrapolation and -1 slope was assumed for the pessimistic extrapolation*.

It was now decided to construct predicted creep curves at low stresses for 125 deg. C in order to compare with the other methods. The procedure followed was that outlined in Fig. 12. It was anticipated that stresses between 5 and 9 tons per sq. inch would be of interest so far as the 30 000 hour creep performance was concerned and Table 4 shows, at 1 ton per sq. inch intervals, the times for 0·1 per cent creep by each of the $t^{1/3}$, t^1 and t^3 components alone as obtained from the optimistic and pessimistic extrapolations at 125 deg. C.

By the further procedure shown in stage 4 of Fig. 12, the optimistic and pessimistic curves at each stress were now derived and are shown in Fig. 6.

In order to obtain the (optimistic) 30 000 hour predictions these curves were crossplotted at the standard strains (Fig. 7) and interpolated at 30 000 hours. The results are given in the appropriate columns of Table 3.

These optimistic curves will be used later as a measure of the upper limit of performance in order to assess the credibility of extrapolated values by the other methods.

The Graham-Walles procedure is perhaps unique in requiring a knowledge of the whole shape of each creep curve rather than spot values of the time to achieve a given strain or of the time to rupture. An attractive feature is that once the optimistic and pessimistic curves have been constructed at a certain stress, a confirmatory test can be run and it should be possible to see quite quickly whether the actual curve corresponds to one or other of the extremes or whether it is falling between the two. The results of

*Up to this point the analysis was carried out by Mr. K. F. A. Walles of the National Gas Turbine Establishment, using special templates for the analysis of the curves.

such a confirmatory test at 125 deg. C 7 tons per sq. inch are included in Fig. 4. Up to 5000 hours the result falls almost equidistant from the optimistic and pessimistic lines.

4.4. *Time-Temperature Parameters.*

(a) *The Dorn Parameter.*

Using Table 2, graphs of $\frac{1}{T}$ against $\log_e t$ were plotted for the standard strains and for rupture. It was apparent that for each strain a set of parallel lines could reasonably be drawn, indicating that the Dorn parameter $t.e^{-\Delta H/RT}$ was likely to give a good representation of the data. The slope of the parallel lines gives in each case the relevant value of $\frac{\Delta H}{R}$ and these are listed in Table 5.

The derived master curves of stress against $te^{-\Delta H/RT}$ are plotted in Fig. 8, the tests lasting more than 1000 hours to reach the stated values of strain being given an identifying mark.

The 30 000 hour predictions using these master curves are entered in Table 3. Their probable accuracy will be discussed later.

The value of ΔH increases through the test from a value of 32 600 cal/mol at 0·1 per cent strain to 41 500 cal/mol at rupture. These figures are in fact the average apparent activation energy up to the particular strain. This change of activation energy during the test disagrees with the result given by Dorn[9] for pure aluminium where the value of ΔH at a given temperature was insensitive both to stress and strain.

However, it must be borne in mind that the values of ΔH quoted in Table 5 are, in effect, average values for the range of temperature covered by the tests namely 373 deg. K to 528 deg. K. Over this range of temperature Dorn's results for ΔH vary between limits of 28 000 and 35 000 cal/mol, about the same percentage variation as reported here.

A possible reason for the observed variation in ΔH lies in the fact that the index of time is unity in the Dorn parameter. If t^K is substituted for t in Equation (5A) of the Appendix, the slope of the $\frac{1}{T}$, $\log_e t$ graph becomes $\left(\frac{1}{K}\right)\left(\frac{\Delta H}{R}\right)$. Thus the apparent variation in ΔH may in fact mask a change in the average K from one strain level to another.

(b) *The Larson-Miller Parameter.*

The Dorn and Larson-Miller Parameters are mutually exclusive in that Dorn predicts parallel lines on the $\frac{1}{T}$, $\log t$ plot whereas Larson-Miller predicts convergence. Since, as mentioned above, the graphs indicated parallelism it seemed pointless to try to obtain a value of C_{LM} in Equation 5.

However, one of the advantages claimed for the Larson-Miller method is that a standard value of 20 for C_{LM} fits a wide range of data. For example the recently published sheets by the Creep Information Centre at N.E.L.[10] are compiled on this basis. A standard value of C_{LM} if valid, enables a master curve to be constructed quickly with a minimum amount of testing because the intermediate $\frac{1}{T}$ *vs* $\log t$ graph can be omitted.

Consequently it was thought worthwhile to try the standard value for the present tests and the results are shown in Fig. 9 for the 3 standard strains and for rupture, the tests lasting more than 1000 hours being identified as before. It was found that a family of parallel lines could be drawn through the data between stresses of 5 and 21 tons per sq. inch. The consequent 30 000 hour values are listed in the appropriate row of Table 3 for later comment. No doubt it would be possible to discover an optimum value of C_{LM}, but in view of the apparent validity of the Dorn parameter this question was not pursued. It may be remarked however, that a value of $C_{LM} = 35$ would be a reasonable first estimate.

(c) *The Manson-Haferd Parameter.*

The data for the standard strains and for rupture contained in Table 2 were plotted at T against $\log_e t$. The convergence of the isostress lines produced the values of To and $\log_e t_o$, itemised in Table 6. It is interesting to find that To is practically constant for all strains and in fact a constant value of 285 deg. K could easily be used throughout. The value of $\log_e t_o$ however increases with increasing strain.

The master curves are shown in Fig. 10, again with identifying marks for tests lasting more than 1000 hours and the consequent 30 000 predictions appear in the relevant row of Table 3.

5. *Effectiveness of the Methods for Interpolation and Extrapolation.*

5.1. *Interpolation.*

The most obvious method of interpolation is to use one of the six types of diagram shown in Fig. 1. This simple procedure can be perfectly adequate as evidenced by Figs. 4 and 5. For temperatures other than those of the actual tests, another set of diagrams of the $T-t$ type (Fig. 1 (vi)) would need to be plotted, one for each of a number of stress levels, so that intermediate curves could be constructed on the stress-time axes.

The Graham-Walles method as usually applied does not appear to offer significant advantages over the simple use of such cross-plots for interpolation of data. The parametric master curves do, however, possess advantages because all the results are conveniently pooled onto one curve for each strain and the prediction of performance at any required temperature is consequently rapid and requires no further graphical work. Also, this pooling of data often means that a smaller number of tests need be performed.

For purposes of interpolation there is little to choose between the Dorn, Larson-Miller and Manson-Haferd parameters; all give points closely grouped about the respective master curves.

5.2. *Extrapolation.*

The over-riding requirement for a method of extrapolation is accuracy; it is hardly an exaggeration to say that all the other considerations are subsidiary to this.

There are two aspects of extrapolation to be considered. The first is the use of the shorter time results in order to forecast performance in the 1000–5000 hour range so that comparison can be made with the actual 1000–5000 hour results. The second aspect is to use all the available data for tentative forecasts well outside the available test range, say up to 30 000 hours, using indirect evidence to discriminate between one method and another.

(a) *Forecasts in the* 1000–5000 *hour range.*

Long-time tests lasting more than 1000 hours are marked by arrows on the master curves of Figs. 8, 9 and 10. The number inserted next to each arrow indicates the thousands of hours run.

It is clear that these long-time tests fall reasonably close to the mean master curves. Consequently the master curves could have been established reasonably well by tests up to about 300 hours and then used to predict the 5000 hour performance. Any of the three methods could reasonably be employed but, for choice, the Dorn parameter appears to give rather more consistent results than either Larson-Miller or Manson-Haferd in terms of the stress difference between the arrowed points and the mean curves.

(b) 30 000 *hour forecasts.*

These forecasts, made with particular reference to 125 deg. C are listed in Table 3 for each of the methods under discussion. They are also marked on the stress-time charts of Fig. 5.

Since no 30 000 test results were available, a definite final judgement cannot yet be made but it is nevertheless possible to draw certain conclusions about the likelihood of each of the methods giving reliable forecasts of 30 000 hour behaviour.

It will be convenient first to consider the Graham-Walles results with the aid of the derived optimistic curves for 5 to 9 tons per sq. inch which have been transferred from Fig. 6 to Fig. 4. Judging from the earlier parts of the curve at 10 tons per sq. inch and the deduced curve at 9 tons per sq. inch it seems clear that the Graham-Walles 'optimistic' curves are aptly named. (This deduction is confirmed by a comparison of the actual and predicted curves at 7 tons per sq. inch in Fig. 4).

Hence a reasonable line of action is to accept the Graham-Walles optimistic curves as upper limits on performance and to regard the other methods as suspect if they predict yet higher values of stress to achieve a given strain in a given time. Almost certainly such predictions would be on the unsafe side for design purposes.

At 0·1 per cent strain by this criterion, only the Dorn method is credible. The Graham-Walles optimistic forecast is 6·9 tons per sq. inch and in fact the position of the lower curves in Fig. 4 strongly suggests that a stress of less than 6 tons per sq. inch is necessary to produce 0·1 per cent creep in 30 000 hours. This indicates that the Dorn figure of 5·7 is a very reasonable forecast on the available data. All the other methods predict stresses above 6·9 tons per sq. inch.

Fig. 11 shows the relationship between stress and temperature to produce 0·1 per cent creep in 30 000 hours based on the Dorn master curve. In the range of practical interest, 100 deg. C–130 deg. C, the relationship is linear.

Turning to strains higher than 0·1 per cent the position is not so clear. All the methods give predictions higher than the Graham-Walles optimistic forecast, so all are probably dangerous. Again, however, Dorn gives the lowest prediction at 0·15 per cent and is therefore likely to be the least unsatisfactory. At 0·2 per cent it is impossible to give a decision because the predicted values at 10 tons per sq. inch is 25 per cent higher than the upper limit due to Graham-Walles. At fracture (30 000 hours, 120 deg. C) there is a more consistent picture. Both the Dorn and the Larson-Miller parameters agree closely at 14·4 and 14·1 tons per sq. inch respectively, although they were widely different at the lower strains. This is interesting because the Larson-Miller method has hitherto been used mainly for the analysis of rupture data. The present work tends to justify its use in that connection, but not for smaller strains.

To sum up it appears that no single method is completely satisfactory for extrapolation to 30 000 hours of creep data for DTD 5070A but that, of the methods investigated, the Dorn parameter $t.e^{-\Delta H/RT}$ is the least unsatisfactory. It can be used with a fair degree of confidence in the range of 100 deg. C–130 deg. C for interpolation and extrapolation provided that the value of ΔH has been evaluated for the appropriate creep strain.

6. *Conclusions.*

(1) The creep curves of the aluminium alloy DTD 5070A between temperatures of 100 deg. C and 255 deg. C are of the usual form but there are indications of anomolous behaviour in the region of 12 to 13 tons per sq. inch at 125 deg. C.

(2) For purposes of interpolation at any one of the test temperatures, the direct use of stress-time cross plots is adequate both for small creep strains and for rupture.

(3) For purposes of interpolation at the test temperatures or between them the Dorn, Larson-Miller and Manson-Haferd parameters are all fairly satisfactory. On the question of limited extrapolation, the 1000–5000 hour results can be reasonably well forecast from the shorter time data with perhaps the Dorn parameter giving the smallest errors.

(4) For the range of temperature investigated, the apparent activation energy ΔH in the Dorn parameter increases from 32 600 cal/mol at 0·1 per cent strain to 41 100 at 0·2 per cent strain and to 41 500 at rupture. In the Manson-Haferd parameter the value of To is constant at 285 deg. C but $\log_e t_o$ increases from 16·0 at 0·1 per cent strain to 21·2 at rupture.

(5) For extrapolations to 30 000 hours the Graham-Walles method of constructing 'optimistic' creep curves at low stresses seems to provide a consistent guide to the upper limit of performance. Such curves can be used as a yardstick against which the credibility of other methods can be assessed.

(6) On this basis none of the proposed non-parametric or parametric methods can be rated as satisfactory for extrapolation to long times at all values of strain. The Dorn parameter however, is the least unsatisfactory. It gives predictions for the 30 000 hour strength which appear to be reliable for 0·1 per cent strain and for rupture but, in common with the other methods, is optimistic at 0·15 per cent and 0·2 per cent strain.

(7) The 30 000 hour creep strengths as given by the Dorn parameter are as follows:

Temperature	Stress in tons per sq. inch	
	0·1 per cent creep	Rupture
100 deg. C	10·4	18·1
120 deg. C	6·7	15·3
125 deg. C	5·7	14·4
130 deg. C	4·9	10·5

LIST OF SYMBOLS

σ	Stress
$\hat{\sigma}$	Arbitrary value of stress
σ_{EXT}	Stress at which extrapolation is required
ε	Creep strain
$\hat{\varepsilon}$	Arbitrary value of creep strain
t	Time in hours
T	Temperature in deg. K
ΔH	Activation energy for creep
R	Gas constant
ϕ	Function of time and temperature
β	Index of stress
K	Index of time
f, F	Functions
P	Value of time-temperature parameter
C	Constant
C_D	Dorn constant
C_{LM}	Larson-Miller constant
t_o, T_o	Manson-Haferd constants
A, C', T'	Graham-Walles constants
l, m, n	Coefficients in Andrade equation

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APPENDIX

Derivation of the Methods

A.1. *The Graham-Walles method.*
Writing the creep strain as a summation of terms of the form $C\,\sigma^{\beta}.\Phi^{K}$ Graham and Walles suggest that Φ can be taken of the form $t\,(T'-T)^{-A}$ where T' and A are constant for one term of the summation. The complete creep law is then:

$$\varepsilon = \Sigma[C.\sigma^{\beta}\,\{t(T'-T)^{-A}\}^{K}] \tag{1.A}$$

C, β, T', A and K being constants which vary from term to term. The number of terms to be used in any particular analysis can, theoretically, be very large but in practice a small number are usually dominant. (For purposes of analysis the value of A can be standardized at 20).

The method consists of analysing the creep curves on the basis that (following Andrade) K takes values from the sequence —— $\frac{1}{3}$, 1, 3 —— and further that $\frac{K}{\beta}$ takes values from the sequence —— $+\frac{1}{32}$, $+\frac{1}{16}$, $+\frac{1}{8}$ ——

At constant temperature the equation becomes,

$$\varepsilon = \Sigma \quad C'\,\sigma^{\beta}\,t^{K} \tag{2.A}$$

where C' now contains the temperature. Analysis and extrapolation can now be described in four stages with the aid of Fig. 12.

Stage 1. The first stage is to plot the strain-time curves on double logarithmic co-ordinates and with aid of special templates, to split each curve into three (or more) components as represented by the straight lines labelled $t^{1/3}$, t^{1} and t^{3}. At any time t, the sum of the strains on these three lines equals the total creep strain so that in effect the creep equation has been expressed at the Andrade type equation:

$$\varepsilon = lt^{1/3}+mt+nt^{3} \tag{3.A}$$

Stage 2. To evaluate the constants* in each term of equation (3.A) the second stage is to choose an arbitrary creep strain ε (usually 0·1 per cent) and cross-plot isothermal results for the $t^{1/3}$, t^{1} and t^{3} data separately on log σ, log t co-ordinates. Each of these diagrams shows the stress-time combinations to achieve 0·1 per cent creep strain by *one* of the terms of (3.A).

Graham and Walles's further analysis is based on the observation that each of the log σ *vs* log t curves could be closely represented by a series of straight line segments with slopes in the sequence —— $-\frac{1}{32}$, $-\frac{1}{16}$, $-\frac{1}{8}$ ——. Now, for any one term of the form

$$\varepsilon = C'\,\sigma^{\beta}\,t^{K}$$

$$\log\varepsilon = \log C'+\beta\log\sigma+K\log t \tag{4.A}$$

Hence for a constant value of ε the graph of log σ *versus* log t is a straight line of slope $-K/\beta$ i.e. K/β takes the values —— 1/32, 1/16, 1/8, ——. A second set of templates is available to help analyse the curves into these segments.

Stage 3. The third stage aims at obtaining the best overall fit to the data by graphical evaluation of the optimum values of T'. Since T' is constant in any term of (1.A) the actual value of T' can be computed

*The constants are not usually evaluated numerically. As indicated in Fig. 12, the analysis and subsequent synthesis can be entirely graphical.

from analysis at the arbitrary strain $\hat{\varepsilon}$ and an arbitrary stress $\hat{\sigma}$.

The $t^{1/3}$, t^1 and t^3 charts of Stage 2 are taken separately and each set of parallel segments are cross-plotted at the selected stress $\hat{\sigma}$. This cross plotting gives a series of $\log t - T$ curves, one for each set of values of K and K/β. The values of T' are obtained by using a third set of calibrated templates representative of the assumed form of temperature dependence $(T'-T)^{-20}$*. The position of the segments of standard slope in Stage 2 and the curves of Stage 3 are then progressively modified to obtain the best overall fit to both stages. (The $\log t - T$ curves are not shown in Fig. 12).

Stage 4—Extrapolation. In order to extrapolate the data to a low stress σ_{EXT} not covered by the available test results, the Stage 2 curves are produced to σ_{EXT} and then the data are recombined to give the creep curve at σ_{EXT}.

It is possible to obtain both optimistic and pessimistic predictions according to the method adopted for extending the Stage 2 curves. The 'optimistic' procedure is depicted in the lower part of Fig. 12.

At Stage 2 the data are continued by the upper broken line in each figure at the greatest value of K/β identified by the data. The intersection of these broken lines with σ_{EXT} gives predicted values of time to cause a strain of $\hat{\varepsilon}$ independently by each of the terms in the expression $lt^{1/3}+mt+nt^3$.

Points can now be inserted at the appropriate values of t on the Stage 1 graphs at the strain level $\hat{\varepsilon}$ and straight lines of appropriate slope 1/3, 1 or 3 drawn through these points. These lines are the components of the required creep curve which can now be found simply by adding the three component strains together at each value of time.

The 'pessimistic' curve is constructed in a similar manner except that the Stage 2 curves are extended at a greater slope (usually $-\frac{1}{2}$ or -1) than actually observed in the experiments. These extensions are indicated by the lower broken lines in each of the Stage 2 graphs of Fig. 12.

A.2. *Time-Temperature Parameters.*

The statement of a time-temperature parameter implies a creep law expressible as a function of the four main variables and it is instructive to associate each parameter with such a law in order to examine other implications of that law and to assess possible advantages and limitations.

(a) *Dorn.*

A creep law implied by this parameter is:

$$\varepsilon = F(e^{-\Delta H/RT}.t).f(\sigma) \tag{5.A}$$

where ΔH is the activation energy of the creep process and R is the gas constant.

At a constant strain

$$te^{-\Delta H/RT} = f(\sigma) = P \tag{6.A}$$

If now a series of tests is conducted at a constant stress σ_1, the value of $f(\sigma)$ is constant so:

$$\log_e t = \frac{\Delta H}{RT} + C_D \tag{7.A}$$

where C_D contains the stress and is constant for a given stress. A graph of $\log_e t$ against $\frac{1}{T}$ for constant strain and constant stress should therefore produce a straight line of slope $^{\Delta H}/R$ as indicated in Fig. 13a.

If a family of parallel lines, one for each stress, is obtained on the $\log_e t$, $\frac{1}{T}$ axes, all the data can now

---

*Or if $T' < T$ the form $(T-T')^{20}$ may be used.

be pooled by plotting σ against P still keeping the strain constant. This gives the master curve shown in Fig. 13b and there will be a different such curve for each strain as well as one for rupture—on the assumption that the rupture ductility is constant.

The appropriate master curve now allows the stress to produce the stated strain to be read off for any combination of time and temperature.

(b) *Larson-Miller.*

A creep law corresponding to this parameter is

$$\varepsilon = F(e^{f(\sigma)/T}.t) \tag{8.A}$$

which differs from the previous law in that the function of stress $f(\sigma)$ is brought into the temperature term and becomes a stress-modified activation energy. Proceeding by the same route as before:

At constant strain

$$e^{\frac{-f(\sigma)}{T}}.t = \text{const.} \tag{9.A}$$

or

$$\log_e t = \frac{f(\sigma)}{T} - C_{LM} \tag{10.A}$$

where C_{LM} is the Larson-Miller constant and contains the strain but not the stress. Hence if $\log_e t$ is plotted against $\frac{1}{T}$ at a constant strain the results for different stresses should give a set of straight lines converging of the $\log_e t$ axis at an intercept equal to $-C_{LM}$ (Fig. 14). If these lines do so converge, the data can be pooled by noting that from (10.A).

$$T(C_{LM}+\log_e t) = f(\sigma) = P \tag{11.A}$$

and so the master curve is obtained by plotting stress against $T(C_{LM}+\log_e t)$ as in Fig. 14b.

In practice the Larson-Miller parameter is often employed using logs to base 10 in combination with a standard value of 20 for C_{LM}. As pointed out in the text, this procedure, if valid, enables a master curve to be drawn using the results of a very few tests without the intermediate graph of $\frac{1}{T}$ *vs.* $\log_e t$.

(c) *Manson-Haferd.*

A creep law compatible with this parameter is:

$$\varepsilon = F(e^{(T_o-T)/f(\sigma)}.t) \tag{12.A}$$

where a new constant T_0 is introduced. As before, for constant strain,

$$e^{(T_o-T)/f(\sigma)}.t = \text{const.} = t_o \text{ (say)} \tag{13.A}$$

$$\frac{T_o-T}{f(\sigma)} + \log_e t = \log_e t_o \tag{14.A}$$

$$f(\sigma) = \frac{T-T_o}{\log_e t - \log_e t_o} = P \tag{15.A}$$

For a series of tests at the same stress and analysed at the same strain this equation predicts a straight line when T is plotted against $\log_e t$, a variant of Fig. 1(vi).

As shown in Fig. 15a the constants To and $\log_e t_o$ are located by the co-ordinates of the intersection of the lines of constant stress. The master curve is then obtained, as indicated in Equation (15.A) by plotting against $(T - To)/(\log_e t - \log_e t_o)$ for the stipulated value of strain as shown in Fig. 15b.

TABLE 1

Tensile Data at Room Temperature DTD 5070A, 16 S.W.G. Sheet

0·1 per cent Proof Stress	24·7 tons/in^2
0·2 per cent Proof Stress	25·4 tons/in^2
Ultimate Tensile Stress	27·7 tons/in^2
Youngs Modulus (Initial)	$10{\cdot}7 \times 10^6$ lb/in^2
Elongation on 2 in.	7 per cent

TABLE 2

Times for 0·1 per cent, 0·15 per cent, 0·2 per cent Creep and for Rupture

Temperature deg. C	Stress Tons/in^2	Time in Hours			
		0·1 per cent	0·15 per cent	0·2 per cent	Rupture
100	22	0·5	1·2	2·0	105
	21	9·2	18	30	800
	20	15	42	80	2700
	19	74	220	400	—
	18	250	560	1000	—
125	21	0·4	0·7	1·0	22
	20	1·2	2·5	3·7	85
	19	2·1	4·8	8·0	300
	18	9·0	21	35	900
	17	17	52	100	2500
	16·5	31	84	155	—
	16	42	140	250	—
	15	120	440	1050	—
	14	230	720	1550	—
	13	600	1600	2700	—
	12	400	2000	4000	—
	11	2300	5600	—	—
	10	4000	—	—	—
140	20	0·1	0·18	·25	6·2
	18	1·3	2·9	5·0	150
	17	5·0	13	23	600
	16	14	35	50	1200
	15	20	70	150	2800
150	20	0·15	0·22	0·29	4
	19	0·18	0·30	0·42	—
	18	0·30	0·56	0·84	25
	17	0·58	1·8	3·3	110
	16	3·0	9·4	18	—
	15	10·5	30·5	60	—
	14	20	75	140	—
	13	30	100	230	—
160	15	3·0	10·5	21·5	—
	13	12·5	41	89	350
200	15	0·18	0·29	0·4	3
	10	3·5	8·4	15	240
	9	5·0	12·5	22	500
	7	17	35	60	1600
	6	23	60	96	2800
255	10	0·09	0·13	0·16	2·4
	7	0·52	0·90	1·3	26
	6	1·6	2·8	4·1	68
	5	2·7	5·6	8·4	170

TABLE 3

30 000 *Hour Predictions*

Stress in tons per sq. inch

Method of Extrapolation	100°C				120°C				125°C				150°C			
	0·1%	0·15%	0·2%	R	0·1%	0·15%	0·2%	R	0·1%	0·15%	0·2%	R	0·1%	0·15%	0·2%	R
Direct (By Eye)	13·5	14·4	15·0	18·0	—	—	—	—	7·0	8·4	9·7	14·0	4·3	5·6	7·4	11·0
'Optimistic' Graham-Walles	—	—	—	—	—	—	—	—	6·9	7·7	8·1	—	—	—	—	—
Dorn	10·4	13·1	14·8	18·1	6·7	9·3	11·6	15·3	5·7	8·1	10·5	14·4	—	3·9	6·2	10·5
Larson-Miller	13·5	14·4	14·4	17·2	10·6	11·4	11·6	15·2	10·0	11·0	11·0	14·1	6·6	7·4	7·6	10·6
Manson-Haferd	—	10·4	11·8	17·6	—	—	6·4	13·9	—	—	—	13·0	—	—	—	8·6

R = Rupture

TABLE 4

Graham-Walles Method

Extrapolated Times to Produce 0·1 *per cent Creep by each of the Components*

Stress Tons/sq. in.	Creep Component	Time in Hours for 0·1 per cent Creep	
		Optimistic	Pessimistic
5	$t^{1/3}$	500 000	42 000
	t^1	640 000	24 000
	t^3	200 000	10 000
6	$t^{1/3}$	260 000	34 000
	t^1	250 000	19 000
	t^3	100 000	9 000
7	$t^{1/3}$	130 000	28 000
	t^1	120 000	15 000
	t^3	56,000	7 800
8	$t^{1/3}$	66 000	23 000
	t^1	66 000	13 000
	t^3	34 000	6 900
9	$t^{1/3}$	40 000	17 000
	t^1	37 000	11 000
	t^3	22 000	6 300

TABLE 5

Values of ΔH for Dorn Parameter

Strain	$\frac{\Delta H}{R}$ Measured as in Fig. 13a	ΔH Cal/mol
0·1 per cent	16 400	32 600
0·15 per cent	18 100	36 000
0·2 per cent	20 700	41 100
Rupture	20 830	41 500

TABLE 6

Values of T_o and $Log_e\, t_o$ for Manson-Haferd Parameter

	Creep Strain			
	0·1 per cent	0·15 per cent	0·2 per cent	Rupture
T_o (deg. K)	288	288	285	235
$\mathrm{Log}_e\, t_o$	16·0	18·0	18·3	21·2

(t_o in hours)

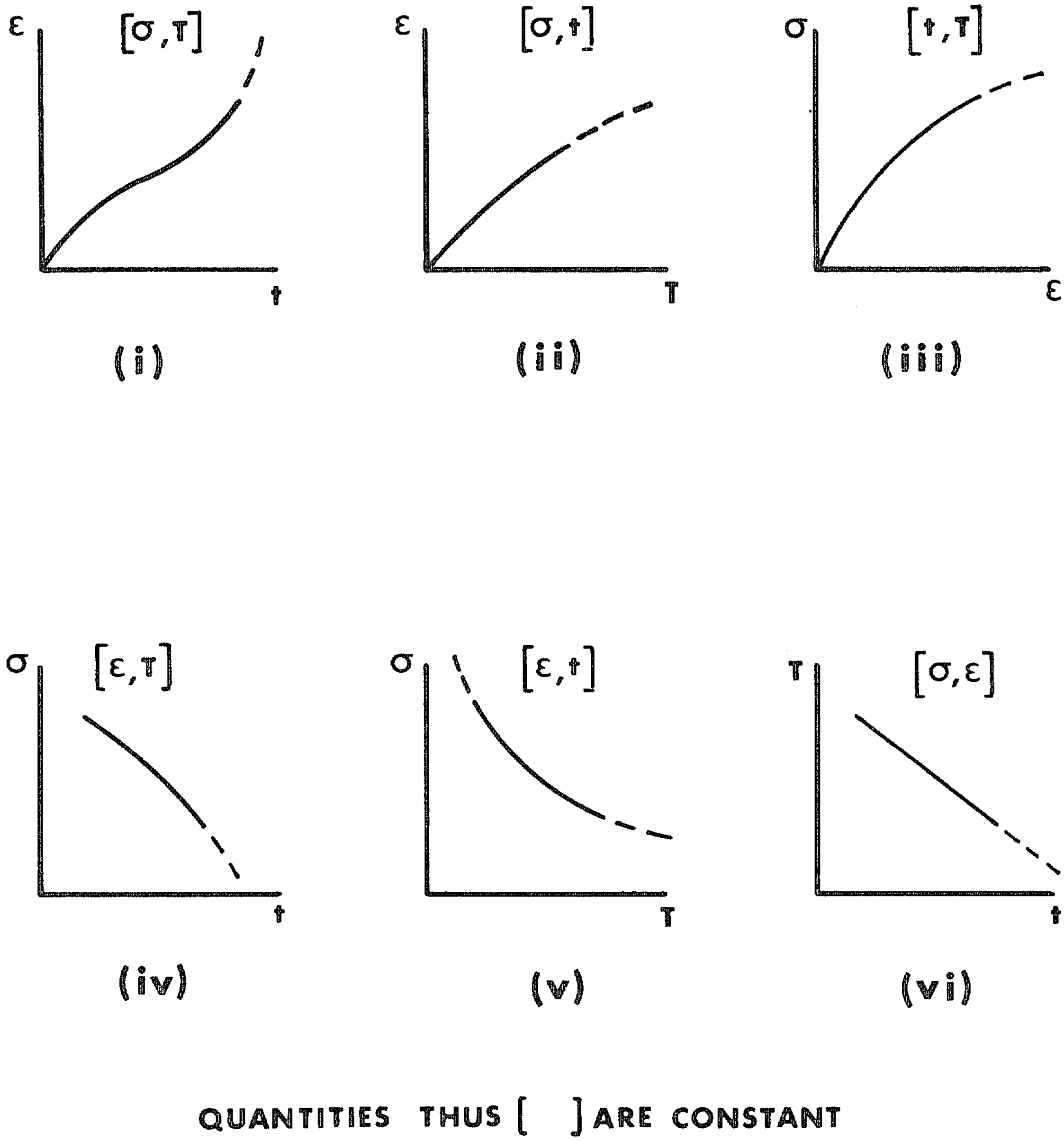


FIG. 1. Basic methods of presenting creep data.

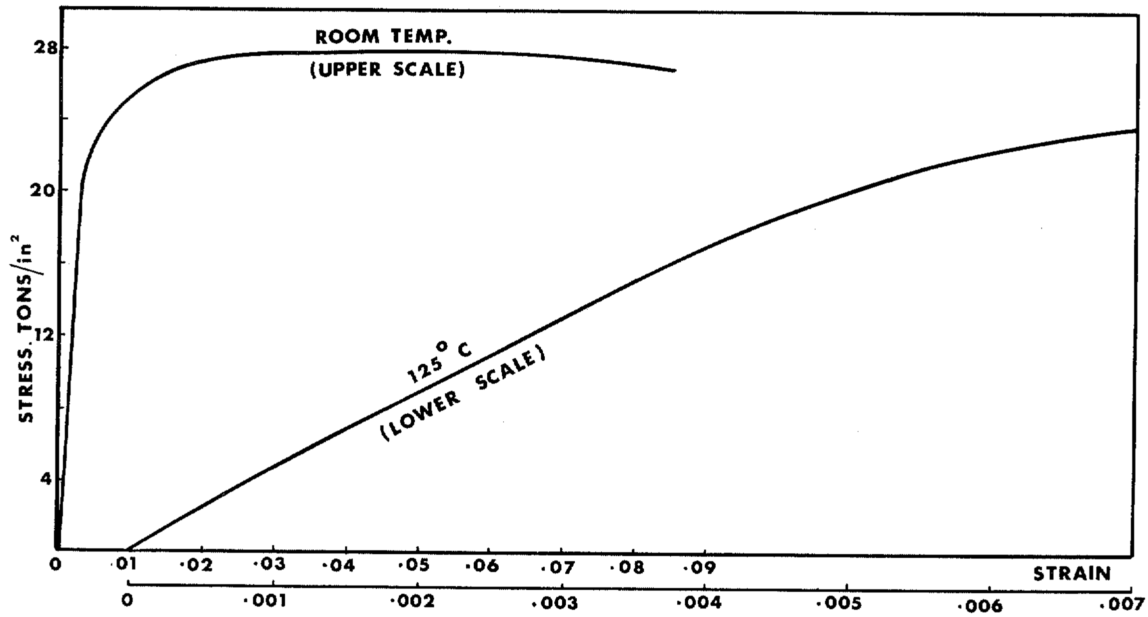


FIG. 2. Tensile stress/strain curves DTD 5070A.

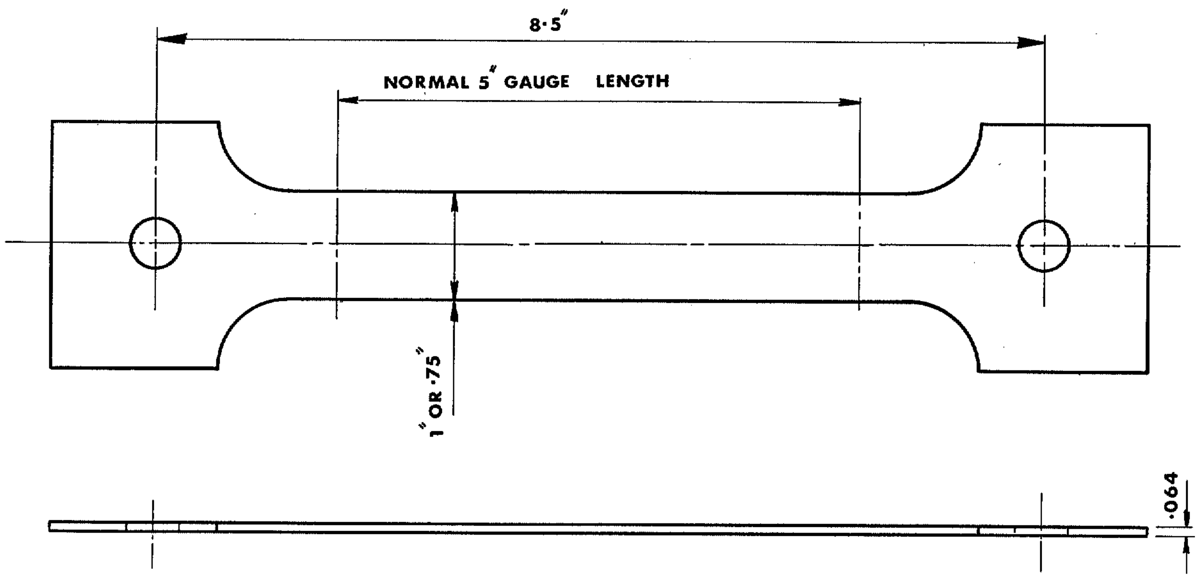
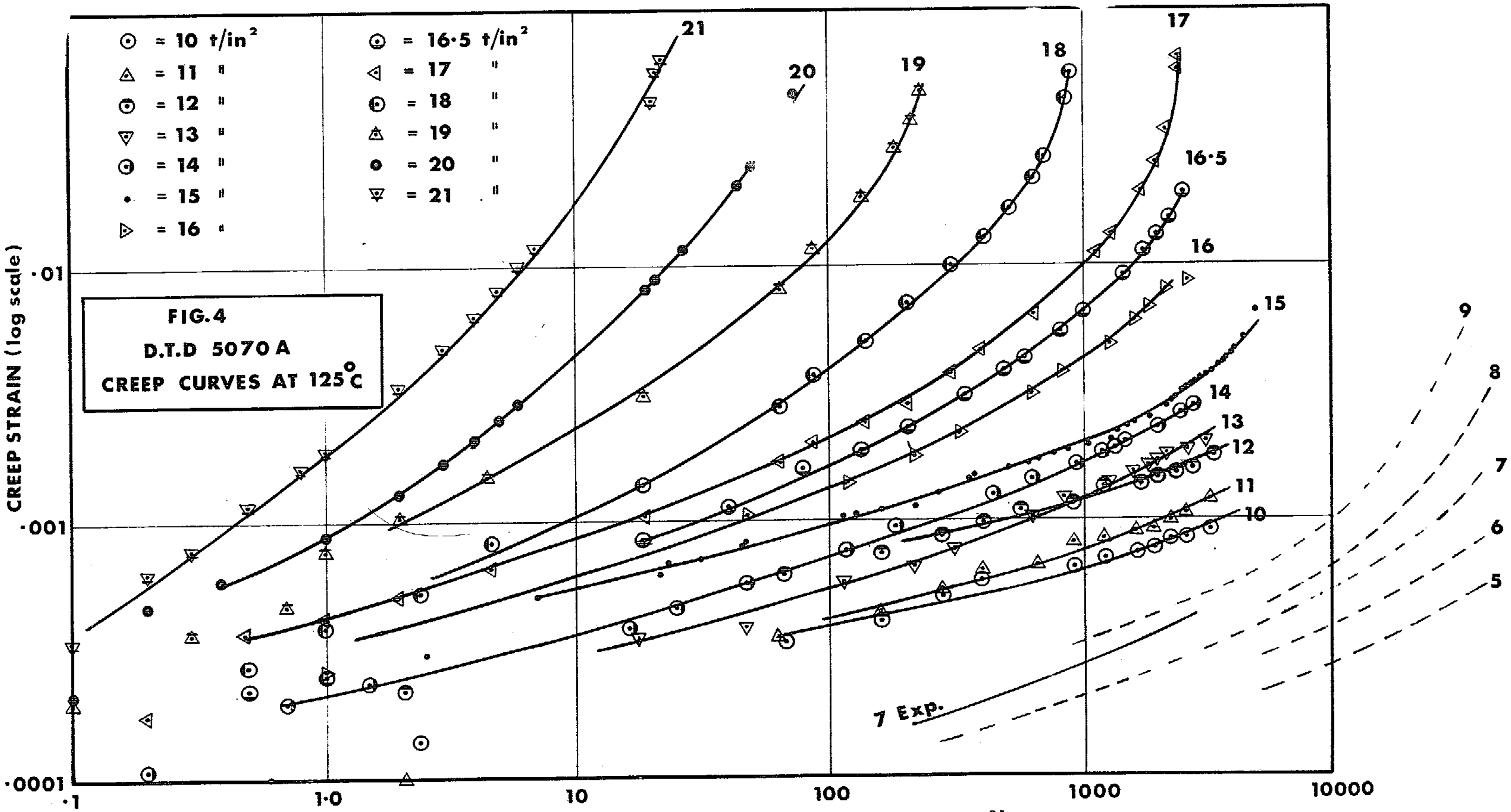


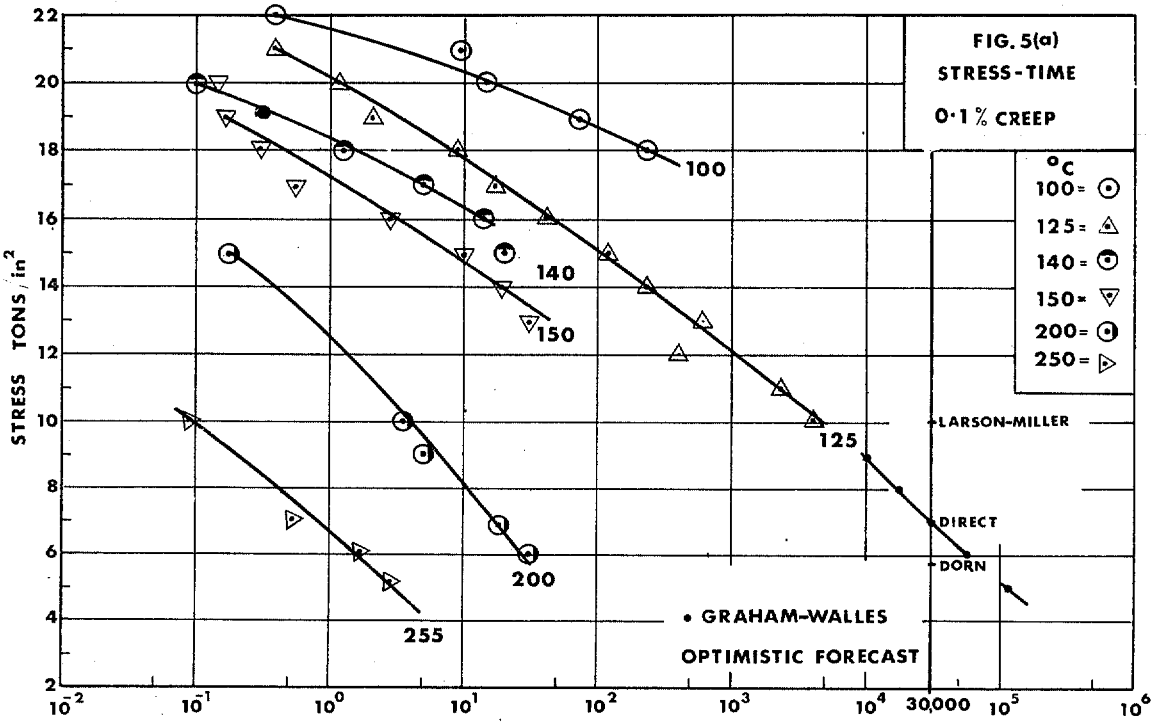
FIG. 3. Creep specimen DTD 5070A.



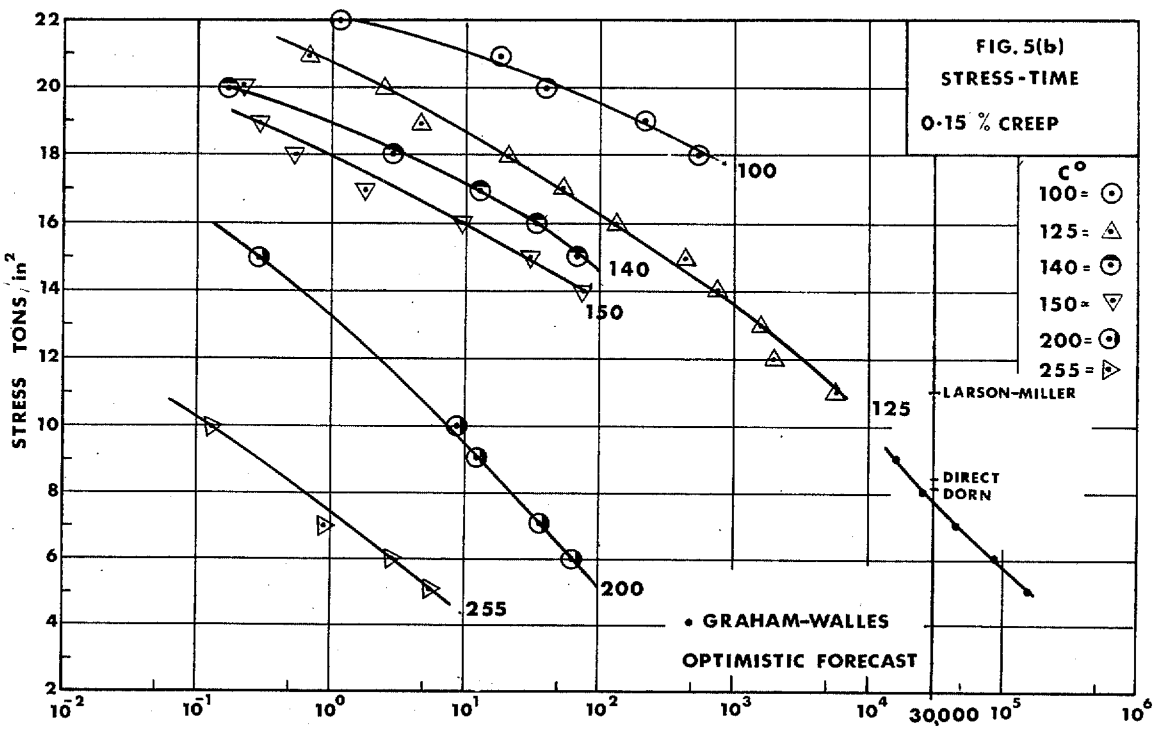
Time in hours (log scale).

FIG. 4.

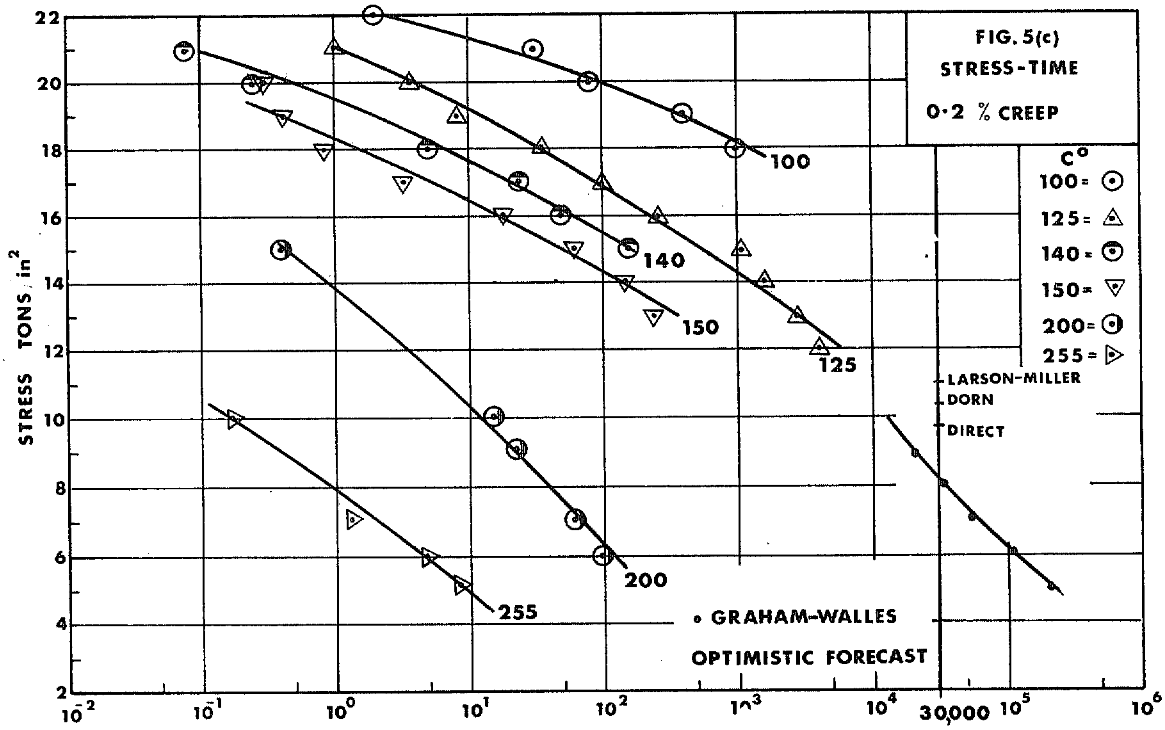
Note
Broken curves for 5, 6, 7, 8 & 9 t/in² are the optimistic Graham-Walles forecasts.



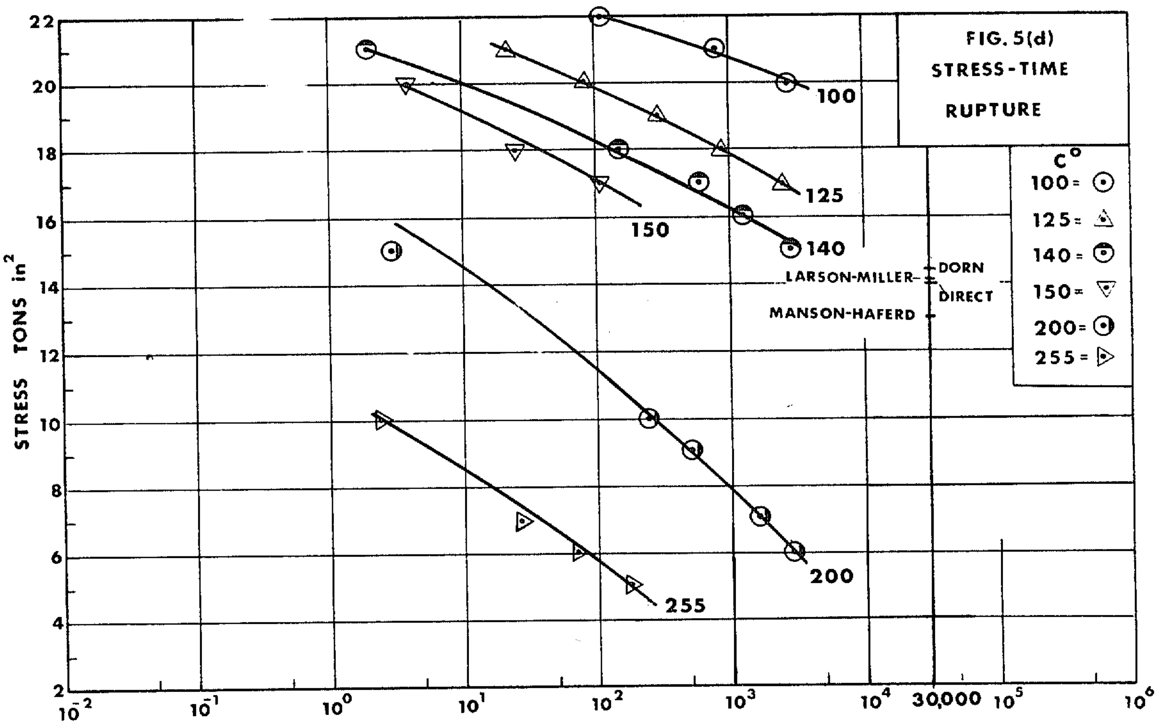
Time in hours (log scale).
FIG. 5a.



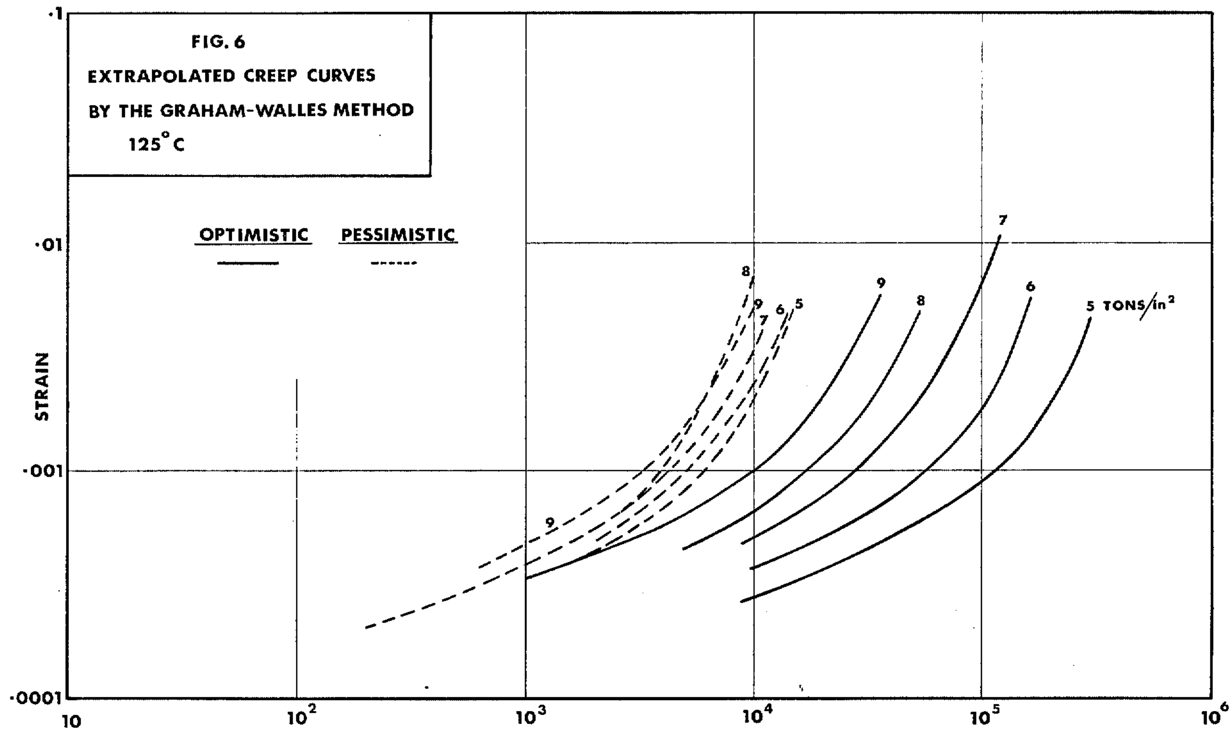
Time in hours (log scale).
FIG. 5b.



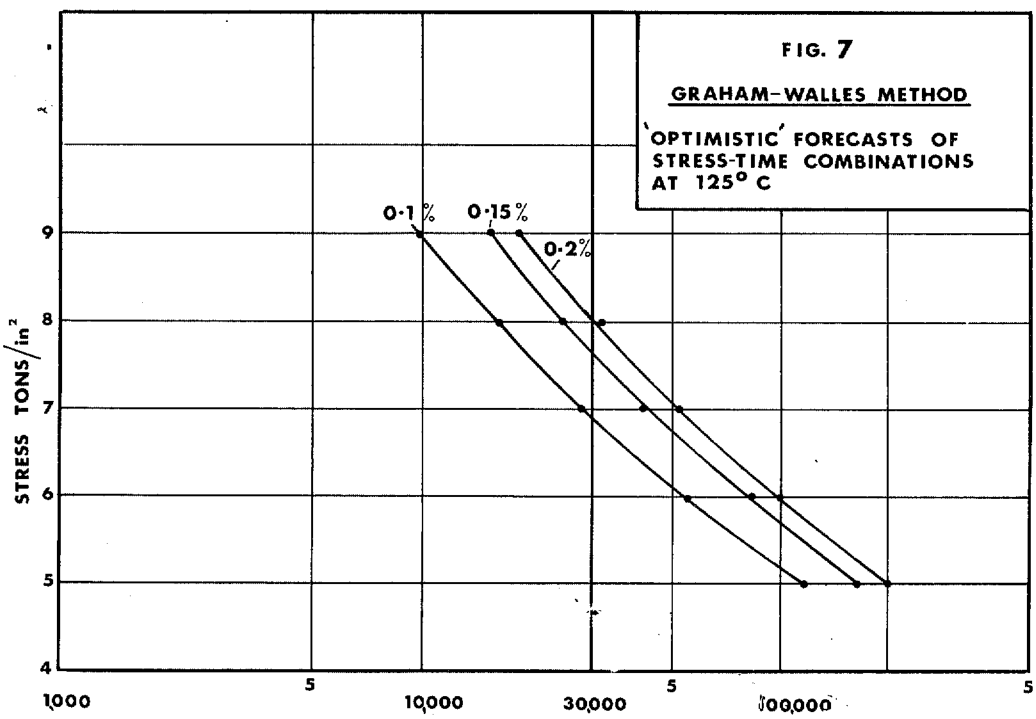
Time in hours (log scale).
FIG. 5c.



Time in hours (log scale).
FIG. 5d.



Time in hours (log scale).
FIG. 6.



Time in hours (log scale).
FIG. 7.

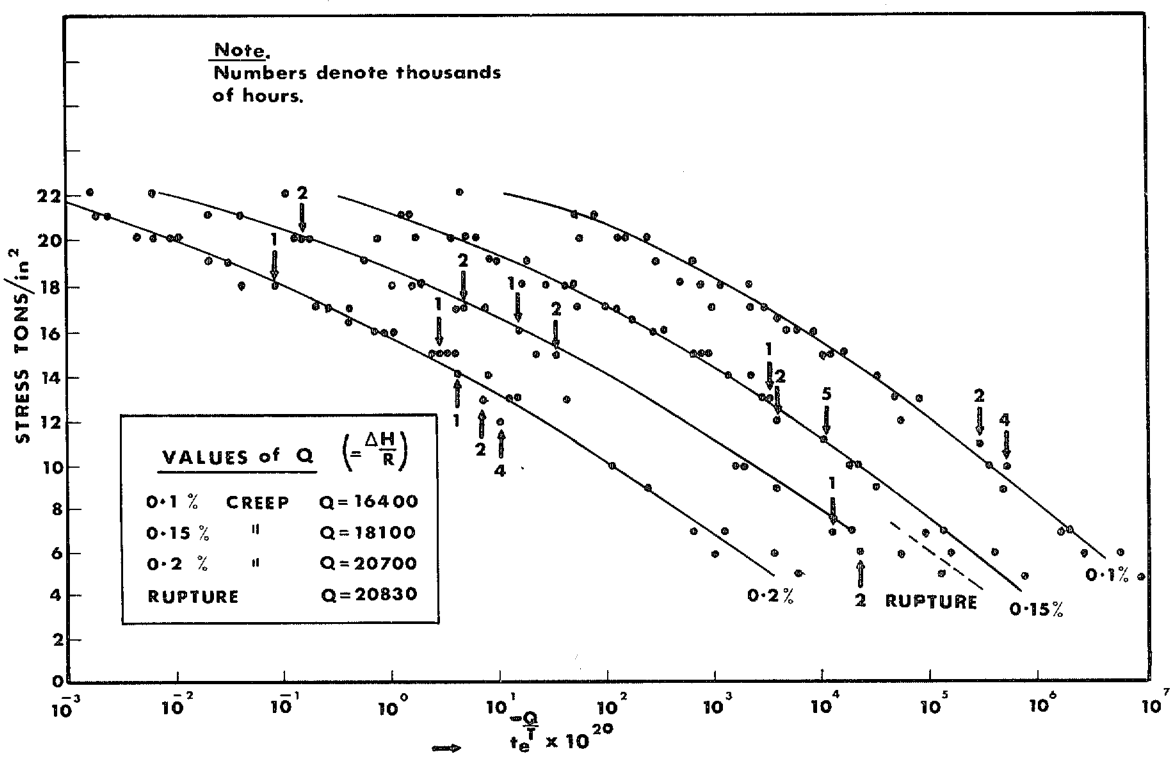


FIG. 8. Dorn master curves.

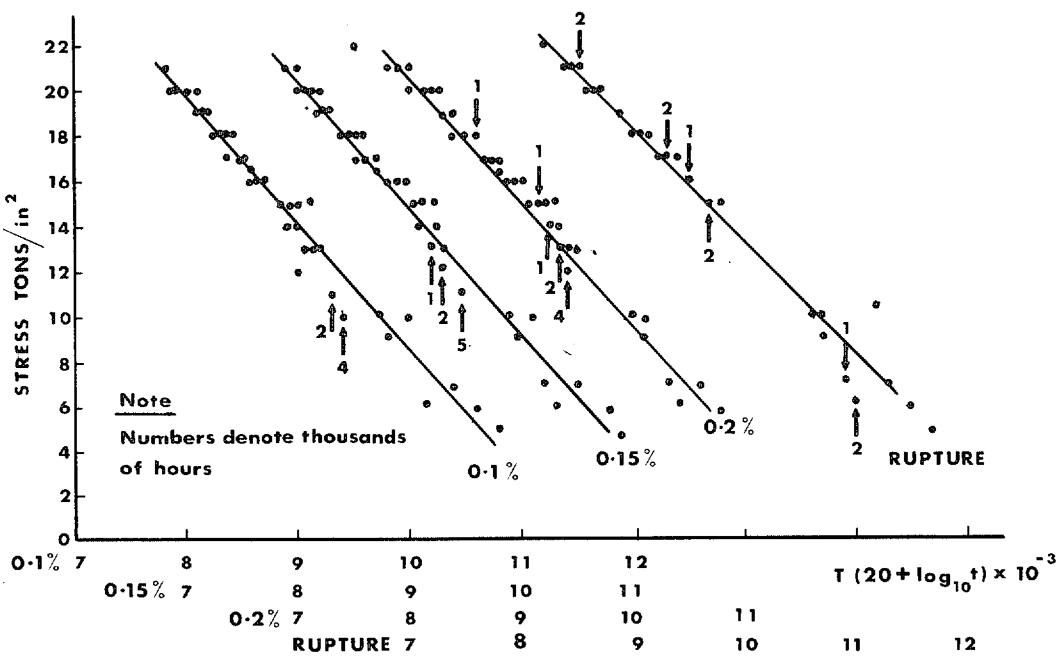


FIG. 9. Larson-Miller master curves.

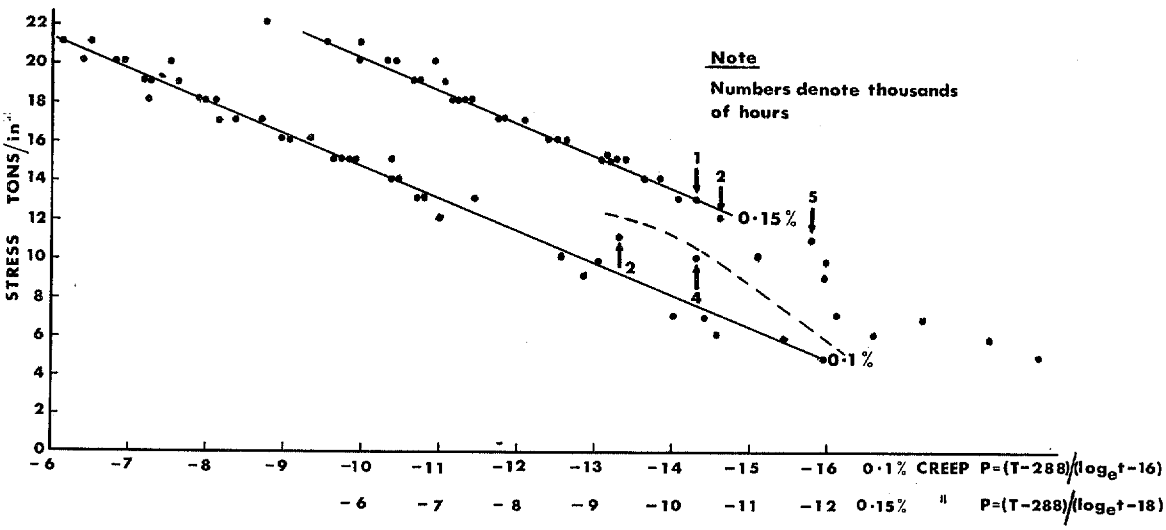


FIG. 10a. Manson-Haferd master curves 0·1 per cent and 0·15 per cent creep.

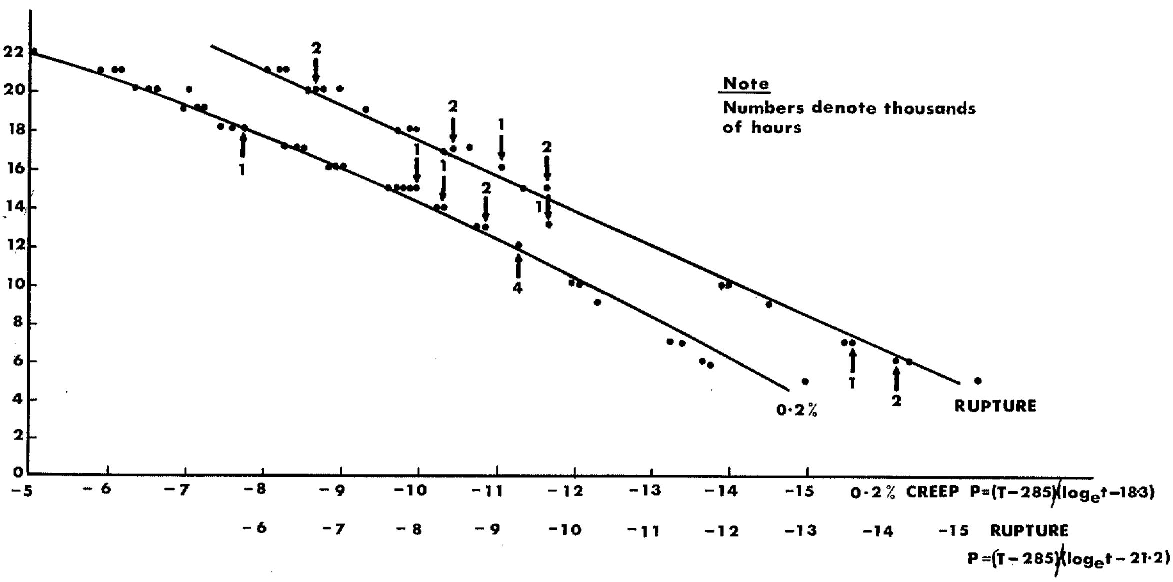


FIG. 10b. Manson-Haferd master curves 0·2 per cent creep and rupture.

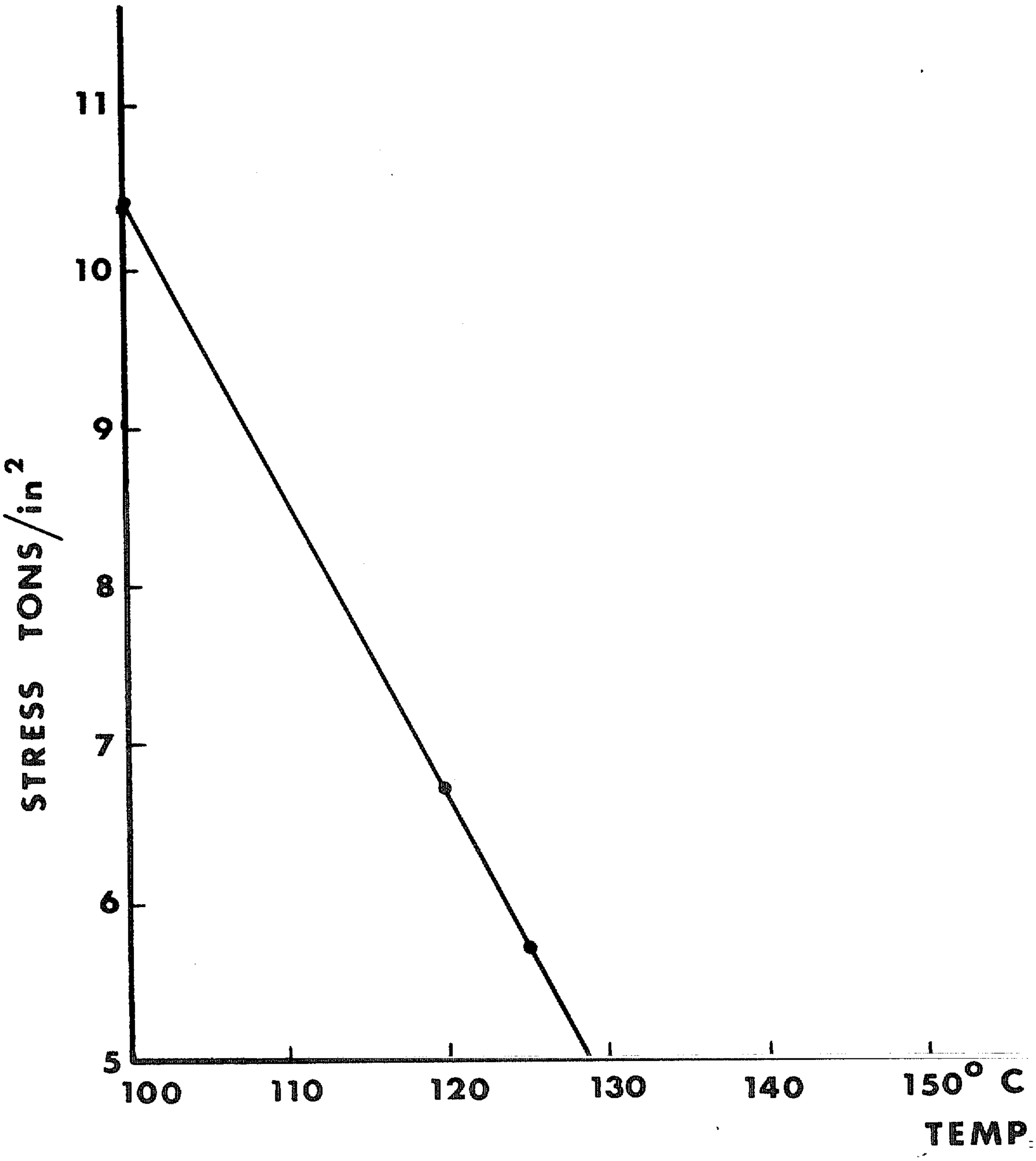


FIG. 11. Prediction given by Dorn parameter for 0·1 per cent creep in 30 000 hours.

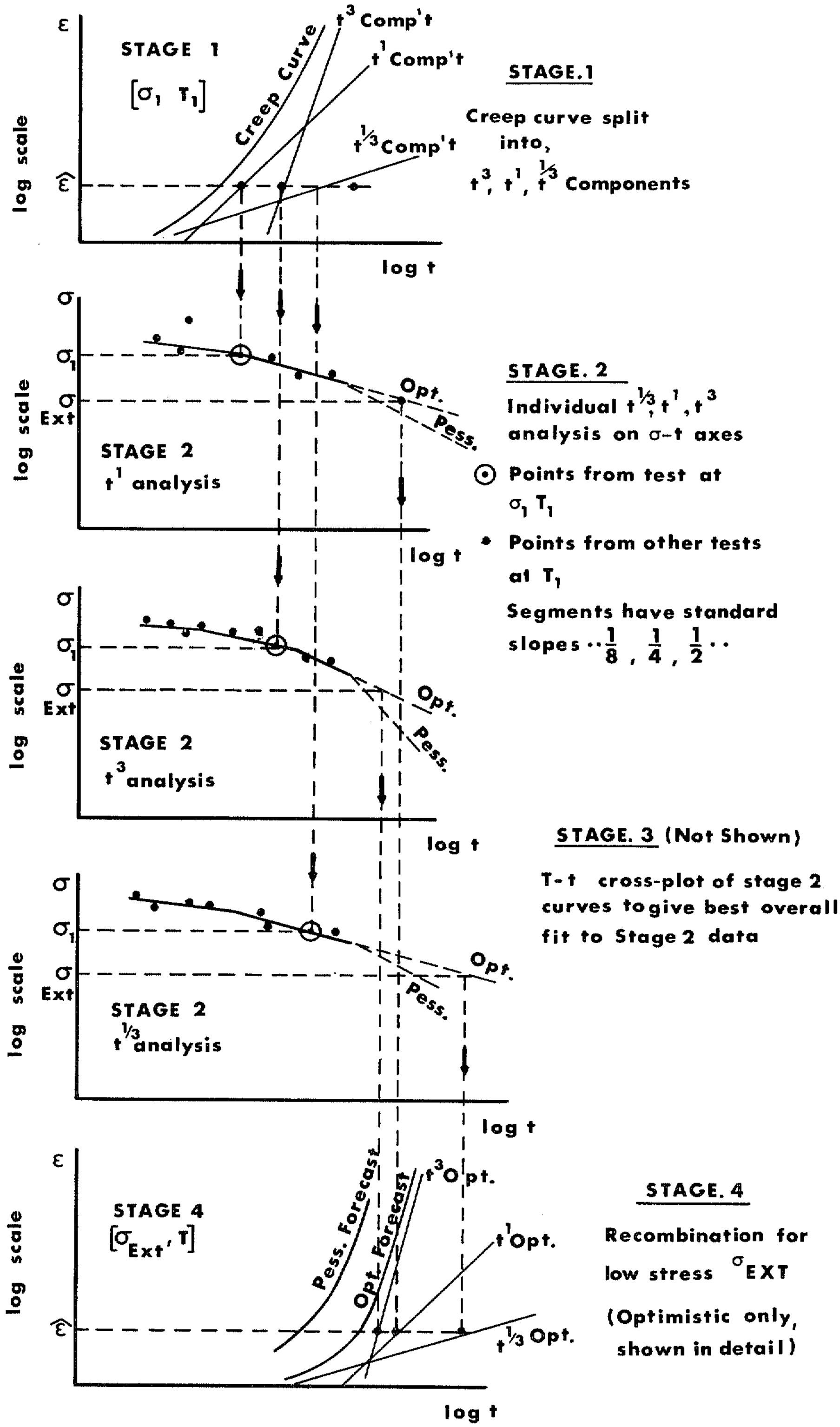


FIG. 12. Graham-Walles method.

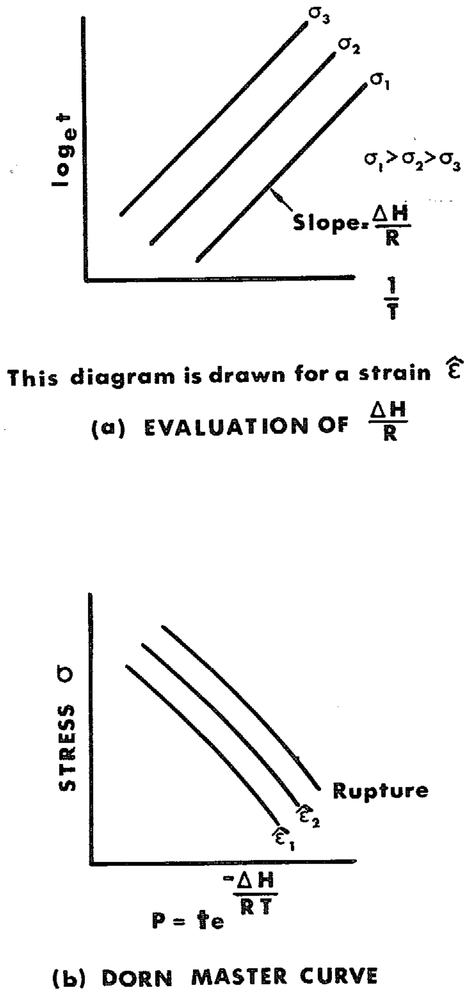


FIG. 13. Dorn parameter.

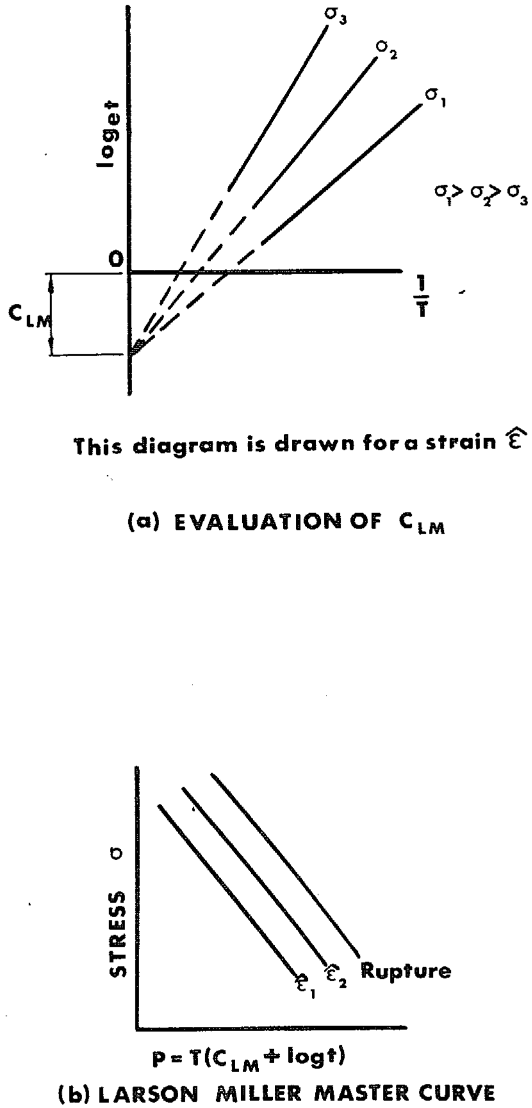


FIG. 14. Larson-Miller parameter.

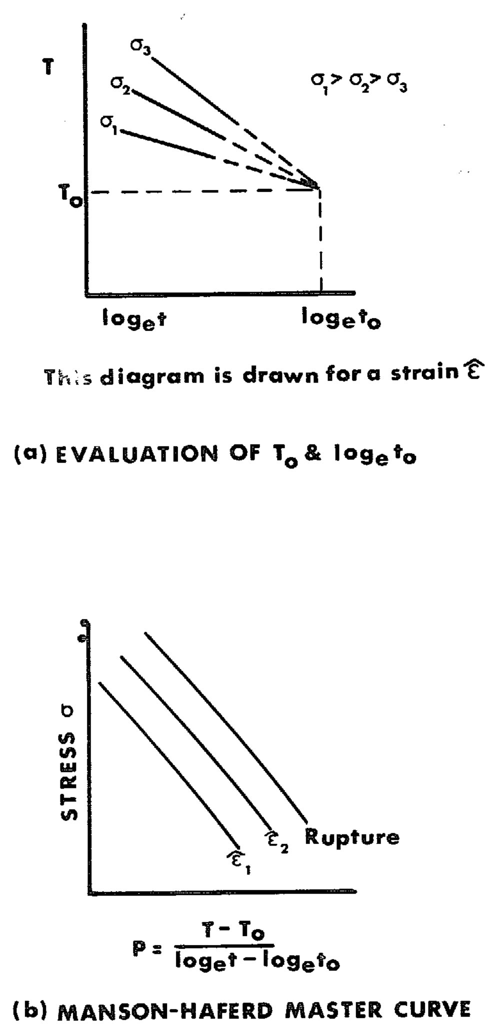


FIG. 15. Manson-Haferd parameter.

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