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The Dynamics of Aircraft Rotation and Lift-off and its Implication for Tail Clearance Requirements, Especially with Large Aircraft

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Summary.

Ground clearance requirements are considered in relation to take-off, treating lift-off as a dynamic manoeuvre. It is shown that ground clearance is defined by conditions about one to two seconds after lift-off. Simple calculations are presented which permit a rational assessment of the proper tail and wing tip clearance margins arising from the immediate post lift-off motion of an aircraft. These effects are potentially more severe for the larger aircraft and are aggravated on tailless designs by adverse elevator lift. Recommendations are made for the use of directors and autostabilisation in this context.

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1. *Introduction.*

Lift-off is that instant during the take-off of an aircraft at which the main wheels leave the ground and the aircraft becomes fully airborne. When assessing the performance during departure from an airfield, lift-off is only treated in passing and not usually studied in great detail. One main requirement to be satisfied at lift-off is that there must be enough clearance of the rear extremities of the aircraft from the ground to permit the appropriate lift-off incidence to be safely reached. This condition becomes more critical in the demonstration of minimum take-off speed demanded during certification trials. In many instances ground clearance may in fact be the limiting factor for this case.

One normally considers this condition to be satisfied if wind-tunnel data suggests that, with the tail just scraping the ground, there will be enough C_L at least to support the weight of the aircraft at the chosen lift-off speed. This must then be verified on the airfield.

In this relatively simple approach the dynamics of the aircraft motion are entirely ignored and moreover the C_L value appropriate to this manoeuvre is not very sharply defined. However, there is no evidence to suggest that this approach has so far failed to provide adequate operational margins. Nevertheless, it will be shown, that for large aircraft also with tailless designs, it may no longer be permissible entirely to ignore the various dynamic effects acting during lift-off if later disappointment during flight trials is to be avoided. The present Report attempts to consider all the factors, both of aerodynamic and dynamic origin, which one ought to take into account in a more complete assessment of the lift-off manoeuvre.

First we shall consider the effect of elevator lift and unsteady flow on the C_L available for lift-off, and this will be followed by a discussion of the post-lift-off motion of the aircraft and the various condition which might lead to ground contact after lift-off proper is achieved.

2. *The Lift Available for Lift-off.*

2.1. *Steady State Lift.*

When an aircraft is proceeding at a constant attitude along a level runway, the lift coefficient is a function of three parameters: incidence attitude, flap configuration and elevator angle. The appropriate aerodynamic data must of course be obtained with the ground represented, and as the relationships are not necessarily linear we simply write:

$$C_L = f(\alpha, \eta_F, \eta) \tag{1}$$

for a given take-off, flap is selected before the start of the take-off run and can therefore be treated as constant, so that for our purposes equation (1) reduces to

$$C_L = f(\alpha, \eta), \tag{2}$$

incidence and control deflection being the only two parameters under pilots' control. In Fig. 1 $C_L(\alpha)$ is plotted with elevator angle as a parameter for two aircraft types, a conventional high aspect ratio tailed aircraft and a slender tailless delta. The basic untrimmed lift is shown, and for comparison the trimmed lift for a typical centre of gravity range. Also shown is the lift coefficient available with the stick either fully aft or forward. The trimmed lift does not take into account the elevator required on the ground to balance that portion of the weight unsupported by lift, so Fig. 1 is strictly only applicable at lift-off or thereafter when $L \geqq W$. To indicate more clearly the significance of these data, appropriate lift-off speeds for the two cases are also presented.

The picture for the conventional design shows no particularly startling features. As one would expect trimmed lift is somewhat lower than the untrimmed lift and, with the stick fully aft, the effect of additional adverse elevator lift is to reduce further the C_L available in this condition. Though not negligible, these effects would not be expected to use up more than part of the margins in lift-off speed or tail clearance, which compliance with the air-worthiness requirements normally provide.

With the tailless delta, this is, however, no longer so obvious. Because of the relatively short moment arm at which the elevons acts, very substantial control surfaces are needed and these are then capable of generating lift, especially in the negative direction, which is almost comparable in magnitude with that generated by the wing itself. Moreover the ground induces a very substantial nose down pitching moment which adds to these elevator demands. There is now a marked loss of lifting capability when aerodynamic trim is taken into account, and this becomes progressively more significant as the centre of gravity is moved forward. This phenomenon is of course allowed for in take-off calculations and needs no further discussion here. What is, however, not generally realised is the potential power of the pilot, through elevator application beyond that required for static trim, to alter substantially the overall lift acting on the aircraft. If we assume for instance that this aircraft is scheduled for lift-off at 180 knots, with aft centre of gravity this would be expected to occur at 10·5 incidence. If the pilot were to pull the stick right back, lift-off would be delayed until 13·2 incidence is reached. In other words 2·7 of the nominal ground clearance are eaten up by this phenomenon. In the case where the tail would touch the ground before the appropriate stick-aft lift-off incidence is reached, lift-off will be delayed until the required higher speed is reached. Immediate recovery is obviously possible by simply relaxing elevator. It is perhaps worth noting that this effect has actually been observed during a take-off with the H.P. 115, a small slender wing research aircraft, where the aircraft eventually became airborne only after the stick was firmly pushed forward.

As well as delaying lift-off with excessive aft elevator application, the elevator, if pushed forward say to check too rapid rotation, can equally effect premature lifting of the main wheels, resulting possibly in a rebound when elevator is reversed.

Whether any of these events are in fact likely to occur in practice, at least in their more extreme form, is, of course, a matter for conjecture. All that theoretical analysis can show is that they are physically not impossible, if one ignores pilots' intelligence and experience as sufficiently powerful inhibiting factors. Take-off calculations, such as those presented in Ref. 1, which are based on an assumed smooth and progressive control 'law', will not be able to predict the likelihood of such form of control abuse, although they should give a realistic picture of normal usage. The only really meaningful information would come from observations obtained during actual operation. Unfortunately the majority of aircraft in existence today having a configuration generating large elevator lift, i.e. tailless deltas, are physically small. As small aircraft have a fast pitch response, on them 'out of trim' elevator can only occur during brief transient periods, too short to show up the effects discussed above. Large aircraft, on the other hand, for which adequate operational data are available, do not possess the adverse control characteristics under discussion, so that their record cannot yield any useful clues.

Ideally one would wish aircraft to be able to absorb the consequences of piloting abuses unless these can be shown to be potentially improbable. On the other hand, to clear the more extreme configurations for an conceivable control usage would result in quite unacceptable performance penalties.

The problem, like those discussed later in this Report, is essentially one of usage. As such, it can only be resolved by experiments with a pilot at the controls. In the case of aircraft designs with properties not found in existing aircraft, this condition can only be studied on simulators.

There is another possible consequence of the effect of exaggerated adverse elevator lift, which is readily apparent from Fig. 1. If full aft stick is applied to initiate nose lifting on the tailless aircraft, negative lift is generated equivalent to perhaps up to 2/3 of the aircraft weight, i.e. the load on the main wheels will be increased momentarily to $1\frac{2}{3}$ times the take-off weight, a condition which may have to be considered as a loading case.

To demonstrate these phenomena more clearly Fig. 2 has been prepared where, for the slender aircraft considered in Fig. 1, aerodynamic lift has been plotted as a fraction of the aircraft weight against incidence for a speed of 180 knots. Three cases are represented:

(i) Elevator corresponding to trim in flight. This condition only applies sensibly for $L/W > 1$.

(ii) Elevator corresponding to trim on the ground taking account of nose lifting requirements. This condition does not apply for $L/W > 1$.

(iii) Elevator corresponding to full aft stick.

It has been shown that elevator lift, especially for tailless designs, can substantially reduce the lift coefficient available for take-off and that cases of elevator abuse must be taken into account when defining operational requirements.

In the discussion so far, the relationship of lift to incidence is taken to be time invariant, in other words quasi-steady flow is assumed. As lift-off occurs normally when the aircraft is rotating, i.e. $\alpha = f(t)$ the development of lift will be subject to unsteady aerodynamic phenomena and these will be discussed in the following Section.

2.2. *Unsteady Lift.*

The condition relevant to lift-off is pitching motion of the aircraft in the presence of the ground. With slender wings there is the additional complication that the flow will be separated from the leading edges and a large part of the lift is generated by the consequent vortex. No solutions, either theoretical or experimental, exist to cover this case. The best one can do is to consider the general nature of the solutions available to date, for essentially much more simple cases, and to make some speculative estimates for lift-off, assuming that basically similar effects operate in that condition as well.

Ref. 2 gives a collection of results obtained by various authors, a selection of which is presented in Fig. 3. These represent indicial functions, i.e. time histories of lift following a sudden change in incidence on a wing in free air, which is not strictly the case applicable to the present inquiry, where incidence changes are a result of rotation and not of plunging. Corresponding experimental results are not available, so that the theory cannot be tested for accuracy and should not be relied upon to give more than an indication of the order of magnitude of the unsteady effects.

Slender wings with separated leading-edge flow present even greater difficulties for analysis, but recently some theoretical results again for sudden increase in incidence have become available[3-7]. The principal results given in Ref. 3 are shown in Fig. 4. Again no direct comparison with experiment can be made, but the theory had a measure of success in predicting the movement of the vortex cores observed during experiments reported in Ref. 8 and can therefore claim a degree of credibility.

Nevertheless, none of the theoretical data can be considered as final until it is confirmed by experiment. Furthermore the presence of the ground during rotation to lift-off is unaccounted for in these results. Also the theories quoted above treat only the case where incidence is changed by a sudden plunging motion whereas the case in which we are interested is that where incidence changes as a result of rotation about a point generally close to the centre of gravity of the aircraft.

In view of the uncertainty of the available aerodynamic data it is thought more prudent to estimate the effects of varying amounts of unsteady lag on the lift-off capability of aircraft, rather than to make calculations for specific cases based on doubtful data. This will outline the potential influence of unsteady aerodynamics on the manoeuvre, and indicate the need or otherwise, for more effort in establishing reliable unsteady aerodynamic data.

For this purpose the indicial lift function is expressed in the idealised form sketched in Fig. 5:

$$k_1(D) = k_{1_0} + (1 - k_{1_0})(1 - e^{-D/c\tau}) \qquad (3)$$

where D = distance travelled

$k_{1_0} = k_1$ at $t = 0$

τ = time constant of lift lag in chords

c = reference chord.

With the transformation $D = Vt$, equation (3) can be expressed as a function of time:

$$k_1(t) = k_{1_0} + (1 - k_{1_0})(1 - e^{-t/t_L}) \qquad (4)$$

where $t_L = \dfrac{\tau c}{V}$

Considering the simplest case where the aircraft is rotating with a constant rate of pitch q_0, the unsteady part of the lift $(1-k_{1_0})$ will lag behind the incidence, as shown in Appendix D, with the time constant t_L and as a consequence lift-off will require the additional incidence:

$$\Delta\alpha = (1-k_{1_0})\, q_0 \frac{\tau c}{V}. \tag{5}$$

This function has been plotted against q_0 for parameters considered typical for current transport aircraft with 10 and 15 ft mean chord (c) in Fig. 6a and for a possible range of parameters applicable for small aspect ratio slender wings in Fig. 6b. As suggested by Fig. 3 the time constants τ are assumed to have values of 3 and 2·5 chords respectively. It is seen that unsteady flow will hardly make a significant difference for the conventional design. With small aspect ratio wings on the other hand, this is only true if k_{1_0} is really very close to unity, as predicted in Fig. 4. If, however, the lift delay is more like the results indicated in Fig. 3, there is a possibility of needing something of the order of an extra $\frac{1}{2}$ deg incidence or even more to make up for the delay in the development of unsteady lift. Clearly this is not an insignificant amount, and it would seem that tunnel tests should be made to obtain a clearer indication of the order of magnitude of this effect.

2.3. *Loss of Incidence due to Vertical Movement During Unloading of Main Undercarriage.*

Unless the aircraft has been checked in its motion prior to lift-off, at the instant of lift-off it will not only be rotating in pitch but also have acquired a vertical velocity resulting from the extension of the main undercarriage. This vertical velocity $(-w)$ will then reduce the incidence by

$$\Delta\alpha = \frac{w}{V} \tag{6}$$

and again this will add to the attitude required for lift-off. Making simple assumptions about the nature of the rotation and the characteristics of the undercarriage in Appendix A, a simple expression has been derived to permit the vertical velocity w during rotation to be determined from:

$$w(t) = -\left(\frac{T_0}{t_R}\right)^2 \frac{2}{g}\sqrt{\frac{g}{T_0}}\sin\left(t_R\sqrt{\frac{g}{T_0}}\,\frac{t}{t_R}\right) + 2\frac{T_0}{t_R}\left(\frac{t}{r_R}\right) \tag{7}$$

where T_0 = undercarriage stroke to static compression

t_R = rotation time.

At lift-off $t = t_R$ and we get

$$w(t_R) = -\left(\frac{T_0}{t_R}\right)^2 \frac{2}{g}\sqrt{\frac{g}{T_0}}\sin\left(t_R\sqrt{\frac{g}{T_0}}\right) + 2\frac{T_0}{t_R}. \tag{8}$$

Equation (8) has been computed and plotted in Fig. 7 for the relevant range of the parameters T_0 and t_R. For easier interpretation the relationship given in equation (6) has been presented in Fig. 8. The results shown in Fig. 7 are to a large extent determined by the nature of the assumed rotation manoeuvre, and as a consequence not too much notice should be taken of details. However, the general order of the effect should be well enough represented. It can be seen that for reasonable values, say rotation times between 2 to 3 sec and $T_0 \sim 1$ ft, the aircraft will acquire a vertical velocity of the order of 1 ft/sec and that this represents a loss of incidence of say 1/5 deg. This is a relatively small amount, but as it is additive to other negative contributions it cannot be entirely ignored. Also vertical velocity at lift-off enters quite substantially into the question of tail strike after lift-off to be discussed in the next Section and must be considered in this context.

3. *Aircraft Motion after Lift-off.*

There is a temptation to assume that once the main wheels have left the ground and provided of course that the aircraft has adequate performance to sustain flight, ground clearance is no longer relevant.

The vertical velocity of the tail having a distance of l_T from the centre of gravity of the aircraft is:

$$\frac{dH_T}{dt} = -q\, l_T + \frac{dH}{dt} \tag{9}$$

where q is the rate of pitch and dH/dt the vertical velocity of the centre of gravity of the aircraft. Clearly at lift-off, this quantity will be negative, i.e. the tail will move down towards the ground, if

$$q\, l_T > \frac{dH}{dt}.$$

Normally at lift-off the aircraft will be rotating with a significant pitch rate q whereas the vertical velocity of the centre of gravity will be relatively small, only the contribution discussed in the previous Section, and shown in Fig. 7, coming into the picture. So it appears that in this case the point of lift-off is not generally the most critical instant to determine ground clearance for take-off, and more thought has to be given to the details of the aircraft motion immediately following lift-off proper.

The type of motion resulting from this phenomenon is illustrated in Fig. 9. When considering the possibility of ground contact after lift-off, it is worth noting that, in this condition, the part of the aircraft exposed to the ground may be different from that in the more usual case of ground clearance with the aircraft's main wheels on the ground. This is illustrated in the sketch in Fig. 10.

We shall now treat the post lift-off dynamics as it would be affected by the nature of the rotation, and by the initial flare-up control manoeuvre adopted by the pilot. First the case is considered where only the basic kinematics of the aircraft motion are involved and finally the influence of adverse elevator lift arising from pilot's control will be also discussed.

3.1. *Motion not Involving Elevator Action immediately after Lift-off.*

To simplify the discussion we shall first consider the post-lift-off motion ignoring elevator lift. To reduce further the number of variables it is proposed to specify the pitching motion $q(t)$ rather than the control input $\eta(t)$. As time histories of $q(t)$ for the lift-off manoeuvre are available from many sources from both simulations and flight experiments, these can be easily related to actual practice.

The method used for the response calculations is given in Appendix B. Two cases are considered:

(i) $q = q_0 = \text{const.}$

(ii) $q = q_0 \cos\left(\frac{\pi}{2}\frac{t}{t_0}\right)$, i.e. pitch rate decreasing from a maximum value q_0 at lift-off to zero after t_0 sec.

A time history of the immediate post-lift-off motion with constant q is illustrated in Fig. 11, showing the motion of the main wheels and that of the tail, or more generally that part of the aircraft l_T feet aft of the main wheels, which constitutes the ground clearance hazard. The assumption of constant pitch rate clearly becomes unrealistic when applied for more than the first second or so after lift-off, but this is the only part of the manoeuvre of interest here.

It is shown in Appendix B that the problem is fully defined by the aircraft parameters:

V = speed

$\mathscr{L} = \frac{g\rho}{2}\frac{V}{W/S}\left(\frac{\partial C_L}{\partial \alpha}\right)_{\text{TRIM}}$ the effective trimmed lift slope

l_T = the distance from the main wheels of the rear ground-contact point and by the initial conditions

$\dot{H}$ = vertical velocity at lift-off

q_0 = pitch rate at lift-off

Fig. 12 shows solutions obtained for $q_0 = 2$ deg/sec = const and $V = 300$ ft/sec for a range of values for the lift slope $\mathscr{L}$ and for two values of the tail arm l_T and also for two values of H_0. Being the only quantity of interest, only the movement of the tail ΔH_T is presented. The range of $\mathscr{L}$ represented in these calculations cover the full range of interest from the conventional high aspect ratio design ($\mathscr{L} = 0{\cdot}5$) to a slender delta ($\mathscr{L} = 0{\cdot}3$). It is seen to have a relatively minor effect, so that in the further analysis a mean value ($\mathscr{L} \simeq 0{\cdot}4$) is used.

Similar calculations have been made for a range of lift-off manoeuvres, covering a range of pitch rates, different variations of pitch rate after lift-off, and a range of initial values of vertical velocity, the results are summarised in Fig. 13. Only the maximum downwards stroke of the tail ΔH_{max} has been plotted in terms of the equivalent pitch attitude $\Delta\theta_m = \dfrac{\Delta H_{max}}{l_T}$ which has to be allowed for at lift-off to avoid ground contact during the consequent manoeuvre.

It can be seen that this phenomenon may demand a margin in tail clearance at lift-off of up to 3 deg for the shorter aircraft and perhaps up to 4 deg for the aircraft with the 70 ft tail unless it can be shown that pitch rates will not exceed say 4 deg/sec. These margins correspond roughly to those available on current civil aircraft.

It can be concluded that the post lift-off motion makes increasing demands on tail clearance, the longer the rear part of the fuselage and the faster the pitch rate used at lift-off. The problem is therefore more likely to become acute with the larger aircraft, and also if pilots are attempting rapid rotation. Pitch rate used during rotation will tend to increase if lift-off requires a relatively large attitude; consequently the tailless design using small aspect ratio wings should be more prone to tail-strikes than more conventional configuration, unless appropriate ground clearance is provided or piloting technique is suitable modified.

3.2. *The Effect of Elevator Manipulation.*

In the previous Section we have considered the post-lift-off motion principally as a kinematic phenomenon without specifically studying the longitudinal control action involved in such a manoeuvre. From the earlier discussion of the effect of elevator on lift-off, it is apparent that for tailless aircraft this control has a powerful direct effect on the lift itself, and that control activity after lift-off may therefore also influence the motion of the aircraft immediately following lift-off. This case will now be investigated.

The condition in which a pilot is most likely to apply elevator in excess of that required just to maintain the aircraft in trimmed steady or quasi-steady motion is perhaps lift-off from a checked rotation. By this is understood a take off, where the aircraft has been rotated to take off incidence and held there for a moment with zero pitch rate and then elevator is applied to expedite climb out. The response of an aircraft to an instantaneous aft elevator application at this instant is shown in Fig. 14. If one ignores in these calculations the restraint of the ground on the main wheels, the motion depicted in Fig. 14a would be obtained. Clearly the downwards movement of the *CG*, and thus of the main wheels implied in this solution, is physically impossible and instead the case illustrated in Fig. 14b must be considered, where lift-off is suppressed until an attitude is reached to give $L > W$. The situation is changed if one assumes an initial vertical velocity at $t = 0$ which allows lift-off to occur although at that instant $L < W$. In the particular case shown in Fig. 15 the main wheels would recontact the ground about 1 second after the first lift-off, and the further loss of height will be so small that it can be readily absorbed by the undercarriage. To obtain a realistic result in this case, the undercarriage characteristics ought to be represented in the calculations.

In addition to the parameters considered in the earlier analysis of Section 3.1 we must now also allow for the effect of the elevator on the aircraft response. For this the elevator is represented by a lift generated at a point l_η ft aft of the centre of gravity and aircraft pitch inertia is defined by the radius of gyration k_y. Static stability m_w and pitch damping m_q of the aircraft are ignored, but this should have little consequence, especially for conditions with small static margin, during the first few seconds of the response. This allows the elevator application to be expressed simply as an effective acceleration in pitch, i.e. we can as in Section 3.1 define a pitching manoeuvre rather than a control input which allows more easily the plausibility or otherwise of a given case to be assessed.

Fig. 16 gives an example of a response to an assumed manoeuvre with a constant acceleration in pitch $dq/dt = 2$ deg/sec^2, i.e. within the simple assumptions made, constant elevator, instantaneously applied at $t = 0$, also assuming that at that instant the aircraft pitch rate is zero. To obtain a physically viable solution and to ensure that the main wheels do not strike the ground immediately after lift-off an initial vertical velocity is assumed as $H_0 = 1$ ft/sec. The sums have been made for two tailless aircraft with the relevant characteristics as defined in this figure. For comparison results are also shown with elevator lift ignored and it is clear that this would give entirely misleading answers.

Calculations have also been made for the same type of manoeuvre, but covering a range of the relevant aircraft and control parameters. The results are shown in Fig. 17, where again only the maximum down stroke of the tail is plotted in terms of the equivalent additional tail clearance pitch angle:

$$\Delta\theta_m = \frac{\Delta H_{T_{max}}}{l_T}$$

It is apparent that these requirements get more severe the larger the tail length and also the shorter the effective elevator arm l_η in relation to the radius of gyration in pitch k_y.

A more realistic manoeuvre is perhaps one in which the post lift-off rotation is defined by an incremental change in pitch angle θ_M starting from a rotation checked at lift-off incidence which is achieved in a given 'manoeuvre time' T_M. Corresponding results obtained for the two aircraft defined in Fig. 16 are given in Fig. 18. It is clear from these results that this type of manoeuvre can make even greater demands on ground clearance than the case of a continuous rotation through lift-off (Fig. 13) discussed previously.

It must be concluded that to arrive at a realistic assessment for ground clearance requirement, the motion of the aircraft following lift-off must be considered in some detail, covering a range of piloting techniques and not ignoring the effect of elevator lift.

4. *Ground Clearance Requirements for Simultaneous Pitching and Banking after Lift-off.*

Ground clearance must be provided not only to allow the aircraft freedom for pitching but also for banking at or immediately after lift-off. The principal cause for the aircraft to commence rolling at lift-off will be the existence of a crosswind at that instant. Unless an appropriate amount of lateral control is applied before lift-off, the effect of the crosswind will appear suddenly on the aircraft, when the main wheels lift off the ground. This condition is distinctly different from that existing in an approach, where the pilot is obliged continuously to maintain lateral trim and the basic crosswind component does not occur as a sudden disturbance, only gusts have this effect but they are generally of smaller magnitude.

Before we consider the lateral response of an aircraft in a crosswind lift-off, we must first determine the ground clearance envelope available. The extremities relevant to this are generally the tail, as considered in the main body of this report, and secondly the wing tips and/or engine nacelles to define bank clearance.

With a straight wing design, the aircraft pitches about an axis which in plan view coincides practically with the axis of the main wheels and as a consequence pitch clearance and bank clearance are independent of one another, giving a clearance envelope as indicated in Fig. 19a. If the wings are swept back the tips are located aft of the pitch axis and now the available bank clearance is reduced with increased pitch attitude. At the same time beyond a certain bank angle, bank angle itself will reduce the available pitch clearance of the wing tip. See Fig. 19b.

In the extreme case of a slender tailless design, the wing tips (or the engine nacelles) may form both the rear and the lateral extremity of the aircraft and the ground clearance envelope assumes the configuration indicated in Fig. 19c. Here the interaction between pitch and bank clearance becomes even more powerful. In this case a take-off right up to the tail clearance limit would allow no bank angle margin at all.

As we are interested not only in the conditions existing at lift-off but also in those immediately thereafter, the relevant ground clearance envelopes ought to be determined for a range of wheel heights. Such a ground clearance envelope is shown in Fig. 20 for an aircraft of a Concorde type configuration. When considering the development of bank during or immediately following lift-off, it must be realised that pilot's recovery action will invariably apply ailerons in the sense to reduce the available wing tip clearance. To account for this, Fig. 20 also contains contours with ailerons half or fully deflected.

For a given take-off, we can now determine the available bank angle clearance through the lift-off manoeuvre by using the results for pitch attitude and wheel height in combination with the boundaries of the appropriate ground clearance envelope.

For the case used as an example in Fig. 20 this gives a time history of bank clearance as shown in Fig. 21. It is interesting to observe that during the first second after lift-off the available bank clearance decreases and that the freedom one expects from the climb out is only beginning to materialise after about 2 seconds. If it were possible now to predict a likely roll response of the aircraft during this period for instance to a crosswind, and to superimpose this over the bank angle clearance time history, the adequacy or otherwise of this clearance could be assessed.

This has been attempted for a number of assumed cases with the results shown in Figs. 22 and 23. The conditions treated in these examples are as follows. The aircraft takes off in a steady crosswind of 10 knots or 20 knots respectively. Up to the instant of lift-off ($t = 0$) the wheels restrain the aircraft on the ground so that no rolling occurs. At lift-off, the rolling moment generated by the crosswind causes the aircraft to roll and after a short delay the pilot applies aileron to counteract the aircraft response. Delays of from 0·5 to 1·0 sec are considered. Although the pilot is assumed at that instant to apply ailerons instantaneously, the power controls actuators will only be capable of moving the surfaces at a limited rate, 40 deg/sec has been assumed in the present case. In each of the two cases the application of either $\frac{1}{2}$ or full aileron is considered.

Before discussing the results of this exercise, it must be clearly understood that they are entirely at the mercy of a number of assumptions and that their plausibility must be carefully assessed. The assumed pitching manoeuvres, i.e. take-off with constant rate of pitch is considered a realistic approximation to a real case at least during the first second after lift-off, which is the only part of this response of interest, because it is then, or shortly after, that the potential ground impact hazard exists. A practical commercial aircraft must be capable of operating in crosswinds of up to 30 knots, so the two cases considered (10 and 20 knots) are far from extreme. However, if one considers that to maintain track on the runway the aircraft is usually allowed by the pilot to drift during the latter stages of the take-off run (so that the aerodynamic sideforce on the airframe due to crosswind is balanced by a lateral force on the wheels), this will reduce the actual sideslip of the aircraft by perhaps 15 per cent compared to that arising from the full crosswind $\beta = \sin^{-1}\left(\frac{V_c}{V}\right)$. Consequently one could argue that the nominal 10 knots case corresponds perhaps to 12 knots and the nominal 20 knots case to a true crosswind of say 23 knots.

The most critical assumption, however, is that concerning pilots' control. If the pilot were to apply the appropriate amount of aileron before lift-off, the aircraft response obtained in the present calculation would be entirely suppressed. Whether pilots will in fact do this or only react when the aircraft begins to roll as the wheel constraint is removed at lift-off is difficult to predict. Enquiries with pilots have been made indicating that piloting technique apparently varies from aircraft to aircraft. In some aircraft aileron is applied well before lift-off, whereas in others no such action is required or exercised. It is likely that undercarriage characteristics play an important part in this. No obvious criterion has been found so far to indicate what particular features make the pilot adopt one or the other technique. This would be a useful subject for further research. The results presented in Figs. 22 and 23, reflect, of course, the case where prior to lift-off no control is applied and they are only valid therefore, if this form of piloting technique actually applies.

The results would, however, be fully representative in either case if the crosswind is interpreted as a sudden gust but then one will have to consider the most remote probability of such an extreme gust occurring precisely at the instant of lift-off. Alternatively one can interpret the nominal crosswind component as the sum of a steady crosswind plus a contribution from turbulence. Thus, for instance, the nominal 20 knots case can be interpreted to arise from say a steady 15 knots crosswind, reduced by aircraft drift to 13 knots plus a gust in the same direction of 7 knots etc.

The results presented in Fig. 22 show that in a 10 knots crosswind, in this particular pitching manoeuvre, ground contact will only be avoided if the pilot applies aileron not later than 0·75 sec after lift-off. It is interesting to note that it makes little difference whether the pilot applied 1/2 aileron or full aileron because the reduction in roll response with the larger aileron application is largely eaten up by the corresponding reduction in ground clearance.

The pitching manoeuvre assumed in Fig. 22a and b is, perhaps, rather extreme with 4 deg/sec contact pitch rate. The effect of reducing pitch rate is shown in Fig. 22c. It is seen that in the manoeuvre, the available bank clearance is somewhat more favourable and also the aircraft's roll response is slightly less rapid because, as a result of slower pitching, the incidence and therefore the rolling-moment coefficient l_v during the crucial part of the flare up is reduced. The combined effect is to make the aircraft now just miss ground contact even if no aileron is applied. It is interesting to note that ground contact will, however, occur if aileron is applied at 1·25 sec, simply because in this particular case the control tip moves faster towards the ground than the wing moves up in response to the control action.

In the 20 knots case (Fig. 23) the situation is, however, much more serious. If only half aileron is used, ground contact can only be avoided if the pilot reacts earlier than 0·25 sec from lift-off and even if full aileron is used, this must be applied within approximately 0·35 sec.

As with the pitching response discussed earlier the potential danger is after lift-off, because due to the continued downwards motion of the rear portion of the aircraft after lift-off, the available ground clearance is reaching its minimum well after lift-off.

It should be noted that with a straight wing design or if the wings are only moderately swept, this does not occur (*see* Fig. 19) because clearance of the wing tips (or the engine nacelles) increases with wheel height and is hardly affected by pitch. At the same time with such an aircraft the rolling moment due to sideslip, l_v, responsible for the aircraft response to the crosswind, is likely to be much less and this would further relieve ground clearance requirements or alternatively the demands made on the pilot. This can be seen from Fig. 24 where the response of a typical subsonic jet transport has been calculated in conditions exactly identical to those assumed for the slender transport in Fig. 23. The application of full aileron can now be delayed $1\frac{3}{4}$ sec before there is a danger of ground contact. Although perhaps no indication for complacency this condition is clearly not critical.

5. *Discussion.*

The analysis presented in this Report has revealed a number of potential hazards to safe take-off, which are not apparent in a more conventional assessment of lift-off requirements. These, although inherent to a degree in practically every design, appear to become noticeably more severe for the large and for the tailless aircraft.

One can distinguish two distinct problem areas.

(i) Loss of lift due to the application of an undue amount of elevator during rotation and consequent delay of lift-off until either the appropriate increased incidence and/or a proportionally greater speed is reached. This problem, if it is real, obviously demands close control over piloting techniques and this may be achieved either by the compulsory use of a suitable take-off director or ultimately by making the lift-off manoeuvre fully automatic.

(ii) Danger of ground contact after lift-off. The obvious solution would be lengthening of the undercarriage so as to increase the available ground clearance. In the more extreme cases considered in the text such a solution would clearly require an extraordinary size of undercarriage, which the designer will be unwilling or even unable to accept. Another solution must be found. As the pitch case and the roll case arise from essentially different causes, they will be discussed separately.

(a) The possibility of ground impact of the rear extremities of an aircraft after lift-off proper arises if too high a pitch rate is maintained during this manoeuvre, or if elevator is used incautiously in an an aircraft having strongly adverse elevator lift. As in the previous case, the problem seems to resolve into proper control of piloting technique and the answer as before would be the use of a take-off director or full automation of lift-off. Another solution which has been suggested is to protect the rear end of the aircraft by a tail bumper, designed to be able to absorb an appropriate amount of energy. It must be realised that such a device will have to protrude from the existing ground clearance contour and thereby basically reduce the available pitch clearance.

(b) The danger of ground impact of the lateral extremities of an aircraft derives from the roll response to crosswinds existing at lift-off. To a certain extent the bank angle clearance is affected also by the pitching manoeuvre executed by the pilot, and as a consequence closer control over this manoeuvre will be beneficial for bank clearance too. This by itself, however, is unlikely to remove the problems in other than just marginal conditions. It is obvious from the discussion in Section 4 of this Report that the answer lies in ensuring that aileron is applied as early as possible, i.e. not later than the instant of lift-off.

Directors might help in this situation, but as conceived to date, they will only respond if there is already an aircraft motion to be detected; thus, if the aircraft does not respond significantly in roll as long as the wheels are on the ground, a director sensing back angle or roll rate will then not be of much help in the present problem as it will only make a demand for control when the pilot himself is becoming aware of the aircraft response. The only effective answer is to sense sideslip and to send an appropriate demand into the director so that it forces the pilot to trim the rolling moments acting on the aircraft before they actually affect its motion. It must be emphasised, that this is open loop control, a technique which itself introduces considerable hazards.

Another solution may be obtained by autostabilisation. As this technique does not depend on pilots reaction it has a greater potential than a director to ensure a reduction in the delay in aileron application immediately after lift-off. However, to get the full benefit of this ability, the stabiliser must be able to sense roll acceleration, rather than roll rate, and also it must have sufficient control authority to contribute significantly to a problem involving a major upset.

6. *Conclusions.*

The ground clearance requirements for take-off have been considered by treating lift-off as a dynamic manoeuvre rather than as a quasi-static condition as is assumed in conventional performance analysis. The enquiry has highlighted two main problem areas.

(i) The lift available at lift-off is reduced by elevator lift if more control is applied than is strictly necessary for trim. This makes take-off performance sensitive to piloting technique.

(ii) The potential danger of ground contact both in pitch and in bank during lift-off is greatest during the period 1 to 2 secs after the main wheels have left the ground, so that consideration of the situation existing at the lift-off point only is insufficient to determine ground clearance requirements.

Although not negligible these phenomena do not appear critical for conventional high aspect ratio designs where they are apparently covered by existing margins. However, these effects become more powerful for large tailless aircraft, where their careful study appears imperative.

Lengthening of the main undercarriage would be an obvious means of improving available ground clearance, but this solution may be unacceptable to the designer and alternative answers have been suggested.

The most promising would appear to be the use of a suitable take-off director to reduce the variability in pilots elevator control. This method should be able to guarantee a more predictable take-off manoeuvre and moreover would allow the designer to specify the form of lift-off most suitable for a given aircraft.

Take-off directors do, however, not appear very promising as means for controlling roll response in crosswind conditions. Only autostabilisation offers some potential in this area or the enforcement of a piloting technique to anticipate the crosswind response by early aileron application.

LIST OF SYMBOLS

$a = \frac{\pi}{2t_0}$ (sec^{-1})	Coefficient defining duration of pitching manoeuvre (Appendix B)
B	Constant in Appendix A
c(ft)	Reference chord
C_L	Lift coefficient
D(ft)	Distance travelled
F(lb)	Force acting on undercarriage
g(ft/sec^2)	Gravitational acceleration
H(ft)	Height of main wheels above ground
H_T(ft)	Height of rear extremity of aircraft
k_1	Indicial function
k_{10}	Value of k_1 at $t = 0$
k_y(ft)	Radius of gyration in pitch
L(lb)	Lift
$L_\alpha = \frac{dL}{d\alpha}$	Dimensional lift slope
$\mathscr{L} = \frac{L_\alpha}{mV}$	
l_T	Distance of rear extremity of aircraft from $C.G.$
l_η	Distance of centre of pressure of elevator from aircraft $C.G.$
$m = \frac{W}{g}$	Aircraft mass
q	Rate of pitch
S(ft^2)	Wing area
t	Time
t_0(sec)	Duration of post lift-off pitching manoeuvre
$t_L = \frac{\tau c}{V}$	Time constant of unsteady lift
t_R(sec)	Duration of rotation to lift-off
t_{L_0}(sec)	Time to lift-off
T(ft)	Compression of undercarriage
T_0(ft)	Stroke of undercarriage from static load condition to fully extended
T_M(sec)	Manoeuvre time (Section 3.2)
V(ft/sec)	Speed

W(lb)	Weight
$w = \frac{dH}{dt}$ (ft/sec)	Vertical velocity
α	Incidence
γ	Flight path angle
θ	Pitch attitude
$\Delta\theta_m$	Pitch-attitude margin
θ_M	Total pitch angle covered in flare up
ρ	Air density
$\sigma = \rho/\rho_0$	Relative air density
τ	Time constant of unsteady lift in root chords travelled
η	Elevator angle

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APPENDIX A

Determination of Vertical Velocity at the Moment of Lift-off.

During rotation the aircraft attitude increases and the consequent build up in lift will unload the undercarriage and the aircraft will rise until, with the undercarriage fully extended, lift-off occurs. Undercarriage characteristics are highly non-linear so that, to obtain a quick assessment of the aircraft motion in this manoeuvre, some crude simplifications have been made. First, as only the extension stroke is involved, oleo-damping is ignored and secondly the spring-characteristics are assumed to be linear. Especially the latter is a fairly gross distortion of the truth and the result can therefore be only taken as a rough estimate. When the wheels are in contact with the ground the vertical movement of the aircraft is identical to the undercarriage extension T and is described by the equation

$$L+\frac{dD}{dT}\Delta T = m\frac{d^2T}{dt^2} \qquad \text{(A.1)}$$

where

L = aerodynamic lift

F = undercarriage reaction acting as the aircraft

ΔT = undercarriage extension relative to static load position

$\frac{dF}{dT}$ = spring constant of oleo

m = aircraft mass

To specify the development of lift $L(t)$ during rotation we assume that the aircraft is subject to constant angular acceleration $d^2\,\theta/dt^2$ = const and ignoring the small reduction in α due to the vertical velocity developed on the aircraft during the manoeuvre, we can write

$$L(t) = B\,t^2 + L_0 \tag{A.2}$$

where L_0 is the lift appropriate to ground attitude prior to rotation and B a constant. Assuming ground attitude to be small, L_0 can be ignored, and equation (A.1) can be written

$$\ddot{T} - \frac{1}{m}\frac{dF}{dT}T = \frac{B}{m}t^2 \tag{A.3}$$

If T_0 is the stroke of the undercarriage from the static load condition to fully extended we get

$$\frac{1}{m}\frac{dF}{dT} = -\frac{W}{T_0}\frac{g}{W} = -\frac{g}{T_0}.$$

Further if t_R is the duration of rotation, i.e. the time required for the incidence to reach a value giving $L = W$, from equation (A.2)

$$W = B\,t_R^2; \frac{B}{m} = \frac{g}{t_R^2}.$$

With the initial condition $\Delta T = 0$ and $\frac{dT}{dt} = 0$ at $t = 0$ we can now write equation (A.3) in Laplace notation

$$s^2\,\bar{T} + \frac{g}{T_0}\bar{T} = 2\frac{g}{t_R^2}\frac{1}{s^3}.$$

This can be solved to give

$$T(t) = \left(\frac{T_0}{t_R}\right)^2 \frac{2}{g}\left\{\cos\left(\sqrt{\frac{g}{T_0}}t\right) - 1\right\} + T_0\left(\frac{t}{t_R}\right)^2. \tag{A.5}$$

Lift-off occurs when $T = T_0$, i.e.

$$T_0 = \left(\frac{T_0}{t_R}\right)^2 \frac{2}{g}\left\{\cos\left(\sqrt{\frac{g}{T_0}}t_{Lo}\right) - 1\right\} + T_0\left(\frac{t_{Lo}}{t_R}\right)^2. \tag{A.6}$$

from which $t_{L.O.}$, the instant of lift-off can be determined. (It should be noted that $t_{L.O.}$ is not strictly identical to t_R, because the aircraft will lift-off in this dynamic manoeuvre before $L = W$.) However, it was found by exploratory analysis that in all practical cases $t_{L_0} \simeq t_R$ and the subtle difference between the two can be ignored. This allows the vertical velocity of the aircraft, i.e. the rate of change of oleo compression at the instant of lift-off to be computed from

$$\frac{dT}{dt} = -\left(\frac{T_0}{t_R}\right)^2 \frac{2}{g}\sqrt{\frac{g}{T_0}} \sin\left(t_R\sqrt{\frac{g}{T_0}}\right) + 2\frac{T_0}{t_R} \qquad \text{(A.7)}$$

which is obtained by differentiating equation (A.5) and substituting $t = t_R$.

This expression has been computed for the relevant range of the parameters T_0 and t_R and the results are plotted in Fig. 7.

APPENDIX B

Aircraft Response Following Lift-off, Ignoring Elevator Lift.

From Fig. 11 we see that the height of the rear extremity of an aircraft H_T, having a distance l_T from the centre of gravity is

$$H_T = H_{T_0} + H - \theta\, l_T \qquad \text{(B.1)}$$

where H_T is the tail-clearance from the ground with the aircraft at zero pitch attitude θ and with the (fully extended) undercarriage just on the ground ($H = 0$). The increment in tail height after lift-off is then

$$\Delta H_T = H - \Delta\theta\, l_T. \qquad \text{(B.2)}$$

Ignoring elevator lift and other minor terms

$$m\ddot{H} = L_\alpha\, \alpha \qquad \text{(B.3)}$$

with

$$\alpha = \theta - \frac{\dot{H}}{V}. \qquad \text{(B.4)}$$

This gives

$$\ddot{H} + \frac{L_\alpha}{mV}\dot{H} = \frac{L_\alpha}{m}\theta(t) \qquad \text{(B.5)}$$

Assuming constant speed (V), this equation can be solved if $\theta(t)$ is known. Normally the pitching motion of the aircraft would be described by the pitching-moment equation, thereby introducing a host of additional parameters, m_w, $m_{\dot{w}}$, m_q, m_η and of course a definition of the elevator application $\eta(t)$.

Such an approach would only permit the presentation of a few typical examples and not lend itself readily to the desired generalisation. As only the first two seconds of the aircraft motion after lift-off are of interest, (as the results of this analysis will shortly show) the details of the aircraft response are not of great consequence, and it would appear permissible to specify the pitching motion $\theta(t)$ as such and cover a range of lift-off manoeuvres by a suitable defined family of such time histories $\theta(t)$. This is the approach chosen here. Inspection of a large number of recently recorded take-off tests suggests that a good approximation to this aircraft pitching motion for this part of the take-off is either simply to assume constant pitch rate q_0, i.e.

$$\theta(t) = q_0\, t \tag{B.6}$$

or more realistically, maximum pitch rate q_0 at lift-off decaying in the form of a cos-function

$$q = q_0 \cos\left(\frac{\pi}{2}\frac{t}{t_0}\right)$$

where t_0 is the time after lift-off when rotation is completed, i.e. $q = 0$. Then

$$\theta(t) = \frac{q_0}{\frac{\pi}{2t_0}} \sin\left(\frac{\pi}{2t_0}t\right). \tag{B.7}$$

Introducing the initial condition $dH/dt = \dot{H}_0$ at $t = 0$, equation (B.5) has been solved for the two forms of $\theta(t)$ to give:

(i) $\underline{\theta(t) = q_0\, t}$

$$H(t) = V q_0 \left\{ -\frac{t}{\mathscr{L}} + \frac{t^2}{2} + (1-e^{-\mathscr{L}t})\frac{1}{\mathscr{L}^2} \right\} + \frac{\dot{H}_0}{\mathscr{L}}(1-e^{-\mathscr{L}t}) \tag{B.8}$$

and with equation (B.2)

$$\Delta H_T(t) = V q_0 \left\{ -t\left(\frac{1}{\mathscr{L}} + \frac{l_T}{V}\right) + \frac{t^2}{2} + \frac{1}{\mathscr{L}^2}(1-e^{-\mathscr{L}t}) \right\} + \frac{\dot{H}_0}{\mathscr{L}}(1-e^{-\mathscr{L}t}) \tag{B.9}$$

where

$$\mathscr{L} = \frac{L_\alpha}{mV} = \sigma\frac{\rho_0 g}{2}\frac{\partial C_L}{\partial \alpha}\frac{V}{W/S} \tag{B.10}$$

(ii) $\underline{\theta(t) = \frac{q_0}{\frac{\pi}{2t_0}} \sin\left(\frac{\pi}{2t_0}t\right) = \frac{q_0}{a}\sin(a\,t)}$

where

$$a = \frac{\pi}{2t_0}$$

$$H(t) = \frac{\mathscr{L} V q_0}{\mathscr{L}^2 + a^2}\left\{ \frac{\mathscr{L}^2 + a^2}{\mathscr{L}\, a^2} - \frac{1}{\mathscr{L}}e^{-\mathscr{L}t} - \frac{\mathscr{L}}{a^2}\cos at - \frac{1}{a}\sin at \right\} + \frac{\dot{H}_0}{\mathscr{L}}(1-e^{-\mathscr{L}t}) \tag{B.11}$$

$$H_T(t) = \frac{V q_0}{\mathcal{L}^2 + a^2}\left\{ \frac{1}{\mathcal{L}}(1 - e^{-\mathcal{L}t}) + \frac{\mathcal{L}}{a^2}(1 - \cos at) - \frac{1}{a}\sin at \right\} - q_0 \frac{l_T}{a}\sin at + \frac{\dot{H}_0}{\mathcal{L}}(1 - e^{-\mathcal{L}t}) \qquad \text{(B.12)}$$

Equation (B9) has been computed for $q_0 = 2°/\text{sec} = \text{const.}$ for a range of values of $\mathcal{L}$, l_T and H_0 and the results given as time histories in Fig. 12. It is seen that in all these manoeuvres the tail moves downwards during the period immediately following lift-off until at about one second after lift-off this downstroke has a maximum. This of course, is the instant at which tail clearance has its minimum and it is the critical condition for which tail clearance margins during take off must be considered. This maximum incremental downwards stroke ΔH_T max can be expressed as an equivalent pitch angle margin

$$\Delta\theta_m = \frac{\Delta H_{T_{\max}}}{l_T} \qquad \text{(B.13)}$$

which must exist at lift-off, to allow for the post lift-off motion of the rear end of the aircraft. For a large number of lift-off manoeuvres this margin $\Delta\theta_m$ has been computed with the results shown in Fig. 13.

APPENDIX C

Aircraft Response to Elevator Control during Flare-up.

If elevator lift is included equation (B3) becomes

$$m\ddot{H} = L_\alpha \alpha + L_\eta \eta . \qquad \text{(C.1)}$$

Again, as in Appendix B, the pitching moment equation is drastically simplified, ignoring m_w, m_q and $m_{\dot{w}}$; this allows elevator η to be uniquely related to pitching acceleration θ by

$$m k_y^2 \ddot{\theta} = L_\eta(-l_\eta)\eta \qquad \text{(C.2)}$$

where k_y is the inertia radius in pitch and l_η the effective elevator moment arm (i.e. the distance of the centre of pressure of the elevator lift from the centre of gravity of the aircraft). Then

$$L_\eta \eta = -m k_y^2 \frac{\ddot{\theta}}{l_\eta}. \qquad \text{(C.3)}$$

Equations (C.1), (C.3) and (B.4) give

$$\ddot{H} + \frac{L_\alpha}{mV}\dot{H} + \frac{L_\alpha}{mV} V\,\theta(t) - \frac{k_y^2}{l_\eta}\ddot{\theta}(t). \qquad \text{(C.4)}$$

We are considering the case where at lift-off the aircraft has an initial pitch rate q_0 and elevator is applied instantaneously to give for $t > 0$

$$\ddot{\theta}(t) = \dot{q}_0 = \text{const.}$$

Then

$$\theta(t) = q_0 \quad t + \frac{\dot{q}_0}{2} t^2 . \tag{C.5}$$

A further initial condition to be considered is

$$\dot{H}(t = 0) = \dot{H}_0$$

With these assumptions equation (C.4) can be solved to give

$$H(t) = V q_0 \left\{ \frac{t}{\mathscr{L}} + \frac{t^2}{2} + (1 - e^{-\mathscr{L}t}) \frac{1}{\mathscr{L}^2} \right\} + \frac{\dot{H}_0}{\mathscr{L}} (1 - e^{-\mathscr{L}t})$$

$$+ V \dot{q}_0 \left\{ \frac{t^3}{6} - \frac{1}{\mathscr{L}} \frac{t^2}{2} + \frac{1}{\mathscr{L}^2} t - \frac{1}{\mathscr{L}^3} (1 - e^{-\mathscr{L}t}) \right\} - \frac{k_y^2}{l_\eta} \frac{1}{\mathscr{L}} \left\{ t - \frac{1}{\mathscr{L}} (1 - e^{-\mathscr{L}t}) \right\} \tag{C.6}$$

and

$$H_T(t) = V q_0 \left\{ -t \left(\frac{1}{\mathscr{L}} + \frac{l_T}{V} \right) + \frac{t^2}{2} + \frac{1}{\mathscr{L}^2} (1 - e^{-\mathscr{L}t}) \right\} + \frac{\dot{H}_0}{\mathscr{L}} (1 - e^{-\mathscr{L}t})$$

$$+ V \dot{q}_0 \left\{ \frac{t^3}{6} - \frac{1}{\mathscr{L}} \frac{t^2}{2} + \frac{1}{\mathscr{L}^2} t - \frac{1}{\mathscr{L}^3} (1 \overset{\text{x}}{-} e^{-\mathscr{L}t}) \right\}$$

$$- \frac{k_y^2}{l_\eta} \frac{1}{\mathscr{L}} \left\{ t - \frac{1}{\mathscr{L}} (1 - e^{-\mathscr{L}t}) \right\} - \frac{t^2}{2} l_T . \tag{C.7}$$

Equation (C.7) has been computed for a range of manoeuvres with $q_0 - 0$ and $H_0 = 1$ ft sec and the corresponding tail clearance margins according to equation (B.13) are shown in Fig. 17. A more complex manoeuvre representative of a flare-up through a given pitch angle θ_M involving elevator reversal to check pitching has been computed by superimposing appropriate time histories as determined from equation (C.7) and the results are shown in Fig. 18.

APPENDIX D

Development of Unsteady Lift during Aircraft Rotation.

The idealized indicial lift function equation (4)

$$k_1(t) = k_{10} + (1 - k_{10})(1 - e^{-t/t_L})$$

is the solution of the first order equation

$$\bar{k}_1(s) = \frac{\partial C_L}{\partial \alpha} \frac{\bar{\alpha}_{(s)}}{C_{L\infty}} \left\{ k_{10} + (1-k_{10}) \frac{1/t_L}{s+1/t_L} \right\} \tag{D.1}$$

to a step disturbance in incidence:

$$\frac{\alpha(t)}{\alpha_\infty} = 0 \text{ for } t < 0$$

and

$$\frac{\alpha(t)}{\alpha_\infty} = 1 \text{ for } t > 0$$

so that

$$k_1(s) = \frac{\partial C_L}{\partial \alpha} \frac{\bar{\alpha}_\infty}{C_{L\infty}} \frac{1}{s} \left\{ k_{10} + (1-k_{10}) \frac{1/t_L}{s+1/t_L} \right\} \tag{D.2}$$

where

$$k_1(t) = C_L(t)/C_{L\infty}.$$

Multiplying both sides of equation (D–1) by $C_{L\infty}$ we get

$$C_L(s) = \frac{\partial C_L}{} \bar{\alpha}(s) \left\{ k_{10} + (1-k_{10}) \frac{1/t_L}{s+1/t_M} \right\} \tag{D.3}$$

During rotation about the main wheels on the ground $\theta = \alpha$ or $d\alpha/dt = q$. Assuming a steady pitch rate $q_0 = \text{const.}$

$$\alpha(t) = q_0\, t$$

or

$$\bar{\alpha}(s) = q_0 \frac{1}{s^2}$$

Substitution in equation (D.3) gives

$$\bar{C}_L(s) = \frac{\partial C_L}{\partial \alpha} q_0 \frac{1}{s^2} \left\{ k_{10} + (1-k_{10}) \frac{1/t_L}{s+1/t_L} \right\} \tag{D.4}$$

which has the solution

$$C_L(t) = \frac{\partial C_L}{\partial \alpha} q_0 \left\{ k_{10}\, t + (1-k_{10})\,(t - t_L\,[1-e^{-t/t_L}]) \right\}. \tag{D.5}$$

After the initial transient, i.e. after $t >> t_L$ this becomes

$$C_L(t) = \frac{\partial C_L}{\partial \alpha} q_0 \left\{ k_{10}\, t + (1-k_{10})\,(t - t_L) \right\}. \tag{D.6}$$

It is seen that the unsteady lift contribution $(1-k_{10})$ lags behind $\alpha = qt$ by t_L sec.

If the lift were to follow incidence in a quasisteady manner equation (D.6) would read

$$C_L(t) = \frac{\partial C_L}{\partial \alpha} q_0 t \quad \text{(D.7)}$$

Equating (D.6) and (D.7) for a given value of C_L we get

$$t_u - (1-k_{10})\, t_L = t_s . \quad \text{(D.8)}$$

where

t_u = time required to obtain a given value of C_L in unsteady flow

t_s = time required to obtain the same value of C_L in quasisteady flow.

Hence, if lift develops in an unsteady manner during aircraft rotation with constant q_0, to obtain a given value of C_L one will need the extra rotation time

$$\Delta t = t_u - t_s = (1-k_{10})\, t_L \quad \text{(D.9)}$$

and a corresponding increase in incidence (or attitude)

$$\Delta\alpha = q_0\, \Delta t = q_0 (1-k_{10})\, t_L . \quad \text{(D.10)}$$

When compared with steady aerodynamic data, the incidence required to achieve lift off whilst the aircraft is in steady rotation has to be increased by the amount defined in equation (D.10) and as a consequence ground clearance must be provided to allow for this additional demand in attitude.

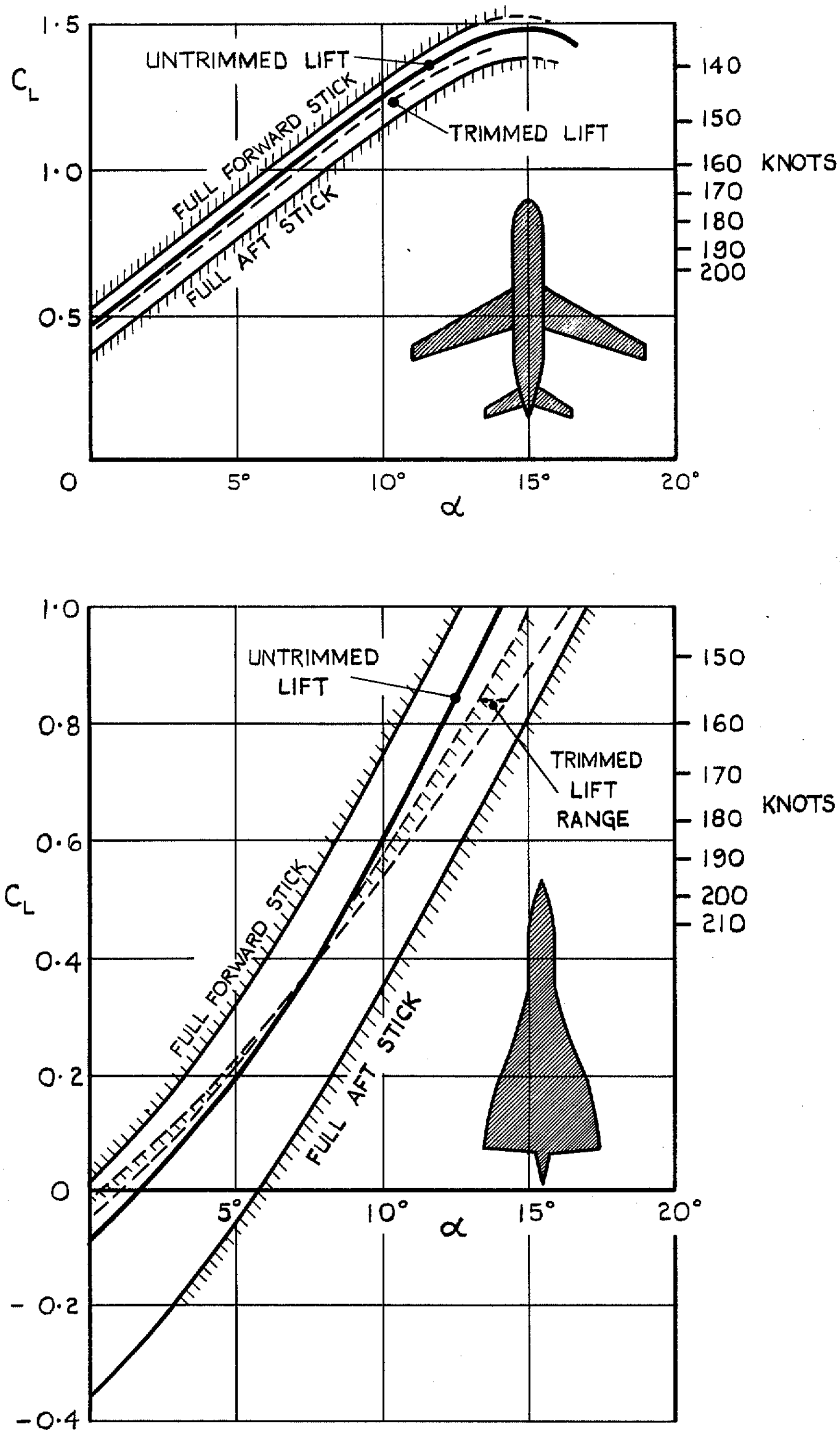

FIG. 1. Lift coefficient as a function of incidence and elevator deflection for a conventional and a slender aircraft.

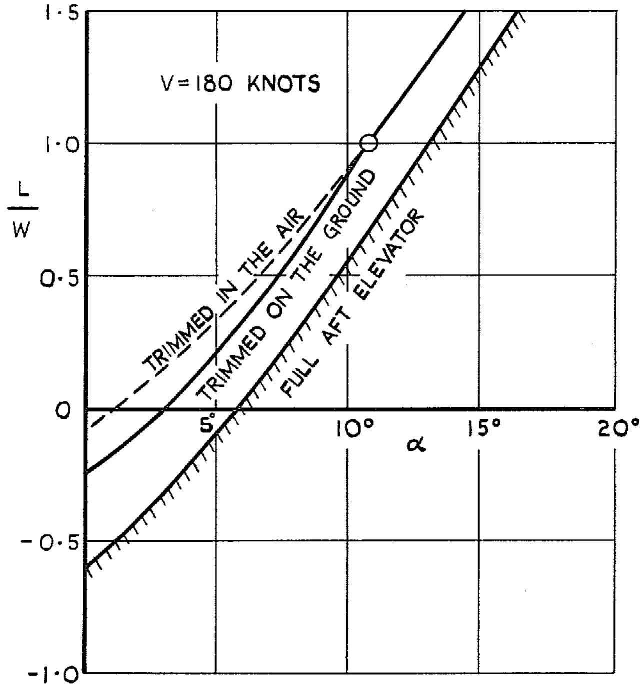

FIG. 2. Lift as a fraction of take-off weight *versus* incidence for various elevator applications for a typical slender-wing aircraft at lift-off speed.

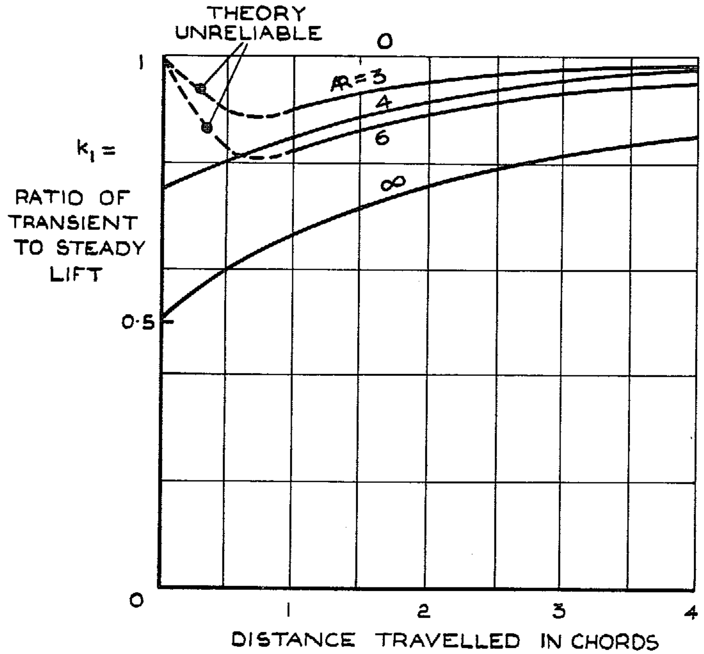

FIG. 3. Indicial lift on rectangular wings following sudden change in incidence. (From Ref. 2).

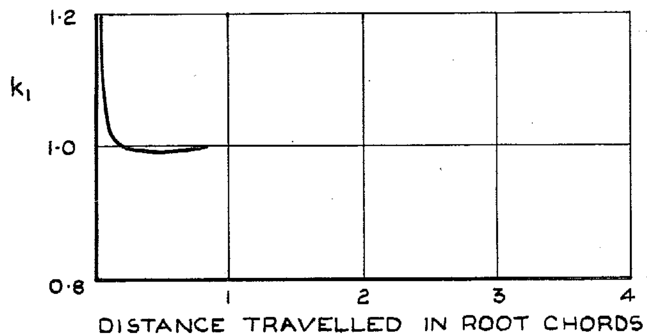

FIG. 4. Indicial lift on slender wings following sudden change in incidence. (From Ref. 3).

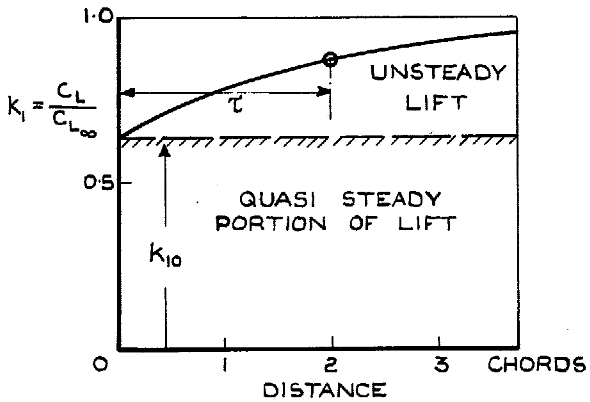

FIG. 5. Simplified representation of indicial-lift function for aircraft-response calculations.

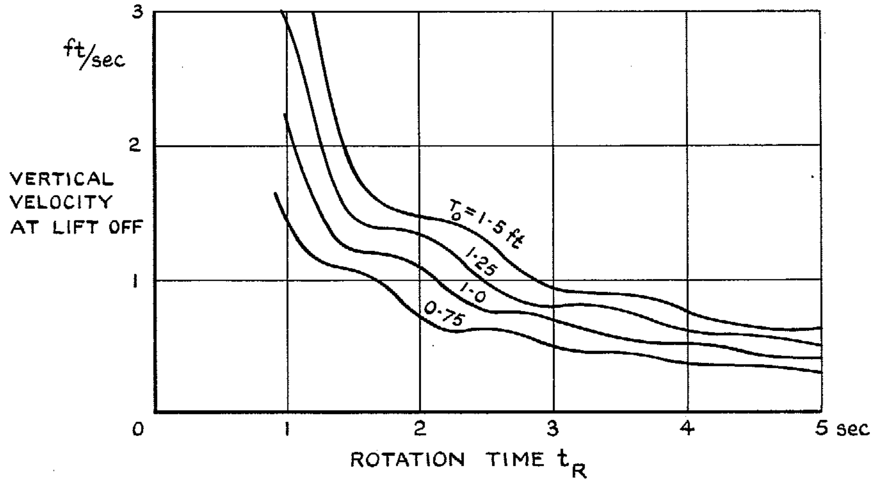

FIG. 7. Vertical velocity at lift-off as a function of undercarriage stroke T_0 and rotation time t_R.

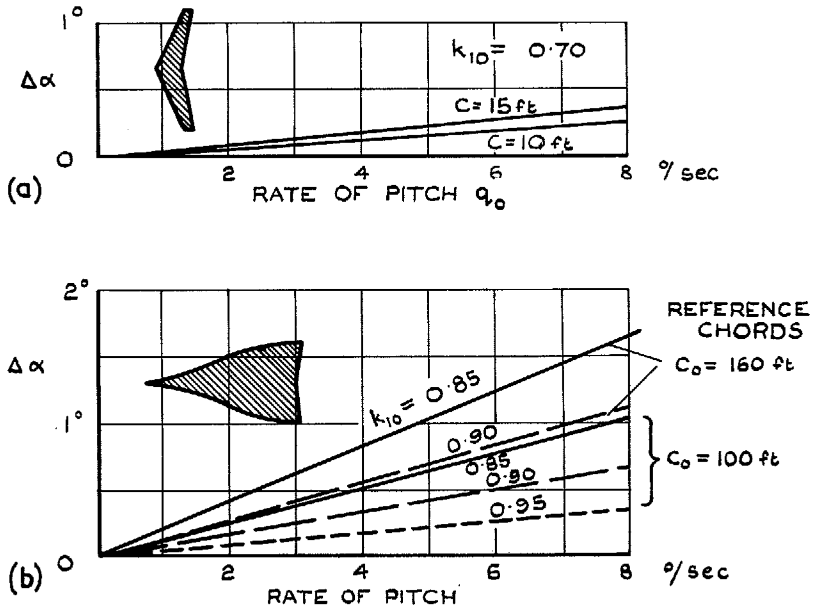

FIG 6 a & b. Extra incidence required for lift-off at 300 ft/sec to make up for unsteady lift.

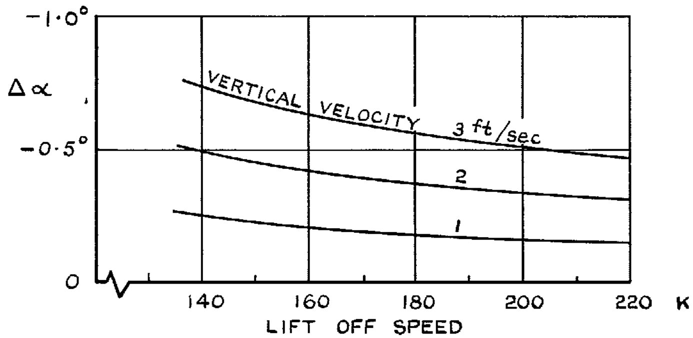

FIG. 8. Loss of incidence Δα at lift-off due to vertical velocity of aircraft.

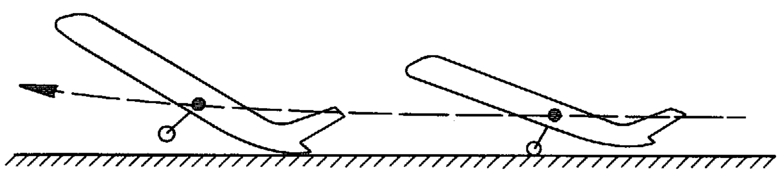

FIG. 9. Tail strike after lift-off.

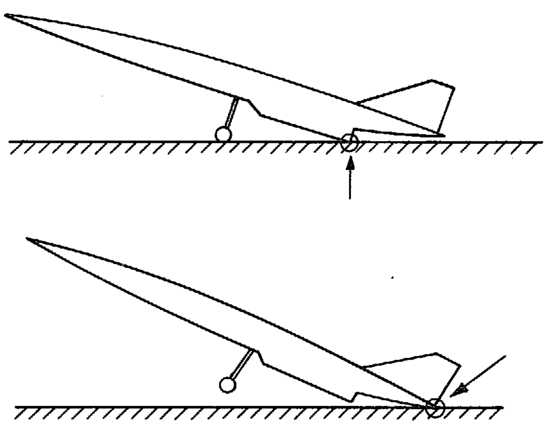

FIG. 10. Design where different part of fuselage is limiting ground clearance with wheels on the ground and off the ground.

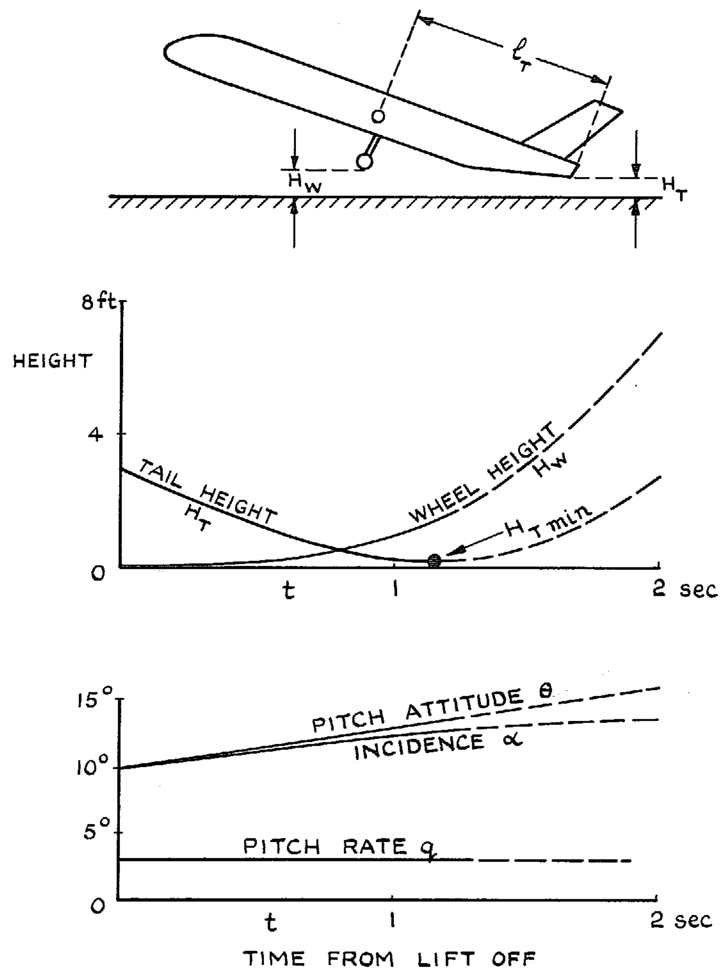

FIG. 11. Time history of aircraft motion immediately after lift-off assuming pitch rate is constant (3 deg/sec) in this part of the manoeuvre.

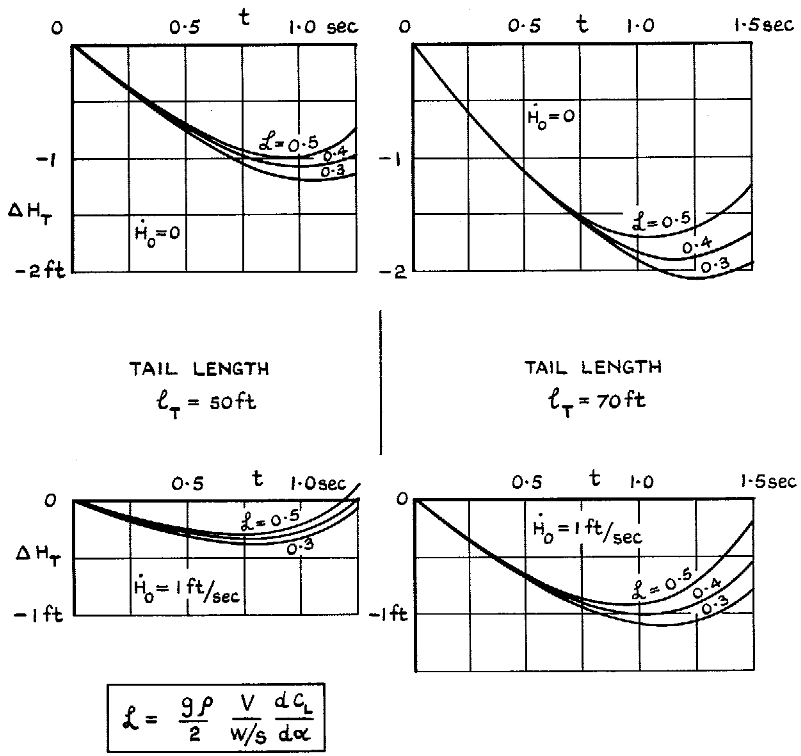

FIG. 12. Movement of tail immediately after lift-off assuming constant rate of pitch $q = 2$ deg/sec and $V = 300$ ft/sec as influenced by tail length (l_T), vertical velocity at lift-off ($\dot{H}_0$), and effective lift slope $\mathscr{L}$.

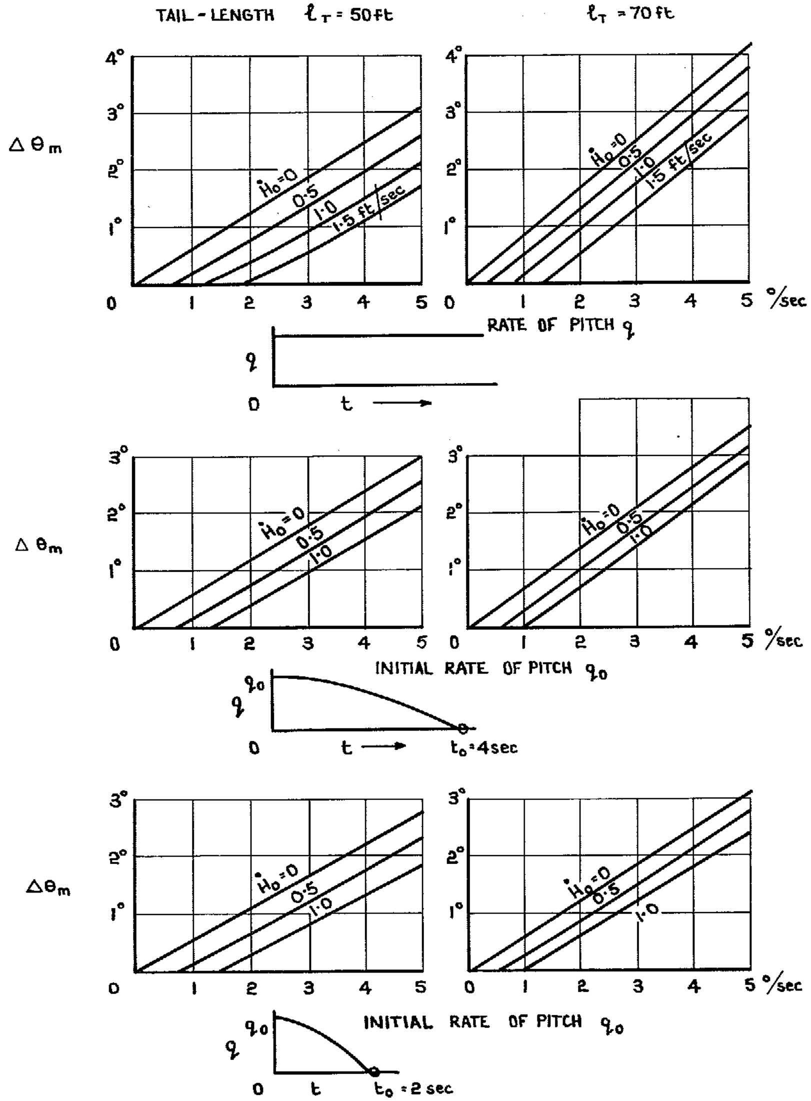

FIG. 13. Additional tail clearance $\Delta\theta m$ required to allow for aircraft motion immediately following lift-off ($\mathscr{L} = 0{\cdot}411$, $V = 300$ ft/sec).

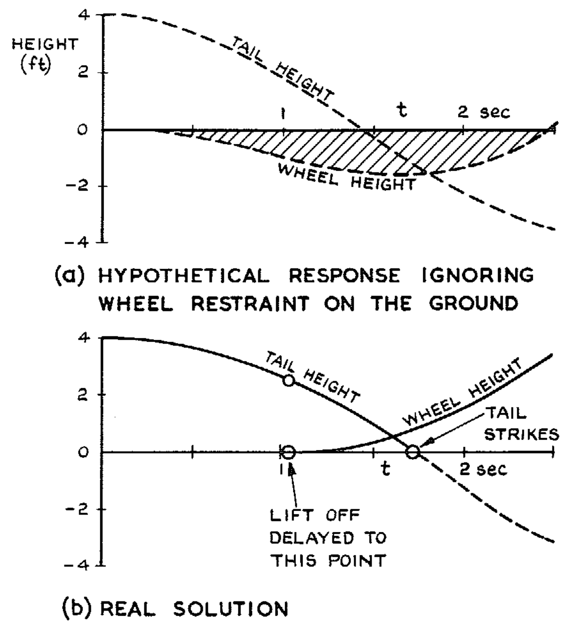

FIG. 14a & b. Lift off from checked rotation by instantaneous elevator application at $t = 0$. Slender aircraft with adverse elevator lift.

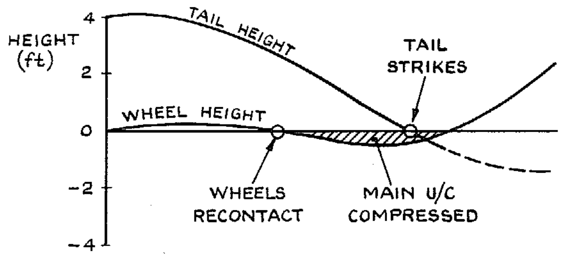

FIG. 15. As Fig. 14 but assuming an initial vertical velocity $\dot{H}_0 = 1$ ft/sec.

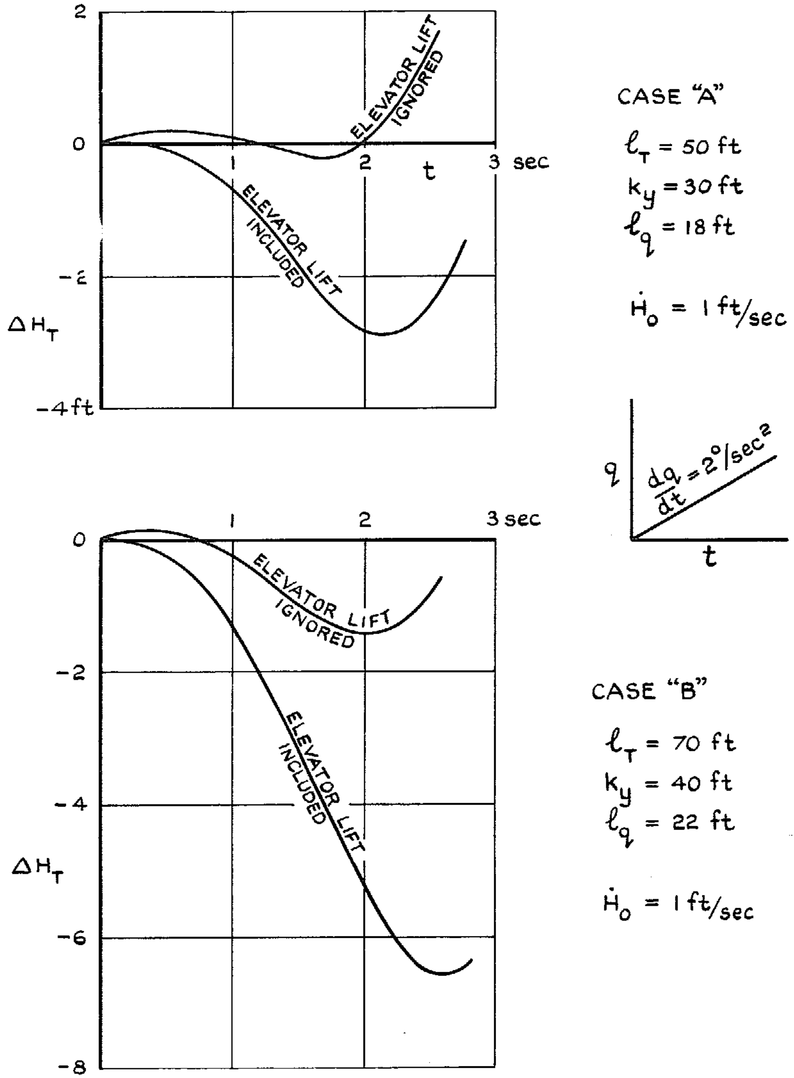

FIG. 16. Tail movement after lift-off from checked rotation with elevator applied at lift-off to produce constant angular acceleration for two slender transport aircraft.

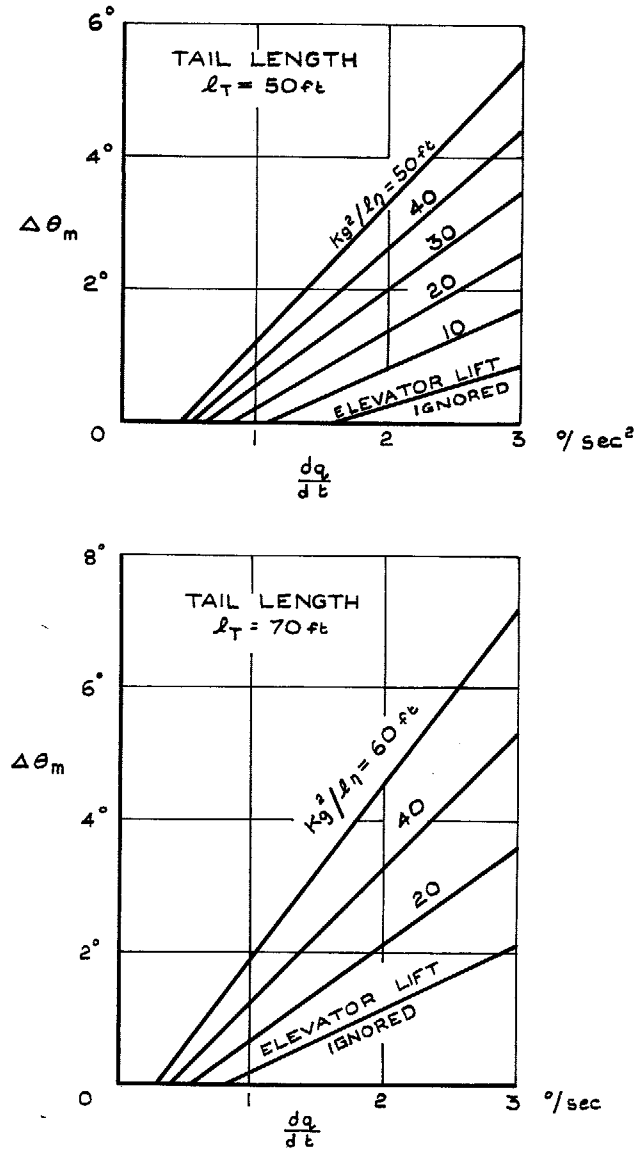

FIG. 17. Additional tail clearance $\Delta\theta_m$ required to allow for aircraft motion after lift-off from checked rotation. Elevator applied to give constant acceleration in pitch. dq/dt $\dot{H}_0 = 1$ ft/sec.

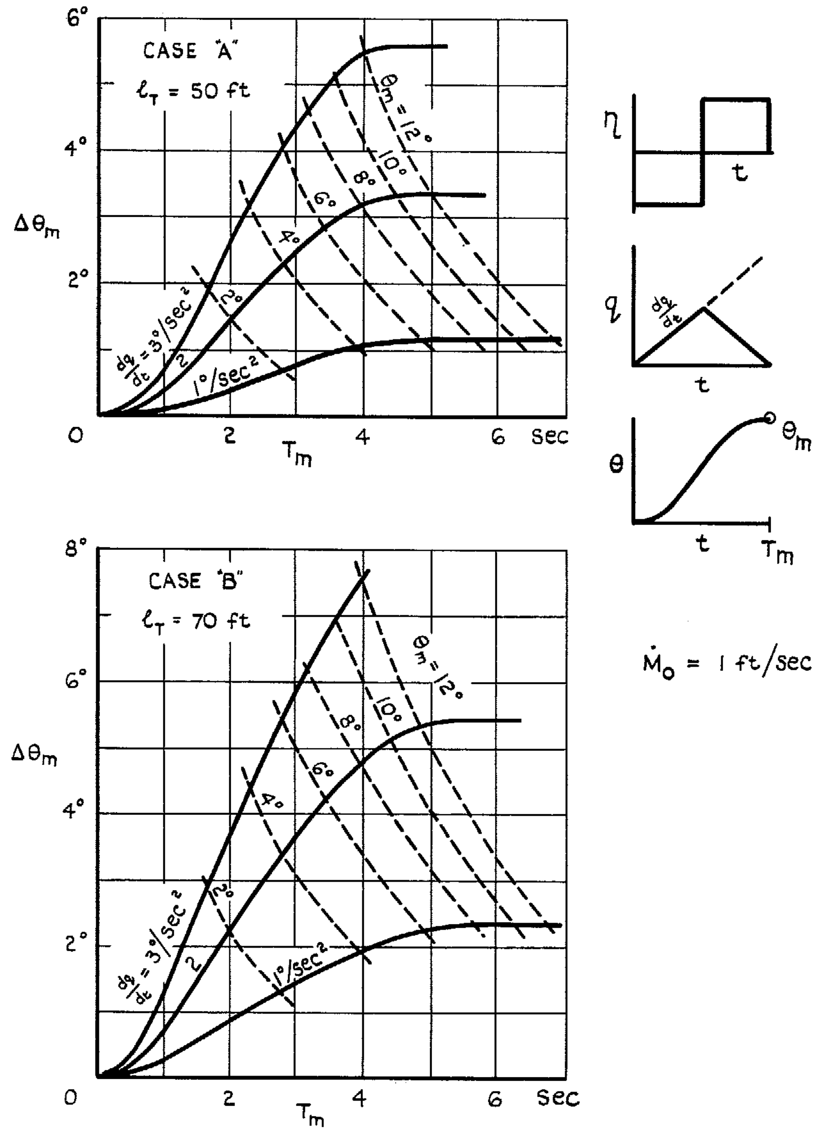

FIG. 18. Tail clearance $\Delta\theta_m$ required to allow for post lift-off motion from checked rotation for a range of pitching manoeuvres for aircraft defined in Fig. 16.

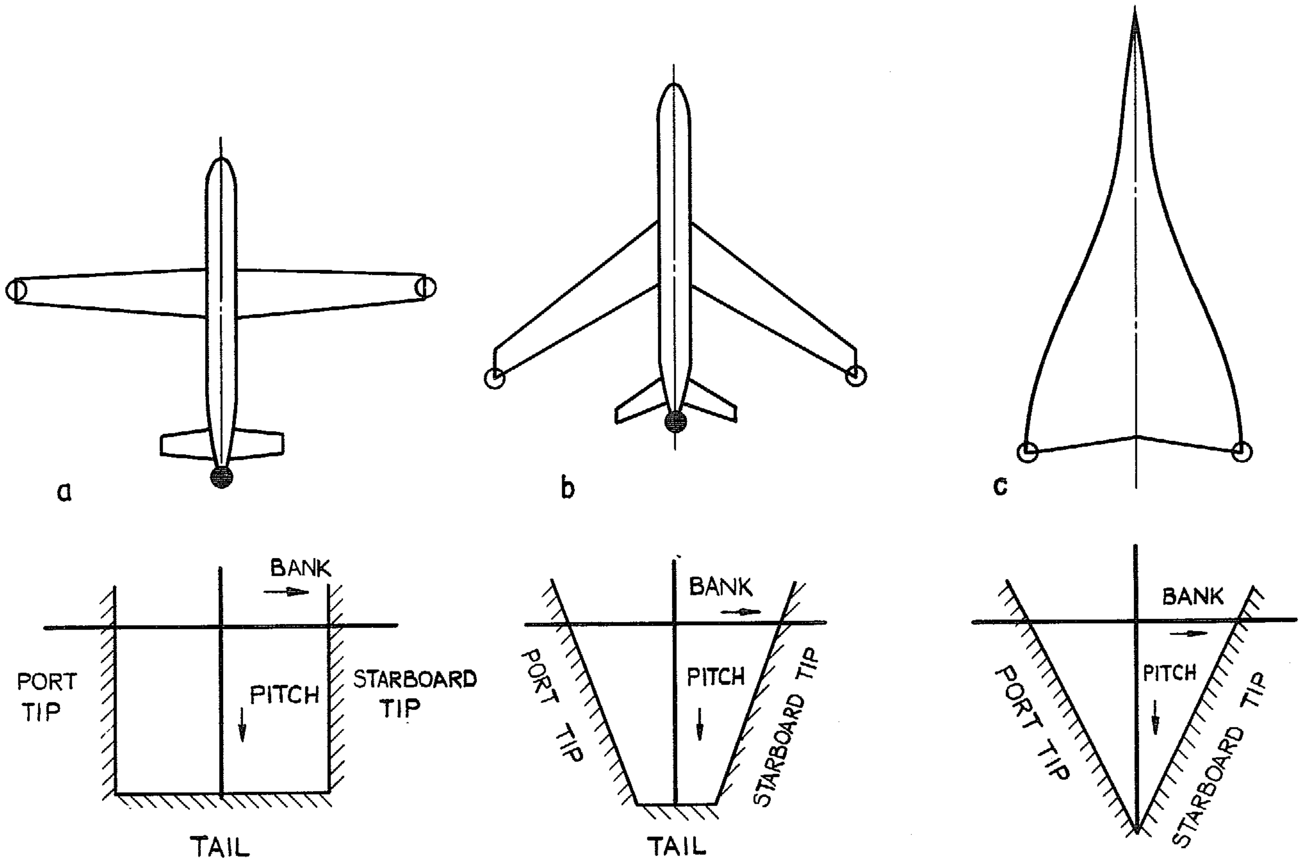

FIG. 19. Ground clearance envelopes for typical straight wing, swept wing and slender wing designs.

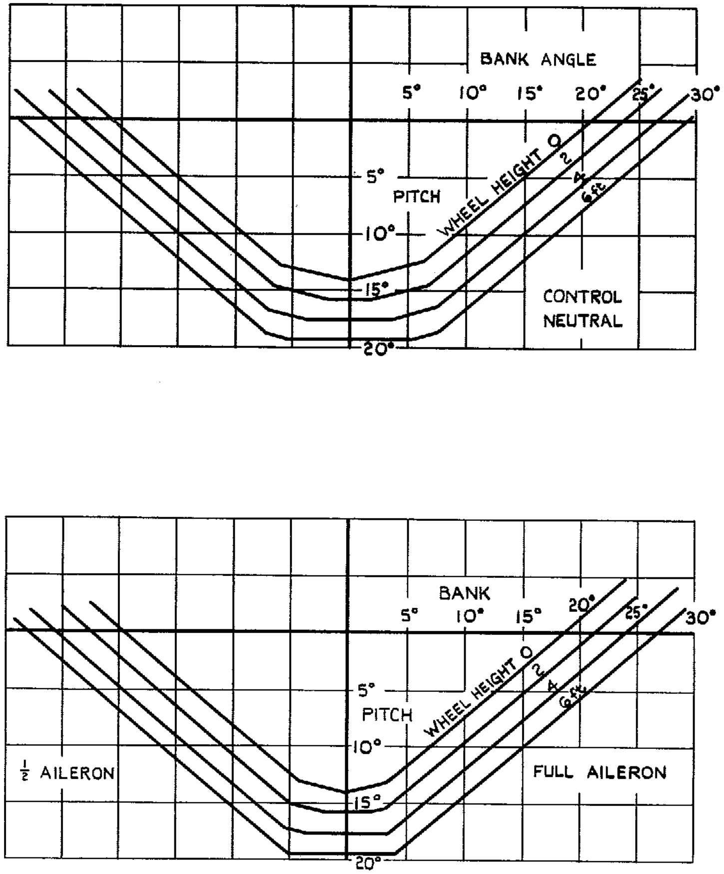

FIG. 20. Ground clearance envelope for slender wing transport aircraft as a function of height of main wheels above ground.

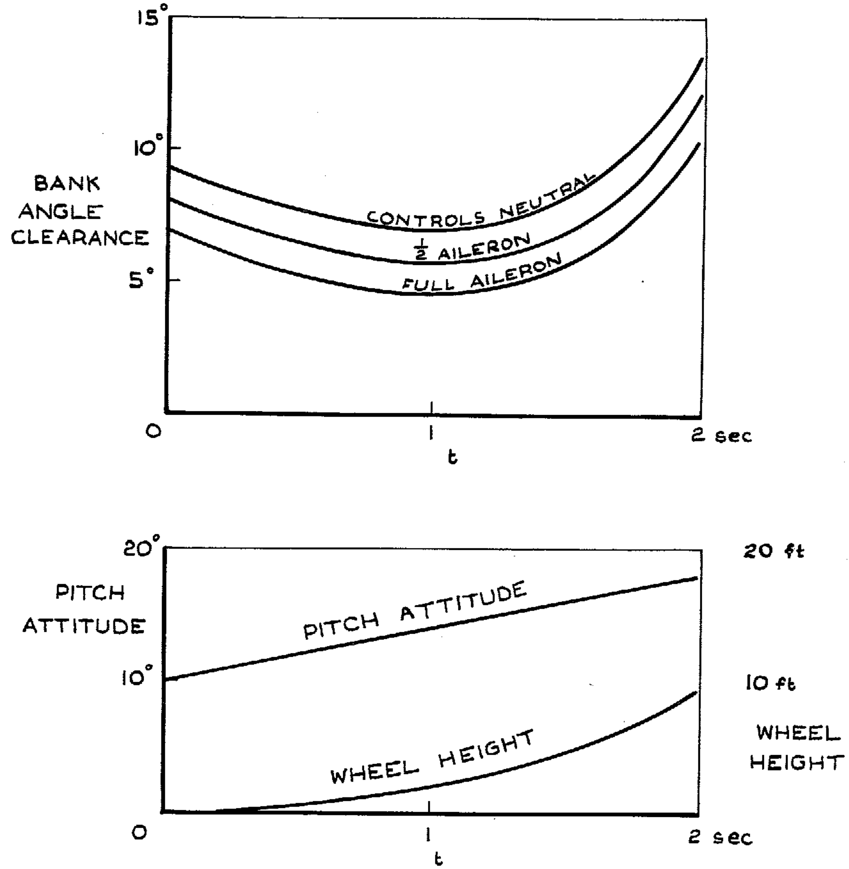

FIG. 21. Bank angle clearance available during a particular take-off manoeuvre for a large slender aircraft. Lift-off with constant pitch rate $q = 4$ deg/sec.

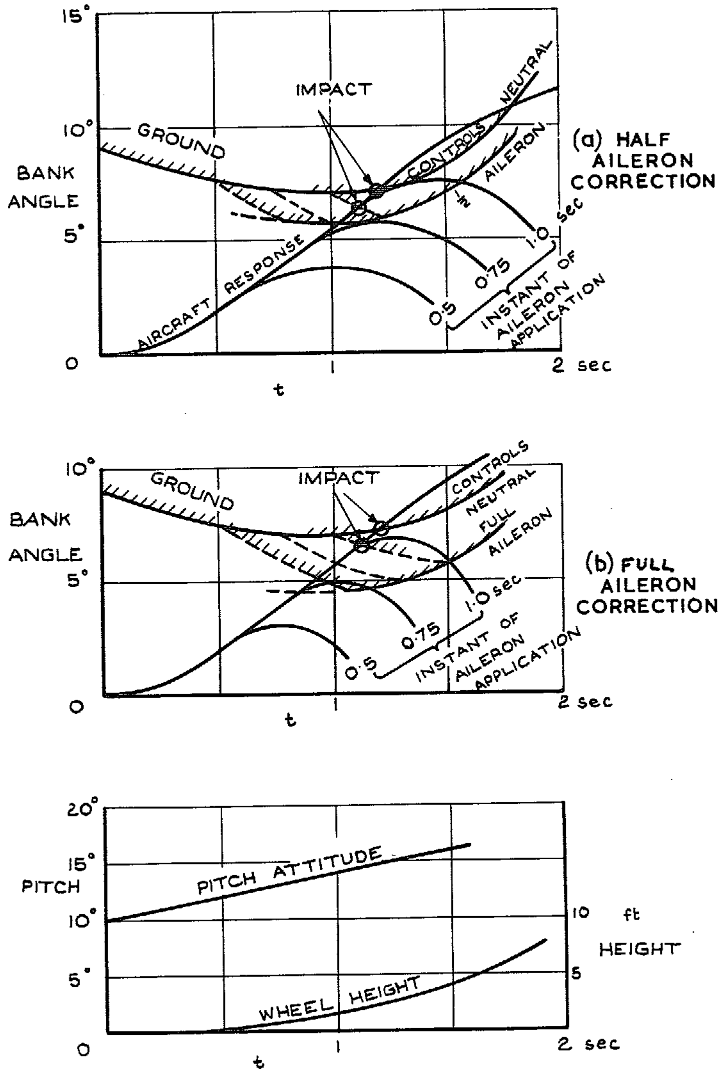

FIG. 22a & b. Roll response after lift-off in 10 knots crosswind and available ground clearance of slender transport.

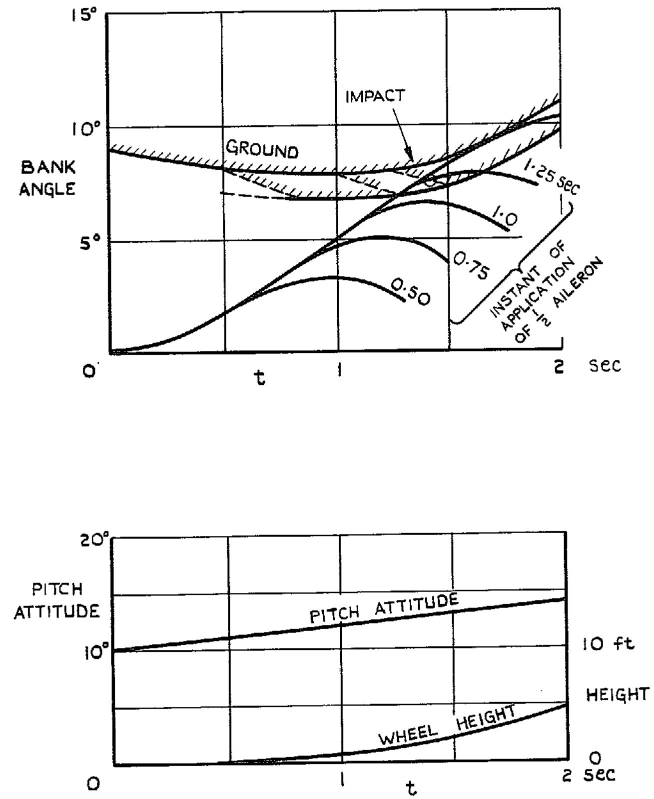

FIG. 22c. Effect of reducing pitch rate to 2 deg/sec on roll clearance in lift-off with 10 knots crosswind.

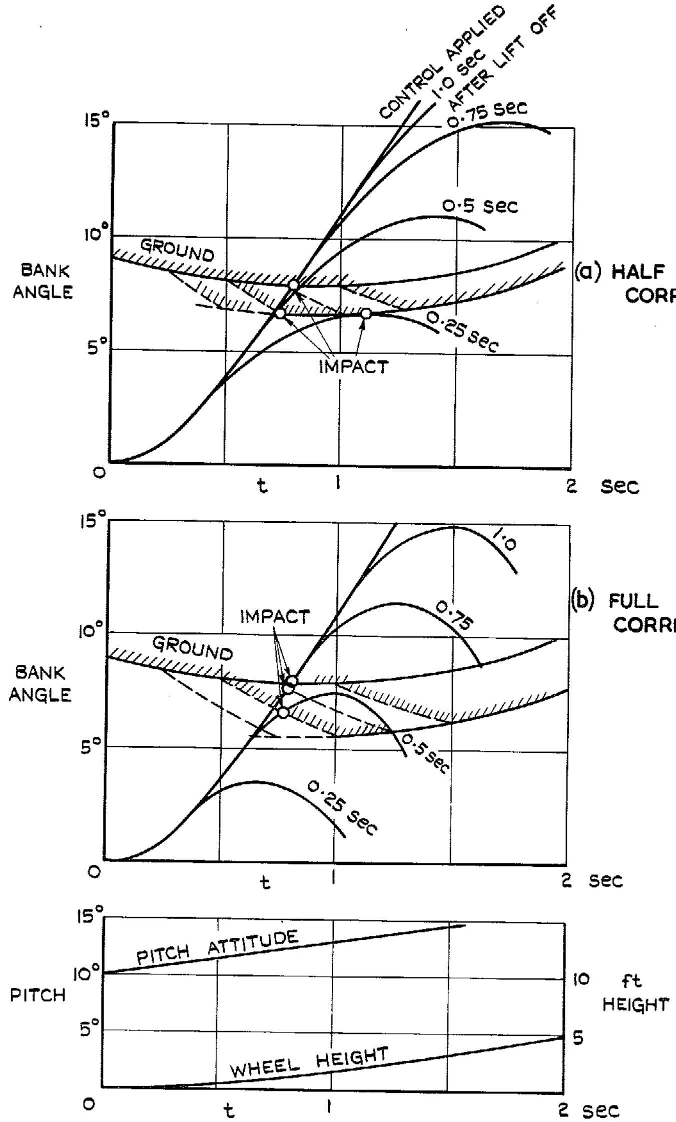

FIG. 23a & b. Roll response after lift-off in 20 knots crosswind and available ground clearance.

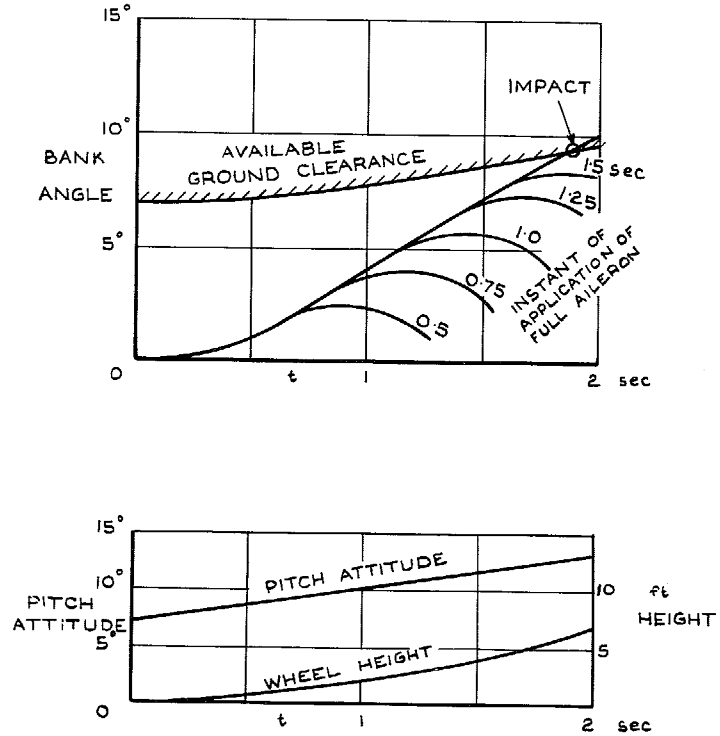

FIG. 24. Roll response after lift-off in 20 knots crosswind and available ground clearance for typical subsonic transport aircraft.

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