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CONTENTS

(93022)

Spring Tabs on Frise Ailerons

By

F. B. Baker, M.A.
of Messrs. Handley Page, Ltd.

This paper was published in *Aircraft Engineering*, Vol. XIX, No. 224, pp. 314–319, October, 1947, Vol. XIX, No. 225, pp. 353–358, November, 1947, and Vol. XIX, No. 226, pp. 380–389, December, 1947.

When designing spring tab lay-outs for control surfaces, it is usual to assume an average value of b_1 and b_2. In the case of Frise ailerons, the hinge-moment curve is often so irregular that the assumption of a constant b_2 may lead to serious error.

Before using the hinge-moment curves obtained in a wind tunnel, they must be modified for the effect of response and twist. Across these curves are drawn certain straight lines, whose equations are determined from the parameters of the control surface circuit and the torsion bar or spring to be used. The intersection of line and curve determines the control surface and tab angles corresponding to a given setting of the pilot's control.

The paper considers the effect of neglecting c_0 and c_3, in calculations; and also discusses the effect of changes in follow-up ratio, spring rate, and b_3. Since " lost motion " has the effect of reducing the actual maximum aileron angle, formulae are determined for deciding what should be the maximum angle with the spring locked; and also explains why separate trim tabs are preferable to those which are combined with spring tabs.

The latter part of the paper deals with the question of what rate of roll can be expected in practice from a given aileron setting at various speeds. Once again, the curve for l_ξ is not assumed to be a straight line, since its slope varies considerably at large aileron angles. The liability of "peaking" is discussed; and also the effect of friction, and stretching of the control rods or wires. The formulae developed are equally applicable to elevators and rudders, and have been successfully used in practice for designing spring tabs for actual aircraft.

Why Shear Webs?

By

H. L. Cox

This paper was published in full in the *Journal of the Royal Aeronautical Society,* Vol. 52, p. 759, November, 1948.

The use of progressive taper as a means to free the web of a cantilever from shear stress due to bending loads is described; but attention is drawn to the second function of the web to stabilise the compression flange. The limitation imposed in respect of this second function by the liability of the web to buckle under lateral compression is examined, and a means to avoid this limitation by curving the whole cantilever is indicated.

The theoretical conclusions are supported by a series of tests on cardboard and paper models; it is shown that by suitably curving a cantilever instability of the compression flange may be prevented, so that the strength of the cantilever, limited now only by the tensile strength of the tension flange, may be at least doubled.

A Boundary Value Problem for a Hyperbolic Differential Equation Arising in the Theory of the Non-uniform Supersonic Motion of an Aerofoil

By

O. Todd, M.A., D.Phil.

This paper was published in full in the *Courant Anniversary Volume.* Interscience Publishers Inc. New York. 1948.

A solution of the linearised equation for the velocity potential of a non-uniform supersonic two-dimensional motion is obtained in terms of its normal derivative along the aerofoil. The problem is treated as a Cauchy problem since the values of the potential along the aerofoil can be eliminated by a reflexion. The expression coincides with a formula of C. Possio. The treatment is a generalisation of a procedure used by G. Temple and H. A. John in the case of harmonic motion.

A Study by a Double-refraction Method of the Development of Turbulence in a Long Circular Tube

By

A. M. Binnie and J. S. Fowler

(Communicated by Sir Geoffrey Taylor F.R.S.—Received to 5th March, 1947.)

This paper was published in full in *Proceedings of the Royal Society*, A, Volume 192, 1947.

A streaming double-refraction method was employed to examine the flow in a long glass tube of a very weak solution of benzopurpurin in water. Two kinds of turbulent entry were used: with one, laminar flow at a Reynolds number of about 1900 was observed at cross-sections more than 120 diameters from the entry; with the other the corresponding distance was 90 diameters. The nature of the breakdown of laminar flow at a cross-section was found to depend upon the kind of entry and upon the distance of the cross-section from the inlet. The development of complete turbulence at various cross-sections was also investigated.

Notes on the Linearised Equation for the Velocity Potential of the Supersonic Flow of a Compressible Fluid

By

R. K. Tempest and L. Rosenhead

This paper was published in full in the *Proceedings of the London Mathematical Society,* Vol. **51,** pp. 197–214 (1949).

The paper deals with solutions of the linearised differential equation for the velocity potential of a compressible fluid in steady supersonic motion.

The equation itself is identical in form with that associated with two-dimensional wave motion. Thus known solutions of the two-dimensional wave equation may be interpreted in terms of supersonic flow, certain ones emphasizing particular characteristics of flows more readily than others. Some of the solutions which have been used by various investigators are brought together and are shown to be special cases of more general solutions. Two new types of normal solution, which are of special interest and are appropriate to special systems of co-ordinates, are developed.

The linearised equation for the velocity potential of a compressible fluid in supersonic flow is

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \alpha^2 \frac{\partial^2\phi}{\partial z^2} . \qquad (1)$$

It is assumed that the departure from a uniform stream, of Mach number M, in the z-direction is small, and that $\alpha^2 = M^2 - 1$. For supersonic flow $\alpha^2 > 0$. A general solution of (1) is

$$\phi = f(lx + my + nz) \qquad (2)$$

where f is an arbitrary function and $l^2 + m^2 - \alpha^2 n^2 = 0$. This was the basis of Schlichting's attempt (1) to put forward a theory of supersonic flight in which he considered a plane-wave contribution together with a circulation round a wing profile to be the correct interpretation of streamlined supersonic flow.

In cylindrical co-ordinates (r, z, ψ), normal solutions exist in the forms

$$\left.\begin{aligned} \phi &= \sum_{m,n} A_{m,n} \cosh\left(\frac{\beta_n}{\alpha} z + \xi_n\right) \cos(k_m\psi + \eta_m) Z_{k_m}(i\beta_n r) \\ \phi &= \sum_{m,n} A_{m,n} \cos\left(\frac{\beta_n}{\alpha} z + \xi_n\right) \cos(k_m\psi + \eta_m) Z_{k_m}(\beta_n r) \end{aligned}\right\} \qquad (3)$$

where $Z_{km}(\gamma r)$ is a linear combination of the two Bessel functions $J_{km}(\gamma r)$ and $Y_{km}(\gamma r)$. Normal solutions in terms of Legendre functions exist as

$$\phi = \sum_{p,q} A_{r,q}(R^{n_q} + B_{n_q} R^{-(n_q+1)})(P_{n_q}{}^{m_r}(\mu) + C_{m_r} Q_{n_q}{}^{m_r}(\mu)) \cos(m_r\psi + \xi_p) \qquad (4)$$

where

$$R = \alpha^{-1}(z^2 - \alpha^2 r^2)^{\frac{1}{2}}, \quad \mu = z(z^2 - \alpha^2 r^2)^{-\frac{1}{2}}.$$

In the case of line-symmetry, with velocity constant on the cone μ = constant, the solution is

$$\phi = R\{aP_1(\mu) + bQ_1(\mu)\} \qquad (5)$$

which is recognisable as the Karman-Moore potential (2) for high-speed flow past a cone.

Ward[3] used a variation of Whittaker's solution of Laplace's equation (Ref. 4, p. 388) to obtain an expression for the potential for flow over wings bounded by straight lines. This solution is

$$\phi = \int_{-\cosh^{-1}(p-1)}^{\cosh^{-1}(p-1)} \sum_{n=0}^{\infty} (z/\alpha)^n (1 - \rho \cosh t)^n f_n(t)\, dt\,, \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

where $\rho = \alpha r/z$. Karman and Moore's solution[2] corresponds to the special case of (6) for which

$$\left.\begin{aligned} f_1(t) &= -\tfrac{1}{2}a\alpha \\ f_n(t) &= 0, \quad n \geqslant 2 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

Reference is also made to the well-known source and doublet potentials which have been defined by Prantl[5] and Tsien[6] and also to Robinson's adaptation of a solution of the linearised equation in terms of curvilinear co-ordinates[7]. The basis of this type of solution will be found in a work by Hobson[8].

If the potential is independent of ψ, the linearised equation in co-ordinates $\left(z,\ \theta = \tan^{-1}\dfrac{r}{z}\right)$ is

$$\cos^2\vartheta(\cos^2\vartheta - \alpha^2\sin^2\vartheta)\phi_{\vartheta\vartheta} + \cos^3\vartheta(\sin\vartheta)^{-1}(\cos 2\vartheta - 2\alpha^2\sin^2\vartheta)\phi_\vartheta$$
$$+ 2\alpha^2 z(\sin\vartheta\cos\vartheta)\phi_{\vartheta z} = \alpha^2 z^2\phi_{zz}\,. \quad \ldots \quad \ldots \quad (8)$$

Solutions of (8) are useful in considering potentials which are symmetrical about the z-axis, and particularly simple normal solutions exist for certain eigen-values. If $t = \dfrac{\cot\vartheta}{\alpha}$, normal solutions of (8) exist as

and

$$\left.\begin{aligned} \phi &= z^n t^{-n} F\{[-\tfrac{1}{2}n, -\tfrac{1}{2}n;\ \tfrac{1}{2};\ t^2\} \\ \phi &= z^n t^{-(n-1)} F\{-\tfrac{1}{2}(n-1), -\tfrac{1}{2}(n-1);\ \tfrac{3}{2};\ t^2\} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

These are regular functions if $|t| < |$, and therefore represent flows outside the Mach cone, $t = 1$. Correspondingly, if $\alpha\tan\vartheta = k$, a solution suitable for power series approximation for small ϑ is found to be

$$\phi = z^n F\{-\tfrac{1}{2}n, -\tfrac{1}{2}(n-1)\ 1;\ k^2\}\,. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

The expansion of the solution in terms of Legendre functions (4) is equivalent to Murphy's expansion (Ref. 4, p. 311) and the Whittaker integral solution is equivalent to Barne's integral representation of the hypergeometric function (Ref. 4, p. 286).

Solutions are developed which converge rapidly in the region of the Mach cone. For restricted values of n these solutions exist in the form of a product of a power of $(1 - t^2)$ and a hypergeometric function of argument $(1 - t^2)$; in which case

and

$$\left.\begin{aligned} \phi &= (1 - t^2)^{1/2} F\{\tfrac{1}{2}, \tfrac{1}{2};\ \tfrac{3}{2};\ (1 - t^2)\} \\ \phi &= z(1 - t^2)^{3/2} F\{\tfrac{3}{2}, \tfrac{3}{2};\ \tfrac{5}{2};\ (1 - t^2)\} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

Other solutions are obtained in the form

$$\phi = z^n t^{-c} F\{\tfrac{1}{2}c, \tfrac{1}{2}c;\ (\tfrac{1}{2} - n);\ (1 - t^2)\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

when $c = n$ or $n - 1$; the degenerate solutions

$$\phi = \sin^{-1} t \quad \text{and} \quad \phi = z\{t^{-1}\sqrt{1 - t^2} - \cos^{-1} t\} \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

are also noted.

In the region immediately inside the Mach cone, where $(1 - k^2)$ is small and positive, solutions involving functions of small argument are

$$\left.\begin{aligned} \phi &= z^n F\{-\tfrac{1}{2}n, -\tfrac{1}{2}(n-1); (\tfrac{1}{2}-n); (1-k^2)\} \\ \text{and} \quad \phi &= z^n(1-k^2)^{n+1/2} F\{\tfrac{1}{2}(n+1), \tfrac{1}{2}(n+2); (n+\tfrac{3}{2}); (1-k^2)\} \end{aligned}\right\} \quad \ldots \quad (14)$$

Finally, solutions of the linearised equation are obtained in terms of wedge co-ordinates $(z, \vartheta = \tan^{-1} y/z)$ for two-dimensional flow, in which case the linearised equation assumes the form

$$\cos^2\vartheta\,(\cos^2\vartheta - \alpha^2\sin^2\vartheta)\phi_{\vartheta\vartheta} - 2(1+\alpha^2)\,(\sin\vartheta\cos^3\vartheta)\phi_\vartheta + 2\alpha^2 z\,(\sin\vartheta\cos\vartheta)\phi_{\vartheta z} = \alpha^2 z^2\phi_{zz}\,. \quad \ldots \quad \ldots \quad (15)$$

With the same notation as before, solutions exist in the forms

$$\left.\begin{aligned} \phi &= z^n t^n F\{-\tfrac{1}{2}n, -\tfrac{1}{2}(n-1); \tfrac{1}{2}; t^2\} \\ \text{and} \quad \phi &= z^n t^{(n-1)} F\{-\tfrac{1}{2}(n-1), -\tfrac{1}{2}(n-2); \tfrac{3}{2}; t^2\} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

outside the Mach angle, and as

$$\left.\begin{aligned} \phi &= z^n F\{-\tfrac{1}{2}n, -\tfrac{1}{2}(n-1); \tfrac{1}{2}; k^2\} \\ \text{and} \quad \phi &= z^n k\, F\{-\tfrac{1}{2}(n-1), -\tfrac{1}{2}(n-2); \tfrac{3}{2}; k^2\} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

inside the Mach angle.

As in the case of flow symmetrical about a line, solutions are developed for series approximation in the immediate neighbourhood of the Mach angle.

The latter solutions enable mathematical models to be built, of supersonic flow past bodies with two-dimensional symmetry.

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Technique of the Step-by-Step Integration of Ordinary Differential Equations

By

W. J. DUNCAN, D.Sc., F.R.S.

Professor of Aerodynamics at the College of Aeronautics, Cranfield.

This paper was published in full in the *Philosophical Magazine*, Ser. 7, Vol. 39, p. 493, July, 1948 (substantially the same as Report No. 4 of the College of Aeronautics).

In Part I step-by-step methods for the solution of one-point boundary problems are examined critically and emphasis is placed on the dependence of the error on the number n of equal steps used for a given range of the independent variable. When the dominant term in the error at the end of the range is proportional to the inverse k^{th} power of n the process of integration is said to be of *index k*. The index of currently used processes ranges from 1 in the original method of Euler to 4 in that due to Runge and Kutta. When the index of the process used is known and when results have been obtained for the same range of integration with 2 or more values of n, the errors can be assessed and partially corrected. This process of extrapolation towards the limit of the step-by-step calculation appears to have been given first by L. F. Richardson. The method of assessment and correction is illustrated by examples. It is suggested that an estimate or guess at the number of intervals required for the attainment of results of given accuracy should be made, and that the calculation should first be made with say half this number of intervals and then with the full number. It is thought that processes having the index 2 or 3 will be found usually to be most advantageous. Attention is drawn to the advantages in certain cases of using an analytical representation of the solution in each interval for the interval can then be substantially lengthened without loss of accuracy. This is true, in particular, when the differential equation or equations can be treated as linear with constant coefficients within each interval. All the methods described are applicable to linear and non-linear equations of any order and to sets of these.

In Part II methods for the numerical integration of ordinary differential equations are classified and very briefly reviewed. The first division is into purely numerical or digital and non-digital methods, the latter including analogic and graphical methods. Digital methods are classified as progressive and holic or unitary. In the latter the whole range of integration is considered at once; examples are provided by the methods of Rayleigh-Ritz and of Galerkin. The classification is carried further in the paper.

Control Reversal Effects on Sweptback Wings

By

H. TEMPLETON

This paper was published in full in *The Aeronautical Quarterly*, May, 1949.

Aileron reversal effects on sweptback wings in general and elevon reversal effects on tailless sweptback wings in particular are discussed on a non-mathematical basis, attention being confined to the orthodox flap type of control. The main purpose of the paper is to convey in the simplest terms possible a clear physical picture of the conditions producing loss of control power, emphasis being naturally laid upon the part played by structural wing distortion.

The nature of the structural distortions occurring on a sweptback wing are discussed, with particular reference to alternative rib arrangements. For the purpose of the paper, attention is confined to the overall wash-out produced by flexure and torsion, assuming the ribs to be infinitely stiff, and to the change of camber produced by flexure when the ribs are normal to the spar.

As a general introduction to the discussion on aileron reversal effects, the definition of "aileron power" in relation to the actual dynamic condition of rolling is discussed at some length. Three conditions are postulated; the initial condition, when rolling velocity is zero and acceleration a maximum; the static condition, in which the wing is restrained at the centre and both velocity and acceleration are therefore zero; and the steady rolling condition, when the rolling velocity is a maximum and the acceleration zero. At reversal all three conditions become one and the same. The loads and distortions occurring in the static condition are considered in some detail, and it is shown that the ratio of flexural to torsional distortion decreases with speed. Additional effects discussed are those relating to flexural axis position, the additional inertia and damping loads in the initial and steady rolling conditions, and the camber change occurring with ribs normal to the spar. It is anticipated that flexural distortion should have less influence in the initial and steady rolling conditions than in the static condition.

Elevon reversal effects on a tailless sweptback wing, defined as the control reversal effects associated with the use of the elevon as an elevator, are discussed along similar lines. As with the aileron, a complete representation of elevon power would be based on a realistic dynamic condition, but for the present purpose it is sufficient to consider the elevon as a pitching moment producer in a static condition analogous to that used for the representation of aileron power. The loads and distortions consequent upon elevon application in the static condition are considered and interesting comparisons drawn with the aileron case. It is deduced that for a given tailless aeroplane on which the same control is used as aileron and elevator, the elevon reversal speed must be greater than the aileron reversal speed. Also, the effect of flexural distortion near the reversal condition is to augment elevon power and therefore to delay reversal. It is therefore conceivable that if the flexural stiffness is low enough reversal might be delayed indefinitely. The effects of flexural axis position, camber change, and inertia loads in the dynamic condition are briefly touched upon, as in the case of the aileron.

Apart from elevon power in relation to manoeuvrability there is also the question of providing trim in the steady flight condition. Though the elevon reversal speed may be well above the maximum speed, it is possible that the elevon angle required for trim may be seriously affected by distortion of the structure. This involves consideration of all the loads acting upon the structure, not merely those occasioned by elevon application. Loads due to initial wing camber and washout (that is, of the undistorted wing) may have a major effect on the trim condition and upon the distortions produced.

For the quantitative estimation of control reversal effects a method is presented in the Appendix to the paper. The net wash-in due to distortion is expressed as a function of the non-dimensional spanwise parameter z in the form of alinear term plus a Fourier series

$$\theta = a_0 z + \sum_n a_n \sin n\pi z \, ,$$

the linear term giving the total wash-in (root to tip) and the Fourier series modifying the spanwise distribution. For a given speed and control surface angle in the static condition the coefficients a_n (including a_0) are determined from a set of equations linear in a_n representing the equilibrium conditions at a number of sections along the wing. For the trim condition on a tailless aeroplane an additional equation representing overall equilibrium in pitch provides a solution for the coefficients a_n and the elevon angle. In the same way, overall equilibrium in roll provides an additional equation for net rolling moment coefficient and rolling velocity respectively in the initial and steady rolling conditions for the aileron. For the reversal condition appropriate to either aileron or elevon the equations are non-linear in a_n and speed V, but are reducible to a polynomial in V^2 of degree equal to the number of coefficients a_n. If direct solution of the polynomial proves too cumbersome an indirect solution can be obtained by solving for the static condition over a range of arbitrary speeds, from which the reversal condition follows by interpolation.

Calculated results for a hypothetical wing are given to illustrate certain features of interest. Convergence with respect to the number of terms taken in the series expression for θ, obviously an important feature of the method, is demonstrated by calculated values of aileron reversal speed and elevon angle required to trim using one to five terms in the series for θ. Convergence in respect of aileron reversal speed is good, two terms (a_0 and a_1) being sufficient : for elevon angle to trim, the two-term solution is about 10 per cent in error and three or possibly four terms are required for a close approximation. Values of the rolling moment coefficient in the static condition calculated by the one-term (a_0) solution, which for aileron reversal speed is only 4 per cent in error, are compared with the values calculated by a more standard semi-rigid method using a single linear mode for the wash-in due to distortion. The semi-rigid method gives a good approximation over the first two-thirds of the speed range, but the reversal speed is underestimated by about 17 per cent.

The effects of certain design parameters on the elevon reversal characteristics are illustrated by calculated values of elevon angle to trim and of elevon reversal speed for a range of values of the parameters in question. A forward flexural axis, low aspect ratio, small sweepback, low wing loading, high overall stiffness, and a high ratio of torsional to flexural stiffness are all shown to be beneficial. Certain conclusions arrived at qualitatively in the main part of the paper are confirmed by the calculations.

The Radial Focusing Effect in Axially-symmetrical Supersonic Flow

By

R. E. MEYER

The Department of Mathematics, The University of Manchester.

This paper was published in full in the *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. 1, Part 4, December, 1948.

The mathematical significance of the characteristics lies in the fact that they are the lines on which normal derivatives of the velocity components can be discontinuous. Physically speaking, they are the carriers of disturbances. The paper is a contribution to the study of the laws of growth and decay of the disturbances along the Mach lines.

It is shown that in the case of an irrotational, isentropic, steady supersonic flow of a perfect gas, involving two independent variables, the first order derivatives, in the direction normal to a Mach line, of the velocity components satisfy each a first order, ordinary, non-linear differential equation along that Mach line. The variation of these derivatives is closely connected with the convergence and divergence of the Mach lines. A system of generalised orthogonal co-ordinates, α, β, is introduced, with $h_\beta \, d\beta$ denoting the element of length along the lines $\alpha =$ const. If the lines $\beta =$ const. are identified with one of the Mach line families, then h_β is proportional to the normal distance between two neighbouring Mach lines of that family and satisfies a second order, ordinary, linear, homogeneous differential equation along any one Mach line of the family.

These focusing equations are integrated in the special case of a straight Mach line in axially-symmetrical flow on which the velocity is constant. Such a Mach line occurs, *e.g.*, at the entry of a diffuser with uniform axial flow upstream, provided the slope of the wall is initially zero. It is found that

$$h_\beta = (c + \sqrt{r})/(1 + c) \quad (\text{if } h_\beta = 1 \text{ when } r = 1)$$

where r is the distance from the axis, and c a constant which can be calculated from the initial radius of curvature, R_{s1}, of the meridian section of the wall. Now, $h_\beta = 0$ implies a limit line and hence a shockwave, and thus a necessary condition for shock-free flow in the entry of a diffuser can be deduced. It is

$$R_{s1} > (\gamma = 1) \; M_0^4/(M_0^2 - 1),$$

where M_0 denotes the upstream Mach number (the initial radius of the diffuser is chosen as unit of length). For a two-dimensional diffuser of width $2b$, the corresponding condition is

$$R_{s1} > (\gamma = 1) \; M_0^4 \, b/2(M_0^2 - 1).$$

The integration of the focusing equations also leads to the result that the rate of change of any velocity component per unit length in the direction normal to the Mach line is proportional to

$$r^{-1/2}(c = \sqrt{r})^{-1} .$$

On the other hand, the change along the normal, from the Mach line to a neighbouring one, is proportional to $r^{-1/2}$. This distinction is the main improvement which Non-Linear Theory brings about in comparison with Linear Theory which leads to the $r^{-1/2}$-law but neglects the curvature of the Mach lines.

The focusing laws are used to calculate the velocity distribution in the entry of a diffuser and it is shown that Focusing Theory explains the surprising features of this distribution which were first noticed by Tupper at the instance of his numerical investigation based on Linear Theory.

The focusing laws show also that a discontinuity of the stream-line curvature at the entry of a diffuser leads to a singularity of the field of flow where the leading Mach line of the diffuser meets the axis. This singularity is, however, not connected with the appearance of a limit line and its nature is investigated on Linear Theory. The perturbation potential which describes the field of flow can be written in the form

$$\phi = \frac{2}{3\pi} \int_0^\pi (x + \alpha r \cos u)^{3/2} \, \delta \, du,$$

wnere x is measured downstream from the singular point, $\alpha^2 = M_0^2 - 1$, $\delta = 1$ when $(x + \alpha r \cos u) \geqslant 0$, and $\delta = 0$ otherwise.

The most striking feature of the singularity is that the second derivatives of ϕ are proportional to

$$r^{-1/2} \log p, \quad \text{near } p = 0,$$

where $p = (\alpha r - x)/2\alpha r$. That means, on Linear Theory the disturbance, *i.e.*, discontinuity of the velocity derivatives, is reflected from the axis not as a discontinuity but as a logarithmic singularity.

On Source and Vortex Distributions in the Linearised Theory of Steady Supersonic Flow

By

A. Robinson

This paper was published in full in the *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. 1, 1948; College of Aeronautics Report No. 9, October, 1947.

Various particular solutions of the lineared equation of steady supersonic flow are considered with a view to their application to the flow round aerofoils and other bodies. General line, surface, and volume distributions of supersonic sources and doublets are considered, as well as similar distributions of vorticity. Results are given corresponding to the theorems of Gauss (on total normal intensity) and of Poisson in classical potential theory. Formulae are derived for the field of flow due to an isolated re-entrant vortex travelling at steady supersonic velocity; they constitute a counterpart to the law of Biot-Savart.

Two mathematical ideas are required to carry out the analysis. One is the formal introduction of the operator $\nabla h\beta$ ('hyperbolic nabla of index β') defined by $\left(-\beta^2\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ which is used in conjunction with the ordinary vector operator $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$. The second concept is that of the finite part of an infinite integral due to Hadamard and others. It is pointed out that in spite of the apparent artificiality of the definition of the finite part, this concept can be used to describe real physical quantities, such as the flow across a surface surrounding an isolated source.

An application of the theory is given in College of Aeronautics Report No. 10.

Assessment of Errors in Approximate Solutions of Differential Equations

By

W. J. Duncan, D.Sc., F.R.S.

Professor of Aerodynamics at the College of Aeronautics, Cranfield.

This paper was published in full in the *Quarterly Journal of Mechanics and Applied Mathematics*, Vol 1, p. 470, December, 1948. (Substantially the same as Report No. 13 of the College of Aeronautics.)

The paper discusses three particular methods, two of which apply to linear partial differential equations, while the third applies to linear or non-linear ordinary differential equations and is treated more fully in the paper A.R.C. 10,526 of which an abstract is given separately. When the Green's function of a linear partial differential problem is one-signed, as is true in many important cases, strict upper and lower bounds to the error in an approximate solution can be assigned when the absolute maximum and minimum values of the residual have been found. The residual is defined as the value of the differential expression for the approximate solution which is zero for the exact solution. It is shown that the Green's function is necessarily negative for Poisson's equation in the region interior to a closed surface or curve on which the solution vanishes. This method of assessment has been successfully applied to a number of problems arising in the theory of elasticity. The error for a linear partial differential problem can also be expressed approximately as an integral containing the residual when an approximation to the Green's function is known.

Notes on The Linear Theory of Incompressible Flow round Symmetrical Swept back Wings at Zero Lift

By

F. Ursell

This paper was published in full in *The Aeronautical Quarterly*, Vol. 1 (1949) p. 101.

A derivation is given of the linearised equations of motion Green's theorem on potential functions leads from these equations to the method of sources, whereby the pressure is expressed as a surface integral extended over the wing. For points on the wing the principal value in some sense of the surface integral must be taken. To this must be added a line integral depending on the definition of the principal value. When the swept back wing consists of two equal cylindrical surfaces joined at an angle an integral relation can be derived connecting the pressure distribution along the centre section and the profile; this relation is due to Neumark. When the pressure distribution is given an integral equation for the profile is obtained which can be solved explicitly, but care is required in prescribing the pressure near the leading edge if geometrically impossible wings are to be avoided. When the wing consists of two equal cylinders joined at an angle the wing plan has a discontinuity in gradient near the nose; but if the wing plan is assumed to have large but finite curvature near the nose the pressure on the wing is not greatly changed.

It is next shown that it is not possible to design wings with isobars kinked in the centre, unless suction slots or similar devices are used. Finally it is proved that there may be no wing or one wing with a given pressure distribution but that there cannot be more than one.

Flutter of Systems with Many Freedoms

By

Professor W. J. Duncan, D.Sc., F.R.S.

This paper was published in full in *The Aeronautical Quarterly*, Vol I, page 59, May, 1949.

The aim of the paper is to discuss methods for calculating critical flutter speeds and the nature of the motion at these speeds for systems with a large number of degrees of freedom. This problem is becoming increasingly important since it is now recognised that reliable estimates of critical speeds can, in many instances, only be made when many independent kinds of motion of the structure are admitted. However, the labour in the calculations increases exceedingly rapidly as the number of degrees of freedom is increased. Hence two principal problems arise:—

(a) The choice of a minimum set of dynamical co-ordinates or degrees of freedom which leads to calculated results of adequate accuracy.

(b) The choice of the methods of conducting the calculations after the dynamical co-ordinates have been chosen.

These problems are considered separately in Parts I and II of the paper.

It is concluded that a particular freedom F must be retained when the balance of energy at a critical flutter speed is sensitive to its inclusion, unless it can be shown that the amplitude of F is very small. This amplitude will be very small when one or both of the following conditions is satisfied:—

(a) The coupling terms in the Lagrangian dynamical equation corresponding to F are all very small.

(b) The direct impedance for F at the critical flutter speed and for the flutter frequency is very large.

The equation of energy for any number of freedoms is reduced to a convenient form showing explicitly the influence of the phase relations of the motions. It is shown that large skew-symmetric components in the aerodynamic stiffnesses will result in a large intake of energy when the phase relations of the motion are favourable for this. Hence dynamical co-ordinates giving rise to such aerodynamic stiffnesses must be scrutinised with special care.

Some simple illustrative applications given in the paper support the validity of the proposed method of selecting co-ordinates, but its general usefulness can only be judged on the results of extended trials. In any event, it is hoped that the analysis of the flutter problem given in this paper will contribute towards a better understanding of the physics of flutter.

Attention is drawn to the usefulness of comparing the various aerodynamic couplings with the corresponding inertial couplings, since this may suffice to show that the former are small.

Inverse methods appear to be the most advantageous for the calculation of critical flutter speeds when there are many freedoms.

There are a few known special cases where "exact" calculations of critical flutter speeds can be made for elastic continuous systems having infinitely many degrees of freedom. Such systems throw much light on the general problem of the choice of freedoms and they are considered briefly in the paper.

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