C.P. No. 247
(17,568)
A.R.C. Technical Report

C.P. No. 247
(17,568)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
CURRENT PAPERS

Tests of Humidity Effects on Flow in a Wind Tunnel at Mach Numbers between 2.48 and 4

By

R. J. Monaghan. M.A.

LONDON: HER MAJESTY'S STATIONERY OFFICE

1956

FIVE SHILLINGS NET

C.P. No. 247

U.D.C. No. 533.6.071.4.011.5 : 536.423.4

Technical Note No. Aero 2358

January, 1955

ROYAL AIRCRAFT ESTABLISHMENT

Tests of humidity effects on flow in a wind tunnel at Mach numbers between 2.48 and 4

by

R.J. Monaghan, M.A.

SUMMARY

Static and pitot pressure distributions were measured in the working section of a 5 in. × 5 in. supersonic wind tunnel at nominal Mach numbers of 2.48, 3.25 and 4, over ranges of absolute humidity at the inlet from 5×10^{-5} to 3×10^{-3}. For these conditions, previous work would indicate that a condensation shock would occur in the nozzle.

For a stagnation pressure of 1 atmosphere and stagnation temperatures giving zero heat transfer conditions at the walls, no humidity effects were discernible if the absolute humidity was less than 2×10^{-4} at $M = 2.48$, 3×10^{-4} at $M = 3.12$ and about 5×10^{-4} at $M = 3.8$. Above these critical values there was a gradual deterioration in flow distribution, but no localised disturbances were found.

Tests at nominal $M = 3.25$ showed no effect of relative humidity if the absolute humidity was less than the critical values quoted above. (In a typical case of an absolute humidity of 2×10^{-4}, the relative humidity was varied between 6×10^{-3} and 5×10^{-2} without showing any effect).

LIST OF CONTENTS

Page

1 Introduction 4

2 Experimental equipment, range and accuracy of measurements 5

- 2.1 Pressure measurements 6
- 2.2 Humidity control and measurement 7

3 Results and discussion of tests at 1 atmosphere stagnation pressure, fixed stagnation temperature and varying absolute humidity 8

- 3.1 Static pressures 8
- 3.2 Pitot pressures 9
- 3.3 Comparison of static and pitot pressures 9
- 3.4 Tentative conclusions 10

4 Results and discussion of tests at constant absolute but varying relative humidity 10

- 4.1 Tests at p_0 = 1 atmosphere and varying stagnation temperature 11
- 4.2 Tests with varying stagnation pressure 11

5 Conclusions 12

List of Symbols 13

References 14

APPENDIX

Simplified treatment of the condensation shock I

LIST OF ILLUSTRATIONS

Fig.

Location of pressure measuring points 1

Details of pitot rake 2

Ranges of humidity covered by the tests 3

Mach number distributions indicated by ratios of static to stagnation pressures

- Nominal M = 2.48 4a
- Nominal M = 3.25 4b
- Nominal M = 4 4c

LIST OF ILLUSTRATIONS (Contd)

Fig.

Mach number distributions indicated by ratios of pitot to stagnation pressures

Nominal $M = 3.25$ 5a

Nominal $M = 4$ 5b

Comparison of Mach numbers indicated by static and pitot pressures

Nominal $M = 2.48$ 6a

Nominal $M = 3.25$ 6b

Nominal $M = 4$ 6c

Variation of critical absolute humidity with Mach number for $p_o = 1$ atmosphere and $T_{Ho} = 35^\circ C$ 7

Effect on indicated Mach number of varying stagnation temperature 8

Effect on indicated Mach number of varying stagnation pressure 9

Errors in calculated Mach number associated with errors in pressure measurement 10

---

1 Introduction

When a mixture of air and water vapour undergoes an adiabatic expansion, its pressure and temperature are reduced. The reduction in temperature leads to a fall in the saturation pressure of the vapour and this fall is more rapid than that of the partial pressure of the expanding vapour so that at some stage the air becomes saturated. From that stage onwards the vapour can condense, with consequent liberation of its latent heat, so that the expansion is no longer along a dry adiabatic line.

The mechanism of the condensation process seems to depend on the rate of expansion. If this is high, as in the nozzle of a supersonic wind tunnel, considerable supersaturation may occur before the vapour condenses, but the condensation then takes place rapidly giving a discontinuity in the flow or "condensation shock". A good account of the theory and mechanism of condensation processes is given by Lukasiewicz in Ref.1 and will not be repeated here.

There are two relevant measures of the humidity of the airstream. The first is the "absolute humidity" (Ω) which is defined as the mass of water vapour contained in unit mass of air. This gives a measure of the amount of heat which can be liberated during condensation, and prior to condensation it will obviously remain constant during the expansion process.

From the general gas laws the absolute humidity is related to the total and partial pressures of the air-water vapour mixture by the formula

$$\Omega = \frac{\text{Mol wt of water}}{\text{Average mol wt of air}} \times \frac{p_V}{p - p_V}$$

$$= 0.622 \frac{p_V}{p - p_V}$$

where p is the total pressure of the mixture

and p_V is the partial pressure of the vapour.

The vapour pressure is usually small by comparison with the total pressure, so that approximately

$$\Omega = 0.622 \frac{p_V}{p} \quad . \qquad (1)$$

The second measure is that of "relative humidity" (φ), which is defined by

$$\varphi = \left(p_V / p_{SV} \right)_T \qquad (2)$$

where p_{SV} is the saturation vapour pressure at the temperature under consideration. As the temperature decreases the saturation vapour pressure decreases and hence the relative humidity increases. (The mixture becomes saturated when $\varphi = 1$). Thus knowledge of the relative humidity under, say, stagnation conditions means that we can determine at what stage in the expansion the mixture will become saturated. The amount of supercooling which can then occur before the vapour condenses can be obtained experimentally and values around 50°C are quoted in Ref.1.

In a wind tunnel it would be desirable to avoid condensation altogether. This would be achieved by having a sufficiently low value of relative humidity in the settling chamber and one method of doing this would be to increase the stagnation temperature. However the temperatures involved increase rapidly with Mach number in the supersonic range and the condensation shock, if it occurred, would still be strong. A more satisfactory method is to dry the incoming air and hence reduce its absolute humidity. Equations (1) and (2) show that this will also reduce the relative humidity in the settling chamber to values dependent on the stagnation pressure (i.e. for constant absolute humidity, the relative humidity will vary directly with the stagnation pressure). The reduced relative humidity postpones the occurrence of the condensation shock, while the reduced absolute humidity ensures that its strength is reduced.

Raney and Beastall have made tests in a 9 in. × 9 in. supersonic wind tunnel to determine the dryness necessary to ensure condensation free flow in the working section over a range of Mach numbers from 1.4 to 1.9 [2]. At a stagnation pressure of 1 atmosphere and stagnation temperature of 35°C they found that the absolute humidity had to be less than 2×10^{-3} lb/lb at $M = 1.4$ and less than 1.4×10^{-4} lb/lb at $M = 1.9$. At higher stagnation pressures or lower stagnation temperatures the requirements were more stringent.

At Mach numbers above 2 it rapidly becomes impossible to provide low enough values of the absolute humidity to avoid condensation completely. The basis for judgement of whether the humidity is low enough must therefore be changed to one less absolute in form : the judgement would depend both on the general Mach number level and on the uniformity of flow in the working section. Thus in all cases there would probably be a condensation shock in the nozzle which by itself would alter the Mach number level in the working section and its reflections might penetrate into the working section, giving local non-uniformities.

The present tests were made at nominal Mach numbers of 2.48, 3.25 and 4 in a 5 in. × 5 in. wind tunnel and the intention was to obtain a rough check on humidity requirements rather than to make a detailed study of the problem. Beforehand one might specify a requirement that, say, static pressure should not change anywhere by more than $\frac{1}{2}$ per cent, but in practice the conclusions had to be drawn in a much more qualitative fashion since the measuring accuracy (particularly at $M = 4$) was insufficient for a quantitative analysis. Unfortunately, Schlieren apparatus was not available for observation of the flow, so all the results had to be based on static and pitot pressure measurements alone.

The majority of the tests were made at a stagnation pressure of one atmosphere and a stagnation temperature of 35°C, but checks were also made of the effects of variations in relative humidity obtained from varying the stagnation pressure or the stagnation temperature.

The tests were made on dates between November 1953 and January 1954.

Acknowledgements

The author wishes to acknowledge the considerable assistance received from Miss R. Hensby, who analysed the results, and from Mr. J.F.W. Crane, who designed the pitot rake.

2 Experimental equipment, range and accuracy of measurements

The tests were made in the R.A.E. No.5 supersonic Wind Tunnel (5 in. × 5 in. working section) at nominal Mach numbers of 2.48, 3.25 and 4 in the working section. Stagnation pressure could be varied from 1 to 5 atmospheres and stagnation temperature could be held steady to within 1/10°C for several hours at values up to 40-50°C (the upper limit is set by the use of wood in the liner construction). The tunnel is of the non-return type.

2.1 Pressure measurements

All the nozzles were single sided and static pressures were measured along the centre line of the flat wall at positions shown in Fig.1. Pitot pressures were measured at the positions shown in Fig.1, using the pitot rake shown in Fig.2 (except at $M = 2.48$, when pitot pressure was measured only at the single point K, using a standard pitot tube). All the pitot measurements were made at a single longitudinal position as shown in Fig.1.

Stagnation pressure and temperature were measured in the upstream ducting where the maximum speed was 25 ft/sec (at $M = 2.5$).

All pressures were taken through Teneplas plastic tubing to liquid manometers. Absolute measurements were made of one static pressure (R, Fig.1) and of one pitot pressure (K, Fig.1) using a large bore (9 mm) mercury manometer with vernier and a barometer. The remainder (1-14 and A-H) were measured as differences from the reference pressures, using manometer banks filled with Butyl-Phthalate. Estimates of the maximum reading errors which might have occurred are:

Stagnation Pressure:	
1 atmosphere operation	1.25×10^{-2} in.Hg
5 atmosphere operation	$\pm 5 \times 10^{-2}$ in.Hg
Pitot and Static reference pressures	$\pm 1.25 \times 10^{-2}$ in.Hg
Pitot and Static differences	$\pm 1.5 \times 10^{-3}$ in.Hg.

These are only of interest by comparison with the absolute magnitudes of the various pressures and possible percentage errors are shown in the following table, for 1 atmosphere operation, where the absolute values are least. (The errors in stagnation pressure reading are of the order of 0.04 per cent in all cases.)

M		2.48	3.25	4
p in.Hg		1.81	0.57	0.20
$\pm \frac{\Delta p}{p}$ per cent	(Reference	0.69	2.2	6.25
	(Differences	0.083	0.26	0.75
p_o' in.Hg		15.2	7.9	4.2
$\pm \frac{\Delta p_o'}{p_o'}$ per cent	(Reference	0.082	0.16	0.30
	(Differences	0.0099	0.019	0.036

In the above table, p is static and p_o' is pitot pressure.

The values illustrate the difficulty there would be in detecting small changes in static pressure, particularly at the higher Mach numbers. It should be stressed that these are estimates of reading error: the actual reading accuracy obtained may have been higher. On the other hand the overall measuring accuracy may have been lower: for example, difficulties were experienced in obtaining and maintaining a completely leak-proof system at $M = 4$.

The pitot results should be more accurate, but against this must be set unknown errors from possible re-evaporation of water during the compression in front of a pitot tube.

All the test results are compared later on the basis of Mach number distributions calculated from the pitot and static measurements. Fig. 10 shows the errors in calculated Mach number associated with errors in pressure measurement: if all these errors were of equal magnitude then at the higher supersonic Mach numbers it would be desirable to calculate Mach number from the ratio of either static or pitot pressure to stagnation pressure rather than from the ratio of pitot to static pressure. However the various reading errors are not of equal magnitude and the true stagnation pressure (following the condensation shock) cannot be measured, and in calculations it is assumed to be equal to the stagnation pressure in the settling chamber. Some discussion of the errors in "indicated" Mach number (using stagnation pressure measured in the settling chamber) is given in Appendix I.

For all these reasons it was considered best to rely on general trends shown up by the tests rather than to attempt to make a detailed quantitative analysis. The degree of consistency achieved in the tests is illustrated, for example, by Fig. 5a ($M = 2.48$). Considering the Mach numbers calculated from the ratio of static to stagnation pressure (symbol "x", $\hat{x}$ and $\tilde{x}$) then at low humidities the scatter is within the estimated reading error, but the scatter increases considerably with increasing humidity. The reason for this increased scatter is not understood, but it is possible that it is a real effect caused by some flow instability introduced by the condensation shock and may not necessarily be caused by inaccuracy of measurement.

2.2 Humidity control and measurement

In the tests at a stagnation pressure of 1 atmosphere, the air was dried by refrigeration, being supplied to the tunnel through one of the air-cycle refrigeration units ("Butterley cold-air machine") of the High Altitude Test Plant. By this means the humidity could be controlled over a considerable range. The pressure in the pipe line was kept at about $\frac{1}{2}$ in.Hg above atmospheric (by means of a booster fan) to avoid any leaks of "wet" atmospheric air into the system after the drier. (Without the booster fan, the pressure in the pipe line is slightly below atmospheric and frost points measured at the settling chamber were about 8°C above those obtained when the booster fan was running. It is not certain, however, whether this was the result of a leakage into the pipe line or a leakage into the hygrometer.)

After this drying by refrigeration, the air was heated electrically and the majority of the tests were made at a stagnation temperature of about 35°C.

The usual check on humidity is from the measurement of the air temperature at the exit from the cold-air unit. This is a considerable distance upstream from the tunnel entry, so in the present tests the humidity was determined from frost points measured with a Brewer and Dobson hygrometer at the settling chamber. Three measurements with this instrument were made during each test, often with different operators and the results were always in agreement to within ±1°C, even at the lowest humidities.

Fig. 3 shows the ranges of absolute and relative humidities covered by the tests, which correspond to a range of dew or frost points from about +5°C down to about -50°C.

For the tests at stagnation pressures above 1 atmosphere, air was supplied through the Jaeger compressors and dried by beds of silica gel. Thus controlled variations of humidity were no longer possible. However at the time of the tests the plant was undergoing proving trials and the driers were not functioning as well as they should have. This was useful for the present purposes and absolute humidities between 10^{-3} and 5×10^{-5} were obtained.

3 Results and discussion of tests at 1 atmosphere stagnation pressure, fixed stagnation temperature and varying absolute humidity

These tests were made at stagnation temperatures (T_H) of the order of those which would be required for zero heat transfer conditions at the tunnel walls, i.e.

$$T_H/T_{wo} = \frac{1 + \frac{\gamma - 1}{2} M^2}{1 + 0.9 \frac{\gamma - 1}{2} M^2} \qquad (3)$$

where T_{wo} is the wall temperature for zero heat transfer, taken as being equal to the ambient temperature in the tunnel room.

3.1 Static pressures

Mach number distributions indicated by the ratio of static to stagnation pressure are given in Figs. 4a-4c. These are plotted as carpets of Mach number against position (see Fig.1) and humidity.

Fig.4a gives the results for nominal $M = 2.48$ and the longitudinal distributions remain similar for humidities up to about 2×10^{-4} lb/lb, above which there is a gradual deterioration, particularly at the upstream end.

Fig.4b, for $M = 3.25$, shows the same trends, the deterioration occurring above a humidity of about 3×10^{-4}. At lower humidities the average Mach number is about 3.15 instead of the design value of 3.25, but this probably means that the boundary layer correction to the liner profile was inadequate. A pronounced trough occurs at humidities between 3×10^{-4} and 6×10^{-4}. Repeat tests verified the existence of this trough and made it seem unlikely that it could be explained solely by errors in measurement of reference pressure, but no other explanation can be offered at present.

Fig.4c gives the results for nominal $M = 4$. These are likely to be affected considerably by errors in measurement of reference pressure, since the probable reading error of 6.25 per cent quoted in section 2.1 would give over 1 per cent error in M (Fig.10). As a result a considerable amount of faith has to be placed on the shapes of the distributions (probable error 0.15 per cent), which remain similar over the range of humidity from 1.5×10^{-5} up to 3×10^{-4}. Slight changes are discernible at humidites of 5.5×10^{-4} and 10^{-3} and a single set of measurements made at a humidity of 3×10^{-3} gave an indicated mean Mach number of 3.4, which could not be plotted on Fig.4c. A plausible value for critical humidity could therefore be about 5×10^{-4}.

It is of interest that in all three illustrations there is no evidence of any localised disturbance varying with humidity level which might be attributed to downstream reflections of a condensation shock in the nozzle.

Instead there is only a gradual deterioration in Mach number distribution as the humidity is increased above a certain level*. Taking this level as defining the critical humidity then Figs. 4a-4c give the following approximate values for it.

Nominal M	2.48	3.25	4
Critical absolute humidity	2×10^{-4}	3×10^{-4}	about 5×10^{-4}

3.2 Pitot pressures

Figs. 5a, 5b give the Mach number distributions indicated by the ratios of pitot to stagnation pressure at $M = 3.25$ and 4. (Only a single pitot position was available at $M = 2.48$.)

Apart from a slight reduction in average value, the distributions at either Mach number remain similar over the whole range of humidity. No disturbances were detected which might have come from reflections of the condensation shock in the nozzle (except for a single disturbance at pitot C, $\Omega_o = 10^{-4}$ in Fig.5b, but this may not be reliable).

3.3 Comparison of static and pitot pressures

These are compared in Figs.6a-6c on the basis of Mach number in the region of static 14 and pitot K, computed from the experimental values (at static 14 and pitot K) of

(1) the ratio p/p_o

(2) the ratio p_o'/p_o

and (3) the ratio p_o'/p

where p_o is the stagnation pressure measured in the settling chamber upstream of the nozzle. Of these, the third should give the true Mach number, provided both that there is no re-evaporation in front of the pitot tube and also that the static pressure does not vary between the wall and position K. In fact it seems that the static pressure may have varied in this region (except at $M = 4$) since even at the low humidities the Mach numbers calculated by the three methods do not agree with each other. That this is unlikely to be a humidity effect is evidenced by the fact that the individual values appear to be essentially independent of humidity in this region.

The lower graph in each case is of absolute values of Mach number plotted against absolute humidity. The upper graphs are of the ratio of Mach number at given humidity to the appropriate value at low humidity, once again plotted against absolute humidity. This latter plot collapses the values calculated by the three methods at low humidities and emphasises the increasing displacements at high humidities. (Note that the ordinate scale of Fig.6c is half that of Figs. 6a and 6b.)

The curves drawn on Figs.6a-6c are completely theoretical and were calculated from equations I.11 (for true Mach number), I.15 (for Mach number indicated by the ratio p/p_o) and I.16 (for Mach number indicated by the ratio p_o'/p_o) of Appendix I, assuming that there was 45°C supercooling before the condensation shock occurred and that there were no downstream reflections. In the lower graphs the curves were fitted to the experimental values of M at low humidity.

---

* As a possible qualification of these statements it should be noted that there is a spacing of 1 inch between the static pressure points (Fig.1) so that a relatively stationary local disturbance might escape detection.

The assumption of 45°C supercooling is not very important since the order of the theoretical results would not be altered if the cooling were increased to 70°C (see last paragraph of Appendix I). Also the theoretical approach would predict only a change in level of the Mach number distribution in the working section as the humidity is increased and would not account for any deterioration in the distribution itself.

However in spite of these shortcomings, the theoretical curves show the same trends as the experimental values and there is some measure of agreement between them as regards the disturbing effect of high humidity. For these reasons it was thought worthwhile to apply the theoretical equations to determine quantitative humidity requirements. The results of doing this are given in the next section.

3.4 Tentative conclusions

The curves of critical absolute humidity in Fig.7 have been drawn in accordance with the equations of Appendix I.

The limiting curve at Mach numbers between 1.5 and 2 is for condensation free flow in the working section, assuming 45°C supercooling, and this is in good agreement with the results of Raney and Beastall[2] for p_0 = 1 atmosphere. These limiting values would vary roughly in proportion with the stagnation pressure.

For higher Mach numbers the criterion has been changed to that of the pressure rise (above the value for zero humidity) allowable in the working section and the curves of Fig.7 show that theoretically the critical humidity increases slowly with increase in Mach number. The experimental estimates from Mach number distributions derived from measured static pressures (section 3.1) are plotted and show a similar trend, but increase more rapidly with Mach number than the theoretical curves for constant pressure rise.

However the experimental comparisons of section 3.1 were of Mach number distributions, so further theoretical curves are given in Fig.7 corresponding to changes in Mach number of 0.14 and 0.2 per cent. The former could be taken as being in fair agreement with the experimental results.

Also drawn is a boundary imposed by the reading accuracy of the static pressure differences in the present tests and this could be taken to explain in part the divergence in trend between theoretical and experimental values.

Thus the experimental results could reasonably be said to give some support to the theoretical curves and it is apparent that absolute humidities of less than 2×10^{-4} lb/lb should not be necessary for normal operation at the higher supersonic Mach numbers, for one atmosphere stagnation pressure. Effects of varying relative humidity will be considered in the next section.

4 Results and discussion of tests at constant absolute but varying relative humidity

Varying the relative humidity in the settling chamber would be expected to change the position of the condensation shock in the nozzle and hence it might have some effect on the Mach number distributions in the working section. If the absolute humidity is held constant, the relative humidity can be changed by varying the stagnation temperature (section 4.1) or the stagnation pressure (section 4.2).

The tests described in this section were made at nominal $M = 3.25$ and the comparisons are based on Mach number distributions calculated from the static pressure readings along the centre line of the flat wall (Fig.1) in conjunction with the stagnation pressure in the settling chamber.

4.1 Tests at $p_o = 1$ atmosphere and varying stagnation temperature

Fig.8 gives the results of tests made at a stagnation pressure of one atmosphere, three levels of absolute humidity and over ranges of stagnation temperature between 36° and -4°C.

Fig.8c is for a low absolute humidity of 4.6×10^{-5} and shows that a tenfold change in relative humidity caused no significant change in Mach number distribution. The change in level between the two distributions is mostly within $\frac{1}{2}$ per cent, which is within the estimated order of accuracy of pressure measurement (section 2.1 and Fig.11, giving $\frac{\Delta p}{p} = \pm 2.2$ per cent and $\frac{\Delta M}{M} = \mp 0.46$ per cent).

Fig.8b is for an absolute humidity of 2×10^{-4}, which is near to the critical value (3×10^{-3}) suggested in section 3, when the stagnation temperature was 35°C. Once again the effects of changing the relative humidity are hardly significant, except at the most forward position in the working section (static hole 1).

Fig.8a is for a high absolute humidity of 3.8×10^{-3} and in this case changing the relative humidity from 8.9×10^{-2} to 3.3×10^{-1} causes an appreciable change in distribution over the front half of the measuring stretch (static holes 1-7).

Thus it seems that changes in relative humidity by varying the stagnation temperature will only affect the Mach number distribution in the working section if the absolute humidity is near or above the critical values suggested in section 3. As regards the centre line distributions of these tests, any changes are probably gradual, spreading backwards from the front of the working section as the relative humidity is increased.

4.2 Tests with varying stagnation pressure

Fig.9 gives the results of some tests made at different levels of stagnation pressure, holding the stagnation temperature near to that for zero heat transfer conditions. In this case comparisons are made more difficult by the fact that an increase in stagnation pressure leads to a proportionate decrease in the effects of reading errors. Differences may also be caused by changes in boundary layer development along the walls of the nozzle, but these should be of smaller order (since the boundary layer changes would vary inversely only with the one-fifth power of the stagnation pressure).

The decrease in the effects of measuring errors is probably reflected in the fact that the distributions in Fig.9 become smoother as the stagnation pressure is increased. If this is accepted, then it would seem that changes in stagnation pressure may have little effect on the Mach number distributions in the working section, even at high values of absolute humidity. However this is necessarily a very tentative conclusion, but at least it would seem that the humidity requirements do not become more stringent when the stagnation pressure is increased.

It should be emphasised that the tests of this section were made only at $M = 3.25$. It might be expected that the effects of relative humidity would be more marked at a lower Mach number (when the condensation shock would be closer to the working section) and less marked at a higher Mach number.

5 Conclusions

Consideration of Mach number distributions obtained from static and pitot pressure measurements in the working section has indicated the following conclusions.

(1) For a stagnation pressure of 1 atmosphere and stagnation temperatures giving zero heat transfer conditions at the walls (equation 3), no humidity effects were discernible if the absolute humidity in the settling chamber was less than 2×10^{-4} at $M = 2.48$, 3×10^{-4} at $M = 3.12$ and about 5×10^{-4} at $M = 3.8$.

(2) Above these critical values there was a gradual deterioration in distribution, but no localised disturbances were found.

(3) The experimental results were in qualitative agreement with trends calculated in Appendix I assuming one dimensional conditions and no reflections of the condensation shock in the nozzle. Fig.7 illustrates the extent of this agreement, includes results from earlier tests by Raney[2] for $M < 2$ and gives tentative curves for critical humidity for $M > 2$.

(4) Varying the relative humidity by varying the stagnation temperature at $M = 3.25$ had no effect if the absolute humidity was less than the critical value quoted in conclusion (1).

(5) Varying the relative humidity by varying the stagnation pressure at nominal $M = 3.25$ had no effect over the whole humidity range, but in this case trends may have been obscured by the improvement in measuring accuracy as the stagnation pressure was increased. However it can probably be said that the humidity requirements do not become more stringent when the stagnation pressure is increased.

(6) Relative humidity would probably have a more marked effect at Mach numbers less than $M = 3.25$ and vice versa.

LIST OF SYMBOLS

M	Mach number
p	pressure (static)
p_v	partial pressure of water vapour
p_{sv}	saturation vapour pressure
p_o	stagnation pressure
p_o'	pitot pressure
T	temperature
T_H	stagnation temperature
T_{wo}	wall temperature for zero heat transfer
γ	ratio of specific heats
φ	relative humidity (equation 2)
Ω	absolute humidity (lb/lb and equation 1)
φ_o, Ω_o	values in settling chamber

Additional symbols occurring only in Appendix

c_p	specific heat of air
h	latent heat of evaporation (sublimation) of water (ice)
J	mechanical equivalent of heat
q	heat input per unit mass of air
Q	q/c_pT
u	velocity
ρ	density

Suffices

1, 2	conditions immediately before and after heat addition
c	conditions in working section when there is a condensation shock in the nozzle
i	indicated values of Mach number (e.g. from $\frac{p_c}{p_{o1}}$)

LIST OF REFERENCES

No.	Author	Title, etc.
1	J. Lukasiewicz	Humidity effects in supersonic flow of air R & M 2563 July, 1947
2	D.J. Raney and D. Beastall	Criteria for condensation free flow in the R.A.E. No.18 (9 in. × 9 in.) supersonic tunnel CP.164 June, 1953

APPENDIX I

Simplified treatment of the condensation shock

If, as in Ref.1, we regard the overall effect of condensation to be equivalent to the addition of latent heat to the flow and assume that one dimensional theory is applicable, then we have the following relations between the states of the gas before (subscript "1") and after (subscript "2") the heat addition, (neglecting viscosity and thermal conductivity).

$$\frac{p_2}{\rho_2 T_2} = \frac{p_1}{\rho_1 T_1} \qquad \text{(state)} \qquad \text{I.1}$$

$$\rho_2 u_2 = \rho_1 u_1 \qquad \text{(mass flow)} \qquad \text{I.2}$$

$$\rho_2 u_2^2 + p_2 = \rho_1 u_1^2 + p_1 \qquad \text{(momentum)} \qquad \text{I.3}$$

$$J c_p T_2 + \tfrac{1}{2} u_2^2 = J c_p T_1 + \tfrac{1}{2} u_1^2 + J q \qquad \text{(energy)} \qquad \text{I.4}$$

where q is the heat input per unit mass of gas.

Now under normal wind tunnel conditions, q will be small compared with the energy of the air and the resulting changes in the other quantities may also be small. To the first order, solutions of the above equations become

$$p_2/p_1 = 1 + \frac{\gamma M_1^2}{M_1^2 - 1} Q_1 \qquad \text{I.5}$$

$$\frac{\rho_1}{\rho_2} = \frac{u_2}{u_1} = 1 - \frac{1}{M_1^2 - 1} Q_1 \qquad \text{I.6}$$

$$\frac{M_2}{M_1} = 1 - \frac{1}{2} \frac{1 + \gamma M_1^2}{M_1^2 - 1} Q_1 \qquad \text{I.7}$$

and

$$\frac{p_{o2}}{p_{o1}} = 1 - \frac{\gamma M_1^2}{2} Q_o \qquad \text{I.8}$$

where

$$Q_1 = \frac{q}{c_p T_1}$$

and

$$Q_o = \frac{q}{c_p T_{H1}}$$

with

$$T_{H1} = T_1 + \frac{u_1^2}{2 J c_p} \qquad \text{I.9}$$

(note that $\frac{T_{H2}}{T_{H1}} = 1 + Q_o$).

We now assume that equations I.5 to I.8 will give the changes in gas quantities across the condensation shock. If the shock occurs in the expanding portion of the nozzle and if we neglect its reflections, we may also calculate its approximate effect on the flow quantities in the working section, as follows.

Suppose the cross section of the nozzle is A_1 at the shock position and A in the working section. Then making the sweeping assumption that one-dimensional conditions apply, we have

$$\left.\begin{aligned} \rho_1 u_1 A_1 &= \rho u A \\ \rho_2 u_2 A_2 &= \rho_c u_c A \end{aligned}\right\} \qquad \text{I.10}$$

where subscript "c" denotes the disturbed conditions in the working section when a condensation shock is present in the nozzle. Assuming that isentropic flow conditions exist between A_1 and A, we find that

$$\frac{M_c}{M} = 1 - \frac{1}{2}\,\frac{\left(1+\gamma M_1^2\right)\left(1+\frac{\gamma-1}{2}M^2\right)}{M^2-1}\,Q_o \qquad \text{I.11}$$

or

$$\left(1-\frac{M_c}{M}\right) = \left(1-\frac{M_2}{M_1}\right)\frac{1+\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M_1^2}\,\frac{M_1^2-1}{M^2-1} \qquad \text{I.12}$$

Inspection of equation I.12 will show that for given M_1 and $M > M_1$

$$\left(1-\frac{M_c}{M}\right) < \left(1-\frac{M_2}{M_1}\right)$$

and its value decreases as M increases. Thus this crude argument would indicate that the overall effect in the working section of a condensation shock of given strength in the nozzle, decreases as the working section Mach number increases. Hence, for example, the humidity requirements at $M = 4$ may be less stringent than those at $M = 3$.

The effect on static pressure is given by

$$\frac{p_c}{p} = 1 + \tfrac{1}{2}Q_o\left[\frac{\gamma M^2}{M^2-1}\left(1+\gamma M_1^2\right) - \gamma M_1^2\right] \qquad \text{I.13}$$

and on pitot pressure by

$$\frac{p'_{oc}}{p'_o} = 1 + \tfrac{1}{2}Q_o\left[\frac{M^2-1}{M^2-\frac{\gamma-1}{2\gamma}}\left(1+\gamma M_1^2\right) - \gamma M_1^2\right] \qquad \text{I.14}$$

(neglecting any re-evaporation in front of the pitot tube).

Indicated Values

The above represent the true changes which would occur with this simplified model. In experimental work p_c, p'_{oc} and p_{o1} will be known, but not p_{o2}, so it is of interest to determine the "indicated" Mach number, M_i.

If determined from the ratio ${}^{p_c}/{}_{p_{o1}}$, it can be shown that

$$\left.\begin{aligned} \frac{M_i}{M} &= 1 - \varepsilon_i \\ \text{where} \qquad \varepsilon_i &= \frac{\left({}^{p_c}/{}_{p} - 1\right)\left(1 + \frac{\gamma - 1}{2} M^2\right)}{\gamma M^2} \end{aligned}\right\} \qquad \text{I.15}$$

where ${}^{p_c}/{}_{p}$ is given by equation I.13.

On the other hand if M_i is determined from the ratio ${}^{p'_{oc}}/{}_{p_{o1}}$ then

$$\left.\begin{aligned} \frac{M_{io}}{M} &= 1 - \varepsilon_{io} \\ \text{where} \qquad \varepsilon_{io} &= \frac{\left(\frac{p'_{oc}}{p'_o} - 1\right)\left(1 + \frac{\gamma - 1}{2} M^2\right)\left(M^2 - \frac{\gamma - 1}{2\gamma}\right)}{(M^2 - 1)^2} \end{aligned}\right\} \qquad \text{I.16}$$

Finally the true Mach number (M_c) should be given by $\left({}^{p'_{oc}}/{}_{p_c}\right)$, but additional errors may arise in the measured pitot pressure if the compression in front of the pitot tube causes some re-evaporation of the water present in the airstream. For this reason it might seem better to rely on static pressure measurements when evaluating humidity effects.

Values of Q

In previous experimental work at lower Mach numbers it has been found that supercooling takes place before the condensation shock and equations I.5 to I.8 would give reasonable estimates of pressure change etc., if the amount of vapour condensed is chosen to give saturation conditions after the shock.

Now

$$Q_o = \frac{q}{C_p \, T_{H1}} \qquad \left(Q_1 = \frac{q}{C_p \, T_1}\right)$$

where q is the heat input per unit mass of air. Thus

$$q = h \, n \, \Omega_o \qquad \text{I.17}$$

where h is latent heat

Ω_o is absolute humidity

and $o < n < 1$.

Assuming 45° supercooling and values of Ω_o between 5×10^{-5} and 10^{-3} (a range of humidities found in wind tunnel work) then application of the charts of Ref.1 shows that "n" is very nearly unity under these conditions. Hence in a high Mach number nozzle a second condensation shock is most unlikely to occur and the main uncertainty will arise from reflections of the original shock.

Values of M_1

The curves which have been drawn in Figs. 6a-6c were calculated from equations I.11, I.15 and I.16 assuming a supercooling of 45°C. For a stagnation temperature of 35°C, this amount of supercooling gives the following values of M_1 in terms of absolute humidity Ω_o,

$\Omega_o = 5 \times 10^{-5}$	10^{-4}	2	4	6	10^{-3}	3×10^{-3}
$M_1 =$ 1.96	1.86	1.80	1.70	1.65	1.59	1.43

The choice of 45° supercooling was quite arbitrary in the present case and different amounts would make a slight difference to the positions of the curves in Figs. 6a-6c. For example, a supercooling of 70°C would alter the "true" Mach numbers (equation I.11) by the amounts shown in the following table.

Effects of varying amounts of supercooling on "true" Mach number

M	2.48	3.25	4.0
(a) $\Omega_o = 1 \times 10^{-4}$			
$\left(1-\frac{M_c}{M}\right)$ 45° sc	0.0012	0.0009	0.0008
70° sc	0.0015	0.0012	0.0010
(b) $\Omega_o = 1 \times 10^{-3}$			
$\left(1-\frac{M_c}{M}\right)$ 45° sc	0.0091	0.0072	0.0061
70° sc	0.0119	0.0095	0.0080

Thus there is no change in the order of the "errors" and in view of the drastic assumptions made in formulating the theory and the scatter of the experimental results in Figs. 6a-6c, no conclusions can be drawn concerning probable amounts of supercooling in the nozzle.

Wt.2078. CP247.K3 - Printed in Great Britain.

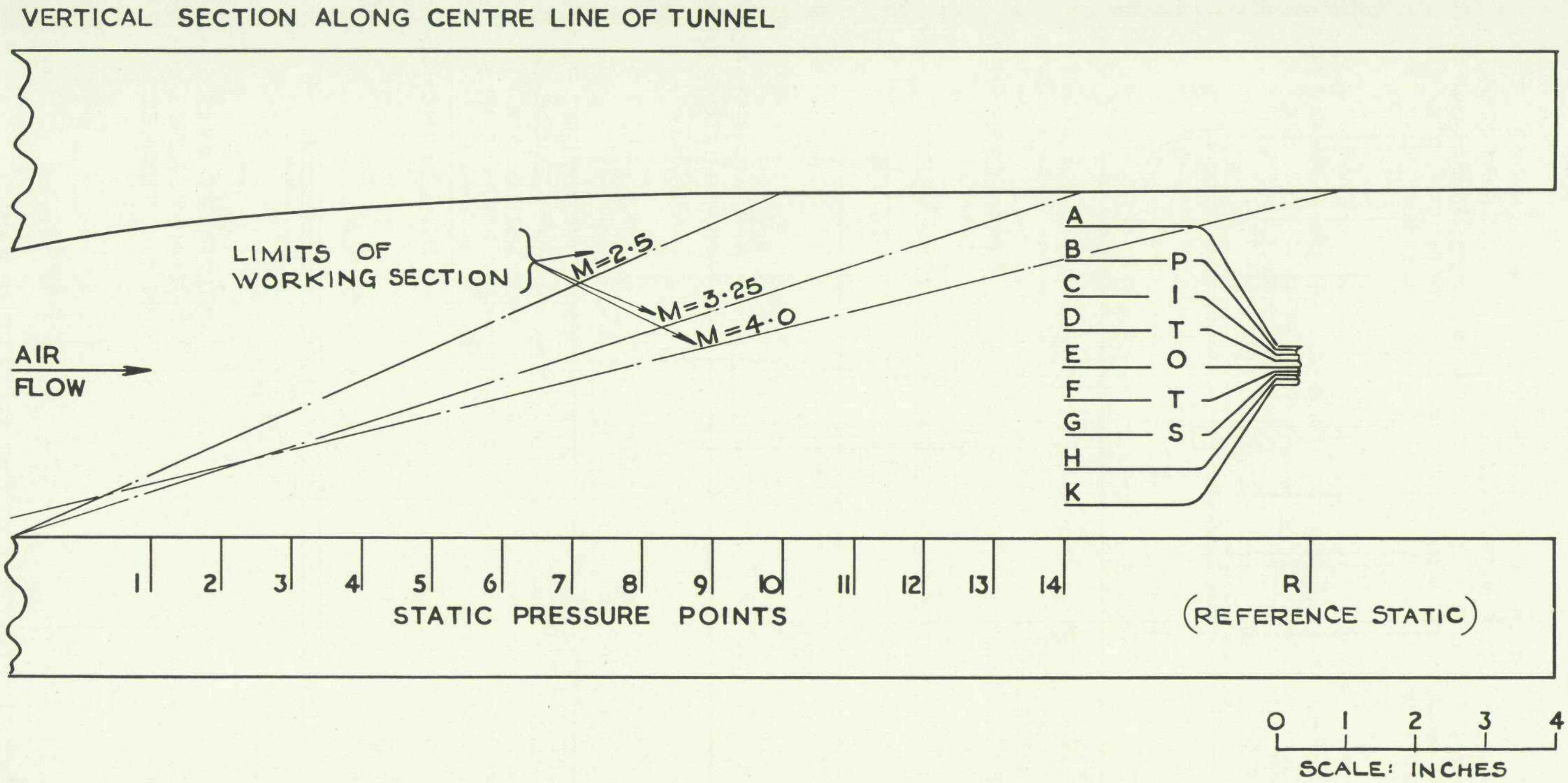


FIG.1. LOCATION OF PRESSURE MEASURING POINTS IN WORKING SECTION OF Nº5 SUPERSONIC WIND TUNNEL.

FIG.1.

FIG. 2.

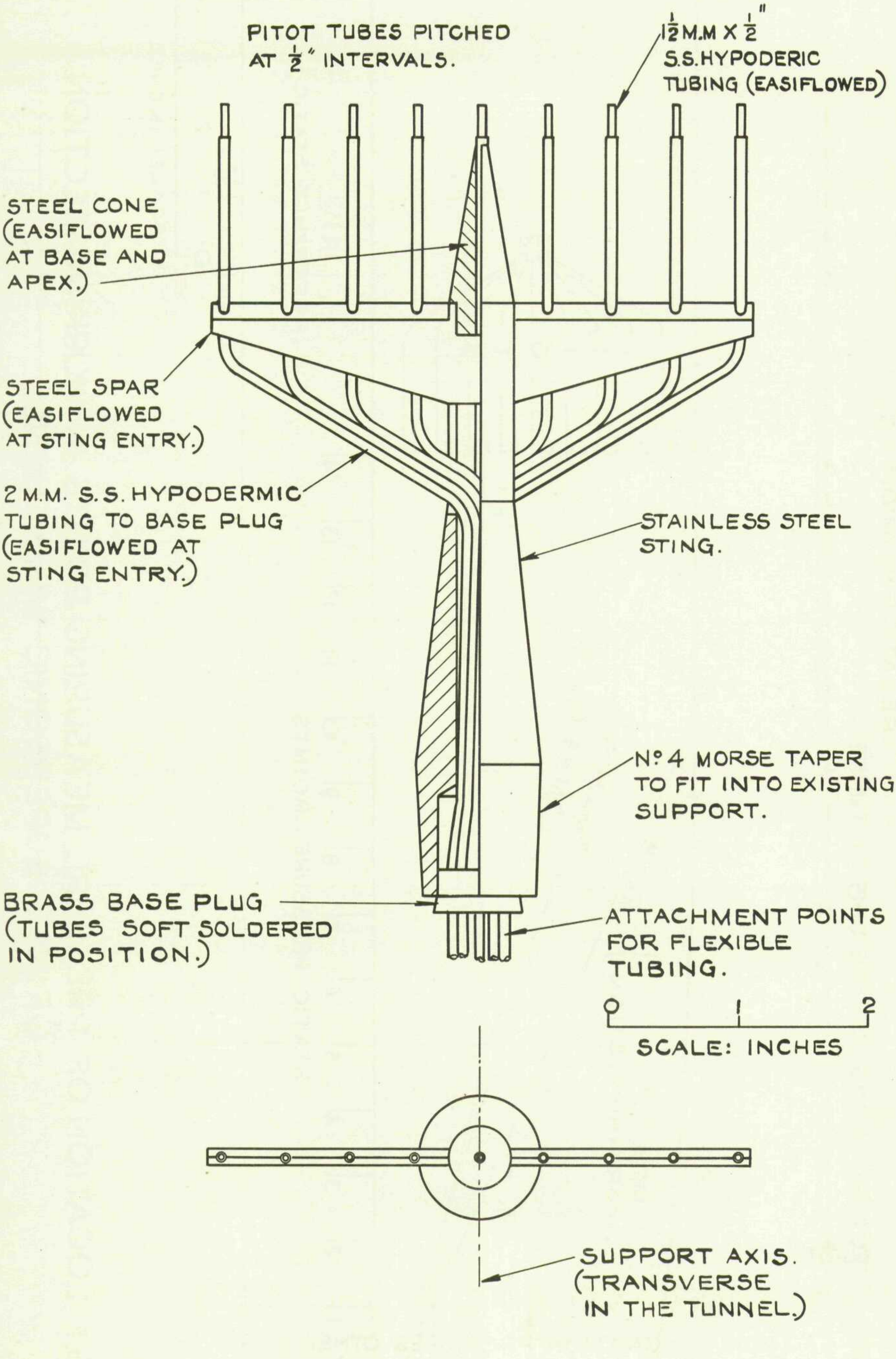


FIG. 2. DETAILS OF PITOT RAKE

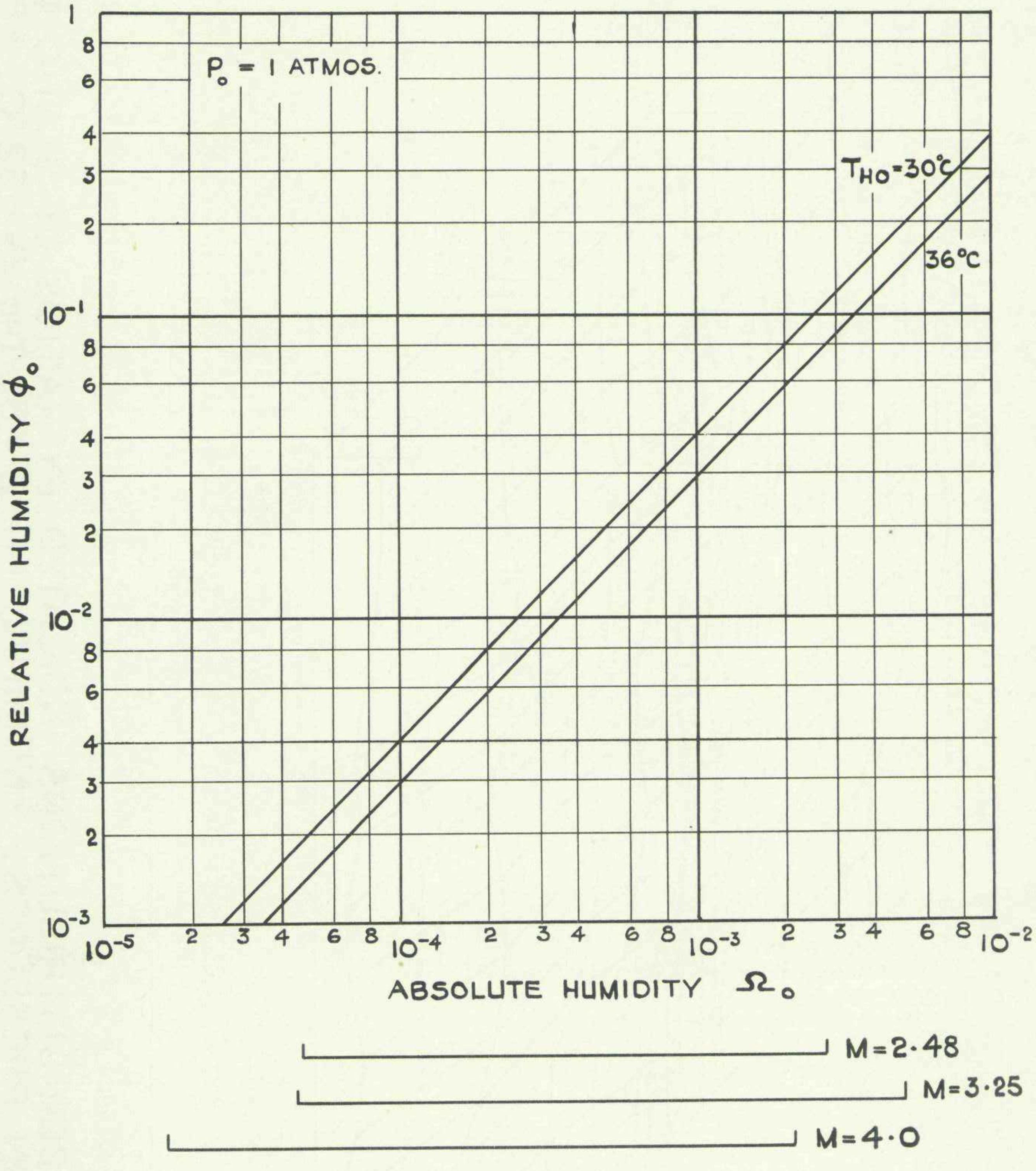


FIG.3. RANGES OF ABSOLUTE AND RELATIVE HUMIDITY COVERED BY THE TESTS.

(FOR STAGNATION PRESSURES OTHER THAN 1 ATMOS. $\phi_o \propto P_o$)

FIG.4a.

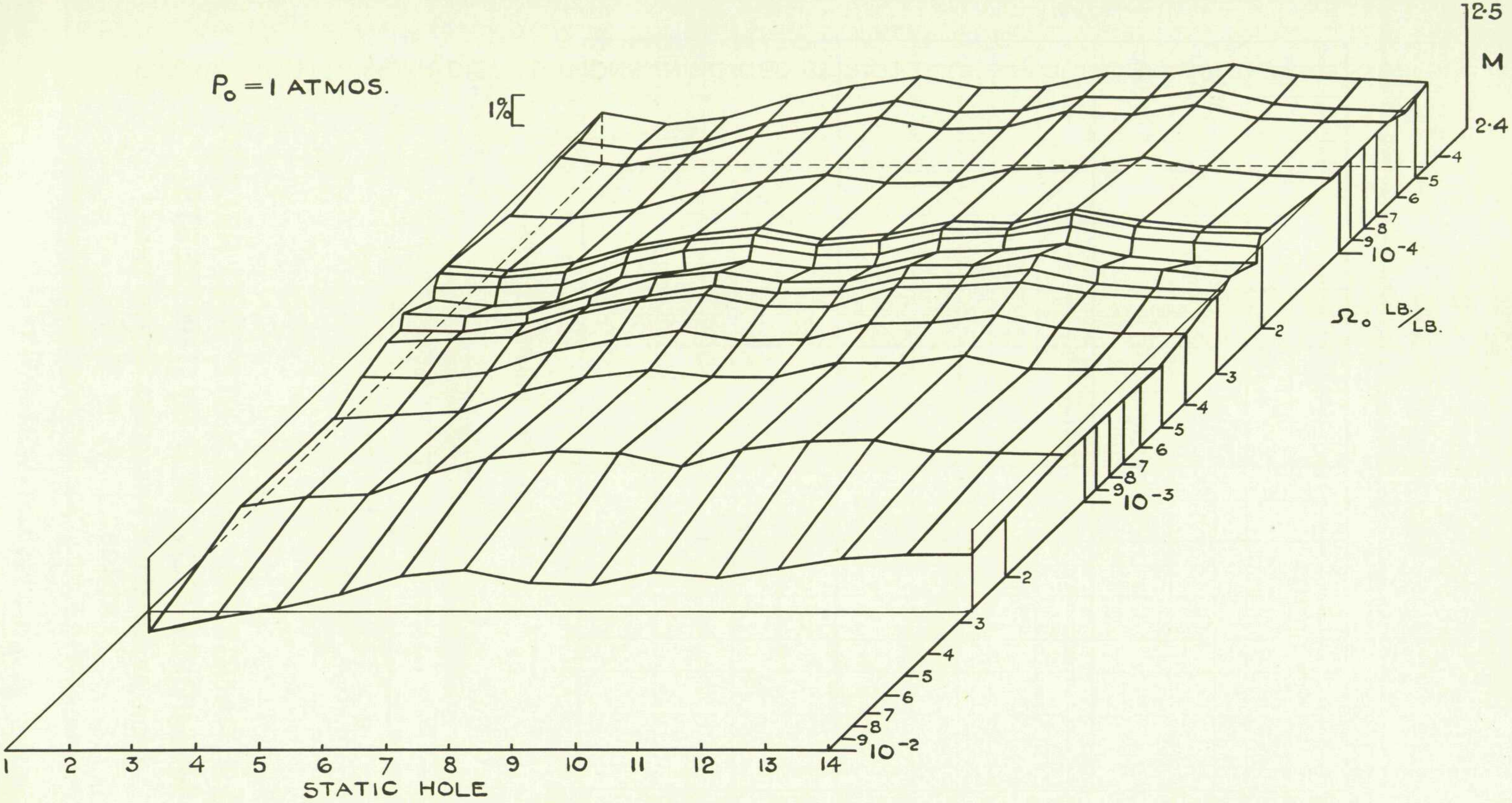


FIG. 4a. MACH NUMBER DISTRIBUTIONS INDICATED BY RATIOS OF STATIC TO STAGNATION PRESSURES. NOMINAL M = 2·48. THO ≏ 32°C.

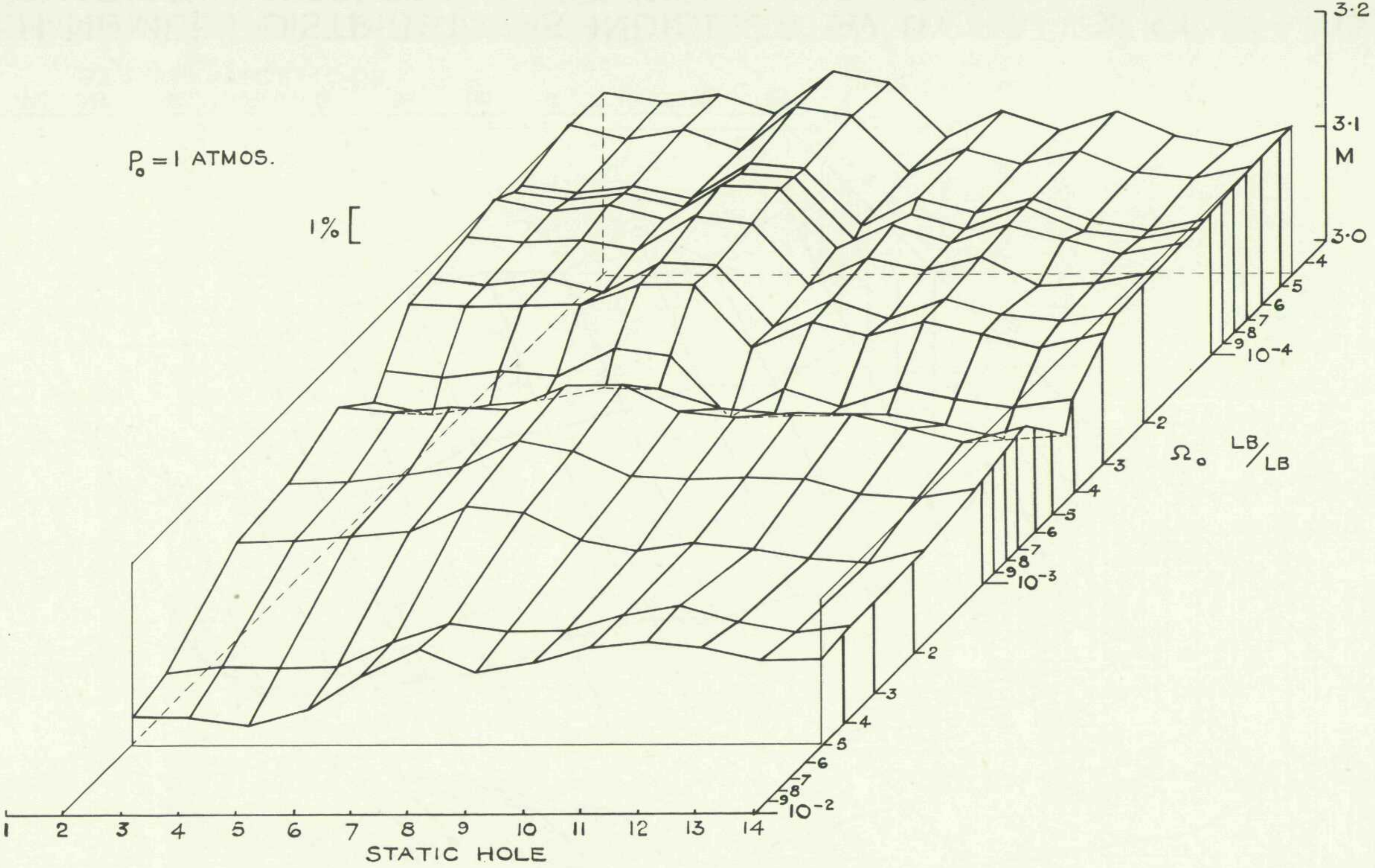


FIG.4b. MACH NUMBER DISTRIBUTIONS INDICATED BY RATIOS OF STATIC TO STAGNATION PRESSURES.
NOMINAL M=3·25 T_{HO} $\underline{\Omega}$ 35°C.

FIG.4 b.

FIG.4C.

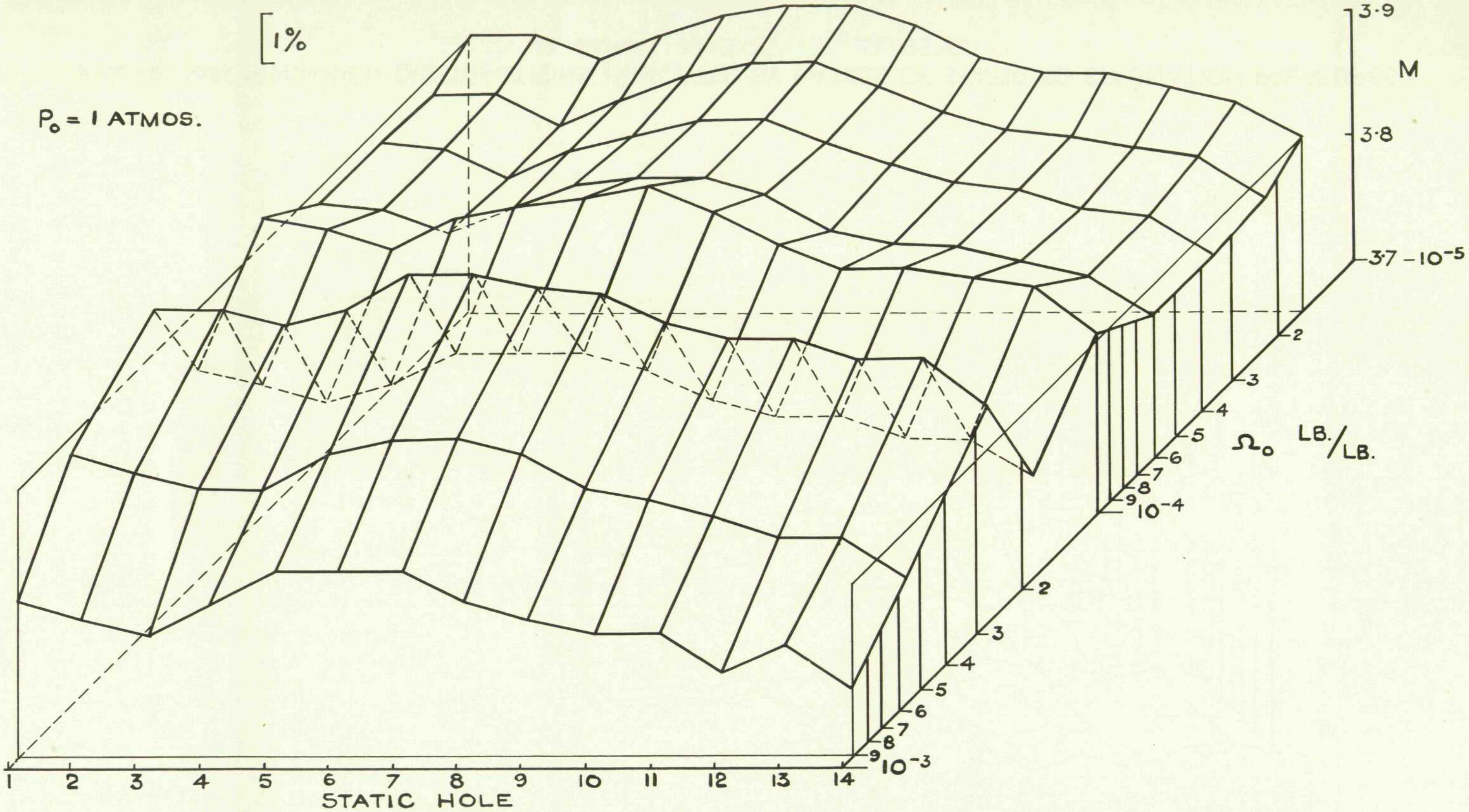


FIG. 4c. MACH NUMBER DISTRIBUTIONS INDICATED BY RATIOS OF STATIC TO STAGNATION PRESSURES. NOMINAL M = 4·0. T_{HO} ≏ 36°C.

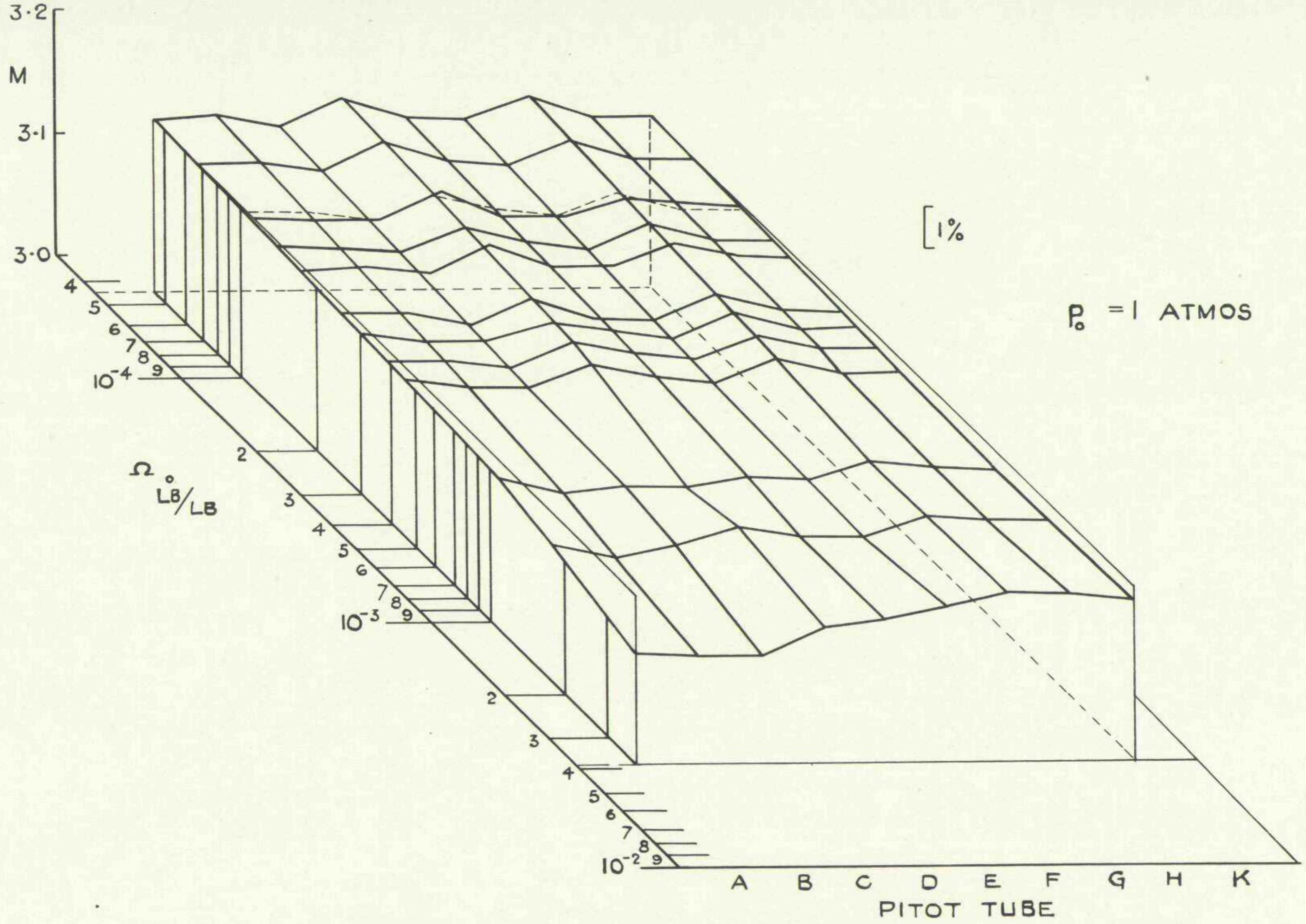


FIG.5a. MACH NUMBER DISTRIBUTIONS INDICATED BY RATIOS OF PITOT TO STAGNATION PRESSURES. NOMINAL M=3·25. T_{H0} Ω 35°C.

FIG.5b.

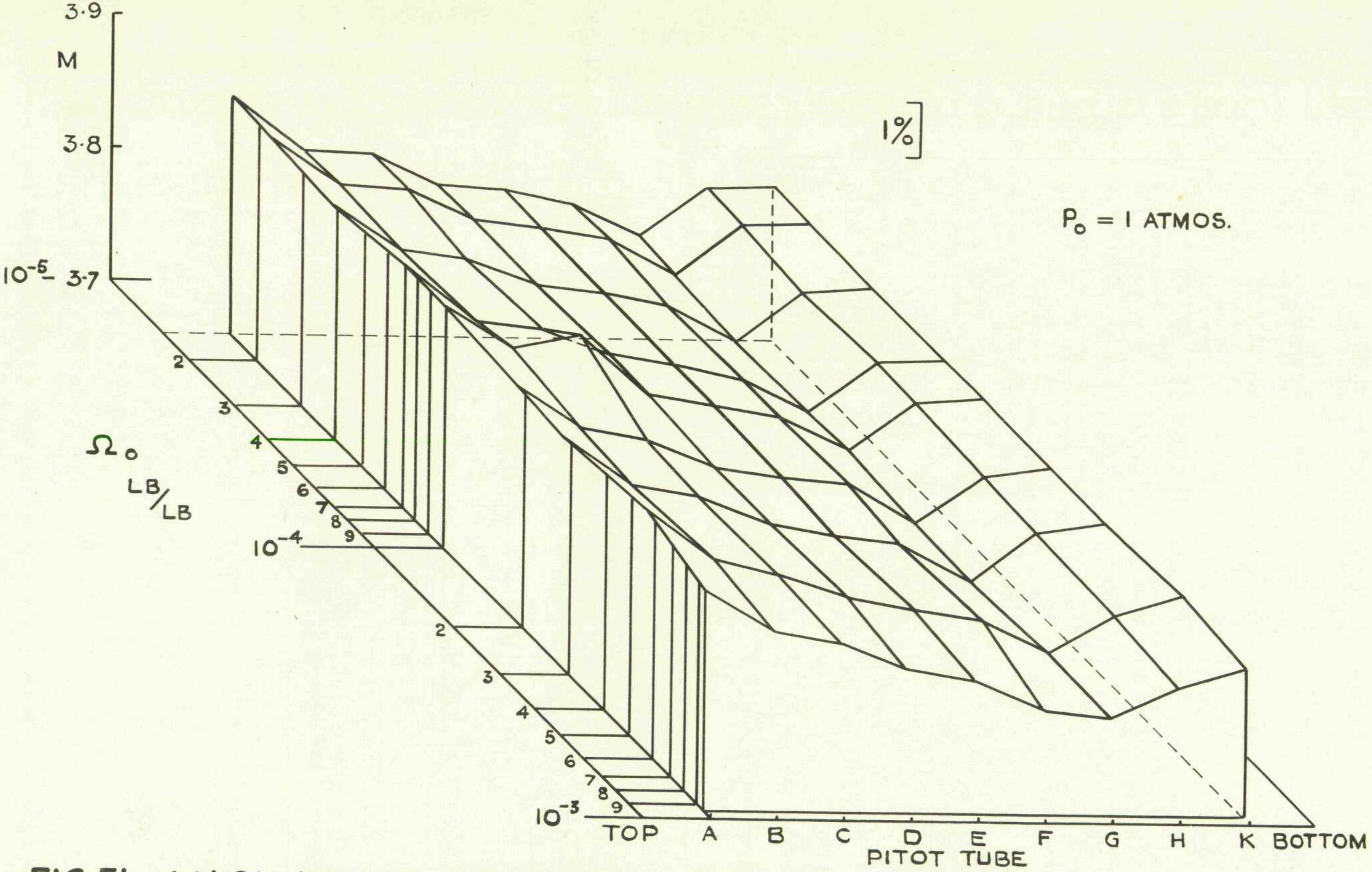
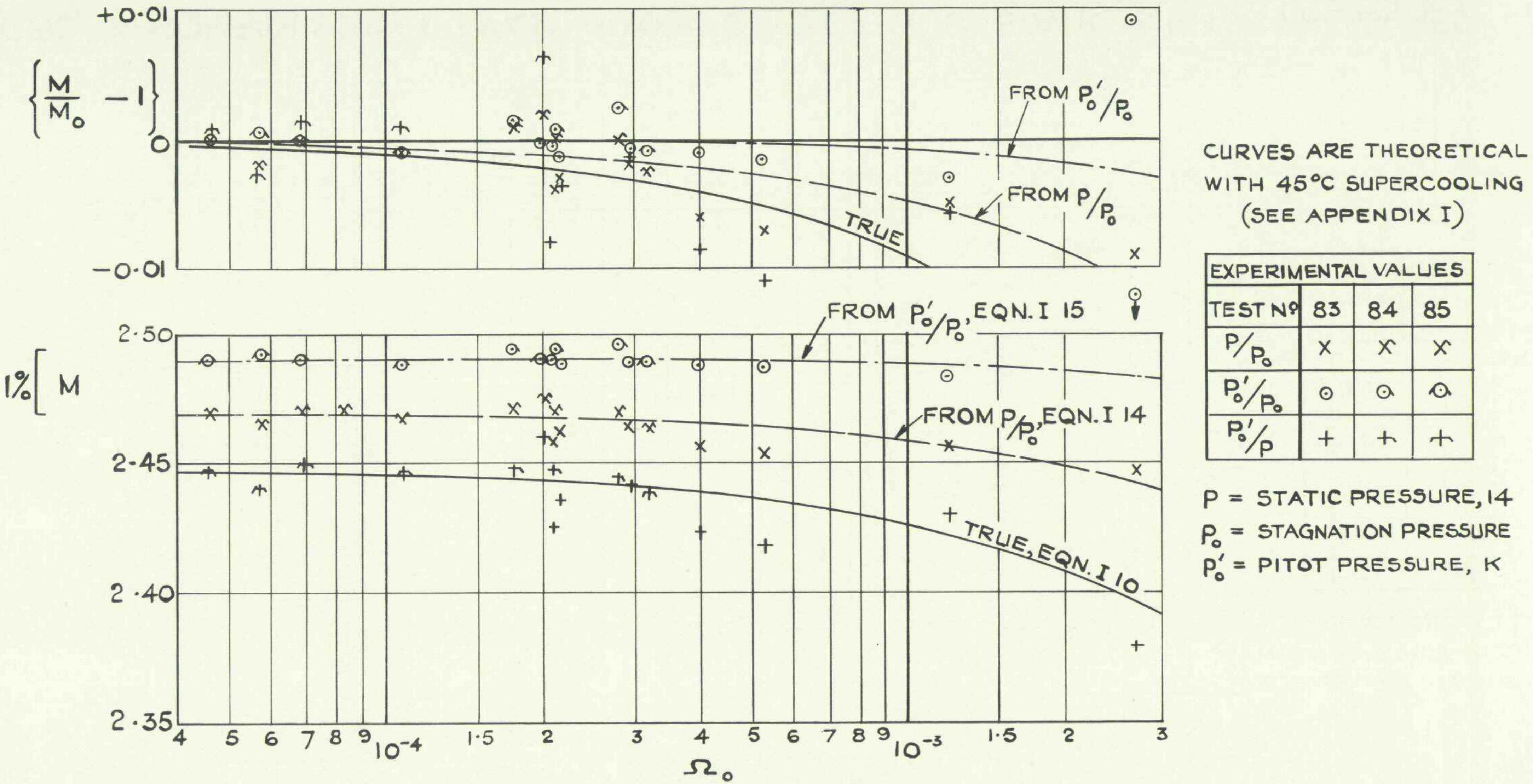


FIG.5b. MACH NUMBER DISTRIBUTIONS INDICATED BY RATIOS OF PITOT TO STAGNATION PRESSURES. NOMINAL $M = 4 \cdot 0$. THO $\underline{\Omega}$ 36 °C.



CURVES ARE THEORETICAL WITH 45°C SUPERCOOLING (SEE APPENDIX I)

EXPERIMENTAL VALUES			
TEST Nº	83	84	85
P/P_0	×	×	×
P_0'/P_0	⊙	⊙	⊙
P_0'/P	+	+	+

P = STATIC PRESSURE, 14
P_0 = STAGNATION PRESSURE
P_0' = PITOT PRESSURE, K

FIG.6a. COMPARISON OF MACH NUMBERS INDICATED BY STATIC & PITOT PRESSURES. $M = 2{\cdot}48$. $p_0 = 1$ ATMOS. $T_{HO} = 32$ °C.
(STATIC HOLE 14 & PITOT TUBE K, SEE FIG.1.)
(CURVES ARE FROM EQUATIONS I.11, I.15 & I.16 OF APPENDIX I)

FIG.6a.

FIG.6 b.

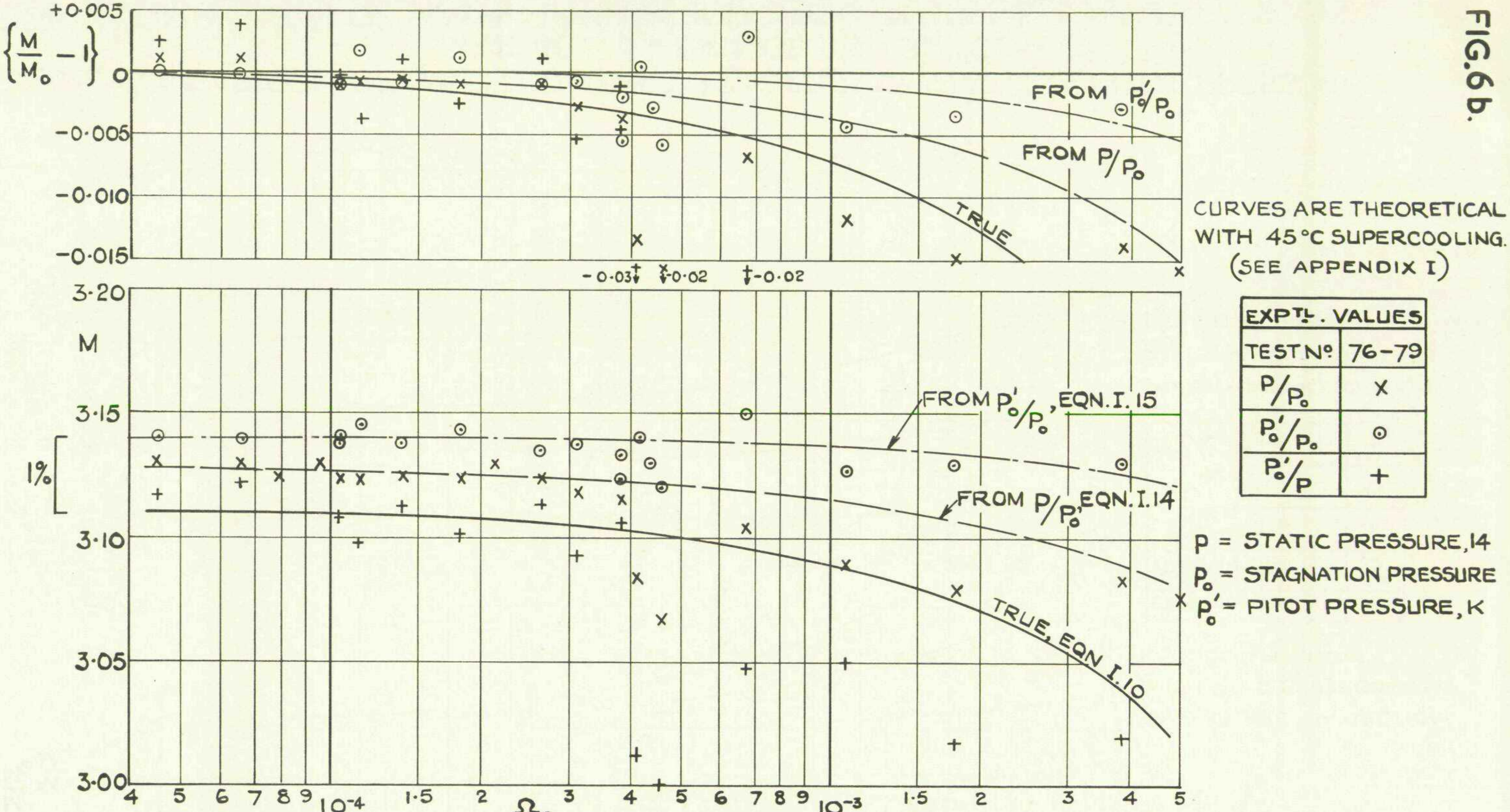


FIG.6 b. COMPARISON OF MACH NUMBERS INDICATED BY STATIC & PITOT PRESSURES.
$M=3\cdot 25$. $p_0 - 1$ ATMOS. $T_{H0} = 35$ °C.

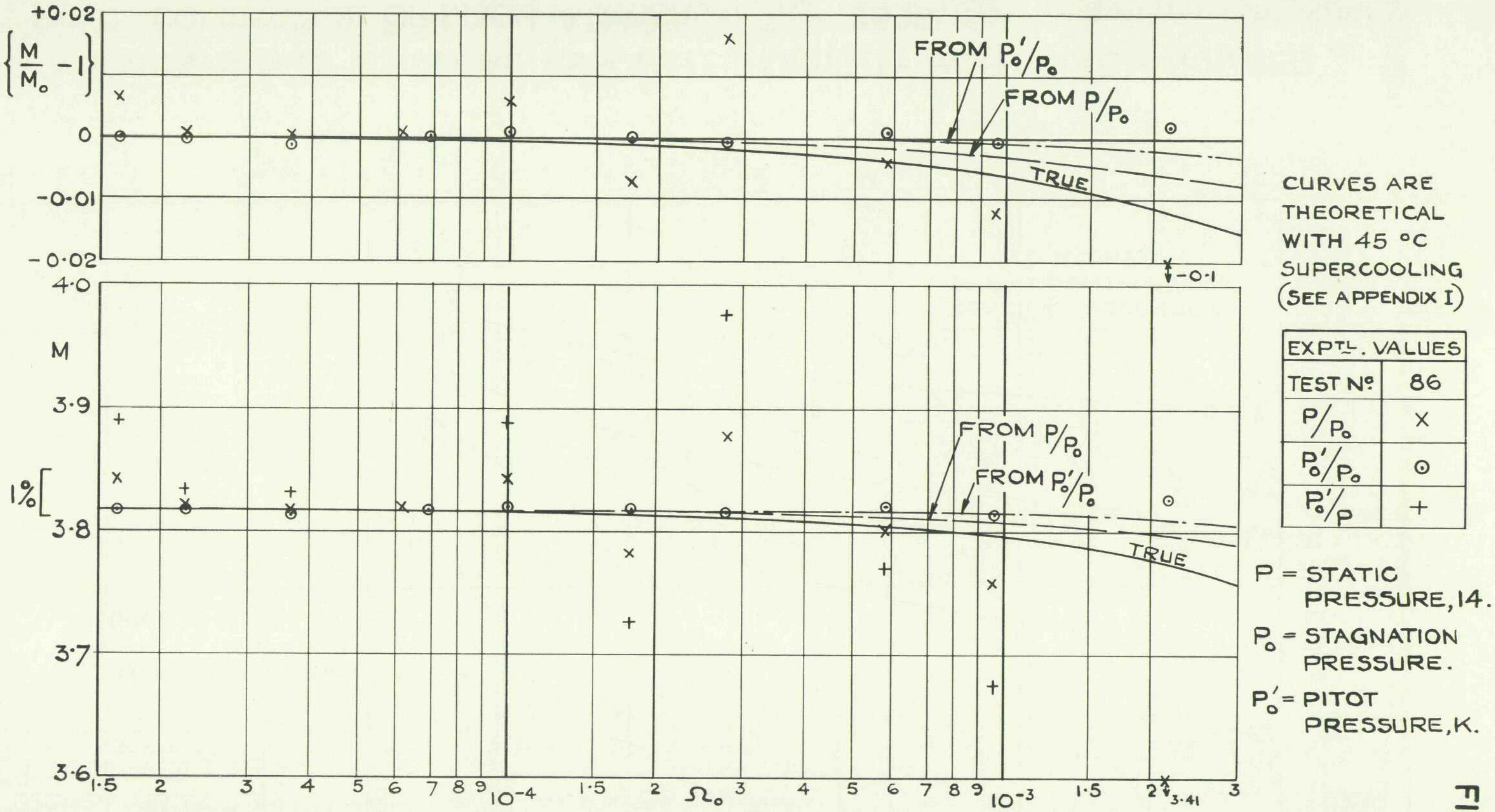


FIG.6c. COMPARISON OF MACH NUMBERS INDICATED BY STATIC & PITOT PRESSURES.
$M=4\cdot0$. $p_o=1$ ATMOS. $T_{HO}=36$ °C.

FIG.6c.

FIG. 7.

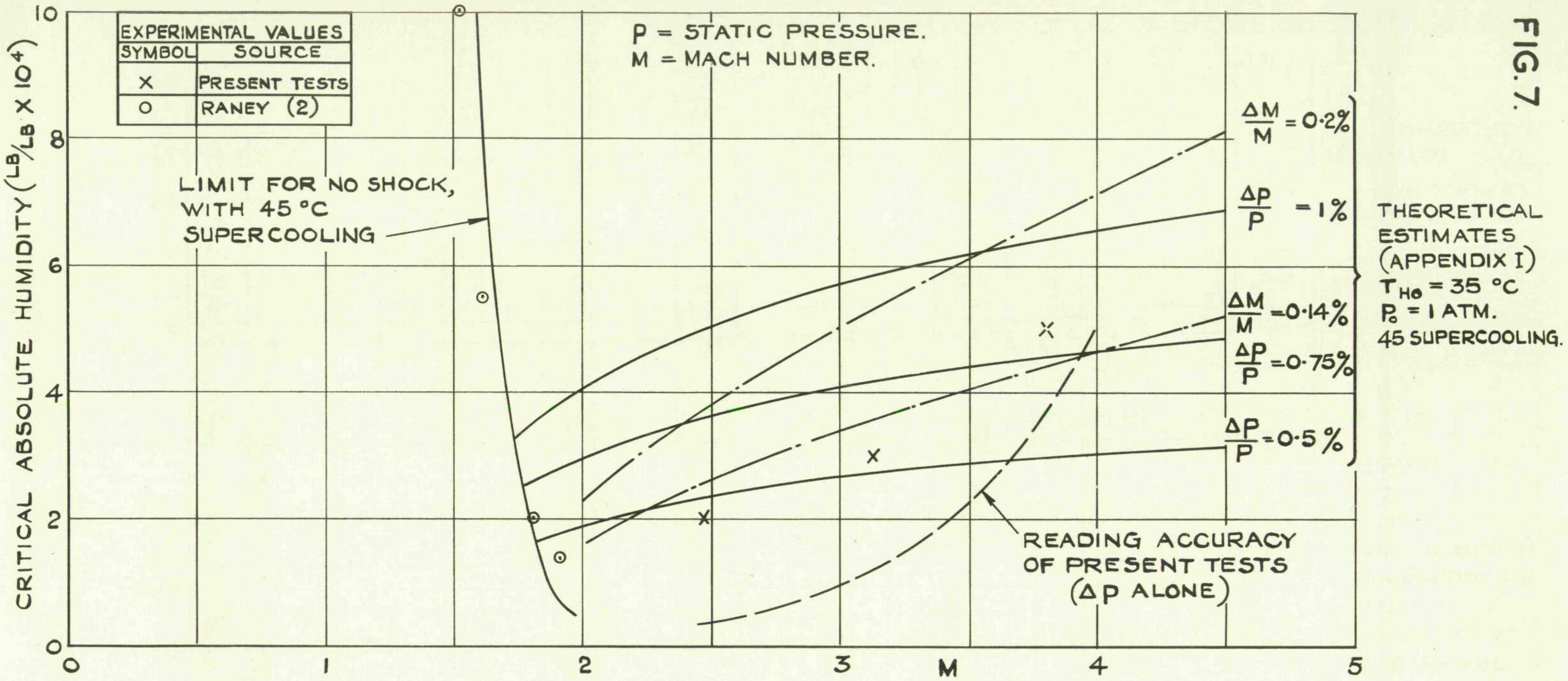
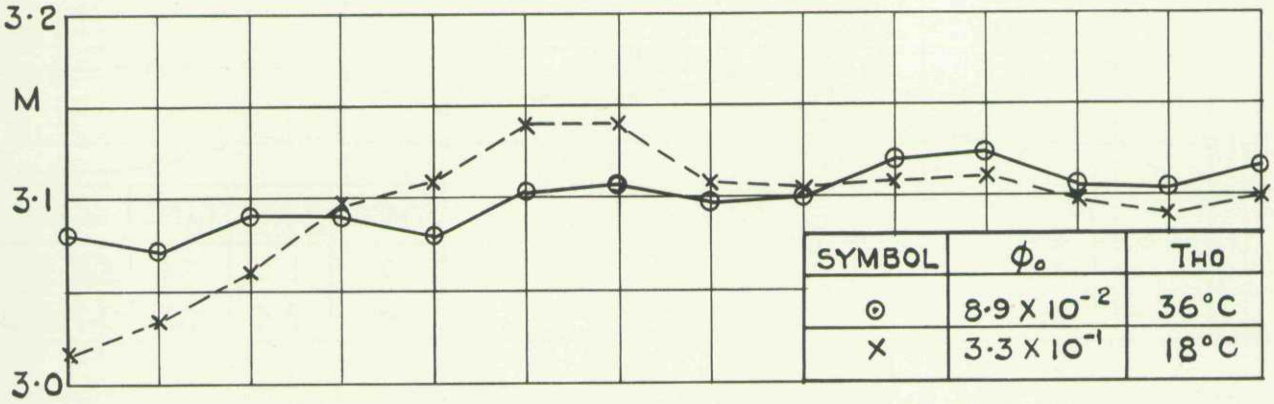
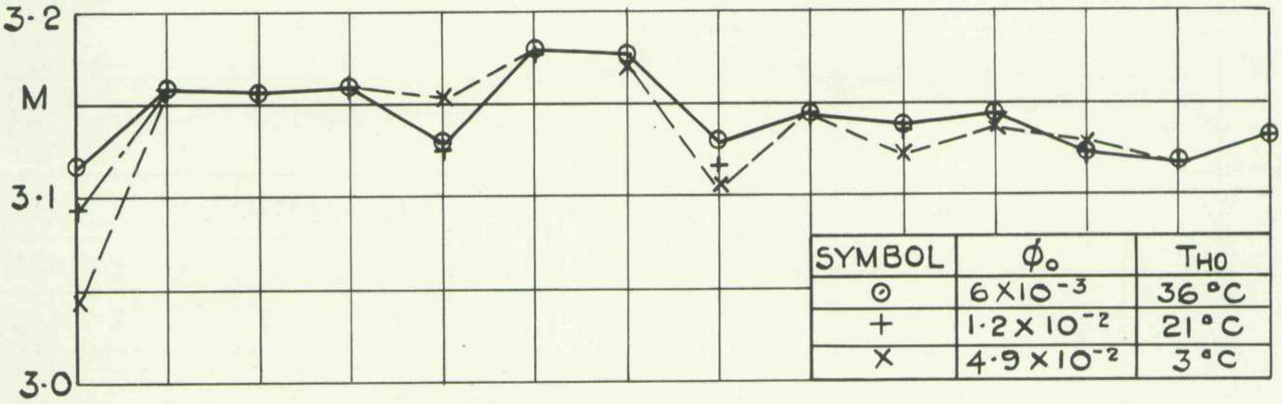


FIG. 7. VARIATION OF CRITICAL ABSOLUTE HUMIDITY WITH MACH NUMBER FOR p_o=1 ATMOS. & T_{HO}= 35 °C.

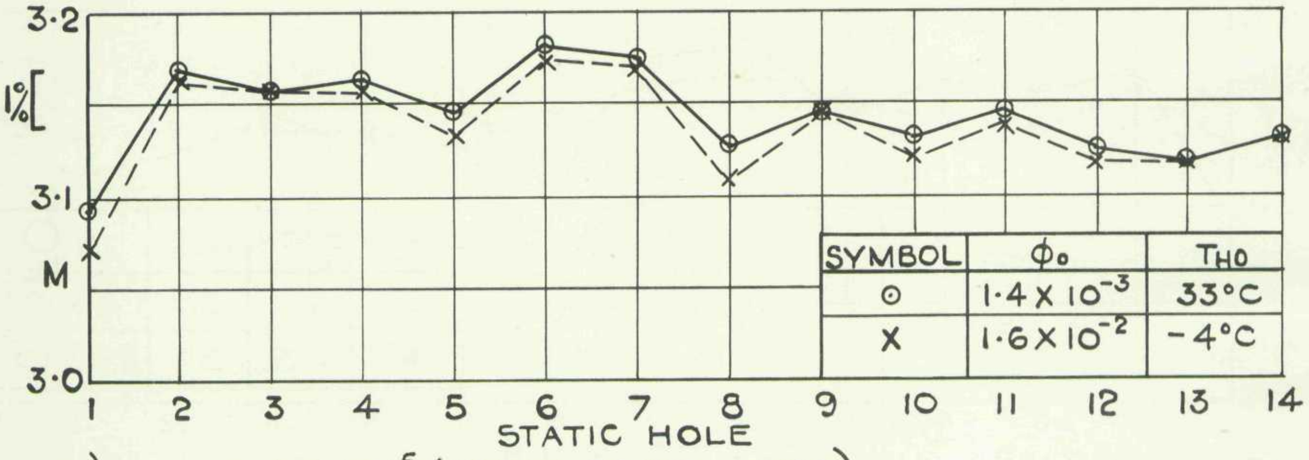
FIG. 8



a) $\Omega_o = 3 \cdot 8 \times 10^{-3}$ (FROST POINT 0°C.)



b) $\Omega_o = 2 \times 10^{-4}$ (FROST POINT -32° C.)



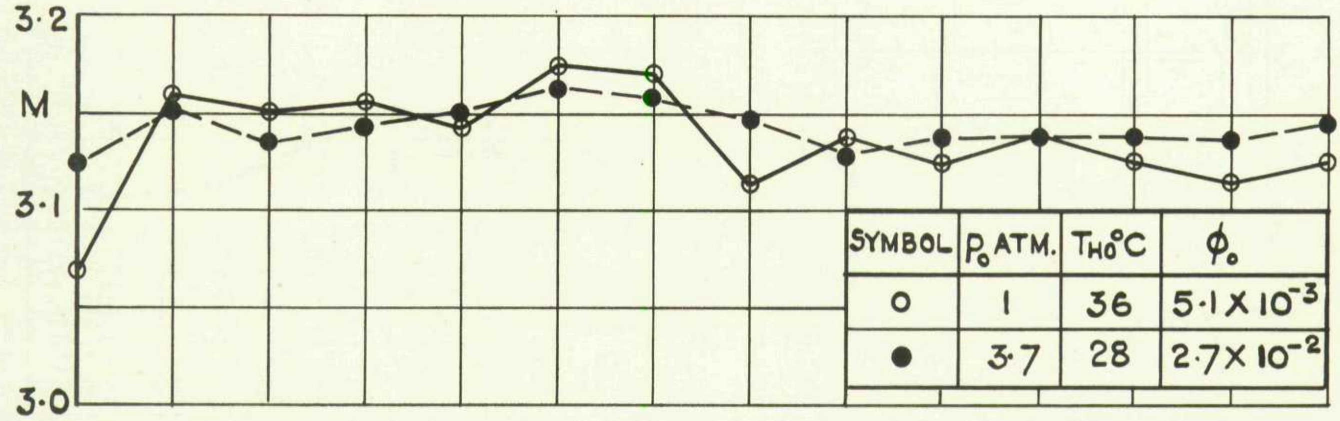
c) $\Omega_o = 4 \cdot 6 \times 10^{-5}$ (FROST POINT -45°C)

FIG.8. EFFECT ON INDICATED MACH NUMBER OF VARYING STAGNATION TEMPERATURE AT CONSTANT ABSOLUTE HUMIDITY(Ω_o). M = 3·25. p_o = 1 ATMOS.

FIG. 9

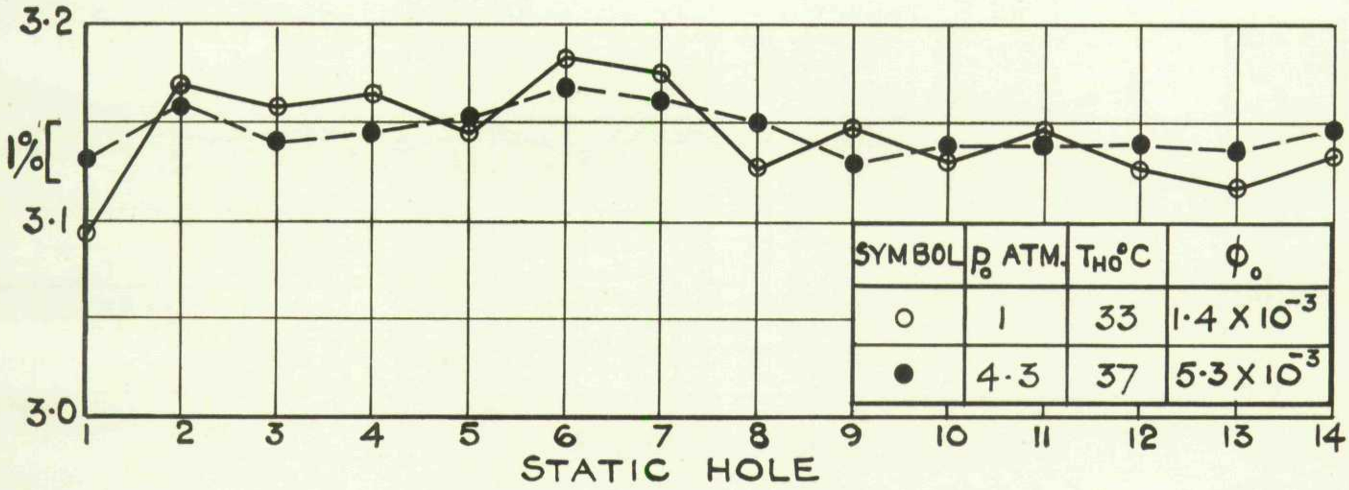
SYMBOL	P_o ATM.	T_{HO} °C	ϕ_o
○	1	34	$3 \cdot 5 \times 10^{-2}$
●	2·6	33	$9 \cdot 7 \times 10^{-2}$

a) $\Omega_o = 1 \cdot 1 \times 10^{-3}$



SYMBOL	P_o ATM.	T_{HO} °C	ϕ_o
○	1	36	$5 \cdot 1 \times 10^{-3}$
●	3·7	28	$2 \cdot 7 \times 10^{-2}$

b) $\Omega_o = 1 \cdot 75 \times 10^{-4}$

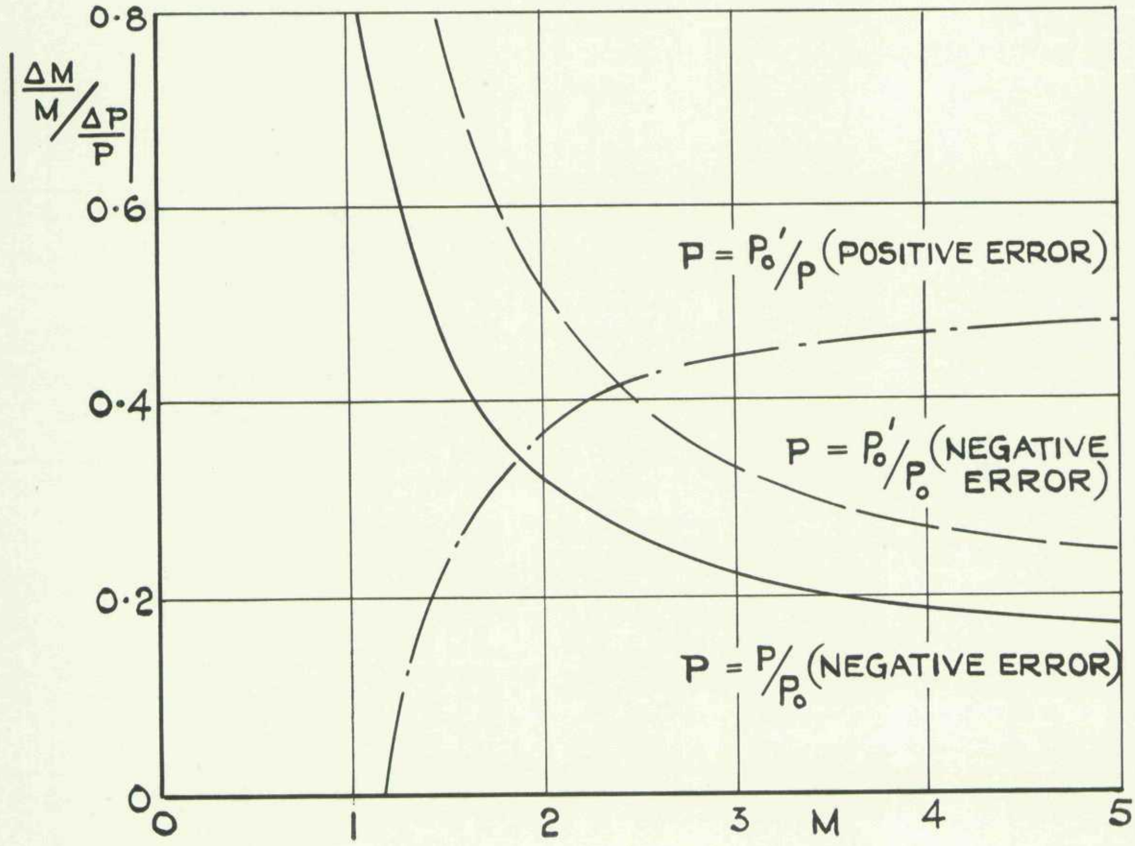


SYMBOL	P_o ATM.	T_{HO} °C	ϕ_o
○	1	33	$1 \cdot 4 \times 10^{-3}$
●	4·3	37	$5 \cdot 3 \times 10^{-3}$

c) $\Omega_o = 4 \cdot 6 \times 10^{-5}$

FIG.9. EFFECT ON INDICATED MACH NUMBER OF VARYING STAGNATION PRESSURE AT CONSTANT ABSOLUTE HUMIDITY. M=3·25.

FIG.10.



p = STATIC PRESSURE.

p_o = STAGNATION PRESSURE.

p_o' = PITOT PRESSURE.

FIG.10. ERRORS IN CALCULATED MACH NUMBER ASSOCIATED WITH ERRORS IN PRESSURE MEASUREMENT.

C.P. No. 247
(17,568)
A.R.C. Technical Report

Published by
HER MAJESTY'S STATIONERY OFFICE

To be purchased from
York House, Kingsway, London W.C.2
423 Oxford Street, London W.1
P.O. Box 569, London S.E.1
13A Castle Street, Edinburgh 2
109 St. Mary Street, Cardiff
39 King Street, Manchester 2
Tower Lane, Bristol 1
2 Edmund Street, Birmingham 3
80 Chichester Street, Belfast
or through any bookseller

PRINTED IN GREAT BRITAIN

S.O. Code No. 23-9009-47

Wt.2078.K3

C.P. No. 247