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Pressure-cabin Design

A Discussion of some of the Structural Problems involved, with suggestions for their solution

By

D. Williams, D.Sc., M.I.Mech.E., F R.Ae.S.

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ADDENDUM

A constructional scheme for making pressure cabins immune from catastrophic failure without undue weight penalty

It seems desirable to make some reference here to a constructional scheme put forward by the writer in a sequel to the present report*. This scheme has its origin in paragraph 6 of Appendix D of the present report, where it is shown (see Fig.2D) that when the former rings are reduced in pitch to 10 in. i.e. about half the conventional pitch, they effectively limit the radial swelling of the inter-ring skin-stringer wall to that suffered by the rings themselves. In other words, the rings reduce the hoop stress in the skin almost as effectively as if the material in the rings were incorporated in the skin to increase its thickness.

In the actual scheme described in the sequel the conventional former rings are retained and the shell wall fitted with flat hoops directly attached to the skin and pitched some 10 in. apart. These hoops have a thickness about four times that of the skin and a width of about 2 in. so that they have a total cross-sectional area (for taking hoop tension) nearly equal to that of the skin itself. They pay for their own weight however by reducing, as explained above, the hoop-tension taken by the skin. Not being integral with the skin they should act as potent barriers to any crack that may start in between a pair of hoops.

The scheme is described at length in the sequel report above mentioned, and experimental work is in hand to prove that the advantages indicated by theory are borne out in practice.

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* R.A.E. Technical Note No. Structures 156 "A constructional method for minimising the hazard of catastrophic failure in a pressure-cabin". March 1955.

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Technical Note No. Structures 155

March, 1955

ROYAL AIRCRAFT ESTABLISHMENT

Pressure-cabin design - A Discussion of some of the Structural Problems Involved, with suggestions for their solution

by

D. Williams, D.Sc., M.I.Mech.E., F.R.Ae.S

SUMMARY

The problems that arise in the design of a pressure cabin are almost entirely due to the numerous structural discontinuities that inevitably break up the otherwise smooth distribution of stress. Some of the main discontinuities, such as are caused by windows, doors, canopies, floors, formers, bulkheads and domes, are discussed here, and design suggestions are made for dealing with them.

Particular attention is called to Appendix D and the Addendum based on it, which refers to a scheme for making pressure cabins safe against catastrophic failure with little weight penalty.

LIST OF CONTENTS

LIST OF APPENDICES

LIST OF APPENDICES (Cont'd.)

1 Introduction

As pressure cabins have only recently come into use it is not surprising that no generally accepted methods for their structural design have so far been developed. It is obvious at once that, with a pressure vessel having the approximate form of a surface of revolution, and a ratio of cross-sectional radius to skin thickness of something like 2000, membrane theory is applicable. There is therefore a tendency to assume that, since membrane theory is well-known to be particularly simple and straightforward, there are no serious structural problems to worry the designer. A little thought however soon disposes of such an assumption, and pressure cabins are seen to require - more than most structures - the utmost care on the part of the designer in balancing the many conflicting factors that enter into their construction.

What gives rise to most of the problems is the necessity for introducing discontinuities and constraints such as are caused by windows, doors, canopies, floors, bulkheads and domes. These interferences break up the smooth distribution of membrane stresses in the skin, and tend to cause stress concentrations that reduce both the static strength and the fatigue life of the structure. The main task of the designer is therefore to minimise the stress concentrations by preserving as far as possible the original membrane stress distribution associated with the unbroken skin.

That it is eminently worthwhile to go to some trouble in seeking an optimum construction is well illustrated by quoting hypothetical figures for a typical size of cabin. In a cabin with a main section 10 ft in diameter and walls of 20 gauge (0.036 in.) the maximum stress is that associated with the hoop tension in the main section, and has a value of 16,600 lb/in^2 (approximately) for an internal operating pressure of 10 lb/in. For an ultimate stress in the sheet of 60,000 lb/in^2 the theoretical maximum pressure is therefore 36 lb/in^2. The introduction of a multiplicity of discontinuities must inevitably weaken the structure, but the maintenance, by good design, of a failing pressure of 30 lb/in^2 would not appear an unreasonable target to aim at in spite of discontinuities. Suppose however that the figure actually achieved is 20 lb/in^2. The upshot is that, for the latter figure, the cabin is loaded to half its ultimate load at every flight, whereas for the target figure it is loaded only to one-third of its ultimate. Having regard to the shape of the typical S-N curve, the value of such a reduction in the ratio of working to ultimate load is obvious, increasing the working life, as it does, some ten-fold.

The purpose of the present paper is to examine some of the salient structural problems, to discuss the design difficulties they give rise to, and to suggest possible ways of overcoming those difficulties. Among the problems touched upon are those connected with:-

(1) The design of frames for windows and similar openings.

(2) The design of doors.

(3) The design of the canopy.

(4) Interference or constraints caused by bulkheads and transverse frames.

(5) The design of pressure domes.

(6) Constraints caused by the main floor and its supports.

They are all treated on the basis of existing shell theory or simple deductions therefrom, and extensive use has been made of the formulae and tables contained in Timoshenko's well known treatise 'Theory of Plates and Shells' (Ref.1).

2 Membrane forces in a pressure cabin regarded as an unbroken surface of revolution

If, as seems legitimate, we regard a pressure cabin as a surface of revolution, and if, further, we assume that only membrane forces are operative, the distribution of these forces is very simple and readily visualised.

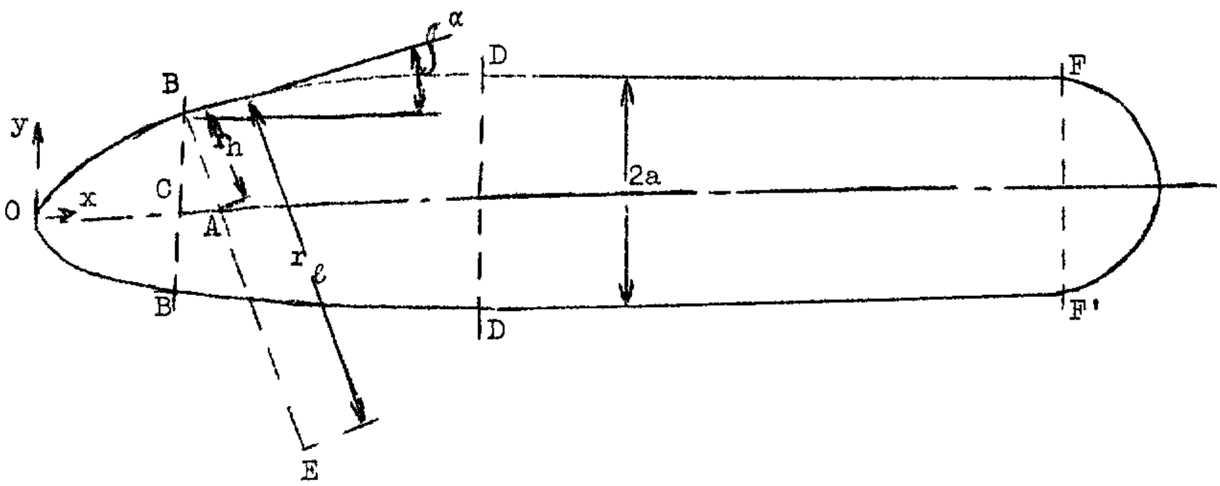

Fig. A

The cabin shown in outline in Fig. A consists essentially of an approximately cylindrical part DF, a rounded rear part and a streamlined forward part. Every element of surface such as that at B, for example, is in equilibrium under the outwardly directed normal force due to the internal pressure and the inwardly directed components of the membrane forces brought into action by the two curvatures - lateral and longitudinal. As mentioned in Appendix C, the centre of lateral curvature must lie on the longitudinal axis, and for the element of area at B, is therefore the point A where the normal cuts the axis. The longitudinal radius of curvature, while coinciding with the same normal, has a length BE depending on the shape of the cabin contour in side view, being infinite at sections aft of DD'. If

T_h = membrane hoop tension

T_ℓ = longitudinal tension

r_h = lateral (or hoop) radius of curvature

r_ℓ = longitudinal or meridianal radius of curvature

p = internal pressure,

we have the standard formula connecting pressure and membrane forces

$$\frac{T_h}{r_h} + \frac{T_\ell}{r_\ell} = p \qquad (1)$$

For a given longitudinal cross-section such as Fig.A, the value of T_ℓ is at once obtained from simple equilibrium considerations (Appendix C), whence knowing r_h and r_ℓ we can write down the hoop tension T_h.

For small variations in the shape of the longitudinal section, T_ℓ varies little, but its contribution to the local containment of the pressure is sensitive to r_ℓ, being nothing over the parallel part of the section and becoming progressively more important as the extreme nose is approached. It follows at once that, so far as resisting the internal pressure is concerned, a cabin having the shape of Fig. A can have its skin thickness progressively reduced from D to O without loss of strength. If, for example, the nose takes the form of a spherical cap with a radius 1/3 that of the main parallel section, the maximum membrane force at the nose is obviously only 1/6 that of the hoop tension aft of DD'. Under simple membrane force, therefore, the skin in the nose region will have either the same strength as the main section at 1/6 the thickness, or 6 times the strength for the same thickness.

It is a fortunate fact that, for reasons of visibility, the canopy is inevitably located well forward in a nose region where the maximum membrane tension is little more than half that in the main cabin. If therefore the same skin thickness is used in the two locations, any loss of strength due to the discontinuity introduced by the canopy is well covered.

The above remarks apply to the unbroken cabin shell: the remainder of this report is concerned with discussing the effects of various kinds of discontinuities.

3 Main-cabin discontinuities - windows etc.

Windows and doors are a principal source of discontinuities in the main cabin, and the designer's problem is to neutralise the rise in stress level that tends to occur whenever a hole is cut in the cabin shell, and the material so removed is replaced by a non-stress-transmitting window-panel. The ideal to aim at is to choose a window frame of such a shape and frame section as to cause no change of stress in the shell outside the frame. A hole with this kind of reinforcement round its margin has been referred to as a 'neutral hole'. The theory of neutral holes has been discussed by Gurney[2] and more recently by Mansfield[3], who has derived some elegant solutions based on the stress-function defining the stress distribution in the uncut plate. The simplest case is that in which the two principal stresses in the plate are equal. Under these conditions it is possible to deduce at once by purely elementary considerations (see Appendix A) that a hole, to be neutral, must be circular in shape and reinforced by a ring of constant cross-sectional area. That area, moreover, must be such that the weight of the ring (of the same material as the plate) is about $2/(1 - \nu)$ times that of the disc it replaces, where ν stands for Poisson's Ratio.

If the principal stresses are unequal, but still of the same sign, the neutral hole becomes an ellipse, with its major axis in the direction of the greater principal stress. The reinforcing elliptical 'ring' is no longer of constant cross-section but has a maximum and minimum at the ends of the major and minor axes respectively.

3.1 Importance of cross-sectional shape of reinforcing member

In the theoretical treatment of neutral holes the problem is considered solved once the shape of the hole and the cross-sectional variation of the boundary member are specified in terms of the thickness of the sheet and the ratio of the principal stresses. It is possible, however, while still nominally satisfying these conditions, to lose much of the expected beneficial effects by indifferent practical design.

As an example, consider again the simple case of equal principal stresses for which the neutral hole is a circle and the reinforcing member a circular ring of constant cross-section.

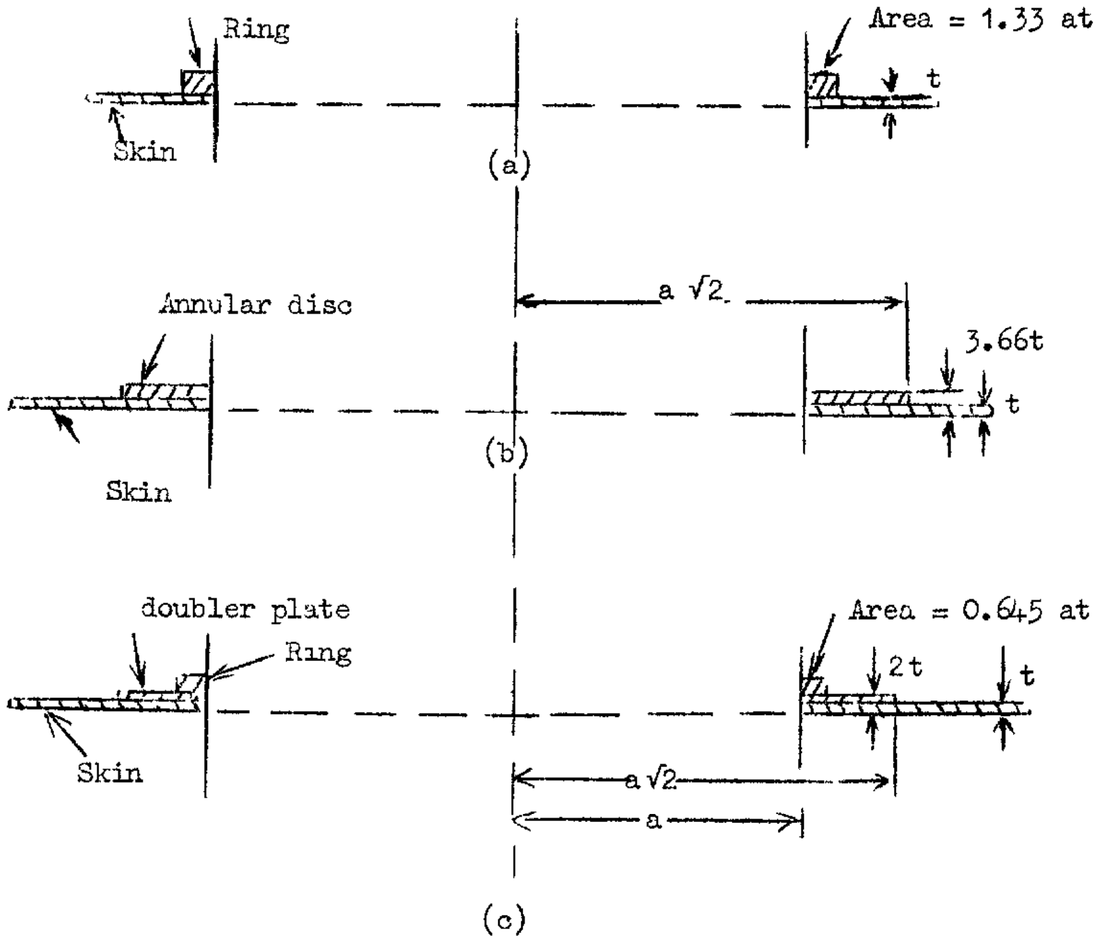

Fig. 1

Figs. 1(a), (b) and (c) show various ways of reinforcing the edge of a circular hole of radius 'a' so as to make the hole neutral. In 1(a) the reinforcement is all concentrated at the edge; in 1(b) it is spread out as an annular disc, and in 1(c) it takes the form of an annular doubler plate plus an inner ring. Scheme 1(a) hardly lends itself to practical design. Scheme 1(b) is better from this point of view, but entails a circumferential stress at the inner edge some 10% above the uniform stress in the outer plate. The sudden and considerable change of section at the outer edge of the reinforcing annular disc is also a disadvantage. A still better scheme is 1(c) where the reinforcement takes the form of an annular disc equal in thickness to the outer skin, supplemented by an inner ring at the edge of the hole.

It may be of interest to quote the values of the radial and circumferential stresses σ_r and σ_θ respectively at the outer and inner edges of the doubler plate and at the edge of the hole assuming σ_o to be the uniform tensile stress in the outer sheet.

	σ_r	σ_θ
Outer edge	$0.5\,\sigma_o$	$0.87\,\sigma_o$
Inner edge	0	$1.05\,\sigma_o$
Edge ring	-	$0.98\,\sigma_c$

From these results, which are deduced from a simple extension of standard formulae (see Appendix B), it is seen that the greatest stress is no more than 5% above that of the uniform sheet stress.

In scheme 1(c) the doubler plate performs the useful function of minimising the otherwise sudden change of radial stress at the skin-to-ring junction, and at the same time constitutes a more flexible type of connection.

Another advantage is that it provides a slightly greater stiffness for a given weight than the simple ring of Fig.1(a). This is because the circumferential stiffness of an annular disc is magnified by the Poisson's Ratio effect of the accompanying radial tension.

It will be noted that owing to the fitting of the edge-reinforcing member on the inside surface only of the skin, rather than symmetrically on both sides, an undesirable couple is introduced by the offset pulls which tends to bend the edge of the hole outwards. Since external reinforcement is hardly acceptable, it is clearly advisable to reduce the offset to a minimum. One way of countering this kind of bending moment is to 'build-in' (i.e. encastre) the edge of the window pane - a device that would also reduce the bending moment in the central region of the pane.

3.2 Ineffective types of reinforcing ring

A type of reinforcing ring that is of little use is that shown in Fig.2.

Fig.2

Here the effective part of the ring consists of little more than the flange AB, the greater part of the upright portion BC of the ring section being rendered useless by the bell-mouthing tendency at B under the radial load.

3.3 Effect of too stiff a reinforcing ring

The correct size of ring provides the same stiffness as the disc it replaces. If the reinforcing ring is less stiff than this, the circumferential stress in both ring and sheet is greater than the uniform stress (σ_0 say) in the uncut sheet. On the other hand, if the ring is stiffer than the original disc, the circumferential stress in the adjoining sheet is, as might be expected, reduced - but only at the expense of an increase in the radial stress above the original σ_0 (since the sum of circumferential and radial stresses must always remain constant at $2\sigma_0$).

In other words excessive stiffness in a reinforcing ring defeats its object by actually attracting loads from the surrounding sheet. For example, a ring of double the proper stiffness induces a radial stress in the adjoining sheet that is 24% above the original stress σ_0.

3.4 An interesting paradox

The problem of the edge-reinforcement of holes in plates raises an interesting paradox, which is not without design importance.

Consider a hole made in an infinite expanse of sheet in which, to fix ideas, the principal stresses are equal and the hole circular. If the uniform tensile stress in the uncut sheet is σ_0, it follows (see Appendix B) from standard theory that the circumferential stress at the edge of the hole is $2\sigma_0$. The actual formula may be quoted in order to show how quickly this stress falls off with distance from the hole. We have

$$\text{circumferential stress} \quad \sigma_\theta = \sigma_0\left(1 + \frac{a^2}{r^2}\right) \qquad (2)$$

(where a = radius of hole

r = distance from hole centre)

which shows that at (say) 4 diameters from the edge of the hole ($r = 9a$) the circumferential stress has dropped from $2\sigma_0$ at the edge to $1.01\,\sigma_0$, or to within 1% of the stress at infinity. At 100 diameters from the hole the stress given by the formula is $1.000025\,\sigma_0$, and therefore practically identical with the stress at infinity. It might be thought therefore that, if the thickness of the plate were doubled over an area extending to 100 diameters from the edge of the hole, the edge stress would be reduced by half. This is not so, however, as the stress at the hole is still $1.24\,\sigma_0$, (for a Poisson's Ratio of $\frac{1}{4}$) a figure below which the edge stress cannot be reduced however wide the area covered by the doubler plate.

The explanation is at once evident if we visualise the infinite plate as (say) 10 ft square and the hole as $\frac{1}{4}$ inch diameter. It is then clear that a doubler plate extending to (say) 10 diameters from the edge of the hole constitutes in effect a local reinforcement of the 10 ft plate by a 5 inch diameter solid disc - since the effect of the $\frac{1}{4}$ inch central hole has a negligible affect on its radial stiffness. The result is that, as shown in the previous paragraph, the 'hard spot' attracts forces from the surrounding area of the 10 ft plate so that the radial stress immediately outside the doubler plate increases from σ_o to 1.24 σ_o. Immediately inside the doubler plate this drops by half to 0.62 σ_o, which, in accordance with formula (1) above (after substituting 0.62 σ_o for σ_o) again increases to double value, i.e. 1.24 σ_o, at the edge of the hole.

It need hardly be pointed out that this paradox, and the paragraph leading up to it are not without relevance to practical design.

3.5 Superiority of neutral holes over holes of arbitrary shape

The great superiority of a correctly designed neutral hole over a hole of arbitrary shape arbitrarily reinforced, is well illustrated by considering again the simple case where, in the uncut sheet, the principal stresses are equal.

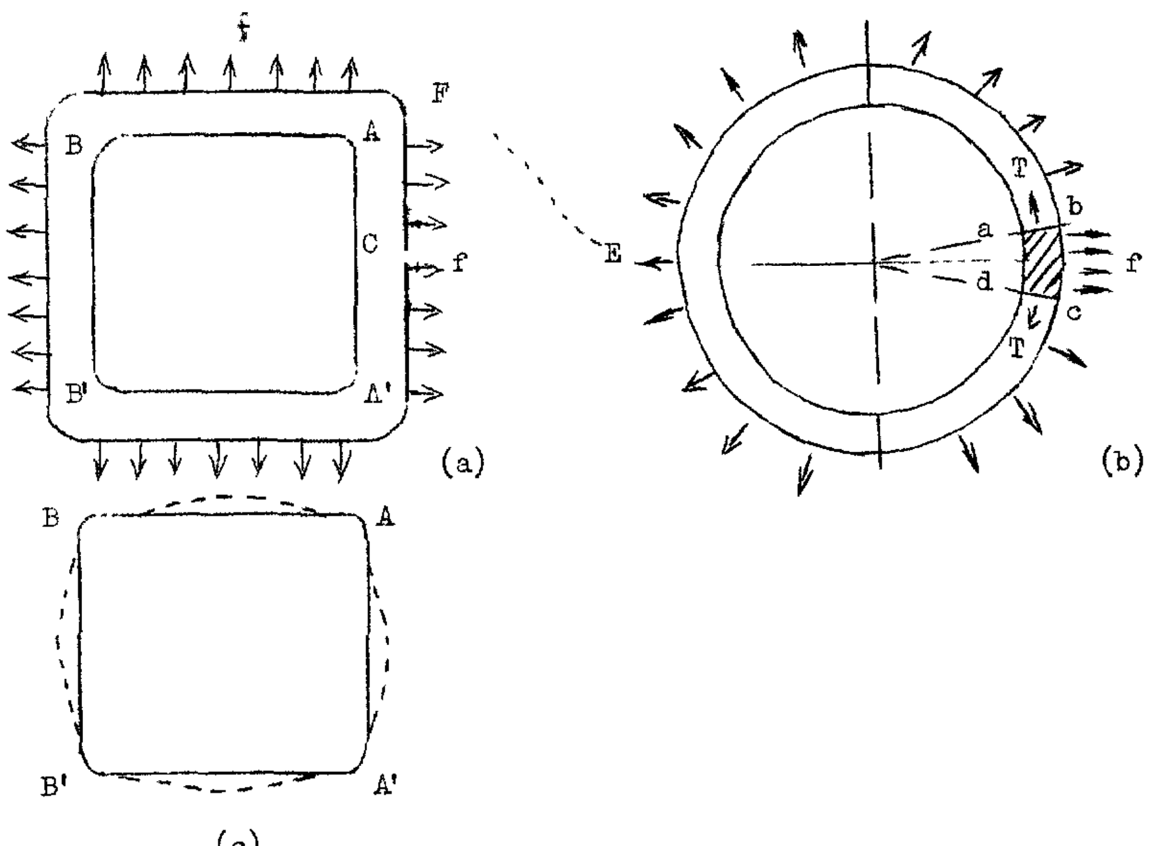

Fig.3

Figs.3(a) and 3(b) show a square and a circular hole, each reinforced at the edge. In comparing the relative effectiveness of the reinforcements it is useful to remember that, when the principal stresses are equal, the pull in the unbroken sheet is the same in all directions, so that the net effect on any element of area is a pure dilatation. Thus the disc that originally occupied the place of the circular hole must have been subjected to a uniform radial pull. That radial pull, in the absence of the disc, is, in Fig.3(b), resisted by the reinforcing ring, whose cross-section, in order to make the hole neutral, has to be such as to give the same overall expansion as the disc it replaces.

The way equilibrium is maintained between the ring and the adjoining sheet is shown by the shaded sector abcd of the ring. The radial forces f are equilibrated by the radial component of the circumferential pull T in the ring.

The important point is that the ring resists the pull in the sheet by a simple tension, without any adventitious aid from its bending stiffness, in the same way in fact as the chain of a suspension bridge.

Compare now the behaviour of the circular ring with the square frame with which the square hole is edge-reinforced. The forces f in the middle region of the side AA' can only be resisted by the bending of the frame, since any direct pull in AA' has no component in the direction of f. It is possible to make the frame strong enough to resist the bending, but it is not possible to make it stiff enough without making it prohibitively heavy. For, to have adequate stiffness, its bending deflection has to be comparable with the stretch of the square of sheet that originally occupied the hole. What actually takes place is that the sides of the frame try to resist the forces f by bending but, being much too flexible to do so effectively take up the shape shown exaggerated in Fig.3(c), from which it can be inferred that excessive tensile stress concentrations are brought into action at the inner edge of the doubler frame in the corner regions. Meeting no resistance from the frame, the forces f must largely change their direction to that indicated in Fig.3(a) by the dotted line EF. The same conditions obtain, of course, for the other pair of sides.

It follows from these remarks that square or rectangular holes for the windows of pressure cabins are to be avoided as structurally inefficient.

3.6 Outline of suggested arrangement

It is clear from what has already been said that the problem facing the designer who wants to use neutral holes for his windows is much simplified if the principal stresses in the uncut sheet are equal. For the proper shape of hole is then circular, and the reinforcing ring is also circular and of constant cross-section - facts that simplify the design and facilitate the constructional problems.

As it happens, however, the two principal stresses induced in the walls of a closed circular cylinder, like a pressure cabin under internal pressure, are not equal. The hoop stress in the main cabin is twice the longitudinal stress, so that it would appear incumbent on a designer wishing to fit his windows into neutral holes to use holes of elliptical shape, with the major axes vertical. This, however, does not necessarily follow, because it is possible, by a simple constructional device that can be justified on other grounds, to reduce the effective hoop forces to half their nominal value, so making the hoop loads in the area that matters equal to the longitudinal loads.

The suggested construction will be understood by reference to Fig.4, which shows a part of the side of a pressure cabin and three windows.

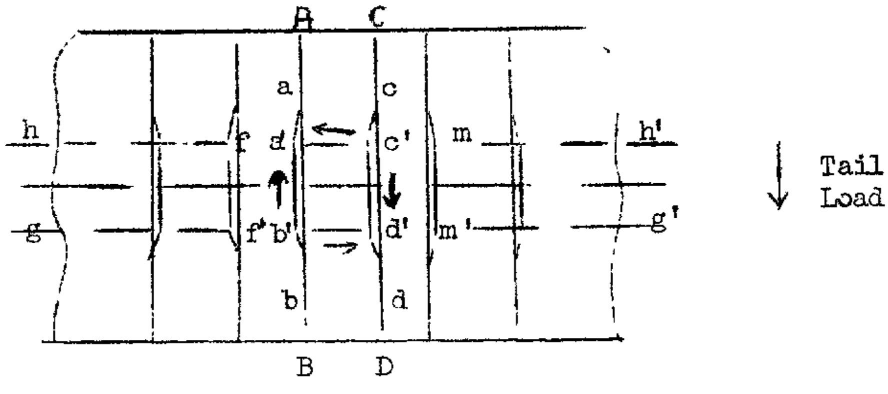

Fig.4

In flight the most important load, other than the internal pressure, that has to be carried by the middle strip of wall hh'g'g is the vertical bending shear. Under a downward tail load the direction of the shear forces, as they affect the square a'b'd'c', are shown by the arrows in the figure. The distribution of bending shear stress round the hole is best visualised as made up of two additive parts - first, the simple distribution appropriate to the continuous uncut walls, and second, that directly induced by the cutting of the hole. The first requires no discussion. The second can be conveniently regarded as four equal forces applied to the square frame a'b'd'c' in the direction of the arrows. In the absence of the material that has been removed to make the hole, these forces have to be liquidated via the surrounding structure, the vertical forces by inducing shear in the rectangular areas ac' and bd', and the horizontal forces by shears in areas a'f' and c'm'. In order to distribute the shearing actions over a reasonably wide area of sheet, and so reduce stress concentrations, it is desirable to introduce stress-distributing members. For the vertical couple these may take the form of reinforcements of the existing fuselage rings AB and CD, and are shown in the figure as ab and cd. Their function is to carry the shearing action deeper into the circular strip AD, and so reduce the shear stress. The complementary horizontal couple is liquidated across the panels a'f' and c'm, well enough by the agency of the existing longitudinal stringers without the use of special reinforcing members.

It is now proposed to rely on these necessary vertical reinforcing members ab, cd to relieve the intervening sheet from some of its hoop load, the intention being that, in the immediate neighbourhood of the hole, this relief should amount to half the original load. Experience indicates that, in order to ensure the required redistribution of hoop load as between sheet and ring-reinforcement members, the latter should extend a distance a'a, about equal to the cross distance a'c', along the ring BA. The cross-sectional area of each tapered member as it approaches the hole should, for the present purpose, be equal to the cross-sectional area of the intervening sheet, so that in this way half of the load is taken by the reinforcing members and the remaining half by the sheet.

Such a construction makes the effective hoop load per unit width of section equal to the longitudinal load, and therefore justifies the use of circular windows.

There is, of course, always the alternative course of using elliptical windows and of taking care of shear due to tail loads by reinforcing the sheet, rather than the frames, adjacent to the windows. Choice between the two methods is largely a matter of manufacturing convenience.

3.7 Summary of main points covered in 3 above

The following is a summary of the main points discussed in section 3.

1 Holes cut in a pressure-cabin wall should be properly shaped and reinforced so as to qualify as neutral holes under pressure loads, for a neutral hole leaves the stresses in the surrounding sheet unaffected by its presence.

2 The simplest shape of hole and the simplest type of edge-reinforcement go with a sheet in which the principal stresses are equal, and hence cause uniform dilatation throughout the sheet. The hole is then circular and the edge-reinforcing member is of constant cross-section.

3 A suitable type of edge-reinforcing member might well be an annular disc, or doubler plate, equal in thickness to the surrounding sheet and further stiffened by an edge ring as in Fig.1(c).

4 A stand-up type of ring (as in Fig.2) is unsuitable as its real stiffness under loads applied in the plane of the sheet is (due to bell-mouthing) much less than its nominal stiffness.

5 Reinforcing a sheet by another of equal thickness extending beyond the edge of the hole by several diameters will not halve the stress at the edge - a paradox that is discussed in the text.

6 Stiffening the edge of a hole beyond what is required to make it neutral makes the reinforcing member stiffer than the original sheet it replaces. The result is to create a 'hard spot' that 'attracts' load to itself by magnifying the radial pull in the surrounding sheet.

7 Square or rectangular holes for windows are not good design as they produce stress concentrations however substantially reinforced.

8 Since the principal loads in a pressure cabin are not equal - the hoop loads being twice the longitudinal loads - a method is described for halving the hoop loads in the immediate vicinity of the windows, thus making the principal loads effectively equal and justifying the use of the simple circular hole.

4 Cabin doorway

Since, for practical convenience, the cabin doorway must take the form of a vertically elongated hole, the fact that the hoop tension is twice the longitudinal tension may be turned to advantage. For the correct shape of a neutral hole under such conditions is an ellipse, with a ratio of major to minor axis of $\sqrt{2}$. As for the size of the edge-reinforcing member, Mansfield[3] has shown that the cross-sectional area A_m, which in this case must be variable, is given by the formula

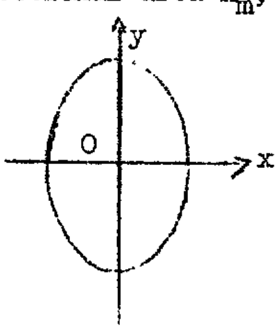

Fig.5

$$\frac{A_m}{b_t} = \frac{\sqrt{2}\,(1 + x^2/b^2)^{3/2}}{1 - 2\nu + 3\,x^2/b^2} \qquad (3)$$

in terms of the co-cordinates x and y of Fig.5.

As already indicated in the case of circular holes, it is desirable for the reinforcing edge-member to take the form of a flat disc-like ring, so as to lie as nearly as possible in the plane of the shell wall.

As it is desirable to strengthen the surrounds of the door frame against accidental damage etc., this should be arranged in such a way as not to increase the effective cross-sectional area of the edge-reinforcing member proper. A convenient way is indicated in Fig.6 by a rough diagram. This is intended to convey the idea that it is possible to introduce a box-like surrounding structure for the door frame which,

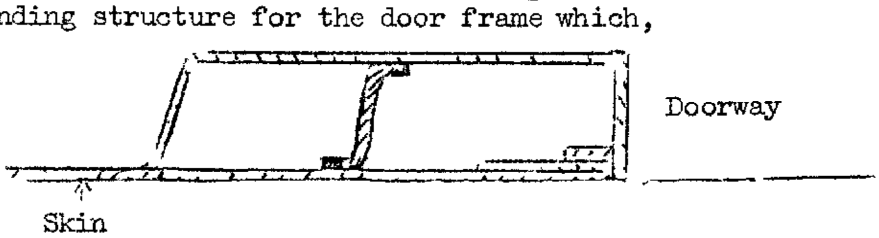

Fig.6

due to its high depth-width ratio cannot pick up any appreciable fraction of the tension T in the skin.

5 Pilot's canopy

The pilot's transparent canopy offers a more awkward obstacle to shell continuity than either windows or doors; it is much too wide to be effectively bypassed by any system of edge-reinforcing members alone. Nothing less than its division into panels by stress-carrying members that bridge the gap will here suffice.

Consider a long narrow rectangular gap in a plane sheet in which the principal stresses are unequal but neither of them zero. If the principal stresses coincide in direction with the sides of the rectangle it is natural to place the bridging members straight-across the gap as in Fig.7(a), the narrowness of the gap making it unnecessary to have complementary members at right angles to these, other than the longitudinal edge members themselves.

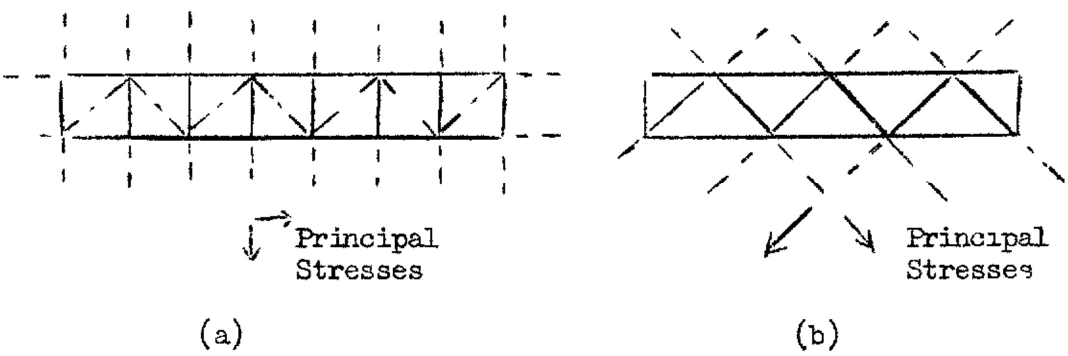

Fig.7

The size of the bridge-members will be such as to make the cross sectional area of each equal to that of the sheet between adjacent members, thus maintaining the stiffness of the original unbroken sheet. Transference of load from sheet to bridge-members is by shear action between the sheet and extensions of the members into the body of the sheet, as shown by the dotted lines in the figure. The usual shear-reinforcement at the corners, where the extensions meet the edge of the gap, is of course necessary.

A point to be noted is that such an arrangement of bridge-members is satisfactory only so long as the directions of the principal stresses are immutable. The slightest change of direction puts the slot under shear forces which the upright bridge-members are powerless to resist, and which, if the bridge-members constitute the supports for a non-stress carrying material (such as perspex), puts the latter under unfair strain. To cater for small changes of direction of the principal stresses it is therefore essential to introduce a diagonal bracing of some kind such as that indicated by chain-dotted lines in the figure.

In the case where the main load across the gap involves (unequal) principal stresses that are oblique to the edges of the rectangle, as in Fig.7(b), ease of load transference from sheet to bridge-members demands that these (and their extensions) should lie parallel to the directions of the principal stresses, as shown in the figure. By arranging these members so that their extremities meet to form a kind of Warren-girder bracing, two birds are killed with one stone; for the gap is then adequately braced against minor changes of direction of the principal stresses, and no undue strain can fall on any non-stress-carrying material supported by the members.

If these principles are applied to the canopy, it will be traversed by bridge members that are, in effect, continuations of the hoop frames along the cabin cross-section and of the longitudinal members in the direction of the generators. A plan view of a canopy bridged on these lines is shown in diagrammatic form in Fig.8, where it is seen that the hoop and generator bridge-members make up a well-triangulated braced structure.

What has been said above (in discussing Fig.7) regarding the cross-sectional area of the bridge-members and their extensions again applies, independently of whether the latter coincide with existing cabin hoop-frames and longitudinal members or not.

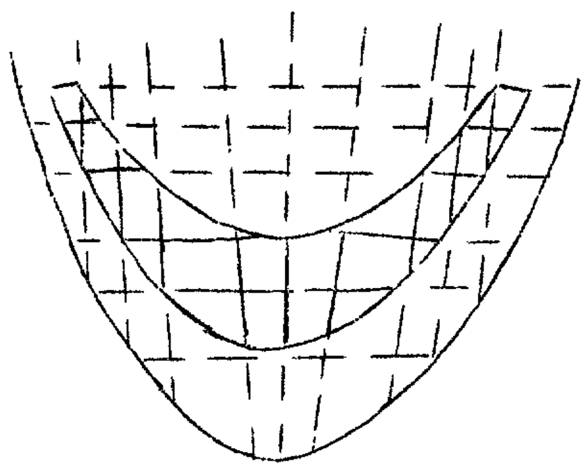

Fig.8

The canopy may strike one as somewhat unorthodox in appearance but it also strikes one as highly functional! This sort of design admittedly requires a greater number of panes in the canopy, but this is offset by their smaller size and the smaller stresses that go with smaller size. Such considerations are, in any case, of secondary importance. What matters is that a canopy bridged in this way is one, if not the only, logical answer to the demand for strength at least cost in weight. Compared on a weight basis with a canopy fitted with more or less unidirectional bridge members, its strength is likely to be several times greater.

6 Interaction between cabin-walls and bulkheads or transverse frames

Cabin transverse frames and bulkheads introduce discontinuities that require the investigation of stresses other than membrane stresses. The importance of such secondary stresses can most conveniently be assessed if they are compared with a datum stress that is basic to the shell considered as a pressure vessel. Such a stress is the nominal hoop stress in the cylindrical part of the cabin. This depends only on the shell radius, the skin thickness and the pressure, and for the typical case we are considering, where the radius $r = 60$ ins., skin thickness $h = 0.036$ in. and pressure $p = 10$ lb/in^2 this stress

$$\sigma_{datum} = pr/h = 16,660 \text{ lb/in}^2 \qquad (4)$$

6.1 Effect of frame-constraint on longitudinal stresses

The constraint exercised by a frame against the free expansion of the cabin walls induces longitudinal bending stresses in the skin-stringer shell, whose magnitude depends on the stiffness of the ring against radial expansion. In the absence of stringers these bending stresses extend no more than a couple of inches each side of the ring, but when stringers are present, as they always are, the extent of the disturbance is more like a couple of feet. The maximum bending stress in the skin-stringer shell - in each case occurring immediately over the frame - is however not very different, as shown in Appendix D, although the reaction between frame and shell is six to eight times greater for the stringer-reinforced skin - a point to remember when designing the rivets or other fastening connecting the two.

One needs to consider two types of transverse frames - ordinary former-frames and frames (including bulkheads) specially stiffened for various purposes. It is necessary to consider not only the bending stresses induced in the shell but also the hoop and other stresses induced in the frames themselves.

6.2 Constraining effect of stiff frames

The free radial expansion of the shell associated with the hoop stress quoted above amounts to

$$w_f = \frac{pa^2}{Eh}(1 - \nu/4) \tag{5}$$

where the Poisson's Ratio term takes account of the added stiffness contributed by the longitudinal tension, which is assumed equally shared between skin and stringers.

Since the radial deflection must be zero at a frame that is completely rigid, such a frame applies in effect a radial deflection equal and opposite to w_f.

Now, at any cross section of the shell the relation between a radial load P per unit periphery and the radial deflection it produces in the shell wall is given (as shown in Appendix D) by the formula

$$w = \frac{Pe^{-\beta x}}{8\beta^3 D}(\sin \beta x + \cos \beta x) \tag{6}$$

where $\beta = Eh/4a^2D$

D = longitudinal bending stiffness of the shell wall per unit of circumference

x = longitudinal distance from the section concerned (7)

It is seen that the quantity β determines both the rate of die-away and the periodicity of the deflection. At the frame $(x = 0)$

$$w = P/(8\beta^3 D) \tag{8}$$

and the bending moment

$$M_x = P/4\beta \tag{9}$$

both maximum values.

A perfectly rigid frame applies to the expanded shell a load P that is obtained at once from (6) by putting w_f (of equation 5) for w; the bending moment M_x follows, and hence the bending stress σ.

For the frame itself the corresponding relation between load and deflection is given by (10).

$$w_{frame} = \frac{Pa^2}{AE} \tag{10}$$

where A = cross-section area of frame.

Thus, if the radial constraint is not rigid, but is provided by a frame of area A, the load P is obtained by equating to w_f the sum of (8) and (10). This gives

$$P\left(\frac{1}{8\beta^3 D} + \frac{a^2}{AE}\right) = w_f \tag{11}$$

With P found, the bending stress in the shell is obtained via (7) and the hoop stress σ in the frame by

$$\sigma_{frame} = Pa/A \tag{12}$$

6.3 Numerical values for stiff-frame case

Some numerical values will put the various quantities above discussed in proper perspective. For this purpose we assume the 20 s.w.g. skin to be reinforced by top-hat stringers at about 6 in. pitch and that the stiff frame (or ring) has $2\frac{1}{2}$ in^2 of cross-sectional area. The numerical values are therefore as follows:-

$p = 10$ lb/in.

$p_1 = 9.5$ lb/in. $[= p(1 - \nu/4)]$

$a = 60$ in.

$h = 0.036$ in.

$A = 2.5$ in^2

$D = 0.006E$ lb in. or $0.012E$ with stringer locally reinforced

$E = 10^7$ lb/in^2

From this $\beta = (Eh/4a^2D)^{\frac{1}{4}} = 0.14$ in.$^{-1}$

It works out that a frame of this stiffness produces a radial deflection (from the free-expansion position) some 80% of that due to a perfectly rigid frame. Also the loading P = 105 lb/in.

Nominal stringer-skin bending stress = 15,500 lb/in^2.

Secondary bending effect due to end-tension reduces this to 14,000 lb/in^2.

To this must be added the overall longitudinal-tension stress of about 4700 lb/in., the resultant stress thus becoming 20,200 lb/in^2.

This stress, if we wish to bring all secondary stresses well below the datum hoop stress, must be reduced. Probably the most effective way is to increase the bending stiffness of the stringers, either by increasing the gauge thickness or the height of the top-hat section.

The radial depression caused by the frame extends to a point $3\pi/4\beta$ either side of it, so that if we stiffen the shell we must do so over that distance at least. Suppose we merely double the stringer gauge. This gives

$$\beta = 0.118$$

$$\text{i.e.} \quad 3\pi/4\beta = 20 \text{ in.}$$

so that the stiffening should extend a couple of feet each side of the frame.

The net result is to reduce the bending stress from 14,000 to 10,000 lb/in^2 - a worthwhile reduction that brings the resultant stress of (10,000 + 4700), = 14,700 lb/in^2 below our arbitrary datum.

The corresponding hoop stress in the frame itself is of little account, amounting as it does to only about 3,000 lb/in^2.

6.4 Constraining effect of former-frames

In the case of ordinary, or former, frames the emphasis in the matter of stress switches over from the shell to the frame. The flimsiness of the frame - of cross-sectional area 0.16 compared with 2.5 in^2 for the stiff frame - allows it to expand with the skin, with a consequent substantial reduction in the constraining force P.

The problem of the former-frame, unlike that of the stiff frame where the adjacent former-frames were neglected, is complicated by the close pitch of the rings. This makes the conditions at one frame dependent on those at the adjacent frames. This is dealt with in Appendix D on the lines of Timoshenko's treatment, according to which the loading P is obtained from the relation

$$P\beta\left(\chi_1 - \tfrac{1}{2}\frac{\chi_2^2}{\chi_3}\right) = p_1 - \frac{Ph}{A} \qquad (13)$$

where the χ's are functions of $(\beta\ell)$ defined in Appendix D, and ℓ = pitch of frames. For any value of β and ℓ, tables given by Timoshenko allow the χ functions to be readily evaluated, whence P is found. The consequent longitudinal maximum bending moment is given by

$$M_{max.} = \left(\frac{p - Ph/A}{2\beta^3}\right)\chi_2 \qquad (14)$$

from which the stress is at once obtained.

6.41 Swelling of skin between frames

The disturbance caused by a single frame is only local, and beyond that local region the shell wall carries its full hoop stress. The pitch of former-frames however is usually close enough to prevent the skin from reaching its full unimpeded expansion, even at a section mid-way between two frames, where obviously the expansion must be a maximum. It is shown in Appendix D that the inward radial deflection (from the position of unimpeded expansion) midway between two rings is given by the formula

$$w = w_1\left(1 - \frac{\cos\theta \sinh\theta + \sin\theta\cosh\theta}{\sinh\theta\cosh\theta + \sin\theta\cos\theta}\right) \qquad (15)$$

where w_1 is the deflection at a frame

and $\theta = \beta\ell/2$ (16)

When the pitch ℓ drops below about 10 in. the second term on the right hand side of (15) becomes negligibly small, which means that the shell wall and the frames have the same radial displacement. It also means that material in the frames is almost as effective as that in the skin itself in reducing skin hoop-stress. The qualification is due to the 5% extra hoop stiffness of the skin (for the same radial displacement) derived from the Poisson's Ratio effect of the longitudinal tension.

6.42 Some numerical values

We assume a Z section former of 20 s.w.g. sheet (0.036 in.) and 0.16 in^2 cross-sectional area as shown in Fig.1D of Appendix D. It is cut away, or notched, to allow the unimpeded passage of the stringers.

Two cases are considered. In one the pitch is 20 in. and the formers have the section just described. In the other, the pitch is 10 in. and the frame gauge is reduced from 20 to 24 (0.022 in.), the amount of material in the frames thus increasing by 22%. For the first of these cases the longitudinal bending stress in the shell wall is 4,200 lb/in^2 which becomes 9,000 on adding the overall longitudinal stress. As the bending stress is lower in the second case, it is clear that we need not be greatly concerned for this type of shell-wall stress, whatever the pitch.

Making use of the above formulae (as described in more detail in Appendix D) we obtain the following results.

TABLE I

	20 s.w.g. frames at 20 in. pitch	24 s.w.g. frames at 10 in. pitch
Radial loading P between frame and shell	32 lb	20 lb
Hoop stress in ring	12,000 lb/in^2	12,400 lb/in^2
Hoop stress in skin (interframe)	14,500 lb/in^2	13,400 lb/in^2
Weight of frame material per ft run of cabin	3.3 lb	4.1 lb

The drop from 14,500 to 13,400 lb/in. in skin maximum hoop stress (whose nominal datum value by (4) is 16,660 lb/in^2) is not in itself impressive, but it is to be remembered that the stress in question is the main stress for the whole cabin. The lower it can be brought, the lower will be all the secondary stresses that vary directly with it, and the longer the fatigue life of the whole structure.

It is seen from the table that the hoop stress in the frame is much the same for the two cases. This is a fundamental feature of a shell structure of the type now considered, in that any acceptable change of pitch or cross-sectional area of former-frames can have little effect on the hoop stress. Increasing the section-area of the frame merely increases the constraint it applies to the shell almost in the same ratio, unless of course an unacceptably large frame section is used. Slender frames must therefore inevitably have hoop stresses little short of that of the skin.

The stress of 12,000 lb/in. quoted is reasonably low in itself, being in fact no greater than that in the skin. It is objectionable only because the frame is deeply notched at every stringer, with the result that stresses perhaps twice that amount are induced at each of the 60 odd notches in every frame throughout the cabin.

One way of reducing this concentration is to use a favourable shape of notch, the stringer section being modified to suit. An alternative, or (preferably) further, step would be to reinforce the frame around the notches.

6.43 Transmission of load between frame and stringer-skin

The notching of former-frames has one curious result, which is to allow comparative freedom for the outer lip of the frame to bend about a circumferential axis, in the way described in Appendix D. Unless the lip is reinforced in some way, the bending stress thus induced reaches something like 20,000 lb/in^2.

The only straightforward way to relieve this stress is to fasten the skin to the frame at a point on the lip as close as possible to the junction between lip and web, with some kind of corner washer for even load distribution. A solution of this problem by changing the frame-section from Z to a deep-catenary shape is discussed in Appendix D (last para.).

A further point to note is the local increase in the skin hoop stress between the flanges of a stringer wherever the stringer passes through a frame notch. This is discussed in relation to Fig.3D in Appendix D, where it is shown that any stress picked up between notches by the frame outer lip is returned in more or less concentrated form to the short skin span across the notch. From this point of view the lighter frame that goes with a closer pitch, as discussed in 6.42, is obviously to be preferred. It is to be remembered however that, for light-gauge close-pitch frames - even more than with the conventional type - it is desirable, in order to minimise bending stresses, to abandon the Z type of frame-section and adopt a lipped U-section having the shape of a deep catenary, as explained at the end of Appendix 3D.

It may be noted here that, from one point of view, the method of fastening a stringer flange by a single line of rivets has advantages over the use of an adhesive for the same purpose. For the adhesive enables the stringer flange, which though narrow is wide in comparison with its thickness, to pick up its full share of hoop load. Considerable shear stresses are thus induced at the edges of all stringer flanges. No such stresses are induced in the riveted flange.

In studying the stress distribution in a former frame it is necessary to take account of the rather unusual way in which the stresses are induced. This is fully discussed in Appendix D, where it is shown that the inner lip of the frame, and the web region lying below the level of the notches, are loaded indirectly by the radial loading P applied at the outer lip. Were it not for the direct fastening between skin and frame-lip the latter would be free of circumferential stress. Such fastening however enables the lip to pick up its share of hoop tension, but only, as already observed, by a concentration of shear force at the two ends of each inter-notch length of frame lip.

7 Rear dome of pressure cabin

On the basis of the argument put forward in Appendix E the best shape for the rear dome of a pressure cabin is the hemispherical. Assuming this to be correct, we are left with the problem of choosing the most efficient method of joining the hemispherical dome to the forward cabin shell and to the rear fuselage. In this we are guided by two basic considerations:-

(i) Owing to the comparatively heavy membrane forces involved, it is desirable to avoid any radial offset between the shell and the dome skins.

(ii) There must not, in the neighbourhood of the joint, be any reduction in the longitudinal bending stiffness of the fuselage wall, on the maintenance of which the elastic stability of the wall depends.

It is not easy to outline a scheme of design that satisfies both these requirements, and the nearest approach put forward here is that shown in Fig.3E of Appendix E, where it is described in detail.

The scheme entails local stiffening of the dome around its base, both on its inner and outer surface. The joint itself is made by sandwiching together (over the region AK of Fig. 2E) the three skins - of shell, dome and rear fuselage - to form a single lap-joint. The dome and the rear fuselage wall are further directly connected by fastening the latter to the outer stub-stringers that form the external reinforcement of the dome.

The problem of determining the forces and moments introduced by the differential free radial expansion of shell dome and rear-fuselage wall is fully discussed in Appendix E. It is also there shown, by reference to a typical numerical example, that the maximum resultant longitudinal stress - from bending and longitudinal tension - is under 8,000 lb/in^2. As this is only about half the datum stress we have set up, it is entirely acceptable.

8 Constraint due to cabin floor

Since the longitudinal tension stretches the cabin in the fore-and-aft direction by an appreciable amount - about $\frac{1}{2}$ in. for a cabin 80 ft long under the longitudinal stress of 4,700 lb/in^2 we have previously assumed - the constraint exercised by a stiff floor cannot be neglected. The problem is one of stress-diffusion and its implications will be understood by reference to Fig. 9, which shows a diagrammatic view of a pressure-cabin in side elevation, and cross-section. The line ADB represents the floor in the side view and the line D_1D in the sectional view.

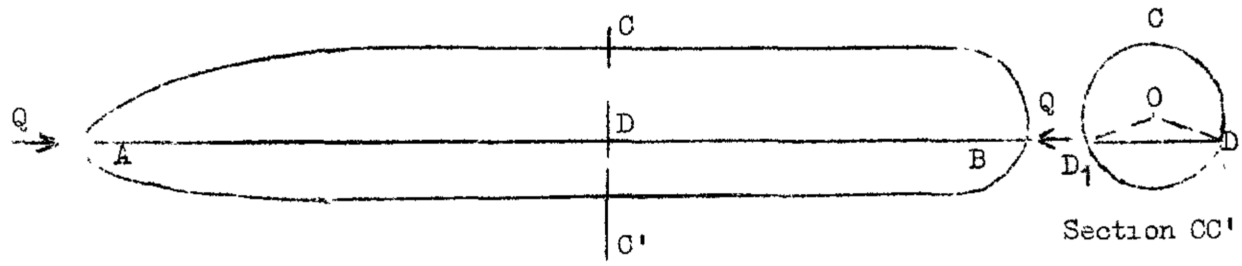

Fig.9

The radial constraint introduced by the floor induces no undue stresses in the cabin and need not be discussed.

The problem of the longitudinal constraint is reduced to its simplest terms if we imagine the floor attached to the cabin wall only at section CC_1 in the first place. While then the cabin is elongated by the pressure, a suitable external agency extends the floor to precisely the same amount. If the cabin were cylindrical in shape from end to end, the external agency would need to apply tensile loads Q (say) only at the two ends A and B of the floor.

If the floor is now fastened to the cabin wall at all sections and the external agency removed, the stress field induced, additive to and superposable on the original longitudinal-tension, is that due to the application of compressive forces Q at A and B to the complete structure (with cabin and floor as integral parts).

Having regard to the length of the arc D_1 C D_2 in relation to the half-length AD of the cabin, we conclude that, at section CC', the compression force Q will be uniformly distributed over floor and cabin wall. This means that considerable shear stresses must be induced in the cabin wall adjacent to the floor in the region of the two ends.

The problem of finding the magnitude and distribution of such shear stresses is one that should, however, not be allowed to arise. The real problem is rather to design the cabin-to-floor connections to allow the differential expansion of floor and cabin-wall to take place unhindered. This is not a difficult problem, for there are many obvious ways in which this object can be achieved without detriment either to the efficiency or economy of the structure. They need not therefore be discussed here.

9 Conclusions

The main conclusions may be summarised as follows:-

1 Main cabin

The highest stress level (directly due to pressurisation) in a pressure-cabin - the datum stress as we may call it - is set by the nominal hoop stress in the main cabin, and, if this is the design case for the fuselage, a prime objective of the designer should be to keep all stresses throughout the cabin below that level. The actual hoop stress can be reduced well below this datum level by taking advantage of the fact that former-frames a little more closely pitched than usual effectively prevent the skin between frames from expanding to a bigger radius than that of the frames themselves. The material in the frames in this way helps the skin to carry its hoop tension. The effect in a typical case is to reduce a nominal stress of 16,000 lb/in^2 to an actual stress of 13,400 lb/in.

2 Forward cabin

In the streamlined forward part of the cabin, where the cabin diameter is smaller and the longitudinal tension helps to contain the pressure by virtue of the longitudinal curvature, the skin membrane stresses are much lower. It is therefore much easier in this region of the cabin to keep the stress concentrations at the various discontinuities from rising above the datum above mentioned. This is fortunate, since the most considerable discontinuity in the whole cabin - the pilot's canopy - occurs in the nose region.

3 Windows

In the main cabin the windows are the chief cause of discontinuity in the smooth surface of the skin, but it has been shown that, by a slight modification in the design, it is possible to make round windows the ideal shape for eliminating stress-concentrations. A window frame of constant section goes with the round window, in contrast with the variable-section frame required for the elliptical window which, in the absence of the modification, would be the ideal shape.

4 Doors

To reduce the stress-concentrations liable to occur around door-frames, it is suggested that these be of ideal elliptical shape with the edge-reinforcing member satisfying the theoretical requirements for least stress-concentration. Incidental structure should not be allowed to interfere with the main skin stresses round the door.

5 Pilot's canopy

In the design of the pilot's canopy the members that brace the transparent canopy should lie in the directions - hoop and longitudinal - of the main stresses. They may well be continuations of existing frames and longitudinal members, and arranged so that, where they cross the canopy, they form a braced tri-angulated structure. A canopy braced in this way is likely to be far stronger than if braced in the conventional way.

6 Frames - stiff and former-frames

Stiff frames are not themselves critically stressed but they cause heavy longitudinal bending stresses in the stringer-reinforced skin. The use of heavier stringers in the neighbourhood of the frame - 2 ft or so to either side - is advocated, in order to reduce the stress well below the set datum.

Former-frames cause no troublesome stresses in the stringers but themselves experience hoop stresses comparable to those of the skin. Being notched to allow passage to the stringers, they are subject to stress concentrations at the notches that may be well above the datum. Using a good shape for the notch and reinforcing the edge is a palliative.

Bending stresses in the lips and web of a Z-section frame can be very high - particularly the outer lip which takes the full radial reaction between skin and frame. A lipped U section in the shape of a deep catenary would obviate bending stresses in all but the outer lip, the fastening between which and the skin should be as close to the spring of the catenary arch as possible.

7 Rear dome

To use membrane strength to advantage, the rear dome should be hemispherical and unreinforced except near its junction with the main shell. Here stub-stringers are used in order to ensure continuity of cabin-wall longitudinal bending stiffness without introducing radial offset of dome and main-cell skin.

8 Floor

Floors should be designed to allow differential longitudinal expansion relative to the cabin-wall, otherwise undesirable shear stresses are set up.

REFERENCES

No.	Author	Title, etc.
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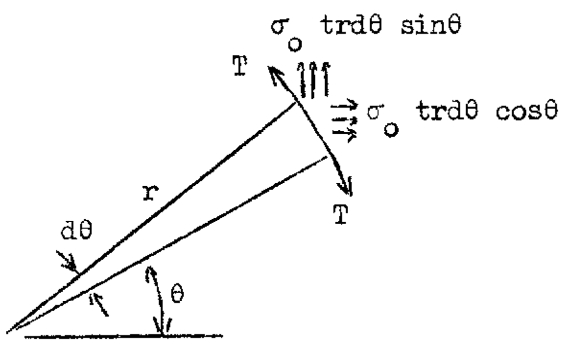
2	C. Gurney	"An analysis of the stresses in a flat plate with a reinforced circular hole under edge forces" R & M 1834, 1938
3	E.H. Mansfield	"Neutral holes in plane sheet - reinforced holes which are elastically equivalent to the uncut sheet" Quarterly Journal of Applied Maths. 1953

APPENDIX 'A'

Neutral hole in plane sheet with equal principal stresses



Let t = thickness of plate

A = cross-section area of reinforcing ring

r = radius of curvature of element of arc of ring

T = tensile force in ring

σ_o = principal stresses

Fig.1A

Consider the equilibrium of the element of arc $rd\theta$ of ring. As shown in Fig.1A, the horizontal and vertical forces exerted by the sheet on the element of arc are respectively $(\sigma_o\ trd\theta)\cos\theta$ and $(\sigma_o\ trd\theta)\sin\theta$ the only other forces are the tensions T at the ends of the arc.

The resultant tangential force in the positive direction of θ is

$$(T + \sigma_o\ trd\theta.\sin\theta.\cos\theta) - (T + \sigma_o\ trd\theta.\cos\theta.\sin\theta)$$

which is identically zero.

For equilibrium in the radial direction the outward resultant is

$$(\sigma_o\ trd\theta)\ \sin^2\theta + (\sigma_o\ trd\theta)\ \cos^2\theta - T\ d\theta \quad = \quad 0$$

so that

$$T \quad = \quad \sigma_o\ tr \tag{1A}$$

which shows that, if the tension T is to be constant in the reinforcing member,

$$r \quad = \quad \text{const.} \tag{2A}$$

and therefore the hole must be circular.

For compatibility of displacement in the tangential direction the strain in the ring arc must equal that of the sheet, i.e.

$$\frac{T}{AE} \quad = \quad \frac{\sigma_o}{E}(1 - \nu)$$

or

$$A \quad = \quad \frac{T}{\sigma_o\ (1 - \nu)}$$

$$= \quad \frac{tr}{1 - \nu} \text{ by (1A)} \tag{3A}$$

Since the circumferential strain of the ring is thus equal to that of the adjacent sheet their diametral strain must also be the same. In other words the radial stretch of the ring is identical with that of the disc that previously occupied the hole.

APPENDIX 'B'

Design of reinforcing rings for circular holes - useful data derived from standard formulae

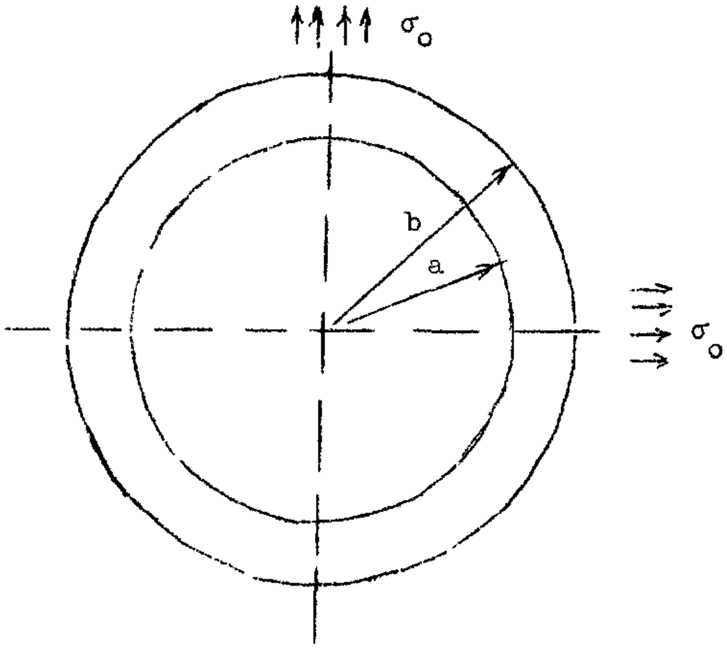


Fig. 1B

A circular hole of radius 'a' is supposed cut in a large expanse of sheet in which the principal stresses have the same value σ_o.

Unreinforced hole

For an unreinforced hole with free edge the radial and circumferential stresses are respectively (as given by standard formulae)

$$\left.\begin{aligned} \sigma_r &= \sigma_o\left(1 - \frac{a^2}{r^2}\right) \\ \sigma_\theta &= \sigma_o\left(1 + \frac{a^2}{r^2}\right) \end{aligned}\right\} \qquad (1B)$$

It follows that the sum of the two stresses is constant at $2\sigma_o$. Where therefore the radial stress σ_r is zero at the edge of the hole, the circumferential stress rises to $2\sigma_o$.

If, at the hole, there is an inward pull of p_i per unit arc

$$\left.\begin{aligned} \sigma_r &= \sigma_o\left(1 - \frac{a^2}{r^2}\right) + \frac{p_i}{t}\frac{a^2}{r^2} \\ \sigma_\theta &= \sigma_o\left(1 + \frac{a^2}{r^2}\right) - \frac{p_i}{t}\frac{a^2}{r^2} \end{aligned}\right\} \qquad (2B)$$

Hole reinforced by annular disc

If the hole is reinforced by an annular disc of outer radius b, the sheet beyond b may be regarded as having a hole of radius 'b' subjected to an inward radial pull per unit arc of amount p_o. The annular disc, correspondingly, may be regarded as having zero radial stress at the edge of the hole and an outward pull p_o at its outer periphery. The value of p_o is determined by the necessity for identical circumferential displacement (which ensures equal radial displacement also) of annular disc and outer sheet at the common radius b.

For an annular disc of thickness nt (where t is the thickness of the sheet) under radial pulls per unit arc specified as

$$\left.\begin{aligned} p_o &= \text{outward pull per unit arc at outer radius } b \\ p_i &= \text{inward pull per unit arc at inner radius } a \end{aligned}\right\}$$

the stresses (from standard formulae) are

$$\left.\begin{aligned} \sigma_r &= \left(\frac{1}{b^2 - a^2}\right)\left\{\frac{a^2b^2}{r^2}(p_i - p_o) + (p_o b^2 - p_i a^2)\right\}\frac{1}{nt} \\ \sigma_\theta &= \left(\frac{1}{b^2 - a^2}\right)\left\{\frac{a^2b^2}{r^2}(p_o - p_i) + (p_o b^2 - p_i a^2)\right\}\frac{1}{nt} \end{aligned}\right\} \quad (3B)$$

For the annular disc σ_i $(= p_i/t) = 0$, so that we have

$$\left.\begin{aligned} \sigma_r &= \left(\frac{b^2}{b^2 - a^2}\right)\left(1 - \frac{a^2}{r^2}\right)\frac{p_o}{nt} \\ \sigma_\theta &= \left(\frac{b^2}{b^2 - a^2}\right)\left(1 + \frac{a^2}{r^2}\right)\frac{p_o}{nt} \end{aligned}\right\} \quad (4B)$$

At radius b the circumferential strain is

$$(\varepsilon_\theta)_b = \frac{1}{E}(\sigma_\theta - \nu\sigma_r)$$

$$= \frac{1}{E}\cdot\frac{p_o}{nt}\left(\frac{b^2}{b^2 - a^2}\right)\left\{(1 - \nu) + \frac{a^2}{b^2}(1 + \nu)\right\} \quad (5B)$$

by substitution from (4B).

For the outer sheet (with hole of radius b under pull p_o)

$$(\varepsilon_\theta)_b = \frac{1}{E}(\sigma_\theta - \nu\sigma_r)_b$$

$$= \frac{1}{E}\left\{2\sigma_o - \frac{p_o}{t}(1 + \nu)\right\} \qquad (6B)$$

by (2B), (after substituting b for a).

Equating (5B) to (6B) gives

$$\frac{p_o}{t} = \frac{2n\sigma_o}{\left(\frac{1}{b^2 - a^2}\right)\left\{b^2(1 - \nu) + a^2(1 + \nu)\right\} + n(1 + \nu)} \qquad (7B)$$

It may be noted (in relation to the paradox mentioned in the text) that if n = 2 (which corresponds to doubling the original sheet) and $\nu = \frac{1}{4}$, the radial stress p_o/t in the sheet, when b is very large compared with a, becomes equal to $1.24\ \sigma_o$.

Equation (7B) applies to any thickness of reinforcing member. For the original stresses σ_o in the sheet to be unaffected by the hole, the annular disc must have such a thickness nt that p_o/t, the radial stress in the sheet at the edge of the annular disc, is still equal to σ_o. Putting p_o/t equal to σ_o in (7B) gives

$$n = \left(\frac{b^2}{b^2 - a^2}\right)\left\{1 + \frac{a^2}{b^2}\left(\frac{1 + \nu}{1 - \nu}\right)\right\} \qquad (8B)$$

Neutral hole reinforced by annular disc plus inner ring

When the inner edge of an annular disc, itself insufficient to neutralise the hole, is reinforced by an inner ring of cross-sectional area A, the disc is then subjected to radial pulls at both outer and inner edges. Equations (3B) therefore still apply, and in place of (5B) we have

$$(\varepsilon_\theta)_b = \frac{1}{E}\left(\frac{1}{b^2 - a^2}\right)\left\{a^2(p_o - p_i)(1 + \nu) + (p_o b^2 - p_i a^2)(1 - \nu)\right\}\frac{1}{nt} \qquad (9B)$$

The corresponding strain in the outer sheet is again given by (6B). By equating (6B) and (9B) for compatibility of displacement, we obtain

$$\left[\left(\frac{1}{b^2 - a^2}\right)\left\{a^2 p_o(1 + \nu) + b^2 p_o(1 - \nu) - 2a^2 p_i\right\} + np_o(1 + \nu)\right]\frac{1}{t} = 2n\,\sigma_o \qquad (10B)$$

and since p_o/t must equal σ_o if the hole is to be neutral, this gives

$$p_i/t = \frac{\sigma_o}{2}\left\{(1+\nu) + \frac{b^2}{a^2}(1-\nu)(1-n) + n(1-\nu)\right\} \qquad (11B)$$

For equal strain of ring and annular disc at the edge of the hole $(r = a)$

$$(\varepsilon_\theta)_{ring} = (\varepsilon_\theta)_a \text{ for annulus}$$

or

$$\frac{p_i a}{AE} = \frac{1}{E}\{(\sigma_\theta)_a - \nu(\sigma_r)_a\} \qquad (12B)$$

Using (3B), and substituting σ_o for p_o/t, we find the cross-sectional area of the ring to be

$$A = nat. \frac{(b^2-a^2)\{(1+\nu)+\frac{b^2}{a^2}(1-\nu)(1-n)+n(1-\nu)\}}{4b^2-\{b^2(1+\nu)+a^2(1-\nu)\}\{(1+\nu)+\frac{b^2}{a^2}(1-\nu)(1-n)+n(1-\nu)\}} \qquad (13B)$$

The case where there is no annular disc, and the sheet is reinforced by the inner ring alone, is represented in (13B) by making $n = 1$. The value of A is then $at/(1-\nu)$ as already given by equation (3A). An annular disc of the same thickness as the sheet $(n = 2)$ gives, for $b^2/a^2 = 2$ and $\nu = \frac{1}{4}$,

$$A = 0.63 \text{ at} \qquad (14B)$$

which is the value appropriate to the arrangement of Fig.(10).

APPENDIX C

Membrane forces in a surface of revolution under internal pressure

A pressure cabin may be regarded as approximately a surface of revolution about the fuselage longitudinal axis, and consequently the standard membrane theory for thin shells can be applied. For a shell under internal pressure the membrane forces are very simply derived, and, as applied to a pressure cabin are briefly given here for convenience.

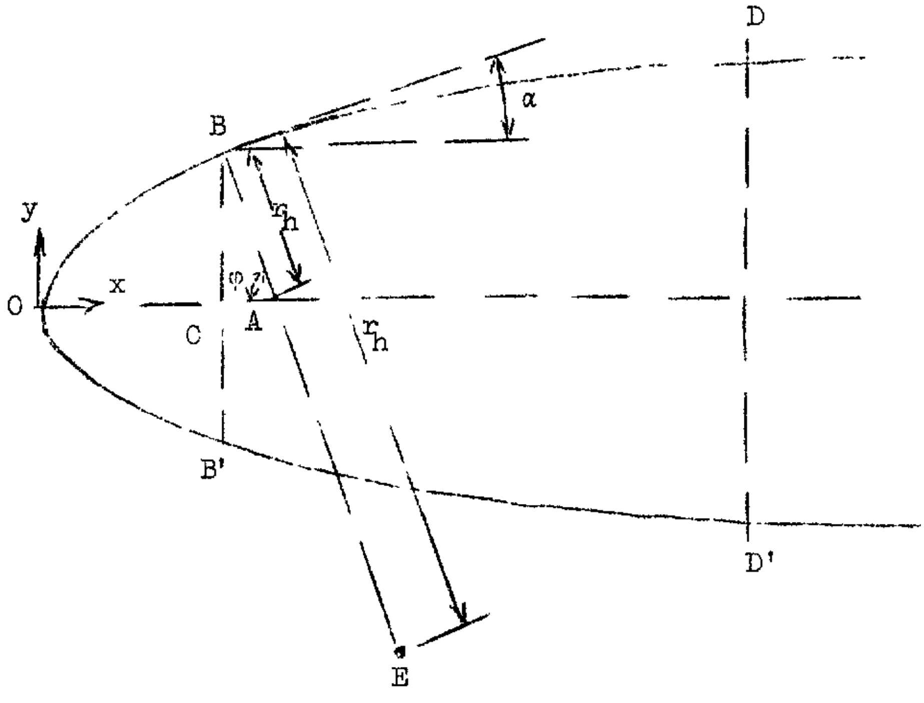


Fig.1C

Fig.1C shows a surface of revolution for which the axis of symmetry is the longitudinal axis x.

In membrane theory it is well known that the outwardly-directed pressure p on any elementary unit area is equilibrated by the inward components of the pull which the membrane exerts by virtue of its principal curvatures.

In any pressure vessel, if the principal planes of curvature are the X and Y planes, and if N_x, N_y and r_x, r_y are the corresponding membrane forces and radii of curvature, it is well known that

$$\frac{N_x}{r_x} + \frac{N_y}{r_y} = p \qquad (1C)$$

where p is the internal pressure.

From this we see at once that, if both the radii of curvature are known together with either force N_x or N_y, the other force is at once known in terms of p. This fact is made use of here.

Consider, for example, an element of unit area at B in Fig.1C, where the meridian line OB cuts the cross-sectional circle BB'. By symmetry, the normal to the element, on which both radii lie, must cut the axis of revolution, as at A in the figure. Also by symmetry, the radius of curvature in the plane AB perpendicular to the meridian must be equal to AB or $y/\sin\varphi$, where φ is the supplement to the slope α of the tangent at B, and y is the radius of the cross-sectional circle BB'. The other radius of curvature is that of the meridian at B with centre at E (say).

If T_ℓ = membrane pull in longitudinal direction

T_h = hoop tension perpendicular to T_ℓ

r_h = transverse, or hoop, radius of curvature ($= y/\sin\varphi$)

r_ℓ = longitudinal, or meridian, radius of curvature EB

we have, from (1C),

$$\frac{T_\ell}{r_\ell} + \frac{T_h}{r_h} = p \qquad (2C)$$

For any section BB' the value of T_ℓ is at once written down by using the equation of overall equilibrium for the nose portion OBB'. Thus

$$2\pi y \cdot T_\ell \cos\alpha = \pi y^2 p$$

or

$$T_\ell = py/2\cos\alpha \qquad (3C)$$

It follows that the longitudinal membrane pull is not sensitive to the precise shape of the curve OBD. Substituting for T_ℓ in (2C) we have

$$T_h = r_h\left(p - \frac{T_\ell}{r_\ell}\right) \qquad (4C)$$

from which we see that the hoop force T_h is very sensitive to the precise curvature of the meridian curve. It can therefore be changed quite violently in a short distance in the longitudinal direction by changes of meridian curvature that have little effect on the general appearance of the meridian curve. If, for example, the slope α is kept constant for a short distance along the curve, the local radius of curvature r_ℓ, being then infinite, makes the T_ℓ term in (4C) disappear, leaving T_h to resist the pressure alone. The consequence is a sudden change in the hoop tension, which produces local bending of an amount depending on the skin bending stiffness.

In the shape shown in Fig.1C the longitudinal curvature falls off gradually from the nose O until at D, where the meridian becomes parallel to the x axis, it becomes zero. Beyond D the pressure is resisted entirely by the hoop tension. At the nose, not only is the resistance to the pressure well shared between the two membrane forces T_{ℓ} and T_h but, owing to the smaller radii of curvature, their absolute values are also much reduced. If the extreme nose is spherical in shape, for example, with a radius 1/3 that of the main parallel section (beyond DD') the membrane stresses at the nose O - hoop and longitudinal - have a value only 1/6 that of the main hoop stresses beyond DD'.

APPENDIX D

Constraining effect of cabin-frames on shell expansion

1 Free radial expansion of the cabin shell is restricted at the frames, or formers, and the amount of restriction depends on the radial stiffness and pitch of the frames.

In his "Theory of Plates and Shells" Timoshenko shows that, for a cylindrical shell, the relation connecting the inward radial deflection and any inwardly-directed radial loads Z per unit area (both assumed constant circumferentially) is given by the differential equation:-

$$D \frac{d^4 w}{dx^4} + \frac{Ehw}{a^2} = Z \tag{1D}$$

where h = skin thickness

D = longitudinal bending stiffness of the shell per unit width, including the effect of stringers, if any.

The equation can be regarded as governing the deflection of a longitudinal strip of unit width under a longitudinally variable load Z, the radial support for the strip deriving from the stiffness of the skin against radial displacement and its accompanying circumferential stretch. Following Timonshenko's treatment, we write this in the form:-

$$\frac{d^4 w}{dx^4} + 4\beta^4 w = \frac{Z}{D} \tag{2D}$$

where

$$\beta^4 = \frac{Eh}{4a^2 D} \tag{3D}$$

Equation (2D) can be used to solve any problem in which internal frames interfere with the free expansion of the shell and its stringer reinforcements.

Equation (2D) as it stands applies only when longitudinal tension is absent. In a typical cabin shell the longitudinal stress σ_ℓ in the skin (owing to the stringers taking approximately half the load) is $\frac{1}{4}$ of the hoop stress and to take account of this, (2D) is written in the form:-

$$D \frac{d^4 w}{dx^4} + \frac{Eh_w}{a^2} = Z + \nu \frac{h}{a} \sigma_\ell \tag{4D}$$

In the cabin problem the only externally applied normal force is the pressure p and therefore:-

$$\sigma_\ell = pa/4h \tag{5D}$$

whence equation (4D) is finally written:-

$$D \frac{d^4 w}{dx^4} + \frac{Eh}{a^2} w = -p\,(1 - \nu/4)$$

$$= -p_1 \qquad (6D)$$

where $$p_1 = p(1 - \nu/4) \qquad (7D)$$

Thus the constraint exercised by the longitudinal tension is allowed for by taking a reduced pressure p_1 instead of the actual internal pressure.

2 Single frame in long expanse of shell

A single frame in an otherwise uniform cylindrical shell applies what is effectively a concentrated inward radial load P per unit length along the circumference when the shell is under internal pressure, and the corresponding inward radial deflection, as deduced from the appropriate solution of (2D), (with Z put equal to zero and with a shear P/2 at x = 0) is given by Timoshenko in the form:-

$$w = \frac{P e^{-\beta x}}{8\,\beta^3 D} \quad (\sin \beta x + \cos \beta x) \qquad (8D)$$

where x is the longitudinal distance from the loaded section.

The corresponding bending moment is given by:-

$$M_x = \frac{P}{4\beta} e^{-\beta x} \quad (\sin \beta x - \cos \beta x) \qquad (9D)$$

Both w and M_x have their maximum values at the loaded section, x = 0, where:-

$$(w)_{max.} = \frac{P}{8\,\beta^3 D} = \frac{P}{8}\left(\frac{4a^2}{Eh}\right)^{\frac{3}{4}} \left(\frac{1}{D}\right)^{\frac{1}{4}} \qquad (10D)$$

$$(M_x)_{max.} = \frac{P}{4\beta} \qquad (11D)$$

$$= \frac{P}{4}\left(\frac{4a^2 D}{Eh}\right)^{\frac{1}{4}} \qquad (12D)$$

It is seen from these equations that, for a given shell diameter and skin thickness, the weaker the stringer reinforcement the more local the disturbance and the greater the bending stress.

3 Rigid frame with unreinforced skin

Taking typical figures - a shell radius of 60 in., a skin thickness of 0.036 in. and a pressure of 10 lb/in^2 - we find the unimpeded radial expansion p_1a^2/Eh of the skin to be 0.094 in.

If at a given section the skin, unreinforced by stringers, is rigidly held against expansion, the effect is to cause an inward local deflection of 0.094 in., which by (10D) requires a distributed force P of 21 lb/in. We then have:-

$$\beta = 0.9 \text{ in}^{-1}$$

which, by (8D), means that the width of the circumferential groove is $2 \times 3\pi/4\beta$ or 5.2 in.

The maximum bending stress at the bottom of the groove is, by (11D):-

$$\sigma_{max.} = 28{,}500 \text{ lb/in}^2$$

4 Effect of stringer reinforcement

The longitudinal stiffness of the shell wall is greatly increased by the stringer reinforcement and, for a typical stringer section and pitch:-

$$\beta = 0.14 \text{ in}^{-1}$$

and the width of the dip in the skin increases from 5.2 in. to 33 in. The force necessary to prevent radial expansion is now 132 lb/in. and

$$(\sigma_{max.})_{skin\ +\ stringer} = 19{,}000 \text{ lb/in}^2$$

Thus, although the radial force necessary to prevent expansion of the shell is much greater for a stringer-reinforced skin than for the skin alone, the increased stiffness that brings this about also reduces the bending stress, which is of the same order in both cases.

5 Stiffness of typical actual frames specially stiffened

The ordinary typical frame is not stiff enough to provide the kind of constraint required to produce the above loads and stresses, but a stiffened ring with (say) 2.5 in^2 of cross-sectional area would, for the size cabin here considered, have a radial expansion of only 0.019 in. i.e. 1/5 of the free shell expansion under the 125 lb/in. loading. The amount of frame flexibility this implies reduces the value of P to 105 lb/in. and the stringer maximum bending stress by some 20% to 15,500 lb/in^2 under the 10 lb/in^2 operating pressure.

The above figures neglect the relieving effect of the secondary bending moment due to the overall longitudinal tension, but this moment will be less than 10% of the total. A resultant stress of 14,000 lb/in^2 for the above case is therefore near the mark.

One way, and possibly the most convenient, of making a substantial reduction in the stringer stress is to double the gauge of the stringer over the disturbed region - here covering a distance of about 18 in. to each side of the frame. By doing this, we obtain the following values:-

$\beta = 0.118 \text{ in}^{-1}$

I (of stringer cum skin) per unit length $= 0.012 \text{ in}^3$

Radial force between shell and frame $= P = 120$ lb

Stringer bending stress (max.) (after allowing for end tension relief) $= 10{,}000 \text{ lb/in}^2$

6 Ordinary former frames

The problem of the constraining effect of the ordinary former-frame is many-sided, in that the effect on the frame itself is just as important as the effect on the skin-stringer shell wall. Perhaps the best way to demonstrate this is again to consider a typical case. For this purpose we choose the type of skin-former fastening in which the formers are notched to allow the passage of the stringers, rather than the type where the stringers ride over the formers.

Bearing in mind the extent of the disturbance caused by the constraining effect of a frame - about 18 in. to each side of the frame for a typical stringer-reinforced skin - one perceives that, with a typical frame-pitch of 20 in., the effect of any one frame cannot be considered independently of that of its neighbours.

A typical section used in the past for fuselage formers is that shown in Fig.1D(a).

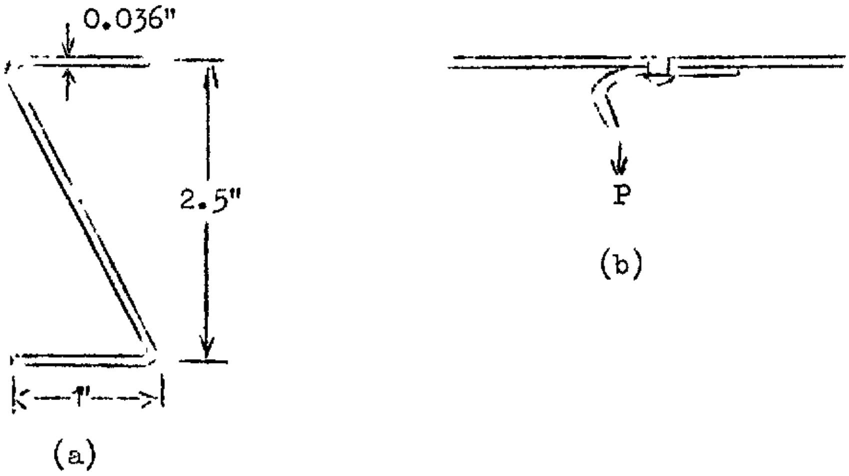


Fig.1D

in which the sheet thickness is 0.036 in. (20 s.w.g.).

Again following Timoshenko's* treatment, and using his notation, let

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* A reference to the original work by I.G. Bocbnov on this subject is given by Timoshenko.

P	=	reaction force between ring and shell-wall per inch of arc
ℓ	=	pitch of rings
A	=	cross-sectional area of ring
a	=	radius of cylindrical shell
h,β	=	same as used already
p_1	=	$p(1 - \nu/4)$
(2α)	=	$\beta\ell$
$\chi_1(2\alpha)$	=	$(\cosh 2\alpha + \cos 2\alpha)/(\sinh 2\alpha + \sin 2\alpha)$
$\chi_2(2\alpha)$	=	$(\sinh 2\alpha - \sin 2\alpha)/(\sinh 2\alpha + \sin 2\alpha)$
$\chi_3(2\alpha)$	=	$(\cosh 2\alpha - \cos 2\alpha)/(\sinh 2\alpha + \sin 2\alpha)$

The radial loading P on the shell wall is then given by the relation:-

$$P\beta \left\{ \chi_2(2\alpha) - \frac{\chi_2^2(2\alpha)}{\chi_3(2\alpha)} \right\} = p_1 - \frac{Ph}{A} \tag{13D}$$

which, by means of Table 46 in Timoshenko's book where the χ functions are given in tabulated form for various values of (2α), can be easily solved to give P.

The maximum skin-stringer bending moment at the frame is given by

$$M_{max.} = \frac{p_1 - Ph/A}{2\beta^2} \chi_2(2\alpha) \tag{14D}$$

from which the maximum stringer stress is at once found. The corresponding radial deflection w_f at the frame is given by:-

$$w_f = \frac{P\beta a^2}{Eh} \left\{ \chi_1(2\alpha) - \tfrac{1}{2} \frac{\chi_2^2(2\alpha)}{\chi_3(2\alpha)} \right\} \tag{15D}$$

Here the hoop stress in the frame is obtained directly from P, and the bending stress in the skin-stringer cabin wall from M_{max}. Both these stresses are therefore affected by the pitch ℓ of the frames via the quantity $2\alpha(= \beta\ell)$. By reducing the pitch, not only are both these stresses reduced, but another important advantage is gained. This is a reduction in the maximum hoop stress itself, which comes about from the fact that, if the frames are not too far apart, the skin in between is effectively held down against radial expansion. By symmetry, the skin will have its greatest radius midway between two frames, and the nearer this radius is to the radius at the frames the more effective are the frames in limiting the maximum skin hoop-stress.

Now the local inward radial deflection caused by a frame or ring measured from the freely expanded skin (minus frames) is given by (15D) in which P is given by (13D), but to obtain the corresponding deflection midway between two rings it is necessary to consider again the general solution of equation (6D). This gives:-

$$w = \left(\frac{-p_1 a^2}{Eh}\right) + C_1 \sin \beta x \sinh \beta x + C_2 \sin \beta x \cosh \beta x$$

$$+ C_3 \cos \beta x \sinh \beta x + C_4 \cos \beta x \cosh \beta x \qquad (16D)$$

Taking the origin for x midway between two adjacent frames, and assuming for the moment that the frames are completely rigid, we see that, since by symmetry w must be an even function, C_2 and C_3 are both zero. The remaining two constants C_1 and C_4 are determined by the conditions that both w and dw/dx are zero at $x = \pm \ell/2$. We thus obtain finally the outward radial deflection curve (-w) of the skin beyond the surface defined by the frames:-

$$-w = \frac{p_1 a^2}{Eh}\left[1 - \frac{(\sin\theta \cosh\theta - \cos\theta \sinh\theta)\sin\beta x \sinh\beta x + (\cos\theta \sinh\theta + \sin\theta\cosh\theta)\cos\beta x \cosh\beta x}{\sinh\theta\cosh\theta + \sin\theta\cos\theta}\right] \qquad (17D)$$

where $\theta = \beta\ell/2$.

At x = 0, midway between the rings, the swell of the skin above the level of the rings is:-

$$-(w)_{x=0} = \frac{p_1 a^2}{Eh}\left[1 - \frac{\cos\theta \sinh\theta + \sin\theta\cosh\theta}{\sinh\theta\cosh\theta + \sin\theta\cos\theta}\right] \qquad (18D)$$

where the expression in the square brackets approaches zero as ℓ approaches zero and unity as ℓ becomes large.

If the coefficient $p_1 a^2/Eh$ in (17D) is regarded as the inward radial deflection caused in the freely expanded cabin wall by the rigid frames, it is clear that for a constant frame-pitch ℓ, (and therefore constant θ) the corresponding skin deflection at any inter-frame point defined by x is a constant fraction of the deflection at the frame. The shapes of the curves in Fig.2D are therefore independent of the stiffness of the frames, and so we can make use of (18D) to obtain the inter-frame skin deflection if the deflection at the frame is known.

The argument put forward here is again best illustrated by considering typical numerical values. Thus let:-

p = internal pressure = 10 lb/in^2

$p_1 = p(1 - \nu/4)$ = 9.5 lb/in^2

a = cabin radius = 5 ft

h = skin thickness = 0.036 in.

D = bending stiffness of skin-stringer cabin wall per unit width = 0.006 $in^3 \times E$

q = height of stringer crown from neutral axis = 0.5 in.

A = cross-sectional area of frame (or ring) = 0.16 in^2 or 0.098 in^2 (according to pitch)

ℓ = pitch of frames (or rings), given various values

$\beta = (Eh/4a^2D)^{\frac{1}{4}} = 0.14$

Consider first the case where the frame-pitch ℓ has the typical value of 20 in. This makes:-

$$2\alpha = \beta\ell = 2.8$$

From equation (13D) we find:-

$$P = 32 \text{ lb/in.}$$

From (14D), the stringer bending moment

$$M_{max.} = 50.5 \text{ lb/in.},$$

which makes the bending stress

$$\sigma = \frac{Mq}{I} = \frac{50.5}{0.006} \times 0.5 \text{ lb/in}^2$$

$$= 4,200 \text{ lb/in}^2$$

This is additive to the overall longitudinal tension stress of 4,700 lb/in^2, making a total of about 9,000 lb/in^2.

The corresponding average stress in the frame is:-

$$\frac{pa}{A} = 12,000 \text{ lb/in}^2$$

and something a good deal higher than this at the notches - a point considered again later.

Consider next the effect of halving the frame-pitch and reducing the frame gauge from 20 to 24 (i.e. 0.036 to 0.022 in.). This gives

$$P = 20.2 \text{ lb/in.}$$

$$\text{stringer } M_{max.} = 16.6 \text{ lb/in}^2$$

$$\text{stringer } \sigma_{max.} = 1370 \text{ lb/in}^2$$

$$\text{frame stress} = 12,400 \text{ lb/in}^2$$

Thus, by increasing the total amount of material in the frames in the ratio 2 × 0.022/0.036, i.e. by 23%, the stringer longitudinal bending stress is reduced from 4,200 to 1370 lb/in^2, i.e. by 67%.

This however is not the whole story, for, by reducing the frame pitch from 20 in. to 10 in. the material in the frames becomes nearly as effective as the skin itself in reducing the basic hoop stress (the absence of a Poisson's Ratio stiffening effect on the frame accounting for the difference). The reason for this is illustrated by Fig.2D (a), (b) and (c) which shows how the character of the skin displacement changes with frame pitch.

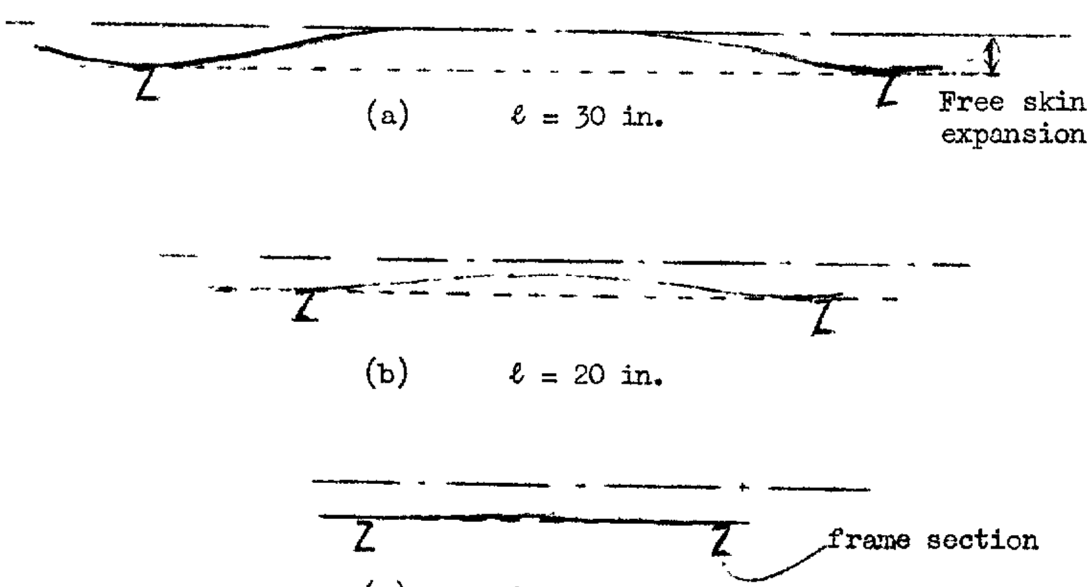

Fig.2D

In each sub-figure the dotted line marks the level of the constraining frames, the chain-dotted curve marks the position the expanded skin would take if unconstrained by the rings, and the full curve represents the actual shape it takes up.

In Fig.2D (a) where the pitch is 30 in., it is seen that the displacement of the skin midway between adjacent frames is unaffected by the constraining frames.

In Fig.2D (b), with the conventional pitch of 20 in., the skin still swells out between frames to about 44% of the swell associated with a very long frame-pitch. In (c) however the frames are close enough together to prevent anything more than a negligible amount of swell between frames. In other words, the hoop strain is the same in skin and frames, and all the material put into the frames is, in the matter of hoop stress, nearly as effective as if it had been used to increase the skin thickness - not quite as effective because, as mentioned above, for the same strain the hoop stress in the skin is greater than that in the frame by ν times the longitudinal stress, i.e. by 1000 lb/in^2.

Table I summarises the situation as between 20 gauge frames at 20 in. pitch and 24 gauge frames at 10 in. pitch.

	20 in. Pitch	10 in. Pitch
Radial pull between skin and frame per inch of arc	32 lb	20.2 lb
Hoop stress in frame	12,000 lb/in^2	12,400 lb/in^2
Maximum hoop stress in skin (midway between frames)	14,500 "	13,400 "
Stringer bending stress at rings	4,200 "	1,370 "
Stringer resultant stress from bending and longitudinal tension (4,700 lb/in)	8,900 "	6,000 "
Weight of frame material per ft run of cabin length	3.3 lb	4.1 lb

It is seen from this that, at the cost of an extra weight of 1 lb per ft run of cabin, a worthwhile reduction in skin and stringer stress is achieved. Unfortunately there seems to be no way to reduce the hoop stress in the frame, either by reducing frame pitch or increasing the frame cross-sectional area. Any acceptable increase of frame cross-section merely increases the radial constraint on the shell, which immediately reacts to cause a higher hoop stress on the frame itself. Although the frame hoop stresses quoted in the table are a little lower than the maximum interframe skin stresses, the frame notches cause a further concentration that must make the resultant stress around the rotches considerably higher.

A possible way of reducing the notch stresses is to use a more favourable shape of notch. This implies a shape of stringer section wider and with a crown less sharp than that normally used.

6 Interaction between former-frames and cabin walls - frame-lip stresses etc.

To understand the stress distribution in a former-frame it is necessary to consider the precise way in which load is transmitted to it by the skin-stringer cabin wall.

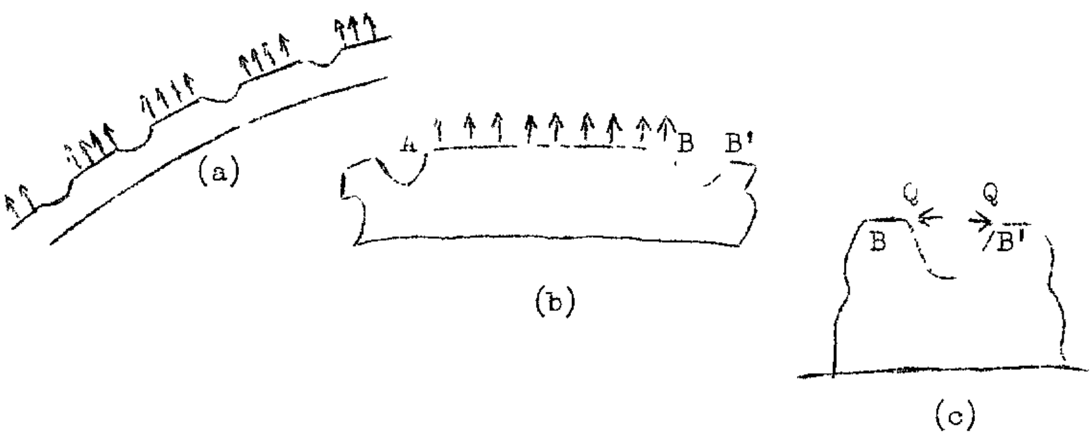

Fig.3D

Fig.2D(a) shows part of a notched frame and the primary forces applied to it by the skin. An enlarged inter-notch part of the frame is shown in Fig.2D(b) under radial forces P per unit length.

6.1 Frame hoop-stresses

If the load applied by the expanding skin were purely radial, the frame would be able to resist expansion by virtue only of its unbroken inner region lying below the notches, the circumferential tension having little chance of diffusing into the projecting portion AB lying directly between the notches. This would entail heavy stress concentration around the bottom of the notches and practically zero circumferential stress along the outer edge of the ring. Actually, however, owing to the direct fastening to the frame over the region AB, the skin tends to stretch the outer lip of the frame over that region. Thus the ring is extended in two distinct ways - over its inner part indirectly by radial extension, and over its outer part directly by contact with the expanding skin.

In considering the stress distribution in the ring the easiest way is to superpose two systems of stress distribution. In the first we assume that the outer lip (BA of Fig.3D) is continuous over the notches. The whole ring section is then extended in the first of the two distinct ways mentioned in the previous paragraph, and the skin is not called upon to stretch the outer lip directly. Under this condition the load P is obtained from equation (13D) and this gives the tensile stress in the ring. The stress concentrations round the bridged notches are treated as if they occurred in a straight bar under end tension with the same bridged notches. The bridging of course considerably reduces these concentrations.

In the second system the continuity of the outer lip over the notches is assumed to be broken, which is equivalent to removing the bridge portion and applying the loads Q that the bridge previously carried, as shown in Fig.3D(c). The loads Q are now carried in two ways:-

(i) By compression of the combined former-lip, stringer-flange and skin over the inter-notch portion AB, and

(ii) By tension of the skin across BB'.

The stringer is too springy to carry any load and is therefore negligible.

Although the thickness of the skin over BB' is smaller than the combined thicknesses of skin and lip over AB it still carries the greater part of the load Q by reason of the greater stiffness it derives from its much shorter length. This means that the end rivet connecting the ring lip to the skin and stringer flange takes a considerable amount of shear as well as direct radial tension. The consequent local stress in the skin around the rivet (or bolt) is greatly reduced if skin and stringer-flange are continuously connected, such as by a metal adhesive of some type. It is to be appreciated however that the use of a continuous adhesive introduces an adverse effect that is practically absent when each flange of a stringer is connected to the skin by a single line of rivets. With an adhesive, each stringer flange within the confines of its own width picks up and again discards its appropriate share (depending on the relative thicknesses of skin and flange) of the hoop tension in the skin, so introducing heavy shear stresses in the adhesive between skin and stringer-flange at the flange edges. The local reduction in the skin hoop stress offers no benefit since the inter-stringer skin is subjected to the full cabin hoop stress in any event. With a single line of rivets per flange this kind of hoop-tension pick-up cannot take place.

6.2 Frame lip-bending stresses

One important effect that has to be considered is the bending of the outer lip under the radial forces from the shell wall. Owing to the notches, radial displacement of the lip can be greater than that of the ring as a whole, because of the bending of the lip as a short beam in the way indicated by the lip shown in Fig. 1D(b). If the ring lip or flange has a width f, this bending moment amounts to (Pf/2) × 6/5 per inch periphery (assuming the notches to occupy 1/6 of the ring periphery) and the corresponding bending stress σ is given by

$$\sigma = \frac{18\,Pf}{5h^2} \tag{19D}$$

With P = 32 lb., and f = 0.8 in. this gives a value of 70,000 lb/in^2 for σ. The flexibility of the lip largely relieves the situation, but even so the stress amounts to over 20,000 lb/in^2.

Stresses of this magnitude are unacceptable, but they are difficult to avoid in frames with a Z or similar section, particularly as the inner lip suffers from similar, if smaller, bending stresses from the same cause. There is no doubt that, from this point of view, a more suitable section is the lipped U section shown in Fig.4D. Here, since the resistance to radial expansion of each element of cross-section is proportional to the area of that element; the appropriate shape of the section is a deep catenary. The lips may be any convenient width, but the fastening between lip and skin, or stringer-flange, should naturally be as close as possible to the spring of the catenary arch, in order to cut down bending stresses.

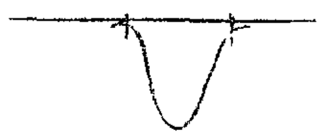

Fig.4D

As the bending 'modulus' (q/I) of the sheet constituting the frame-lip drops more rapidly than its thickness, the bending stresses above discussed vary inversely with the thickness and hence more or less inversely with the frame pitch. The closer the frame-pitch therefore, the more desirable it is to adopt the kind of catenary section above advocated.

APPENDIX E

Rear dome of pressure-cabin

In considering the design of the rear dome of a pressure cabin, the objective is to achieve a minimum weight for the dome itself and a minimum amount of interference stresses at the junction of dome and cabin walls.

As a matter of academic interest it may be proved that a circular opening with completely rigid edges is most exonomically closed by a 60° spherical cap as shown in Fig.1E. Here the rigid circular edge AA' applies the tension T that is the constant membrane tension in the cap. The cabin wall is adequate to contribute the horizontal component of T but the vertical component must be provided by an outside agency if the membrane stresses are to be preserved in the cap. The true loads in dome and wall are obtained by adding the stresses just noted to those produced by an inwardly applied radial load T cos α around the circle AA'. Unless a very heavy ring is fitted at A, the stresses and displacements that such an inward load would produce, are quite unacceptable - hence the academic character of this type of solution. A hemispherical dome, as shown in Fig.2E does away with the unbalanced vertical component of the membrane stress, and weighs only a little more. The only problem left is that of connecting together the dome, the cylindrical shell, and the aft portion of the fuselage to the best advantage.

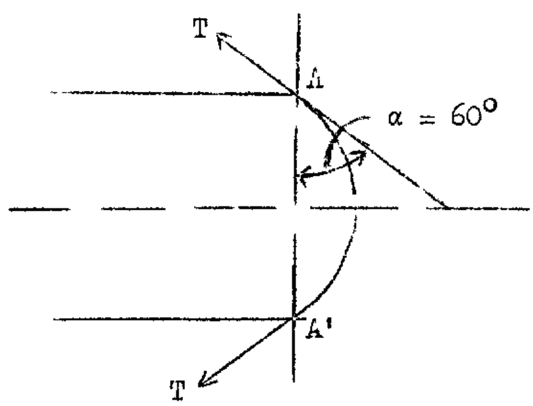

Fig.1E

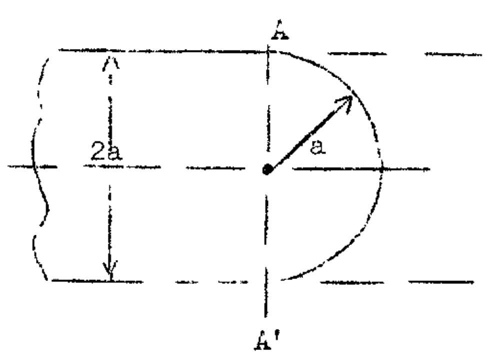

Fig.2E

One essential condition of the joint is that it shall allow no radial offset between these three components. Another is that there must not be any reduction in the elastic stability of the aft fuselage (i.e. the unpressurised fuselage aft of section AA' of Fig.2E).

The arrangement visualised is that shown roughly in Fig.3E.

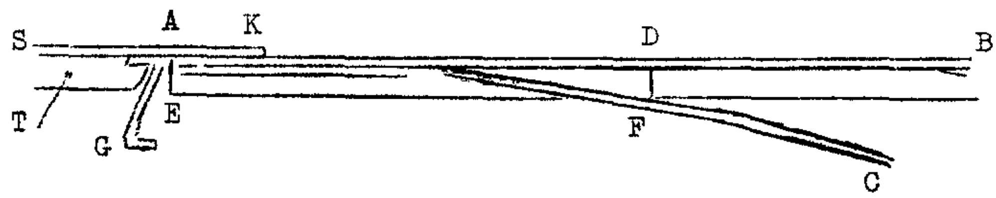

Fig. 3E

Here the cabin skin SK with its reinforcing stringer T and aftermost ordinary former-frame G projects a little beyond the frame in order to pick up the dome skin AFC and the aft-fuselage skin ADB. Short stringers AEF, similar in pitch and section to the forward stringers, are fastened to the inner face of the dome. The inner edges EF of these stub-stringers are parallel to the longitudinal axis of the cabin, so that they taper from full-section at AE to nothing at F.

Another set of stub-stringers ADF, their outer edges in line with the forward skin, are fastened to the outer face of the dome and terminate at D, where they attain full depth of section after starting from nothing in the neighbourhood of K. Aft of D the stringer-reinforcement for the fuselage skin is of the standard type. Connection between aft fuselage-skin and the curved dome - additional to the direct fastening over AK - is made by finally attaching the otherwise unreinforced length of skin KD to the outer faces of the stringers KFD.

The object of the arrangement is to ensure that the aft fuselage skin over the region KD is adequately stabilised against buckling under end-loads, without introducing any radial offsets at the joint AK, thus satisfying the two conditions already set down. The size of the stub-stringers should be such that the longitudinal bending stiffness aft of section A is maintained at approximately the same value as that forward of A.

Numerical values

If we imagine the cabin cut through at A while the internal pressure and longitudinal tension are maintained, the forward shell at A will have a greater radial expansion than the dome and rear fuselage at A.

For the forward shell, the radial deflection (in the absence of a frame) is

$$w_1 = \frac{pa^2}{Eh}\left(1 - \frac{\nu}{4}\right) = 0.095 \text{ in.} \tag{1E}$$

as already found.

For the dome unconnected to the aft-fuselage skin

$$w_2 = \frac{pa^2}{2Eh}\left(1 - \frac{\nu}{2}\right) \tag{2E}$$

($\nu/2$ being taken instead of ν because about half the longitudinal tension is accounted for by the stub-stringers).

For the aft-fuselage, supposing it resisted the pressure by itself,

$$w_3 = \frac{pa^2}{Eh} \qquad (3E)$$

Thus,

$$\frac{\text{Radial stiffness of aft fuselage + dome}}{\text{radial stiffness of dome alone}}$$

$$= 1 + (1 - \nu/2)/2 = 1.44 \qquad (4E)$$

With dome and aft-fuselage connected, therefore, the deflection w_4 is given by

$$w_4 = \frac{w_2}{1.44} = \frac{pa^2}{2Eh}\left(1 - \frac{\nu}{2}\right) \div 1.44 = 0.03 \text{ in.} \qquad (5E)$$

(assuming the dome skin thickness to be the same as that of the cabin walls i.e. 0.036 in.).

In the above we have assumed (justifiably for the ratio a/h here considered) that the longitudinal bending deflections of the dome in the region AD are nearly the same as they would be if the dome were replaced by a cylinder.

To re-establish continuity of displacement at A, a radial loading and a moment per unit length of arc has to be applied to the forward shell at A and equal but opposite loading and moment to the dome aft of A.

For the forward shell

$$\beta_s^4 = \frac{Eh_o}{4a^2D_o} = (0.14)^4 \qquad (6E)$$

where h_o and D_o refer to the forward shell.

For the dome (and aft fuselage)

$$\beta_d^4 = \frac{E\,2h_o}{4a^2D_o} = 2\,\beta_s^4 \qquad (7E)$$

Since D is the same for both cases.

If now

w_s = inward deflection of shell relative to w_1

w_d = outward deflection of dome relative to w_4

we must make $$(w_s + w_d) = (w_1 - w_4) \qquad (8E)$$

Also if

i_s = inward slope of shell edge from horizontal (approaching joint)

i_d = outward slope of dome edge from horizontal (" ")

$$i_s = i_d \qquad (9E)$$

Now the deformations of the open ends of a shell and dome under the loading shown in Fig.4E is given* by

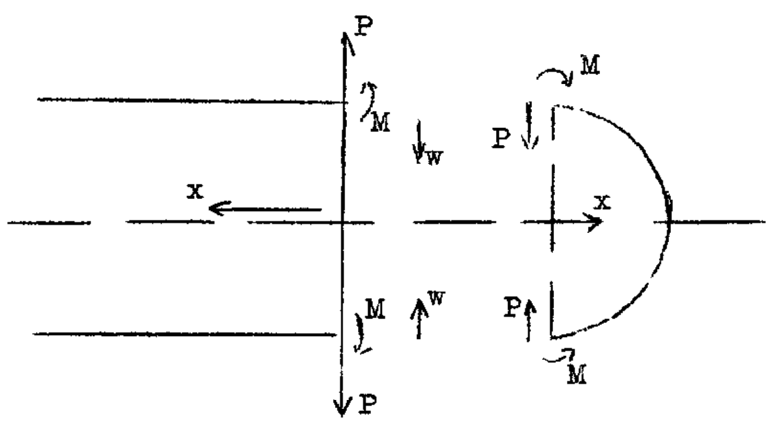

Fig.4E

$$w = \frac{e^{-\beta x}}{2\beta^3 D}\{\beta M(\sin\beta x - \cos\beta x) - P\cos\beta x\} \qquad (10E)$$

for the shell, and the same expression but with the sign of P reversed for the dome. From this it follows that, at x = 0,

$$w = -\frac{1}{2\beta^3 D}(\beta M + P) \qquad (11E)$$

(positive inward)

$$\frac{dw}{dx} = \frac{1}{2\beta^2 D}(2\beta M + P) \qquad (12E)$$

for the forward shell.

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* See Ref.1 equation 232

Thus, using (6E) and (7E) with (11E) and (12E) we have

$$w_s = -\frac{1}{2\beta_s^3 D_o}(\beta_s M + P) \qquad (13E)$$

$$w_d = \frac{1}{2\beta_d^3 D_o}(\beta_d M - P) \qquad (14E)$$

and therefore, from (8E),

$$w_s + w_d = \frac{1}{2D_o}\left\{\left(\frac{1}{\beta_d^2} - \frac{1}{\beta_s^2}\right) M - \left(\frac{1}{\beta_s^3} + \frac{1}{\beta_d^3}\right)P\right\}$$

$$= w_1 - w_4 = 0.095 - 0.03 = 0.065 \text{ in.} \qquad (15E)$$

Putting $D_o = 0.006E$, $\beta_s = 0.14$, $\beta_d = 0.168$,

we have

$$1.3\ M + 48\ P = -650 \qquad (16E)$$

Taking the slope dw/dx next, we have

$$i_s = -\frac{dw_s}{dx} = -\frac{1}{2\beta_s^2 D}(2\beta_s M + P) \qquad (17E)$$

$$i_d = \frac{dw_d}{dx} = \frac{1}{2\beta_d^2 D}(2\beta_d M - P) \qquad (18E)$$

From (9E) therefore

$$-(2\beta_s M + P)/\beta_s^2 = (2\beta_d M - P)/\beta_d^2 \qquad (19E)$$

which, on substituting for β_d and β_s gives

$$26.2M + 15.5\ P = 0$$

or

$$P = -1.7\ M \qquad (20E)$$

Substituting this in (16E), we find

$$\left.\begin{aligned} M &= 8.2 \text{ lb in/in.} \\ P &= -4.8 \text{ lb/in.} \end{aligned}\right\} \qquad (21E)$$

Thus the bending moment at the common section is the reverse of a hogging moment.

The bending moment distribution in shell and dome near the common section (hogging moment positive) is given* for the forward shell by

$$D \frac{d^2 w}{dn^2} = - M \varphi (\beta x) - \frac{P}{\beta} \cdot \zeta (\beta x) \qquad (22E)$$

where

$$\left.\begin{aligned} \varphi(\beta x) &= e^{-\beta x} (\cos \beta x + \sin \beta x) \\ \zeta(\beta x) &= e^{-\beta x} (\sin \beta x) \end{aligned}\right\} \qquad (23E)$$

and by the same equation, with the sign of P reversed, for the dome. The functions φ and ζ are conveniently tabulated in ref.1.

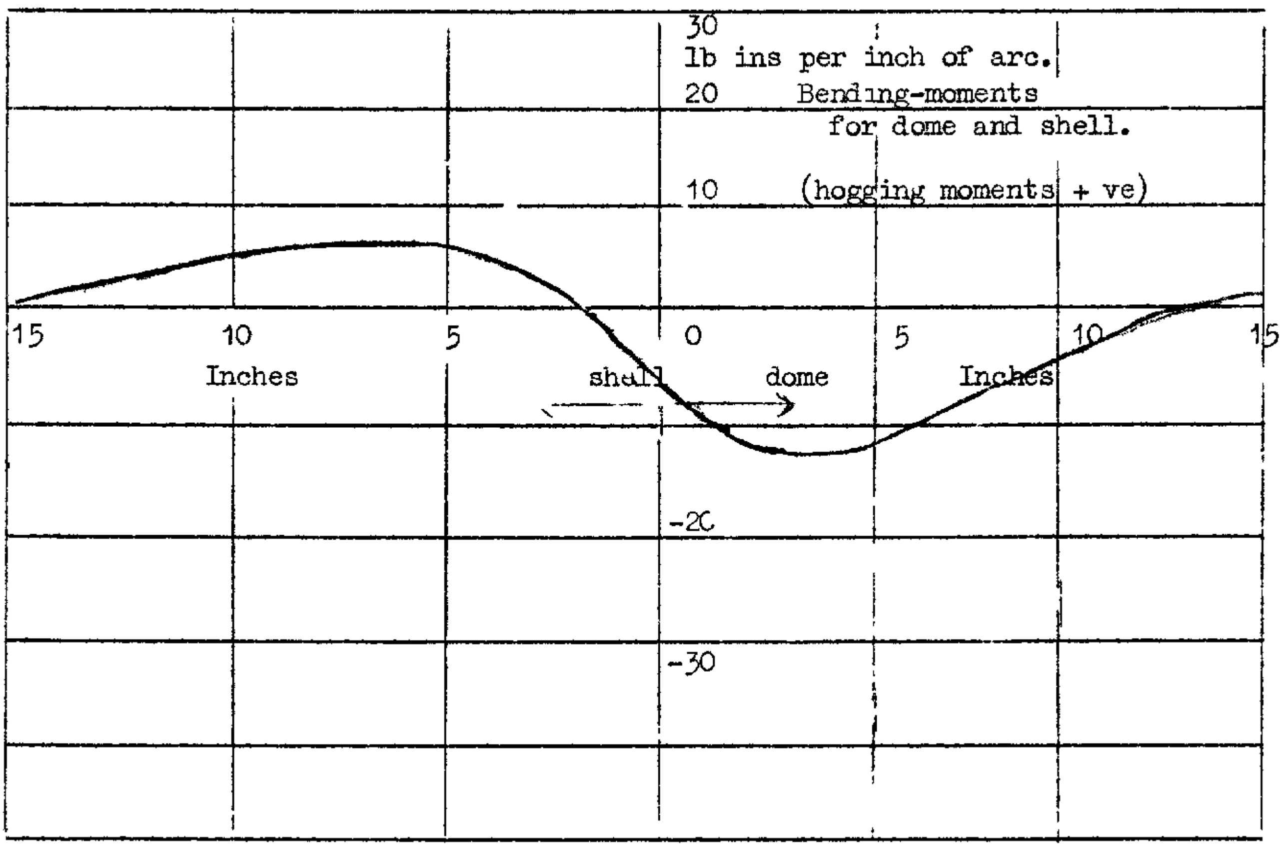

Fig.5E

Fig.5E shows how the bending moment varies each side of the common section. It is seen that the greatest bending moment occurs in the dome - 15 lb in. per inch of circumference - and, since the stiffness D has been assumed the same for shell and dome, the greatest bending stress also. This is satisfactorily small, having the value

$$\sigma_{max} = 1{,}250 \text{ lb/in.},$$

* Ref.1 equation (236)

the corresponding maximum stress in the shell being 550 lb/in^2. The addition of the longitudinal tension stress of 4,700 lb/in^2 brings these stresses up to 5,950 and 5,250 lb/in^2 respectively.

Effect of frame at common section

Nothing more than an ordinary former frame seems to be required at the common section of shell and dome.

It slightly increases the inward radial deflection of the shell and reduces the outward-deflection of the dome. Its effect is small and can easily be found by the method discussed in Appendix D.

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