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Calibration of the R.A.E. No. 18 (9 in. x 9 in.) Supersonic Wind Tunnel

Part I. Preliminary Investigations

By

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SUMMARY

A detailed account is given of the investigations performed in the R.A.E. No.18 (9" × 9", continuous flow, variable density) Supersonic Wind Tunnel prior to an extensive calibration of the tunnel. The variables which have an important effect on the behaviour of the flow are discussed, and preliminary experiments to determine their significance are described. The results of the investigations serve to define the course of the complete calibration, and may provide a useful guide to future calibrations of similar supersonic tunnels. The calibration programme is outlined: Part II will deal with tests at atmospheric stagnation pressure, and further tests at various stagnation pressures are proposed.

Note (August 1953)

Since the accounts of the calibration contained in Parts I and II were written (Part I - December 1951; Part II - March 1952), the tunnel equipment has been greatly improved and its limitations are no longer as severe as those described here; the improvements do not invalidate the present results, but no further detailed calibration measurements have been made.

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1 Introduction

The aims of the calibration of the R.A.E. No.18 (9" × 9") Supersonic Wind Tunnel are to establish criteria for ensuring repeatability of results to within a stated accuracy under the same basic conditions, and to provide accurate illustrations of the flow in the empty working section, for the full ranges of nominal Mach number and Reynolds number, giving explanations of the results wherever possible. Though much of the work will be peculiar to this particular tunnel, information relevant to the operation and performance of all supersonic wind tunnels of similar type should be obtained.

The flow in the empty working section will be expressed in the form of the Mach number distributions in several longitudinal traverses spaced across the working section to give an adequate representation of the behaviour of the flow. No measurements of flow direction are contemplated at present.

The accuracy of a calibration determines the limiting accuracy of tests performed in the tunnel, and thus calibration requires a close scrutiny of all the factors which may affect the results. Since there are many such factors, preliminary experiments must be made to discover their significance. This report describes the investigations which have been made (up to September 1951) in developing the technique for obtaining the highest accuracy with the present equipment.

In Section 2, the tunnel and equipment are briefly described. In Section 3, the factors which are known to influence the behaviour and measurement of the flow in the empty working section are discussed, and preliminary experiments which have been made to investigate the effects of these factors are described. A summary of the important features of the preliminary investigations and the suggested programme for an extensive calibration are included in the Conclusion; an initial reading of the Conclusion would help as a guide to the scope of the main text.

2 Description of tunnel and equipment

2.1 Tunnel

The R.A.E. No.18 (9" × 9") Supersonic Wind Tunnel is a continuous flow, closed circuit, variable density tunnel with a 9" square working section, bounded by parallel side walls, a flat top wall and one shaped liner. The tunnel is powered by a 1000 k.w. D.C. motor driving a two-stage centrifugal compressor. Liners are available to give nominal Mach numbers $\bar{M}$ of 1.3, 1.4, 1.5, 1.6, 1.8, 1.9 and 2.0*, though use of the tunnel at $\bar{M} = 1.3$ is restricted by the small model area required to produce choking, and compressor limitations may prevent the establishment of flow at $\bar{M} = 2.0$. The Reynolds number Re in the working section, which may be expressed in terms of $\bar{M}$ and the stagnation pressure and temperature in the reservoir, has a range of roughly 0.1 to 0.7 millions per inch. The nominal stagnation pressure $\bar{p}_0$ may be varied between about 5" and 45" of mercury absolute, and the nominal stagnation temperature $\bar{T}_0$ is usually controlled between 15°C and 45°C. It is possible to attain an absolute humidity $\bar{\Omega}$ of 0.0001 lb water per lb air at atmospheric stagnation pressure, corresponding to a dew-point of about -40°C.

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* Consideration is being given[1] to the design of a liner to give $\bar{M} = 1.7$.

The control of the above parameters may be outlined as follows. $\bar{M}$ is determined by the choice of shaped liner. $\bar{p}_0$ is adjusted by balancing the effect of two pumps, used either as exhausters or compressors, with the leak into or out of the tunnel through regulator valves. $\bar{T}_0$ is controlled by varying the flow of cooling water through a battery of tubes installed upstream of the reservoir. $\bar{\Omega}$ is maintained as low as possible by evacuating the tunnel of wet air and introducing fresh air through two silica gel driers; then, to complete the drying and to counteract the effect of leaks into the tunnel during operation, a proportion of the air is continuously by-passed through the driers, which require periodic reactivation.

Schlieren and shadowgraph systems are available for flow observation.

The tunnel circuit and working section are shown diagrammatically in Figs.1 and 2.

2.2 Pressure traversing gear

2.21 Pitot shower

The instrument used in measuring the pressures throughout the working section is a pitot* shower, consisting of nine tubes placed symmetrically along the sides and at the centre of a square of side 6" (see Figs.3 and 4). Thus, a single longitudinal traverse provides representative distributions of the pitot pressure throughout the working section. The pitot shower is detachable from the traversing gear. The reference numbers of the tubes are shown in Fig.3. It is possible to traverse in intervals of any desired size over a length of 9" in a single traverse, and by mounting the gear in the tunnel in two positions 9" apart a total length of 18" of the working section may be investigated. The scheme of the pressure traverses is shown diagrammatically in Fig.5.

In the original pitot shower the central (No.5) tube was made $\frac{3}{4}$" longer than the others to prevent the pressure measured by it from being influenced by the disturbance from the central conical support. Thus the No.5 tube recorded the pressure $\frac{3}{4}$" upstream of the outer tubes. For the actual calibration, the shower has been redesigned so that all pressures are measured in the same plane.

The pitot tubes are made from 2 m.m. hypodermic tubing sawn off perpendicular to the axis, the external diameter being 0.078" and the internal diameter being 0.058". (Check tests showed that the readings of this tubing, which has an internal to external diameter ratio of $\frac{3}{4}$, were identical with the readings of 2 m.m. hypodermic tubing with an internal to external diameter ratio of $\frac{1}{2}$, which is the usual criterion). The pressure holes are 1" ahead of the supports.

2.22 Micrometer pitot tube

An instrument which has been used for measurements in the boundary layer on the side wall in the working section is the micrometer pitot tube. This consists of a pitot tube constructed from a length of 1 m.m. hypodermic tubing, with a head flattened to an external width of 0.017" and internal width of 0.009", which is supported inside a double wedge of

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* Originally a static shower was also designed, but an analysis of the relative merits of pitot and static tubes for $1.4 < \bar{M} < 1.9$ (given in full in Section 3.3) showed that pitot tubes were to be preferred; since no measurements have been made with a static shower, descriptions of it will be omitted to avoid confusion.

semi-angle $10\frac{1}{2}^{\circ}$ (see Fig.6). The micrometer pitot tube is mounted eccentrically in a movable circular wall plate, containing suitably placed static holes of 0.024" diameter, so that, by rotating the plate and pitot tube, pressures traverses at several points may be made (see Fig.7). It may be projected in steps of the order of 0.001", and has now been extended to have a total traversing length of 9" so that it can reach to the opposite side wall. The instrument may therefore be used for measurements in the boundary layer on the near wall, when projecting from the wall, and in the boundary layer on the far wall, when projecting from the free-stream, or for free-stream measurements.

2.3 Manometers and arrangements for pressure measurement

2.31 Manometers

The present equipment for pressure measurement consists of:- two recently calibrated U-tube mercury-in-glass barometers, reading absolute pressure from 0 to 50" to an estimated accuracy of ± 0.02" of mercury; two banks of ten mercury manometers each, reading differences of pressure from a specified standard to within ± 0.03" of mercury; one bank of ten water manometers reading differences of pressure from a specified standard to within ± 0.05" of water; one U-tube water manometer measuring pressure difference to within ± 0.05" of water; six U-tube mercury manometers, reading pressure differences to within ± 0.02" of mercury.

2.32 Pressure measurement with pitot shower

In using the pitot shower, the various pressures are measured as indicated in Fig.8. If p_o denotes the stagnation pressure, and p'_{o_n}, $n = 1 \ldots\ldots 9$, denotes the pressure of the n^{th} pitot tube, then p_o and p'_{o_5} are measured on the U-tube mercury barometers, and p'_{o_n}, $n = 1.4, 6 \ldots\ldots 9$, are balanced against p'_{o_5} on the bank of water manometers. The estimated measuring errors are ± 0.02" of mercury in p_o and p'_{o_5}, and ± 0.05" of water in $(p'_{o_n} - p'_{o_5})$.

2.33 Pressure measurement with micrometer pitot tube

In using the micrometer pitot tube, the stagnation pressure and pitot pressure are measured on the U-tube mercury barometers, and the static pressure at the wall, assumed constant through the boundary layer, is measured on a mercury manometer balanced against atmospheric pressure. No errors have been estimated of the results to be quoted from some rough tests made with the micrometer pitot tube.

3 Discussion of important factors and preliminary experiments

We have stated the aims of the calibration to consist of the establishment of criteria for obtaining results which may be repeated to within a specified accuracy under the same basic conditions, and the production of detailed Mach number distributions throughout the empty working section, with explanations of the results wherever possible. A preliminary discussion may therefore be divided into three parts*: an investigation of how much explanation it is sensible to attempt, by considering some of

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* The sequence of the preliminary investigations was not as outlined here, but this methodical presentation seems to have advantages over a chronological list of experiments.

the sources of the disturbances which contribute to the non-uniformity of the flow in the working section of this particular tunnel, in fact a tentative determination of the upper limit of the accuracy of the calibration; an examination of the basic variables in any test performed in the tunnel and the establishment of their critical values to ensure the repeatability of the results of any test which is carried out with sufficient care; a discussion of the details involved in producing comprehensive Mach number distributions to within an estimated error.

3.1 Some causes of non-uniform flow

We shall distinguish between the words "nozzle" and "liner": "nozzle" will be used to denote the shape which produces a theoretical uniform inviscid isentropic irrotational flow, whilst "liner" will refer to the actual manufactured article for simulating that flow in practice; the co-ordinates of a liner are obtained from the co-ordinates of the corresponding nozzle by allowing for the growth of the boundary layers along the liner and the walls.

The liners for the 9" × 9" tunnel were obtained by directly scaling up corresponding liners designed for and tested in a 3" × 3" tunnel; this was possible since the boundary layer correction was assumed to be linear. The basic liners were designed as follows[2,3]. The two-dimensional method of characteristics was used to draw a nozzle for a Mach number $\bar{M}$, say. By suitable scaling to fit the 3" × 3" tunnel and by making an appropriate allowance for the growth of the boundary layer, a liner shape for Mach number $\bar{M}$ was deduced.

This liner would not be expected to give a constant measured Mach number $\bar{M}$ in the working section of the 3" × 3" tunnel, since: the method of design of the nozzle was only approximate and based on two-dimensional theory whereas the actual flow was three-dimensional; the applied linear boundary layer correction was unlikely to be correct and, moreover it was applied for one value of Reynolds number only whilst the same liner was to be used over the full range of Re; the manufactured liner would not conform exactly to the shape of the designed liner: the pressure measurements on which the values of Mach number depended probably involved an appreciable measuring error. If the measured Mach number distribution was still unsatisfactory when, presumably, allowance for the measuring error was made, the liner was redesigned[2,3] to improve the distribution as much as possible.

However, some of the unwanted disturbances which were reduced in the redesign may have been peculiar to the 3" × 3" tunnel alone. Since the flow in the 9" × 9" tunnel probably possesses its own peculiarities, the Mach number distribution produced there by the scaled-up redesigned liner may differ from the accepted one in the 3" × 3" tunnel, and whatever connections between the liner shape and the measured Mach number distribution did exist may be lost. Such three-dimensional effects as disturbances from the window junctions and inaccuracies in the top and side walls may be different in the 9" × 9"-tunnel from the 3" × 3" tunnel and the measuring error may also be different.

We may therefore infer from this discussion of some of the causes of the non-uniform flow in the working section of the 9" × 9" tunnel that it is a hopeless task to try to trace the history of "small" disturbances; the order of "smallness" is not clear yet, but an idea of it should emerge from the calibration. The lower limit of "smallness" is obviously defined by the accuracy of measurement, but there may be small but measurable disturbances which may not be traced back even to their physical source (the appropriate bump or dent) and certainly not to their design origin. Thus, in particular, an attempt to correlate the shape of a manufactured liner with the measured pressure distribution is unlikely to succeed; only relatively large disturbances, such as should not occur, will be identifiable.

The ideal procedure in providing a liner for a given supersonic tunnel would be to extract the corresponding nozzle from an accepted catalogue of supersonic nozzles covering a large range of Mach number*, adapt the size of the nozzle to fit the particular tunnel, apply the boundary layer correction to give the co-ordinates of the designed liner and from them construct the basic manufactured liner. This would then be tested under the expected standard working conditions, and a method of redesign employed to improve the measured Mach number distribution, although some of the disturbances that would be reduced might not, in fact, be due to faults in the basic liner itself.

The bearing of this discussion on the present calibration is to restrict the aim of providing explanations of the results to indicating the sources of "comparatively large" disturbances only; we shall try to measure what is actually happening in the working section under a given set of conditions, without trying to explain precisely why it happens. Nevertheless, the calibration should bring to light information which may be useful in attempting such an explanation in future.

3.2 Criteria for repeatability of results

3.21 Basic variables

The important variables which define the basic conditions of any test are the nominal Mach number $\bar{M}$, the nominal stagnation pressure $\bar{p}_0$, the nominal stagnation temperature $\bar{T}_0$ and the dryness of the air expressed in terms of the nominal absolute humidity $\bar{\Omega}$. Here, $\bar{p}_0$ is measured in inches of mercury, $\bar{T}_0$ in $^{\circ}$C, and $\bar{\Omega}$ in lb of water content per lb of air. (The Reynolds number Re is defined by the relation

$$\frac{Re}{d} = 4.05\ \bar{p}_0 \left[\frac{\bar{T}_0+393}{(\bar{T}_0+273)^2}\right] \left[1 + \frac{24\ \bar{M}^2}{(\bar{T}_0+393)}\right] \left[\bar{M}(1+0.2\ \bar{M}^2)^{-\frac{5}{2}}\right] \times 10^6 \qquad (1)$$

where $\frac{Re}{d}$ is the Reynolds number per inch; it is shown in Appendix I that for $15 \leq \bar{T}_0 \leq 45$ and $1.4 \leq \bar{M} \leq 1.9$ the approximation

$$\frac{Re}{d} = 0.0204\ \bar{p}_0\ (1 - 0.004\ \bar{T}_0)\ (1 - 0.2\ \bar{M}) \times 10^6 \qquad (2)$$

is correct to two figures.) Other variables which may have some effects are the compressor speed ω r.p.m. and the room temperature T_R $^{\circ}$C.

Repeatability of results may reasonably be expected only for the same values of each of these basic variables. In the case of the calibration, we are seeking the distributions of measured Mach number M for specified nominal values $\bar{M}$, $\bar{p}_0$, $\bar{T}_0$ and $\bar{\Omega}$; we must, therefore, consider the possible influence on $\bar{M}$ of the method of fixing the liner, and examine how the measurement and control of the stagnation pressure p_0, the stagnation temperature T_0 and the absolute humidity Ω vary with their nominal values.

3.22 Method of liner installation

The nominal Mach number $\bar{M}$ is governed by the choice of liner, and it is essential to ensure that each liner is fixed in its correct position each time it is used. The method employed in this calibration is to scribe fine lines on the side wall of the tunnel to denote the design positions of the throats and the ends of each liner, and to shim up the liners where necessary until the desired positions are attained

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* Such a catalogue would be a tremendous asset; a more systematic treatment of the problem of nozzle design is desirable[1].

(see Fig.9). Occasional checks on this simple procedure are sufficient to allow for possible shrinkage of the liners; the possible shrinkage of the top wall may be allowed for only by a much more complicated procedure.

We may make a quantitative estimate of the error involved in an inaccurate setting of a liner by assuming the one-dimensional flow relation between the throat area and the working section area. For an error of $\pm 0.005''$ in the positions of the throats and the ends of the liners, the corresponding error in mean free stream Mach number is $\pm \Delta M'$, where $\Delta M'$ is given in Table I. The analysis is set out in

TABLE I

$\bar{M}$	$\Delta M'$
1.4	0.0028
1.5	0.0021
1.6	0.0020
1.8	0.0018
1.9	0.0018

Appendix II. Provided that the longitudinal position of the liner is fixed (see Fig.9), the errors in the throat positions are of the order of $\pm 0.002''$ in this method of liner installation, and so the corresponding errors in Mach number may be taken to be ± 0.001 at all Mach numbers.

3.23 Measurement and control of stagnation pressure

The stagnation pressure p_o is measured by a static hole in the wall of the reservoir. We have no knowledge of the variation of total head across the reservoir. In evaluating the Mach number at a point in the working section from the pitot pressure there, we assume that the total head at the point is equal to that measured in the reservoir. This is standard practice, and is necessitated by our inability to measure the total head at a point in a supersonic flow or the pitot and static pressures simultaneously at one point. Estimates of the possible loss of total head between the reservoir and the working section, and the effect of a variation of total head across the reservoir, would be useful.

p_o may be controlled at any value between 5" and 45" of mercury by balancing the effect of two pumps working either as exhausters or compressors with the leak into or out of the tunnel. It is easy to keep p_o constant at atmospheric pressure, but at other values there may be considerable difficulty, since it is not always possible to adjust the leak to atmosphere with sufficient delicacy to obtain an exact balance, even when the compressor speed is constant, and small fluctuations in the compressor speed upset the balance temporarily.

The variation of p_o during a test at $\bar{p}_o$ other than atmospheric is illustrated by a pitot shower traverse at $\bar{p}_o = 20$ with $\bar{M} = 1.6$. Fig.10 shows the variation of p_o through the test, (a typical variation at atmospheric stagnation would be ± 0.03 about the mean), and gives the distributions of measured Mach number over several inches of the working section for each tube of the pitot shower. Similar disturbances occur at the same times and positions for each tube. To analyse these distributions, let us denote the measured Mach number at the n^{th} tube by M_n, $n = 1.....9$. Then we may write $M_n = M_5 + (M_n - M_5) = M_5 + m_n$, where $m_n = (M_n - M_5)$ and it is shown in Section 3.35 and Appendix III that, to a sufficient order of accuracy, m_n is directly proportional to $(p'_{o_n} - p'_{o_5})$, measured in inches of water, and is independent of small changes of p_o of the order of $\pm 0.20''$ of mercury. Hence, any effects of changes in p_o will be confined to M_5 and transmitted to M_n through M_5. Since no flow disturbances would appear in all tubes at the same time and position, we conclude that the similar disturbances repeated in all the distributions are due to sudden changes in p_o. The results are thus invalid. The effects of these moderate changes in p_o could be eliminated in the Mach number distributions if p'_{o_5} and p_o were steady sufficiently long for their simultaneous values to be clamped; however, this is not possible at stagnation pressures other than atmospheric. Henceforth, for convenience the phrase "atmospheric stagnation pressure" will be abbreviated by $\bar{p}_o = 30$.

Apart from the influence of small changes, of the order of $\pm 0.20''$ of mercury, in p_o, two effects of large changes, of the order of $5''$ of mercury, in $\bar{p}_o$ are important.

It may be seen from equation (2) that over the range of operating conditions, $\bar{p}_o$ has a dominating influence on the Reynolds number. Thus, a large change in $\bar{p}_o$ will change Re and alter the thickness δ of the boundary layer on the tunnel walls, and consequently may affect the mean Mach number in the working section. An example of these effects is provided by a series of rough tests (performed during May 1950) on the boundary layer on one side wall at one point in the working section. At the junction of the contraction with the wooden top and bottom liners and the steel side walls is a joint (AA in Fig.2) which at $\bar{p}_o > 20$ was sufficient to cause transition from laminar to turbulent flow in the boundary layer. For lower values of $\bar{p}_o$ it was possible to maintain a laminar boundary layer well downstream of the joint, with a consequent thinning of the boundary layer in the working section, although the decreased Reynolds number indicated an increase of boundary layer thickness. A graph of δ, measured by the micrometer pitot tube, against $\bar{p}_o$ and Re is shown in Fig.11, and the corresponding states of the boundary layer, indicated by the azo-benzene evaporation technique, are shown in Fig.12. It is clear that the change of transition line is responsible for the contrary variation of δ with Re. To eliminate the uncertainty caused by a transition line which varied with Reynolds number, a length of thin cotton thread was fixed round the end of the contraction just upstream of the junction (see Fig.2) and this was enough to cause transition to turbulence there for all values of Re. A graph of δ against $\bar{p}_o$ and Re with the boundary layer completely turbulent downstream of the contraction junction is given in Fig.13. It is suggested that some means of fixing the position of transition on the tunnel walls should be a feature of all variable density supersonic wind tunnels.

The second important effect of large reductions of $\bar{p}_o$ below atmospheric is that it becomes possible for wet air to leak into the tunnel circuit during operation, with a consequent increase in the humidity. It is necessary to regulate the by-pass opening to the driers in order to keep the air sufficiently dry; the normal opening of 1 turn at $\bar{p}_o = 30$ must be increased to $1\frac{1}{2}$ turns for $\bar{p}_o = 10$, for instance.

In view of the inadequacy of the present tunnel equipment to guarantee that stagnation pressures other than atmospheric may be maintained constant enough for the purposes of the calibration, and the large field of investigation which is opened up by the variation of p_o through the full range, we shall restrict the immediate scope of the calibration to the case of $\bar{p}_o = 30$. Strictly, this discussion of repeatability criteria now applies only to this case. The effect of variation of $\bar{p}_o$ through the range $5 \leqslant \bar{p}_o \leqslant 45$ will be considered later.

3.24 Measurement and control of stagnation temperature

The stagnation temperature T_o is measured by a remote reading mercury thermometer installed near the wall of the reservoir and just downstream of the cooling system.

T_o is controlled by a water cooler, and by suitable regulation of the cooling water it is possible to maintain T_o constant to within $\pm 1^\circ C$ at any value of $\bar{T}_o$ between $15^\circ C$ and $45^\circ C$, at all stagnation pressures, though only for $\bar{T}_o \geqslant 25$ may this constant temperature be maintained throughout a long test.

Equation (2) shows that the variation of Re through this range of $\bar{T}_o$ is small, for constant $\bar{p}_o$ and $\bar{M}$, and we should therefore expect little scale effect to be noticeable in a change of $\bar{T}_o$ through this range. However, such a variation of $\bar{T}_o$ may have an important effect on the formation and strength of condensation shocks. The importance of $\bar{T}_o$ depends on the value of the humidity, so it is preferable to postpone a discussion of its effects until the control and measurement of the humidity have been outlined.

3.25 Measurement and control of humidity

The humidity Ω is measured by a frost-point hygrometer designed for the 9" × 9" tunnel but which should be of use in all supersonic wind tunnels[4].

The air in the tunnel is dried by first evacuating the tunnel of wet air and then introducing fresh air which is dried by entering and being continuously by-passed through two silica gel driers, which require to be reactivated after roughly twelve hours running. The by-pass opening must be adjusted to compensate for any extra leaks into the tunnel at $\bar{p}_o < 30$. The minimum absolute humidity $\bar{\Omega}$ which may be attained at $\bar{p}_o = 30$ is about 0.0001 lb water per lb air, and the variation of Ω during a test is of the same order, except when the driers require reactivation.

With air which is too wet, for a given $\bar{T}_o$, condensation shocks may form in or upstream of the working section. Criteria for the avoidance of condensation shocks in the 9" × 9" tunnel have been deduced from a series of tests on humidity effects performed as part of this calibration and the work is of sufficient merit to warrant publication[5] separate from this main account of the preliminary investigations. The results which are of particular interest in ensuring a repeatable calibration at $\bar{p}_o = 30$ are presented here in Fig.14, which shows the corresponding critical values of maximum $\bar{\Omega}$ and minimum $\bar{T}_o$ for the prevention of condensation shocks for the full range of $\bar{M}$. It may be seen that with $\bar{T}_o = 35$, say, there should be no difficulty in preventing the occurrence of condensation shocks for $\bar{M} = 1.4$, 1.5 and 1.6, but that it may be difficult to avoid humidity effects entirely at $\bar{M} = 1.8$ and 1.9.

The effects of the presence of condensation shocks are illustrated by four separate pitot shower traverses. In Fig.15 are plotted the distributions of measured M_5 and m_n for $\bar{M} = 1.6$, $\bar{p}_o = 30$ and Position I (see Fig.5), in one case when the presence of condensation shocks was suspected and one where the flow was free from condensation shocks; there is little agreement between the distributions of M_5 and m_n; no attempt has been made to trace in detail the condensation effects. In Fig.16 are plotted the distributions of measured M_5 and m_n for $\bar{M} = 1.5$, $\bar{p}_o = 30$ and Position II, under apparently identical conditions free from condensation shocks; the agreement is remarkable.

3.26 Influence of other variables

There appear to be no other variables which have significant effects on the flow in the working section. The variation of compressor speed ω about the appropriate mean alters p_o temporarily and may also affect Ω at $\bar{p}_o < 30$, but since the installation of an automatic speed control (August 1951) which keeps ω constant to within ± 50 in 4000 r.p.m., its effect is negligible at $\bar{p}_o = 30$ and small, though possibly not negligible, at other values of $\bar{p}_o$. Changes of the order of $\pm 2^oC$ in the room temperature T_R are likely to have little influence, though we have no knowledge of the effects of heat transfer through the side walls of the tunnel; whenever an absolute pressure is stated, it is given corrected to $T_R = 15$ for measurement on mercury-in-glass-on-brass.

3.27 Summary

It will help to clarify the general trend of the argument to summarise the important points of this section.

Before defining a procedure for performing an experiment so that we may have confidence in the results, it is necessary to fix the basic conditions under which the test is to be made, that is to specify $\bar{M}$, $\bar{p}_o$, $\bar{T}_o$ and $\bar{\Omega}$, and to know how accurately the actual values of Mach number M, stagnation pressure p_o, stagnation temperature T_o and absolute humidity Ω may be controlled under these conditions. To define the limits of variation of M throughout the empty working section is the purpose of the calibration, and requires knowledge of the limits of control of p_o, T_o and Ω.

$\bar{M}$ depends on the choice of liner, which must be fixed in the same position for each test. The method of liner installation employed here ensures that the error in liner position is of the order of $\pm 0.002''$, which corresponds to a variation of mean Mach number of about ± 0.001 for all $\bar{M}$.

The present equipment of the tunnel does not enable p_o to be controlled with sufficient precision except at the prevailing atmospheric pressure $\bar{p}_o = 30$. Consequently, we shall restrict our immediate attention to the case of $\bar{p}_o = 30$, where the variations in p_o are small, of the order of $\pm 0.03''$ of mercury, and are slow enough for p_o to be measured leisurely. The effects of large variations, of the order of $5''$ of mercury, in $\bar{p}_o$ will be considered later when the tunnel has been calibrated satisfactorily under the simpler conditions associated with $\bar{p}_o = 30$. Two such effects which may be noted now are the variation of the transition lines on the tunnel walls unless transition is forced to take place somewhere well upstream of the working section, and the possibility of increases in Ω due to leakage of wet air into the tunnel at $\bar{p}_o < 30$.

At $\bar{p}_o = 30$, T_o may be controlled to within $\pm 1^\circ C$ for $15 < \bar{T}_o < 45$ for a length of time depending on T_o and which is of the order of two hours for $\bar{T}_o > 25$. The minimum attainable value of Ω is about 0.0001 at $\bar{p}_o = 30$, and the variation of Ω during a test is of the same order, except when the driers are in need of reactivation. It is vital for the repeatability of results that $\bar{T}_o$ and $\bar{\Omega}$ should lie within the critical values governing the formation of condensation shocks at the test values of $\bar{M}$ and $\bar{p}_o$.

The effects of changes of the order of ± 50 in 4000 r.p.m. in the compressor speed ω and slight variations of $\pm 2^\circ C$ in the room temperature T_R , are neglected at $\bar{p}_o = 30$.

We should now expect repeatability of the results of any tests performed under the same basic conditions of $\bar{M}$, $\bar{p}_o = 30$, $\bar{T}_o$ and $\bar{\Omega}$ which conform with the derived criteria; the closeness of the repeatability will clearly depend on the amount of care taken in the tests.

3.3 Technique for detailed calibration at atmospheric stagnation pressure

We shall now discuss the technique for obtaining Mach number distributions in the empty working section at atmospheric stagnation pressure, paying particular attention to the relative merits of pitot and static tubes*. It is likely that most of the discussion will apply to pressure traverses at other stagnation pressures. We do not expect to be able to account for "small" variations in Mach number, but "comparatively large" disturbances should easily be traced. The present task is to decide the technique which will restrict the "small" variations we cannot measure to the estimated minimum.

3.31 Some sources of error

In addition to the effect of slight changes in the basic conditions (basic error), errors might arise from the dimensions of the pitot and static showers (instrument error), from inaccuracies in setting the positions of the showers (position error), and from the errors in measuring the pressures when applied in the Mach number formulae (measuring error).

We shall distinguish three values of the Mach number at a point under the same basic conditions at a nominal Mach number $\bar{M}$: $\tilde{M}$, the mean Mach number throughout any number of runs; M' , the actual Mach number in one particular run; M , the measured value of M' in one particular run, the total head being assumed constant between the reservoir and the working section. All values of M for several distinct runs should be related with $\tilde{M}$ by $|M - \tilde{M}| < \Delta\tilde{M}$, say, where $\Delta\tilde{M}$ is to be kept as low as possible. The difference between $\tilde{M}$ and M', expressed by $|\tilde{M} - M'| < \Delta M'$, is due to the basic error. The difference between M' and M , expressed by $|M' - M| < \Delta M$, is due to the combination of the instrument, position and measuring errors. It is clear that the lowest value of $\Delta\tilde{M}$ we may take is given by $\Delta\tilde{M} = \Delta M' + \Delta M$.

3.32 Basic error

The only contribution to the basic error at $\bar{p}_o = 30$ is the variation of $\tilde{M}$ caused by inaccuracies in the method of liner fixing, since we can show that the variations of p_o , T_o , Ω and ω are negligible.

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* This section provides the justification for using only a pitot shower in the calibration.

We consider the Mach number to be indicative of the flow at any point, and provided the stagnation pressure p_o and the corresponding pitot pressure p'_o and static pressure p are measured simultaneously, the calculated Mach number should be independent of the small fluctuations in p_o during a run at $p_o = 30$.

Variations of T_o of the order of $1^{\circ}C$ are unlikely to affect the pressure measurements provided they do not cause the onset of condensation shocks.

No calibration tests will be performed under conditions where condensation shocks are likely to appear before the end of the test because the driers require reactivation, though it may be difficult to avoid condensation shocks altogether at $\bar{M} > 1.8$. The effects of small changes of the order of 0.0001 in Ω below the critical value are negligible.

Small variations of ω are related with those of p_o and Ω, but have no effect at $p_o = 30$ if corresponding pressures are measured simultaneously.

Hence, the basic error $\pm \Delta M'$ is due solely to the error in liner position and is about ± 0.001 for all $\bar{M}$.

3.33 Instrument error

Some recent systematic tests[6,7] on the influence of shape and size on the readings of pitot tubes at $\bar{M} = 1.6$ and 1.8 and on static tubes at $\bar{M} = 1.6$ indicate that the reading of a pitot tube is independent of the ratio of internal to external diameter if it lies between $\frac{1}{2}$ and $^1/16$, and that the reading of a static tube is independent of the nose shape and of the position of the static holes if the holes are more than ten diameters behind the shoulder of the nose and provided that the boundary layer does not thicken within one or two diameters behind the holes. It is therefore possible to design pitot and static showers so that each tube reads the true pitot or static pressure at its position. Since the tubes in the pitot shower described in Section 2.21, which have an internal to external diameter ratio of $\frac{3}{4}$, give the same readings as tubes of internal to external diameter ratio of $\frac{1}{2}$, we conclude that we may have faith in their readings. The instrument error of the pitot shower may therefore be taken as zero.

3.34 Position error

The points where pressures are to be measured are defined nominally, and for the calibration to be exact the pressure holes should be placed precisely at those points and with the tubes pointing directly upstream. There are thus two possible sources of error here in an actual test: the pressure holes may not be placed precisely at their nominal positions, and the tubes may be yawed with respect to the direction of flow or may be vibrating when the pressure is measured.

The error due to the inaccurate position of the tubes will be small in regions where the flow is approximately uniform, and where large disturbances of the flow do occur the effect of a slight error in position will be obvious and the error in measured Mach number thus accounted for. However, to eliminate the necessity to correct for inaccuracies in position, it is better to take care to note the true positions of the tubes.

An idea of the error due to yaw was given by some tests on the pressures recorded by yawed pitot and static tubes, which showed, in agreement with previous tests[6,7], that for angles of yaw up to 12° the effect on the reading of a pitot tube was negligible, whilst the effect at an angle of yaw of 8° on the reading of a static tube at $\bar{M} = 1.6$ was as much as 0.20" of mercury. When tubes are vibrating slightly during the measurement of pressures, it is assumed that the steadiness of the readings on the water manometers indicates that the tubes are, in fact, recording the mean pressures at the appropriate points.

We may therefore take the position error of a pitot tube to be zero, but the position error due to a yaw of 8° on a static tube is of the order of 0.020 in Mach number at $\bar{M} = 1.6$.

3.35 Measuring error

When absolute pitot and static and stagnation pressures may be measured to within ± 0.02" of mercury, which would be the case in this calibration, it follows[8] that the measuring errors $\pm \Delta M$ to be expected in M at $\bar{p}_o = 30$ are of the order given by Table II. Thus the measuring error in using static pressures is about half that in using pitot pressures for $\bar{M} = 1.4$ and 1.5 , whilst for $\bar{M} \geq 1.6$ there is nothing to choose between them.

TABLE II

$\bar{M}$	ΔM using pitots	ΔM using statics
1.4	0.005	0.002
1.5	0.004	0.002
1.6	0.003	0.002
1.8	0.003	0.003
1.9	0.003	0.003

These estimated errors apply to measurements of absolute Mach number only. However, in using the pitot shower, the Mach numbers M_n may be written $M_n = M_5 + m_n$, and the above errors will apply to M_5 only and m_n may be evaluated simply and with a much smaller error. The pressure differences $(p'_{o_n} - p'_{o_5})$ may be measured to within ± 0.05" of water, and it is shown in Appendix III that pressure differences $(p'_{o_n} - p'_{o_5})$ of the order < 4" water may be converted into Mach number differences m_n to within ± 0.001 by multiplying by a constant factor K for each nominal Mach number and stagnation pressure. For $\bar{p}_o = 30$, the variation of K with $\bar{M}$ is given in Table III.

TABLE III

$\bar{M}$	K
1.4	-0.0098
1.5	-0.0077
1.6	-0.0066
1.8	-0.0055
1.9	-0.0053

3.36 Comparison of merits of pitot and static tubes

The basic error is the same for both pitot and static tubes, and the instrument error of well-designed tubes is zero for both. However, the position error is much less for a pitot tube than a static tube because of the latter's greater susceptibility to yaw, and the order of the position error for a static tube is so large as to swamp the advantages of using a static tube at $\bar{M} = 1.4$ and 1.5, where the measuring error for a static is about half that for a pitot. Since we have no measurements of flow direction, and it is difficult to maintain each separate tube of a static shower absolutely rigid, it is clear that pitot pressure measurements should provide the better basis for calculating Mach numbers - certainly for $\bar{M} \geqslant 1.6$, and probably also for $\bar{M} = 1.4$ and 1.5. Consequently, the calibration is confined to the measurement of pitot pressures alone at all values of $\bar{M}$. It may be possible in the future to make a sufficiently sturdy static shower which combined with measurements of the flow direction will provide results for comparison with those obtained by using the pitot shower.

3.37 Summary

We distinguish three values of the Mach number at a point under the same basic conditions at a nominal Mach number $\bar{M}$: $\tilde{M}$, the mean Mach number throughout any number of runs; M', the actual Mach number in one particular run; M, the measured value of M' in one particular run, the total head being assumed constant between the reservoir and the working section. All values of $\tilde{M}$ for several distinct runs should be related with $\tilde{M}$ by $|M - \bar{M}| \leqslant \Delta\bar{M}$, where $\Delta\bar{M}$ is to be kept as low as possible.

The difference $\pm\Delta M'$ between $\tilde{M}$ and M' is due to lack of exact control over the basic conditions. We consider that at $\bar{p}_o = 30$ the only variation in basic conditions likely to contribute to $\Delta M'$ is the inaccuracy in the fixing of the liner; this gives $\Delta M' = 0.001$ for all $\bar{M}$.

The difference $\pm\Delta M$ between M' and M, for a well-designed instrument, is due to the errors in setting the positions of the pressure tubes and those inherent in the formulae from which the Mach number is deduced. For a pitot tube, the position error is negligible and the only contribution to ΔM is the measuring error which varies from 0.005 at $\bar{M} = 1.4$ to 0.003 at $\bar{M} = 1.9$. For a static tube, the position error due to a yaw of 8^o at $\bar{M} = 1.6$ is about 0.020 in Mach number, whilst the measuring error varies from 0.002 at $\bar{M} = 1.4$ to 0.003 at $\bar{M} = 1.9$. Thus the large position error of a static tube far outweighs the advantages of its smaller measuring error at $\bar{M} = 1.4$ and 1.5, and for this reason the calibration is confined to the measurement of pressures with a pitot shower. Comparable measurements with a reliable static shower would be useful.

The total error $\pm\Delta\tilde{M}$ between M and $\tilde{M}$ is given by $\Delta\tilde{M} = \Delta M' + \Delta M$, and for an error of $\pm 0.02''$ of mercury in the readings of absolute pitot and stagnation pressures at $\bar{p}_o = 30$, the value of $\Delta\tilde{M}$ for a pitot tube is given by Table IV. Although this maximum error in measuring an absolute

TABLE IV

$\bar{M}$	$\Delta\tilde{M}$
1.4	0.006
1.5	0.005
1.6	0.004
1.8	0.004
1.9	0.004

value of Mach number is rather large, especially at $\bar{M} = 1.4$, the error in measuring Mach number differences m_n by the pitot shower is much smaller, being simply $0.001 + 0.001 = 0.002$ for all $\bar{M}$. Thus, the correspondence between two runs under the same basic conditions should be best illustrated by the distributions of m_n, though the distributions of M_n provide a clearer picture of the flow in the working section.

Strictly, the above error analysis applies only to the case of $\bar{p}_o = 30$, but could be modified to apply equally well at all values of $\bar{p}_o$ if p_o could be controlled with enough precision.

The size of the total error $\pm\Delta\tilde{M}$ between M and $\tilde{M}$, given in Table IV, is an estimate of the minimum "smallness" of the disturbances we can expect to measure with the pitot shower.

4 Conclusions

The original aims of the calibration were to establish criteria for ensuring repeatability of results to within a stated accuracy under the same basic conditions, and to provide detailed illustrations of the flow in the working section in the form of Mach number distributions along several representative lines, for the full working ranges of nominal Mach number and Reynolds number, giving explanations wherever possible. However, the preliminary investigations described in this report show that such ambitious aims must be somewhat curtailed.

The important features of the investigations may be briefly summarised.

(1) Because of the large number of causes which may contribute to the non-uniformity of the flow in the empty working section, and the difficulties involved in obtaining accurate measurements of the flow, it is impossible to measure and trace the history of disturbances of the order of 0.005 in Mach number.

(2) Repeatability of results in any test may only reasonably be expected if the nominal values $\bar{M}$, $\bar{p}_o$, $\bar{T}_o$ and $\bar{\Omega}$ of the Mach number M , stagnation pressure p_o , stagnation temperature T_o and absolute humidity Ω are chosen to conform with established criteria, and if the limits of control of M , p_o , T_o and Ω at these nominal values are known.

The value of $\bar{M}$ is defined by the choice of liner, and the present method of liner installation introduces an error of the order of ± 0.001 in M for all $\bar{M}$.

The present equipment of the tunnel does not enable p_o to be controlled with sufficient precision except at the prevailing atmospheric pressure, denoted by $\bar{p}_o = 30$. Accordingly, the immediate scope of the calibration is restricted to the case $\bar{p}_o = 30$, where the variations in p_o are slow and of the order of $\pm 0.03''$ of mercury. The effects of large variations, of the order of $5''$ of mercury, in $\bar{p}_o$ will be considered later. Two such effects which may be noted now are the variation of transition to turbulence in the boundary layers on the tunnel walls and the possibility of increases in Ω due to leakage of wet air into the tunnel at $\bar{p}_o < 30$; it is suggested that some means of fixing the position of transition on the tunnel walls should be a permanent feature of all variable density supersonic wind tunnels.

At $\bar{p}_o = 30$, T_o may be controlled to within $\pm 1^\circ$C for $15 \leqslant \bar{T}_o \leqslant 45$ for a time depending on $\bar{T}_o$ and which is about two hours or more for $\bar{T}_o \geqslant 25$. The minimum attainable value of $\bar{\Omega}$ is about 0.0001 at $\bar{p}_o = 30$, and the variation of Ω during a test is of the order of 0.0001 except when the driers are in need of reactivation. It is vital that the values of $\bar{T}_o$ and $\bar{\Omega}$ should conform to the criteria governing the formation of condensation shocks at the test values of $\bar{M}$ and $\bar{p}_o$.

The effects of changes of the order of ± 50 in 4000 r.p.m. in the compressor speed ω , and slight variations of $\pm 2^\circ$C in the room temperature T_R , are neglected at $\bar{p}_o = 30$.

(3) The accuracy of an individual test under correct basic conditions depends on the care with which it is performed. The estimated errors in obtaining the Mach number distributions throughout the working section at $\bar{p}_o = 30$ with pitot or static pressures used in conjunction with the stagnation pressure show that the much greater susceptibility of static tubes to yaw outweighs their advantage in having smaller measuring errors at $\bar{M} = 1.4$ and 1.5. Consequently, the calibration is confined to the measurement of pitot pressures throughout the working section at $\bar{p}_o = 30$ for $1.4 \leqslant \bar{M} \leqslant 1.9$. The use of static tubes may be investigated later. The total error between the mean Mach number $\tilde{M}$ at a point and the value M measured by a pitot tube varies between ± 0.006 at $\bar{M} = 1.4$ and ± 0.004 at $\bar{M} = 1.9$. By using a pitot shower to measure pressures at nine symmetrical points simultaneously, differences in Mach number between the outer tubes and the central tube may be measured to within an error of about ± 0.002 at all values of $\bar{M}$ and for $\bar{p}_o = 30$.

Thus the results of the preliminary investigations restrict the immediate scope of the calibration to the measurement of Mach number distributions, deduced from pitot pressures, throughout the working section at atmospheric stagnation pressure, with explanations of the causes of Mach number variations > 0.005 from the mean (this estimate may turn out to be rather pessimistic). More investigations are required to decide the possibilities of an accurate calibration at various stagnation pressures and the use of static tubes.

The Mach number distributions throughout the working section at atmospheric stagnation pressure and for the full working range of nominal Mach numbers, under conditions free from condensation shocks, will be presented in Part II of this report.

5 Acknowledgement

We should like to express our appreciation of the help we have received from Mr. J. Ireson, who has made the pitot showers used in Parts I and II and has contributed many valuable suggestions during the course of the calibration.

LIST OF SYMBOLS

m_n	: Mach number difference (see below)
p	: static pressure
p_o	: stagnation pressure
p'_o	: pitot pressure
M	: Mach number
Re	: Reynolds number
T_o	: stagnation temperature
T_R	: room temperature
δ	: boundary layer thickness
ω	: compressor speed
Δ	: denotes "absolute error in ...."
Ω	: absolute humidity

Superscripts

$-$	: denotes nominal value
$\bar{M}$	: nominal value of Mach number; defined by liner
$\tilde{M}$	: mean value of Mach number at a given point in the empty working section over any number of tests
M'	: actual value of Mach number at a given point in the empty working section in one particular test
M	: measured value of Mach number at a given point in the empty working section in one particular test

Subscripts

n $(1 \leqslant n \leqslant 9)$	: number of tube in pitot shower

For example

p'_{o_n}	: pitot pressure	at a given position of the n^{th} tube of the pitot shower
M_n	: measured Mach number	at a given position of the n^{th} tube of the pitot shower
M_5	: measured Mach number at a given position of central (No.5) tube of pitot shower	
$m_n = M_n - M_5$	: Mach number difference between positions of n^{th} tube and central (No.5) tube with shower in a fixed position	

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APPENDIX I

An approximate formula for Reynolds number

The Reynolds number per foot, $\frac{Re}{d}$, may be expressed exactly in the form

$$\frac{Re}{d} = \left(\frac{\gamma}{R}\right)^{\frac{1}{2}} p_o \frac{1}{\mu_o T_o^{\frac{1}{2}}} \left[\frac{T_o+120\left(1+\frac{\gamma-1}{2}M^2\right)}{T_o+120}\right] \left[M\left(1+\frac{\gamma-1}{2}M^2\right)^{1-\frac{\gamma}{\gamma-1}}\right] \qquad (1)$$

where p_o is the stagnation pressure in lb per sq ft, M the Mach number, T_o the stagnation temperature in ^oK, μ_o the coefficient of viscosity at temperature T_o given by

$$\mu_o = 0.0309 \frac{T_o^{\frac{3}{2}}}{T_o + 120} \times 10^{-6} \qquad (2)$$

and γ is the ratio of the specific heats and R is the gas constant for unit mass of the gas. Taking $\gamma = 1.4$ and $R = 3092$ ft^2 per sec^2 per ^oC for air, this may be written

$$\frac{Re}{d} = 48.6\, p_o \left[\frac{T_o+120}{T_o^2}\right] \left[1 + \frac{24\, M^2}{T_o+120}\right] \left[M(1 + 0.2\, M^2)^{-\frac{5}{2}}\right] \times 10^6 \qquad (3)$$

where now p_o is expressed in inches of mercury.

To find an approximation to this complicated formula for $1.4 \leqslant M \leqslant 1.9$ and $283 \leqslant T_o \leqslant 323$, let us write

$$M = 1.65\,(1 + m) \quad , \quad |m| \leqslant \frac{0.25}{1.65} = 0.152 \quad ,$$

$$T_o = 303\,(1 + t_o) \quad , \quad |t_o| \leqslant \frac{20}{303} = 0.0660 \quad .$$

Since $m^2 \leqslant 0.0231 = 0(t_o)$, we shall retain terms $0(m^2)$ in the initial expansion of $\frac{Re}{d}$ in powers of m and t_o. We have

$$\frac{T_o + 120}{T_o^2} = 0.00461\,(1 - 1.284\, t_o) + 0(t_o^2) \quad ,$$

$$1 + \frac{24\, M^2}{T_o + 120} = 1.154\,(1 + 0.267\, m - 0.0962\, t_o + 0.133\, m^2) + 0(m\, t_o),$$

$$M(1 + 0.2\, M^2)^{-\frac{5}{2}} = 0.556\,(1 - 0.763\, m - 0.469\, m^2) + 0(m^3) \quad .$$

Hence

$$\frac{Re}{d} = 0.144\, p_o\, (1 - 0.496\, m - 1.38\, t_o - 0.540\, m^2) + 0(m\, t_o)$$

We see now that the term containing m^2 is in fact much smaller than the term containing t_o, so we shall take the simpler approximation

$$\frac{Re}{d} = 0.144\, p_o\, (1 - 0.496\, m - 1.38\, t_o) + 0(m^2)$$

which may also be written in the form

$$\frac{Re}{d} = 0.144\, p_o\, (1 - 0.496\, m)\, (1 - 1.38\, t_o) + 0(m^2)$$

In terms of M and T_o we then have the approximation

$$\frac{Re}{d} = 0.513\, p_o\, (1 - 0.201\, M)\, (1 - 0.00191\, T_o) \qquad (4)$$

If we now revert to the notation of the main text and let the superscript $-$ denote the nominal values of p_o, M and T_o, and further if T_o is expressed in ^oC, then the exact formula (3) becomes

$$\frac{Re}{d} = 4.05\, \bar{p}_o \left[\frac{\bar{T}_o + 393}{(\bar{T}_o + 273)^2} \right] \left[1 + \frac{24\, \bar{M}^2}{\bar{T}_o + 393} \right] \left[\bar{M}(1 + 0.2\bar{M}^2)^{-\frac{5}{2}} \right] \times 10^6 \qquad (5)$$

and the approximation to it may be written

$$\frac{Re}{d} = 0.0204\, \bar{p}_o\, (1 - 0.004\, \bar{T}_o)\, (1 - 0.2\, \bar{M}) \times 10^6 \qquad (6)$$

which should give the value of $\frac{Re}{d}$ correct to two significant figures.

APPENDIX II

The error induced by inaccurate liner position

We may get an idea of the error involved in the fixing of a liner by assuming the one-dimensional theory relation

$$\frac{A}{A^*} = \frac{1}{\tilde{M}} \left[\frac{1 + \frac{\gamma-1}{2}\tilde{M}^2}{\frac{\gamma+1}{2}} \right]^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)} = 6^{-3} . 5^3 . \tilde{M}^{-1} (1 + 0.2\,\tilde{M}^2)^3 \qquad (1)$$

to hold between the mean Mach number $\tilde{M}$ and the areas A^* and A at the throat and working section respectively. Neglecting the difference between the boundary layers at the throat and the working section, we may write $A = 9h$, $A^* = 9h^*$ so that

$$\frac{A}{A^*} = \frac{h}{h^*} = \lambda \text{ , say , } = 6^{-3} . 5^3 \tilde{M}^{-1} (1 + 0.2\,\tilde{M}^2)^3 \qquad (2)$$

If h and h^* are subject to errors $\pm \Delta h$ and $\pm \Delta h^*$ respectively, then $\tilde{M}$ is subject to an error $\pm \Delta M'$, in the notation of the main text, given by

$$\Delta M' = \frac{1}{h^*} (\Delta h + \lambda\, \Delta h^*) \left| \frac{d\tilde{M}}{d\lambda} \right|$$

where from (2) we have

$$\frac{d\lambda}{d\tilde{M}} = 5^3 . 6^{-3} . \tilde{M}^{-2} (1 + 0.2\,\tilde{M}^2)^2 \quad (\tilde{M}^2 - 1)$$

If we put $\Delta h = a\,\Delta h^*$ and $h = 9''$, then

$$\Delta M' = 6^3\, 5^{-3}\, \tilde{M}^2 (1 + 0.2\,\tilde{M}^2)^{-2} \quad (\tilde{M}^2 - 1)^{-1} \frac{\lambda}{9} (a + \lambda)\, \Delta h^* \qquad (3)$$

Since λ is a function of $\tilde{M}$, we may tabulate $\Delta M'$ for various values of $\tilde{M}$, a and Δh^* . Table A gives the values of $\Delta M'$ for $1.4 \leq \tilde{M} \leq 1.9$, $a = 0$, 1 , 2 and $\Delta h^* = 0.01''$.

TABLE A

$\tilde{M}$	$\Delta M'$		
	$a = 0$	$a = 1$	$a = 2$
1.4	0.0030	0.0056	0.0083
1.5	0.0023	0.0042	0.0061
1.6	0.0022	0.0039	0.0056
1.7	0.0021	0.0037	0.0053
1.8	0.0021	0.0036	0.0051
1.9	0.0022	0.0036	0.0050

APPENDIX III

The conversion of pressure differences to Mach number differences

We have used water manometers to measure the difference between the pitot pressure p'_{o_n} at the n^{th} tube and the pitot pressure p'_{o_5} at the corresponding point on the tunnel centre-line. It is possible to deduce the difference of the corresponding Mach numbers M_n and M_5 directly from $(p'_{o_n} - p'_{o_5})$ without evaluating each separately from the values of $\frac{p'_{o_n}}{p_o}$ and $\frac{p'_{o_5}}{p_o}$, where p_o is the stagnation pressure.

If we write $\frac{p'_{o_n}}{p_o} = t_n$ and $\frac{p'_{o_5}}{p_o} = t_5$, then we have

$$t_n = 6^6 M_n^7 \left[M_n^2 + 5\right]^{-\frac{7}{2}} \left[7 M_n^2 - 1\right]^{-\frac{5}{2}} = F(M_n) \text{ , say ,}$$

$$t_5 = F(M_5) \quad ,$$

and we write the inverses of these relations as

$$M_n = f(t_n) \quad ,$$

$$M_5 = f(t_5) \quad .$$

Now

$$t_n = \frac{p'_{o_n}}{p_o} = \frac{p'_{o_5}}{p_o} + \frac{(p'_{o_n} - p'_{o_5})}{p_o} = t_5 + \frac{\varepsilon_n}{p_o}$$

where $\varepsilon_n = (p'_{o_n} - p'_{o_5})$ is the quantity we measure in inches of water.

Since $\frac{\varepsilon_n}{p_o}$ is small compared with t_5 we may expand $f(t_n)$ by Taylor's theorem and obtain

$$M_n = f(t_n) = f\left(t_5 + \frac{\varepsilon_n}{p_o}\right) \doteq f(t_5) + \frac{\varepsilon_n}{p_o} \cdot f'(t_5) + \frac{\varepsilon_n^2}{2p_o^2} \cdot f''(t_5) + \ldots.$$

where the dashes denote differentiation. Putting

$$m_n = M_n - M_5$$

we then have

$$m_n = \frac{\varepsilon_n}{p_o} \cdot f'(t_5) + \frac{\varepsilon_n^2}{2p_o^2} \cdot f''(t_5) + 0\left(\frac{\varepsilon_n^3}{p_o^3}\right)$$

If we further write

$$t_5 = \bar{t} + \delta t_5 \quad , \qquad f(\bar{t}) = \bar{M} \quad ,$$

$$p_o = \bar{p}_o + \delta p_o \quad ,$$

where δ denotes the variation of a quantity from its nominal value (δ is not an error), and if we assume that δp_o and $\bar{p}_o \, \delta t_5$ are of the same order as ε_n, then we have

$$m_n = k_1 \cdot \frac{\varepsilon_n}{\bar{p}_o} + k_2 \cdot \frac{\varepsilon_n^2}{\bar{p}_o^2} + O\left(\frac{\varepsilon_n^3}{\bar{p}_o^3}\right)$$

where

$$k_1 = f'(\bar{t})$$

$$k_2 = \left(\tfrac{1}{2} + \frac{\bar{p}_o \cdot \delta t_5}{\varepsilon_n}\right) f''(\bar{t}) - \frac{\delta p_o}{\varepsilon_n} \cdot f'(\bar{t})$$

If we choose $\varepsilon_n < \pm 0.3$ inches of mercury, then for atmospheric stagnation pressure $\bar{p}_o = 30$ inches of mercury we have $\frac{\varepsilon_n}{\bar{p}_o} < \pm 0.01$, so that $\frac{\varepsilon_n^2}{\bar{p}_o^2} = O(0.0001)$. Hence, unless $k_2 = O(10)$, then $k_1 \cdot \frac{\varepsilon_n}{\bar{p}_o}$ will give m_n correct to $O(0.001)$. In fact k_2 is $O(10)$ only for $\bar{M} < 1.4$ or $\delta p_o < -1.0$, so that we may in other cases write

$$m_n = k_1 \cdot \frac{\varepsilon_n}{\bar{p}_o} + O\left(\frac{\varepsilon_n^2}{\bar{p}_o^2}\right)$$

and expect this approximation to give us m_n correct to ± 0.001.

The value $\varepsilon_n = \pm 0.3''$ mercury corresponds to $\varepsilon_n = \pm 4''$ water which is a considerable difference of pressure for an empty working section, and hence the simple procedure of multiplying the pressure differences $\varepsilon_n = p'_{o_n} - p'_{o_5}$ by a constant $K = \frac{k_1}{\bar{p}_o}$, which depends only on the nominal Mach number $\bar{M}$ and the nominal stagnation pressure $\bar{p}_o$, to give the corresponding Mach number differences $m_n = M_n - M_5$ will be applied throughout the calibration. In the cases where it is not valid, near $\bar{M} = 1.4$, for $\bar{p}_o - p_o > 1.0''$ mercury and for $|p'_{o_n} - p'_{o_5}| > 0.3''$ mercury, say, at $\bar{p}_o = 30$, the error will still be small, though $> \pm 0.001$, and in the latter case will be entirely insignificant since the disturbance causing the large pressure difference will be so large as to be obviously artificial and easily traced.

To summarise the results of this Appendix, we state that we shall calculate the difference m_n between the Mach numbers at a point on the n^{th} line of traverse and the corresponding point on the centre-line from the formula

$$m_n = K \varepsilon_n$$

where K is given in Table B as a function of $\bar{M}$ for atmospheric stagnation pressure, and ε_n is the difference between the pitot pressures at those points, measured in inches of water. Since the error in measuring ε_n is of the order of $\pm 0.05''$ water, then the value of m_n will be correct to within ± 0.001 unless $\bar{M} \leq 1.4$ or $p_o < 29''$ mercury, when the error in m_n will still be $O(0.001)$, or unless $|\varepsilon_n| > 4''$ water, when the error will be insignificant compared with m_n.

TABLE B

$\bar{M}$	K
1.4	-0.0098
1.5	-0.0077
1.6	-0.0066
1.8	-0.0055
1.9	-0.0053

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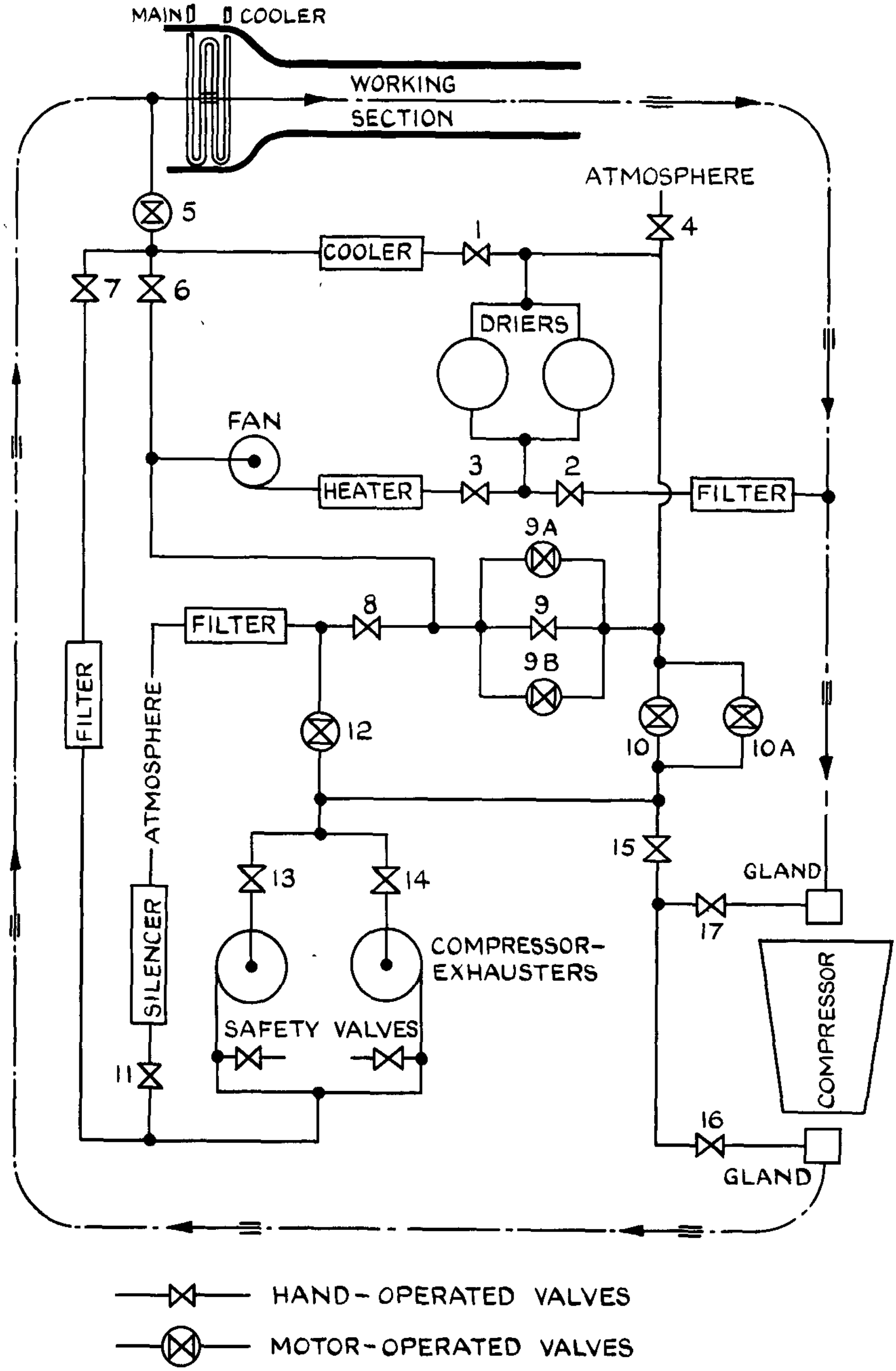


FIG. 1. TUNNEL CIRCUIT

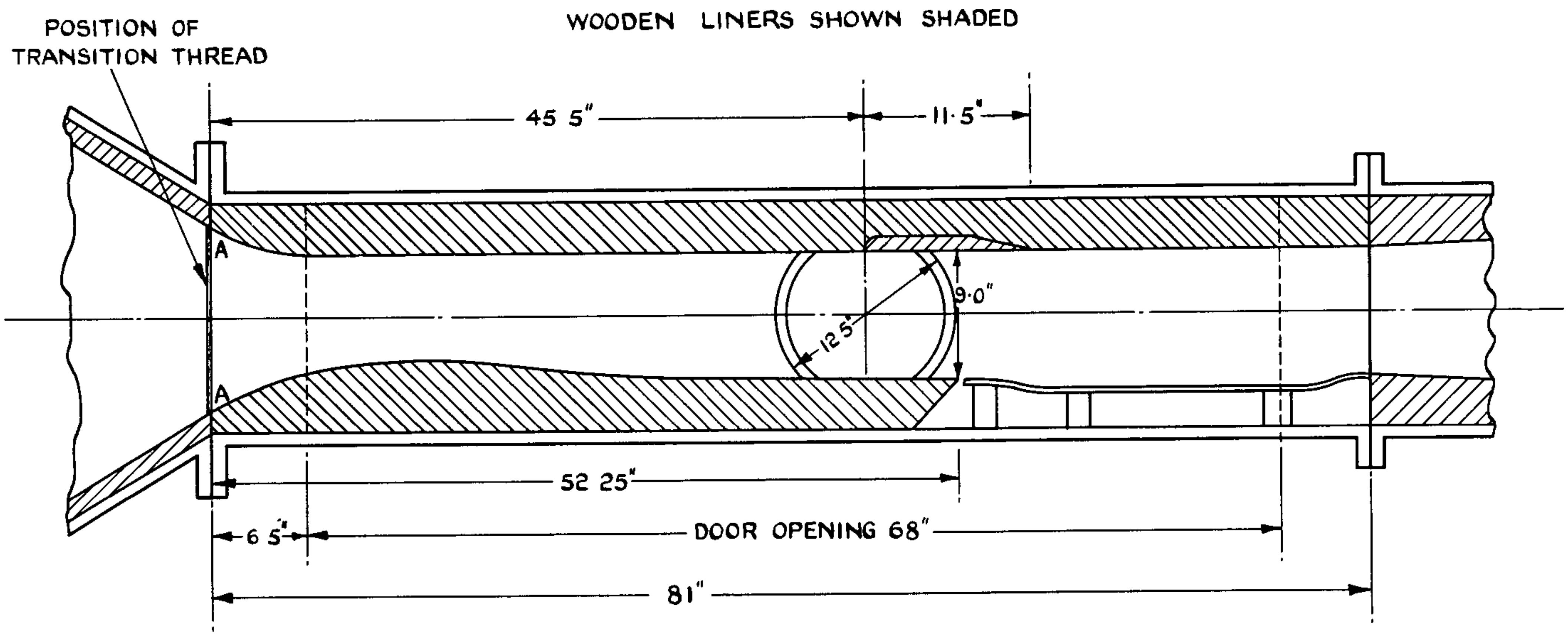
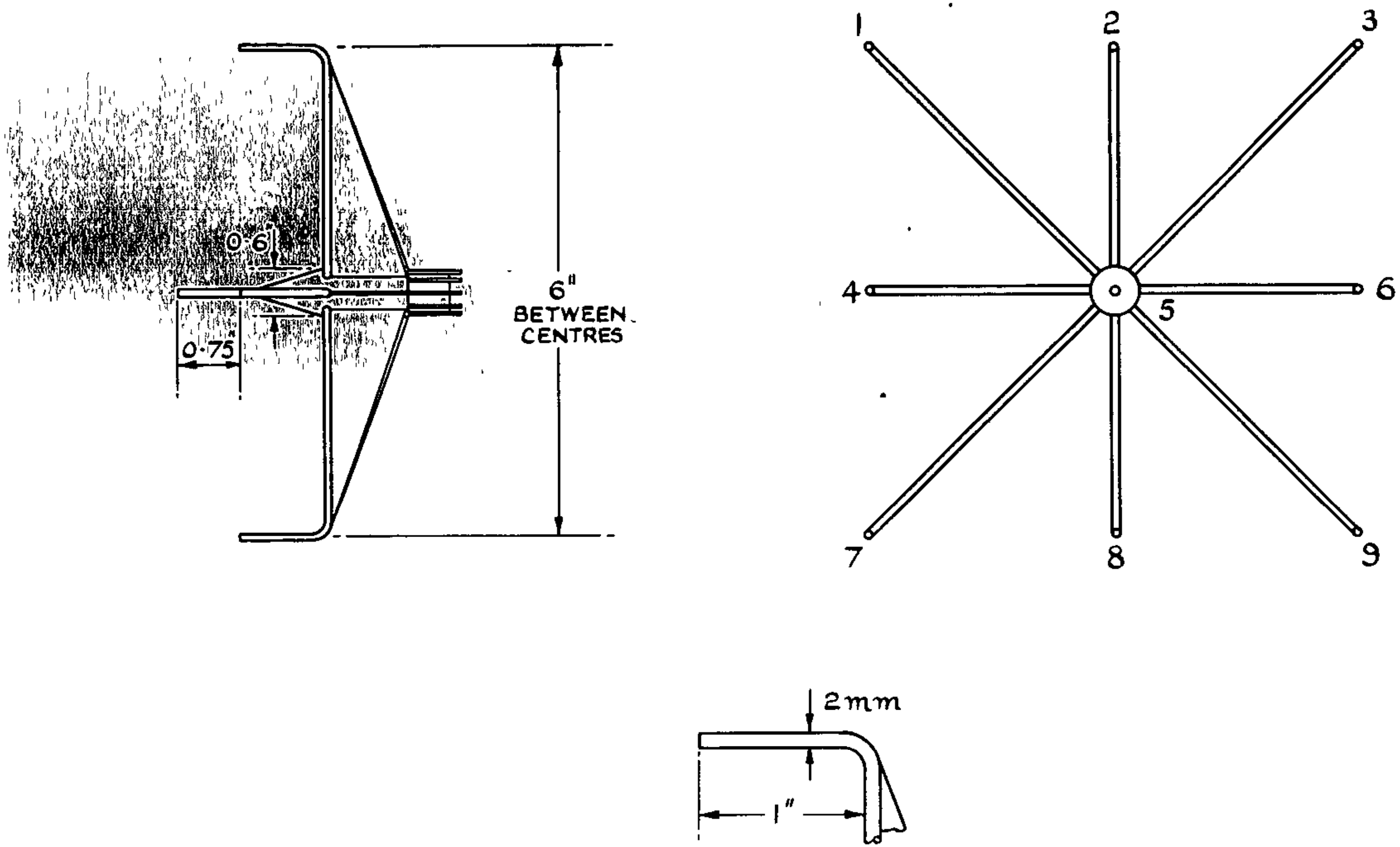


FIG. 2. WORKING SECTION



DETAIL OF HEADS

FIG. 3. PITOT SHOWER

FIG. 3.

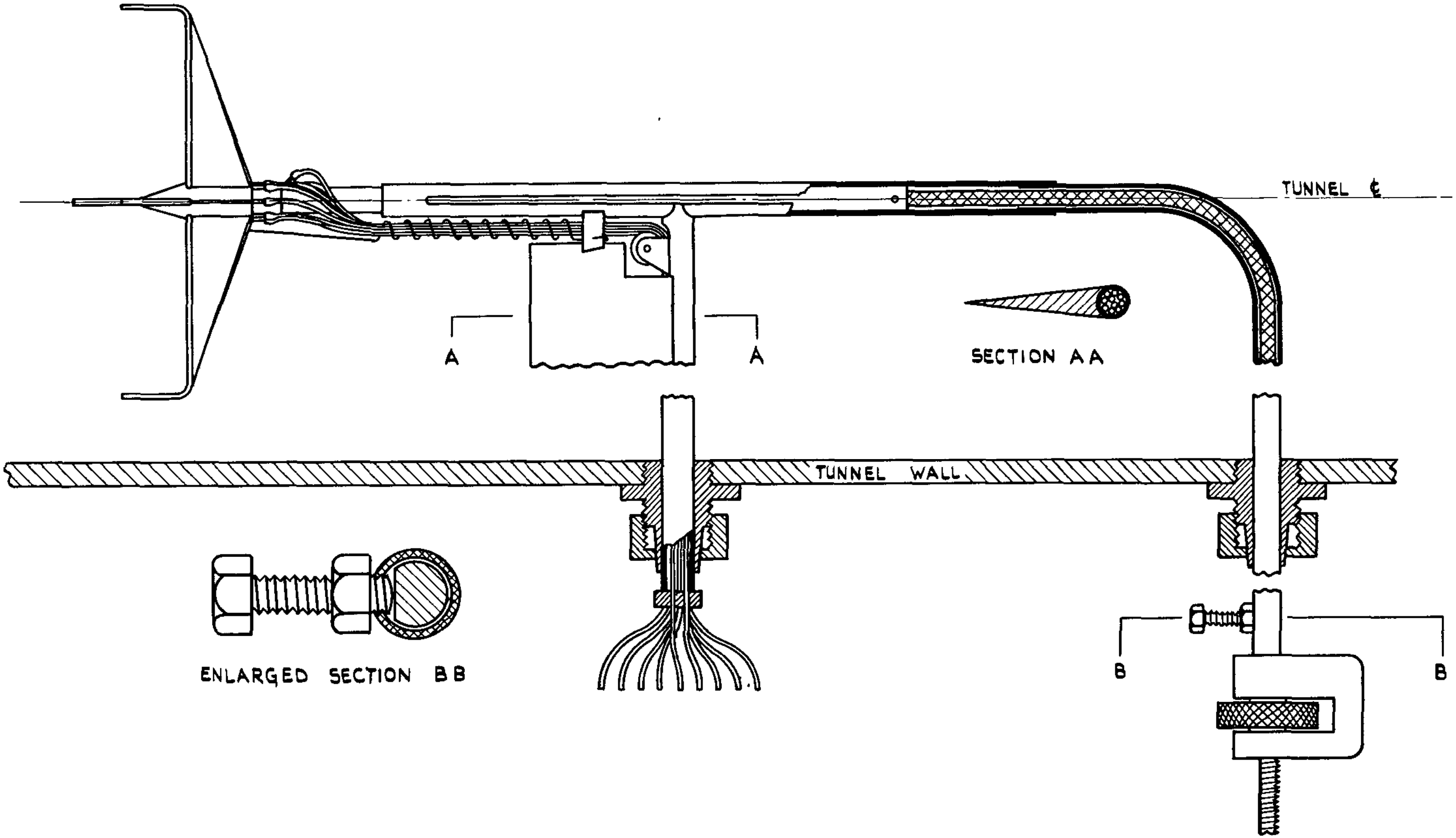


FIG. 4. PITOT SHOWER TRAVERSING GEAR.

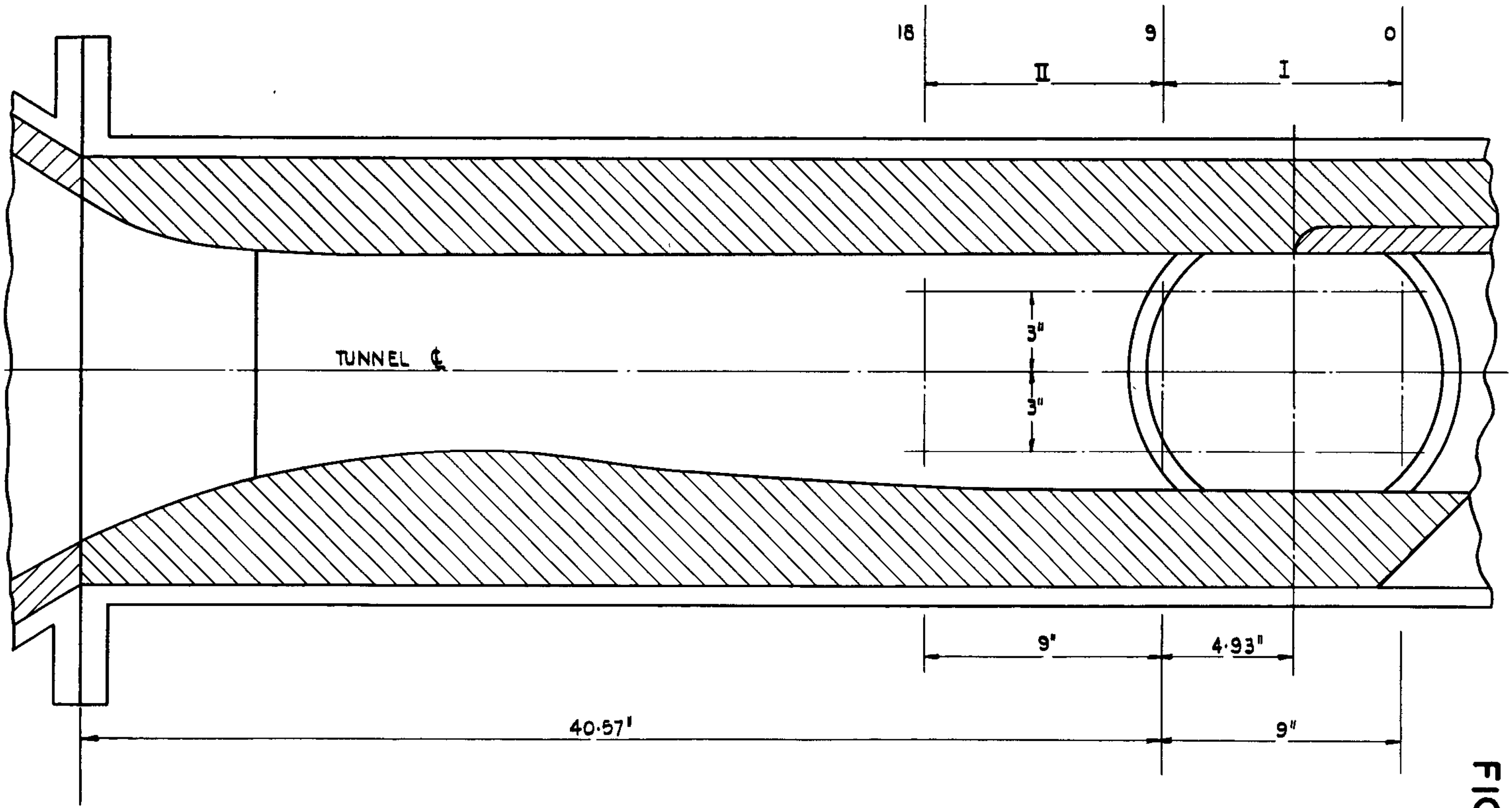


FIG. 5. POSITIONS OF TRAVERSES WITH PITOT SHOWER.

FIG. 5.

FIG.6.

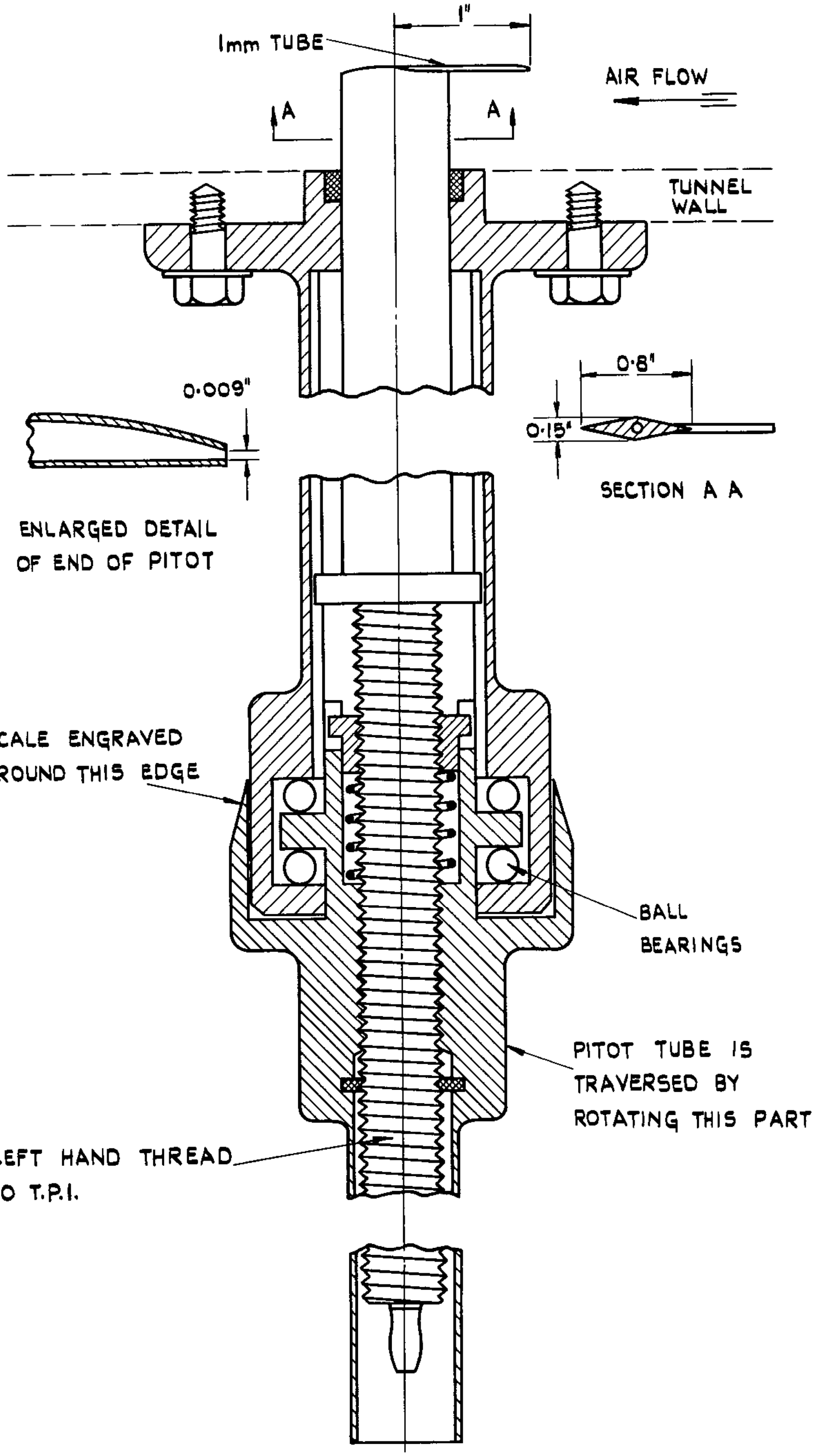
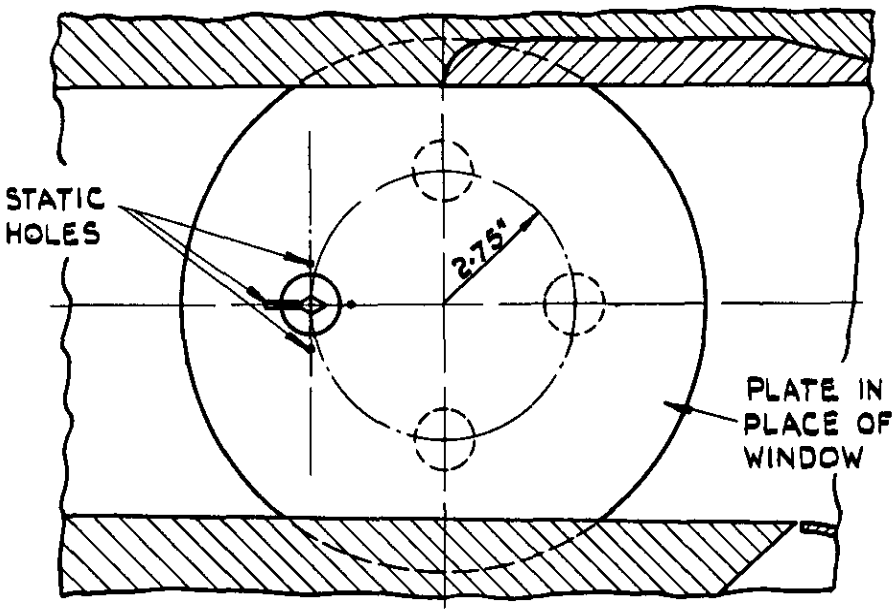
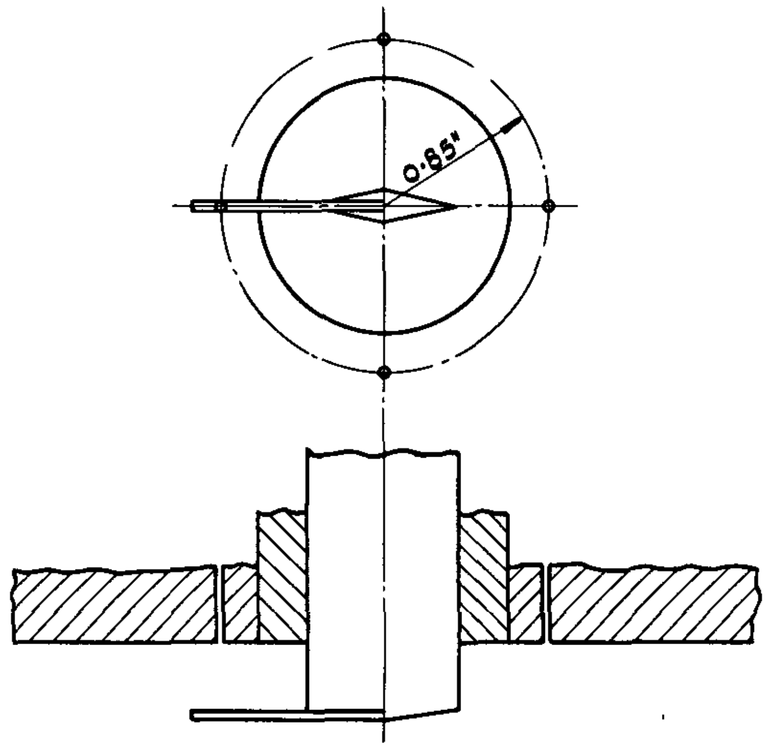


FIG.6. MICROMETER PITOT TUBE.

FIG.7.



ELEVATION SHOWING PITOT IN TUNNEL (OTHER POSITIONS, SHOWN DOTTED, OBTAINED BY ROTATING WALL PLATE)



DETAIL SHOWING POSITIONS OF STATIC HOLES

FIG.7. POSITIONS OF TRAVERSES WITH MICROMETER PITOT TUBE.

FIG.8.

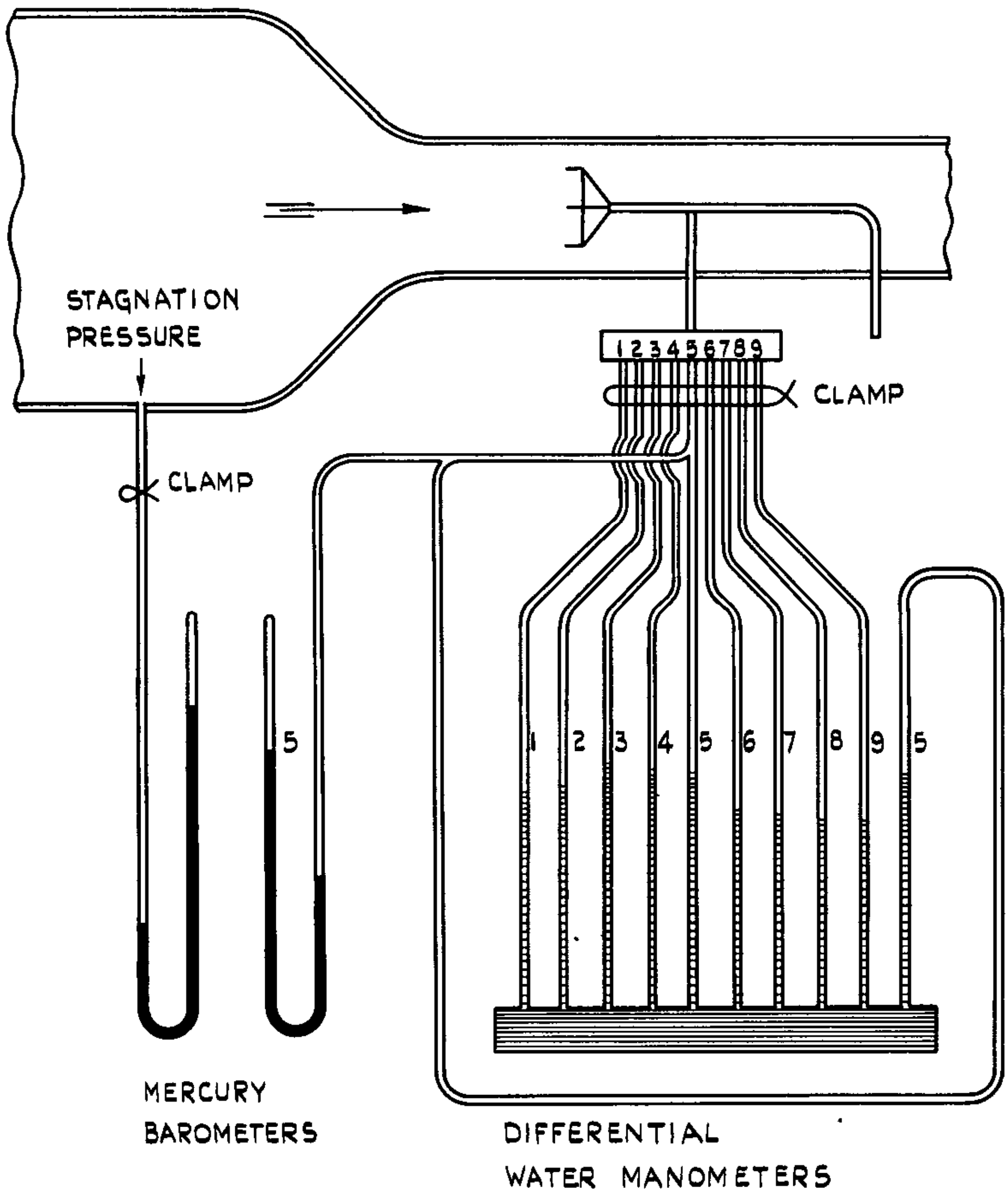


FIG.8. ARRANGEMENT FOR PRESSURE MEASUREMENT WITH PITOT SHOWER.

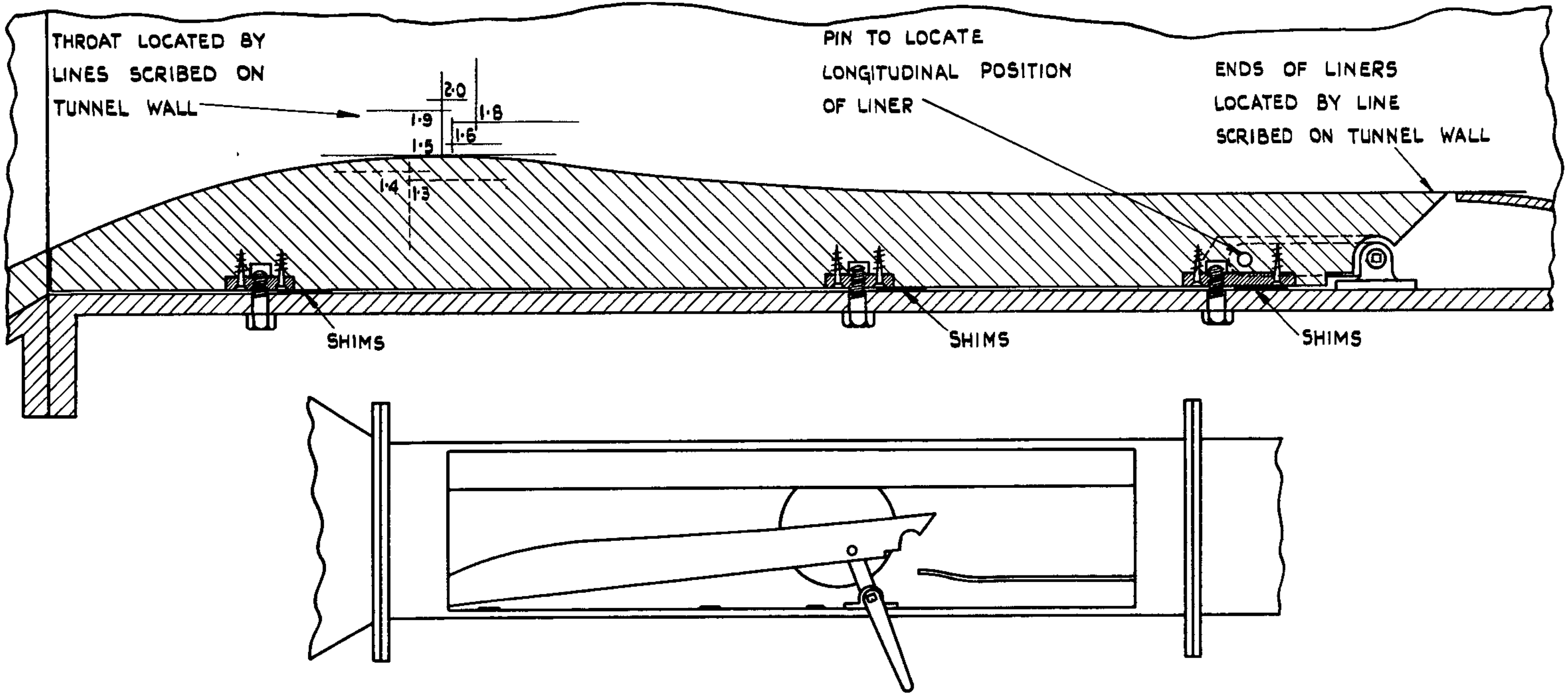


FIG.9. METHOD OF LINER INSTALLATION.

FIG.9.

FIG. 10.

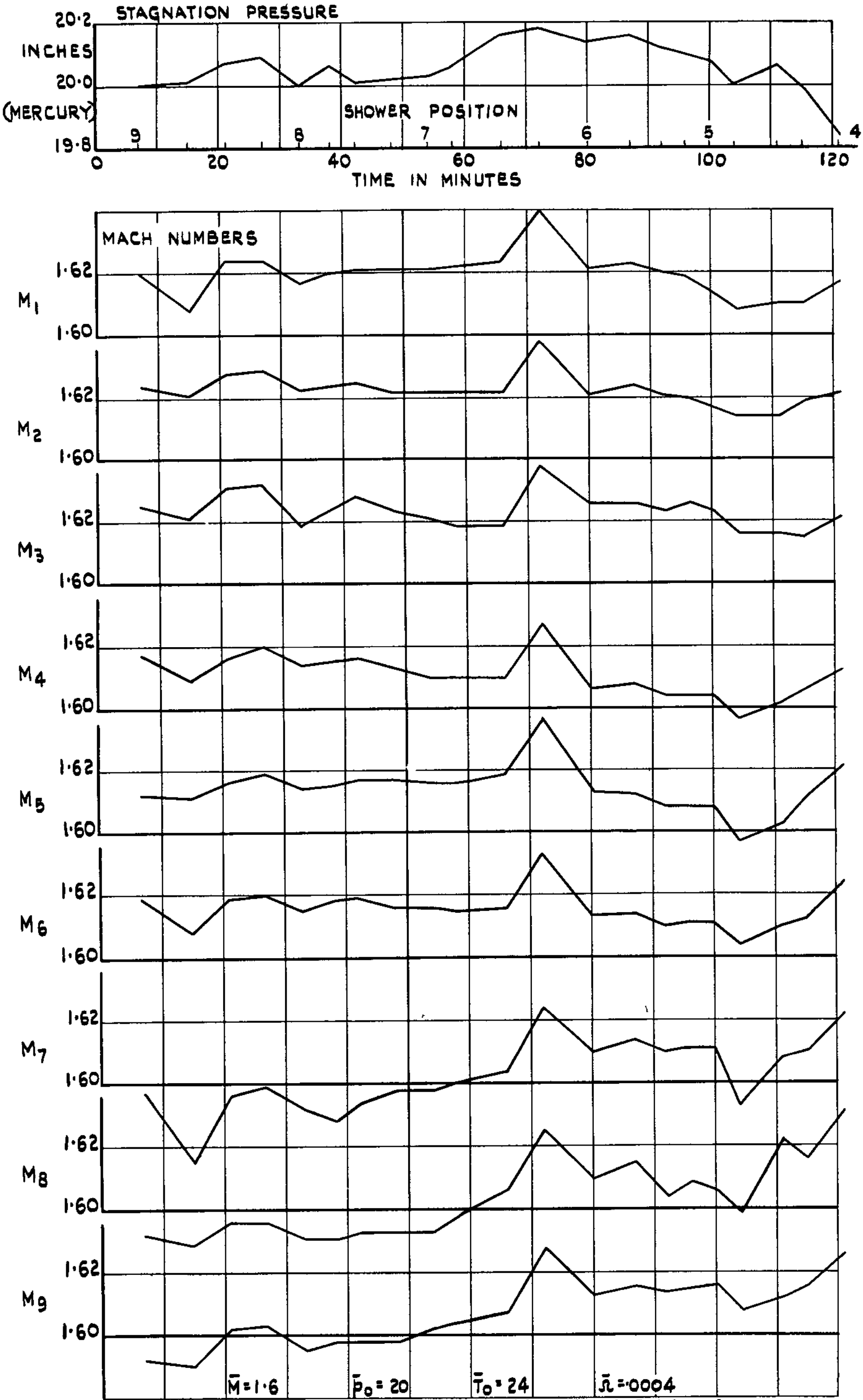


FIG.10. DIFFICULTY OF CALIBRATION AT STAGNATION PRESSURE OTHER THAN ATMOSPHERIC.

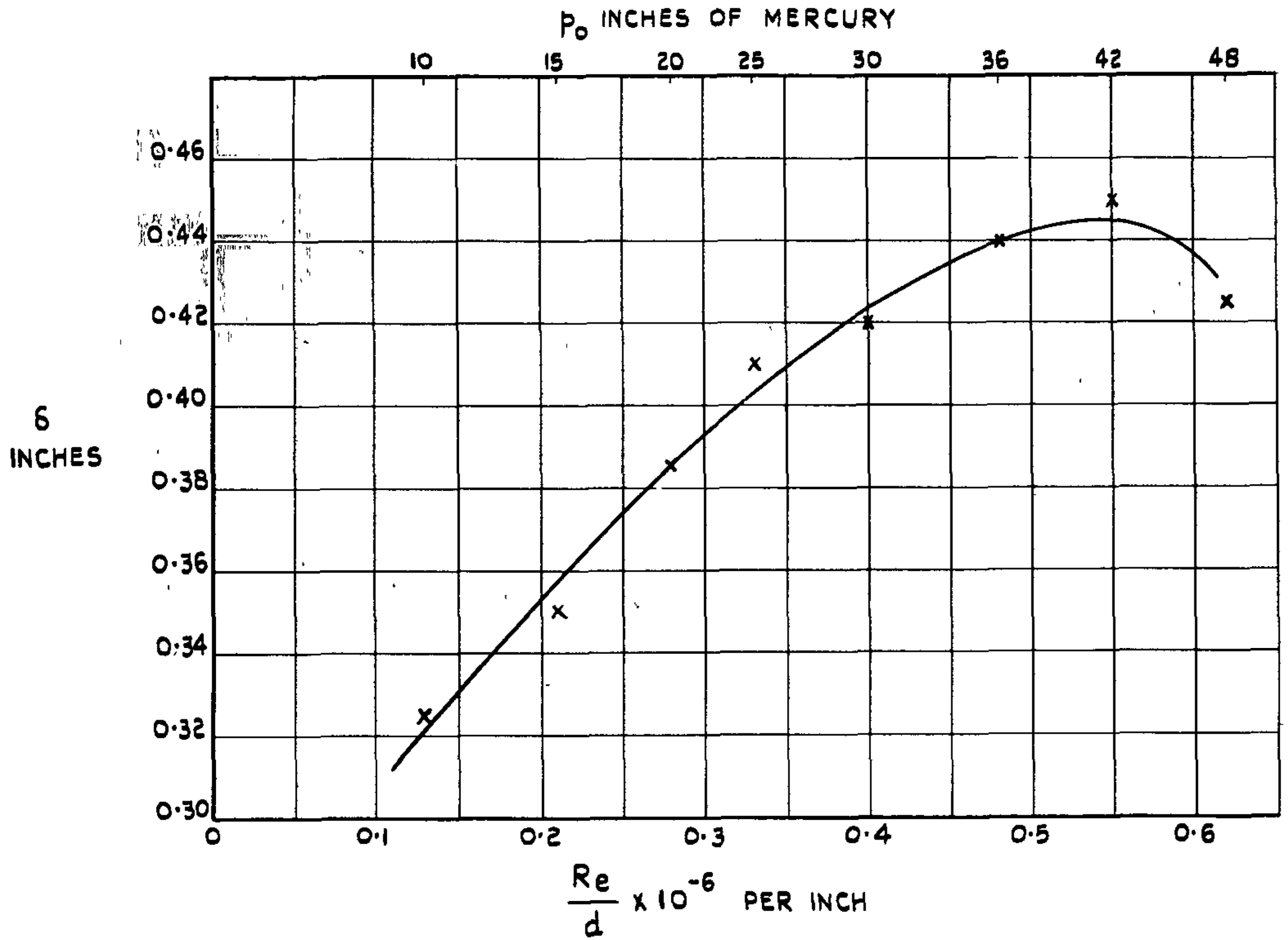


FIG.11. VARIATION OF BOUNDARY-LAYER THICKNESS WITH REYNOLDS NUMBER FOR SOME EXTENT OF LAMINAR FLOW.

FIG.11.

FIG.12.

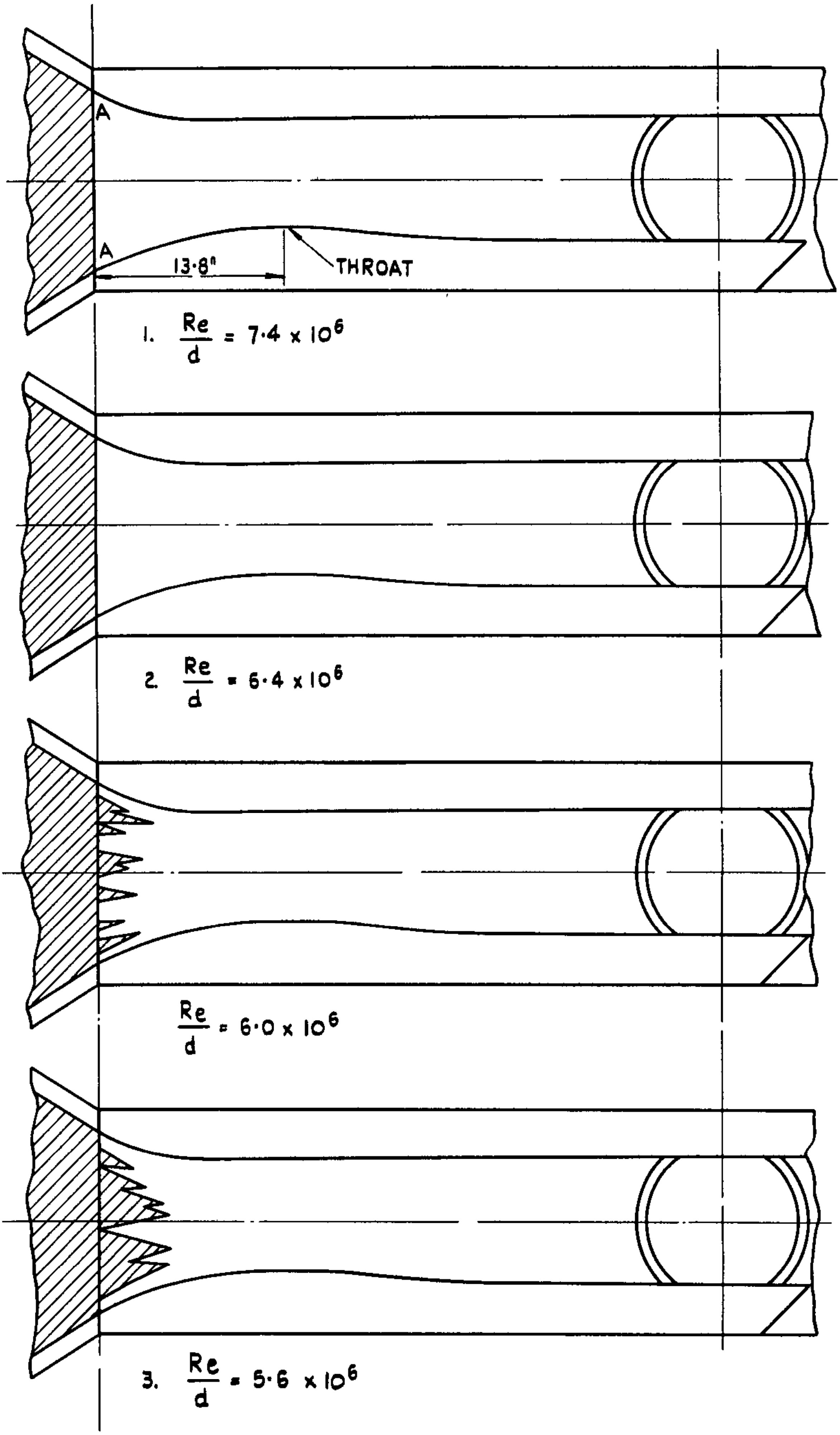


FIG.12. SHAPES OF TRANSITION LINES FOR SOME EXTENT OF LAMINAR FLOW ($\overline{M} = 1{\cdot}4$).

SCALE 1/10

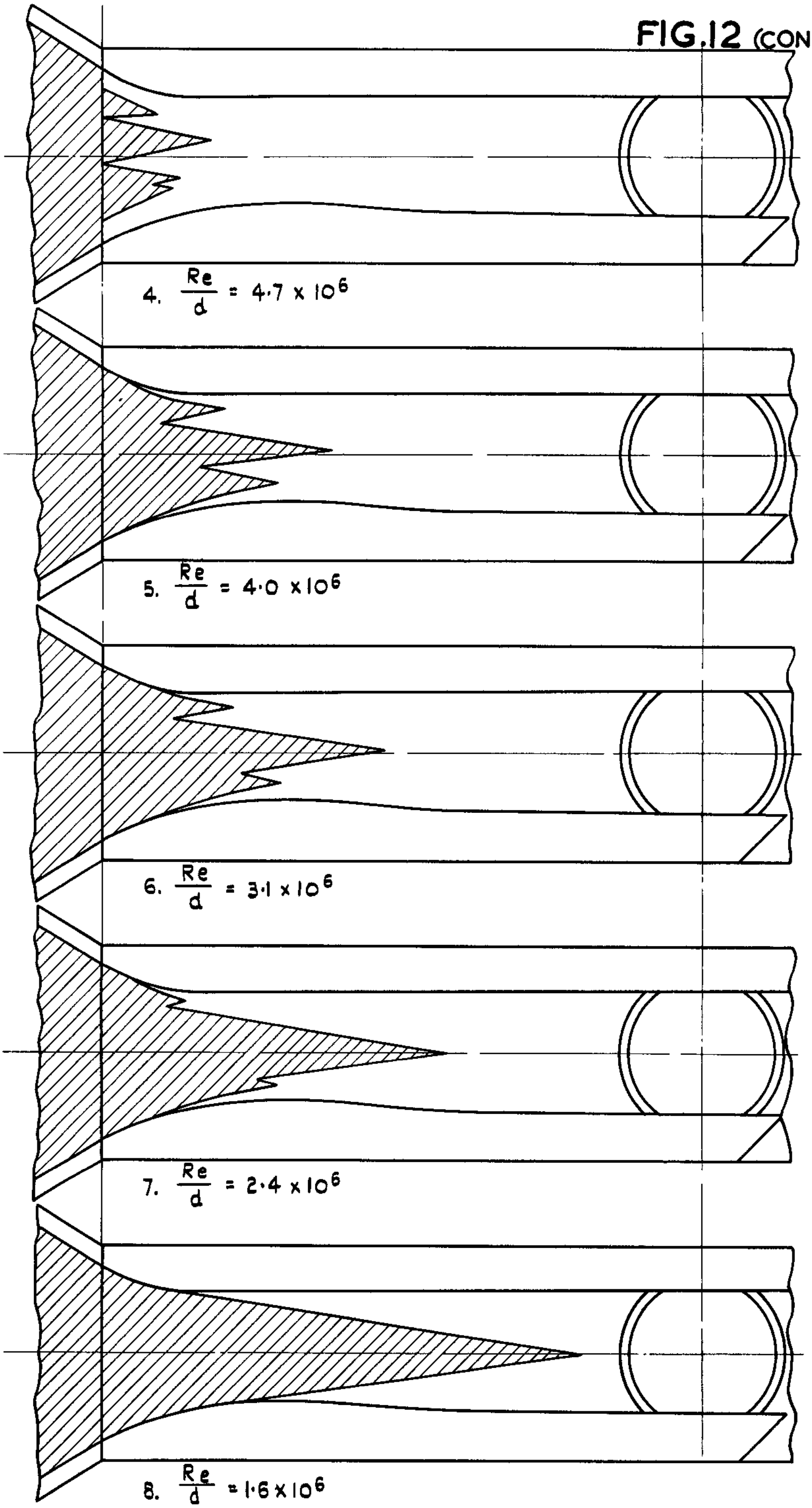


FIG.12 (CONTD). SHAPES OF TRANSITION LINES FOR SOME EXTENT OF LAMINAR FLOW ($\bar{M} = 1 \cdot 4$).

SCALE 1/10

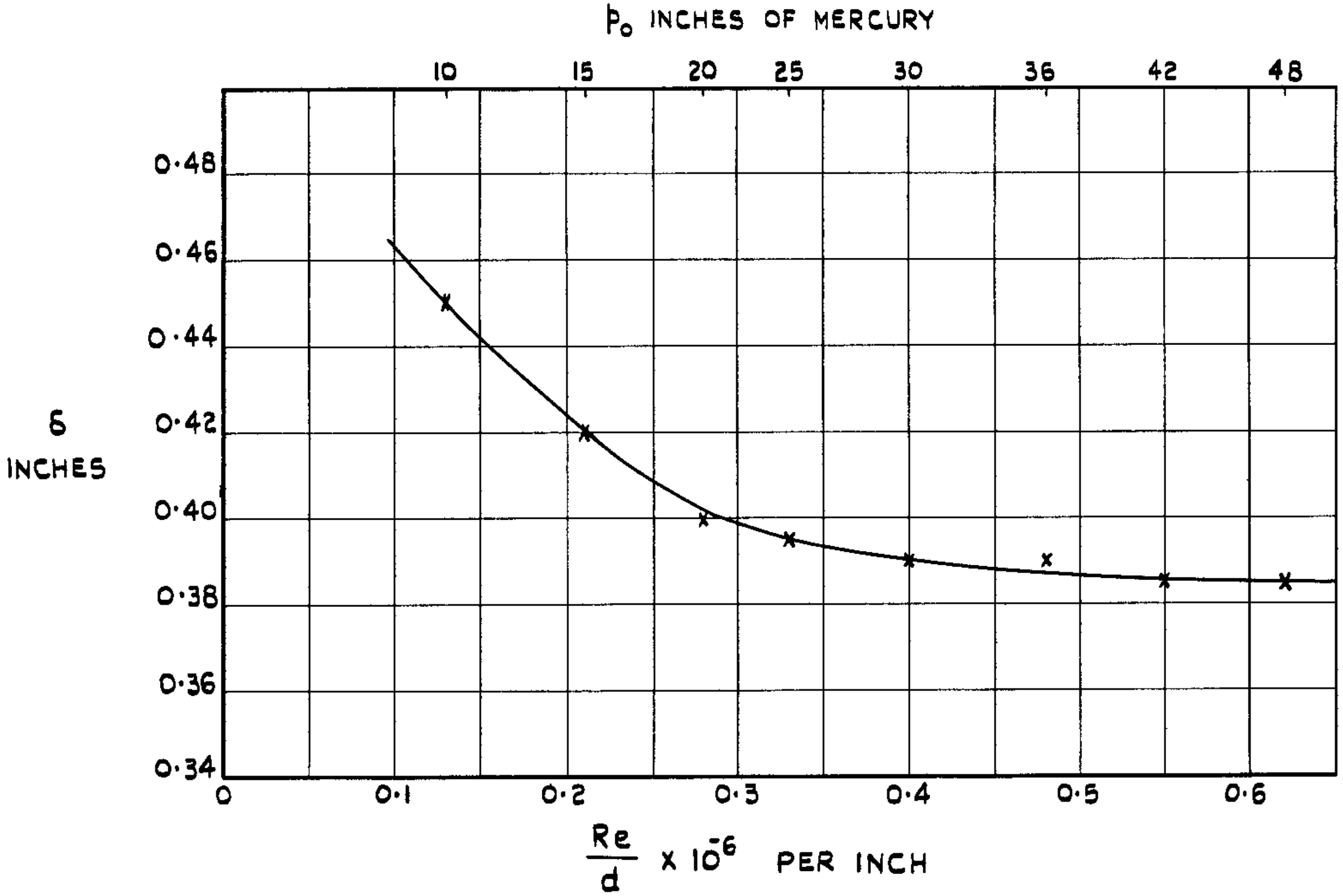


FIG.13. VARIATION OF BOUNDARY LAYER THICKNESS WITH REYNOLDS NUMBER FOR COMPLETELY TURBULENT FLOW.

FIG.14.

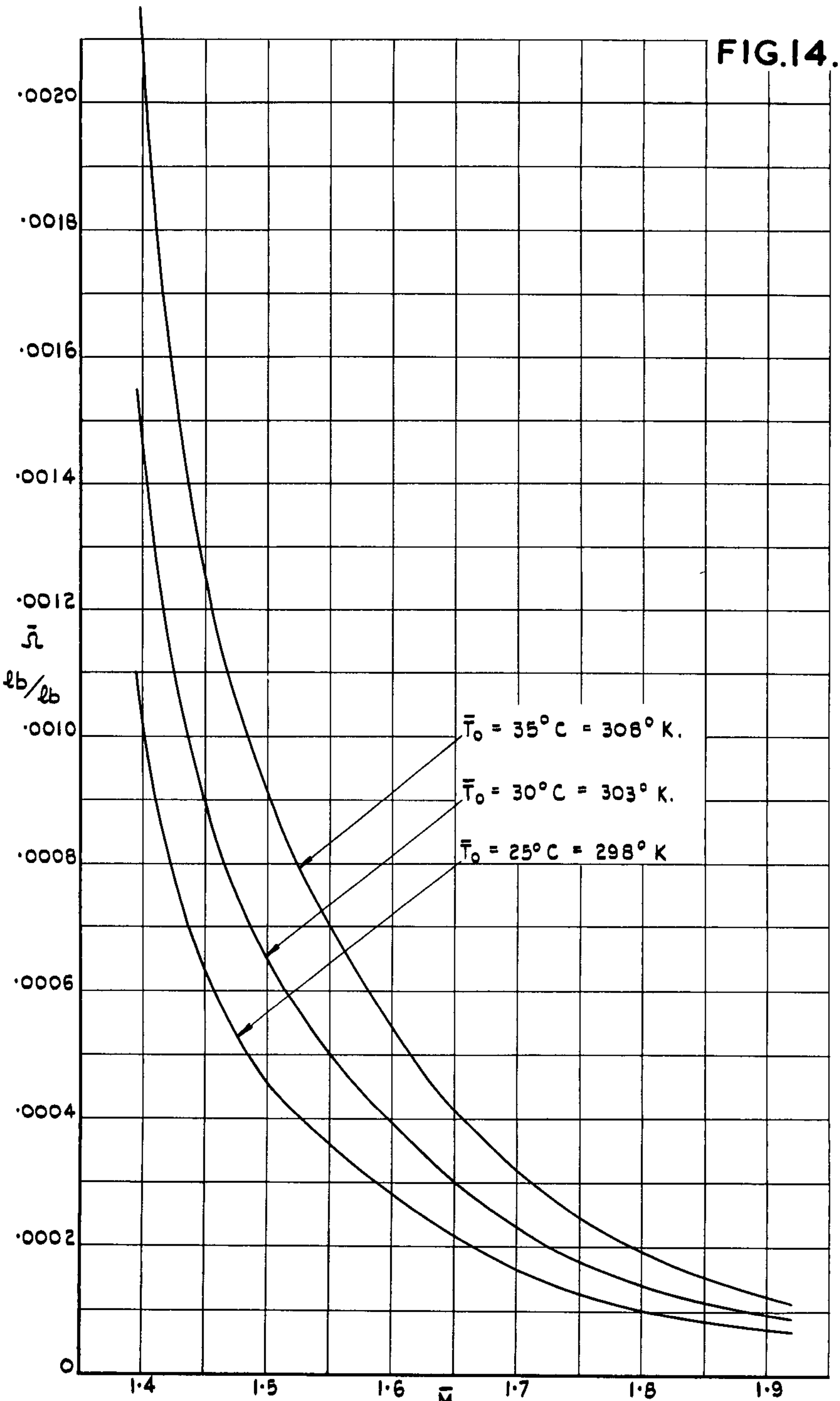
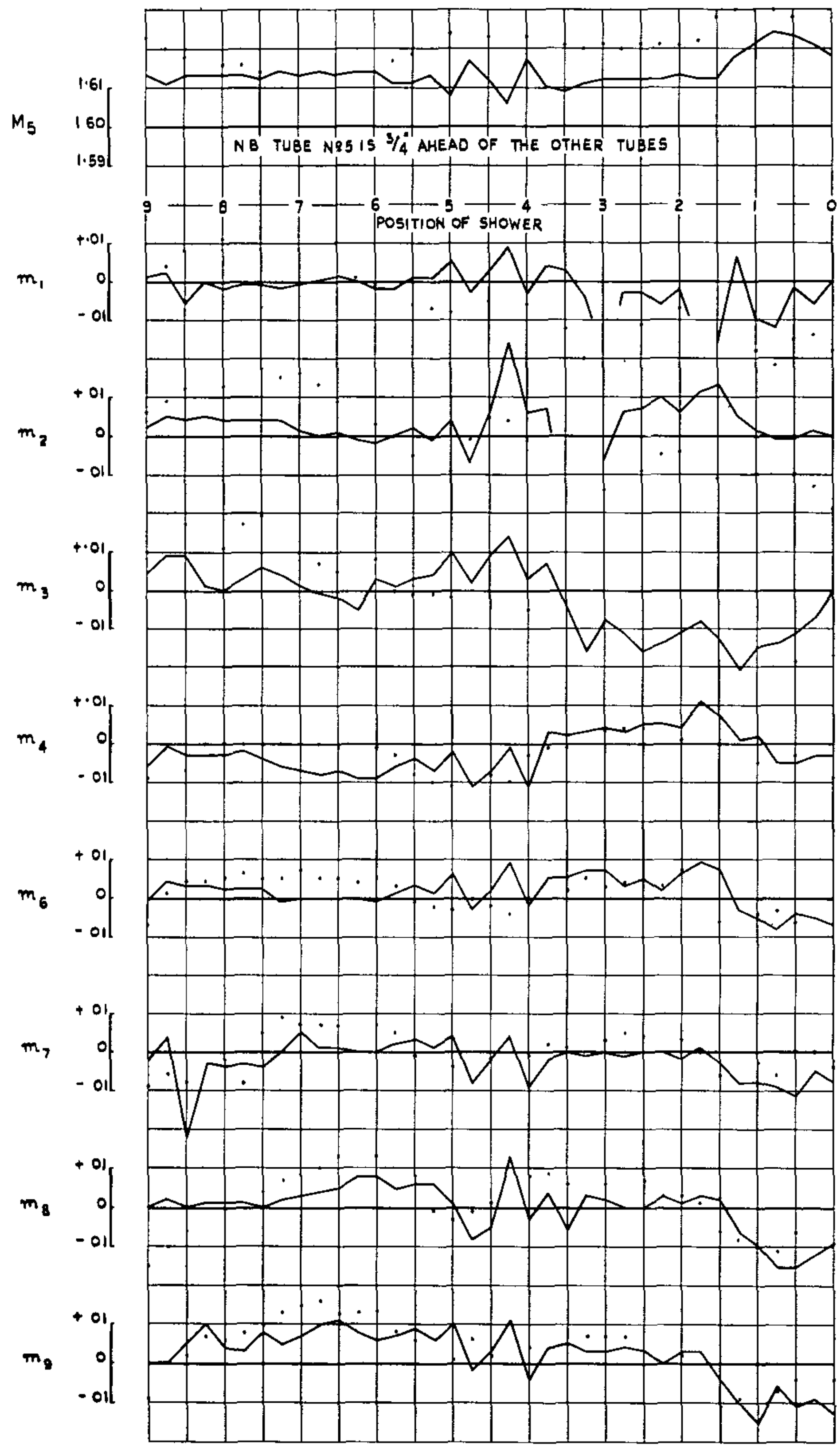


FIG.14. CRITERIA FOR FLOW FREE FROM CONDENSATION SHOCKS AT ATMOSPHERIC STAGNATION PRESSURE.

FIG.15.

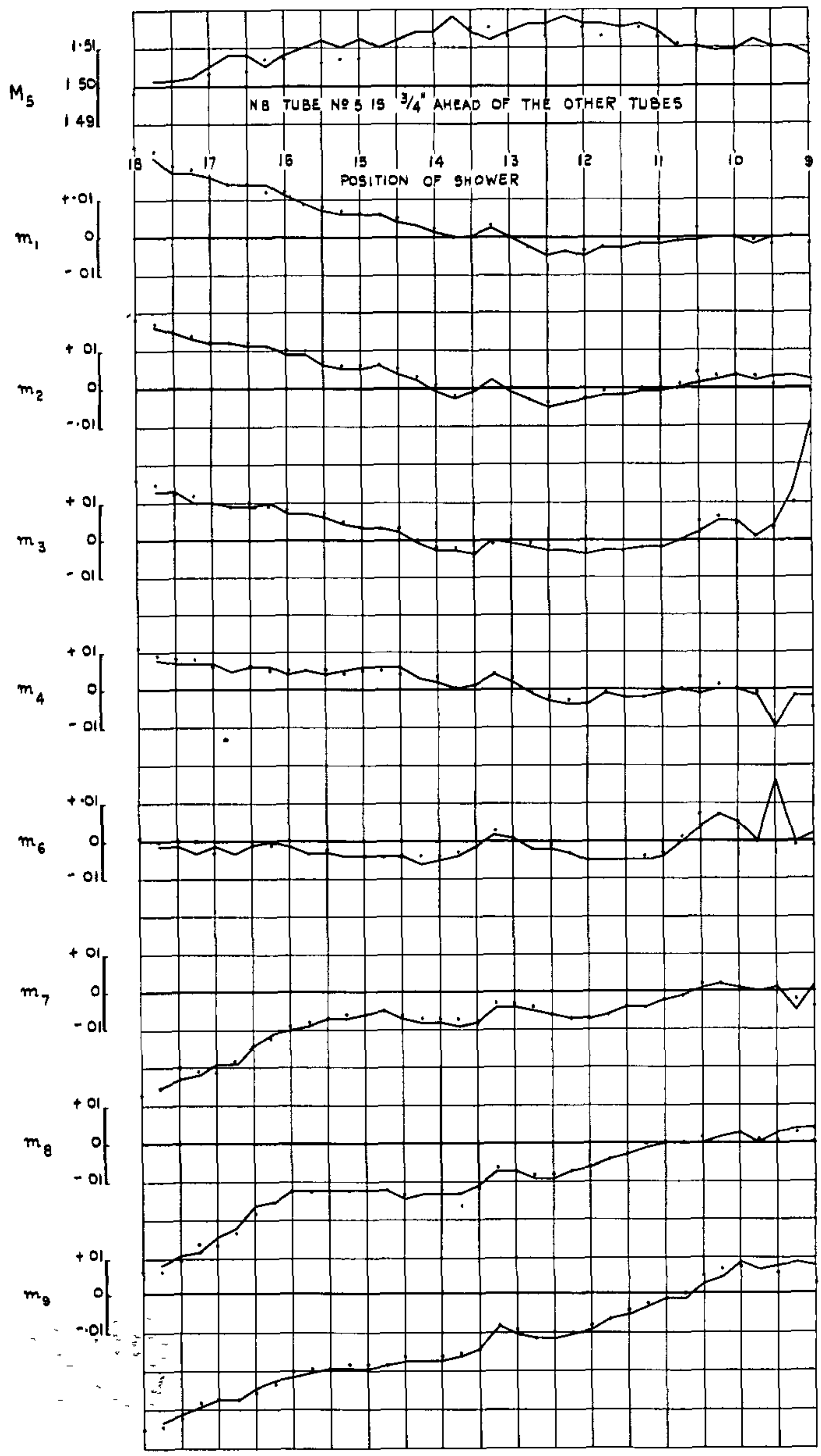


$\bar{M}$ = 1·6 , $\bar{p}_0$ = 30 , POSITION I

— $\bar{T}_0$ = 28 , $\bar{\Omega}$ = ·0002 , NO CONDENSATION SHOCKS

· $\bar{T}_0$ = 28 , $\bar{\Omega}$ = ·0005 ; CONDENSATION SHOCKS PRESENT

FIG.15. DISAGREEMENT BETWEEN RESULTS OF SIMILAR TESTS WHEN CONDENSATION SHOCKS PRESENT IN ONE CASE.



$\bar{M} = 1{\cdot}5$, $\bar{P}_0 = 30$, POSITION II

:- $\bar{T}_0 = 28$, $\bar{\Lambda} = \cdot 0001$; NO CONDENSATION SHOCKS

- $\bar{T}_0 = 28$, $\bar{\Lambda} = \cdot 0001$, NO CONDENSATION SHOCKS

FIG. 16. AGREEMENT BETWEEN RESULTS OF SIMILAR TESTS WHEN CONDENSATION SHOCKS ABSENT IN BOTH CASES.

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