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The Effect of Variation of Gear Ratio on the Performance of a Variable-pitch Airscrew for a High-speed Aeroplane.

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October, 1941

Summary.—The effect of change of gear ratio has been examined in the case of a four-bladed airscrew of 14 ft. diameter absorbing 2,000 b.h.p. at 37,000 ft. at a forward speed of 450 m.p.h. with a given engine speed (3,700 r.p.m.). Using the limited data at present available for the lift and drag of an airscrew blade section at high Mach number, it appears that for a sufficiently low gear ratio the higher working lift coefficient of each blade element may increase the induced power loss so much that it is not off-set by the reduction in compressibility loss brought about by the decreased Mach numbers of the blade sections.

The purpose of this investigation was to determine how the performance of a given airscrew operating under prescribed conditions could be improved by alteration of gear ratio. The given design conditions specified the absorption of 2,000 b.h.p. by a four-bladed variable-pitch airscrew of 14 ft. diameter at a height of 37,000 ft. and a forward speed of 450 m.p.h., giving a value of V/a of $0 \cdot 682$. The engine r.p.m. were quoted as 3,700. Writing the airscrew revs. per second as

$$n = \frac{3700}{60} \times (G.R),$$

where $G.R.$ is the gear ratio,

we have

$$\begin{aligned} J &= \frac{V}{nD} \\ &= \frac{450 \times 88}{3700 \times (G.R) \times 14} \\ \text{or } G.R &= 0 \cdot 766/J, \end{aligned}$$

and

$$\begin{aligned} k_Q &= \frac{\text{Power}}{2 \pi \rho n^3 D^5} \\ &= 0 \cdot 00459 \times J^3 \\ \text{using } \rho &= 0 \cdot 000677 \text{ slug per cu. ft. at 37,000 ft.} \end{aligned}$$

Also

$$\begin{aligned} \frac{\Omega R}{a} &= \pi \frac{V}{a} \cdot \frac{1}{J} \\ &= 2 \cdot 143/J. \end{aligned}$$

In the performance calculations the range of J covered was $2{\cdot}75$ to $3{\cdot}3$, corresponding to gear ratios of $0{\cdot}278$ to $0{\cdot}232$ and resultant tip Mach numbers of $1{\cdot}036$ to $0{\cdot}942$. These are given in Table 1.

Details of the airscrew blade are listed in Table 2. The plan form, thickness-chord distribution, zero-lift angles and pitch distribution are the same as those of airscrew B of R. & M. 2021[1], in which the blade section was Clark Y modified. As the high engine horsepower and the low air density at high altitude produce a high torque coefficient k_Q, an increase of solidity is necessary. The present calculations therefore refer to a four-bladed airscrew so that the values of solidity are 33 per cent greater than those of Reference 1. Apart from the increased number of blades, therefore, the airscrew is fairly representative of an existing design.

The method of computation was that described in R. & M. 2035[2]. A range of blade setting was covered for each J in order to obtain the setting required to absorb the given horsepower in each case ; these design settings are included in Table 1. The working values of sC_L and incidence for each of the standard radii are given in Table 3 and Fig. 1, together with the induced power loss grading coefficient (p_{C1}) and the low-speed and shock-stall components of the profile drag loss (p_{C0} and p_{CS} respectively). The integrated results relating to the whole airscrew are tabulated in Table 4 and plotted in Fig. 2.

A decrease in J at the given forward speed corresponds to an increase of gear ratio and tip speed, and results in an increase in the compressibility loss of efficiency (k_{PS}/k_Q, Table 4 and Fig. 2). It can be seen from Table 3 that at the same time the increased sectional speed is accompanied by a fall in the working values of sC_L and α, with a consequent reduction in the induced loss (k_{P1}/k_Q). The low-speed component of the drag efficiency loss shows a small increase. The overall effect is that the efficiency reaches a maximum in the region of $J = 3{\cdot}0$, corresponding to a gear ratio of $0{\cdot}255$ and a resultant tip Mach number of $0{\cdot}99$. It appears from Fig. 2 that the gear ratio can be varied between $0{\cdot}27$ and $0{\cdot}25$ ($J = 2{\cdot}8$ and $3{\cdot}1$) with practically no change in efficiency.

It is to be emphasised that all these calculations were based on the provisional lift and drag data of 4872[3] and 4944[4]*. There is evidence that the compressibility-increase of sectional drag coefficient has been somewhat underestimated. The effect of an increase in the drag coefficient would be to move the position of maximum efficiency to a higher value of J, with a lower gear ratio and lower rotational speed ; it would also sharpen the peak of the efficiency curve so that a given change from maximum efficiency would result from a smaller deviation from the optimum gear ratio. The values assumed for the low-speed drag coefficient may also be a little optimistic, and no allowance has been made for blade root losses nor for body interference ; all the values of efficiency are therefore consistently in excess of those which will probably be obtained in practice. There is also evidence from 4872[3] that at the high working lift coefficients and high Mach numbers the values used for the lift coefficient may be too high, which would have the effect of further overestimating all the calculated efficiencies.

In the case of J equal to $3{\cdot}0$, at which the curve of efficiency against gear ratio reaches its maximum value for the design conditions, calculations were made covering a range of blade setting of $4{\cdot}5$ deg. The torque grading curves and the power loss gradings are plotted in Fig. 3, and the integrated results for the efficiency and efficiency losses in Fig. 4. The variation of blade setting may be interpreted in three ways :

(1) If the values of diameter, J and ρ are kept constant, a change in blade setting requires simply a change in engine horsepower, the horsepower being directly proportional to k_Q.

(2) For a given engine horsepower, diameter and J, the variation of blade setting can be taken as representing a change of air density or of operating altitude, ρ being inversely proportional to k_Q. In the stratosphere, where the temperature remains constant, the corresponding values

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*These data are very nearly the same as those given in Part II of R. & M. 2020[5], these values being smoothed versions of the data of References 3 and 4.

of height (h) are exactly those for which the atmospheric density has the appropriate values ρ; this relation will, however, be only approximate at lower altitudes, as the variation of temperature there will affect the local velocity of sound and hence the Mach numbers for the airscrew blade sections. Since the changes in the velocity of sound are relatively much less than the changes in density, the approximate scale of height gives a fairly good indication of the effect of reduced altitude.

(3) For a given horsepower and altitude, the range of blade setting shows the effect of changes in blade diameter, provided that the airscrew r.p.m. are altered in inverse proportion so that J is kept constant. Subject to this condition the values of diameter are given by

$$D \propto (k_Q)^{-1/2}.$$

In Fig. 4, scales are included giving the values of horsepower, altitude and diameter corresponding to the variation of blade setting and subject to the conditions of (1), (2) and (3) respectively.

Comparison of the efficiency curve of Fig. 4 with the efficiency obtained for airscrew B at the same forward speed at 21,000 ft. (Table 2 of R. & M. 2021[1]) shows that it is the change of density with height that is the main source of the difference between the value 0·88 at 21,000 ft. and the maximum (0·78) at 37,000 ft. in the present report, the change in the number of blades, sectional Mach numbers and J being of secondary importance. The increase in solidity due to the additional blade is not alone sufficient to raise the torque coefficient to the value corresponding to the increased altitude ; the higher blade loading necessary increases the induced and compressibility losses.

It should also be mentioned that as the blade setting is further reduced in Fig. 4 the compressibility loss k_{PS} will decrease less rapidly (and eventually increase for very low C_L values). Meanwhile the torque coefficient is decreasing steadily ; therefore the efficiency loss will increase so that the efficiency curve will reach a maximum.

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LIST OF SYMBOLS

a	Speed of sound at height h.
c	Chord of blade element at radius r.
C_L	Lift coefficient of blade element.
D	Airscrew diameter.
$G.R.$	Gear ratio.
h	Operating altitude of aircraft.
J	Advance ratio (V/nD).
k_P	Total power loss coefficient (Power loss$/2\pi\rho n^3D^5$).
k_{P0}	Low-speed component of the profile drag power loss coefficient.
k_{P1}	Induced power loss coefficient.
k_{PS}	Compressibility component of the profile drag power loss coefficient.
k_Q	Torque coefficient (Torque$/\rho n^2D^5$).
M_t	Mach number of airscrew blade tip.
n	Airscrew rotational speed (r.p.s.).
N	Number of blades.
p_{C0}	Grading coefficient of the low-speed component of the profile drag power loss: $dk_{PO}/d(r_c{}^2)$.
p_{C1}	Grading coefficient of the induced loss: $dk_{P1}/d(r_c{}^2)$.
p_{CS}	Grading coefficient of the compressibility component of the profile drag power loss: $dk_{PS}/d(r_c{}^2)$.
q_c	Torque grading coefficient : $dk_Q/d(r_c{}^2)$.
r	Radius at blade element.
r_c	Fractional radius at blade element (r/R).
R	Tip radius.
s	Solidity $(Nc/2\pi r)$.
t	Thickness of blade section.
V	Forward speed.
α	Incidence of blade element.
η	Airscrew efficiency.
ρ	Air density.
Ω	Airscrew rotational speed (radians per second).

TABLE 1

Corresponding Values of Blade Setting, J, Gear Ratio, etc.

J	2·75	2·825	2·9	3·0	3·1	3·2	3·3
Gear ratio . .	0·278	0·271	0·264	0·255	0·247	0·239	0·232
k_Q	0·0951	0·1031	0·1115	0·1234	0·1362	0·1498	0·1643
$\Omega R/a$	0·779	0·758	0·738	0·714	0·691	0·670	0·649
M_t	1·036	1·020	1·006	0·987	0·970	0·955	0·942
Design blade setting	2·7°	3·6°	4·5°	5·7°	6·8°	8·0°	9·0°

TABLE 2

Details of Airscrew

r_c	0·3	0·45	0·6	0·7	0·8	0·9	0·95	0·975
c ins.	11·51	12·51	12·10	11·07	9·59	7·60	6·38	5·66
s	0·291	0·211	0·153	0·120	0·091	0·064	0·051	0·044
t/c per cent	16·6	11·0	8·5	7·5	6·7	6·0	5·5	5·2
Basic blade angle . .	71·9°	64·7°	57·6°	53·1°	48·8°	44·7°	42·7°	41·7°

TABLE 3

Working Values of sC_L, α and Grading Coefficients.

	J \ r_c	0·3	0·45	0·6	0·7	0·8	0·9	0·95	0·975
sC_L ..	2·75	0·223	0·177	0·121	0·088	0·058	0·031	0·020	0·014
	2·825	0·234	0·183	0·124	0·090	0·061	0·033	0·022	0·016
	2·9	0·246	0·189	0·127	0·093	0·063	0·036	0·024	0·018
	3·0	0·263	0·198	0·133	0·097	0·066	0·039	0·026	0·020
	3·1	0·278	0·206	0·137	0·100	0·069	0·041	0·028	0·021
	3·2	0·296	0·216	0·143	0·105	0·072	0·043	0·030	0·023
	3·3	0·310	0·224	0·147	0·107	0·074	0·045	0·032	0·024
C_L ..	2·75	0·77	0·84	0·79	0·73	0·63	0·49	0·39	0·33
	2·825	0·80	0·87	0·81	0·75	0·67	0·52	0·43	0·36
	2·9	0·84	0·90	0·83	0·78	0·69	0·56	0·47	0·40
	3·0	0·90	0·94	0·87	0·81	0·73	0·61	0·52	0·45
	3·1	0·95	0·97	0·89	0·83	0·75	0·64	0·56	0·49
	3·2	1·02	1·02	0·93	0·87	0·79	0·68	0·60	0·52
	3·3	1·07	1·06	0·96	0·89	0·81	0·70	0·62	0·54
α° ..	2·75	0·65	1·48	1·91	1·92	1·76	1·47	1·13	0·82
	2·825	0·95	1·68	2·03	2·00	1·81	1·48	1·12	0·78
	2·9	1·26	1·88	2·17	2·11	1·87	1·49	1·11	0·75
	3·0	1·72	2·20	2·39	2·28	2·01	1·56	1·14	0·74
	3·1	2·12	2·46	2·57	2·42	2·10	1·61	1·15	0·72
	3·2	2·62	2·82	2·84	2·64	2·30	1·77	1·27	0·82
	3·3	3·00	3·08	3·01	2·78	2·39	1·83	1·32	0·85
p_{c1} ..	2·75	0·0171	0·0179	0·0138	0·0106	0·0070	0·0036	0·0023	0·0018
	2·825	0·0202	0·0202	0·0153	0·0118	0·0081	0·0043	0·0029	0·0023
	2·9	0·0238	0·0229	0·0170	0·0131	0·0091	0·0051	0·0036	0·0030
	3·0	0·0296	0·0270	0·0196	0·0149	0·0105	0·0064	0·0046	0·0038
	3·1	0·0360	0·0315	0·0223	0·0170	0·0120	0·0075	0·0056	0·0047
	3·2	0·0443	0·0374	0·0260	0·0197	0·0140	0·0088	0·0067	0·0058
	3·3	0·0525	0·0430	0·0292	0·0219	0·0155	0·0099	0·0076	0·0065
p_{c0} ..	2·75	0·0052	0·0039	0·0032	0·0028	0·0024	0·0020	0·0017	0·0015
	2·825	0·0056	0·0042	0·0034	0·0030	0·0025	0·0020	0·0017	0·0015
	2·9	0·0062	0·0045	0·0036	0·0031	0·0026	0·0021	0·0018	0·0016
	3·0	0·0070	0·0050	0·0039	0·0034	0·0028	0·0022	0·0019	0·0017
	3·1	0·0079	0·0055	0·0043	0·0036	0·0030	0·0024	0·0020	0·0018
	3·2	0·0089	0·0061	0·0046	0·0039	0·0032	0·0025	0·0021	0·0018
	3·3	0·0100	0·0067	0·0050	0·0042	0·0034	0·0027	0·0022	0·0019
p_{cs} ..	2·75	0·0036	0·0078	0·0110	0·0124	0·0144	0·0145	0·0132	0·0121
	2·825	0·0047	0·0087	0·0111	0·0125	0·0140	0·0139	0·0127	0·0113
	2·9	0·0062	0·0102	0·0119	0·0130	0·0138	0·0135	0·0122	0·0107
	3·0	0·0086	0·0125	0·0131	0·0135	0·0139	0·0130	0·0115	0·0099
	3·1	0·0114	0·0148	0·0140	0·0141	0·0139	0·0124	0·0107	0·0090
	3·2	0·0169	0·0184	0·0165	0·0156	0·0148	0·0123	0·0104	0·0086
	3·3	0·0202	0·0223	0·0181	0·0164	0·0148	0·0123	0·0102	0·0082
q_c ..	2·75	0·107	0·139	0·139	0·127	0·103	0·070	0·051	0·039
	2·825	0·119	0·150	0·150	0·136	0·112	0·076	0·056	0·044
	2·9	0·131	0·163	0·160	0·146	0·121	0·084	0·062	0·049
	3·0	0·149	0·181	0·176	0·159	0·133	0·095	0·071	0·056
	3·1	0·168	0·200	0·191	0·173	0·144	0·104	0·079	0·063
	3·2	0·191	0·223	0·211	0·191	0·159	0·115	0·088	0·070
	3·3	0·212	0·244	0·229	0·205	0·170	0·124	0·095	0·076

TABLE 4

Power Losses and Efficiency.

J	2·75	2·825	2·9	3·0	3·1	3·2	3·3
k_{P1}	0·0088	0·0100	0·0113	0·0133	0·0154	0·0182	0·0206
k_{P0}	0·0025	0·0027	0·0028	0·0031	0·0033	0·0036	0·0039
k_{PS}	0·0106	0·0106	0·0109	0·0114	0·0119	0·0133	0·0144
k_Q	0·0952	0·1032	0·1116	0·1238	0·1356	0·1503	0·1626
k_{P1}/k_Q	0·092	0·097	0·101	0·108	0·114	0·121	0·127
k_{P0}/k_Q	0·026	0·026	0·025	0·025	0·024	0·024	0·024
k_{PS}/k_Q	0·111	0·102	0·097	0·092	0·088	0·089	0·088
η	0·770	0·775	0·776	0·775	0·774	0·766	0·761

TABLE 5

Power Losses and Efficiency for a Range of Blade Setting at $J = 3·0$

Blade setting.	3·5°	4·5°	5·7°	7·0°	8·0°
k_{P1}	0·0073	0·0098	0·0133	0·0178	0·0216
k_{P0}	0·0029	0·0030	0·0031	0·0032	0·0034
k_{PS}	0·0050	0·0077	0·0114	0·0161	0·0203
k_Q	0·0881	0·1041	0·1238	0·1453	0·1621
k_{P1}/k_Q	0·083	0·094	0·108	0·122	0·134
k_{P0}/k_Q	0·033	0·028	0·025	0·022	0·021
k_{PS}/k_Q	0·057	0·074	0·092	0·111	0·125
η	0·827	0·803	0·775	0·744	0·720

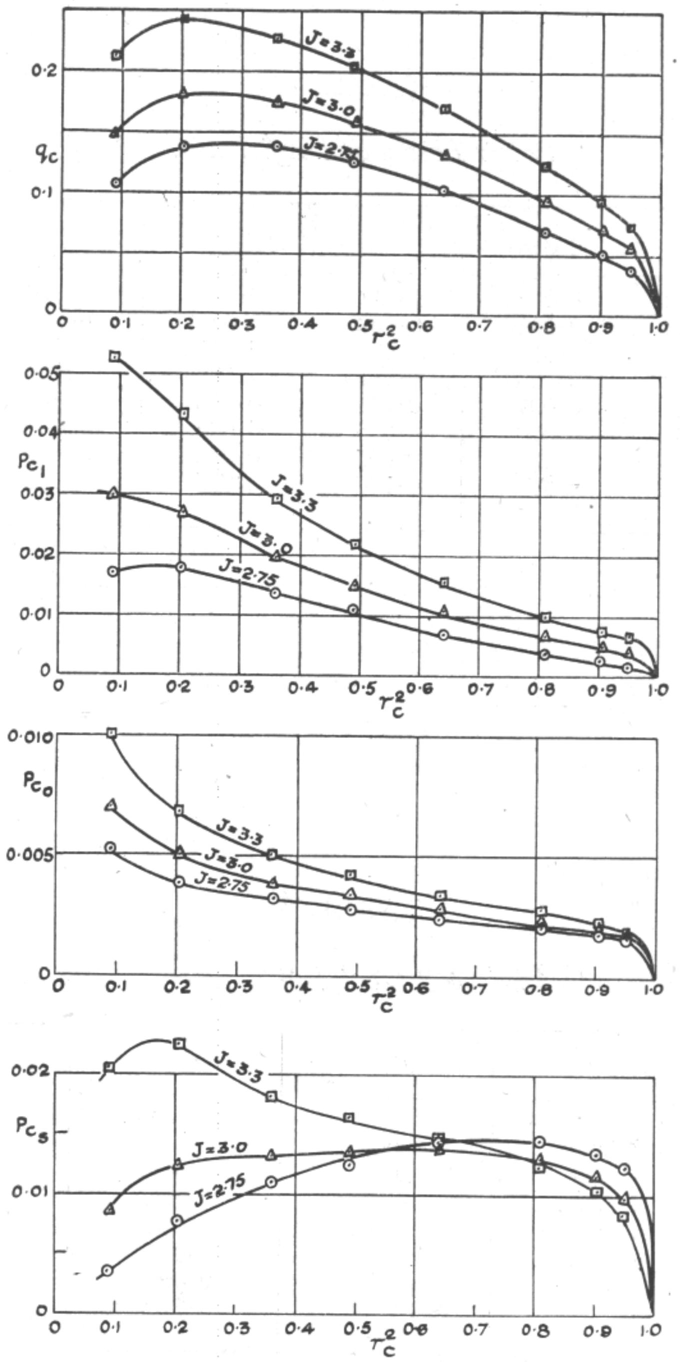

Fig. 1. Torque and Power Loss Grading Curves for Range of *J*.

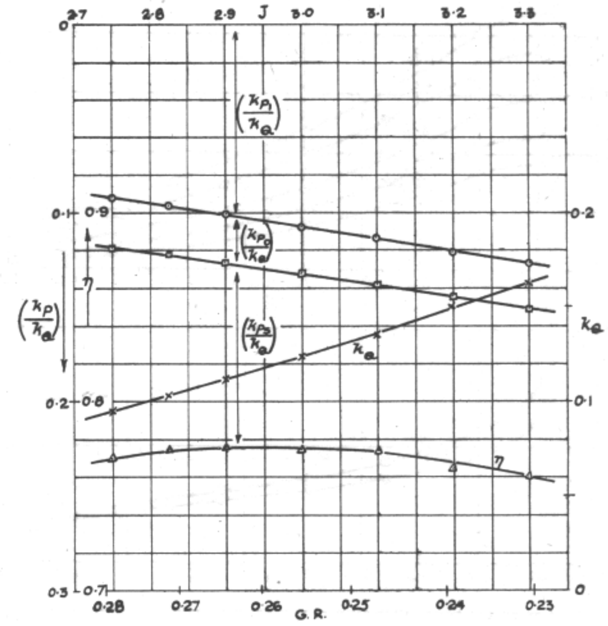

Fig. 2. Curves of Efficiency and Power Losses for Range ot *J*.

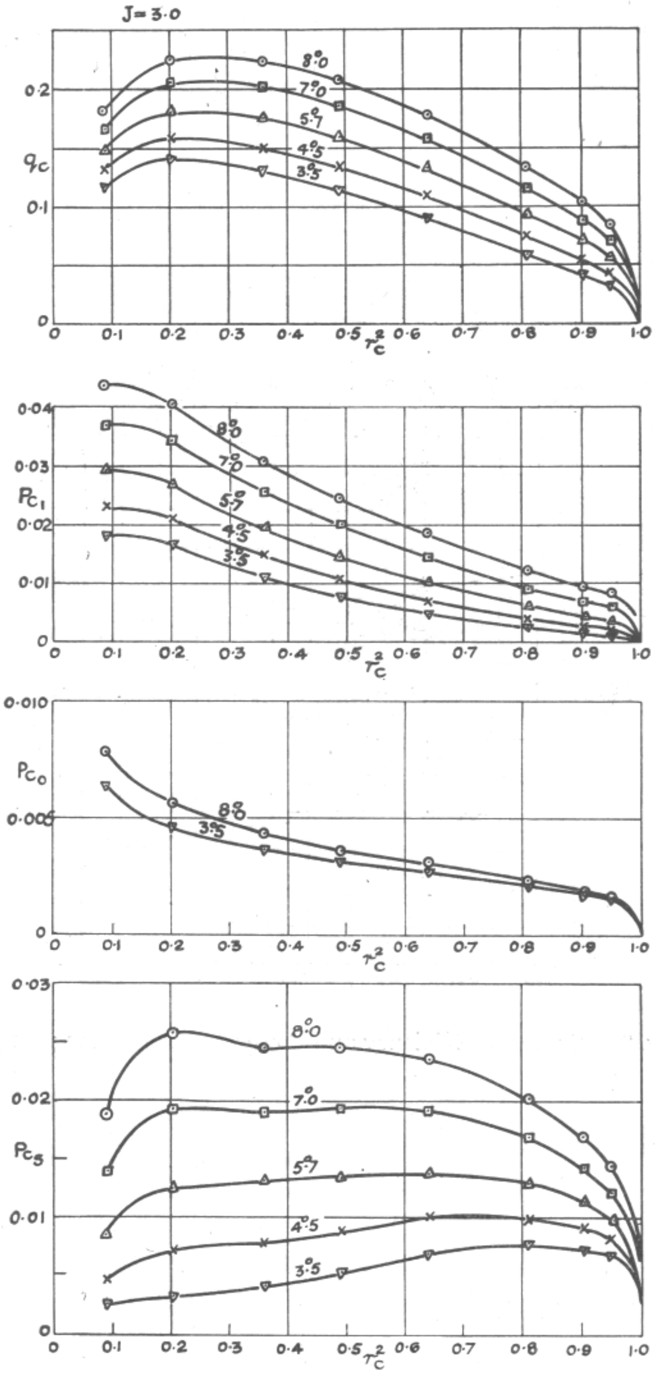

Fig. 3.—Torque and Power Loss Grading Curves for Range of Blade Setting at $J=3\cdot0$.

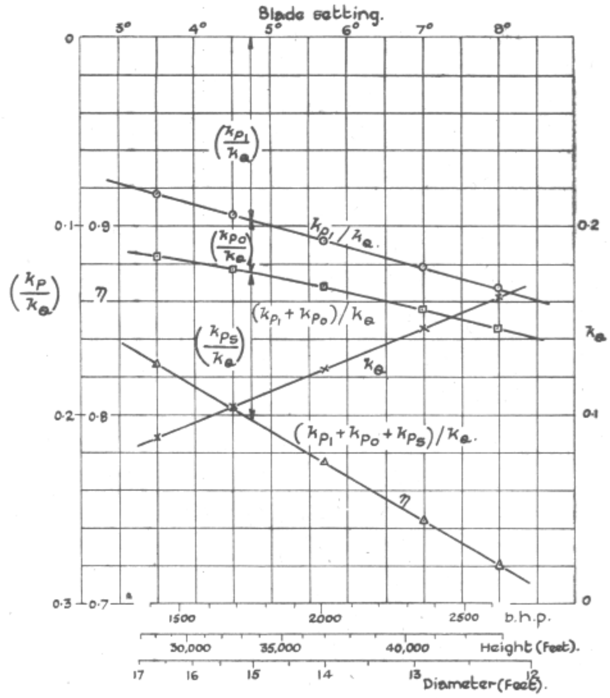

Fig. 4. Curves of Efficiency and Power Losses for Range of Blade Setting at $J = 3{\cdot}0$.

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