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Summary.—The present report gives a correlation of the results of earlier researches into the prevention of flutter of spring tabs. The restrictions on the way in which tab mass-balance must be applied, which are given in the earlier work, are shown to be very simply derivable from the conditions necessary for the elimination of elastic and inertia couplings; and from these considerations an optimum length of tab balancing arm is deduced. The recommendations for avoidance of spring tab flutter are summarised in §9. Two Appendices deal respectively with the optimum length of arm when the aerodynamic actions are taken into account, and the relation between the results of the earlier work and the recommendations of §9.

1. *Introduction.*—In view of the growing interest attaching to the use of spring tabs, a review of investigations into the flutter characteristics of these systems, and a summary of recommendations for the avoidance of flutter, appears desirable.

Spring tabs were first used many years ago, being then normally called servo tabs; at that time, however, aircraft sizes and speeds were not such that control forces called urgently for some form of assistance to the pilot, and when cases of severe flutter of the spring tabs occurred, the system was dropped for some years. In the intervening period, geared tabs became common; nowadays, however, the advantages of the spring tab are thought to be so great that the system is receiving more and more attention, and the prevention of flutter is an urgent matter.

2. *Investigations relating to the Flutter of Spring Tabs.*—The researches relating to the flutter of spring tabs, by British investigators, which are considered in the present paper, are as follows, in the chronological order of their appearance:

(1) Binary servo-rudder flutter. W. J. Duncan and A. R. Collar. R. & M. 1527, February, 1933.

(2) Experiments on servo-rudder flutter. W. J. Duncan, D. L. Ellis and A. G. Gadd. R. & M. 1652, September, 1934.

(3) Wing-aileron-tab flutter, Parts I and II. R. A. Frazer and W. P. Jones. A.R.C. Report 5668 (O.251), March, 1942. (Unpublished.)

(4) Binary aileron-spring tab flutter. G. A. Naylor and Anne Pellew. A.R.C. Report 5828 (O.264), (R.A.E. Report S.M.E. 3209), April, 1942. (Unpublished.)

(5) Experiments on binary aileron-tab flutter. C. Scruton, J. Williams and C. J. W. Miles. A.R.C. Report 5917 (O.275), July, 1942. (Unpublished.)

* R.A.E. Report S.M.E. 3249, received June, 1943.

(6) Graphical treatment of binary mass-balancing problems. R. A. Frazer. A.R.C. Report 6059 (O.282), August, 1942. (Unpublished.)

(7) Wing-aileron-tab flutter, Parts III, IV and V. W. P. Jones. A.R.C. Report 6290 (O.251a), November, 1942. (Unpublished.)

The scope of these investigations, and the principal conclusions drawn from them, are very briefly summarised below.

Reference 1.—This reference contains a theoretical investigation into binary rudder-tab flutter, with applications to a particular case. It is concluded that :—

1 (*a*) ordinary mass-balance of the tab would suppress the flutter in the case considered ;

1 (*b*) there are not sufficient grounds for asserting that ordinary mass balance of the tab will in general suppress flutter ; it is usually difficult to cater for the rudder bar fixed and free cases simultaneously.

Reference 2.—This describes a fairly comprehensive set of wind tunnel experiments covering binary rudder-tab flutter, rudder flutter involving lateral fuselage flexure, and the combined ternary motion. The results were such that the authors would not make specific recommendations for the avoidance of flutter ; for example, they found that :—

2 (*a*) mass-balance of the tab would suppress binary rudder-tab flutter when there was no elastic constraint ;

2 (*b*) when the elastic constraints, particularly that of the control circuit, had certain values, flutter occurred even when the tab was mass-balanced.

Reference 3.—This investigation is very general, and covers wing flexure, wing torsion, aileron motion and tab motion. Preloaded spring tabs are briefly considered, and the mechanism may be such that, when the spring is effectively rigid, the tab acts as a geared tab (gearing n). The more detailed parts of the investigation relate to *binary* aileron-tab flutter where the gearing mentioned above is absent ($n = 0$) and the following important conclusions are drawn :—

3 (*a*) The tab density, relative to the aileron density, should be small : this is equivalent to asking that the tab inertia should be as small as possible ;

3 (*b*) any addition of mass to the aileron alone, such as ordinary aileron mass-balance, is advantageous (this evidently accords with 3 (*a*)) ;

3 (*c*) the tab balancing mass, if present, must be placed at a distance forward of the tab hinge less than $D/(N + 1)$, where D is the distance between aileron and tab hinges, and N is the ratio of tab angle to aileron angle when the system is displaced but the control bloater is held fixed*.

3 (*d*) If condition 3 (*c*) cannot be satisfied, tab balancing should not be attempted.

It will be seen that condition 3 (*c*) imposes a most important limitation on the way in which tab mass-balance must be effected ; Reference 3 goes on to show that conclusion 1 (*a*) was justified for the balance arm assumed, which was less than the limiting length laid down above, but would not have been justified if the balance arm chosen had been greater than the limiting length.

Reference 4.—This reference contains an investigation into the same binary aileron-tab system dealt with in Reference 3 ; the aerodynamic constants used are rather different, and general theory is not attempted. A number of numerical cases are worked out, with special emphasis on the amount of the tab balancing mass and length of balancing arm. It is concluded that :—

4 (*a*) if the critical length of balance arm $D/(N + 1)$ laid down above is exceeded, flutter of the system considered occurs for any added tab balance weight ;

* The gearing N, which involves extension of the spring, must not be confused with the gearing n mentioned above.

4 (*b*) if the arm is only slightly less than the limiting length $D/(N+1)$, very large balance weights are necessary to eliminate flutter ;

4 (*c*) static balance of the tab will eliminate flutter if the length of arm is less than about 0·75 of the limiting length ;

4 (*d*) slight over-balance of the tab is necessary to avoid flutter if any backlash is present in the linkages.

Reference 5.—In the main, this paper describes an experimental verification of the conclusions of References 3 and 4, and the conclusions are borne out by the experiments with remarkable accuracy. The effects of fluid and solid friction are briefly considered. There is also a short theoretical discussion on the effect of offsetting the tab balancing arm from the plane of the tab *; the conclusions are verified by experiment. It is found that :—

5 (*a*) the recommendations of Reference 3 are borne out by experiment ;

5 (*b*) when the line from the tab hinge to the balance mass makes an angle θ with the plane of the tab, the limiting length of arm, projected on the plane of the tab, becomes $D \cos^2\theta/(N+1)$.

It thus appears that in practice it will be uneconomical to use an offset arm, and the balancing mass should be kept in the plane of the tab.

Reference 6.—The recommendations of Reference 3 were based on a stability diagram in which points representing inertia were plotted in relation to a stability boundary which was found to be nearly linear. Reference 6 gives a theoretical discussion from a graphical viewpoint of this stability boundary, and shows that it is in fact nearly linear, being part of a very flat hyperbola when the constant derivatives of classical flutter theory are assumed. It is concluded that :—

6 (*a*) the recommendations of Reference 3 are justified ;

6 (*b*) the recommendations are still valid when the gearing ratio n is not zero.

Reference 7.—This reference considers also the case where n is not zero, and studies ternary flutter involving aileron and tab and either wing flexure or torsion. Approximate methods of calculation are also given. It is concluded that :—

7 (*a*) the recommendations for the avoidance of binary aileron-tab flutter are valid even when n is not zero ;

7 (*b*) ternary flutter is very improbable if binary types have been eliminated ; *i.e.*, if the aileron has been mass-balanced against wing flexure and torsion, and if the precautions against binary aileron-tab flutter have been taken, then flutter is unlikely.

3. *Comments on the Conclusions.*—It will be seen that there are no mutually contradictory conclusions given above ; but as knowledge of the subject has increased, the underlying principles have been more clearly elucidated and the methods to be adopted for flutter prevention progressively clarified.

It will also be observed that the problem involves some restrictions of a nature which are apparently unusual in flutter prevention. In the following sections a physical explanation of these restrictions is given ; it must be understood that the explanation aims at simplicity and is not put forward in a form which will bear rigorous examination.

4. *General Remarks on Flutter Prevention.*—Flutter (except stalling flutter, for which dampings may become negative) essentially involves coupling between two or more degrees of freedom ; and the problem of flutter prevention is therefore that of simultaneous elimination of all couplings.

The principal couplings in a fluttering system are three : aerodynamic, elastic and inertial. In rare cases gravity or other couplings play a part. In general, however, if the three principal couplings could be eliminated, the danger of flutter would be removed.

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* By the plane of the tab is meant here a plane fixed in the tab containing the tab hinge and, in the neutral position, the control surface hinge.

The magnitude of the individual couplings obviously depends on the choice of the co-ordinates used to define the motions. Consider for example a fluttering semi-rigid wing having one degree of freedom in flexure and one in torsion. It is possible to define its motion in terms of its normal modes of vibration in vacuo, and then, by definition, elastic and inertia couplings are absent. But the wing flutters, and the aerodynamic coupling must therefore be severe when normal co-ordinates are used. On the other hand, it is equally possible to choose a system of co-ordinates to define the motion for which the aerodynamic coupling is slight, while one of the remaining couplings vanishes ; the other coupling must then be severe.

Now it is a fact of experience that when the co-ordinates are so chosen that elastic coupling is absent, the aerodynamic coupling is usually small. This is a most fortunate circumstance, since alteration of the aerodynamic characteristics by themselves would usually be very difficult. It is this fact which explains why mass-balancing alone is usually sufficient to suppress flutter. For example, consider the fluttering wing mentioned above; its motion may be defined by flexure of, and torsion about, some spanwise axis. If the axis is chosen to be near the quarter chord, the aerodynamic coupling is a minimum and is so slight that it is not capable alone of promoting flutter ; other couplings must also be present. But it fortunately happens that the elastic (*i.e.*, flexural) axis is seldom far from the quarter chord. If therefore the flexural axis is adopted for reference, the aerodynamic coupling will not be much greater than at the quarter chord. If the inertia coupling is eliminated by applying mass-balance, only this small aerodynamic coupling is left, and this is usually still insufficient to promote or maintain flutter.

It is usually the case that co-ordinates for which elastic coupling is absent are quite obvious : as above, flexure of, and torsion about, the elastic axis of a wing are obvious. Again, for flutter involving a control surface the choice is usually obvious ; *e.g.*, for wing-flexure-aileron flutter, flexure of the wing and rotation of the aileron relative to the wing plane are co-ordinates for which elastic coupling is absent.

It follows, therefore, that when for any given system co-ordinates have been adopted for which elastic coupling is absent, the problem of flutter prevention is usually reducible to that of mass-balancing the system with respect to these co-ordinates. It will now be shown that, even in simple cases, there are, in general, limitations to the way in which mass-balance may be applied to eliminate the inertia coupling.

5. *Elimination of Inertia Couplings.*—Consider the simple system shown in Fig. 1. It may be regarded as an idealisation of a wing section carrying an aileron, the wing section twisting about

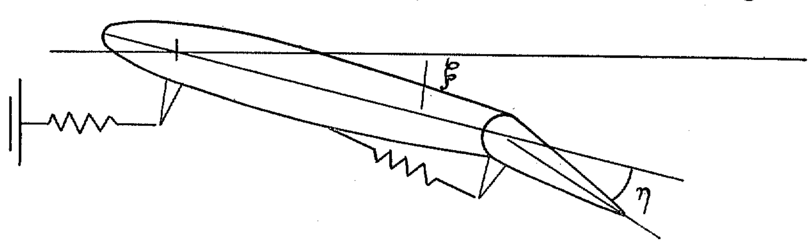

FIG. 1.

its flexural axis, or as a control surface carrying a geared or trimming tab: for definiteness it will be regarded as the latter. The natural co-ordinates to assume for defining the motion are the rotation of the control surface about its hinge, ξ, relative to the plane of the main surface (not shown) and the rotation of the tab about its hinge, η, relative to the plane of the control surface. Elastic constraints (which will normally be control cables or rods) are shown, but it is obvious that there is no elastic coupling, since a pure torque applied to the control surface induces rotation ξ but does not induce tab displacement.

Now suppose an acceleration $\ddot{\xi}$ is imposed on the control surface. This implies acceleration of the tab hinge in a direction normal to the plane of the tab. Suppose that the C.G. of the tab lies aft of the hinge, then the body of the tab will tend to lag behind the accelerating hinge, and a rotation η (of negative sense) will result. An inertia coupling is therefore present.

Next consider the effect of adding to the tab, in its own plane, a mass M mounted on an arm. Quite obviously, if the arm is directed backward, the inertia coupling is increased ; only the case where the arm is directed forward will therefore be considered. Two possibilities are shown in Fig. 2, where A is the main hinge and B the tab hinge. In the first case a short arm BM is

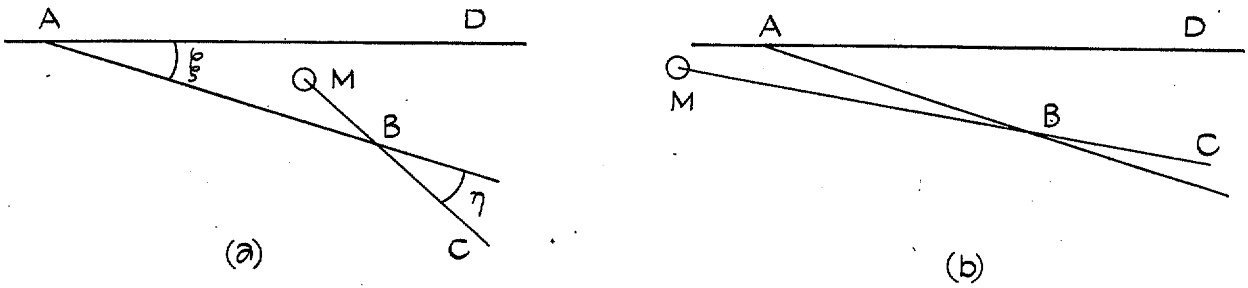

FIG. 2.

shown, so that in the undeflected position AD, M lies between A and B. In the second case a very long arm BM is used so that M lies ahead of A.

In the first case, when an acceleration $\ddot{\xi}$ is imposed, the mass M tends to remain behind on the line AD, and hence to impose a positive rotation η on the tab ; the tab by itself would acquire a negative η, and it is therefore possible to adjust M so that no rotation η results ; the coupling is then removed.

In the second case, M still tends to remain on DA produced and therefore to impose a negative rotation η on the tab ; it thus increases the inertia coupling of the tab, and has the same effect as a mass on an arm directed backwards.

It follows that, for this common case, there is a limit to the length of tab balance arm, namely the length AB ; however, the use of a tab balancing arm longer than AB would in practice be out of the question.

The limitation is, of course, implicit in the usual formula for the product of inertia, which is

$$P = \Sigma x(x + D)\delta m, \quad \ldots \quad (1)$$

where x is the distance of the element of tab mass δm aft of the tab hinge and D is the length AB. For the tab alone

$$\Sigma x(x + D)\delta m = \Sigma x^2 \delta m + D\Sigma x \delta m, \quad \ldots \quad (2)$$

and both terms are positive when the tab C.G. is aft of the hinge. If P is to be made zero, therefore, M must be added forward of the tab hinge ; if the length of its arm is λ, its contribution to P is

$$(-\lambda)(-\lambda + D)M = M\lambda^2 - M\lambda D. \quad \ldots \quad (3)$$

Obviously this can only be negative provided λ is less than D.

The expression (3) has its maximum negative value when

$$\frac{d}{d\lambda}(\lambda^2 - \lambda D) = 0, \quad \text{or} \quad \lambda = \tfrac{1}{2}D. \quad \ldots \quad (4)$$

Thus, to make P zero with minimum mass, the length of the arm should be half the length of the distance between the hinges ; in practice, however, it is often expedient to use a shorter arm, since it is difficult to make a long arm sufficiently rigid.

6. *Couplings in the Case of the Spring Tab.*—The essential physical difference between the case of spring tab flutter and most other flutter problems is that, if the ordinary "obvious" co-ordinates (*see* §4) are adopted to define the motion, an elastic coupling exists. Consider the simple spring tab system shown in Fig. 3.

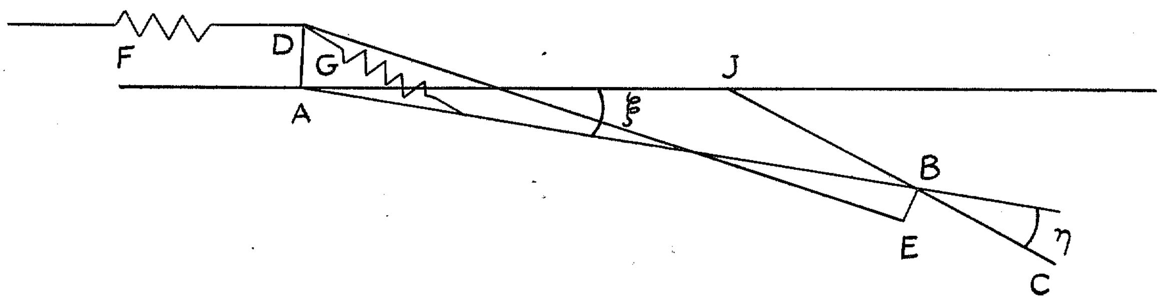

FIG. 3.

AB is the control surface and BC the tab ; the tab is operated by the rod DE. AD is the tab bloater, hinged at A and connected to AB by the spring G. F, shown diagrammatically as a spring, represents the stiffness of the control circuit.

It is at once apparent that for the usual co-ordinates shown, namely ξ and η, an elastic cross-stiffness exists. For, suppose the control surface AB is held fixed (ξ zero) and BC is displaced through η, then DA is displaced through $\eta\ BE/DA$, and the spring F is compressed (for η positive). This compression produces a moment tending to increase ξ, *i.e.*, displacement of the tab produces a moment on the control surface.

Thus the cross-stiffness is due to the spring F, and if this elastic coupling is to be avoided, the spring F must not be brought into play. This requires that, in an otherwise general displacement, AD shall not move ; the condition of no coupling therefore obtains when BC tends to rotate with AB so that $\eta BE = \xi AD$. In this condition the centre of rotation of BC is at J. Now by definition $AD/BE = N$; hence

$$\frac{BJ}{\xi} = \frac{AJ}{\eta} = \frac{AJ}{N\xi},$$

whence

$$\frac{AB}{BJ} = N + 1, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

since the angles are all assumed small.

Now if the inertia couplings for these rotations are also to be eliminated, acceleration of the system must produce no force from the tab tending to move AD. This implies that there must be no tension or compression in the rod DE, *i.e.*, the tab must have a natural tendency to rotate about J, even if DE were absent. This can only be achieved by loading the tab so that its C.G. is forward of the hinge B by an appropriate amount. Moreover, it follows exactly as in §5 that the added mass must lie between B and J, *i.e.*, on an arm of length λ not greater than $1/(N+1)$ of the distance D between tab and control surface hinges ; further, the minimum mass is required when the arm is of length $D/2(N+1)$.

7. *Comments on the Aerodynamic Coupling.*—In the foregoing section, the limiting length of arm for the balance mass of a spring tab has been deduced purely from consideration of the elastic and inertia couplings. Since the same rule is deduced in References 3 and 4 above, when the aerodynamic actions are taken into account, it appears that, in so far as prevention of flutter by mass-balancing is concerned, aerodynamic coupling is not important in this rather unusual case as well as in the more normal cases mentioned earlier. This view is further supported by the fact that some of the aerodynamic terms assumed in References 3 and 4 were quite considerably different.

8. *Effect of Offset in the Balance Mass.*—By the simple arguments already put forward it is possible to deduce the same conclusion as that of Reference 5 in relation to an offset balance mass.

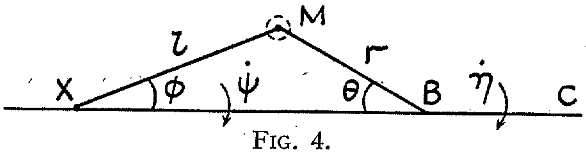

FIG. 4.

In Fig. 4 let BC be the tab, with hinge at B, and let X be the point which it is desired shall be the centre of rotation of the tab: thus X corresponds to A in Fig. 2 and to J in Fig. 3. Let the velocities of rotation about X and B be $\dot{\psi}$ and $\dot{\eta}$ respectively. Then the mass M has components of velocity normal to XB of

$$l\,\dot{\psi}\cos\phi - r\dot{\eta}\cos\theta$$

and parallel to XB

$$l\,\dot{\psi}\sin\phi + r\dot{\eta}\sin\theta.$$

Now from the geometry of the system, if $XB = a$,

$$l\cos\phi = a - r\cos\theta$$

$$l\sin\phi = r\sin\theta.$$

Hence if T is the kinetic energy of the mass M,

$$T = \tfrac{1}{2}M\{(l\,\dot{\psi}\cos\phi - r\,\dot{\eta}\cos\theta)^2 + (l\,\dot{\psi}\sin\phi + r\,\dot{\eta}\sin\theta)^2\}$$

$$= \tfrac{1}{2}M\{(a - r\cos\theta)\dot{\psi} - r\cos\theta\,\dot{\eta}\}^2 + \tfrac{1}{2}M(\dot{\psi} + \dot{\eta})^2 r^2\sin^2\theta \quad \ldots \quad (6)$$

The coefficient of $\dot{\psi}\dot{\eta}$ in this expression, namely, the contribution of M to the product of inertia, is

$$M\{r^2\sin^2\theta - r\cos\theta\,(a - r\cos\theta)\} = M\,r\,\{r - a\cos\theta\}.$$

This is only negative, provided

$$r < a\cos\theta$$

or

$$\lambda = r\cos\theta < a\cos^2\theta. \quad \ldots \quad (7)$$

Thus, whether the limiting distance for the ordinary case is D or $D/(N+1)$ (*i.e.*, plain tab of §5 and Fig. 2 or spring tab of §6 and Fig. 3), when the tab arm is offset by θ from the plane of the tab, the limiting distance λ (measured in the plane of the tab) is reduced by the factor $\cos^2\theta$.

9. *Conclusions.*—The present treatment confirms in an extremely simple way many of the conclusions of the more thorough investigations listed in §2 and gives a clear picture of the physical meaning of the recommendations for the avoidance of flutter of spring tabs. To sum up, these conclusions from all the investigations may be set out as follows:—

(*a*) The tab inertia should be as small as possible.

(*b*) Ordinary mass-balance of the control surface is necessary.

(*c*) The tab must be mass-balanced, and the mass should be placed at a distance forward of the tab hinge not greater than $D/(N+1)$ when the mass is in the plane of the tab.

(*d*) If the line joining the balance mass to the tab hinge makes an angle θ with the plane of the tab, the radial distance from the mass to the hinge must not be greater than $D\cos\theta/(N+1)$.

(*e*) The optimum length of the tab balance arm is about half the maximum length given in (*c*) and (*d*).

(*f*) In view of (*d*), the least added mass will be necessary when placed in the plane of the tab; offset balance masses are not therefore recommended.

(*g*) It appears from the more complete analyses that, in the absence of backlash, flutter will usually be eliminated by static balance of the tab in the manner laid down above ; to cover the possible development of backlash, however, it is recommended that a balance mass 20 per cent. in excess of that necessary to give static balance of the tab be used. This extra mass will be a valuable safeguard when backlash is absent.

APPENDIX I

10. *The Optimum Length of Balance Arm.*—In §6 it was shown that, from consideration of the elastic and inertia couplings only, the optimum length of arm for the tab balance is one half of the limiting length. It will now be shown that this conclusion is valid also when the aerodynamic actions are taken into consideration : the result follows directly from the work of Reference 3.

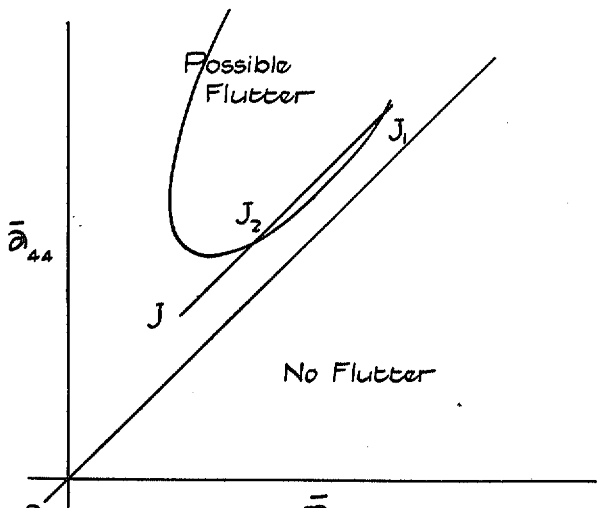

FIG. 5.

Fig. 5 shows diagramatically the stability boundary deduced in Reference 3, in which the limiting length $D/(N+1)$ was first postulated. The ordinate is the quantity $\bar{a}_{44}$, which represents an effective tab moment of inertia in a transformed system of co-ordinates; similarly $\bar{p}$ is the corresponding product of inertia. The stability boundary is very nearly linear and has a slope slightly greater than unity.

If the point J $(\bar{p}, \bar{a}_{44})$ for the unbalanced tab lies above the line, flutter is possible ; if below, binary aileron tab flutter is not possible.

In Reference 5 it is shown that, when a balancing mass is added to the tab on an arm of length $\lambda\sec\theta$, offset at an angle θ to the plane of the tab, the increments in $\bar{p}$ and $\bar{a}_{44}$ are respectively proportional to

$$\delta\bar{p} = \lambda^2 \sec^2\theta\,(N+1)\,(n+1) - \lambda D(N+n+2) + D^2$$

$$= D^2\sin^2\theta + D^2\cos^2\theta\left\{1 - \frac{\lambda(N+1)}{D\cos^2\theta}\right\}\left\{1 - \frac{\lambda(n+1)}{D\cos^2\theta}\right\} \quad \ldots \quad (8)$$

$$\delta\bar{a}_{44} = \lambda^2\sec^2\theta\,(N+1)^2 - 2\lambda D(N+1) + D^2$$

$$= D^2\sin^2\theta + D^2\cos^2\theta\left\{1 - \frac{\lambda(N+1)}{D\cos^2\theta}\right\}^2. \quad \ldots \quad (9)$$

Consider the point J shown ; let it lie in the possible flutter region. Now let a mass be added to the tab on an arm of zero length ; by (8) and (9) J will be displaced both horizontally and vertically by a distance proportional to D^2, to the point J_1 : it will thus move closely parallel to the stability boundary.

If the length of arm is increased from zero to the value given by

$$1 - \frac{\lambda(N+1)}{D\cos^2\theta} = 0, \quad \ldots \quad (10)$$

then J_1 will move to J_2, where the difference between the ordinates and abscissae of J and J_2 is now proportional to $D^2\sin^2\theta$.

In fact, by elimination of λ between (8) and (9) it is easy to show that the locus of the new inertia point, as λ is varied, is a parabola : this parabola must pass through J_1 and J_2. By consideration of the inertia point for λ very large, for which the increments in abscissa and ordinate are respectively proportional to

$$\lambda^2 (N + 1) (n + 1)$$

and

$$\lambda^2 (N + 1)^2,$$

it follows that, since $N > n$, the disposition of the parabola must be as shown in Fig. 5. Only the part of the parabola for λ positive is shown ; it therefore starts from J_1 for $\lambda = 0$, passes through J_2 when (10) is satisfied, and so to infinity.

It is now apparent that only between J_1 and J_2 is the locus nearer the stability boundary than the point J, and the balance arm λ (projected on to the plane of the tab) must therefore lie between the limits

$$0 < \lambda < \frac{D \cos^2 \theta}{N + 1}, \quad \ldots \quad (11)$$

which is the result of Reference 5 and of §8. Moreover, the closest approach to stability (or the deepest incursion into the stability region when the parabola cuts the boundary) occurs when the tangent to the parabola is parallel to the boundary. If the slope of the latter is taken to be unity this gives

$$\frac{d\,(\delta\,\bar{a}_{44})}{d\,(\delta\,\bar{p})} = \frac{d\,(\delta\,\bar{a}_{44})}{d\,\lambda} \cdot \frac{d\,\lambda}{d\,(\delta\bar{p})} = 1,$$

whence by differentiation of (8) and (9) with respect to λ it is found that

$$\lambda = \frac{D \cos^2 \theta}{2\,(N + 1)}. \quad \ldots \quad (12)$$

Thus, as in §8, the optimum length is half the limiting length.

If account is taken of the fact that the slope of the boundary is slightly greater than unity, say $1 + \varepsilon$, then the optimum length is, from Fig. 5, slightly less than half the limiting length. The optimum given by (12) is in fact reduced by the factor

$$1 - \frac{\varepsilon\,(N + 1)}{N - n}.$$

APPENDIX II

11. *Further Remarks on the Bearing of Earlier Work on the Recommendations for Avoidance of Spring Tab Flutter.*—In the present Appendix the results of the References of §2 will be more closely examined in relation to the conclusions of §9.

11.1. *The Effect of Variation of Tab Inertia.*—Recommendation (*a*) of §9, which requires that the tab inertia shall be as small as possible, was first put forward in Reference 3. Support for the recommendation is also provided by work described in certain of the other references. Some experiments described in Reference 2 have an indirect bearing on recommendation (*a*).

Consider the effect of adding a mass m at D in Fig. 3. From the geometry of the system it is seen that the kinetic energy of m will be given by

$$T = \tfrac{1}{2} m\, (\dot{\xi} AD - \dot{\eta} BE)^2,$$

and the contributions of m to the ordinary moment of inertia of the control surface, product of inertia, and tab moment of inertia are thus

$$mAD^2, \quad - mAD.BE = - mAD^2/N, \quad mBE^2 = mAD^2/N^2,$$

respectively. The addition of m thus increases both moments of inertia and decreases the inertia coupling. Now increase in the control surface inertia is beneficial ; so is reduction in the product of inertia. In spite of these beneficial effects, Fig. 5 of Reference 2 shows clearly that when the rudder bar is fixed (spring tab case) increase in inertia of the control lever AD causes a considerable reduction in critical speed until a large control lever inertia is reached. This reduction must be attributable to the increase in effective tab inertia. In fact, the critical speed does not tend to rise, for curves 2 of Fig. 5 and 6 of Reference 2, until the moment of inertia of the control bar reaches 80×10^{-6} slug ft.^{2} approximately. This represents a contribution to the product of inertia of $-80 \times 10^{-6}/N$ slug ft.^{2}, *i.e.*, -35×10^{-6} slug ft.^{2} approximately, since $N = 2 \cdot 3$ (deduced from Fig. 2 of Reference 2). The product of inertia of the unbalanced flap is $6 \cdot 22 \times 10^{-6}$ slug ft.^{2} (Table 2 of Reference 2) so that, in effect, the tab is very greatly over-balanced when the critical flutter speed begins to increase. At the same time, however, the contribution to the tab inertia is $80 \times 10^{-6}/N^2 = 15 \times 10^{-6}$ slug ft.^{2}, and it can be deduced from Table 2 of Reference 2 that the unbalanced tab has an inertia of only $0 \cdot 50 \times 10^{-6}$ slug. ft.^{2}.

Thus, these experiments support recommendation (*a*) strongly.

In Reference 5 variations in tab inertia alone are not recorded ; however, the tab was made heavier, relative to the aileron, by cutting lightening holes in the latter. This had the effect of reducing the critical flutter speed, and thus provides support for recommendation (*a*).

11.2. *The Effect of Variation in Control Surface Inertia.*—Recommendation (*b*) of §9 also originated in Reference 3, but was given there in the form " any addition of mass to the aileron is beneficial in helping to prevent binary aileron-tab flutter." It was, of course, evident that such additions of mass must be in the form of aileron mass-balance weights, if the wing aileron flutter characteristics were not to be adversely affected.

Reference 2 records experiments supporting this recommendation also. Fig. 3 of Reference 2 shows that, for binary servo-rudder flutter, increase in the moment of inertia of the rudder alone always results in an increased critical flutter speed.

Reference 5 quotes a series of experiments which show that in all cases, addition of mass to the control surface alone resulted in an increase in the critical flutter speed.

Reference 7 examines, theoretically, the effect of addition of mass to the aileron for ternary wing-aileron-tab flutter, and shows that elimination of the appropriate wing-aileron product of inertia is necessary if ternary flutter is to be avoided. This work led to the formulation of recommendation (*b*) as given in §9.

11.3. *The Effects of Tab Balance Weight and Position.*—References 3, 4 and 5, all consider fully the effects of position of the tab balancing mass in relation to the limiting distances laid down, and the results do not need comment here. Reference 3, as has been stated, also examines the conclusions of Reference 1 and shows that they are consistent with the more recent work.

It is of interest to consider the experimental results of Reference 2 in the light of recommendations (*c*) and (*d*) of §9.

From Reference 2, p. 6, it may be deduced that the distance between rudder and tab hinges is

$$D = 0 \cdot 307 \text{ ft.};$$

also, from Fig. 2, approximately

$$N = 2 \cdot 3$$

$$\lambda \sec \theta = 0 \cdot 05 \text{ ft.}$$

$$\theta = 40 \text{ deg.}$$

and

$$\lambda = 0 \cdot 039 \text{ ft.},$$

where $\lambda \sec \theta$ is the radial distance from the tab hinge to the balancing masses.

The limiting (radial) length of arm allowed by (*d*) is

$$\frac{0\cdot307 \times 0\cdot766}{(2\cdot3 + 1)} = 0\cdot071 \text{ ft.},$$

so that the arm was 0·75 of the limiting length, approximately. The improvement in the flutter characteristics due to the addition of balance weights, shown by curves 2 and 3 of Fig. 5 of Reference 2, is therefore to be expected.

The values of the inertia couplings present are also of interest. The formula for the product of inertia for two axes distant a apart is, for the tab and balance masses (*see* §8),

$$P(M, a) = a\sum_t x\delta m + \sum_t x^2\delta m - 2aM\lambda + 2M\lambda^2 \sec^2\theta,$$

where $\sum_t$ denotes summation over the unbalanced tab. Thus,

$$P(M, D) = D\sum_t x\delta m + \sum_t x^2\delta m - 2DM\lambda + 2M\lambda^2 \sec^2\theta,$$

$$P(M, \frac{D}{N+1}) = \frac{D}{N+1}\sum_t x\delta m + \sum_t x^2\delta m - 2\frac{D}{N+1}M\lambda + 2M\lambda^2 \sec^2\theta,$$

and $$P(M, \frac{D}{N+1}) = P(M, D) - \frac{N}{N+1} D (\sum_t x\delta m - 2M\lambda).$$

In particular $$P(0, \frac{D}{N+1}) = P(0, D) - \frac{N}{N+1} D\sum_t x\delta m.$$

Table 2 of Reference 2 gives the following data :

(*a*) Tab unbalanced :

$$P(0, D) = 6\cdot22 \times 10^{-6} \text{ slug ft.}^2$$

$$\sum_t x\delta m = \bar{x}\sum_t \delta m = 0\cdot024 \times 7\cdot75 \times 10^{-4} = 18\cdot6 \times 10^{-6} \text{ slug ft.}$$

Hence

$$P(0, \frac{D}{N+1}) = (6\cdot22 - \frac{2\cdot3}{3\cdot3} \cdot 0\cdot307. \; 18\cdot6)\, 10^{-6} = 2\cdot24 \times 10^{-6} \text{ slug ft.}^2$$

(*b*) Tab slightly overbalanced :

$$P(M_1, D) = -1\cdot42 \times 10^{-6} \text{ slug ft.}^2$$

$$\sum_t x\delta m - 2M_1\lambda = -0\cdot006 \times 16\cdot1 \times 10^{-4} = -9\cdot7 \times 10^{-6} \text{ slug ft.}$$

Hence

$$P(M_1, \frac{D}{N+1}) = (-1\cdot42 + \frac{2\cdot3}{3\cdot3}. \; 0\cdot307. \; 9\cdot7)\, 10^{-6} = +0\cdot65 \times 10^{-6} \text{ slug ft.}^2$$

(*c*) Tab overbalanced :

$$P(M_2, D) = -4\cdot61 \times 10^{-6} \text{ slug ft.}^2$$

$$\sum x\delta m - 2M_2\lambda = -0\cdot011 \times 19\cdot6 \times 10^{-4} = 21\cdot6 \times 10^{-6} \text{ slug ft.}$$

Hence

$$P(M_2, \frac{D}{N+1}) = (-4\cdot61 + \frac{2\cdot3}{3\cdot3}. \; 0\cdot307. \; 21\cdot6)\, 10^{-6} = 0.$$

It will be seen that case (*b*) corresponds to a reduction of the product of inertia, in the modified co-ordinates for which the elastic coupling is absent, to about 0·3 of the value for the unbalanced tab, given in case (*e*). Since the inertia coupling is still present, however, the occurrence of flutter is not unexpected.

For case (*c*), however, the modified product of inertia happens to be just zero, so that in this case neither elastic nor inertia coupling is present ; no flutter was obtained for this condition.

Thus, it is seen that, in these experiments, a fairly considerable overbalance of the tab is needed to eliminate flutter when the balance arm is about three-quarters of the limiting length It seems justifiable to assume that elimination of flutter could have been achieved with less balancing mass, if the arm had been in the plane of the tab and of half the limiting length.

11.4. *Effects of Variation of Elastic Stiffnesses*.—The principal source of information on the effects of variation in elastic stiffnesses is Reference 2, Figs. 3, 4 and 5. The first figure shows that when a spring *G* (Fig. 3 of the present report) is added, *i.e.*, when the system is converted from a pure aerodynamic servo to a spring tab—a rise in critical speed results; the percentage rise remains roughly constant as the control surface inertia is varied. This result is obtained in the absence of tab mass balance.

Fig. 4 of Reference 2 contains some useful information. It shows that, whether the circuit stiffness is operative or not (rudder bar fixed or free), increase in the stiffness of the spring *G* (present report, Fig. 3) results in increased critical speed. Fig. 4 of Reference 2 contains another very interesting curve ; it relates to the condition of slight mass overbalance of the tab, with respect to the true hinges, i.e., to condition (*b*) of §11.3. As is shown in §11.3 the tab is underbalanced with respect to the axes for which elastic coupling is absent, and the curve of Fig. 4, Reference 2, shows that flutter occurs. There is, moreover, only one elastic constraint, namely, that of the control cables, so that the critical speed should be proportional to the square root of this stiffness. This is closely true, except where the stiffness becomes very small, when the critical speed jumps to infinity. This behaviour appears to be due to the fact that, with the vanishing of the main stiffness, the elastic coupling also vanishes, and for the usual co-ordinates which would be used in the absence of elastic coupling the system is overbalanced and stable. The same physical system is, in fact, the limiting case of the spring tab as the stiffness tends to zero (system underbalanced and critical speed also tending to zero), and of the ordinary geared tab as the stiffness of the constraints tends to zero (system overbalanced and critical speed infinite). It seems probable that some small variation in the condition assumed, *e.g.*, a small non-linearity in the equations of motion, effectively changes the unstable spring tab system to the stable system when the stiffness becomes very small.

Fig. 5 of Reference 2 (curves 2 and 5) shows that the addition of a spring constraint *G* raises the critical flutter speed ; this remains true as the control bar moment of inertia is varied.

Fig. 2 of Reference 4 gives a theoretical variation of critical speed with the stiffness of the spring *G* for a number of cases where the balance arm is longer than the minimum for the avoidance of flutter. The critical speed rises as the stiffness is increased, though the rise is only slight.

All the evidence, therefore, points to the fact that, if flutter occurs, increase in the spring stiffness of a spring tab is beneficial and there is no evidence to suggest that, if flutter has been eliminated, change in the spring stiffness may be dangerous.

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