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Part I:—Simple, Approximate Formulae for General Application

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PART I

Simple, Approximate Formulae for General Application.

Summary.—It is shown that for an elongated body of revolution of any shape in a closed rectangular tunnel of any cross section between square and duplex, the blockage correction is given approximately by

$$\frac{\Delta u}{U_0} = 0 \cdot 68 \frac{V}{h^2 b},$$

where V is the volume of the body, b is the length of the longer side and h is the length of the shorter side of the tunnel cross section. For a wing of any section spanning the tunnel

$$\frac{\Delta u}{U_0} = 0 \cdot 62 \frac{A}{h^2} = 0 \cdot 62 \frac{V}{h^2 b},$$

where b is the length of the side of the tunnel parallel to the wing span, h is the length of the other side, and A is the area of the section of the wing, and V is the volume of the wing. These formulae are accurate to within about $\pm$ 5 per cent.

To cover both cases, and therefore the intermediate case of a complete aircraft model of span less than or equal to the width of the tunnel, we may write

$$\frac{\Delta u}{U_0} = 0 \cdot 65 \frac{V}{h^2 b},$$

provided that the wing is parallel to the longer side of the tunnel. This formula is estimated to be accurate to within $\pm$ 10 per cent. which will be sufficient for most cases.

These formulae do not include wake blockage effects.

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* R.A.E. Technical Note No. Aero 1286 (Misc.), received 6th January, 1944.

† R.A.E. Technical Note No. Aero 1258 (H.S.T.), received 20th October, 1943.

Introduction.—Lock[1] has shown that the blockage correction for a body of revolution in a closed tunnel is given, to a close order of approximation, by

$$\frac{\Delta u}{U_0} = \tau \, \Lambda \left(\frac{S}{C}\right)^{3/2}, \qquad \qquad (1)$$

where

U_0 is the velocity in the empty tunnel,

Δu is the increase in the axial velocity in the neighbourhood of the body due to the blockage,

τ is a factor depending only on the shape and cross section of the tunnel,

Λ is a factor depending only on the meridian shape of the body,

S is the maximum cross sectional area of the body,

and C is the cross sectional area of the tunnel.

Lock[1] evaluated the factor Λ for a comprehensive range of fineness ratios for spheroids and Rankine ovoids; in Part II, the factor has similarly been determined for streamline shapes with pointed tails. In every case Λ has been found to be a nearly linear function of the fineness ratio, which suggests that the blockage correction can be written as proportional to the volume of the body, and the factor of proportionality is then independent of fineness ratio. A proof of this is given in para. 2 and it is shown that the factor of proportionality is also independent of the shape of the body. A general formula is derived which gives to a close order of approximation the blockage correction for an elongated body of revolution of any shape in a closed tunnel of any rectangular section ranging from square to duplex.

Similarly, for symmetrical wing sections spanning the tunnel the blockage correction takes the form

$$\frac{\Delta u}{U_0} = \tau \, \Lambda \left(\frac{t}{h}\right)^2, \qquad \qquad (2)$$

where τ is again a function of the shape of the cross section of the tunnel,

Λ is now a function of the shape of the wing section,

t is the maximum thickness of the section,

and h is the length of the tunnel side normal to the wing span.

Lock[1] has evaluated the value of Λ for the following types of sections :—

(*a*) Ellipses

(*b*) Generalised Joukowski sections (Fage's sections)

(*c*) Simple Joukowski sections.

In every case Λ was found to be a nearly linear function of the fineness ratio, which again suggests that the blockage correction can be simply written as proportional to the volume of the body and independent of the fineness ratio. The results for the above three types of sections have been examined with this in view, and, as in three dimensions, the factor of proportionality has been found to be nearly independent of shape of section. Further, by arranging the derived formulae for two and three dimensions to involve the dimensions of the tunnel cross section in the same manner, the two formulae are found to be in fair agreement. A single mean formula is therefore suggested giving the blockage correction in both two and three dimensions for all shapes of cross section of the tunnel ranging from square to duplex; the accuracy of this formula is believed to be within $\pm$ 10 per cent. This formula can be applied to evaluate the blockage correction for complete models in a wind tunnel.

2. *Derivation of General Formula for the Blockage Correction for Bodies of Revolution.*—It is shown in Part II that to a close order of approximation the source distribution equivalent to an elongated body of revolution is given by $I\,(z)$, where

$$I\,(z) = \pi U_0 \frac{d}{dz}(r_0^2), \qquad (3)$$

where z is measured along the axis of the body from its mid-point, and r_0 is the radius of cross section of the body. In Part II it is further shown that the error involved in deriving the equivalent doublet from this formula for the source distribution is negligible since it is less than, and of opposite sign to, the overall error involved in Lock's method for obtaining the blockage correction by replacing the images with equivalent doublets.

If the body length is $2l$, then the equivalent doublet is

$$\begin{aligned} Q &= \int_{-l}^{l} I(z)\, z\, d\, z = \int_{-l}^{l} \pi U_0 \frac{d}{dz}(r_0^2)\, z\, dz \\ &= \pi U_0 \left[{}_{-l}^{\,l}\; r_0^2\right] - \pi U_0 \int_{-l}^{l} r_0^2\, dz \\ &= -\,U_0 V, \end{aligned} \qquad (4)$$

where V = volume of the body. The factor Λ is defined by

$$\frac{|Q|}{U_0} = \frac{\Lambda \pi\, t^3}{4},$$

where t is the maximum thickness of the body, and hence

$$\Lambda = \frac{4}{\pi}\frac{V}{t^3}, \qquad (5)$$

It follows that

$$\begin{aligned} \frac{\Delta u}{U_0} &= \tau\, \Lambda \left(\frac{S}{C}\right)^{3/2} \\ &= \frac{\tau\sqrt{\pi}}{2}\,\frac{V}{C^{3/2}} \end{aligned} \qquad (6)$$

For a square tunnel $\tau = 0{\cdot}809$, and for a duplex tunnel $\tau = 1{\cdot}03$. Hence, for a square tunnel

$$\frac{\tau}{C^{3/2}} = \frac{0{\cdot}809}{h^2 b},$$

and for a duplex tunnel ($b = 2h$)

$$\begin{aligned} \frac{\tau}{C^{3/2}} &= \frac{1{\cdot}03}{h^2 b\sqrt{2}} \\ &= \frac{0{\cdot}729}{h^2 b}; \end{aligned}$$

we here define b as the length of the longer side of the cross section. A mean relation for both tunnels is therefore

$$\frac{\tau}{C^{3/2}} = \frac{0{\cdot}77}{h^2 b}. \qquad (7)$$

For the extreme cases of square and duplex tunnels it will be seen that this relation is accurate to within about 5 per cent., but since most tunnels have shapes lying somewhere between square and duplex the accuracy of this relation will in general be considerably closer than 5 per cent.

From (6) and (7) we then have

$$\frac{\Delta u}{U_0} = \frac{0\cdot 77\sqrt{\pi}}{2}\,\frac{V}{h^2 b}$$

$$= 0\cdot 68\,\frac{V}{h^2 b}\,. \qquad \ldots \qquad (8)$$

This simple relation therefore gives the blockage correction for an elongated body of revolution of any shape in a rectangular tunnel of any cross section between square and duplex to an accuracy that is within ± 5 per cent.

3. *Derivation of General Formula for the Blockage Correction in Two Dimensions.*—By an argument, precisely similar to that given above, and based on the approximate source distribution given by thin aerofoil theory, it may be shown that the equivalent doublet is proportional only to the area of cross section of the wing section.* The constant of proportionality thus derived, however, is not as accurate in this case as for three dimensions, and it is better to derive it empirically.

We have, according to Lock,

$$\frac{\Delta u}{U_0} = \tau\,\Lambda\left(\frac{t}{h}\right)^2$$

$$= 0\cdot 822\,\Lambda\left(\frac{t}{h}\right)^2, \qquad \ldots \qquad (9)$$

for a rectangular tunnel of height h. Hence, if A is the area of cross section of the aerofoil,

$$\frac{\Delta u}{U_0}\Big/\frac{A}{h^2} = \frac{\Delta u}{U_0}\Big/\frac{V}{h^2 b}$$

$$= 0\cdot 822\,\Lambda\,\frac{t^2}{A}$$

$$= K, \text{ say} \qquad \ldots \qquad (10)$$

From Lock's results for Λ for elliptical sections, generalised Joukowski sections, and simple Joukowski sections, the value of K has been calculated for values of c/t of 5·0 and 10·0. The results are given in the following table :—

Type of section	$c/t = 5\cdot 0$	$c/t = 10\cdot 0$
Ellipse	0·628	0·575
Generalised Joukowski ..	0·638	0·585
Simple Joukowski	0·649	0·590

* Thus it can be easily shown that the source distribution is given approximately by

$$-\,I(z) = 2U_0\,\frac{dy_0}{dz}\,,$$

where z is measured along the chord line from the leading edge, and y_0 is the ordinate of the section. Hence, the equivalent doublet is

$$Q = \int_0^c I(z)\,z\,dz = -\,U_0 A$$

where A is the area of the section.

It will be seen, therefore, that for $c/t = 5 \cdot 0$ the values of K for the various sections are all close to $0 \cdot 64$, whilst for $c/t = 10 \cdot 0$ the values of K are all close to $0 \cdot 59$. A mean formula for the blockage correction in two dimensions is, therefore,

$$\frac{\Delta u}{U_0} = 0 \cdot 62 \frac{V}{h^2 b}, \quad \ldots \quad (11)$$

where $V = Ab$ is the volume of the wing; and the probable accuracy of this formula is about $\pm$ 5 per cent.

4. *General Formula for Two and Three Dimensions.*—Comparing equations (8) and (11) it will be seen that to an accuracy of about $\pm$ 10 per cent. we can write

$$\frac{\Delta U}{U_0} = 0 \cdot 65 \frac{V}{h^2 b} \quad \ldots \quad (12)$$

to cover both the two- and three-dimensional cases. It may be noted that where this formula is likely to be most inaccurate, as in two dimensions for very thin sections (c/t large), the correction will in any case be small.

This formula has the advantage that it can reasonably be applied to derive the blockage correction for a complete aircraft model, provided the wing span is parallel to the longer side of the tunnel.

It is, of course, understood that these corrections do not include the wake blockage corrections which are additive to the corrections discussed in this note.

PART II

Note on the Blockage Correction for Streamline Bodies of Revolution

Summary.—An approximate method for obtaining the source distribution of a body of revolution is used to derive the equivalent doublet whose field is the same as that of the body at large distances from it. The blockage correction factor Λ was then obtained by Lock's method of replacing the image system for the body in a tunnel by a system of equivalent doublets. It is found that for streamline shapes with pointed tails

$$\Lambda = 0 \cdot 148 \; + 0 \cdot 523 \; L/t,$$

where L/t is the fineness ratio.

A discussion of the accuracy of the method is given.

1. *Introduction*.—In R. & M. 1275[1] Lock demonstrated a method of calculating the blockage correction for a body of revolution in a tunnel of rectangular section. The method was based on the fact that the field of a body at distances large compared with its dimensions is the same as that of a doublet. Hence, it is argued, the effect at the body of the image system of the body in the tunnel can be closely approximated to if each image is assumed to be a doublet. Lock applied this process to Rankine ovoids and spheroids, and calculated a blockage correction factor Λ. This factor is a function only of the meridian shape of the body ; for bodies of similar shape it is a function only of fineness ratio. It occurs in the blockage correction factor as follows :—

$$\frac{\Delta u}{U_0} = \tau \, \Lambda \left(\frac{S}{C}\right)^{3/2} \quad \ldots \quad (1)$$

where (as in Part I)

Δu is the increment in the stream velocity at the body due to the blockage,

U_0 is the velocity in the empty tunnel,

τ is a factor depending only on the shape of the cross section of the tunnel,

S is the maximum cross sectional area of the body,

and C is the cross sectional area of the tunnel.

The factor Λ given by Lock for Rankine ovoids and spheroids is shown in Fig. 1 as a function of fineness ratio.

Because of the difficulty of estimating the equivalent source distribution for streamline bodies of revolution, no attempt appears to have been made to evaluate the factor Λ for such shapes. Hitherto, the value has had to be guessed, when required, from the corresponding values for ovoids and spheroids. For tests at low Mach numbers this procedure is reasonable, since the correction is generally very small and does not require to be accurately estimated. For tests at high speeds, however, the blockage correction is increased, as compared with the incompressible flow correction, by the factor

$$\frac{1}{(1 - M^2)^2},$$

where M is the Mach number[2]. For such cases, therefore, it is desirable to have a more accurate estimate of the blockage correction. The object of this note is to provide such an estimate.

2. *Method of Calculation and Results.*—Let the z axis be along the axis of symmetry. Then any system of sources and sinks $I(z)$ distributed along the axis between the points $z = \pm l$ can be expressed in the form

$$I(z) = 2\pi \sum_{1}^{\infty} A_n P_n(\mu_1), \quad \ldots \quad (2)$$

where $\mu_1 = z/l$,

and $P_n(\mu_1)$ is the Legendre polynomial of order n of the first kind.* These polynomials satisfy the relation

$$\int_{-1}^{1} P_n(\mu_1) \cdot P_m(\mu_1)\, d\mu_1 = 0, \text{ if } n \neq m,$$

$$= \frac{2}{2n+1}, \text{ if } n = m.$$

Also $P_0(\mu_1) = 1$, and $P_1(\mu_1) = \mu_1$.

If $I(z)$ is the equivalent source distribution for a body, then the doublet at the origin whose field is the same as that of the body at large distances from it is given by

$$Q = -l^2 \int_{-1}^{1} I(z) \cdot \mu_1\, d\mu_1,$$

where Q is the doublet strength. It follows from the above that

$$Q = -\frac{4}{3} \pi l^2 A_1. \quad \ldots \quad (3)$$

The relation between the blockage correction factor Λ and the strength of each doublet Q of the image system giving rise to the blockage when the body is in a tunnel is shown in Ref. 1 to be

$$\Lambda = \frac{4Q}{\pi} \frac{U_0}{t^3}, \quad \ldots \quad (4)$$

where t is the thickness of the body.

Hence, we can write

$$\Lambda = -\frac{2A_1}{3lU_0} \left(\frac{2l}{t}\right)^3. \quad \ldots \quad (5)$$

We can obtain a close approximation to the equivalent sink-source distribution of a body by assuming that each element of the distribution is responsible for the local distortion of the flow caused by the body. This assumption implies that the disturbance velocities introduced by the body are small compared with the mainstream velocity. The assumption is therefore analogous to that underlying thin aerofoil theory, but is found to give reasonably accurate results for bodies of thickness ranging up to 30 per cent. If v is the component of velocity in the radial direction at the surface of the body, then with this assumption we may write

$$\frac{v}{U_0} = \frac{dr_0}{dz},$$

and $I(z)dz = 2\pi r_0\, v dz,$

where r_0 is the radius of cross section of the body.

Hence $I(z) = U_0 \frac{d}{dz} (\pi r_0^2).$

* For a discussion of the properties of these functions see, for example, Ref. 3.

It readily follows from equation (2) and the orthogonal properties of Legendre polynomials that

$$\frac{A_1}{lU_0} = \frac{3}{4\pi}\frac{V}{l^3},$$

where V is the volume of the body.

Hence, from equation (5)

$$\Lambda = \frac{4V}{\pi t^3} \; * \qquad \qquad (6)$$

The value of Λ has been calculated from equation (6) for a number of streamline shapes with pointed tails. These shapes cover a range of thickness from about 13 per cent. to about 30 per cent., and the maximum thickness positions vary from about 30 per cent. to about 50 per cent. of the body length from the nose. The values of Λ are shown plotted against fineness ratio (L/t, where $L = 2l$) in Fig. 1. The crosses, circles and squares apply to bodies with the maximum thickness at 50 per cent., 40 per cent. and 30 per cent. of the length behind the nose. The results are also given in Table I. It will be seen that, for the range of L/t covered, all the points fall very closely about the straight line drawn through them, and accepting this line the value of Λ for these bodies can be written

$$\Lambda = 0\cdot148 + 0\cdot523\, L/t \qquad \qquad (7)$$

It appears that this value of Λ is about $0\cdot78$ of the value given by Lock for the spheroid of the same fineness ratio.

TABLE I.

Table of values of blockage correction factor for various streamline bodies of revolution with pointed tails.

L/t fineness ratio	Position of maximum thickness (approx.)	$\frac{A_1}{lU_0} = \frac{2A_1}{LU_0}$	Λ
3·76	0·5L	−0·06026	2·14
3·94	0·5L	−0·05315	2·16
4·15	0·5L	−0·04604	2·19
5·56	0·5L	−0·02738	3·13
5·88	0·5L	−0·02403	3·28
6·25	0·5L	−0·02068	3·37
4·17	0·4L	−0·04799	2·31
4·42$_5$	0·4L	−0·04154	2·40
4·74	0·4L	−0·03545	2·51$_5$
6·80$_5$	0·4L	−0·01795	3·77
7·14	0·4L	−0·01610	3·92
7·69	0·4L	−0·01424	4·13
4·06$_5$	0·3L	−0·0525	2·35

3. *Discussion of Accuracy.*—There are three possible sources of error in the above method of deriving Λ for streamline shapes, they are :—

(1) The method of deriving the equivalent sink-source distribution provides a close but not exact estimate of the value of A_1/lU_0, and hence of Λ.

(2) The field due to the images nearest the body will differ somewhat from that due to the equivalent doublets.

(3) The source distribution corresponding to a body in a tunnel will differ slightly from that for the body in free air.

* This approximate formula could have been derived (as in Part I) without expressing the sink-source distribution in the general form given in equation (2). The latter is needed, however, in examining the errors involved in the approximation (see §3 and the Appendix).

To examine the probable magnitude of the error due to the approximations used to derive A_1/lU_0, the corresponding values of Λ for spheroids were determined and are shown in Fig. 1 by the dotted line. Comparing this line with that derived by Lock using the exact source distribution it will be seen that the method of this note underestimates Λ by about $0 \cdot 2$, or about 5 per cent. As a further indication of the magnitude of this error, the source distribution calculated by a more rigorous method by Kaplan[4] for the body with the maximum thickness at 30 per cent. of its length behind the nose was used to determine Λ for this body. The resulting value was found to be $2 \cdot 48$ as compared with the value of $2 \cdot 35$ given by the approximate source distribution. The difference for this case is therefore about the same as that found for spheroids.

When we come to examine the error under item (2) above, however, it is found that the induced velocity at the body due to the images nearest it is less than that estimated by assuming them to be doublets. The difference is, of course, a function of the relative size of body and tunnel. As an indication of what this difference may amount to, the induced velocity at the body with the maximum thickness at $0 \cdot 3L$ due to its image system in a square closed tunnel of height equal to the body length was calculated. The method adopted was fairly rigorous. The source distribution calculated by Kaplan[4] was used, and the induced velocity due to the images within a square of sides of length twelve times that of the tunnel was calculated accurately, the images outside that square were treated as doublets. The details of the calculation are given in the Appendix. The resulting value of Λ was found to be $2 \cdot 06$, as compared with $2 \cdot 35$ given by the approximate method using the approximate source distribution, and $2 \cdot 48$ using the approximate method and the more exact source distribution. It is realised that this example illustrates rather an extreme case of a large body in a tunnel; it is probable, in fact, that the variation in induced velocity with position along the body would be as great as the variation in the induced velocity given by the various methods considered. Nevertheless, the example demonstrates that the error involved under item (2) above is opposite in sign and can easily be of the same magnitude as that involved under item (1). It is therefore suggested that the value given for Λ by the mean line in Fig. 1 is accurate enough for most purposes, except where the body dimensions are large compared with the tunnel dimensions, in which case any method such as Lock's[1] for deriving the blockage correction tends to become inaccurate.

The error involved under item (3) is generally ignored as being extremely small, and the following argument justifies this conclusion. It has been noted that the equivalent source distribution of a body of revolution is given with fair accuracy by

$$I(z) = \pi \, U_0 \, \frac{d}{dz} (r_0^2),$$

where r_0 is the local radius of cross section of the body. For the body in a tunnel, the local velocity is increased from U_0 to $U_0 + \Delta u$, say. It follows that the equivalent source distribution for the body is increased in the ratio

$$\frac{U_0 + \Delta u}{U_0},$$

and hence A_1 and Λ are increased in this ratio. This suggests that a process of successive approximation could be adopted to allow for this change in the source distribution. Thus, having determined Λ from Fig. 1, a first approximation for $\Delta u/U_0$ can be obtained from equation (1); the value of Λ is then increased in the ratio

$$1 + \frac{\Delta u}{U_0},$$

and then a second approximation for $\Delta u/U_0$ can be derived, and so on. It will be obvious that the difference between successive approximations will be extremely small in general, and cases for which this correction can be at all appreciable are such that the correction will be completely swamped by errors under items (1) and (2).

APPENDIX

Details of Rigorous Process for Checking Approximate Formula

For a source distribution given by

$$I(z) = 2\pi \sum_{1}^{\infty} A_n P_n(\mu_1), \text{ for } -1 \leqslant u_1 \leqslant 1$$

and

$$\mu_1 = z/l,$$

it is shown in Ref. 4 that the potential function is given by

$$\phi = \sum_{1}^{\infty} A_n P_n(\mu) \, . \, Q_n(\lambda), \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

where λ and μ are the prolate elliptic co-ordinates derived from the system of confocal ellipses and hyperbolas in any meridian plane having the points $z = \pm l$ as focii. Thus

$$\left.\begin{aligned} z &= l \lambda \mu, \\ r &= l(\lambda^2 - 1)^{1/2}(1 - \mu^2)^{1/2}, \end{aligned}\right\} \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

where r is the distance from the axis. Thus λ = const. and μ = const. define a confocal ellipse and hyperbola, respectively. P_n and Q_n are Legendre polynomials of the first and second kind, respectively.

In the plane $z = 0$, we have

$$\mu = 0, \; r = l(\lambda^2 - 1)^{1/2},$$

or

$$\lambda = \left[\frac{r^2}{l^2} + 1\right]^{1/2}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

The induced velocity at any point in this plane due to the source distribution is given by

$$u_\mu = -\frac{1}{l\lambda}\left(\frac{\partial\phi}{\partial\mu}\right)_{\mu=0}$$

$$= -\frac{1}{l\lambda}\sum_{1}^{\infty} A_n Q_n(\lambda)\left(\frac{\partial P_n}{\partial\mu}\right)_{\mu=0}. \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

Suppose the body corresponding to the source distribution $I(z)$ centrally placed in a closed tunnel of height h and width b. Then there will be an infinite system of images representing the tunnel constraint consisting of exactly similar source distributions situated at the points

$$x = pb, \; y = qh,$$

where x and y are measured from the origin parallel to the sides of the tunnel ; p and q are integer numbers taking all the values from $+\infty$ to $-\infty$, the combination $p = 0$, $q = 0$ being excluded. The distance the body is from an image is therefore

$$r = \left[p^2 b^2 + q^2 h^2\right]^{1/2},$$

and hence relative to the image the position of the centre of the body is given by

$$\mu = 0, \; \lambda_{pq} = \left[\frac{p^2 b^2 + q^2 h^2}{l^2} + 1\right]^{1/2}. \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

It follows that the induced velocity at the body due to the system of images is given by

$$\frac{\Delta u}{U_0} = -\sum_{p-\infty}^{+\infty}\sum_{q-\infty}^{+\infty}\left[\sum_{n\,1}^{\infty}\frac{A_n}{lU_0}\,\frac{Q_n(\lambda_{pq})}{\lambda_{pq}}\left(\frac{\partial P_n}{\partial\mu}\right)_{\mu=0}\right], \quad \ldots \quad \ldots \quad (13)$$

the combination $p = q = 0$ being excluded. It is found that in the expansion for the source distribution only the first five terms need be considered, and since at $\mu = 0$

$$\frac{\partial P_1}{\partial \mu} = 1\cdot 0, \quad \frac{\partial P_3}{\partial \mu} = -1\cdot 5, \quad \frac{\partial P_5}{\partial \mu} = 1\cdot 875, \quad \frac{\partial P_2}{\partial \mu} = \frac{\partial P_4}{\partial \mu} = 0,$$

the expression for $\Delta u/U_0$ becomes

$$\frac{\Delta u}{U_0} = -\sum_{p+\infty}^{\infty} \sum_{q-\infty}^{\infty} \left[\frac{A_1}{lU_0} \frac{Q_1(\lambda_{pq})}{\lambda_{pq}} - 1\cdot 5 \frac{A_3}{lU_0} \frac{Q_3(\lambda_{pq})}{\lambda_{pq}} + 1\cdot 875 \frac{A_5}{lU_0} \frac{Q_5(\lambda_{pq})}{\lambda_{pq}} \right]. \quad \ldots \quad (14)$$

We have the following expressions for

$\dfrac{Q_1(\lambda)}{\lambda}$, $\dfrac{Q_3(\lambda)}{\lambda}$ and $\dfrac{Q_5(\lambda)}{\lambda}$:—

$$\frac{Q_1(\lambda)}{\lambda} = \tfrac{1}{2} \log\left[\frac{\lambda+1}{\lambda-1}\right] - \frac{1}{\lambda}$$

$$= \left[\frac{1}{3\lambda^3} + \frac{1}{5\lambda^5} + \ldots\ldots + \frac{1}{(2n+1)\lambda^{2n+1}} + \ldots\right], \quad \ldots \quad (15)$$

$$\frac{Q_3(\lambda)}{\lambda} = \tfrac{1}{4}(5\lambda^2 - 3)\log\left[\frac{\lambda+1}{\lambda-1}\right] - \frac{1}{6\lambda}(15\lambda^2 - 4)$$

$$= \frac{2}{35\lambda^5} + \frac{4}{63\lambda^7} + \frac{2}{33\lambda^9} + \ldots + \frac{2(n-1)}{(2n+1)(2n+3)\lambda^{2n+5}} + \ldots, \quad (16)$$

$$\frac{Q_5(\lambda)}{\lambda} = \frac{(63\lambda^4 - 70\lambda^2 + 15)}{16} \log\left[\frac{\lambda+1}{\lambda-1}\right] - \frac{63\lambda^2}{8} + \frac{49\lambda}{8} - \frac{8}{15\lambda}$$

$$= \frac{8}{693\lambda^7} + \frac{8}{429\lambda^9} + \ldots + \frac{4(n-2)(n-1)}{(2n+1)(2n+3)(2n+5)} \frac{1}{\lambda^{2n+1}} + \ldots, \quad (17)$$

It is a fairly simple matter by means of the above expressions to evaluate the functions $\dfrac{Q_1(\lambda)}{\lambda}$, $\dfrac{Q_3(\lambda)}{\lambda}$ and $\dfrac{Q_5(\lambda)}{\lambda}$ for values of λ ranging from the smallest required to, say, $\lambda = 10$, and the values can be plotted for interpolation. For values of λ greater than 10, the contributions to the induced velocity of the terms containing $\dfrac{Q_3(\lambda)}{\lambda}$ and $\dfrac{Q_5(\lambda)}{\lambda}$ are negligible and they can be dropped; in addition, we can then write

$$\frac{Q_1(\lambda)}{\lambda} = \frac{1}{3\lambda^3}. \quad \ldots \quad (18)$$

For the body considered, the values of

$$\frac{A_1}{lU_0}, \; \frac{A_3}{lU_0}, \; \frac{A_5}{lU_0}$$

were already known[4]. The process adopted was to calculate by means of equation (13) the contribution to $\Delta u/U_0$ given by the images within and on a rectangle of sides $12b$ and $12h$ having its centre at the body. Outside the rectangle the contribution of the images is

$$\frac{\Delta u}{U_0} = -\sum_p \sum_q \frac{A_1}{lU_0} \cdot \frac{1}{3\lambda_{pq}^{\;3}}$$

and, since λ is very large,

$$\lambda_{pq} \doteqdot \frac{(p^2b^2 + q^2h^2)^{1/2}}{l},$$

and hence

$$\frac{\Delta u}{U_0} = -\sum_p \sum_q \frac{A_1}{lU_0} \frac{l^3}{3\,[p^2b^2 + q^2h^2]^{3/2}}, \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

where combinations of values of both p and q between 0 and 6 are excluded.

The right hand side of (19) can be written as

$$-\frac{4}{3}\frac{A_1 l^3}{lU_0}\left\{\sum_{p\,0}^{\infty}\sum_{q\,0}^{\infty} - \sum_{p\,0}^{6}\sum_{q\,0}^{6} \frac{1}{[p^2b^2 + q^2h^2]^{3/2}}\right\}.$$

The value of the summation in the brackets can be readily evaluated by a slight modification of a process suggested by Lock, whereby the summations are replaced by integrals, i.e., we can write

$$\sum_0^{\infty}\sum_0^{\infty} - \sum_0^{6}\sum_0^{6} \frac{1}{[p^2b^2 + q^2h^2]^{3/2}}$$

$$\doteqdot \frac{1}{bh}\int_0^{\infty}\int_0^{\infty} - \int_0^{6b}\int_0^{6h} \frac{dx\,dy}{(x^2 + y^2)^{3/2}}$$

$$= \frac{1}{bh}\,\frac{\sqrt{b^2 + h^2}}{6bh} = \frac{\sqrt{b^2 + h^2}}{6b^2h^2}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

Hence the contribution of the remaining images to the induced velocity is given by

$$-\frac{4}{3}\,\frac{A_1}{lU_0}\,\frac{l^3\sqrt{b^2 + h^2}}{6b^2h^2}.$$

More generally, the contribution of the images outside a rectangle of sides $2nb$ and $2nh$ centred on the origin is given by this method as

$$-\frac{4}{3}\cdot\frac{A_1}{lU_0}\,\frac{l^3\sqrt{b^2 + h^2}}{nb^2h^2}.$$

For the example considered, b was taken as equal to h, and $l = h/2$, and hence this contribution became for $n = 6$

$$-\frac{\sqrt{2}}{36}\,\frac{A_1}{lU_0} = -0\cdot0392\,\frac{A_1}{lU_0}.$$

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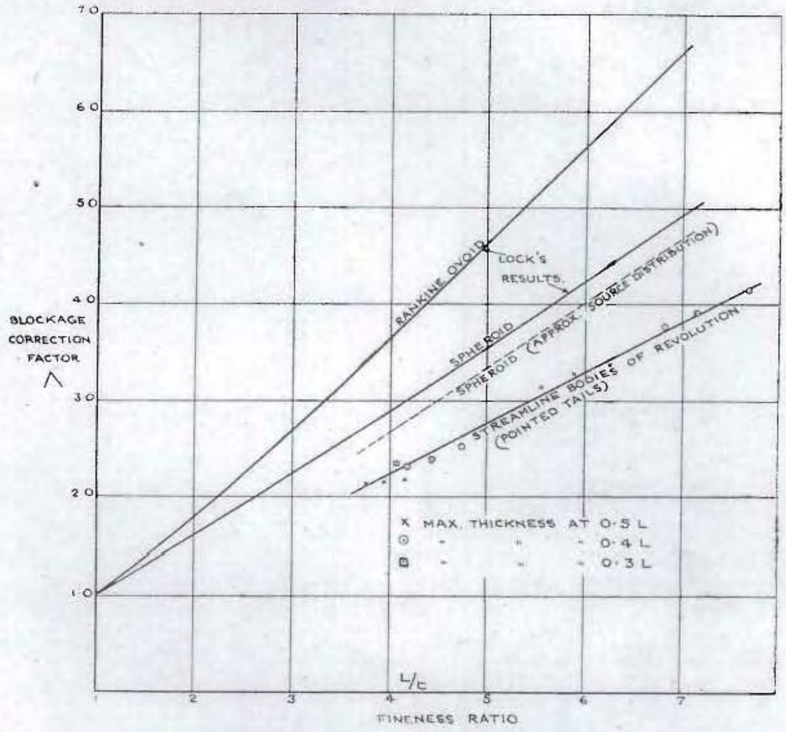

FIG. 1—Blockage Correction Factor for Various Bodies of Revolution.

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