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Abstracts of Papers Published Externally

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AERODYNAMIC SYMBOLS

1. GENERAL

m	Mass
t	Time
V	Resultant linear velocity
Ω	Resultant angular velocity
ρ	Density, σ relative density
ν	Kinematic coefficient of viscosity
R	Reynolds number, $R = lV/\nu$ (where l is a suitable linear dimension)

Normal temperature and pressure for aeronautical work are 15° C and 760 mm.

For air under these conditions $\begin{cases} \rho = 0\cdot002378 \text{ slug/cu. ft.} \\ \nu = 1\cdot59 \times 10^{-4} \text{ sq. ft./sec.} \end{cases}$

The slug is taken to be 32·2 lb.-mass.

α	Angle of incidence
ϵ	Angle of downwash
S	Area
b	Span
c	Chord
A	Aspect ratio, $A = b^2/S$
L	Lift, with coefficient $C_L = L/\frac{1}{2}\rho V^2 S$
D	Drag, with coefficient $C_D = D/\frac{1}{2}\rho V^2 S$
γ	Gliding angle, $\tan \gamma = D/L$
L	Rolling moment, with coefficient $C_l = L/\frac{1}{2}\rho V^2 bS$
M	Pitching moment, with coefficient $C_m = M/\frac{1}{2}\rho V^2 cS$
N	Yawing moment, with coefficient $C_n = N/\frac{1}{2}\rho V^2 bS$

2. AIRSCREWS

n	Revolutions per second
D	Diameter
J	V/nD
P	Power
T	Thrust, with coefficient $k_T = T/\rho n^2 D^4$
Q	Torque, with coefficient $k_Q = Q/\rho n^2 D^5$
η	Efficiency, $\eta = TV/P = Jk_T/2\pi k_Q$

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Note.—All papers discussed by the Aeronautical Research Committee and recommended by them for external publication after 1st April, 1932, have been published in abstract only in the Reports and Memoranda Series.

CONTENTS

The Continuous Beam

By

S. J. E. Moyes, B.Sc.

Communicated by the Director of Scientific Research, Air Ministry

This paper was published in full in " Aircraft Engineering," Vol. X, No. 114, August, 1938.

In this paper, the problem of a continuous beam under any form of irregular lateral loading with end loads is investigated.

It is shown that any form of irregular lateral loading may be easily dealt with by reducing it to a number of concentrated loads and applying systematic tabulation of a very simple nature.

The solution obtained, together with the auxiliary formulae for bending moments and slopes, is shown to be expressible in the standard " Three Moment Theorem " form, the Berry functions associated with the fixing moments remaining unchanged, whilst the terms associated with the lateral loading are shown to lend themselves readily to tabulation for irregular lateral loading, and to integration for lateral loading obeying a definite law.

The Large Deflections of a Thin Circular Ring

By

R. A. Fairthorne, B.Sc.

Communicated by the Director of Scientific Research, Air Ministry

This paper was published in full in the Philosophical Magazine, Series 7, Vol. 26, No. 179, December, 1938, pp. 1006–17.

In order to supply a standard of comparison for engineering approximations, the large deflections of a thin circular ring have been calculated for certain symmetrical load systems.

The ring is assumed thin and uniform and, in the unstressed state, a complete circle. It is further supposed that deformation is inextensional, deformation being due to flexure alone. With these assumptions the exact solutions are given for the following load systems :—

(i) Four equal moments of alternate sign, equally spaced round the circumference.

(ii) Compression or tension along a diameter.

(iii) Compression along a diameter, together with numerically equal tension along a perpendicular diameter.

The results are presented in the form of curves giving the radial deflection at the point of application of the load as a function of a non-dimensional parameter that embodies the load and stiffness.

4

Theoretical Discharge of Air from Ports in a Duct

By

W. S. Brown, M.A.

This paper was published in a slightly condensed form in
" The Engineer ", Vol. CLXVI, No. 4324, November 25th, 1938,
and
" The Engineer ", Vol. CLXVI, No. 4325, December 2nd, 1938

The study of the discharge of air from a series of ports in a duct is of practical importance in the design of ventilating systems and in refrigeration and other work. In this paper the question is treated theoretically. Consideration is first given to the idealised flow of an incompressible frictionless fluid through a system of N equal ports in a uniform duct. Two cases are considered, namely, where there is no guiding, and where there is some guiding of the exit streams. It is shown that in any such system neither the discharge quantities nor the angles of discharge are uniform, but that they show a gradation along the trunkway. The gradation is, however, independent of the total quantity or head. General formulae are given for quantities, velocities, pressures and discharge angles.

Allowance is then made for friction, and it is shown that while this tends to make the discharge quantities slightly more uniform, its effect in most practical instances is negligible. In most ventilating systems it is desirable to make the discharge quantities from the various ports as nearly equal as possible. It is shown that it is theoretically possible to achieve equality of discharge in a uniform trunkway by grading the sizes of the ports. An alternative is to taper the cross section of the trunkway while keeping the ports of equal size. Design formulae are again given. The problem of the discharge from a long narrow parallel slot in a trunkway is also considered and theoretical formulae are given for the velocity and discharge angle at any point. The form of a tapering slot which will give constant discharge per unit length is also deduced.

The conclusions are compared with experimental results obtained in the Physics Department of the N.P.L. In all cases there is good agreement.

The Estimation of Pipe Delivery from Pitot-Tube Measurements

By

A. FAGE, A.R.C.Sc.

This paper was published in full in " Engineering ", Vol. CXLV (1938), pp. 616–617.

The rate of delivery, Q, in a circular pipe is $2\pi \int_0^R r\,U\,dr$, where U is the mean velocity at radius r, and R is the radius of the pipe. When the rate of flow is estimated from total-head measurements taken with a Pitot tube, the standard procedure is to assume that

$$U = \sqrt{\frac{2\,(H - P_w)}{\rho}}$$

where $(H - P_w)$ is the pressure reading given on a manometer connected on one side to the Pitot tube and on the other side to one or more static-pressure holes in the wall of the pipe, at the exploration section. The rate of delivery is then

$$Q = 2\pi \int_0^R r \sqrt{\frac{2\,(H - P_w)}{\rho}}\,dr.$$

The relation

$$U = \sqrt{\frac{2\,(H - P_w)}{\rho}}$$

is derived on the assumptions that the Pitot pressure is $(\frac{1}{2}\rho\,U^2 + \bar{p})$, where U and $\bar{p}$ are the local mean values of the velocity and pressure, respectively, and that $\bar{p}$ is constant across the pipe and equal to P_w, the datum pressure at the wall. These conditions do not hold when the flow is turbulent : for there is reason to believe that the total pressure registered by a Pitot tube is $(\bar{p} + \frac{1}{2}\,\rho\,U^2 + \frac{1}{2}\,\rho\,\overline{q^2})$, where $\overline{q^2}$ is the time average value of the square of the turbulent velocity, and it is known that $\bar{p}$ is not equal to P_w. Furthermore, account should be taken of the fact that the total head registered by a Pitot tube in a region of total-pressure gradient is not associated, in general, with the geometric centre of the mouth of the tube, but with an effective centre, which is displaced from the geometric centre towards the region of higher total head. The simple relation

$$Q = 2\pi \int_0^R r \sqrt{\frac{2\,(H - P_w)}{\rho}}\, dr$$

is therefore not strictly true, and the order of error to be expected when this relation is used to predict the rate of delivery from measurements taken at a section where a state of fully developed turbulence exists is assessed in the analysis given in the paper. The analysis shows that the simple relation

$$Q = 2\pi \int_0^R \sqrt{\frac{2\,(H - P_w)}{\rho}}\, r\, dr$$

overestimates the rate of delivery by about

$$\left[190 \frac{\tau}{\rho\, U_0^2} + 190 \left(\frac{d}{R}\right)^2 + 120 \frac{d d_1}{R^2}\right]$$

per cent., where τ is the intensity of skin friction at the wall, U_0 is the mean speed of flow, and d and d_1 are the external and internal widths of the mouth of the Pitot tube. With small Pitot tubes, d_1 is of the order $0{\cdot}6d$, and then the overestimation becomes

$$\left[190 \frac{\tau}{\rho\, U_0^2} + 262 \left(\frac{d}{R}\right)^2\right]$$

per cent. The table below shows how the overestimation depends on the values of $U D/\nu$ and d/R.

$\frac{U_0 D}{\nu}$	$\frac{\tau}{\rho U_0^2}$	Percentage Overestimation of the Rate of Delivery when $\frac{d}{R} = 0{\cdot}06$.	$\frac{d}{R} = 0{\cdot}04$.	$\frac{d}{R} = 0{\cdot}02$.
3,000	0·0053	1·9	1·4	1·1
25,000	0·0031	1·5	1·0	0·7
100,000	0·0023	1·4	0·9	0·5
200,000	0·0020	1·3	0·8	0·5
400,000	0·0017	1·3	0·7	0·4

The value of d/R is not likely except in pipes of less than 1 in. diameter to exceed 0·04, and since the highest value of $\tau/(\rho\, U_0^2)$ possible is the value at the upper critical, $U_0 D/\nu = 3{,}000$, the overestimation is not likely to exceed 1·4 per cent.

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The Influence of Wall Oscillations, Wall Rotation, and Entry Eddies, on the Breakdown of Laminar Flow in an Annular Pipe

By

A. Fage, A.R.C.Sc.

This paper was published in full in the Proceedings of the Royal Society, Series A, No. 923, Vol. 165 (1938), pp. 501–529

Scope of Work.—The paper describes experiments made to determine the effects of disturbances of known character on the laminar flow of water in a long pipe of annular cross-section. The disturbances considered were those due to axial oscillations of the inner wall of the pipe, to oscillations of the inner wall about its axis, and to both weak and intense entry eddies. Experiments on the breakdown due to a uniform rotation of the inner wall (outer wall fixed) were also made. The work included observation of the breakdown of flow near a plane surface oscillating in a stationary fluid, and the derivation of theoretical relations for the flow of a viscous fluid through an annular pipe, under the influence of a pressure gradient parallel to the axis, with the inner wall oscillating axially and the outer wall fixed.

Conclusions.—The frequency of the axial oscillation of the inner wall when a departure from laminar flow occurs depends on the axial amplitude of the wall and the viscosity of the fluid, and is independent, within the accuracy of measurement, of the velocity of axial flow The Reynolds number of disturbance, defined as the product of the velocity amplitude at the wall and a length $2\pi\sqrt{(\nu/\pi f)}$ (where f = frequency) divided by the viscosity, for which a departure from laminar flow occurs, does not change appreciably over a wide range of amplitude.

With the inner wall of the pipe oscillating about its axis, the flow remains laminar up to the value of the critical Reynolds number of disturbance measured with the inner wall oscillating axially.

Visual observation suggests the presence of rotating bands of fluid, with their axes parallel to the direction of oscillation, at the breakdown.

When the inner wall rotates at a uniform speed (outer wall stationary), the critical rotational speed increases with the axial speed of flow; and the critical number for no axial flow, predicted by extrapolation of the curve drawn through the numbers measured with axial flow, is in close agreement with Taylor's theoretical number.

It is shown that the early breakdown of laminar flow normally associated with intense entry disturbances can be caused by very weak entry disturbances, provided they are in the form of discrete eddies.

8

Thermal Effects on Bodies in an Air Stream

By

W. F. Hilton, Ph.D., B.Sc., A.R.C.Sc., D.I.C.

This paper was published in full in the Proceedings of the Royal Society, Series A, No. 932, Vol. 168, pp. 43–56, October, 1938

A theoretical expression derived from Pohlhausen's formula (1921), giving the temperature due to laminar flow over a flat plate has been tested experimentally in the 1 ft. diameter high speed wind tunnel at the N.P.L. The temperatures were measured by using thermocouples, and good agreement between theory and experiment was obtained with the thermocouple near the leading edge of the flat plate, where the flow was probably laminar. A 26 per cent. discrepancy was found with the thermocouple near to the trailing edge, due probably to the flow being turbulent.

On all models tested the temperature difference was found to increase as the square of the wind speed, until a shock wave was formed, which gave an inflexion in the δT, V^2 curve. The present method can thus be used to detect the formation of a shock wave at a solid boundary, such as an aerofoil, or aeroplane body.

Besides tests on flat plates of four thicknesses, and a streamline bar at four incidences, measurements were made on a circular cylinder and two other shapes. The circular cylinder was found to be a most unsatisfactory shape for the thermometry of moving fluids, unless placed along the wind. A flat plate with a thermocouple near the leading edge is suggested as the best shape for a sensitive element.

An Experimental Determination of the Spectrum of Turbulence

By

L. F. G. Simmons, M.A., and C. Salter, M.A.

This paper was published in full in the Proceedings of the Royal Society, 1938
Series A, Vol. 165, pp. 73–89

The time variation of velocity at a fixed point in a turbulent air stream is analysed into a spectrum. The method adopted involves the use of the ordinary hot-wire technique to produce changes of potential in a Wheatstone bridge circuit, which are magnified by a valve amplifier. The fluctuating voltage drop generated across a resistance in the output circuit of the amplifier is then applied, in turn, to electrical filters having different cut-off frequencies. In each case the output current is measured, with and without each filter in circuit, by means of a thermal milliammeter which indicates the mean value of the square of the current supplied to it. From the ratios of the readings taken with and without each filter, the spectrum curve is calculated by a method described in the Appendix.

All measurements were made in a wind tunnel, at a point in the air stream where the turbulence created by a grid of regular mesh was known to be isotropic ; the wind speeds used were 15, 20, 25, 30 and 35 ft./sec.

10

Sensitivity of Immersed Venturi-Pitot Head at Low Speeds

By

C. Salter, M.A.

This paper was published in full in the Phil. Mag., 1938, Vol. 26, p. 272

Experiments were made on various forms of venturi-pitot head at low speeds, and it was shown how the pressure difference produced depended on the shape of the venturi. Pressure differences several times those of a pitot-static head can be obtained, even at speeds as low as one or two feet per second. The pressures are, however, very critical to certain features of the design, and it was concluded that though the venturi-pitot instrument might be useful as a substandard for use at low speeds, it could not replace the pitot-static tube as a fundamental standard.

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The Influence of the Mean Stress of the Cycle on the Resistance of Metals to Corrosion-Fatigue

By

H. J. GOUGH, M.B.E., D.SC., M.I.MECH.E., F.R.S., and
D. G. SOPWITH, B.SC.TECH., WH.SCH., A.M.I.MECH.E.

This paper was published in full in the Journal of the Iron and Steel Institute, Vol. 135, 1937, No. 1, p. 293P

Whilst much attention has been devoted to the resistance of materials to corrosion-fatigue under cycles of reversed stress, no work has hitherto been carried out on the equally important practical cases of repeated or fluctuating stresses. This paper describes the results of tests under these conditions made on six aircraft materials, the behaviour of which under reversed stresses has previously been reported. These comprised a cold-drawn 0·5 per cent. carbon steel, three stainless steels, duralumin and a magnesium alloy containing $2\frac{1}{2}$ per cent. of aluminium. These were tested in air, also in a spray of 3 per cent. salt solution, under cycles of repeated and of fluctuating stresses.

The results show that, as in air, the fatigue resistance of a material in a corrosive environment is considerably influenced by the mean stress of the applied cycle. As in the case of reversed stresses, no corrosion-fatigue limit was indicated for any of the materials. If the range for any given endurance is plotted against the mean stress, the form of the curve obtained is in general similar to that obtained in air, using the fatigue limit in place of the endurance range.

The Resistance of some Special Bronzes to Fatigue and Corrosion-Fatigue

By

H. J. Gough, M.B.E., D.Sc., and D. G. Sopwith, B.Sc.Tech.

This paper was published in full in the Journal of the Institute of Metals, Vol. 60, No. 1, 1937, p. 143

Fatigue and corrosion-fatigue tests on four types of special bronzes have been carried out to ascertain the suitability of these materials for special aircraft purposes. The materials tested were: phosphor-bronze, aluminium bronze (9 per cent. aluminium), beryllium bronze (2·25 per cent. beryllium), and Superston L189 bronze. The results show that the corrosion-fatigue resistance of the bronzes compares favourably with that of stainless steels, the beryllium bronze in particular having the highest corrosion-fatigue resistance of any material so far investigated by the authors. The fatigue resistance in air of Superston is exceptionally high for a non-ferrous material but the material appears to be somewhat susceptible to stress-concentration effects.

The Effect of Protective Coatings on the Corrosion-Fatigue Resistance of Steel

By

D. G. Sopwith, B.Sc.Tech., Wh.Sch., A.M.I.Mech.E., and
H. J. Gough, M.B.E., D.Sc., M.I.Mech.E., F.R.S.

This paper was published in full in the Journal of the Iron and Steel Institute, Vol. 135, No. 1, 1937, p. 315P

This paper describes the results of tests made to ascertain the suitability of various protective coatings when applied to streamline wire steel subjected to alternating stress in a spray of salt water.

The fatigue resistances, under reversed bending stresses in air and in salt spray, of streamline wire steel have been determined, using the material in both the as-drawn and normalised conditions uncoated and with the following types of coating :—

(1) Zinc, applied by (*a*) galvanising, (*b*) sherardising, (*c*) electrodeposition.
(2) Electrodeposited cadmium (*a*) alone and with supplementary coatings of (*b*) enamel and of (*c*) boiled linseed oil.
(3) Sprayed aluminium (*a*) with enamel and (*b*) without enamel.
(4) Phosphates plus enamel.
(5) Enamel only.

A very satisfactory degree of protection was afforded by galvanising, sherardising and sprayed aluminium plus enamel. Zinc- or cadmium-plating and sprayed aluminium alone gave a fair degree of protection. Phosphate treatment plus enamel and enamel alone gave considerably better results than the uncoated material, but were not nearly so good as the metallic coatings. Enamel was a useful addition to sprayed aluminium, but both enamel and oil reduced the degree of protection afforded by cadmium-plating.

The Constitution of the Magnesium-Rich Alloys of Magnesium and Silver

By

R. J. M. Payne, B.Sc. and J. L. Haughton, D.Sc., F.Inst.P.

This paper was published in full in the Journal of the Institute of Metals, 1937, Vol. 60, p. 351

The addition of small proportions of silver to some magnesium alloys has been found to have good effects, particularly in maintaining the tensile strength at high temperatures, and this research was carried out to confirm and to supplement existing information on the constitution of the binary alloys of magnesium and silver.

The investigation was limited at the outset to include only alloys necessary for checking the composition of the eutectic. The alloy richest in silver which was dealt with contained 59·57 weight per cent. silver.

The form of the liquidus and the values obtained for the autectic and peritectic temperatures by other workers have been checked in magnesium-silver alloys containing up to 60 weight per cent. of silver. It was found that solid magnesium can hold in solution up to 15 weight per cent. of silver at the eutectic temperature, but less than 1 weight per cent. of silver at 200° C. The alloys should, therefore, be capable of precipitation-hardening.

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Oscillatory Motion of a Fluid Along a Circular Tube

By

D. G. CHRISTOPHERSON, B.A., A. GEMANT, A. H. A. HOGG, and R. V. SOUTHWELL, F.R.S.

This paper was published in full in the Proceedings of the Royal Society, Series A, No. 934, Vol. 168, pp. 351–378, November, 1938

This paper is concerned with oscillatory motion (either free or forced) of a viscous fluid along a circular tube; the restoring force comes in both instances from the "gravity head" which results from the passage of fluid. Part I is a theoretical discussion of the problem. Part II deals from a practical standpoint with complicating factors (e.g., turbulence, or meniscus and other end effects) which the theory does not take into account, and briefly discusses a possible application to viscometry. Part III gives the results of experiments made to test the predictions of Parts I and II.

Relaxation Methods Applied to Engineering Problems. III. Problems Involving Two Independent Variables

By

D. G. Christopherson, B.A., and R. V. Southwell, F.R.S.

This paper was published in full in the Proceedings of the Royal Society, Series A, No. 934, Vol. 168, pp. 317–350, November, 1938

In this paper relaxation methods are applied with success to four problems involving Laplace's, Poisson's and similar equations in *two variables*, namely :—

The torsion problem of Saint-Venant for triangular shafts (this, since the solution is known, serves to illustrate and in some degree to test the methods) ;

the same problem for a shaft pierced by axial holes (a multiply-connected cross-section) ;

the torsion problem modified by the imposition of a definite limit to the shear stress (Prandtl's problem) ;

the problem of magnetic induction in a field containing iron.

In all four problems the methods lead without difficulty to solutions of sufficient accuracy for practical purposes, and it seems reasonable to conclude that they will be applicable (suitably modified) to other problems which, like the last three, would present great or insuperable difficulties if treated by orthodox methods.

Torsion of Built-up and Reinforced Tubes

By

W. J. DUNCAN, D.Sc., A.M.I.Mech.E., F.R.Ae.S.,
Wakefield Professor of Aeronautics at University College, Hull

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This paper was published in full in " Engineering ", 7th October and 21st October, 1938

The paper contains an account of the theory of the torsion of thin-walled simple and compound tubes. Emphasis is laid on the physical basis of the theory, and the importance of the displacement w in the direction parallel to the axis of twist. and associated with the distortion of the normal sections of the tube, is pointed out. The following general conclusions are arrived at :—

(1) The stiffnesses of tubes which do not meet, or which are only joined along a single generator, are directly additive.

(2) Torsional stiffnesses are only additive when the displacements w are the same for the tubes at all the points of junction. (This is the condition of the compatibility of the displacements.)

(3) The stiffness of a built-up tube is never less than the sum of the stiffnesses of any components into which it may be supposed divided.

(4) The additional stiffness obtained by attaching an open section to a tube along a single generator is negligible. (But such an open section may add to the stiffness by stabilising the skin when the stress exceeds the buckling stress for a tube without stiffeners.)

(5) The additional stiffness due to an extra internal wall may be zero, and is not large as a rule. (But see remarks under (4).)

(6) The additional stiffness due to an extra plane internal wall is certainly zero if the tube, whether simple or compound, is completely symmetrical about the wall.

The foregoing principles are illustrated by a number of examples of double and triple tubes, for which general formulae are given, and the case of a simple tube provided with a corrugated reinforcement is also worked out. Mechanical analogies of the torsion of compound tubes are described.

The influence of end constraints is not considered in the paper.

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Application of the Galerkin Method to the Torsion and Flexure of Cylinders and Prisms

By

W. J. Duncan, D.Sc., A.M.I.Mech.E., F.R.Ae.S.,
Wakefield Professor of Aeronautics at University College, Hull

This paper was published in full in the Philosophical Magazine, Vol. 25, No. 169, p. 634, April, 1938

The method of Galerkin, which is fully described in R. & M. 1798,[1] can readily be applied to the problems of the torsion and flexure of cylindrical or prismatic bodies. The St. Venant torsion problem is reducible to the formal problem of finding a stress function Ψ which vanishes on the boundary and which satisfies

$$\nabla^2\Psi + 2 = 0 \qquad \text{.........(1)}$$

everywhere within the boundary. Suppose that F_1, F_2, F_3, etc., are linearly independent functions of x and y, which all vanish on the boundary. Then the function

$$c_1F_1 + c_2F_2 + c_3F_3 + \text{etc.} \qquad \text{.........(2)}$$

vanishes on the boundary for all values of the coefficients c. Let the expression (2) be substituted for Ψ in the differential equation (1), and let the left-hand side, which should be zero, actually be equal to ε, which is therefore the error in the differential equation. The error ε is a linear function of the coefficients c, and these can be chosen so as to make ε everywhere small. In accordance with the Galerkin method the coefficients are selected so that the equations typified by

$$\iint \varepsilon F_r \, dx \, dy = 0 \qquad \text{.........(3)}$$

are satisfied when $r = 1$, 2, etc., in succession, and the number of equations is equal to the number of independent functions F in use.

Several examples are worked out by the Galerkin method, and the results are compared with solutions otherwise obtained. In all cases good agreement is obtained when two or three functions F are used, provided that these are properly chosen in the manner indicated in the paper.

When the section of the cylinder is narrow and has OX as axis of symmetry, a good approximation to the torsional stiffness of unit length is given by

$$C = \frac{8\,\mu \int t^3 dx}{3\,(1 + 3K)}, \qquad \text{.......(4)}$$

where

$$K = \frac{\int t^3 \left(\frac{dt}{dx}\right)^2 dx}{\int t^3\, dx}, \qquad \text{.......(5)}$$

t = half thickness of section at the distance x from one end,

and

μ = modulus of rigidity.

The integrals extend over the whole section, and t is supposed to vanish at the two ends, so that the formula (4) cannot be applied to a section with square ends.

The St. Venant flexure problem is concerned with the distribution of the shearing stresses on a normal section of a cantilever beam subject to a load at the tip, and the position of the flexural centre can be deduced from the solution. The Galerkin method is easily applicable with the help of the stress function Ω introduced in R. & M. 1444.[2] A number of examples are worked by the Galerkin method, and the agreement with other methods is satisfactory.

The conclusion may be drawn that the Galerkin method yields approximate solutions of the torsion and flexure problems with good accuracy and with comparatively little labour. Also the method, when once understood, becomes almost mechanical in application.

REFERENCES

1. R. & M. 1798 .. .. "Galerkin's Method in Mechanics and Differential Equations". W. J. Duncan, 1937.
2. R. & M. 1444 .. .. "Torsion and Flexure of Cylinders and Tubes". W. J. Duncan, 1932.

The Elastic Stability of a Curved Plate Under Axial Thrusts

By

S. C. Redshaw, Ph.D., M.Sc., Assoc.M.Inst.C.E., A.F.R.Ae.S.

This paper was published in full in the Journal of the Royal Aeronautical Society, Vol. XLII, No. 330, June, 1938

Theoretical critical stresses have been obtained for four cases in each of which a curved panel has been subjected to a compressive stress, parallel in direction to a generator and uniformly distributed around its top and bottom circumferential edges.

In each case the critical stress may be expressed by an equation of the form

$$p = \{\pi^2 t^2/12\,(1 - \sigma^2)\, b^2\}EK$$

where K is a coefficient which may be split into two parts, K_F and K_C, where these are flat and curved plate coefficients respectively.

Then

$$K = K_F + K_C$$

and if the curvature of the plate is zero, the value of the critical stress obtained will be the flat plate value as obtained by other investigators.

Both coefficients are dependent on the ratio a/nb, that is to say, the ratio axial length of plate *divided by the product* of the circumferential length of plate and the number of half waves in the axial direction.

In addition the curved plate coefficient is dependent on the curvature of the plates which has a most marked effect on its numerical value.

In all the cases the minimum critical stress is obtained when the coefficient K is a minimum. For this condition to be satisfied a particular value for a/nb must be found for each value of the bulge thickness ratio d/t. That is to say, if we have a definite value for a/b the panel will tend to buckle, as nearly as possible, into a number of half waves n which will make the value a/nb agree with the minimum value.

The various edge conditions and expressions for the critical stresses may be summarised in the following table :—

Case.	Condition of Fixing of Edge.				Value of $K_{min.}$ in expression for critical stress $p_{min.} = \{\pi^2 t^2/12\,(1 - \sigma^2) b^2\}\, EK_{min.}$
	AED.	BFC.	AB.	DC.	
a	Simply supported.	Simply supported	Simply supported	Simply supported	$4 + (192/\pi^4)\,(1 - \sigma^2)\,(d^2/t^2)$, for $d/t \leq 1{\cdot}471$ N.B. $p_{min.} = [1/\{3\,(1 - \sigma^2)\}]^{\frac{1}{2}}\; Et/r$ for $d/t \geq 1{\cdot}471$.
b	Simply supported	Simply supported	Fixed	Fixed	$2\,\{(16/3 + 768\, d^2/\pi^4 t^2)^{\frac{1}{2}} + 1\}$
c	Simply supported	Simply supported	Simply supported	Free	$(2/\pi^2)\,\{16\, d\,\sqrt{3}/t + 3\,(1 - \sigma)\}$
d	Simply supported	Simply supported	Fixed	Free	$(1/\pi^2)\,\{2\,(162/13 + 768\, d^2/t^2)^{\frac{1}{2}} + 135\,(6 - 7\sigma)\,/91\}$

With increasing values of the bulge thickness ratio d/t the expression for the critical stress for cases (*b*), (*c*) and (*d*) approaches the value

$$P = (\sqrt{3}/3)\; E\; (t/r)$$

which agrees very closely with the established result for the complete cylinder. When $d/t = 2{\cdot}0$ the error involved in using the above formula does not exceed 2 per cent.

Experimental data have been obtained by other investigators for cases (*a*) and (*b*). In both cases the experimental values may be as low as 40–50 per cent. of the theoretical values.

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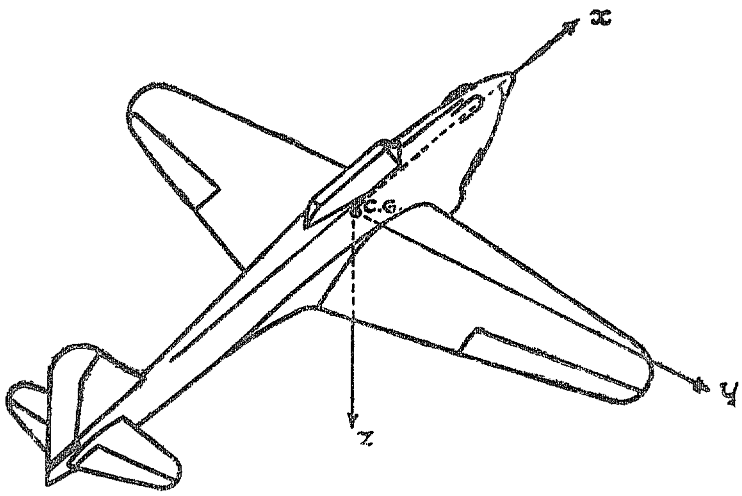
SYSTEM OF AXES



Axes	Symbol Designation Positive direction	x longitudinal forward	y lateral starboard	z normal downward
Force	Symbol	X	Y	Z
Moment	Symbol Designation	L rolling	M pitching	N yawing
Angle of Rotation	Symbol	ϕ	θ	ψ
Velocity	Linear Angular	u p	v q	w r
Moment of Inertia		A	B	C

Components of linear velocity and force are positive in the positive direction of the corresponding axis.

Components of angular velocity and moment are positive in the cyclic order y to z about the axis of x, z to x about the axis of y, and x to y about the axis of z.

The angular movement of a control surface (elevator or rudder) is governed by the same convention, the elevator angle being positive downwards and the rudder angle positive to port. The aileron angle is positive when the starboard aileron is down and the port aileron is up. A positive control angle normally gives rise to a negative moment about the corresponding axis.

The symbols for the control angles are :—

- ξ aileron angle
- η elevator angle
- η_T tail setting angle
- ζ rudder angle

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