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Cooling of Aircraft Engines

With special reference to Ethylene Glycol Radiators enclosed in Ducts

By F W MEREDITH
B A

Communicated by
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AERODYNAMICS SYMBOLS

1 GENERAL

m mass
t time
V resultant linear velocity
Ω resultant angular velocity
ρ density σ relative density
ν kinematic coefficient of viscosity
R Reynolds number, $R = lV/\nu$ (where l is a suitable linear dimension) to be expressed as a numerical coefficient x 10^6

Normal temperature and pressure for aeronautical work are 15° C and 760 mm

For air under these conditions $\begin{cases} \rho = 0.002378 \text{ slug/cu ft} \\ \nu = 1.59 \times 10^{-4} \text{ sq ft/sec} \end{cases}$

The slug is taken to be 32.2 lb-mass.

α angle of incidence
ϵ angle of downwash
S area
c chord
s semi-span
A aspect ratio $A = 4s^2/S$
L lift with coefficient $k_L = L/S\rho V^2$
D drag with coefficient $k_D = D/S\rho V^2$
γ gliding angle $\tan \gamma = D/L$
L rolling moment, with coefficient $k_l = L/sS\rho V^2$
M pitching moment with coefficient $k_m = M/cS\rho V$
N yawing moment, with coefficient $k_n = N/sS\rho V$

2 AIRSCREWS

n revolutions per second
D diameter
nD
ower
rust with coefficient $k_T = T/\rho n^2 D^4$
que with coefficient $k_Q = Q/\rho n^2 D^5$
ficiency $\eta = TV/P = Jk_T/2\pi k_Q$

NOTE ON THE COOLING OF AIRCRAFT ENGINES WITH SPECIAL REFERENCE TO ETHYLENE GLYCOL RADIATORS ENCLOSED IN DUCTS

By F. W. MEREDITH, B.A.

Communicated by the Director of Scientific Research,
Air Ministry

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Summary. (*a*) *Introductory* (*Purpose of investigation*).—The recent increase in the speed of aeroplanes has brought the question of cooling drag into prominence and forced the application of the principle of low velocity cooling. An analysis of the performance of a cooling system enclosed in a duct is required to guide further research and design.

(*b*) *Range of investigation.*—The theory of the ducted radiator is developed and a basis of calculating the drag is provided.

The effects of compressibility are also investigated.

(*c*) *Conclusions.*—It is shown that the power expended on cooling does not increase with speed for a properly designed ducted system but that, owing to recovery of waste heat, a thrust may be derived at speeds of the order of 300 m.p.h.

Attention is drawn to the importance of the momentum of the exhaust gases at high speeds of flight.

1. *Introductory.*—Cooling of aero engines involves the exposure of a large heated surface to a stream of air, a process which involves the expenditure of power owing to the viscosity of air. Until recently, it appeared that this fact imposed an intractable limit to the speed of aircraft since, whereas the heat transfer only varies directly as the speed of the air over the surface, the power expenditure varies as the cube. Thus even though the exposed surface be adjusted until only the required heat transfer is effected, the power expenditure increases as the square of the speed.†

The advent of wing surface cooling appeared, at one time, to offer a solution of this difficulty by effecting the cooling without any additional surface. There is, however, reason to believe that the heat transfer necessarily increases the drag of the wing. Apart from this, the Supermarine S 6 B utilised practically the entire exposed surface for cooling and additional surface inside the wing. Further advance in speed appeared to depend upon raising the temperature of the surface.

* R.A.E. Report, June, 1935.

† If, to be more precise, we take the resistance law to be $V^{1 \cdot 8}$, we get from Osborne Reynolds' law $V^{0 \cdot 8}$ for the heat transfer and $V^{1 \cdot 8}$ for the power expenditure.

It is the purpose of this report to show that, by correct design of low velocity cooling systems, in which the surface (whether in the form of a honeycomb radiator or of fins on the cylinder heads and barrels) is exposed in an internal duct, the power expended on cooling does not increase with the speed of flight, but that, on the contrary, it should diminish to vanishing point at a practicable speed beyond which the cooling system contributes to the propulsion.

2. *Classification of drag.*—The drag of a cooling system may conveniently be divided into three categories—

(1) Skin friction drag of stream flow.

(2) Drag due to eddying arising from separation of a stream from a surface.

(3) Drag due to expansion losses without actual stream separation.

This classification is convenient because the second and third, which are due to imperfect streamlining, are avoidable, whereas the former can only be reduced by reduction of "wetted" area or of the velocity of the stream. Thus the ideal system will be designed,

(*a*) to avoid stream separation or severe expansions,

(*b*) to reduce the stream velocity over the cooling surface, and

(*c*) to reduce the external surface to a minimum.

The additional drag due to additional external surface is easily assessed for any particular installation and need not amount to an appreciable value; in many installations, as, for example, when a radiator is housed in a body or wing, it may be zero.

We are concerned, therefore, primarily with the energy loss suffered by the internal stream.

Since we are supposing that eddying is avoided* we can regard the air as a perfect fluid except for the assumption that it suffers a reduction in pressure in passing the cooling surface and possibly at entry. In the development of the theory we will suppose that the cooling surface takes the form of a honeycomb radiator although it will be obvious that the theory is, in general, equally applicable to the air cooled engine.

3. *The radiator considered as an actuator disc operating on a perfect fluid.*—Before considering this theory it should be noted that the stream through the radiator must eventually reach the normal static pressure of the surrounding air. Thus the total head $(p + \frac{1}{2} \rho V^2)$ cannot be reduced below p_0 the static pressure of the main stream. If the length of the tubes is such that the pressure is so reduced, the flow cannot be induced without eddying and the loss will be unnecessarily great; moreover, the following theory will be inapplicable.

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* The practicability of this assumption will be examined subsequently.

3.1. *The freely exposed radiator.*—Let

A be the area of the radiator considered as an actuator disc,
V be the velocity of the stream in which it is exposed,
V $(1 - a)$ be the velocity through the disc,
F be the volume flow of the stream $= A\,V\,(1-a)$,
V $(1 - b)$ be the ultimate velocity of this stream when the static pressure is again p_0 the static pressure of the main stream,
h be reduction in pressure through the actuator disc, and
H be the dynamic head of the main stream, i.e., $\frac{1}{2}\,\varrho\,V^2$.

The power utilised in the actuator disc is

$$E = Fh$$

or

$$E = h\,AV\,(1 - a) \qquad (1)$$

Also the power expended in overcoming drag is

$$E_D = h\,AV \qquad (2)$$

Thus the efficiency of induction is given by

$$\eta = (1 - a) \qquad (3)$$

and the power expended may be expressed by

$$E_D = F\,h/\eta \quad \text{or} \quad \frac{F\,h}{1 - a} \qquad (4)$$

From consideration of the wake

$$h = H\,\{1 - (1 - b)^2\}$$

or

$$\frac{h}{H} = 2\,b\left(1 - \frac{b}{2}\right) \qquad (5)$$

It may be noted that the maximum value of the right hand side of equation (5) is unity, which occurs when b is equal to unity and the wake velocity is zero. Thus, as was stated previously, this theory is only applicable to values of $\frac{h}{H} < 1$.

We may now obtain the relationship between a and b by equating the drag to the rate of change of momentum. Thus

$$h\,A = A\,\varrho\,V\,(1 - a)\,V\,b$$

or

$$h\,A = 2\,HA\,b\,(1 - a) \qquad (6)$$

From (5) and (6) we get

$$b = 2a \qquad (7)$$

If we debar negative values of $(1 - b)$ as being of purely academic interest, a lies between the limits of 0 and $\frac{1}{2}$, and η lies between the limits 1 and $\frac{1}{2}$.

For a real fluid eddying between the wake and the external stream will be inevitable especially as a approaches the limiting value of $\frac{1}{2}$. In so far as this eddying arises far down the wake, the stream pattern near the radiator and hence the drag will be unaffected by it. If, however, it arises close to the radiator the drag may be modified. Such modification cannot reduce the drag and equation (4) may, therefore, be taken as the minimum drag power which can obtain.

3.2. *Extension of theory to the case of a radiator in a duct.*—In this case, the drag can no longer be taken as h A since the pressures on the internal and external boundaries of the duct will produce an additional resultant force.

We will assume that the exit conditions are such that the stream issues at the pressure p_0.

The drag can still be deduced from the rate of change of momentum and is given by

$$D = 2\,H\,A\,b\,(1 - a) \qquad \text{(8)}$$

Hence the power expended on drag is

$$E_D = 2\,H\,A\,V\,b\,(1 - a) \qquad \text{(9)}$$

The power utilised is still given by equation (1), hence the efficiency of induction of the duct is given by

$$\eta_d = \frac{h}{2\,H\,b} \qquad \text{(10)}$$

Equation (5) is also still applicable. Thus eliminating b from (5) and (10) we get

$$\eta_d = \frac{1}{2}\,\frac{\dfrac{h}{H}}{1 - \left(1 - \dfrac{h}{H}\right)^{\frac{1}{2}}} \qquad \text{(11)}$$

When considering top speed conditions of a modern aeroplane, H must be greater than $2h$ if $\dfrac{h}{H}$ is to be less than unity for the climb condition.

We are thus justified, for the top speed condition, in neglecting terms of a higher order than the second in the expansion of $\left(1 - \dfrac{h}{H}\right)^{\frac{1}{2}}$.

With this approximation we may rewrite (11) in the form

$$\eta_d \approx \left(1 + \frac{1}{4}\frac{h}{H}\right)^{-1} \qquad \ldots \quad (12)$$

Since the power consumed in the actuator disc is still F h, we get for the power expended in overcoming drag

$$E_D \approx F\,h\left(1 + \frac{1}{4}\frac{h}{H}\right) \qquad \ldots \quad (13)$$

It may be noted that, for the top speed conditions we have assumed, the duct efficiency lies between 0·89 and 1·0. For conventional radiators the value of $(1 - a)$ when they are freely exposed is about 0·6. Thus the ideal efficiency of the ducted radiator at high speed is about 50 per cent. greater and the power expended in drag, on the assumptions made, should be about $\frac{2}{3}$ of the power expended with the same radiator freely exposed at the speed at which it would provide the required cooling.

4. *Effects of compressibility of the air.*—These effects are four,

(1) The effective temperature of the air is raised by the kinetic energy of the main stream.

(2) The drop in pressure across the radiator is increased for the same mass flow by the reduction of density resulting from heating the stream.

(3) At altitude, the power necessarily expended in the radiator varies inversely as the square of the density and inversely as the cube of the available temperature difference.

(4) The available energy of the cooling stream is increased by the expansion after the addition of heat.

4.1. *Rise of apparent air temperature with speed.*—If the moving stream be brought to rest by pure adiabatic compression as on the front stagnation point, the temperature is raised according to the equation

$$\frac{\delta\theta}{\theta} = \frac{\gamma - 1}{\gamma}\frac{\delta p}{p_0} \qquad \ldots \quad (14)$$

where θ is the absolute temperature of the approaching stream, p_0 its static pressure, and $\delta p = \frac{1}{2}\varrho V^2$.

Now in the duct most of the kinetic energy of the approaching stream has been so converted and the remainder will be converted to heat by the agency of viscosity before the fluid can come in contact with the surface. We may thus take equation (14) to apply to the whole stream of cooling air.

Thus, $$\delta\theta = \frac{1}{2}\frac{\gamma - 1}{\gamma}\frac{\varrho\theta}{p_0}V^2 \qquad \ldots \quad (15)$$

If V is expressed in miles per hour, the other units being slugs, feet, seconds, this equation reduces to the very convenient form

$$\delta\theta = \left(\frac{\text{V m.p.h.}}{100}\right)^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

Thus the available temperature difference of the cooling surface (in degrees centigrade) is reduced by the square of the speed expressed in units of 100 m.p.h.

4.2. *Increased power absorption of hot radiator.*—Let

v be the stream velocity past the cooling surface, and

T the temperature difference.

Then, for constant cooling, $T\,\varrho v$ must be constant. But E, the power consumed in the radiator, is proportional to ϱv^3.

Thus, $$E \propto \frac{1}{T^3\,\varrho^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

Now the air in passing through a glycol radiator may have its temperature raised by about 40° C.* Thus the mean density of the airstream in the tubes may be decreased by about 7 per cent. and E may be increased by about 15 per cent. by the heating of the stream if the power consumed in the cold radiator for the required mass flow be taken as the datum.

It is convenient to consider here also the combined effects of high speed on radiator loss. Consider a flight speed of 300 m.p.h. Assuming that most of the dynamic head is recovered in front of the radiator, the static pressure is increased by 11 per cent. and the density by 8 per cent. But as we saw in the previous section, the temperature difference T is reduced by 9° C. or about 9 per cent. for a glycol radiator. The combined effect in accordance with equation (17) is an increase of E by about 14 per cent. Since the whole of this effect is proportional to V^2, we may say that *the effect of speed is to increase the loss in a glycol radiator by* $\left(\frac{\text{V m.p.h.}}{100}\right)^2 \times 1\tfrac{1}{2}$ *per cent. To this must be added an increase of* 15 *per cent. for temperature* if the loss in the cold radiator be taken as the datum.

4.3. *Effect of altitude.*—Equation (17) may be written in the form

$$E \propto \left(\frac{1}{T\,\varrho^{\frac{2}{3}}}\right)^3 \times \frac{1}{\varrho^{\frac{1}{3}}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

If the mean temperature of a glycol radiator be taken to be 125° C., the quantity $T\,\varrho^{\frac{2}{3}}$ does not vary by more than one per cent. between ground level and 30,000 ft. for English summer maximum air temperatures.

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* With the further development of ducted radiator systems it is possible that this figure, which is applicable to existing systems, will have to be increased considerably.

Thus the effect of altitude on the radiator loss is given by the factor $\frac{1}{\sigma^{\frac{1}{2}}}$, where σ is the relative density.

It may also be noted that, since $T \varrho v$ must be constant, the constancy of $T \varrho^{\frac{1}{2}}$ involves the constancy of $\varrho^{\frac{1}{2}} v$. *Thus for the same indicated airspeed sufficient cooling, for constant heat flow to the jackets, is given at all heights by one setting of the exit nozzle.*

4.4. *Effect of heat on the available energy of the issuing stream.*—For simplification of the theory, we will consider that all the heat is added at the pressure behind the radiator, thus offsetting the small useful expansion in the tubes against the reduction in pressure due to the increased velocity in the tubes.

The available pressure will, of course, depend upon the size of the radiator provided. The size may conveniently be specified by the flight speed at which the radiator would supply adequate cooling if freely exposed. Let this speed be V_0 and let H_0 and $(1 - a_0)$ be the corresponding values of dynamic head and velocity ratio through the radiator. Now consider the case when the radiator is enclosed in a suitable duct for a flight speed of V. The velocity through the radiator will be $V_0 (1 - a_0)$. From equations (5) and (7) in section 3.1 we get

$$h = H_0 4 a_0 (1 - a_0) \quad \ldots \quad (19)$$

Also the dynamic head of the stream leaving the radiator will be $H_0 (1 - a_0)^2$.

Thus the static pressure of the stream behind the radiator is given by

$$p = p_0 + H \left\{ 1 - \frac{H_0}{H} (1 - a_0)(1 + 3 a_0) \right\} \quad \ldots \quad (20)$$

The factor $(1 - a_0)(1 + 3 a_0)$ will depend upon the radiator used. It has, however, a maximum value of $\frac{4}{3}$ when $a_0 = \frac{1}{3}$, and a minimum value of 1 when $a_0 = 0$. Without danger of overestimating the pressure, we may, therefore, substitute the constant $\frac{4}{3}$ for this factor. Then

$$p = p_0 + H \left(1 - \frac{4}{3} \frac{V_0^2}{V^2} \right) \quad \ldots \quad (21)$$

Now the effect of adding heat at pressure p and expanding the stream to pressure p_0 is to convert a proportion of this heat to kinetic energy in the emergent stream. The efficiency of this conversion is given by the equation

$$\zeta = 1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma - 1}{\gamma}} \quad \ldots \quad (22)$$

From (22) and (21) we get, with sufficient accuracy,

$$\zeta = \frac{\gamma - 1}{\gamma} \mathrm{K} \frac{\mathrm{H}}{p_0} \quad \ldots \quad (23)$$

where

$$\mathrm{K} = 1 - \frac{4}{3} \frac{\mathrm{V}_0^2}{\mathrm{V}^2}$$

Substituting $\frac{1}{2}\varrho \mathrm{V}^2$ for H we get at normal temperature and pressure

$$\zeta = 0 \cdot 00161 \left(\frac{\mathrm{V}}{100}\right)^2 \mathrm{K}$$

and at temperature $\theta°$ absolute

$$\zeta = 0 \cdot 00161 \left(\frac{\mathrm{V}}{100}\right)^2 \frac{288}{\theta} \mathrm{K}.$$

For convenience in use it is preferable to employ miles per hour as the unit of aeroplane speed when

$$\zeta = 0 \cdot 00345 \left(\frac{\mathrm{V\ m.p.h,}}{100}\right)^2 \frac{288}{\theta} \mathrm{K} \quad \ldots \quad (24)$$

It is estimated that the heat transferred to the water jackets for a glycol installation is 0·46 of the B.H.P. Thus the power which will be recovered by the cooling stream from conversion of the radiator heat is, from equation (24)

$$\mathrm{E}' = 0 \cdot 16 \left(\frac{\mathrm{V\ m.p.h.}}{100}\right)^2 \frac{288}{\theta} \mathrm{K} \times \frac{\mathrm{B.H.P.}}{100} \quad \ldots \quad (25)$$

The possible recovery is, however, considerably increased if we can add the exhaust heat as well. This may be achieved by enclosing suitably finned exhaust manifolds in the duct behind the radiator or behind the cylinders in the case of an air cooled engine. Assuming a thermodynamic efficiency of 30 per cent. for the engine, the ratio of waste heat to indicated power is $\frac{7}{3}$. Assuming that all this heat can be transferred to the cooling stream before its pressure is reduced in the exit nozzle and ignoring the increase in this factor when referred to B.H.P. we get the power converted in the duct

$$\mathrm{E}'' = 0 \cdot 8 \left(\frac{\mathrm{V\ m.p.h.}}{100}\right)^2 \frac{288}{\theta} \mathrm{K} \times \frac{\mathrm{B.H.P.}}{100} \quad \ldots \quad (26)$$

5. *Quantitative balance of the power in the duct.*—Although the theory which has been given above relates to a radiator which has been assumed to behave as an actuator disc, of which the drag is less than for an actual radiator, it should serve as a basis for deriving the power consumption in a duct from the measured drag of actual

radiators. The ducted radiator is not flattered on this basis since the losses due to eddying which are ignored will be worse for the free radiator.

It is shown in R. & M. 952[1] that, for a wide range of tube lengths, the power required to overcome the drag at 90 m.p.h. of a radiator when cold is 4 per cent. of the power which it will dissipate at this speed for a temperature difference between the approaching air and the mean of the water of 67° C.

Correcting this figure for a temperature difference of 102° C. for a mean glycol temperature of 125° C. and the English summer maximum figure of 23° C. at ground level and expressing the power as a percentage of the B.H.P. instead of the heat dissipated (0·46 of the B.H.P.) we get 1·2 per cent. at 90 m.p.h.

Since, as we have seen in the introduction, this figure is proportional to the square of the speed (after adjusting the size of the radiator till it suffices for the speed) we may express the power required in terms of the speed V_0 at which the radiator will serve, if freely exposed. Thus the percentage drag power will be $1.5\left(\frac{V_0}{100}\right)^2$. Taking, as in Section 3.2, the figure of 0·6 for the efficiency of the exposed radiator we get $0\cdot 9\left(\frac{V_0}{100}\right)^2$ for the power consumed by the radiator in the duct.

This figure will be increased as we have seen in Section 4.2 by about 15 per cent. by the increased power absorption of a hot radiator and also by a further 14 per cent. if we are considering an aeroplane flying at 300 m.p.h. Furthermore, to apply the results to altitude the figure must be corrected by the factor $\sigma^{-\frac{1}{2}}$ in accordance with Section 4.3.

We thus get for a final estimate of the power dropped in the radiator

$$E = 1\cdot 2\sigma^{-\frac{1}{2}}\left(\frac{V_0}{100}\right)^2 \times \frac{\text{B.H.P.}}{100} \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

Thus from equation (26) and (27) we get for the power gained in the duct

$$E'' - E = \left[0\cdot 8\left(\frac{V\ \text{m.p.h.}}{100}\right)^2 \frac{288}{\theta}\left(1 - \frac{4}{3}\frac{V_0^2}{V\ \text{m.p.h.}^2}\right) - 1\cdot 2\sigma^{-\frac{1}{2}}\left(\frac{V_0}{100}\right)^2\right]\frac{\text{B.H.P.}}{100}$$

or

$$\frac{100\,(E'' - E)}{\text{B.H.P.}} = 0\cdot 8\,\frac{288}{\theta}\left(\frac{V\ \text{m.p.h.}}{100}\right)^2 - \left(1.07\,\frac{288}{\theta} + 1\cdot 2\sigma^{\frac{1}{2}}\right)\left(\frac{V_0}{100}\right)^2 \quad \ldots \quad \ldots \quad (28)$$

Relating this formula to standard conditions at 14,000 feet altitude, we get

$$\frac{100\,(E'' - E)}{B.H.P.} = 0\cdot 88\left(\frac{V\ \text{m.p.h.}}{100}\right)^2 - 2\cdot 67\left(\frac{V_0}{100}\right)^2 \quad .. \quad (29)$$

The percentage gain for ranges of V and V_0 is given in the following table derived from equation (29).

V m.p.h. =	200	300	400
V_0 = 50	2·84	7·23	13·38
100	0·84	5·23	11·38
150	−2·49	1·90	8·05

If the exhaust heat is not utilised, we get for the percentage gain at 14,000 feet altitude,

$$\frac{100\,(E' - E)}{B.H.P.} = 0\cdot 177\left(\frac{V\ \text{m.p.h.}}{100}\right)^2 - 1\cdot 725\left(\frac{V_0}{100}\right)^2 \quad .. \quad (30)$$

The percentage gain in the duct, when only the radiator heat is available, is given in the following table, derived from Equation (30).

V m.p.h. =	200	300	400
V_0 = 50	0·28	1·06	2·40
100	−1·02	−0·23	1·10
150	−3·17	−2·38	−1·05

6. *Applicability of the foregoing theory to an actual installation.*— In the foregoing theory, additional drag, due to an increase of the total surface exposed to the external stream, has been ignored. Where the radiator can be housed inside the fuselage or wing, there may be no such increase of surface. In a large number of cases, however, the fairing round the radiator will increase the surface somewhat. In general, however, this increase should not be more than, say, 1 per cent. of the total surface of the aeroplane. Thus the additional power required to overcome drag so caused should not be more than 1 per cent. of the B.H.P.

Moreover we have assumed that there are no losses due to stream separation. Recent experiments at the R.A.E. have shown that separation of the external stream can be avoided by suitable streamlining, but that, in many cases, the internal stream separates with a resulting loss of total head in front of the radiator. Clearly the result of such a loss is to increase the internal work done on the radiator in the ratio $\frac{h + h_E}{h}$ where h_E is the loss of head at entry. The effect is, therefore, the same as if a smaller radiator were used and may be represented by an increase of the quantity V_0 in the ratio $\left(1 + \frac{h_E}{h}\right)^{\frac{1}{2}}$. It is, therefore, important in any installation to keep the ratio $\frac{h_E}{h}$ fairly small to approach the performances represented by the tables in Section 5.

Although a somewhat academic analysis has been used in discussing the case of a fully exposed radiator the analysis of the ducted case is rigid on the assumption that the loss of total head of the internal stream is h and that the exit is so adjusted that the stream exits at static pressure. This requires, for the calculated performance at top speed, that the exit nozzle should be designed to give parallel flow of the exit stream without disturbing the pattern of the external stream. It also means that the quantity V_0 may differ slightly from the speed at which the radiator would suffice if fully exposed by which it was defined.

If, for either a liquid cooled or air cooled system, the product hF, representing the power expended in cooling in the duct, be known and also the ratio $\frac{h}{H}$, which is the proportion of the dynamic head of the stream required to force this flow, then the necessary cooling drag of the cold system is given by equation (13) and serious addition to this value should be avoidable. It should be noted that if $\frac{h}{H}$ be not greater than $\frac{1}{2}$ its actual value is of little importance until we are concerned with the effects of the added heat.

Further data are required to link up the dimensions of the radiator with the quantity V_0, by which alone they are specified in the above analysis. These data would preferably supply the values of the quantities F and h directly so as to render the analysis independent of the somewhat uncertain theory of an exposed radiator.

It is clear, however, that radiators (as constructed at present) corresponding to $V_0 = 50$ would be prohibitively large unless the mean temperature can be raised above the figure 125° C. assumed. Values of V_0 in excess of 150 are unlikely if adequate cooling is to be provided on the climb. V_0 will, therefore, in general lie between the limits 100 and 150, depending upon the weight which the designer is prepared to allow and upon the space available for the installation. The limitation of frontal area may call for a somewhat greater ratio of tube length to diameter (possibly small bore tubes) than is customary for freely exposed radiators. Theoretical considerations suggest that, for minimum power expenditure, the temperature rise of the air should be about 85 per cent. of the initial temperature difference, but for economy of weight a lower figure must be accepted.

It would, therefore, appear to be desirable to obtain further data on the performance of radiators in ducts particularly in regard to the greatest calibre and smallest tubes which may be practicable. Finned radiators should be considered with a view to minimising the velocity of the stream in the duct.

With regard to air cooled engines, it is clearly desirable to find the values of hF for existing installations and to explore how far this product may be reduced by better use of baffles and fins. The effect of reducing this product is of course equivalent to a reduction of the quantity V_0, the effect of which is brought out in the tables of Section 5. Before, however, the effect of waste heat recovery on an air cooled installation can be assessed the pressure behind the engine must be known since it cannot, as in the case of the radiator, be deduced approximately from the quantity V_0 in the form of equation (21). It is clear, however, that inter-cylinder baffles which reduce this pressure militate against the recovery of the waste heat.

7. *Effect of the momentum of the exhaust gases on the drag of an engine installation.*—Various proposals have been made to utilise the energy of the exhaust gases to assist the induction of the cooling stream, although design to date has apparently been little affected by consideration of the momentum of the issuing gases.

Broadly it may be stated that the effect of this momentum is the same whether it be diffused with the cooling stream or not. It should be noted, however, that some of the benefit in thrust will be lost by a consequent increase of skin friction drag if the exhaust gases scrub an appreciable surface at high velocity. For this reason diffusion of momentum inside the duct may be desirable and this may be a convenient method of diffusing the exhaust heat.

The thrust derivable from the rearward direction of the exhaust gases is given by the product of the mass flow and the velocity of exit and the latter quantity depends upon the internal design of the exhaust system. The thrust power is, however, also proportional to the speed of flight. Thus it becomes increasingly important to utilise this thrust as the speed of flight increases.

No attempt is here made to assess the power which may be available from this source. It is suggested, however, that if, by the use of suitable deflectors for guiding the exhaust gases round the necessary bends and by the avoidance of excessive unguided expansions, an appreciable proportion of the original energy of the exhaust gases can be preserved, this will provide an appreciable increment to the thrust horse power of a high speed aeroplane.

Conclusions.—The employment of the principle of low velocity cooling avoids the necessity for an increasing expenditure of power with increasing speed provided the exit conditions are adjusted to suit the speed.

Further the combined effects of compressibility and heat transfer from the radiator may reduce the power consumption to nothing if the size of the radiator is adequate. By the use of the heat of the exhaust, in addition, an appreciable thrust may be expected from the presence of the cooling stream.

Finally, attention is drawn to the importance of the momentum of the exhaust gases for a high speed aeroplane, although no attempt is made to deal with this point quantitatively.

REFERENCE

No.	*Author.*	*Title, etc.*
1.	Harris & Caygill.	Further experiments on honeycomb radiators. R. & M. 952. November, 1924.

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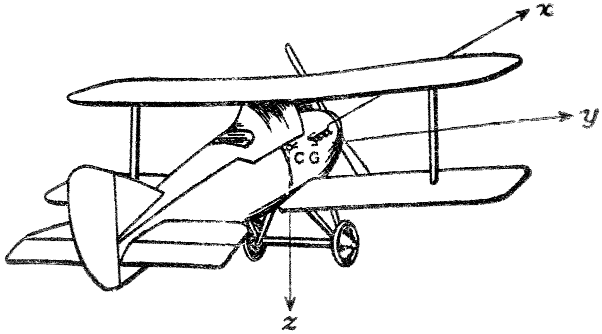
SYSTEM OF AXES



Axes	Symbol Designation Positive direction	x longitudinal forward	y lateral starboard	z normal downward
Force	Symbol	X	Y	Z
Moment	Symbol Designation	L rolling	M pitching	N yawing
Angle of Rotation	Symbol	ϕ	θ	ψ
Velocity	Linear Angular	u p	v q	w r
Moment of Inertia		A	B	C

Components of linear velocity and force are positive in the positive direction of the corresponding axis Components of angular velocity and moment are positive in the cyclic order y to z about the axis of x, z to x about the axis of y, and x to y about the axis of z

The angular movement of a control surface (elevator or rudder) is governed by the same convention the elevator angle being positive downwards and the rudder angle positive to port The aileron angle is positive when the starboard aileron is down and the port aileron is up A positive control angle normally gives rise to a negative moment about the corresponding axis The symbols for the control angles are –

ξ aileron angle
η elevator angle
η_T tail setting angle
ζ rudder angle

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