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TABLES FOR USE IN AN IMPROVED METHOD OF AIRSCREW STRIP THEORY CALCULATION

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Summary.—An improved method of calculating the performance of an airscrew has been described in previous reports (Refs. 1 and 2), which includes an allowance for tip loss. The present report contains tables of a parameter ($\varkappa$) required in the calculation of the blade interference by the new method; these tables being available, the labour involved is no greater than in a calculation by the standard vortex theory. A simplified method of calculation involving the use of charts is described in a separate report (Ref. 6).

The tables given here cover the case of two-, three-, and four-bladed airscrews at a series of standard radii and for all values of pitch. Details of the method of calculation (due to Goldstein, Ref. 3) are given in an Appendix together with a method of interpolation which can be used to extend the results to any other number of blades. The results for three blades were interpolated by this method but are probably quite accurate enough for practical purposes. All formulae required in the present method of strip theory calculation are given here, together with complete details of a specimen calculation.

A preliminary comparison is included between results of calculations by the present method and the new experiments on high pitch model airscrews described in Ref. 5, for two-bladed airscrews of pitch ratio 1.5 and 2.5 and a four-bladed airscrew of 2.5; these show satisfactory agreement (below the stall) and reasons are given to suggest that the agreement should be at least as good over the whole range of pitch ratio.

It is concluded that the present formulae and tables may be used with confidence to calculate the performance of any airscrew below the stalling angle, provided that suitable aerofoil data are available.

List of Symbols

V Forward velocity of airscrew ; feet per second.
Ω Angular velocity of airscrew ; radians per second.
W_0 Resultant of V and $r\,\Omega$ (Fig. 1).
w_1 Total interference velocity (Fig. 1).
W Total velocity relative to blade element ; resultant of W_0 and w_1 (Fig 1).
Γ Circulation round a blade element.
ϕ Inclination of W to plane of rotation (Fig. 1).
α Incidence of blade section (Fig. 1).
θ Blade angle (Fig. 1).
c Chord of blade section.
r Radius of section.
R Tip radius.
N Number of blades.
P_1 " Induced " power loss.
P_2 Profile drag power loss.
κ Coefficient of interference velocity §2, equation 2.
s (Solidity) $= Nc/2\pi r$.
$\mu = \cot\phi$.
$\mu_0 = \mu R/r$.

Coefficients.—x, A, W_c, w_c, T_c', P_{c1}', P_{c2}', T_c, P_{c1}, P_{c2}, Q_c are defined in §3, equations (13–19).

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1. *Introduction.*—In R. & M. 1377[1] and R. & M. 1521[2] a new method of calculating the performance of an airscrew is described which represents an improvement on the standard vortex theory in so far as the latter assumes that the number of blades is infinite while the new method makes an allowance for tip loss which varies with the number of blades. The new method is based on calculations by Goldstein[3] of the velocity field of rigid helicoidal surfaces moving through a perfect fluid. Goldstein's original numerical calculations were limited to two-bladed airscrews of pitch ratio less than 1.5 and to a single four blader. The calculations have now been extended to the case of four-bladed airscrews and to higher pitch values ; they are given in the present report in a form convenient for application to the airscrew problem. By special methods of interpolation the results may be applied to any radius or number of blades for any value of pitch.

The actual strip theory formulae of R. & M. 1377[1] have been simplified by omitting the profile drag coefficient from the formulae for the interference velocity. This omission has been shown to have a negligible effect on the calculated performance of the airscrew below the stalling angle. It is then possible to express the formulae in a form which does not refer explicitly to coefficients of axial and rotational " interference " velocity, but deals with a coefficient w_c of total " interference " velocity w_1 and a coefficient W_c of the total velocity W, relative to a blade element (Fig. 1). The formulae in this form are given in Ref. 2, §7.1, and are repeated here for convenience.

Strictly speaking these formulae are only theoretically correct for a particular distribution of interference velocity with radius (Ref. 1, §6, Ref. 2, §1.24); their use in the general case is justified by their good agreement with the results of theoretically more accurate but more laborious methods described in Ref. 2.

A complete specimen calculation by these formulae is included. A simplified method in which strip theory calculations are made at a single radius (0·7) only, and the thrust and torque coefficients obtained by the use of suitable integrating factors, is described in another report, Ref. 6.

2. *Definition of* ($\varkappa$).—The fundamental relation of the vortex theory, which is based on the assumption of an infinite number of blades[2], may be expressed as a relation between the total interference velocity w_1 (Fig. 1) and the circulation Γ round the blade element at the corresponding radius, in the form

$$w_1 = \mathrm{N}\Gamma/4\pi r \sin \phi, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where N is the number of blades, r the radius, and ϕ the angle of pitch of the resultant relative velocity W (Ref. 2, §1.22 (1)). In the improved theory described in Ref. 1, §6 and Ref. 2, §§1.24 and 7.1, this relation is replaced by the formula

$$w_1 = \mathrm{N}\Gamma/4\pi r \varkappa \sin \phi, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

which differs from (1) only by the presence of the factor $\varkappa$ in the denominator. This factor is assumed be to a function of the following three variables only :—number of blades N, pitch angle ϕ and radius $x(= r/\mathrm{R})$. Thus the assumption of " independence of neighbouring elements " is retained, since the value of $\varkappa$ at any radius does not depend on the conditions over the rest of the blade. At the same time an allowance for tip loss is included, since $\varkappa$ is a function of the number of blades, and tends to zero at the blade tip so that the circulation Γ must tend to zero there. This is true even if the blade has a square tip, since w_1 must still remain finite.

The methods of computing the function $\varkappa$ are discussed in Appendices I and II. Tables of values of $\varkappa$ for two-, three- and four-bladed airscrews* are given in Table 1 for the following standard values of radius :—0·3, 0·45, 0·6, 0·7, 0·75, 0·8, 0·85, 0·9, 0·95, and corresponding curves plotted against sin ϕ are shown in Figs. 3, 4 and 5.† Methods of interpolating to other radii and numbers of blades are given in Appendix I. Table 3 and Fig. 6 give values of $\varkappa$, at a radius of 0·7 only, for two, three, four, six and eight blades interpolated by the method described in Appendix I.

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* The values of $\varkappa$ for four-bladed airscrews for $\sin \phi = 1$ are derived from an exact formula obtained by Mr. F. L. Westwater as yet unpublished. The values for three-bladed airscrews were interpolated by the method of Appendix I.

† In Ref. 1, Fig. 5, K ($= \varkappa/\cos^2 \phi$) is plotted against $\tan \phi$ for two-bladed airscrews only and for a limited range of pitch values.

3. *Detailed Strip Theory Formulae.*—From formula §2 (2) for the total interference velocity together with the fact that the direction of the total *interference* velocity w_1 is normal to that of the resultant relative velocity W (Fig. 1), it is a simple matter to write down all the formulae required in a strip theory calculation of airscrew performance, by reference to the geometry of Figs. 1 and 2, and by making use of the Kutta-Joukowski relation between lift and circulation (equation 4 below).

Taking the incidence α of the blade element at radius r as independent variable, the first formula is

$$\phi = \theta - \alpha \quad \ldots \quad (1)$$

where θ is the blade angle (Fig. 1). Formulae involving the interference velocity may be derived from formula §2 (2) together with the relations

$$W = r\Omega \sec \phi - w_1 \tan \phi \quad \ldots \quad (2)$$

$$V = r\Omega \tan \phi - w_1 \sec \phi \quad \ldots \quad (3)$$

which are derivable from the geometry of Fig. 1. From the Kutta-Joukowski relation,

$$\frac{dL}{dr} = \varrho W \Gamma \quad \ldots \quad (4)$$

together with the definition of k_L

$$\frac{dL}{dr} = \varrho c W^2 k_L \quad \ldots \quad (5)$$

we have

$$\Gamma = cWk_L \quad \ldots \quad (6)$$

and so equation §2 (2) becomes

$$w_1 = \tfrac{1}{2} s k_L \, W/\varkappa \sin \phi \quad \ldots \quad (7)$$

where s (coefficient of solidity at radius r) is written in place of $Nc/2\pi r$. Eliminating W between equations (2) and (7) gives

$$w_1 = \frac{\frac{1}{2} r\Omega \cdot s k_L}{\varkappa \sin \phi \cos \phi} \Bigg/ \left(1 + \frac{\frac{1}{2} s k_L}{\varkappa \cos \phi}\right); \quad \ldots \quad (8)$$

then by substituting w_1 in (2) and (3) it is possible to determine W and V. The elements of thrust and torque are given at once by the geometrical equations (Fig. 2)

$$\frac{dT}{dr} = \varrho N c W^2 (k_L \cos \phi - k_D \sin \phi^*) \quad \ldots \quad (9)$$

$$\frac{dQ}{dr} = \varrho N c r W^2 (k_D \cos \phi + k_L \sin \phi).$$

* This term is generally negligible below the stalling angle.

In practice it has proved convenient to ignore the second equation and to work with power relations derived from the relation :—

Torque power = thrust power + (total)* induced loss
+ profile drag loss . . .

or

$$\Omega \frac{dQ}{dr} = V \frac{dT}{dr} + \frac{dP_1}{dr} + \frac{dP_2}{dr} \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

where

$$\left.\begin{aligned} &\frac{dP_1}{dr} = w_1 \sec \phi \frac{dT}{dr} \text{ (induced power loss)} \\ &\text{and} \\ &\frac{dP_2}{dr} = \varrho N c W^3 k_D \quad \text{(profile drag power loss).} \end{aligned}\right\} \quad (11)$$

Then the thrust, torque and efficiency are given by the relations

$$\left.\begin{aligned} T &= \int^R \frac{dT}{dr}\, dr \\ P_1 + P_2 &= \int^R \left(\frac{dP_1}{dr} + \frac{dP_2}{dr}\right) dr \\ \Omega\, Q &= V\, T + P_1 + P_2 \\ 1 - \eta &= \frac{P_1 + P_2}{\Omega\, Q} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

In numerical calculations it is convenient to work with non-dimensional coefficients defined as follows :—

$$x = r/R$$

$$s = \frac{Nc}{2\pi r}$$

$$\Lambda = V/R\Omega \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

$$W_c = W/R\Omega \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

$$w_c = w_1 \sec \phi / R\Omega\dagger \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

$$T_c' = \frac{1}{\varrho R^2 \Omega^2} \cdot \frac{1}{2\pi r} \cdot \frac{dT}{dr} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

$$P_{c1}' = \frac{1}{\varrho R^3 \Omega^3} \cdot \frac{1}{2\pi r} \cdot \frac{dP_1}{dr} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

* It is no longer necessary to separate the induced power loss into axial and rotational components of power loss as is done *e.g.* in Ref. 8.

† The factor sec ϕ is included in order to simplify equations (22) and (25) below.

$$P_{c2}' = \frac{1}{\varrho R^3 \Omega^3} \cdot \frac{1}{2\pi r} \cdot \frac{dP_2}{dr} \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (18)$$

$$\left.\begin{aligned} T_c^* &= \frac{T}{\pi \varrho R^2 (R\Omega)^2} \\ P_{c(1,2)} &= \frac{P_{(1,2)}}{\pi \varrho R^2 (R\Omega)^3} \\ Q_c &= \frac{Q\Omega}{\pi \varrho R^2 (R\Omega)^3}. \end{aligned}\right\} \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (19)$$

Then equations (1), (8), (3), (2), (9), (12) are replaced by

$$\phi = \theta - a \quad \cdots \quad \cdots \quad \cdots \quad (20)$$

$$w_c = \frac{\frac{1}{2}sxk_L}{\varkappa \sin\phi \cos^2\phi} \Big/ \left(1 + \frac{\frac{1}{2}sk_L}{\varkappa \cos\phi}\right) \quad \cdots \quad \cdots \quad (21)$$

$$\Lambda = x \tan\phi - w_c \quad \cdots \quad \cdots \quad \cdots \quad (22)$$

$$W_c = x \sec\phi - w_c \sin\phi \quad \cdots \quad \cdots \quad \cdots \quad (23)$$

$$T_c' = sW_c^2 (k_L \cos\phi - k_D \sin\phi\dagger) \quad \cdots \quad \cdots \quad (24)$$

$$P_{c1}' = w_c T_c' \quad \cdots \quad \cdots \quad \cdots \quad (25)$$

$$P_{c2}' = sW_c^3 k_D \quad \cdots \quad \cdots \quad \cdots \quad (26)$$

$$T_c = \int_0^1 T_c' d(x^2) \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (27)$$

$$P_{c1} + P_{c2} = \int_0^1 (P_{c1}' + P_{c2}')\, d(x^2) \quad \cdots \quad \cdots \quad (28)$$

$$Q_c = \Lambda T_c + P_{c1} + P_{c2} \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (29)$$

$$1 - \eta = \frac{P_{c1} + P_{c2}}{Q_c}. \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (30)$$

The standard type of coefficients k_T, k_Q, J are given in terms of the coefficients defined in (19) by the relations

$$J = \pi\Lambda \quad \cdots \quad \cdots \quad \cdots \quad (31)$$

$$k_T = \frac{\pi^3}{4} T_c = 7{\cdot}7516\, T_c \quad \cdots \quad \cdots \quad \cdots \quad (32)$$

$$k_{P\,(1,2)} = \frac{\pi^3}{8} P_{c\,(1,2)} = 3{\cdot}8758\, P_{c\,(1,2)} \text{ (Definition)} \quad (33)$$

$$k_Q = (Jk_T/2\pi) + k_{P1} + k_{P2} \quad \cdots \quad \cdots \quad \cdots \quad (34)$$

$$1 - \eta = \frac{k_{P1} + k_{P2}}{k_Q}. \quad \cdots \quad \cdots \quad \cdots \quad (35)$$

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* This symbol has been used in a different sense in some published reports.

† The term $k_D \sin\theta$ may be neglected below the stalling angle.

4. *Specimen Calculation.*—Part of a specimen calculation by the above formulae is given in Table 4, which refers to a two-bladed airscrew of pitch ratio 1.5. Values of ϕ, w_c, Λ, W_c, T_c', P_{c1}', P_{c2}', are calculated by means of equations (20–26) for a series of values of the incidence α ranging here from zero lift to the stall, and for a series of standard radii taken here as :—$x = 0\cdot3$, $0\cdot45$, $0\cdot6$, $0\cdot75$.* $0\cdot9$ and $0\cdot962$. Values of θ and s are known from the design of the screw ; $\varkappa$ is given as a function of x and $\sin\phi$ by Fig. 3 (two-bladed airscrew). k_L and k_D are supposed to be known functions of α for each radius ; in the present case they are taken from Table 3 on p. 15 of R. & M. 892[4] (higher wind speed).

The next step in the calculation is to plot T_c', P_{c1}', P_{c2}' against Λ for each radius (Figs. 7, 8 and 9) and to read off values corresponding to a series of smooth values of Λ (Table 5). These values are then to be plotted against x^2 (Figs. 10, 11 and 12) and integrated graphically according to equations 27 and 28.† The efficiency η and the torque (or power) coefficients are then given by equations (29) and (30) or (34) and (35) (Table 6).

The type of thrust and power grading coefficient as defined by equations (16), (17) and (18) is such that, e.g., the thrust coefficient T_c is equal to the area under a curve of T_c' plotted against radius squared (x^2) (Fig. 10). On the conception of the airscrew as an actuator disc, the ordinate of this curve is proportional to the pressure at a given radius. This has been found more convenient than the more usual type of thrust grading curve, because it is more symmetrical and does not fall so steeply towards the tip, while at the same time it emphasises the greater relative importance of the sections nearer the tip.

A simplified method in which calculations are made at a single radius ($0\cdot7$R) only is described in Ref. 6, together with a graphical method in which the whole of the calculations are replaced by operations with families of curves.

5. *Comparison with Experiment.*—Complete strip theory calculations as described in the last section have so far been made for three of the model airscrews of the family recently tested (Ref. 5) viz., the two-bladed airscrews of P/D $1\cdot5$ and $2\cdot5$ and the four-bladed airscrew of P/D $2\cdot5$, for the range below the stall. The results are plotted in Figs. 13, 14 and 15, and show good agreement with experiment for the screw of P/D $1\cdot5$ though the agreement is not quite so good for the two screws of P/D $2\cdot5$. From results of the calculations in Ref. 4, based on the assumption of infinity blades ($\varkappa = 1$) and of calculations by the approximate method

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* Details are given for $x = 0\cdot75$ only.

† The lower limit of integration corresponds to the surface of the airscrew spinner.

described in Ref. 6 there is little doubt that the agreement will be at least as good (below the stall) for airscrews of lower pitch, and over the whole range of pitch tested in Ref. 5 (P/D 0·3 to 2·5). The appreciable discrepancy which may be observed in Ref. 4, Fig. 12, between theoretical and experimental values of the torque coefficient* for the two-blader of P/D 1·5 (repeated in Fig. 14) has disappeared :—(*a*) for large k_Q, as a result of the correction for tip loss included in the method of the present report ; (*b*) near zero thrust, as a result of the increase of the observed k_Q between the old experiments of Ref. 7 and the new (Ref. 5, Fig. 5). This increase of observed k_Q may be partly attributed to an increase of Reynolds number to a value agreeing closely with that of the aerofoil tests used as a basis for the strip theory calculations (Ref. 4, Table 3, p. 15).

6. *Conclusions.*—The present method of airscrew strip theory calculation gives good agreement with experiment below the stall up to high values of pitch ratio and is superior in this respect as well as on theoretical grounds to the older " vortex theory " which made no allowance for tip loss. The present method is at least as simple in application as the old, and the formulae as here set out appear to reduce the labour of computation to a minimum. The use of coefficients of power loss in computing torque and efficiency shows the relative importance of the two sources of power wastage at various working conditions.

A considerable further simplification in detailed strip theory will be attained by the use of the charts described in Ref. 6, §2, when they are available.

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* The corresponding comparison for the thrust is of little value on account of the uncertainty of the boss correction on the results of Ref. 7.

APPENDIX I

Methods of Interpolation.—The process of calculating $\varkappa$ as a function of ϕ, x and N described by Goldstein in Ref. 3 is laborious, and the infinite series tend to converge less rapidly on approaching the airscrew tip and for large values of the pitch angle ϕ. In order to overcome this difficulty use has been made of the approximate formula due to Prandtl, Ref. 3, §3.4, as a formula of interpolation. Values of $\varkappa$ calculated by this formula will be denoted by the symbol $\varkappa_P$. The formula for $\varkappa_P$ is

$$\varkappa_P = \frac{2}{\pi} \text{ arc cos } e^{-\frac{1}{2}Nf}$$

where

$$f = (1 - x)/\sin \phi_o$$

and

$$\tan \phi_o = x \tan \phi.$$

This equation is solved in practice by writing

$$\varkappa_P = (\theta^\circ/90^\circ)$$

with

$$L \cos \theta = 10 - \tfrac{1}{2}mNf$$

where $m = 0{\cdot}4343$ is the modulus of common logarithms. For small values of sin ϕ and values of x near $1{\cdot}0$ it is convenient to use the approximation

$$f \simeq (1 - x)/x \sin \phi.$$

This implies that the curves (in *e.g.* Fig. 3) cut the lines of constant $\varkappa$ at values of sin ϕ which are approximately proportional to $(1 - x)/x$ (since $\varkappa$ is approximately equal to $\varkappa_P$ in this region).

There is good reason to believe that the ratio $\varkappa/\varkappa_P$ tends to a finite limit at the airscrew tip at which both $\varkappa$ and $\varkappa_P$ separately tend to zero. Both $\varkappa$ and $\varkappa_P$ tend to definite limiting values as ϕ tends to 90° (infinite pitch) and for the two- and four-bladed airscrews simple formulae for the limiting value of $\varkappa$ are available (*see* Appendix III). The limiting value of $\varkappa/\varkappa_P$ at the airscrew tip for $\sin \phi = 1$ is $0{\cdot}5$ for the two-blader, which is presumably the smallest value that can occur ; while both $\varkappa$ and $\varkappa_P$ tend to unity as N tends to infinity. It is true that, as appears from Figs. 3, 4 and 5, both $\varkappa$ and $\varkappa/\varkappa_P$ become greater than unity for high values of pitch and small values of radius, but in practice strip theory calculations are never carried to a radius smaller than $0{\cdot}3$ and it appears from Figs. 3, 4 and 5 that the value of $\varkappa$ at this radius only becomes slightly greater than unity for high values of pitch. It is evident therefore that the ratio $\varkappa/\varkappa_P$ is a suitable function to use in interpolating or extrapolating the limited number of values of $\varkappa$ calculated by the exact formulae. Values of the ratio $\varkappa/\varkappa_P$ are accordingly tabulated in Table 2 over the same range as the values of $\varkappa$ in Table 1 and may be used, in conjunction with the explicit formula for $\varkappa_P$ above, to interpolate the values of $\varkappa$ to intermediate values of radius, sin ϕ, or number of blades, and to extrapolate to radii greater than $0 \cdot 95$ if required.

The largest values of sin ϕ for which it was possible to calculate values of $\varkappa$ directly, correspond to :—tan $\phi = x/1 \cdot 5$, for the two-bladers, and tan $\phi = x/1 \cdot 4$ for the four-bladers (*see* Appendix II and Table 7). Values between these limits and sin $\phi = 1 \cdot 0$, are interpolated by the above method. It is also possible to interpolate values of $\varkappa/\varkappa_P$ for other numbers of blades by plotting against 1/N, assuming that $\varkappa/\varkappa_P$ tends to unity as 1/N tends to zero. The values of $\varkappa$ and $\varkappa/\varkappa_P$ for three blades given in Tables 1 and 2 and Fig. 4 were obtained by this method. Table 3 (for $x = 0 \cdot 7$) also includes results for six and eight blades obtained by the same method. These values are of course less accurate than those for two and four blades but are probably quite good enough for practical purposes.

APPENDIX II

Details of calculations of $\varkappa$ made by Goldstein's formulae additional to those given in his original report[3].

Four-bladed Airscrews.—The relevant formulae are :—Ref. 3, §3.4 (2), §4.1 (9) and §4.2.

$$\left(\frac{\mu^2}{1+\mu^2}\right) \varkappa = G_4(\mu) - \frac{2}{\pi} \sum_{m=0}^{\infty} \left(\frac{\mu_0^2}{1+\mu_0^2} A_m - \varepsilon_m\right) \times \frac{I_{4m+2}[(4m+2)\mu]}{I_{4m+2}[(4m+2)\mu_0]}$$

where

$$\mu = \cot\phi, \quad \mu_0 = \mu/x$$

$$G_4(\mu) = \frac{\mu^2}{1+\mu^2} - \frac{8}{\pi^2} \sum_{m=0}^{\infty} \frac{F_{4, 2m+1}(\mu)}{(2m+1)^2} .$$

$F_{4,1}(\mu)$ is tabulated in Ref. 3, Table 3 (it is there described as $F_{2,1}(\mu)$) ; $F_{4,3}(\mu)$ and later terms are negligible.

The A_m have the values

$$A_0 = 1, \quad 3A_1 = \tfrac{1}{2}, \quad 5A_2 = \frac{1 \cdot 3}{2 \cdot 4}, \text{ etc.}$$

The values of ε_o and ε_1 as functions of μ_o were obtained from curves drawn through the values calculated by Goldstein for the two-bladed airscrews (Ref. 3, p. 449) on the assumption that

$$\varepsilon \text{ (four-bladers) } \mu_o = \varepsilon \text{ (two-bladers) } \tfrac{1}{2}\mu_o .$$

It was first necessary to calculate or obtain values of $I_2(2\mu)$ and $I_6(6\mu)$ over the range $\mu = 0$ to $\mu = 10$.

Methods used for Calculating $I_2(2\mu)$ *and* $I_6(6\mu)$.

$I_2(2\mu)$.—*From* $\mu = 0$ *to* $\mu = 2\cdot 5$, given in Watson,* p. 736.

From $\mu = 2\cdot 5$ *to* $\mu = 8\cdot 0$: by the formula

$$I_2(2\mu) = I_o(2\mu) - \frac{1}{\mu} I_1(2\mu),$$

$I_o(2\mu)$ and $I_1(2\mu)$ being given in Watson,* p. 713.

From $\mu = 8\cdot 0$ to $\mu = 10\cdot 0$; by the formula

$$e^{-x} I_2(x) = \frac{1}{(2\pi x)^{\frac{1}{2}}}\left\{1 - \frac{15}{8x} + \frac{15}{8x}\cdot\frac{7}{16x} + \frac{15.7.9}{8x.16x.24x}\right\}.$$

$I_6(6\mu)$. *From* $\mu = 0$ to $\mu = 1\cdot 0$; given in Gray and Matthews (Bessel functions). *From* $\mu = 1\cdot 0$ *to* $\mu = 2\cdot 67$ by the formula

$$I_6(6\mu) = \left\{1 + \frac{4}{\mu^2}\left(1 + \frac{1}{2\cdot 7\mu^2}\right)\right\} I_o(6\mu)$$
$$- \frac{3}{\mu}\left\{1 + \frac{32}{27\mu^2}\left(1 + \frac{5}{36\mu^2}\right)\right\} I_1(6\mu).$$

From $\mu = 2\cdot 67$ to $10\cdot 0$ by the formula

$$e^{-x} I_6(x) = \frac{1}{(2\pi x)^{\frac{1}{2}}}\left\{1 - \frac{143}{8x} + \frac{143}{8x}\cdot\frac{135}{16x} - \frac{143}{8x}\cdot\frac{135}{16x}\cdot\frac{119}{24x} + \frac{143}{8x}\cdot\frac{135}{16x}\cdot\frac{119}{24x}\cdot\frac{95}{32x}\right\}.$$

The necessity of obtaining $I_{10}(10\mu)$, etc., was obviated by the use of the following approximate formulae. Write

$$S_m = \sum_{o}^{m} u_m$$

where

$$u_m = A_m \frac{I_{4m+2}(4m+2)\,\mu}{I_{4m+2}(4m+2)\,\mu_o}\ .$$

It is required to evaluate S_∞. The approximation used is

$$u_m \simeq v_m$$

* A treatise on Bessel Functions by G. N. Watson.

where

$$v_m = (\cos\theta/\cos\theta_o)^{\frac{1}{2}} X^{4m+2} A_m$$

and $\tan\theta = \mu$

$$X = \exp.\,(y - y_o)$$

$$y = \tan\theta + \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) + \log\tan\tfrac{1}{2}\theta.$$

This formula is due to Nicholson.* Then

$$S_a \equiv \sum_{o}^{\infty} v_m = (\cos\theta/\cos\theta_o)^{\frac{1}{2}} \operatorname{arc\,sin}(X^2).$$

We neglect the difference between u_m and v_m from the third term onwards and write

$$S_\infty = (u_1 + u_2) - (v_1 + v_2) + S_a.$$

Two-bladed Airscrews.—Goldstein's original calculations refer to $\mu_o = 2{\cdot}0, 3{\cdot}0$, etc. up to $10{\cdot}0$. Additional calculations were made for $\mu_o = 1{\cdot}5$ and $2{\cdot}5$ and the original calculations were repeated for $\mu_o = 2{\cdot}0$ and $3{\cdot}0$ and carried nearer to $\mu/\mu_o = 1{\cdot}0$ by the use of Nicholson's formulae as in the case of the four blader.

All results for two and four bladers were afterwards smoothed, interpolated and extrapolated by the method described in Appendix I. In plotting $\varkappa/\varkappa_P$ against μ, it was verified that irregularities in the original points were sufficiently accounted for by the neglect of the terms ε_3, etc., and the curves were accordingly smoothed on the assumption that $\varkappa/\varkappa_P$ tends to a finite limit at the tip $(\mu = \mu_o)$. Values of $\varkappa$ directly calculated by the above formulae are given in Table 7.

APPENDIX III

Rotating Plane Lamina in Two Dimensions.—The solution of this problem is given in Lamb's Hydrodynamics §72.4°. With the notation there used, put

$$z = c\cosh\zeta. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Then on the surface of the lamina,

$$\xi = 0,\ \ y = 0,\ \ x = c\cos\eta. \quad \ldots \quad \ldots \quad (2)$$

Put

$$\Phi + i\Psi \equiv w = ci\,e^{-2\zeta} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

* Phil. Mag. Vol. 20, p. 938 (1910).

and express the condition that

$$\Psi = \tfrac{1}{2}\omega r^2 + \text{const.} \qquad \cdots \quad \cdots \quad \cdots \quad (4)$$

on the surface (Lamb, §72, equation 1). Then, on the surface,

$$\Phi + i\Psi = Ci\,(\cos 2\eta - i \sin 2\eta) \qquad \cdots \quad \cdots \quad (5)$$

$$r = x$$

so that

$$-\tfrac{1}{2}\omega r^2 = C \cos 2\eta - \tfrac{1}{4}\omega c^2\,(1 + \cos 2\eta) \qquad \cdots \quad \cdots \quad (6)$$

and equation (4) is satisfied if

$$C = \frac{1}{4}\,\omega c^2. \qquad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (7)$$

Also

$$u - iv = \frac{dw}{dz}$$

$$= \frac{2C\,i\,e^{-2\zeta}}{c \sinh \zeta} \qquad \cdots \quad \cdots \quad \cdots \quad (8)$$

so that on the surface

$$u - iv = \frac{\tfrac{1}{2}\omega c^2\,(\cos 2\eta - i \sin 2\eta)}{c \sin \eta} \qquad \cdots \quad (9)$$

$$v = \omega c \cos \eta$$

$$= \omega x \qquad \cdots \quad \cdots \quad \cdots \quad (10)$$

and

$$u = \frac{\tfrac{1}{2}\,\omega c \cos 2\eta}{\sin \eta}. \qquad \cdots \quad \cdots \quad \cdots \quad (11)$$

The discontinuity of u across the surface is given by

$$\Delta u = \frac{\omega c \cos 2\eta}{\sin \eta}$$

and

$$\Phi = \frac{1}{4}\,\omega c^2 \sin 2\eta$$

$$\Delta \Phi = \frac{1}{2}\,\omega c^2 \sin 2\eta.$$

In the application to the airscrew problem put

$$c = R,^* \quad x = r^*,$$

$$w_1{}^* = \frac{1}{2}\,v = \frac{1}{2}\,\omega r, \quad \phi^* = \frac{\pi}{2},$$

$$\Gamma^* = \Delta\Phi = \tfrac{1}{2}\omega R^2 \sin 2\eta\,,$$

---

* These symbols refer to the airscrew.

with

$$R \cos \eta = r.$$

Then eliminating ω we get

$$\Gamma = \frac{w_1 R^2 \sin 2\eta}{r}.$$

Comparing with §2 (2)

$$w_1 = \frac{N\Gamma}{4\pi r \varkappa \sin \phi}$$

we have

$$\varkappa = \frac{1}{\pi} \tan \eta$$

with

$$R \cos \eta = r.$$

For the same case of $\phi = \pi/2$, the value of $\varkappa_P$ is

$$\varkappa_P = \frac{2}{\pi} \text{arc} \cos e^{-(1-x)}.$$

As $x \to 1$, $\varkappa \to 0$ and $\varkappa_P \to 0$, and it may be shown that $\varkappa/\varkappa_P$ tends to the finite limit $0{\cdot}5$.

A solution of the corresponding problem for four blades, *viz.*, the rotating cruciform lamina in two dimensions, has been recently obtained by Mr. F. L. Westwater, but is as yet unpublished. His results have been used by permission as a basis of Tables 1 and 2 (four blades) and Fig. 5 near $\sin \phi = 1{\cdot}0$.

APPENDIX IV

A Method of Estimating the Relative Importance of the Interference Correction by the Goldstein and Vortex Theory Formulae.—If we confine our attention to a single section at radius $0{\cdot}7R$ as representing a suitable average for the whole airscrew (*see* Ref. 6) it is possible to obtain a simple formula for the relative importance of the interference velocity. Refer to Fig. 1, but assume that all incidences and blade angles are referred to the zero lift line of the section instead of the chord; write α_0 (the uncorrected incidence) for the angle which W_0 makes with the zero lift line of the section. Then assuming w_1 to be small

$$(\alpha_0 - \alpha) = w_1/W.$$

The relative importance of the interference correction may be defined as the ratio

$$\frac{a_0 - a}{a} = \frac{w_1}{a\mathrm{W}} .$$

Now the fundamental interference formula (§3, equation 7) is

$$\frac{w_1}{\mathrm{W}} = \frac{\frac{1}{2}sk_{\mathrm{L}}}{\varkappa \sin \phi}$$

so that for a lift curve of standard slope for which

$$k_{\mathrm{L}} = a_0 a = \frac{9}{\pi} a$$

we have

$$\frac{a_0 - a}{a} = \frac{w_1}{a\mathrm{W}} = \frac{\frac{1}{2}sa_0}{\varkappa \sin \phi}$$

so that this quantity is a function of :—solidity s, ϕ and number of blades, or of :—ϕ, blade width and number of blades only.

Values of the ratio $(a_0 - a)/a$ for a blade width $c/\mathrm{R} = 0{\cdot}155$ at radius $0{\cdot}7$, equal to that of a standard blade of the family, are plotted against $\sin \phi$ for two and four bladers in Fig. 16. Values of a for two and four blades are distinguished by the suffixes 2 and 4 respectively.

Writing a_∞* for the corresponding value calculated with $\varkappa = 1$ (same solidity, infinity blades), Fig. 16 also shows values of $\frac{a_0 - a_\infty}{a_2}$ and of the difference $\frac{a_\infty - a_2}{a_2}$. A scale of approximate $\mathrm{V}/n\mathrm{D}$ is indicated along the scale of $\sin \phi$, being calculated by the formula :— $\mathrm{J} = \pi x \tan \phi$.

The maximum difference between finite and infinite number of blades occurs for large pitch values for the two blader and amounts to 19 per cent. For $\mathrm{J} = 2{\cdot}5$ it is 16 per cent. and has fallen to 5 per cent. for $\mathrm{J} = 0{\cdot}7$ and to 2 per cent. for $\mathrm{J} = 0{\cdot}5$, so that even for moderate pitch two bladers the error of the assumption of infinity blades is by no means negligible. For the four blader of the same blade width (double solidity) the difference is 10 per cent. at a P/D $2{\cdot}5$ and would have half this value for a screw of the same solidity as the two blader.

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* Actually the values of a_∞ are different for the two- and four-bladed screws since the latter are of double the solidity.

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TABLE 1

Values of κ for two-bladed airscrews

sin φ \ x	0·3	0·45	0·6	0·7	0·75	0·8	0·85	0·9	0·95
0·05	1·000	1·000	1·000	0·999	0·997	0·994	0·985	0·950	0·780
0·1	1·000	0·998	0·997	0·988	0·971	0·937	0·877	0·773	0·586
0·2	0·994	0·991	0·961	0·901	0·852	0·784	0·694	0·578	0·415
0·3	0·978	0·959	0·874	0·774	0·709	0·634	0·548	0·444	0·308
0·4	0·958	0·906	0·783	0·663	0·595	0·520	0·442	0·351	0·243
0·5	0·944	0·848	0·690	0·564	0·501	0·434	0·367	0·289	0·199
0·6	0·930	0·784	0·608	0·492	0·435	0·376	0·316	0·249	0·171
0·7	0·922	0·722	0·547	0·441	0·387	0·333	0·278	0·218	0·149
0·8	0·916	0·677	0·502	0·398	0·347	0·297	0·247	0·193	0·131
0·9	0·935	0·657	0·464	0·360	0·311	0·265	0·220	0·172	0·117
1·0	1·012	0·633	0·425	0·325	0·279	0·238	0·197	0·154	0·105

*Values of κ for three-bladed airscrews**

sin φ \ x	0·3	0·45	0·6	0·7	0·75	0·8	0·85	0·9	0·95
0·05	1·000	1·000	1·000	1·000	0·999	0·998	0·990	0·973	0·863
0·1	1·000	0·999	0·999	0·998	0·994	0·980	0·948	0·872	0·692
0·2	0·997	0·995	0·988	0·964	0·935	0·884	0·810	0·693	0·512
0·3	0·992	0·987	0·955	0·892	0·843	0·774	0·684	0·566	0·406
0·4	0·984	0·966	0·902	0·809	0·746	0·670	0·581	0·471	0·331
0·5	0·975	0·940	0·836	0·725	0·658	0·581	0·496	0·396	0·275
0·6	0·973	0·909	0·771	0·650	0·582	0·508	0·429	0·341	0·236
0·7	0·983	0·880	0·714	0·586	0·520	0·450	0·377	0·299	0·203
0·8	1·010	0·849	0·661	0·533	0·470	0·404	0·337	0·265	0·182
0·9	1·061	0·831	0·622	0·494	0·429	0·367	0·304	0·239	0·164
1·0	1·185	0·832	0·588	0·457	0·393	0·335	0·278	0·218	0·149

* Interpolated.

TABLE 1—*continued*

Values of $\varkappa$ for four-bladed airscrews

x / sin ϕ	0·3	0·45	0·6	0·7	0·75	0·8	0·85	0·9	0·95
0·05	1·000	1·000	1·000	1·000	1·000	0·999	0·998	0·995	0·945
0·1	1·000	1·000	1·000	1·000	0·999	0·993	0·985	0·943	0·770
0·2	0·998	0·997	0·995	0·989	0·973	0·940	0·882	0·777	0·590
0·3	0·996	0·994	0·984	0·945	0·909	0·852	0·774	0·651	0·476
0·4	0·991	0·984	0·950	0·883	0·830	0·759	0·671	0·554	0·396
0·5	0·985	0·971	0·905	0·812	0·750	0·674	0·585	0·476	0·334
0·6	0·986	0·954	0·855	0·745	0·678	0·601	0·517	0·414	0·290
0·7	0·999	0·939	0·803	0·682	0·615	0·541	0·459	0·369	0·255
0·8	1·035	0·924	0·756	0·627	0·562	0·490	0·413	0·329	0·228
0·9	1·103	0·917	0·713	0·581	0·516	0·447	0·375	0·298	0·205
1·0	1·256	0·923	0·681	0·543	0·477	0·412	0·345	0·272	0·187

TABLE 2

$\varkappa/\varkappa_P$: two-bladed airscrews

x / sin ϕ	0·3	0·45	0·6	0·7	0·75	0·8	0·85	0·9	0·95	1·0
0·05	1·000	1·000	1·000	1·000	0·999	0·998	0·996	0·994	0·992	0·990
0·1	1·000	0·998	0·998	0·996	0·994	0·990	0·985	0·983	0·981	0·976
0·2	0·994	0·992	0·984	0·972	0·967	0·959	0·949	0·940	0·933	0·923
0·3	0·978	0·971	0·944	0·921	0·908	0·894	0·881	0·865	0·844	0·827
0·4	0·961	0·941	0·902	0·864	0·845	0·822	0·803	0·781	0·763	0·746
0·5	0·954	0·913	0·852	0·801	0·780	0·757	0·739	0·717	0·699	0·681
0·6	0·955	0·885	0·806	0·758	0·737	0·715	0·697	0·674	0·655	0·635
0·7	0·975	0·865	0·780	0·736	0·710	0·685	0·663	0·639	0·618	0·598
0·8	1·015	0·871	0·777	0·718	0·688	0·659	0·633	0·607	0·584	0·564
0·9	1·128	0·931	0·784	0·704	0·666	0·634	0·605	0·577	0·552	0·531
1·0	1·512	1·040	0·798	0·692	0·646	0·612	0·580	0·550	0·524	0·500

TABLE 2—*continued*

$\varkappa/\varkappa_P$; *three-bladed airscrews**

$\sin\phi$ \ x	0·3	0·45	0·6	0·7	0·75	0·8	0·85	0·9	0·95	1·0
0·05	1·000	1·000	1·000	1·000	1·000	0·999	0·998	0·997	0·997	0·995
0·1	1·000	0·999	0·999	0·999	0·998	0·995	0·993	0·992	0·989	0·987
0·2	0·997	0·995	0·993	0·990	0·988	0·982	0·979	0·972	0·968	0·960
0·3	0·992	0·989	0·980	0·968	0·964	0·955	0·944	0·931	0·921	0·910
0·4	0·984	0·975	0·959	0·938	0·926	0·910	0·895	0·876	0·860	0·844
0·5	0·976	0·963	0·931	0·900	0·883	0·861	0·840	0·815	0·794	0·774
0·6	0·978	0·955	0·906	0·865	0·841	0·816	0·791	0·765	0·745	0·724
0·7	0·999	0·960	0·891	0·835	0·808	0·779	0·750	0·725	0·702	0·680
0·8	1·051	0·975	0·882	0·815	0·784	0·750	0·719	0·689	0·665	0·645
0·9	1·163	1·030	0·897	0·814	0·770	0·730	0·694	0·663	0·637	0·610
1·0	1·535	1·170	0·935	0·817	0·760	0·714	0·675	0·642	0·611	0·583

* Interpolated.

$\varkappa/\varkappa_P$; *four-bladed airscrews*

$\sin\phi$ \ x	0·3	0·45	0·6	0·7	0·75	0·8	0·85	0·9	0·95	1·0
0·05	1·000	1·000	1·000	1·000	1·000	1·000	0·999	0·998	0·998	0·997
0·1	1·000	1·000	1·000	1·000	1·000	0·997	0·997	0·995	0·993	0·992
0·2	0·998	0·997	0·996	0·995	0·995	0·991	0·988	0·983	0·980	0·975
0·3	0·996	0·994	0·992	0·984	0·980	0·973	0·964	0·956	0·948	0·937
0·4	0·991	0·986	0·977	0·962	0·952	0·940	0·928	0·912	0·899	0·878
0·5	0·985	0·979	0·958	0·933	0·918	0·901	0·883	0·863	0·845	0·819
0·6	0·987	0·975	0·940	0·905	0·886	0·864	0·842	0·820	0·799	0·777
0·7	1·004	0·980	0·926	0·880	0·858	0·833	0·807	0·783	0·760	0·737
0·8	1·049	0·999	0·921	0·862	0·836	0·806	0·777	0·748	0·724	0·701
0·9	1·155	1·050	0·929	0·855	0·821	0·786	0·752	0·720	0·693	0·667
1·0	1·493	1·176	0·965	0·862	0·816	0·773	0·735	0·699	0·666	0·637

TABLE 3

Table for $x = 0{\cdot}7$

Values of $\varkappa$

N / $\sin\phi$	2	3	4	6	8
0·1	0·988	0·998	1·000	1·000	1·000
0·2	0·901	0·964	0·989	0·997	0·998
0·3	0·774	0·892	0·945	0·986	0·994
0·4	0·663	0·809	0·883	0·951	0·975
0·5	0·564	0·725	0·812	0·904	0·947
0·6	0·492	0·650	0·745	0·853	0·909
0·7	0·441	0·586	0·682	0·800	0·873
0·8	0·398	0·533	0·627	0·753	0·829
0·9	0·360	0·494	0·581	0·708	0·790
1·0	0·325	0·457	0·543	0·670	0·754

Values of $\varkappa/\varkappa_P$

N / $\sin\phi$	2	3	4	6	8
0·1	0·996	0·997	1·000	1·000	1·000
0·2	0·972	0·990	0·995	0·998	0·998
0·3	0·921	0·968	0·984	0·993	0·996
0·4	0·864	0·938	0·963	0·980	0·986
0·5	0·801	0·900	0·933	0·960	0·972
0·6	0·758	0·865	0·905	0·940	0·955
0·7	0·736	0·835	0·880	0·921	0·945
0·8	0·718	0·815	0·862	0·909	0·932
0·9	0·704	0·814	0·855	0·905	0·929
1·0	0·692	0·817	0·862	0·914	0·937

TABLE 4

Details of Strip Theory Calculation for a two-bladed airscrew of P/D 1·5
Section EE, $x = 0{\cdot}75$, $\theta^\circ = 32^\circ\ 30'$, $s = 0{\cdot}0613$

a°	—6	—4	—2	0	4	8	12	14
ϕ	38° 30′	36° 30′	34° 30′	32° 30′	28° 30′	24° 30′	20° 30′	18° 30′
$\sin\phi$	0·6225	0·5948	0·5664	0·5373	0·4772	0·4147	0·3502	0·3173
k_L	—0·094	0·021	0·137	0·228	0·430	0·615	0·656	0·638
$\varkappa$	0·422	0·438	0·458	0·480	0·527	0·582	0·650	0·688
$\frac{1}{2}sxk_L/\varkappa \sin\phi\cos^2\phi$	—0·0134	0·0027	0·0179	0·0286	0·0509	0·0707	0·0756	0·0747
$1 + (\frac{1}{2}sk_L/\varkappa\cos\phi)$	0·991	1·002	1·011	1·017	1·028	1·036	1·033	1·030
w_c	—0·0135	0·0027	0·0177	0·0281	0·0495	0·0684	0·0731	0·0725
$x\tan\phi$	0·5966	0·5550	0·5155	0·4778	0·4072	0·3418	0·2804	0·2509
Λ	0·6101	0·5523	0·4978	0·4497	0·3577	0·2734	0·2073	0·1784
$x\sec\phi$	0·9585	0·9330	0·9097	0·8895	0·8535	0·8242	0·8010	0·7905
$w_c\sin\phi$	—0·0084	0·0016	0·0100	0·0151	0·0236	0·0284	0·0256	0·0230
W_c	0·9669	0·9314	0·8997	0·8744	0·8300	0·7958	0·7754	0·7675
Tc'	—0·0042	0·0009	0·0056	0·0090	0·0160	0·0217	0·0226	0·0218
$P_{c_1}' = w_cT_c'$	0·00006	0·00000	0·00010	0·00025	0·00079	0·00149	0·00165	0·00158
k_D	0·0346	0·0192	0·0099	0·0069	0·0061	0·0077	0·0287	0·0502
P_{c_2}'	0·00192	0·00095	0·00044	0·00028	0·00021	0·00024	0·00082	0·00139

TABLE 5

Values of T_{c_1}', P_{c_1}', P_{c_2}', *obtained from curves of Figs.* 7, 8 *and* 9 *for two-bladed airscrew of* P/D 1·5.

	Λ	T_{c_1}'	P_{c_1}'	P_{c_2}'
BB	0·2	—	—	—
$x = 0·3$	0·25	0·0049	0·00010	0·00121
$x^2 = 0·09$	0·3	0·0039	0·00004	0·00118
	0·35	0·0062	0·00017	0·00080
	0·4	0·0083	0·00036	0·00037
	0·45	0·0087	0·00041	0·00025
	0·5	0·0067	0·00021	0·00033
	0·55	0·0035	0·00006	0·00047
	0·6	0·0007	0	0·00077
CC	0·2	0·0187	0·00122	0·00054
$x = 0·45$	0·25	0·0208	0·00135	0·00037
$x^2 = 0·2025$	0·3	0·0191	0·00117	0·00026
	0·35	0·0163	0·00086	0·00022
	0·4	0·0134	0·00058	0·00025
	0·45	0·0102	0·00035	0·00032
	0·5	0·0070	0·00016	0·00051
	0·55	0·0035	0·00004	0·00095
	0·6	0·0003	0	0·00175
DD	0·2	0·0252	0·00191	0·00061
$x = 0·6$	0·25	0·0242	0·00171	0·00035
$x^2 = 0·36$	0·3	0·0214	0·00135	0·00022
	0·35	0·0181	0·00096	0·00022
	0·4	0·0146	0·00061	0·00024
	0·45	0·0108	0·00034	0·00030
	0·5	0·0066	0·00013	0·00046
	0·55	0·0021	0·00001	0·00099
	0·6	−0·0037	0·00004	0·00195

TABLE 5—*continued*

Values of T_{c_1}', P_{c_1}', P_{c_2}', *obtained from curves of Figs. 7, 8 and 9 for two-bladed airscrew* of P/D 1·5.

	A	T_{c_1}'	P_{c_1}'	P_{c_2}'
	0·2	0·0224	0·00164	0·00093
	0·25	0·0225	0·00159	0·00035
EE	0·3	0·0199	0·00126	0·00020
	0·35	0·0165	0·00085	0·00021
$x = 0·75$	0·4	0·0128	0·00050	0·00024
	0·45	0·0090	0·00025	0·00028
$x^2 = 0·5625$	0·5	0·0055	0·00009	0·00045
	0·55	0·0011	0	0·00092
	0·6	—0·0034	0·00002	0·00172
	0·2	0·0170	0·00127	0·00081
	0·25	0·0169	0·00126	0·00035
FF	0·3	0·0146	0·00098	0·00022
	0·35	0·0117	0·00061	0·00019
$x = 0·9$	0·4	0·0088	0·00035	0·00021
	0·45	0·0059	0·00015	0·00026
$x^2 = 0·81$	0·5	0·0029	0·00003	0·00036
	0·55	—0·0009	0	0·00079
	0·6	—0·0050	0·00012	0·00132
	0·2	0·0120	0·00119	0·00072
	0·25	0·0111	0·00106	0·00030
FFa	0·3	0·0099	0·00072	0·00018
	0·35	0·0082	0·00047	0·00015
$x = 0·962$	0·4	0·0059	0·00026	0·00015
	0·45	0·0037	0·00010	0·00019
$x^2 = 0·925$	0·5	0·0016	0·00001	0·00028
	0·55	—0·0012	0·00002	0·00056
	0·6	—0·0046	0·00014	0·00098

TABLE 6

Values of T_c, P_{c1}, P_{c2}, *obtained by graphical integration of curves in Figs.* 10, 11, 12; *also deduced values of* J, k_T, k_Q, η, *for two-bladed airscrew of* P/D 1·5.

Λ	J	T_c	Λ T_c	P_{c1}	P_{c2}	Q_c	k_T	k_Q	η
0·2	0·6282	0·0175	0·00350	0·00132	0·00071	0·00553	0·135	0·0215	63·3
0·25	0·785	0·0175	0·00438	0·00128	0·00037	0·00604	0·136	0·0234	72·6
0·3	0·942	0·0155	0·00464	0·00098	0·00025	0·00578	0·120	0·0224	80·2
0·35	1·098	0·0129	0·00453	0·00067	0·00021	0·00541	0·100	0·0210	83·8
0·4	1·257	0·0103	0·00410	0·00042	0·00021	0·00473	0·079	0·0184	86·7
0·45	1·414	0·0075	0·00337	0·00023	0·00024	0·00385	0·058	0·0149	87·6
0·5	1·571	0·0045	0·00226	0·00008	0·00037	0·00272	0·035	0·0105	83·2
0·55	1·728	0·0010	0·00056	0·00001	0·00076	0·00133	0·008	0·0052	42·1
0·6	1·885	−0·0027	0·00165	0·00004	0·00140	−0·00021	−0·021	−0·0008	

(27815)

TABLE 7

Values of $\varkappa \cos^2 \phi$ actually calculated by Goldstein's formulae ((Ref. 3) and Appendix II). Two blades.

$(\cot \phi)/x = 1{\cdot}5$		$(\cot \phi)/x = 2{\cdot}0$		$(\cot \phi)/x = 2{\cdot}5$		$(\cot \phi)/x = 3{\cdot}0$		$(\cot \phi)/x = 4{\cdot}0$		$(\cot \phi)/x = 5{\cdot}0$	
$\cot \phi$	$\varkappa \cos^2 \phi$	$\cot \phi$	$\varkappa \cos^2 \phi$	$\cot \phi$	$\varkappa \cos^2 \phi$	$\cot \phi$	$\varkappa \cos^2 \phi$	$\cot \phi$	$\varkappa \cos^2 \phi$	$\cot \phi$	$\varkappa \cos^2 \phi$
1·4	0·129	1·9	0·153	2·4	0·177	2·8	0·280	3·8	0·319	4·8	0·400
1·3	0·182	1·8	0·222	2·2	0·312	2·5	0·424				
1·2	0·210	1·7	0·267	2·0	0·382						
1·0	0·235	1·6	0·297	1·8	0·423						
0·8	0·226	1·4	0·332	1·6	0·441						
0·6	0·196	1·2	0·341	1·4	0·442						
0·4	0·144	1·0	0·329	1·2	0·426						
0·2	0·076	0·8	0·294	1·0	0·394						
		0·6	0·243	0·8	0·344						
		0·4	0·175	0·6	0·280						
		0·2	0·091	0·4	0·198						
				0·2	0·103						

C

TABLE 7—*continued*

Values of $\varkappa \cos^2 \phi$ *actually calculated by Goldstein's formulae* ((*Ref.* 3) *and Appendix* II). *Four blades.*

$(\cot \phi)/x = 1 \cdot 4$		$(\cot \phi)/x = 2 \cdot 0$		$(\cot \phi)/x = 3 \cdot 0$		$(\cot \phi)/x = 4 \cdot 0$		$(\cot \phi)/x = 5 \cdot 0$	
$\cot \phi$	$\varkappa \cos^2 \phi$	$\cot \phi$	$\varkappa \cos^2 \phi$	$\cot \phi$	$\varkappa \cos^2 \phi$	$\cot \phi$	$\varkappa \cos^2 \phi$	$\cot \phi$	$\varkappa \cos^2 \phi$
1·3	0·213	1·8	0·370	2·8	0·445	3·8	0·488	4·8	0·503
1·2	0·282	1·6	0·469	2·5	0·627	3·5	0·689	4·5	0·714
1·1	0·317	1·4	0·501	2·0	0·706	3·0	0·806	4·0	0·850
1·0	0·329	1·2	0·492	1·8	0·698	2·8	0·821	3·8	0·872
0·8	0·310	1·0	0·449	1·6	0·672	2·5	0·821	3·5	0·887
0·6	0·249	0·8	0·377	1·4	0·630	2·0	0·776	3·0	0·881
0·4	0·159	0·6	0·283	1·2	0·572	1·8	0·744	2·8	0·871
0·2	0·061	0·4	0·173	1·0	0·497	1·6	0·702	2·5	0·848
		0·2	0·064	0·8	0·405	1·4	0·649	2·0	0·786
				0·6	0·297	1·0	0·505	1·6	0·706
				0·4	0·179	0·6	0·299	1·0	0·506
				0·2	0·066			0·6	0·300

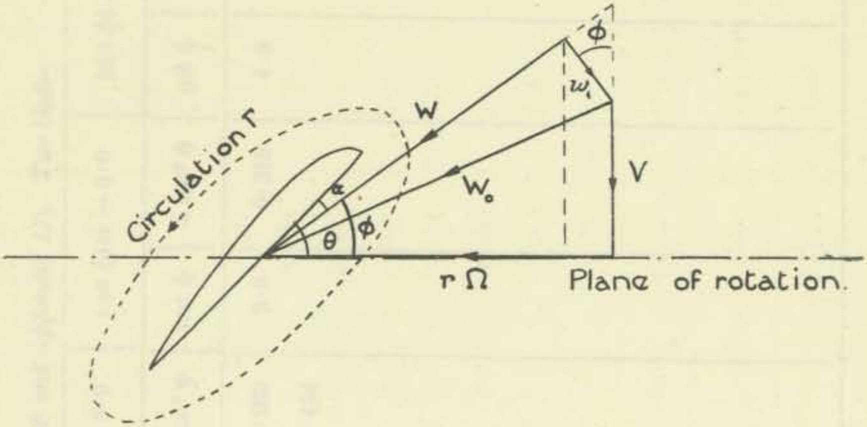


FIG. 1

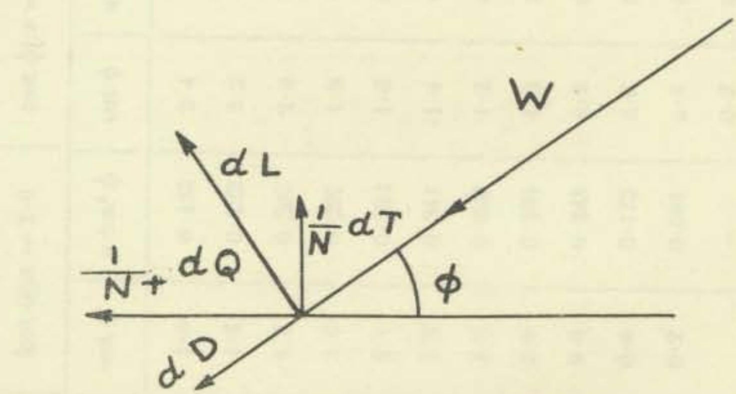
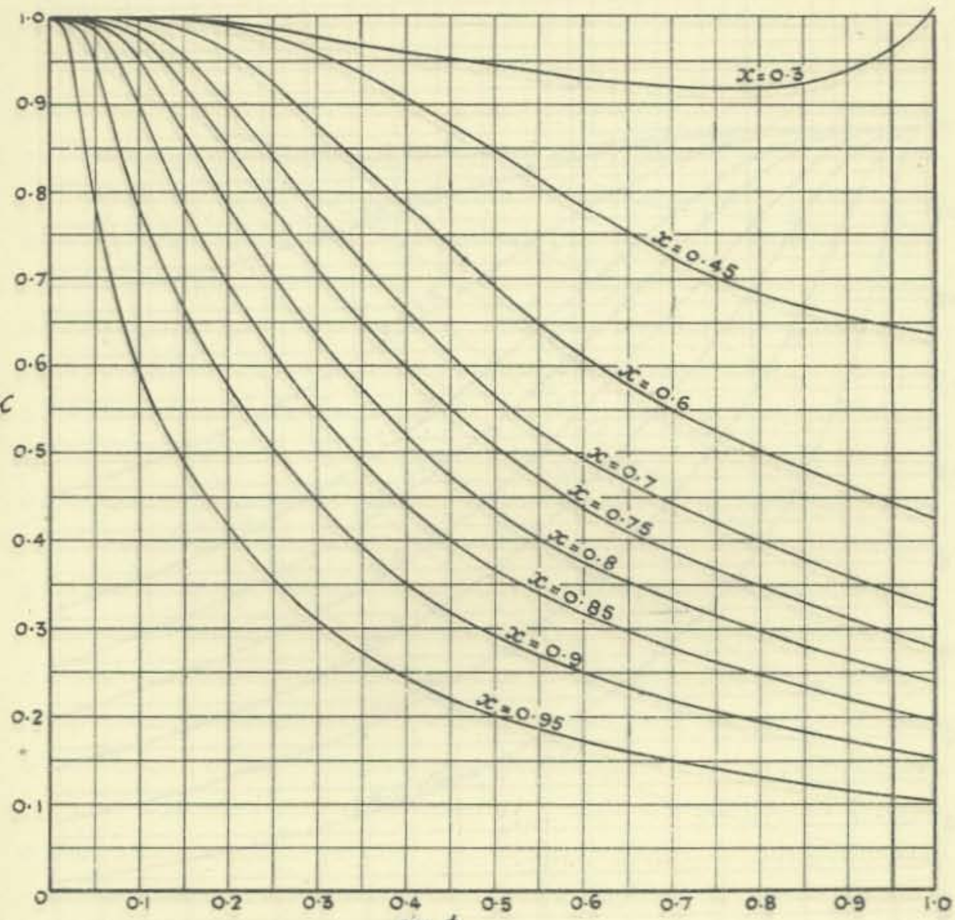


FIG. 2

S.O. Code. No. 23-9999

(27815) Wt. 531/972 500 12/35 Hw G. 377/1

Curves of K against $\sin\phi$ for 2 bladed airscrews.



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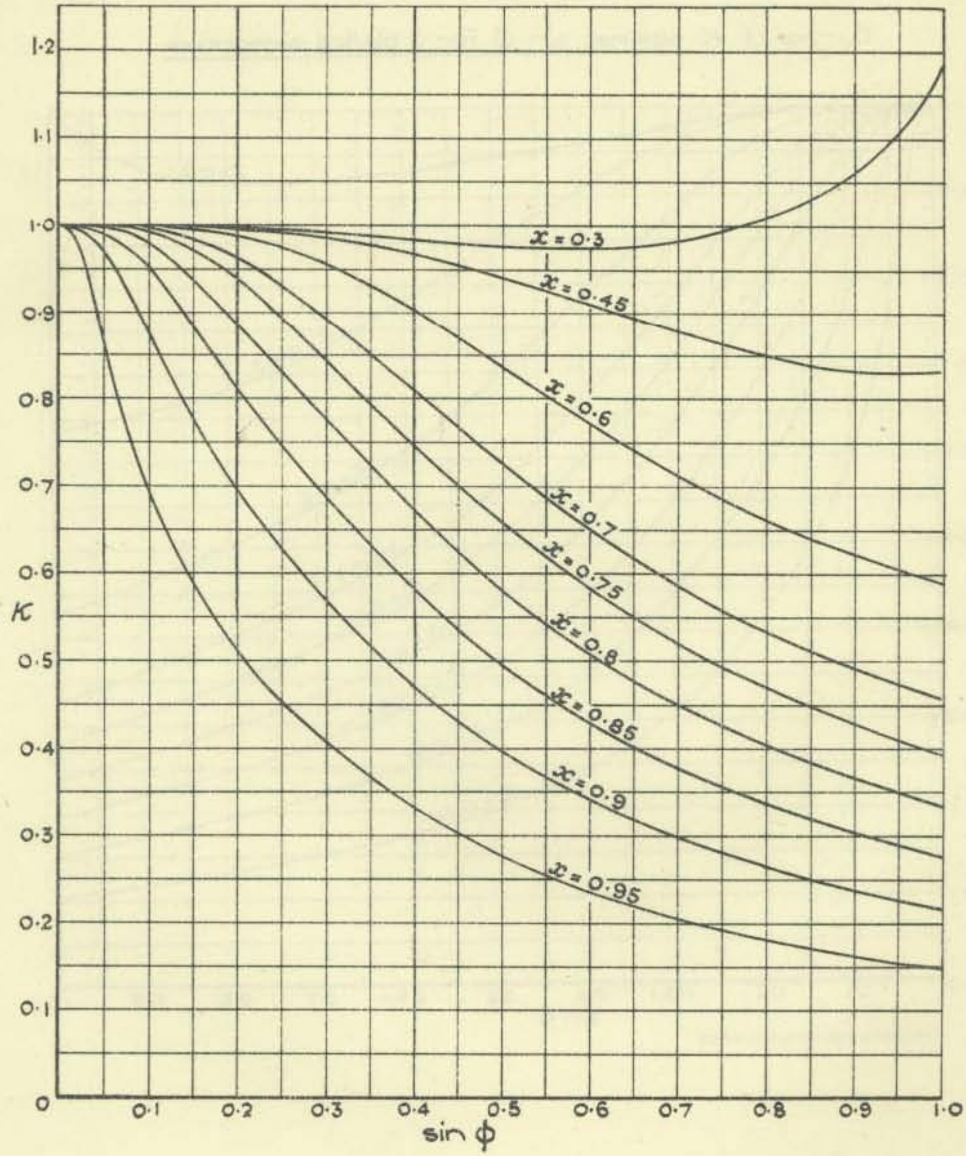
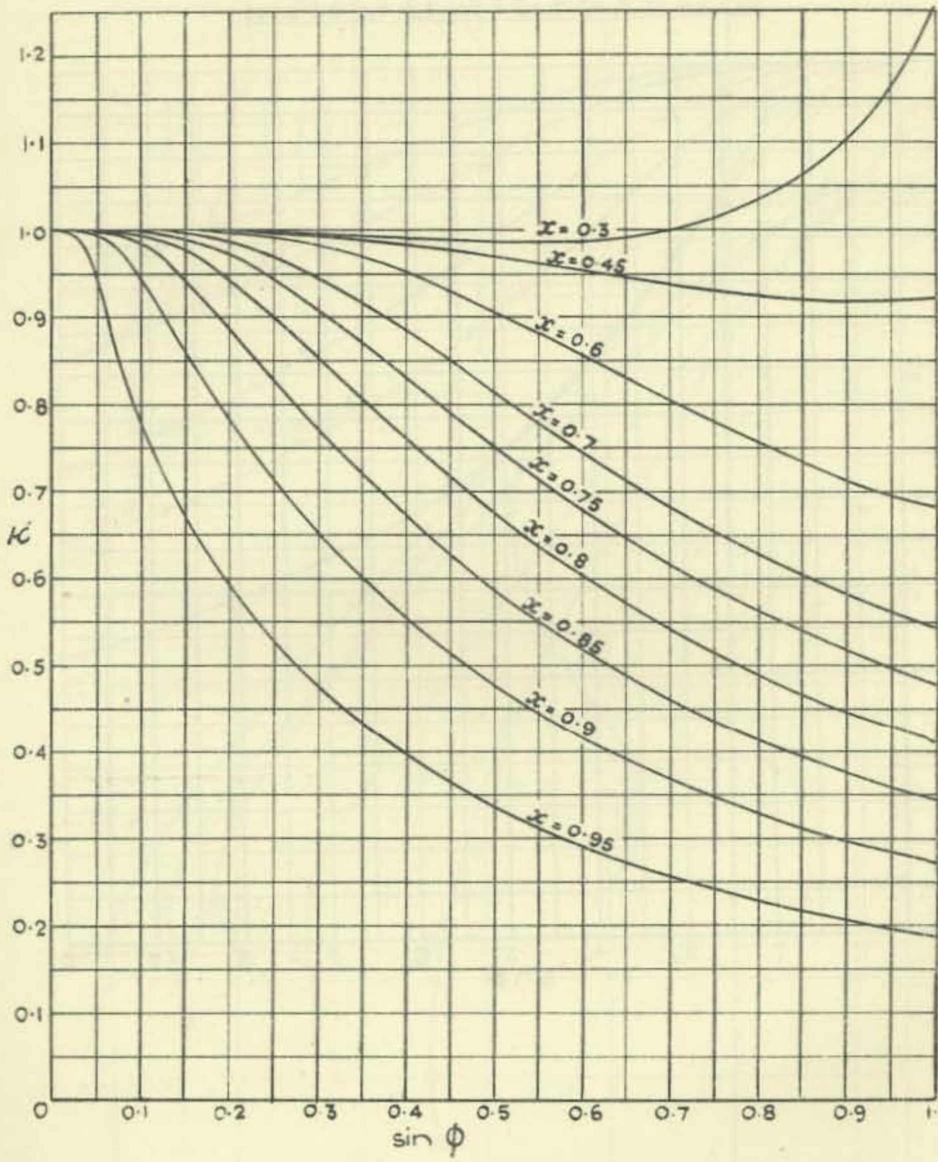
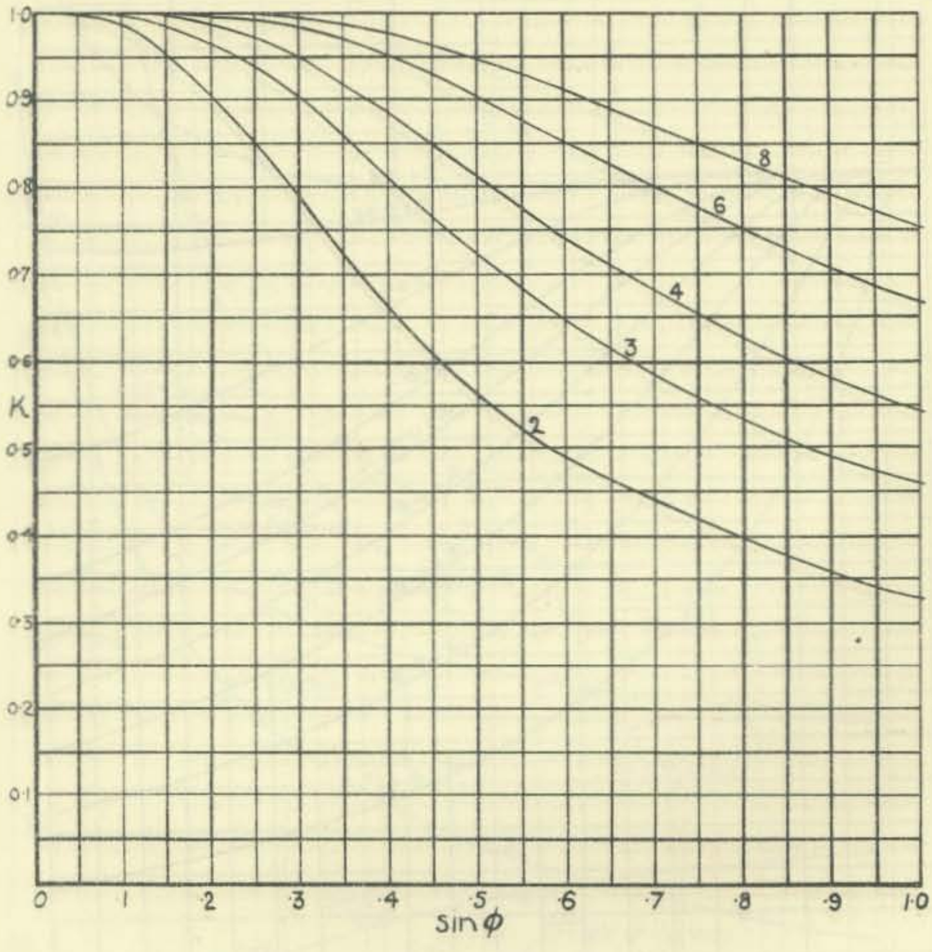
Curves of K against $\sin\phi$ for 3 bladed airscrews. (Interpolated).



Curves of K against sin ϕ for 4 bladed airscrews.



Values of K at x = 0·7 for 2, 3, 4, 6, 8 blades.



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Thrust grading coefficient T_c' plotted against Λ.

FIG. 7.

Two bladed airscrew P/D 1·5

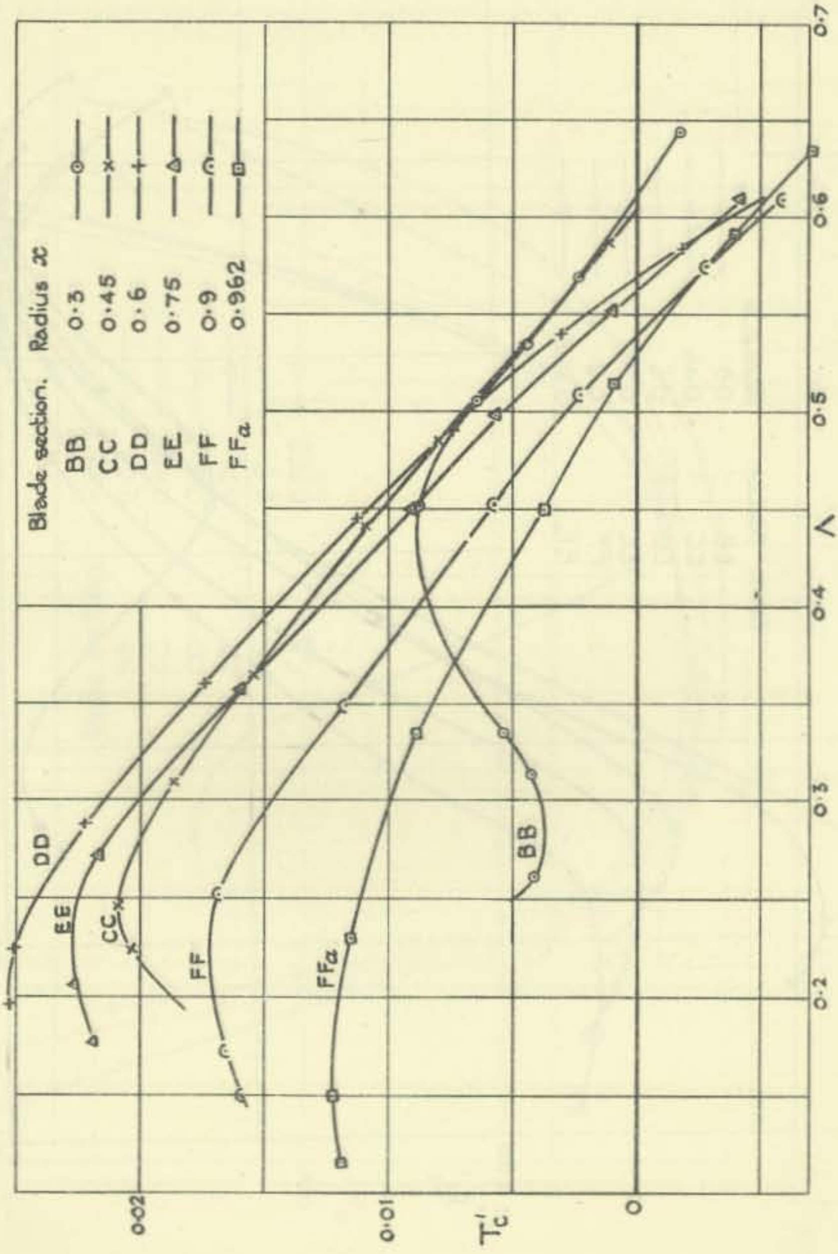

Fig. 8.

'Induced' power loss grading coefficient P_{c_1}' plotted against Λ

Two bladed airscrew P/D 1·5.

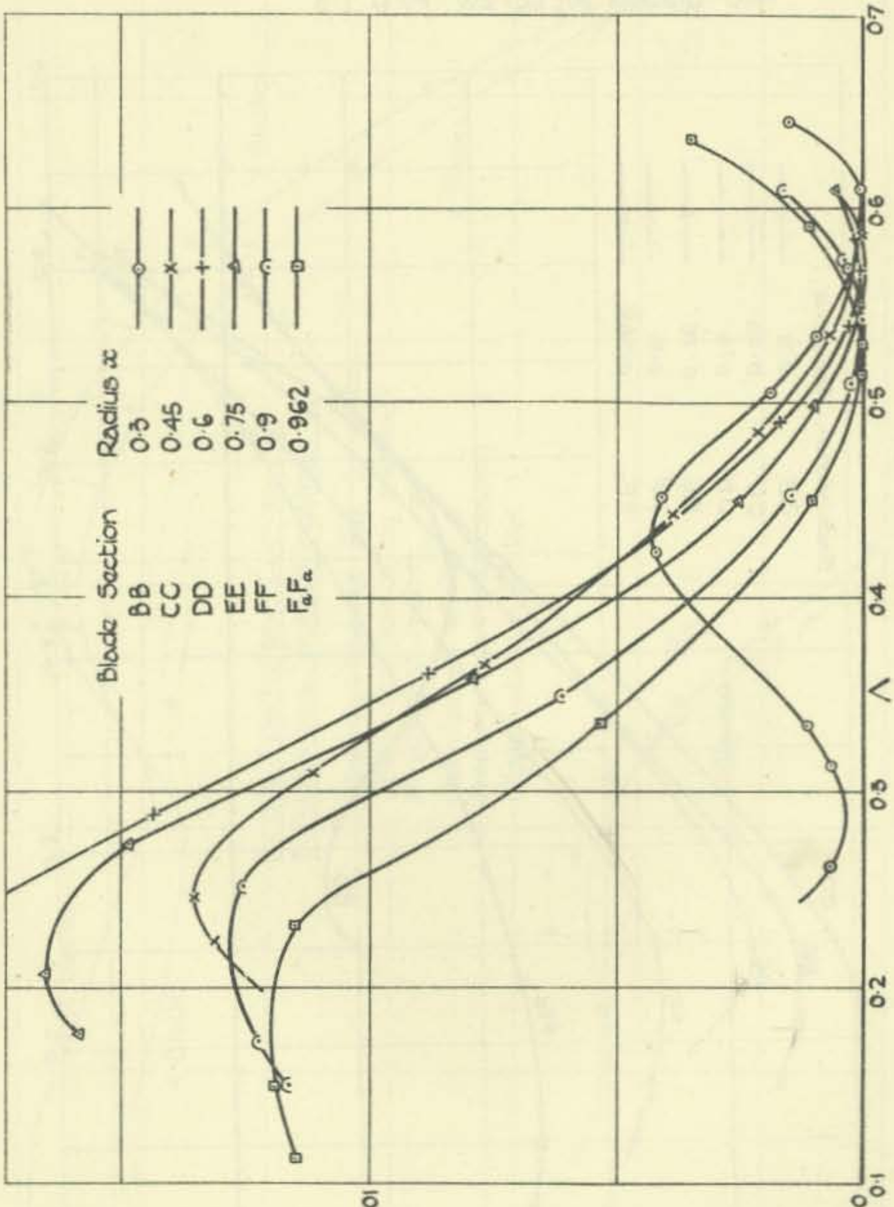

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FIG. 9.

Profile drag power loss grading coefficient P_{c_2}' plotted against Λ

Two bladed airscrew P/D 1·5.

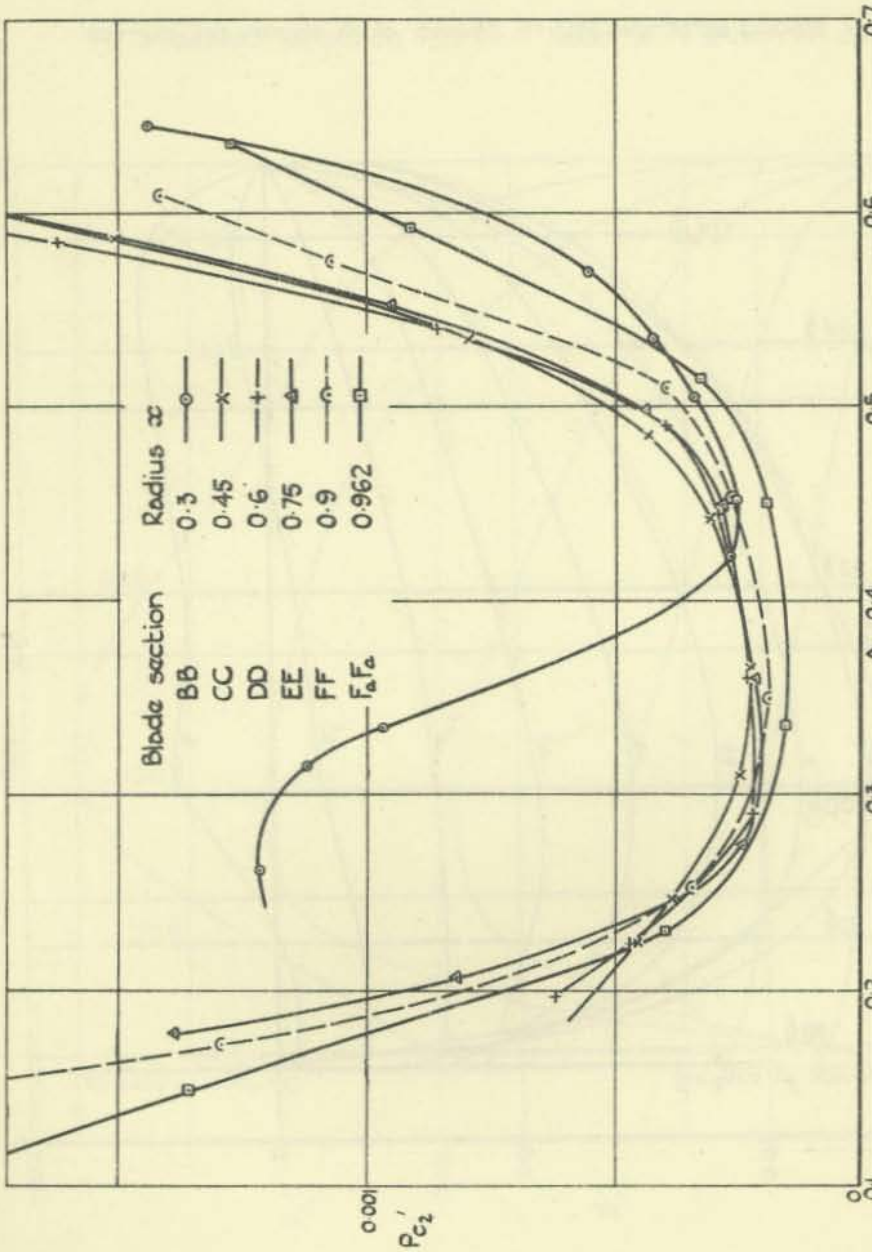

FIG. 10.

Thrust grading coefficient T_c' plotted against x^2 (radius squared).

2 Bladed airscrew P/D 1·5. Values of Λ shown on curves.

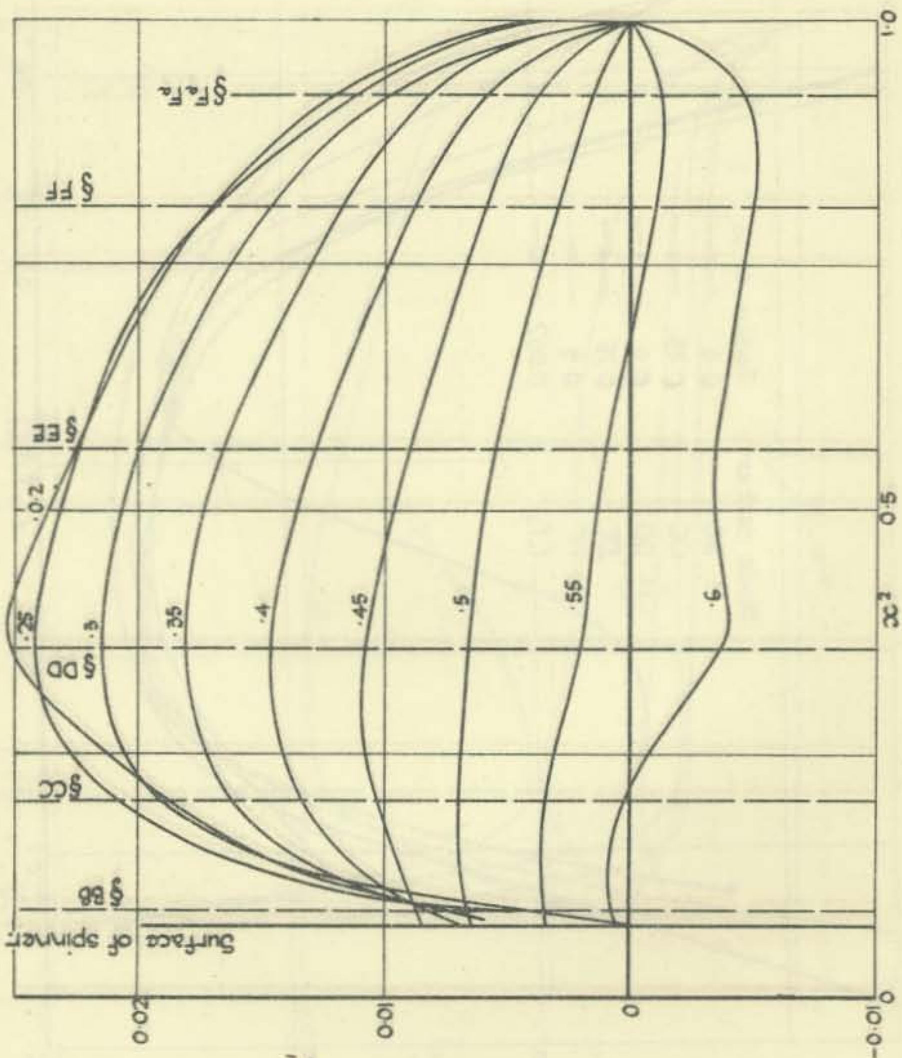

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FIG. 11.

Induced power loss grading coefficient P_{c_1}' plotted against x^2 (radius squared).

2 bladed airscrew P/D 1·5. Values of Λ shown on curves

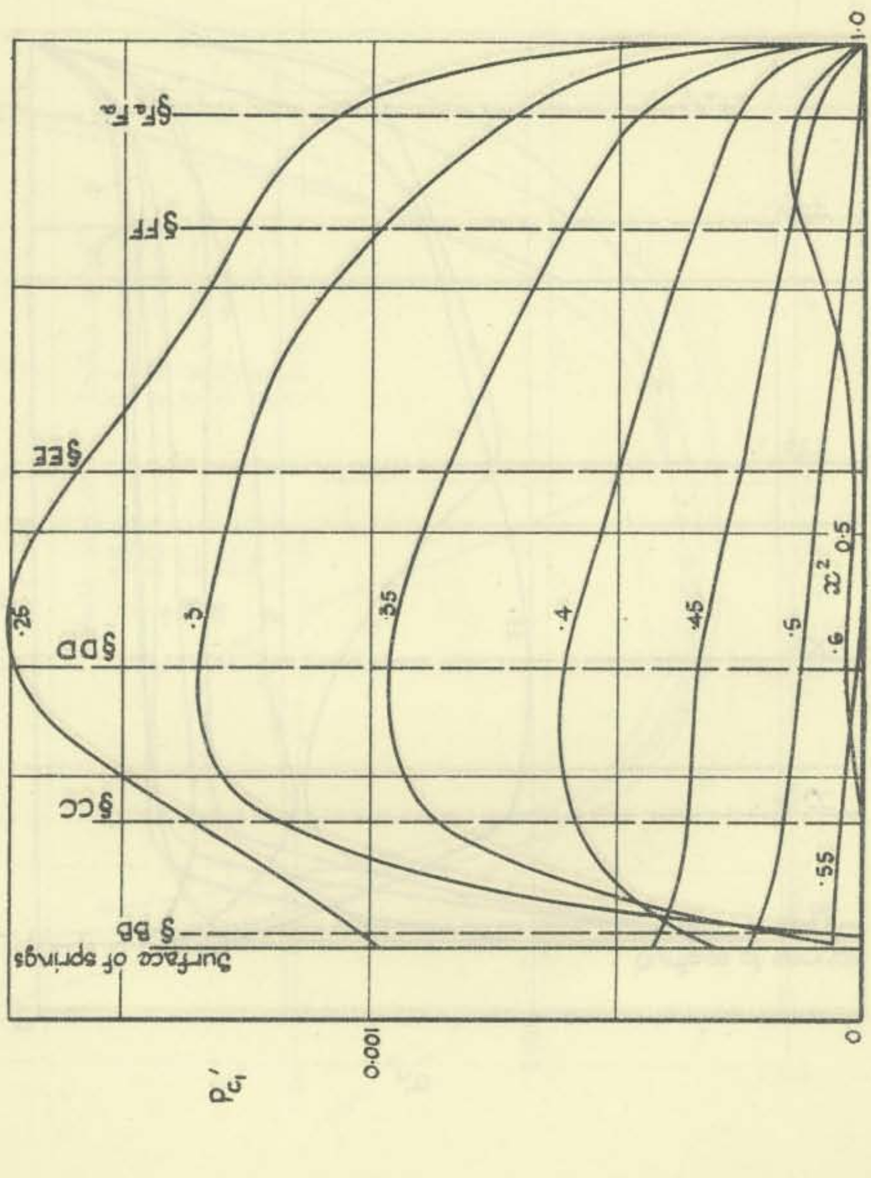

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FIG. 12.

Profile drag power loss grading coefficient P_{c_2}' plotted against x^2 (radius squared).

2 Bladed airscrew P/D 1·5. Values of Λ shown on curves.

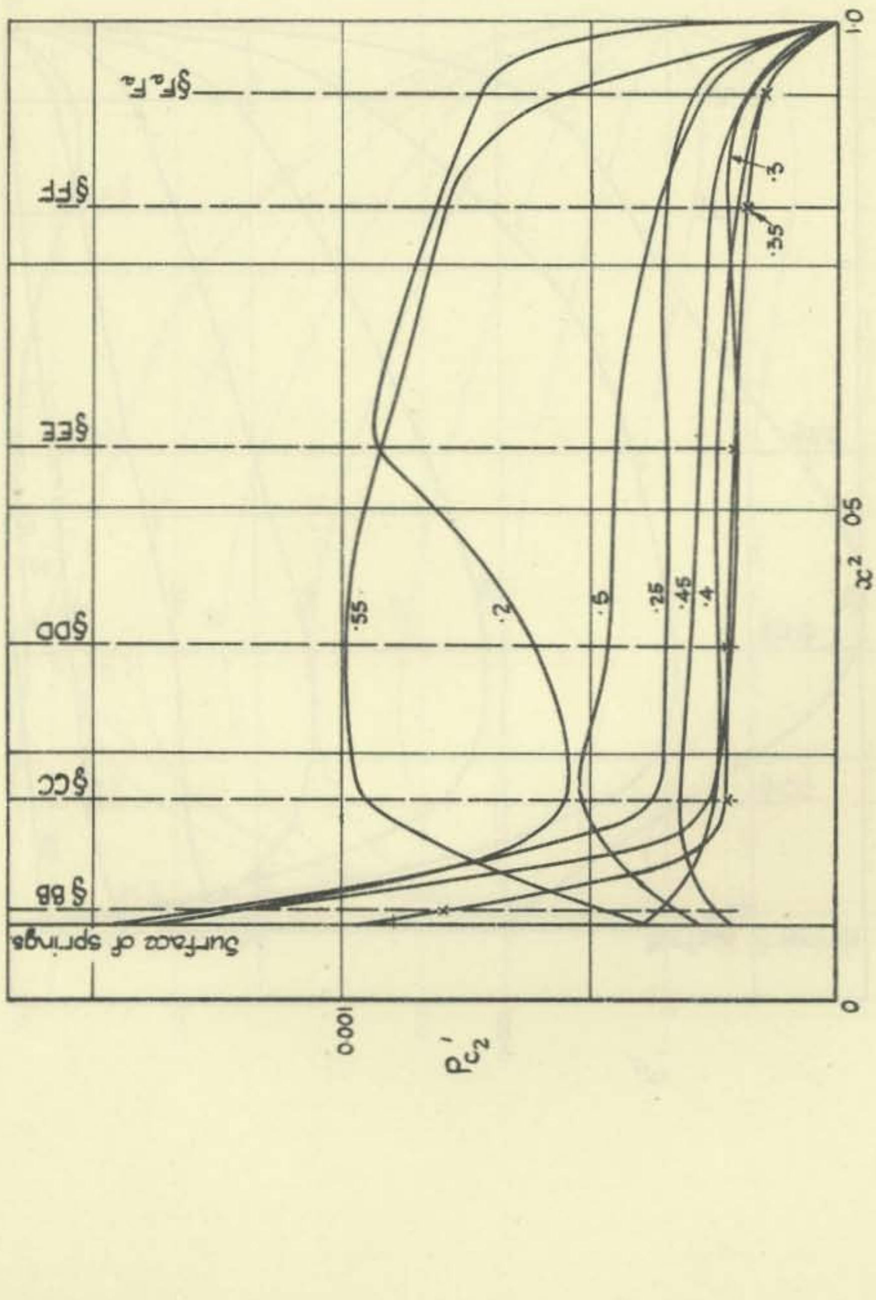
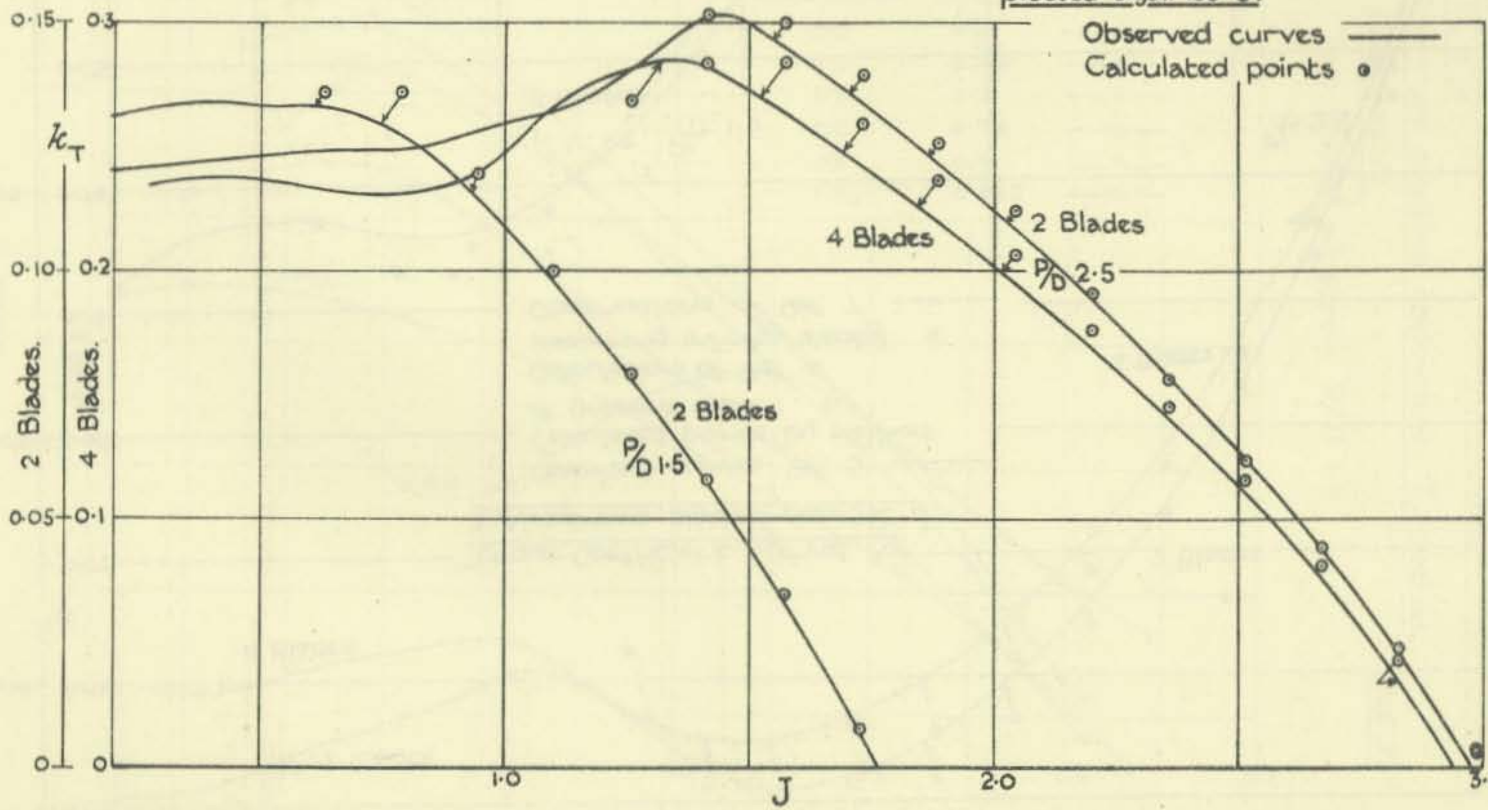
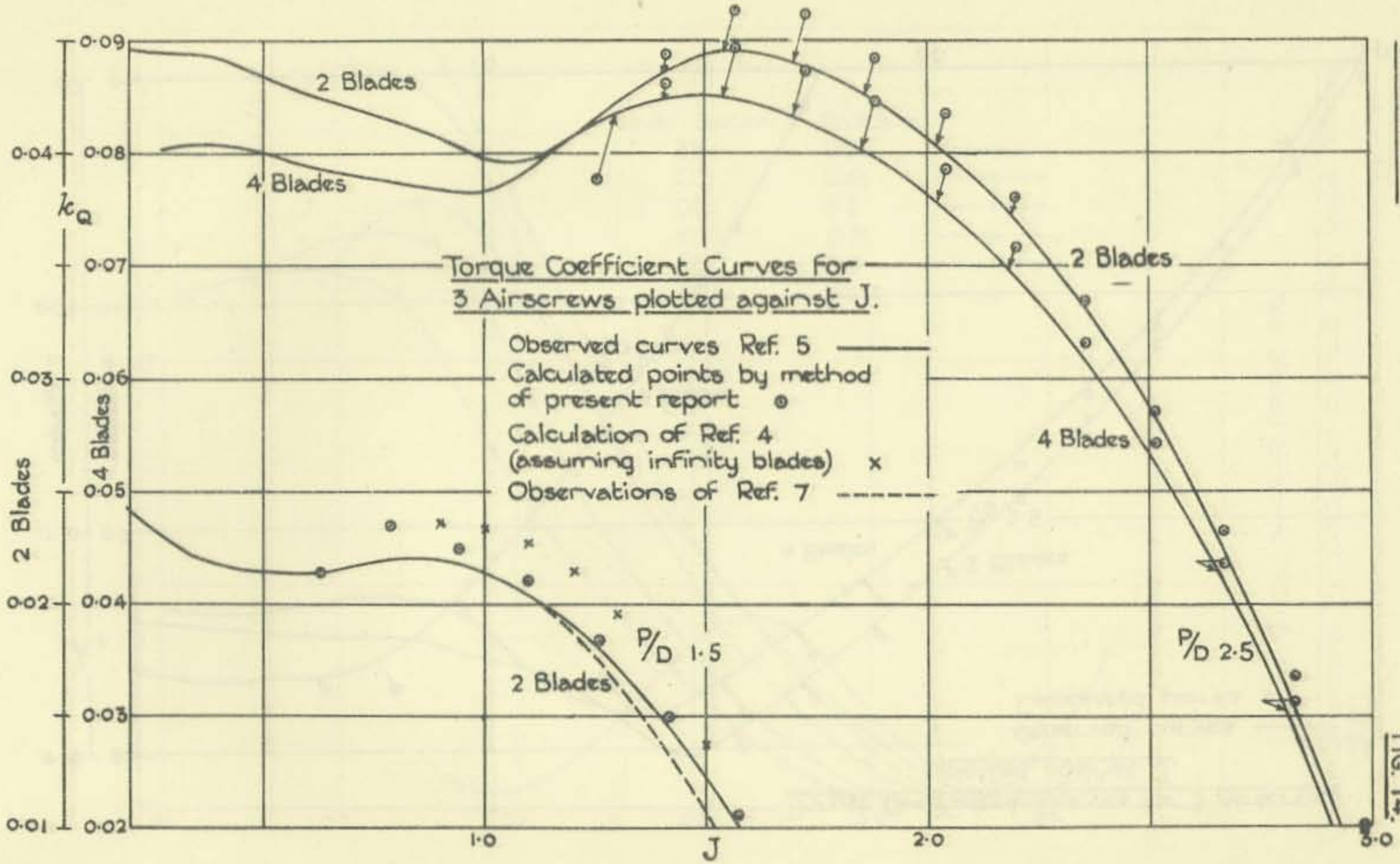


FIG. 13.

Thrust Coefficient Curves for 3 Airscrews plotted against J.





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Torque Coefficient Curves for 3 Airscrews plotted against J.
Observed curves Ref. 5
Calculated points by method of present report
Calculation of Ref. 4 (assuming infinity blades) ×
Observations of Ref. 7
2 Blades
4 Blades
k_Q
P/D 1·5
P/D 2·5
0·09
0·08
0·07
0·06
0·05
0·04
0·03
0·02
0·04
0·03
0·02
0·01
1·0
2·0
3·0
J

FIG. 14.

FIG. 15.

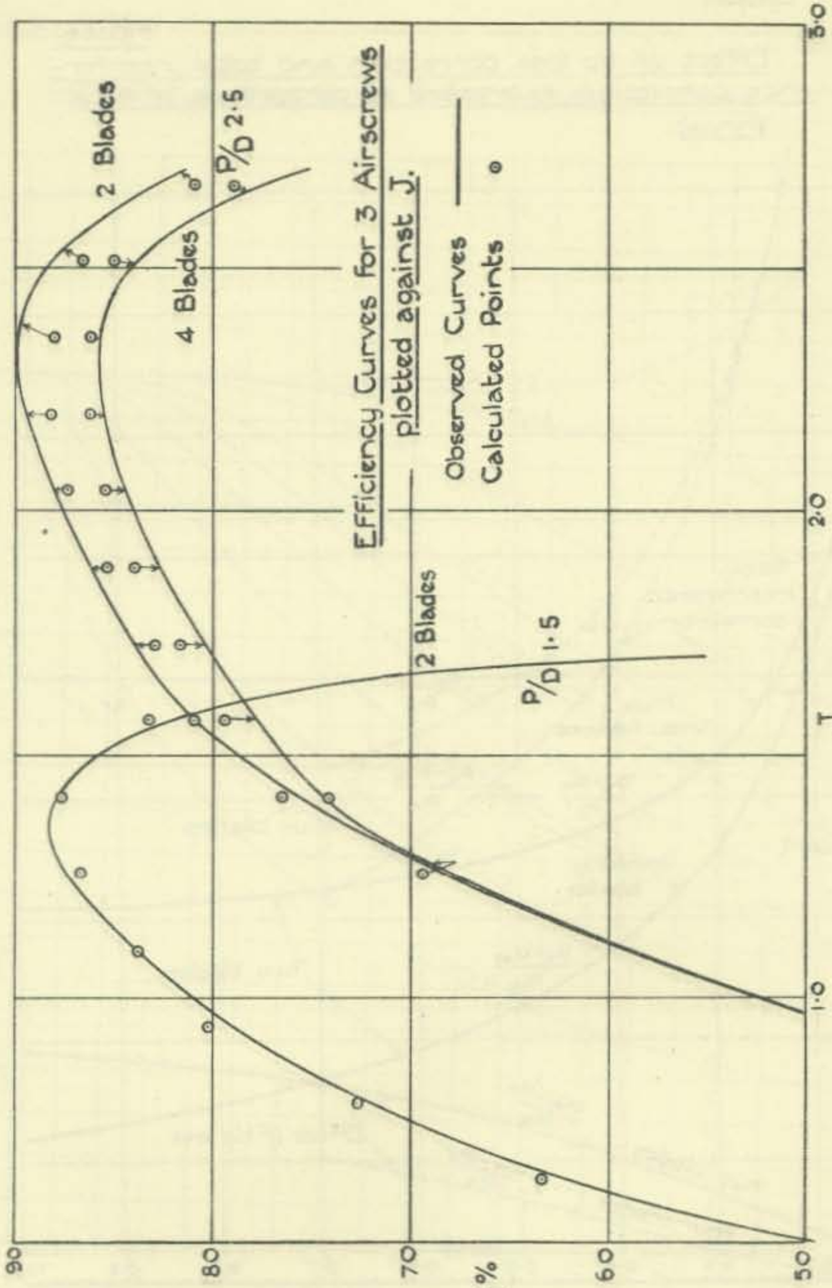
Efficiency Curves for 3 Airscrews
plotted against J.
Observed Curves
Calculated Points
2 Blades
4 Blades
P/D 2·5
2 Blades
P/D 1·5
η %
J
90
80
70
60
50
1·0
2·0
3·0

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FIG. 16.

Effect of tip loss correction and total interference correction expressed as percentage of total thrust.