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Effect of Discs on the Air Forces on a Rotating Cylinder

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AERODYNAMICS SYMBOLS.

1. General

m	mass
t	time
V	resultant linear velocity
Ω	resultant angular velocity
ρ	density, σ relative density
ν	kinematic coefficient of viscosity
R	Reynolds number, $R = lV/\nu$ (where l is a suitable linear dimension), to be expressed as a numerical coefficient x 10^6

Normal temperature and pressure for aeronautical work are 15° C. and 760 mm.

For air under these conditions $\begin{cases} \rho = 0{\cdot}002378 \text{ slug/cu. ft.} \\ \nu = 1{\cdot}59 \times 10^{-4} \text{ sq. ft./sec.} \end{cases}$

The slug is taken to be 32·2 lb.-mass.

α	angle of incidence
ϵ	angle of downwash
S	area
c	chord
s	semi-span
A	aspect ratio, $A = 4s^2/S$
L	lift, with coefficient $k_L = L/S\rho V^2$
D	drag, with coefficient $k_D = D/S\rho V^2$
γ	gliding angle, $\tan \gamma = D/L$
L	rolling moment, with coefficient $k_l = L/sS\rho V^2$
M	pitching moment, with coefficient $k_m = M/cS\rho V^2$
N	yawing moment, with coefficient $k_n = N/sS\rho V^2$

2. Airscrews

n	revolutions per second
D	diameter
J	V/nD
P	power
T	thrust, with coefficient
Q	torque, with coefficient
η	efficiency, $\eta = TV/P =$

ON THE EFFECT OF DISCS ON THE AIR FORCES ON A ROTATING CYLINDER

By ALEXANDER THOM, D.Sc., Ph.D.

Communicated by Professor J. D. Cormack

Reports and Memoranda No. 1623

5th January, 1934

The Potential Flow streamlines past a circular cylinder are as shown in Fig. 1a. The velocity close to the surface is 2 V sin θ where V is the undisturbed velocity and θ is measured from the front generator. If a circulation is superimposed the streamlines become as in Fig. 1b. If the circulation is of strength Γ then the circulation velocity at the surface is $v = \Gamma/2\pi a$, where a is the radius of the cylinder. The stagnation points are evidently given by

$$2V \sin \theta + v = 0 \text{ or } \sin \theta = -v/2V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

As the circulation is increased the stagnation points move together so that when $\theta = -90°$ they have become one as in Fig. 1c. Expression (1) shows that this occurs when $v/V = 2$. The corresponding lift coefficient is then $k_L = 2\pi$, since the relation between lift and circulation is :—

$$\text{Lift} = k_L \varrho l 2a V^2 = \varrho l V \Gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thereafter if the circulation is further increased the stagnation point leaves the surface and the flow pattern becomes as in Fig. 1d.

When we deal with an actual fluid passing a stationary cylinder the streamlines cannot close behind the cylinder because the retardation of the layer passing close to the surface has reduced its kinetic energy so that it cannot force its way against the rising pressure which exists in the theoretical case between $\theta = 90°$ and $\theta = 180°$. In the case shown in Fig. 1b, if the circulation is produced by rotating the cylinder this retardation does not exist, with the result that the dead water region is much reduced. As a result the profile drag of a rotating cylinder is lower than that of a non-rotating cylinder.

Prandtl's theory of the production of circulation about a rotating cylinder requires that the eddies produced at the boundary on the under side should be carried away down stream leaving a general circulation of opposite sign throughout the field. In Fig. 1d the cylinder is surrounded by an annular space from which the air is not extracted, so that this condition can never be produced. The theory is borne out by experiment to this extent, that the maximum lift coefficient actually obtained on a plain rotating cylinder approaches but does not reach 2π, corresponding to Fig. 1c.

It occurred to the writer that if it were possible to produce a flow like that shown at Fig. 1d the drag would be very small as the air near the cylinder does not leave the enclosed region.

The method tried was to fit discs along the length of the cylinder rotating with it. These were found not only to produce (or to permit) a very high k_L but also a very low (and in some cases negative) drag.

In R. & M. No. 1018 the writer has shown that at the higher rotational speeds, in wind channel experiments with the cylinder stretching from wall to wall, the induced drag preponderates. With the discs all along the cylinder the induced drag is naturally small and since values of k_L up to 18 were obtained the condition in Fig. 1d had been reached, so that the profile drag should also be small. This would explain the reduction of k_D to almost zero but does not explain its having become negative. In this connection it should be noted that the experiments were carried out in a closed type channel where there is certainly no room for the full flow pattern to develop. It does not necessarily follow that negative drag would occur in free flow.

1925 *Experiments.*—The first experiments were made in the 2 ft. wind channel in the James Watt Engineering Laboratories, University of Glasgow, during the summer of 1925. The balance used for measuring the forces was that described in R. & M. No. 1018. With this type of balance it is impossible to determine the torque (except for zero air speed) and so some uncertainty remains in the lift determination, as both lift and torque contribute to the balance moment.

The arrangements of discs experimented on are shown in Figs. 2 and 6. The rotors were built up from thin circular aluminium discs mounted on a steel spindle between short lengths of cylinder. Two sizes of discs were used, 3 in. and 6 in. diameter, and two sizes of cylinder 1 in. and 2 in. diameter. The results are given in Table 1, and are shown plotted in Figs. 2, 3, 4, 5, 6 and 7. It will be seen that at the lower values of v/V, k_L and k_D are almost identical with those obtained for the plain cylinder (c.f. R. & M. No. 1018 or Fig. 11 of this report). At higher values of v/V two interesting results are obtained. The lift coefficient becomes very large and the drag becomes small and then rises suddenly.

The irregularity in both Figs. 2 and 3 about $v/V = 3\frac{1}{2}$ is possibly connected with the fact that there was a short length of cylinder outside the end discs which probably behaved as a plain cylinder.

Experiments were also made with end discs only. The results are shown in Figs. 6 and 7. It will be noticed that the drag for the discs all along begins to differ from that for the end discs only, when k_L for the plane cylinder stops rising, that is when the cylinder begins to be surrounded by air not passing off. It is also interesting to note that with discs having a diameter three times that of the cylinder the rise in drag occurs when the diameter of this hollow cylinder of air becomes approximately equal to the disc diameter. For the theoretical field the relation between k_L and the distance y of the stagnation point below the centre of the cylinder is :—

$$k_L = \pi \left(\frac{y}{a} + \frac{a}{y}\right)$$

Putting $y/a = 3$ in the above expression gives $k_L = 10{\cdot}5$ as the value of the lift coefficient when the stagnation point is just at the outer edge of the discs, and when k_L is $10{\cdot}5$ (Fig. 3) the drag begins to rise rapidly. The 3 in. diameter discs on a 2 in. cylinder do not seem to delay the rise in drag.

Later Experiments.—In 1931 a new balance was constructed capable of measuring lift, drag and torque. This balance has already been described in R. & M. 1520. It differs from the previous balance in being more rigid and in having four sets of knife edges arranged in side elevation on the corners of a square as in Fig. 9. Set A is arranged to be almost on the spindle centre line, and is used for measuring torque in conjunction with B or C. For drag D is naturally the best set, and for lift B, but C can be used for both lift and drag as in the old balance.

A preliminary experiment was tried on knife edges C using a plain wooden cylinder. The results do not differ from those previously obtained and so are not given. The 6 in. discs were then mounted on the 2 in. cylinder, as shown in Fig. 8. The arrangement differs from that used in 1925 in that there is a disc at the extreme end instead of a short piece of cylinder. By mounting the balance, first on the A edges and then on the C edges, and measuring the moment with the cylinder rotating in each direction in each case, it is possible to find lift, drag, and torque. This was done for several wind speeds with rotational speeds up to 4,300 r/m. The lift and torque results are given in Table 2 in the form of coefficients defined as follows :—

$$L = \text{Lift} = k_L \rho l d V^2.$$
$$Q = \text{Torque} = k_Q \rho l n d^3 V$$

where d is the diameter of the cylinder, l is its length, and n is revs/sec.

Some of the drag results were very low and some negative. As however the points were rather scattered it was decided to remeasure the drag using the D edges so as to minimise the effects of the large lift forces on the balance moment. Only the latter drag results are included in Table 2.

Various difficulties arose in making these measurements. First, there were vibration troubles, but careful balancing finally enabled speeds up to 5,300 r./m. to be obtained. These high speeds are necessary to get the higher values of v/V. A slow progressive change in the balance zero was traced to the driving belt causing a slight deflection of the motor supports. As the belt stretched the motor settled back, thus changing the balance zero.

To exclude the possibility of the peculiar drag results being due to some such cause, or due to the effect of vibration on the knife edges the following procedure was adopted :—

The cylinder was brought to the required speed with the air speed zero, the balance weight adjusted and the reading taken. Then the air was put on and the weight again adjusted and read. The air was then shut off and if the previous reading was not obtained the process was repeated. This procedure was then gone through with the cylinder rotating at the same speed in the opposite direction.

Now let the mean error in the position of the D edges be a as in Fig. 9. Also let s be the balance reading and W the balance weight. Then we get :—

Positive rotation, wind on	$D\beta - Q_1 - La$	$= Ws_1.$
,, ,, off	$- Q_2$	$= Ws_2.$
Negative rotation, wind on	$D\beta + Q_1 + La$	$= Ws_3.$
,, ,, off	$+ Q_2$	$= Ws_4.$

From these equations we find :—

$$D\beta = \tfrac{1}{2}W\,(s_1 - s_2 + s_3 - s_4)$$

but $s_1 - s_2$ and $s_3 - s_4$ are merely the shifts of the balance weights necessary in the above procedure when the wind is put on. It will thus be seen that the above method removes the possibility of any vitiation of the results by the causes mentioned. It should be stated that in taking the two measurements of drag at $v/V = 7{\cdot}5$ and $8{\cdot}6$ a steady reading could not be obtained. This is possibly due to the very steep nature of the drag curve (Fig. 10) in this neighbourhood.

A by-product of these drag measurements was a determination of the torque on the discs and cylinder in still air. (The torque in an air stream was obtained during the lift measurements.) Plotting log Q on log n gave a straight line of slope $1{\cdot}60$ without any trace of curvature, so that all the still air results are given by

$$Q = 1{\cdot}20\, n^{1{\cdot}6}$$

By dimensional reasoning we obtain from this the form

$$Q_1 = k\, l\, \varrho^{0{\cdot}6}\, \mu^{0{\cdot}4}\, d^{3{\cdot}2}\, n^{1{\cdot}6}$$

where Q_1 is in *dyne cm.* or in *poundal-ft.*, ϱ is gr./cm³ or lbs./ft.³, n is revs./sec. and k is a nondimensional coefficient equal to 930. This gives the torque on any geometrically similar arrangement.

The results of the lift and drag measurements are collected in Fig. 11, along with the results for the smooth and sanded cylinder having no discs from earlier experiments (R. & M. No. 1018). The curves shown are to be taken as representative and not as showing any individual experiment.

Several of the features have already been discussed. In the writer's opinion one of the significant properties is the coincidence of all the lift curves from $v/V \doteqdot \frac{1}{2}$ to $v/V \doteqdot 3$. Sanding the surface or adding discs seems to have no effect on k_L in this region. At higher values of v/V the end discs alone produce very high k_L values, and the intermediate discs have little effect in increasing these, whereas discs all along delay the rise in k_D to a much higher v/V than the end discs alone. A full theory of the Magnus effect must be capable of explaining these points.

In this connection it is worthy of note that the curve of the velocity distribution along the normal to the surface of a cylinder rotating in still air, as given by the writer in R. & M. No. 1410, shows that the velocity drops very rapidly at first and then more slowly. If the hyperbolic curve is drawn showing the velocity distribution in a Rankine vortex having a strength rather less than one-third of what would be present if there were no "slip," the outer part of the experimental curve is fairly well represented. If there were no "slip" the theoretical lift in an air stream would be :—

$$k_L = 2\,\pi\,v/V$$

and the experimental curves in Fig. 11 are throughout the greater part of their length just under one-third of this. If any significance is to be attached to this, it is that Prandtl's eddy shedding theory is not necessary since the required circulation is present, even at zero air speeds. In any case his theory does not explain the fact that the lift coefficient rises far above $2\,\pi$.

In view of the low drag it might be thought at first sight that there was a possibility of some such arrangement being used to form an additional support for a modified type of aeroplane. The high lift coefficient would certainly permit of the machine being very small, but the enormous power which is apparently necessary to give the very high rotational speeds makes the idea impracticable. Thus, if we consider a cylinder one foot in diameter and 12 ft. long fitted with discs three feet diameter, and take

$$v/V = 6,\ k_L = 10,\ k_D = -0\cdot65,\ k_q = 4\cdot1,$$
$$V = 100 \text{ ft./sec.},\ v = 600 \text{ ft./sec. or } n = 191 \text{ revs./sec.}$$

we find

$$\text{Lift} = 2{,}850 \text{ lbs.}$$
$$\text{Drag} = -185 \text{ lbs.}$$
$$\text{Torque} = 4\cdot1\ \varrho\ n\ V\ l\ d^3 = 2{,}220 \text{ ft. lb.}$$
$$\text{Torque horse power} = 4{,}830.$$

The peripheral speed of the edges of the discs would be 1,800 ft./sec. As this is above the velocity of sound it is perhaps unjustifiable to use the results of the present experiments, but it seems likely that in any case the torque horse power would be high.

Using the expression for the torque in still air already given

$$Q_1 = k\, l\, \varrho^{0\cdot 6}\, \mu^{0\cdot 4}\, d^{3\cdot 2}\, n^{1\cdot 6}$$

we obtain $Q_1 = 115{,}000$ poundal feet or 3,600 ft.-lb., indicating that the values as calculated above are of the right order.

TABLE 1

Cylinder 26 in. × 1 in. dia. With 3 in. dia. discs spaced ¾ in. apart all along. Air speed = 17·9 ft./sec. R = 9,400.

n	v/V	k_L	k_D	L/D
0	0	0	0·62	0
1000	0·24	0·11	0·61	0·18
2000	0·37	0·20	0·60	0·34
2500	0·61	0·34	0·59	0·58
3000	0·73	0·41	0·58	0·70
3500	0·85	0·49	0·57	0·86
4000	0·98	0·61	0·56	1·09
4500	1·10	0·73	0·53	1·38
5000	1·22	0·86	0·53	1·64

Cylinder 26 in. × 1 in. dia. With 3 in. dia. discs spaced 1½ in. apart all along. Air speed = 10 ft./sec. R = 5,000.

n r/m	v/V	k_L	k_D	L/D
0	0	0	0·58	0
1000	0·44	0·30	0·55	0·55
2000	0·88	0·67	0·55	1·22
3000	1·30	1·17	0·42	2·8
4000	1·72	1·82	0·36	5·0
5000	2·14	2·60	0·32	8·1
6000	2·51	3·35	0·25	13·5
7000	2·93	4·32	0·34	12·7
8000	3·35	5·16	0·35	14·6

Cylinder 26 in. × 2 in. dia. With 3 in. dia. discs spaced ¾ in. apart all along. Air speed = 11·7 ft./sec. R = 12,200.

n r/m	v/V	k_L	k_D	L/D
0	0	0	0·54	0
500	0·37	0·11	0·56	0·20
1000	0·74	0·35	0·49	0·71
1500	1·12	0·70	0·37	2·9
2000	1·49	1·22	0·38	3·1
2500	1·86	1·93	0·48	4·1
3000	2·23	2·70	0·60	5·9
3500	2·60	3·52	0·67	5·2
4000	2·98	4·3	0·88	4·8

TABLE 1 (*Contd.*)

Cylinder 25 in. × 2 in. dia. With 6 in. dia. discs spaced $1\frac{1}{2}$ in. apart all along. Air speed = 5·35 ft./sec. R = 5,600.

n r/m	v/V	k_L	k_D	L/D
0	0	0	0·70	0
1000	1·63	1·77	0·39	4·6
1500	2·45	3·56	0·40	8·9
2000	3·26	4·91	0·53	9·3
2500	4·1	5·79	0·40	14·4
3000	4·9	6·9	0·30	23·1
3500	5·5	8·5	0·21	40·6
4000	6·5	11·1	0·56	19·7
4200	6·8	12·0	0·83	14·4

Cylinder 25 in. × 2 in. dia. With 6 in. dia. discs spaced $1\frac{1}{2}$ in. apart all along. Air speed = 8·5 ft./sec. R = 8,800.

n r/m	v/V	k_L	k_D	L/D
0	0	0	0·57	0
1000	1·03	0·62	0·45	1·38
1500	1·54	1·41	0·38	3·7
2000	2·06	2·4	0·31	7·7
2500	2·57	3·4	0·35	9·3
3000	3·08	4·4	0·35	12·5
3500	3·59	5·1	0·35	14·4
4000	4·11	5·7	0·25	22·8
4130	4·31	6·0	0·25	24·0

Cylinders 25 in. × 2 in. dia. With 6 in. dia. discs spaced $1\frac{1}{2}$ in. apart all along. Air speed = 12·2 ft./sec. R = 12,500.

n r/m	v/V	k_L	k_D	L/D
0	0	0	0·69	0
1000	0·71	0·33	0·67	0·61
2000	1·43	1·07	0·61	2·15
3000	2·14	2·50	0·48	6·3
3500	2·50	3·35	0·57	7·1
4000	2·86	3·95	0·53	9·1

TABLE 1 (*Contd.*)

Cylinder 26 in. × 1 in. dia. With one 3 in. dia. disc at each end. Air speed = 10·4 ft./sec. R = 5,300.

n r/m	v/V	k_L	k_D	L/D
0	0	0	0·59	0
1000	0·42	0·15	0·65	0·23
2000	0·84	0·51	0·64	0·80
3000	1·26	0·98	0·54	1·83
4000	1·68	1·63	0·43	3·8
5000	2·10	2·53	0·35	7·4

Cylinder 25 in. × 2 in. dia. With one 6 in. dia. disc at each end. Air speed = 5·5 ft./sec. R = 5,800.

n r/m	v/V	k_L	k_D	L/D
0	0	0	0·60	0
1000	1·59	1·08	0·35	3·1
1500	2·38	2·8	0·23	11·1
2000	3·2	4·8	0·43	11·0
2500	4·0	6·5	0·80	8·1
3000	4·8	7·5	0·83	9·1
3500	5·6	8·5	0·87	9·8
4000	6·4	9·2	1·10	8·4
4300	6·8	9·7	0·95	10·2

Cylinder 25 in. × 2 in. dia. With one 6 in. dia. disc at each end. Air speed = 8·4 ft./sec. R = 8,800.

n r/m	v/V	k_L	k_D	L/D
0	0	0	0·59	0
1000	1·02	0·72	0·43	1·67
1500	1·54	1·49	0·36	4·1
2000	2·05	2·55	0·25	10·1
2500	2·56	3·69	0·32	11·7
3000	3·1	4·7	0·46	10·1
3500	3·6	5·5	0·46	12·1
4000	4·1	6·2	0·61	10·2

TABLE 2

Lift, Drag and Torque for a Cylinder with Discs.
Disc dia./Cylinder dia. = 3.

Air speed, m/sec.	Revs. per sec.	v/V	k_L	k_D	k_q	R $=Vd/\nu$
1·34	38·6	4·6	6·8	—0·52	4·5	4,500
1·34	53·0	6·4	—	—0·90	—	
1·34	57·9	6·9	—	—1·02	—	
1·34	62·6	7·5	15·3	0±	5·2	
1·34	72·3	8·6	18·5	+2·6±	5·2	
1·90	0	0	0	+0·57	—	6,300
1·90	19·3	1·6	—	+0·44	—	
1·90	38·6	3·2	—	+0·15	—	
1·90	43·4	3·6	5·5	—	3·1	
1·90	53·0	4·5	—	—0·28	—	
1·90	55·4	4·6	7·0	—	3·5	
1·90	60·2	5·1	—	—0·38	—	
1·90	62·6	5·2	8·2	—	3·8	
1·90	72·3	6·1	9·9	—0·68	4·1	
1·90	87·7	7·3	—	—1·06	—	
2·69	0	0	0	+0·58	—	8,960
2·69	29·0	1·7	—	+0·39	—	
2·69	38·6	2·3	—	+0·29	—	
2·69	43·4	2·6	3·9	—	2·4	
2·69	53·0	3·1	—	+0·10	—	
2·69	53·3	3·3	5·3	—	2·8	
2·69	62·6	3·7	5·8	+0·06	2·9	
2·69	72·3	4·3	6·9	+0·02	2·6	
2·69	87·4	5·2	—	—0·30	—	
2·32	72·3	5·0	—	—0·22	—	7,740
6·00	0	0	0	+0·66	—	20,000
6·65	43·4	1·0	0·55	—	1·2	22,150
6·65	53·0	1·3	0·95	—	1·2	
6·65	62·6	1·5	1·20	—	1·3	
3·78	0	0	0	+0·63	—	12,600
10·0	0	0	0	0·70	—	33,400
14·2	0	0	0	0·70	—	47,300

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Figs. 1a, b, c, d.

Fig. 1a.

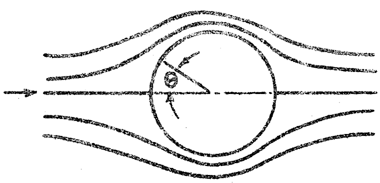


Fig. 1b.

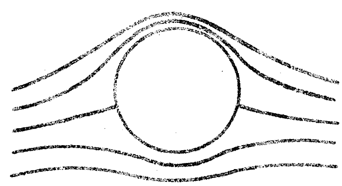


Fig. 1c.

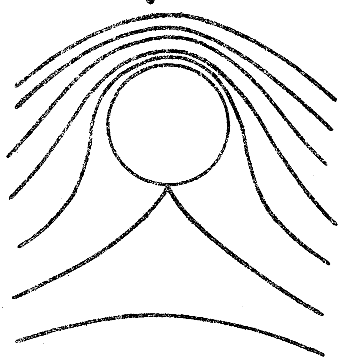
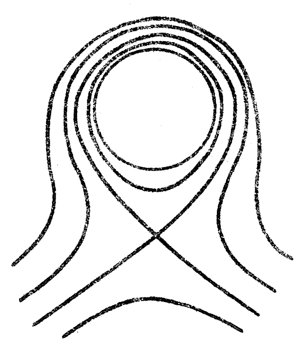


Fig. 1d.



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FIGS. 2 & 3.

Lift and Drag for Cylinder with Discs.
Disc diameter ÷ cylinder diameter = 3.

FIG. 2.

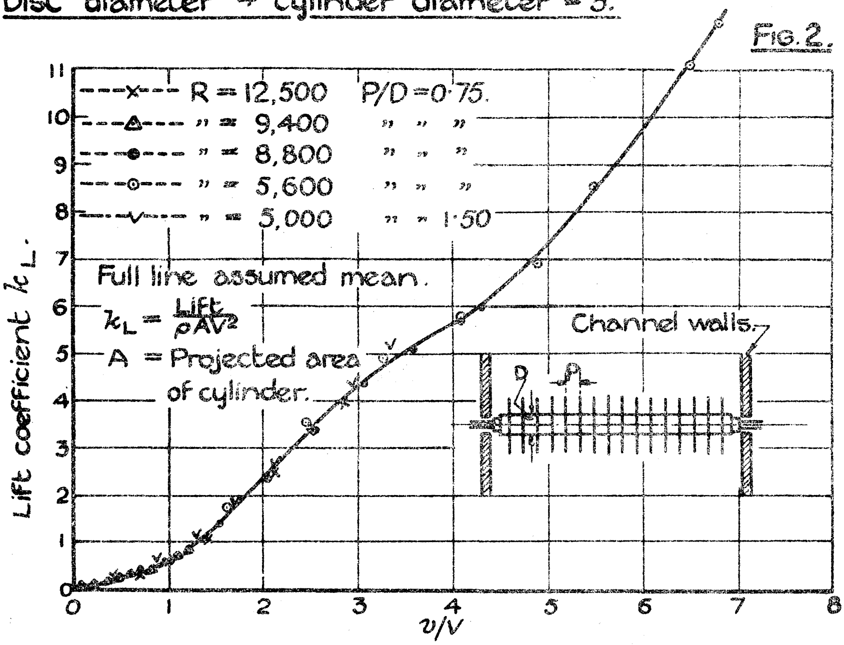
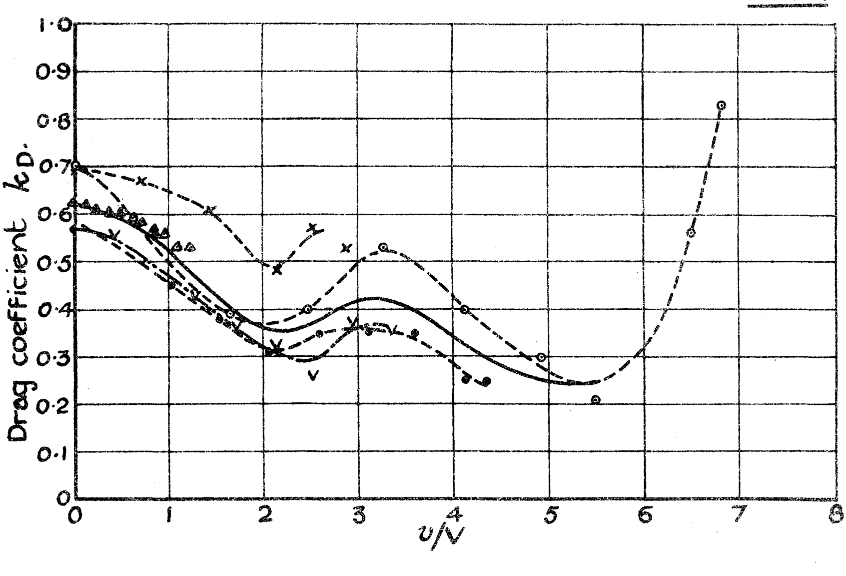


FIG. 3.



Lift and Drag for 2" Diameter Cylinder with 3" Diameter Discs.

Discs spaced ¾" apart. Air speed = 11·7 ft./sec. R = 12,200.

Fig. 4.

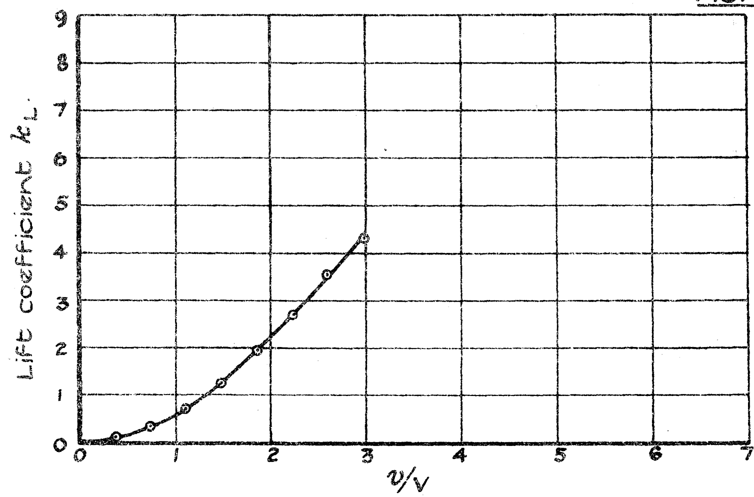
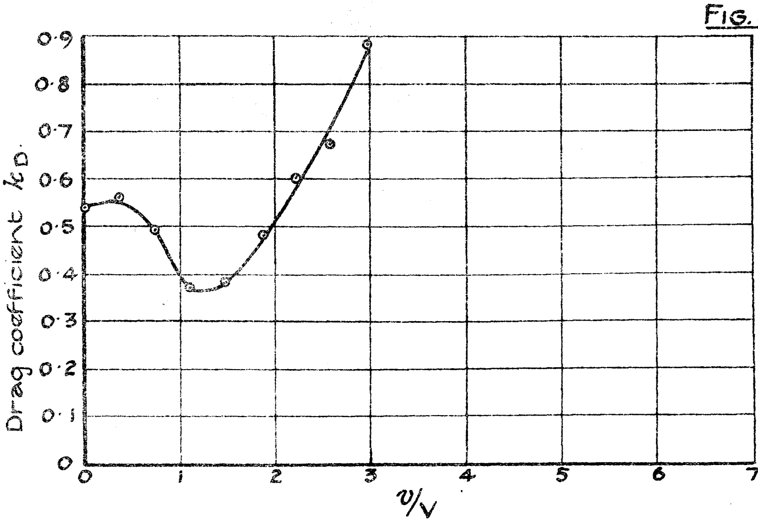


Fig. 5.



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Figs. 6, 7.

Lift and Drag for Cylinder with End Discs.

Fig. 6.

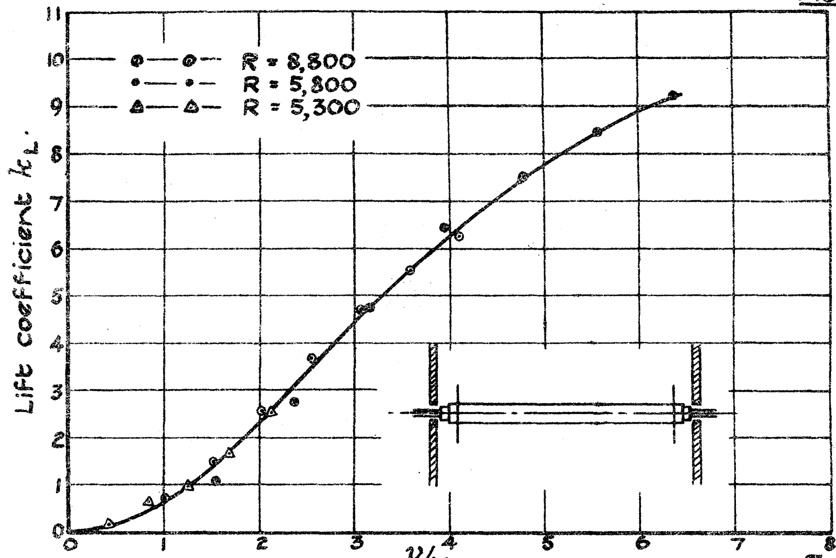
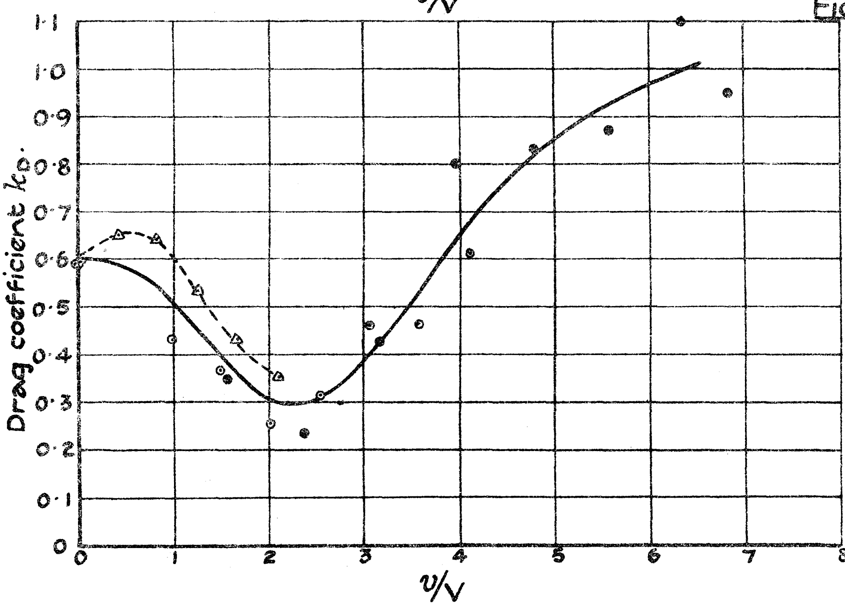


Fig. 7.



FIGS. 8 & 9.

FIG. 8.

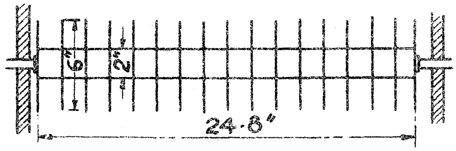


FIG. 9.

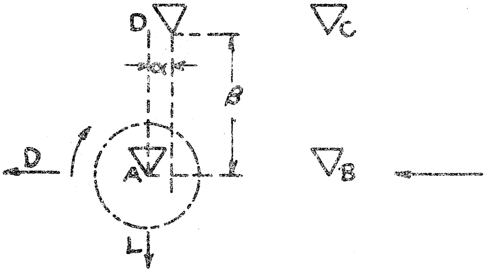


FIG. 10.

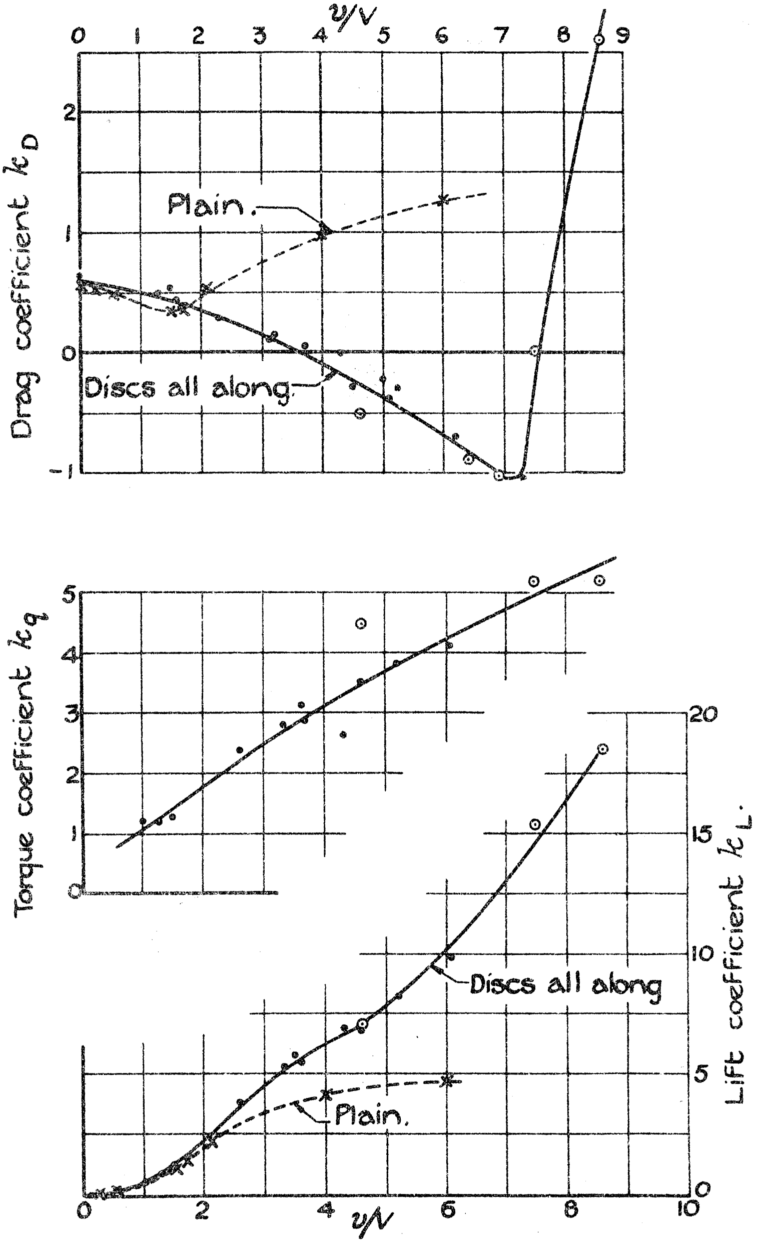
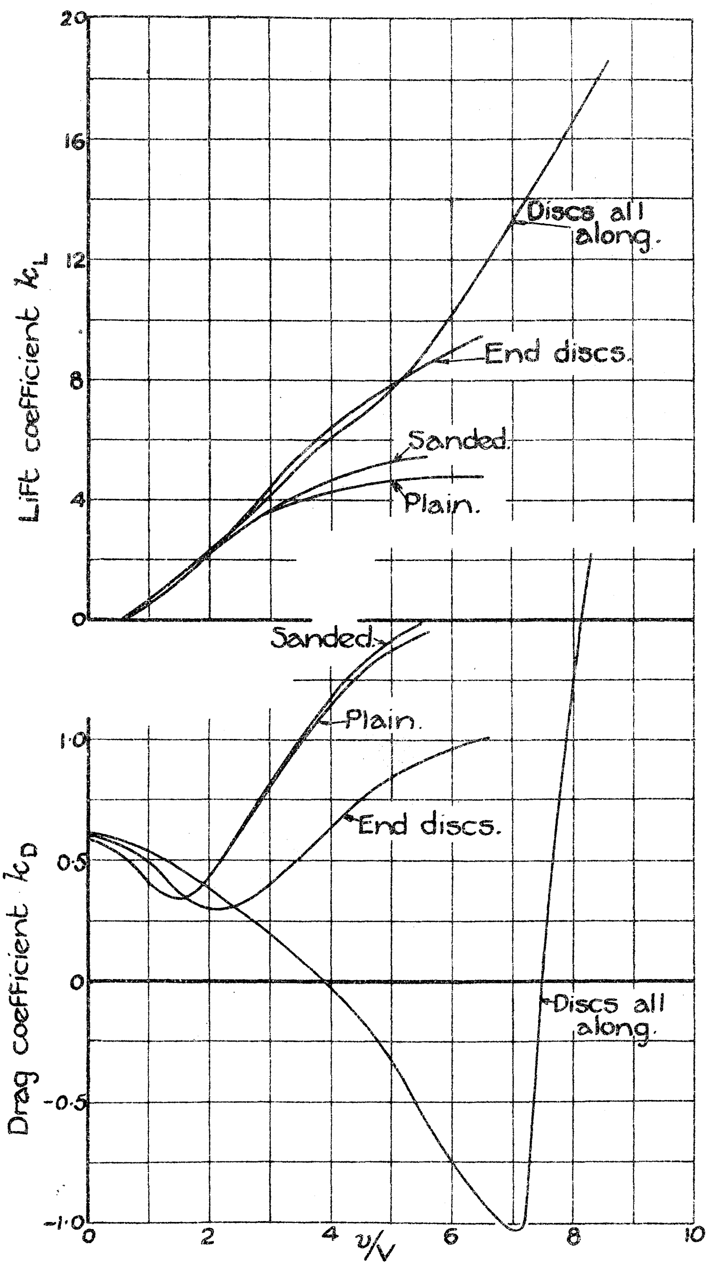
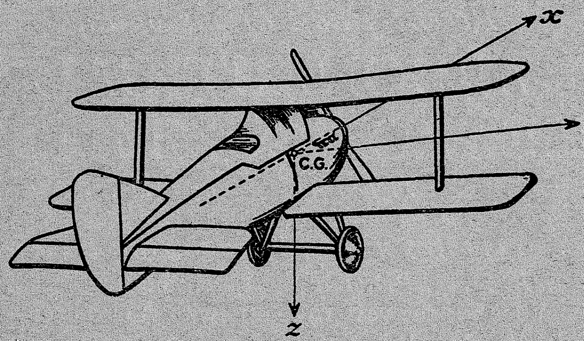

v/V
0
1
2
3
4
5
6
7
8
9
2
1
0
-1
Drag coefficient k_D
Plain.
Discs all along.
5
4
3
2
1
0
Torque coefficient k_q
20
15
10
5
0
Lift coefficient k_L.
Discs all along
Plain.
0
2
4
6
8
10
v/V

FIG. 11.


Lift coefficient k_L
20
16
12
8
4
0
Discs all along.
End discs.
Sanded.
Plain.
Sanded.
Plain.
End discs.
Discs all along.
Drag coefficient k_D
1·0
0·5
0
-0·5
-1·0
0
2
4
6
8
10
v/V

SYSTEM OF AXES.



Axes	Symbol Designation Positive direction	x longitudinal forward	y lateral starboard	z normal downward
Force	Symbol	X	Y	Z
Moment	Symbol Designation	L rolling	M pitching	N yawing
Angle of Rotation	Symbol	ϕ	θ	ψ
Velocity	Linear Angular	u p	v q	w r
Moment of Inertia		A	B	C

Components of linear velocity and force are positive in the positive direction of the corresponding axis. Components of angular velocity and moment are positive in the cyclic order y to z about the axis of x, z to x about the axis of y, and x to y about the axis of z.

The angular movement of a control surface (elevator or rudder) is governed by the same convention, the elevator angle being positive downwards and the rudder angle positive to port. The aileron angle is positive when the starboard aileron is down and the port aileron is up. A positive control angle normally gives rise to a negative moment about the corresponding axis. The symbols for the control angles are :-

ξ aileron angle
η elevator angle
η_T tail setting angle
ζ rudder angle