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Heavy Flexible Cable for Towing A Heavy Body Below An Aeroplane

By **H. GLAUERT**, F.R.S.

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AERODYNAMICS SYMBOLS

1 General

m	mass
t	time
V	resultant linear velocity
Ω	resultant angular velocity
ρ	density σ relative density
ν	kinematic coefficient of viscosity
R	Reynolds number, $R = lV/\nu$ (where l is a suitable linear dimension) to be expressed as a numerical coefficient x 10^6

Normal temperature and pressure for aeronautical work are 15° C and 760 mm

For air under these conditions $\begin{cases} \rho = 0\ 002378 \text{ slug/cu ft} \\ \nu = 1\ 59 \times 10^{-4} \text{ sq ft/sec} \end{cases}$

The slug is taken to be 32 2 lb-mass

α	angle of incidence
ϵ	angle of downwash
S	area
c	chord
s	semi-span
A	aspect ratio $A = 4s^2/S$
L	lift with coefficient $k_L = L/S\rho V^2$
D	drag with coefficient $k_D = D/S\rho V^2$
γ	gliding angle $\tan \gamma = D/L$
L	rolling moment with coefficient $k_l = L/sS\rho V^2$
M	pitching moment with coefficient $k_m = M/cS\rho V^2$
N	yawing moment with coefficient $k_n = N/sS\rho V^2$

2 Airscrews

n	revolutions per second
D	diameter
J	V/nD
P	power
T	thrust with coefficie
Q	torque with coefficie
η	efficiency $\eta = TV/P$

THE FORM OF A HEAVY FLEXIBLE CABLE USED FOR TOWING A HEAVY BODY BELOW AN AEROPLANE

By H. Glauert, F.R.S.

Communicated by the Director of Scientific Research, Air Ministry

Reports and Memoranda No. 1592

9th February, 1934*

Summary.—The mathematical expressions for the form of a heavy cable in a wind have been known for many years, but no systematic numerical results are available. Calculations have been made to derive a family of curves, depending on the weight-drag ratio of the cable, which should suffice to cover all practical problems, involving the towing of a heavy body. The use of the curves is illustrated by a typical numerical example.

1. *Introduction.*—The form assumed by a light flexible cable which is used to tow a heavy body behind an aeroplane at constant speed, has been known for many years, and the stability of the towed body has been investigated in report R. & M. 1312[1]. This knowledge has been used successfully in problems concerning radio aerials and aerodynamic instruments, suspended below an aeroplane, but other problems occasionally arise in which the validity of neglecting the weight of the cable is doubtful. The form assumed by a heavy flexible cable was determined in report R. & M. 554[2], but the analysis did not lead to any simple expressions for the form of the cable and no attempt was made to obtain numerical solutions suitable for general use. Calculations have therefore been made to derive a family of curves which should suffice to cover all practical problems. The physical assumptions are identical with those used in report R. & M. 554, but the notation and some details of the analysis have been modified slightly in order to obtain expressions more suitable for numerical computation.

2. *General analysis.*—Consider a heavy flexible cable of weight w per unit length, and let R be the drag per unit length of the cable when at right angles to a stream of velocity V. This drag will be of the form

$$R = k_D D \rho V^2 \quad \ldots \quad (1)$$

where D is the diameter of the cable and k_D is a non-dimensional drag coefficient. When the cable is inclined at an angle ϕ to the stream, the aerodynamic force F per unit length of the cable will be assumed to be at right angles to the length of the wire and of magnitude

$$F = R \sin^2 \phi \quad \ldots \quad (2)$$

This assumption is a very close approximation[2] to the actual experimental results unless the angle ϕ is very small, and it is the only physical assumption made in the analysis of the problem.

* R.A.E. Report, December, 1933.

It is convenient, in the first place, to assume that the cable is supporting a heavy body of zero drag. The cable will then be vertical at its free end and the general shape will be as shown in Fig. 1 (*a*), where O is the free end of the cable and A is its point of attachment to the aeroplane, assumed to be flying horizontally at a constant speed V. Taking O as the origin of co-ordinates as indicated in the figure, the position of any point P at distance s along the cable from O is specified by the co-ordinates (x, y), and the angle ϕ used in equation (2) is the inclination to the horizontal of the wire at the point P. These four variables are related by the equations

$$\left.\begin{aligned} \frac{dx}{ds} &= \cos\phi \\ \frac{dy}{ds} &= \sin\phi \end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (3)$$

The element ds of the cable at P is in equilibrium under the action of the system of forces comprising its weight $w\,ds$, the aerodynamic force $R \sin^2\phi\, ds$ normal to the cable, and the tension at its ends. Denoting the tension of the cable at any point by T and resolving along the cable, we obtain the equation

$$\frac{dT}{ds} = w \sin\phi$$

which can be integrated at once to give

$$T = T_0 + w\,y \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (4)$$

where T_0 is the tension of the cable at its free end or the weight of the heavy body of zero drag supported by the cable. Resolving also at right angles to the cable, we obtain the second fundamental equation

$$T\frac{d\phi}{ds} = w\cos\phi - R\sin^2\phi \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (5)$$

Before proceeding further with the analysis it is convenient to introduce a non-dimensional system of notation. The tension T is expressed as a multiple of the tension T_0 at the free end of the cable by writing

$$\tau = \frac{T}{T_0} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (6)$$

and all lengths are expressed as multiples of the characteristic length T_0/R, which is the length of cable whose drag, when normal to the relative wind, is equal to the fundamental tension T_0. Thus we write

$$\frac{x}{\xi} = \frac{y}{\eta} = \frac{s}{\sigma} = \frac{T_0}{R} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (7)$$

Finally we write

$$\frac{w}{R} = \mu = 2 \tan \alpha \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

where μ is virtually the weight-drag ratio of the cable and α is a subsidiary parameter replacing μ as a more convenient parameter for use in the subsequent numerical calculations.

Using this non-dimensional notation, the two fundamental equations (4) and (5) become respectively

$$\tau = 1 + \mu\eta \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

and

$$\tau \frac{d\phi}{d\sigma} = \mu \cos \phi - \sin^2\phi \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

After eliminating τ from these two equations, it is possible to perform the necessary integration and to obtain the relationship between η and ϕ. The corresponding values of ξ and σ are then derived by evaluating the integrals

$$\xi = \int_0^\eta \cot \phi \, d\eta \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

and

$$\sigma = \int_0^\eta \operatorname{cosec} \phi d\eta \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

3. *Light cable.*—The solution of the general equations can be obtained in a simple form if the weight of the cable is ignored. Equation (9) then shows that the tension of the cable is constant, and the integration of equation (10) gives simply

$$\sigma = \cot \phi$$

Hence

$$\frac{d\sigma}{d\eta} = \operatorname{cosec} \phi = \sqrt{1 + \sigma^2}$$

and the complete solution is given by the equations

$$\left.\begin{aligned} \sigma &= \sinh \eta \\ \xi &= \cosh \eta - 1 \\ \cos \phi &= \tanh \eta \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

4. *Heavy cable.*—Eliminating τ from equations (9) and (10) and expressing σ in terms of η and ϕ, we obtain the equation

$$(1 + \mu\eta) \sin \phi \frac{d\phi}{d\eta} = \cos^2 \phi + \mu \cos \phi - 1$$

or

$$\int_0^\eta \frac{\mu d\eta}{1 + \mu\eta} = \int_\phi^{\pi/2} \frac{\mu \sin \phi d\phi}{1 - \mu \cos \phi - \cos^2 \phi}$$

The evaluation of these integrals presents no difficulties, but it is convenient to replace μ by 2 tan α as suggested previously. The relationship between η and ϕ is then obtained in the form

$$\log (1 + 2\eta \tan\alpha) = \sin \alpha \log \frac{\cos \alpha + (1 - \sin \alpha) \cos \phi}{\cos \alpha - (1 + \sin \alpha) \cos \phi} \quad \ldots \quad (14)$$

This equation, in a slightly modified form for convenience of computation, has been used to calculate η as a function of ϕ for a series of values of the parameter α. The values of ξ and σ were then derived by graphical integration of the equations (11) and (12), since direct integration was not possible.

5. *Presentation of results.*—A sketch showing the form of the cable in the non-dimensional system is shown in Fig. 1 (*b*), where (ξ, η) are the orthogonal co-ordinates of any point P referred to the origin O, σ is the length along the cable, ϕ is the inclination of the cable to the horizontal, and θ is the angular elevation of the point P as viewed from the origin O. In the following series of figures the weight-drag ratio μ of the cable is expressed by the parameter α in accordance with equation (8), and Fig. 2 shows the relationship between these two parameters.

Numerical values of the length σ and the co-ordinates (ξ, η) for a series of values of α are given in the table at the end of the report, and the corresponding forms of the cable are shown in Figs. 3 and 4, the latter being a large scale drawing of the curves for small values of σ. These curves contain the complete solution of the problem, since the tension can be derived at once from equation (9), but various subsidiary figures have also been prepared to facilitate the solution of any specific problem.

Figs. 5, 6 and 7 show respectively the values of η, ϕ, and θ as functions σ for a series of values of the parameter α, and Fig. 8 shows the limiting value of ϕ or θ as σ tends to infinity. These limiting values can be derived at once from equation (10), since the condition that ϕ has reached a steady value leads at once to the equation

$$\mu \cos \phi = \sin^2 \phi \quad \ldots \quad (15)$$

The use of these curves to solve different problems is considered in a later paragraph.

6. *Drag of the towed body.*—Hitherto it has been assumed that the cable is supporting a heavy body of zero drag and that the cable is vertical at its free end. More generally the towed body will have a weight W and a drag H, the angle of the cable at its free end will be determined by the equation

$$\tan \phi = \frac{W}{H} \quad \ldots \quad (16)$$

and the tension at this point will be

$$T = \sqrt{W^2 + H^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

Knowing the weight-drag ratio of the cable, Fig. 6 can be used to determine the value of σ at the point of suspension of the body corresponding to the calculated value of ϕ, but it is still necessary to determine the value of the fundamental tension T_0 which is used in the general analysis. Now the value of T/T_0 at any point of a cable is known from equation (9), and by using this in conjunction with equation (14), Fig. 9 has been prepared to determine T_0/T in terms of the angle ϕ and the weight-drag parameter α. The actual cable can now be assumed to continue to the point where it becomes vertical and would support a weight T_0, and the solution of the problem from the standard curves presents no further difficulties.

The analysis has been developed on the assumption that the towed body is heavy and that its drag is comparatively small. As the drag of the body increases, for a given weight, the angle of the wire at its free end departs more from the vertical and ultimately a condition is reached in which the form of the wire is simply a straight line inclined at an angle ϕ to the horizontal. This limiting condition is attained when equations (15) and (16) are satisfied simultaneously, and it may be expressed in the alternative forms

$$W^2 = \mu H \sqrt{W^2 + H^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

or

$$\cos \phi = \tfrac{1}{2} (\sqrt{\mu^2 + 4} - \mu) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

If the ratio W/H exceeds the limiting value given by these equations, the cable hangs down in the usual manner. If W/H is less than this value, the cable is concave upwards and it would be necessary to develop a new series of solutions starting from the condition in which the cable is horizontal at its free end.

7. *Numerical example.*—Consider the form assumed by a pipe of weight 0·7 lb. per ft. and of drag 2·0 lb. per ft. supporting a body of weight 40 lb. and of drag 10 lb. As data we have

$$w = 0{\cdot}7 \text{ lb. per ft.} \qquad W = 40 \text{ lb.}$$
$$R = 2{\cdot}0 \text{ lb. per ft.} \qquad H = 10 \text{ lb.}$$

The weight-drag ratio μ of the pipe is therefore 0·35, and from Fig. 2 the corresponding value of α is 10°. At the lower end of the pipe, denoted by the suffix (1) we calculate

$$T_1 = \sqrt{W^2 + H^2} = 41{\cdot}2 \text{ lb.}$$
$$\tan \phi_1 = W/H = 4{\cdot}0$$
$$\phi_1 = 76^\circ$$

and then from Fig. 9,

$$T_0/T_1 = 0{\cdot}916$$
$$T_0 = 37{\cdot}8 \text{ lb.}$$

The characteristic length T_0/R of the pipe is therefore 18·9 ft.

From the known values of α and ϕ_1, and using in turn the Figs. 6, 5 and 4, we deduce that the lower end of the pipe is represented by

$$\sigma_1 = 0{\cdot}32 \qquad \eta_1 = 0{\cdot}30 \qquad \xi_1 = 0{\cdot}04$$

The form of the pipe is therefore represented by the curve ($\alpha = 10°$) of Fig. 3, starting at $\sigma = 0{\cdot}32$ and proceeding to a higher value depending on the length of the pipe.

If the length of the pipe is 150 ft., σ must increase by 7·93 and the upper end of the pipe is represented by $\sigma_2 = 8{\cdot}25$. From Figs. 5, 3 and 6 we then deduce the values

$$\eta_2 = 5{\cdot}28 \qquad \xi_2 = 6{\cdot}10 \qquad \phi_2 = 33°{\cdot}3$$

Hence also

$$\xi_2 - \xi_1 = 6{\cdot}06 \qquad x_2 - x_1 = 114{\cdot}5 \text{ ft.}$$
$$\xi_2 - \eta_1 = 4{\cdot}98 \qquad y_2 - y_1 = 94{\cdot}1 \text{ ft.}$$

Thus the weight hangs 114·5 ft. behind and 94·1 ft. below the aeroplane, and the pipe runs into the aeroplane at 33°·3.

If, on the other hand, we wished to determine the necessary length of pipe in order that the weight should hang 50 ft. below the aeroplane, the calculation would proceed as follows. The value of η or $(\eta_2 - \eta_1)$ must be 2·64, and so we require $\eta_2 = 2{\cdot}94$. From Fig. 5 we obtain $\sigma_2 = 4{\cdot}09$, and hence $\sigma = 3{\cdot}77$ or $s = 71{\cdot}3$ ft. Also from Fig. 3 we obtain $\xi_2 = 2{\cdot}65$, and hence $\xi = 2{\cdot}61$ or $x = 49{\cdot}4$ ft. Thus we require 71·3 ft. of pipe in order that the weight shall hang 50 ft. below the aeroplane, and the weight will then trail 49·4 ft. behind the aeroplane.

8. *Acknowledgment.*—Acknowledgments are due to Messrs. H. R. Fisher, J. O. Hinks, and D. Kimmins for their assistance in the computational work involved in the numerical development of the formulae and for the preparation and checking of the diagrams.

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(1) H. Glauert. The stability of a body towed by a light wire. R. & M. 1312. February, 1930.

(2) A. R. McLeod. On the action of wind on flexible cables, with applications to cables towed below aeroplanes and balloon cables. R. & M. 554. October, 1918.

Notation

ϱ Air density.
V Speed of aeroplane.
W Weight of suspended body.
H Drag of suspended body.
D Diameter of cable.
w Weight per unit length of cable.
R Drag per unit length of cable when normal to the wind.
k_D Drag coefficient of cable.
F Normal force per unit length of inclined cable.
T Tension of cable at any point.
T_0 Tension of cable where vertical at origin O.
s Length of cable from origin O.
x, y Co-ordinates of any point of cable from origin O.
ϕ Angle to horizontal of cable at any point.
θ Angle of elevation from O of any point of cable.
μ Weight drag ratio of cable (w/R).
α Defined by equation, $\mu = 2 \tan \alpha$.
τ $= T/T_0$.
σ $= s\, R/T_0$.
ξ $= x\, R/T_0$.
η $= y\, R/T_0$.

Table 1

Shape assumed by cables

η	$\alpha = 0°$		$\alpha = 2°$		$\alpha = 4°$		$\alpha = 6°$	
	ξ	σ	ξ	σ	ξ	σ	ξ	σ
0·5	0·13	0·52	0·12	0·52	0·12	0·52	0·12	0·52
1·0	0·54	1·18	0·51	1·16	0·48	1·14	0·46	1·13
1·5	1·35	2·13	1·21	2·10	1·08	1·98	1·01	1·87
2·0	2·76	3·63	2·35	3·25	1·93	2·95	1·73	2·75
2·5	5·13	6·05	3·69	4·69	2·95	4·06	2·57	3·73
3·0	9·07	10·02	5·27	6·35	4·09	5·27	3·48	4·78
3·5	15·57	16·54	7·00	8·10	5·30	6·60	4·45	5·87
4·0	—	—	8·82	9·96	6·54	7·98	5·45	6·99
6·0	—	—	—	—	11·65	13·42	9·55	11·50
8·0	—	—	—	—	—	—	13·65	15·98

η	$\alpha = 8°$		$\alpha = 10°$		$\alpha = 20°$		$\alpha = 30°$	
	ξ	σ	ξ	σ	ξ	σ	ξ	σ
0·5	0·12	0·51	0·11	0·51	0·10	0·51	0·09	0·51
1·0	0·44	1·11	0·42	1·10	0·35	1·07	0·29	1·06
1·5	0·93	1·82	0·87	1·77	0·68	1·67	0·54	1·62
2·0	1·56	2·63	1·39	2·53	1·01	2·31	0·82	2·20
2·5	2·29	3·50	2·03	3·34	1·43	2·96	1·12	2·79
3·0	3·08	4·46	2·74	4·18	1·86	3·62	1·43	3·38
3·5	3·90	5·41	3·43	5·05	2·31	4·30	1·75	3·97
4·0	4·74	6·40	4·19	5·94	2·76	4·97	2·09	4·58
6·0	8·20	10·40	7·25	9·60	4·66	7·74	3·44	7·03
8·0	11·72	14·55	10·31	13·50	6·59	10·53	4·81	9·50
10·0	15·22	18·70	13·40	17·40	8·54	13·33	6·22	12·00

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DIAGRAMMATIC SKETCHES OF FLEXIBLE CABLE TOWING HEAVY BODY.

FIG. 1a

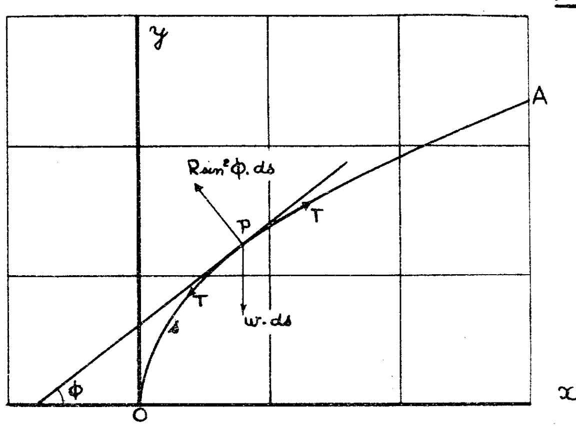
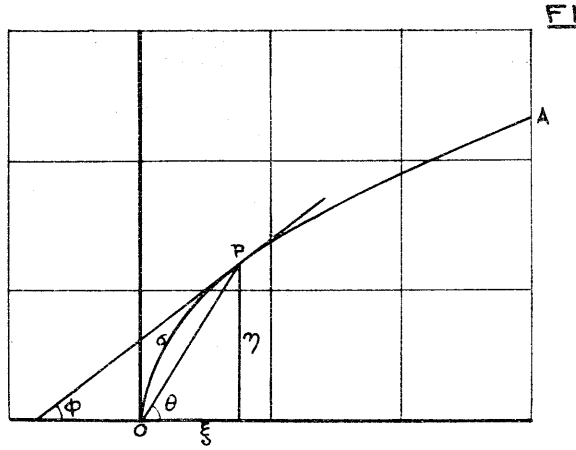


FIG. 1b



(1110) Ps 1864, 1849, 900, 7/34 Gp 921 C&R Ltd.

R&M. 1592. FIG. 2.

CABLE PARAMETERS.

CURVE OF α AGAINST μ

$$2 \tan \alpha = \mu = \frac{w}{R}$$

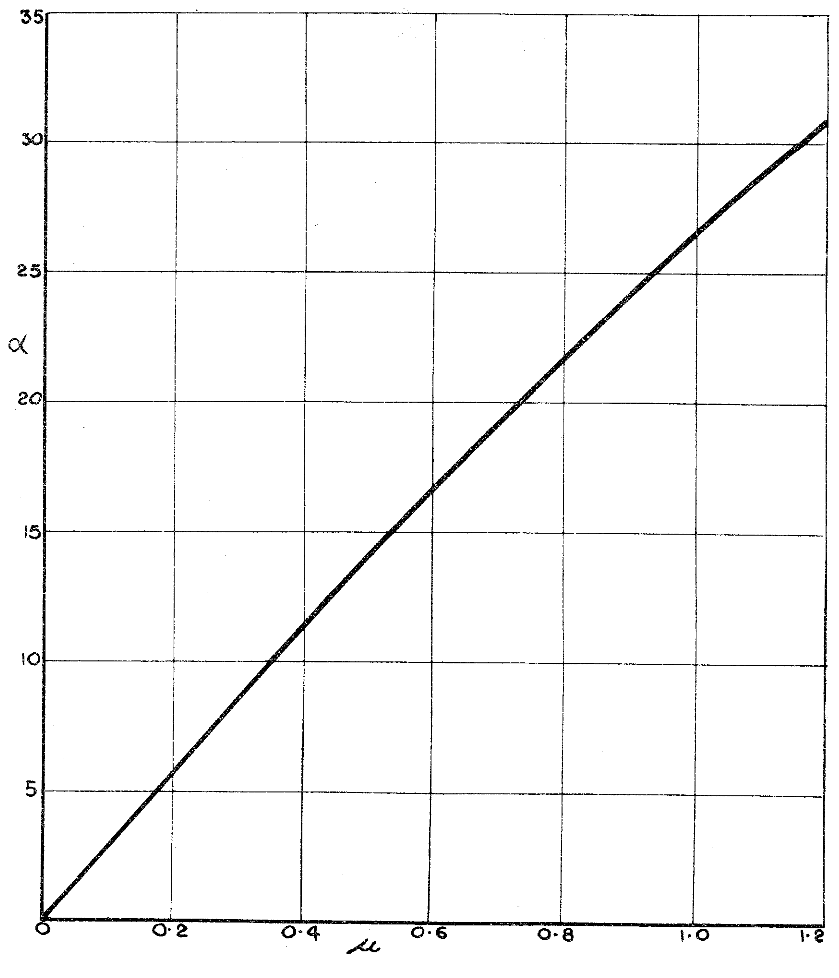


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FIG. 3.

SHAPE OF CABLE.

$\mu = 2 \tan \alpha = \frac{w}{R}$

HORIZONTAL DISTANCE, $x = \xi \frac{T_0}{R}$

VERTICAL DISTANCE, $y = \eta \frac{T_0}{R}$

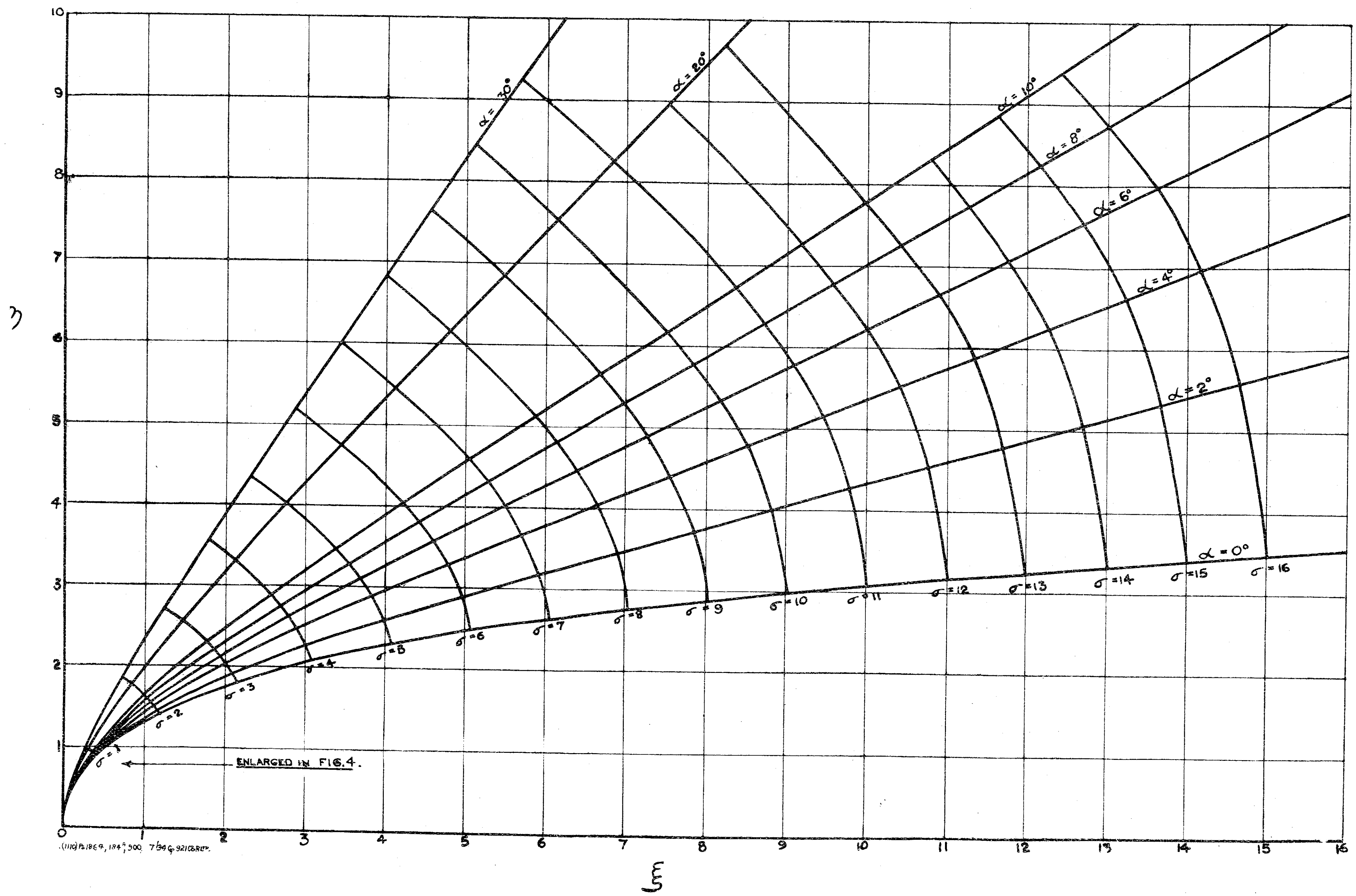

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FIG. 4.

SHAPE OF CABLE ON LARGE SCALE.

$\mu = 2 \tan \alpha = \frac{w}{R}$

HORIZONTAL DISTANCE, $x = \xi \frac{T_0}{R}$

VERTICAL DISTANCE, $y = \eta \frac{T_0}{R}$

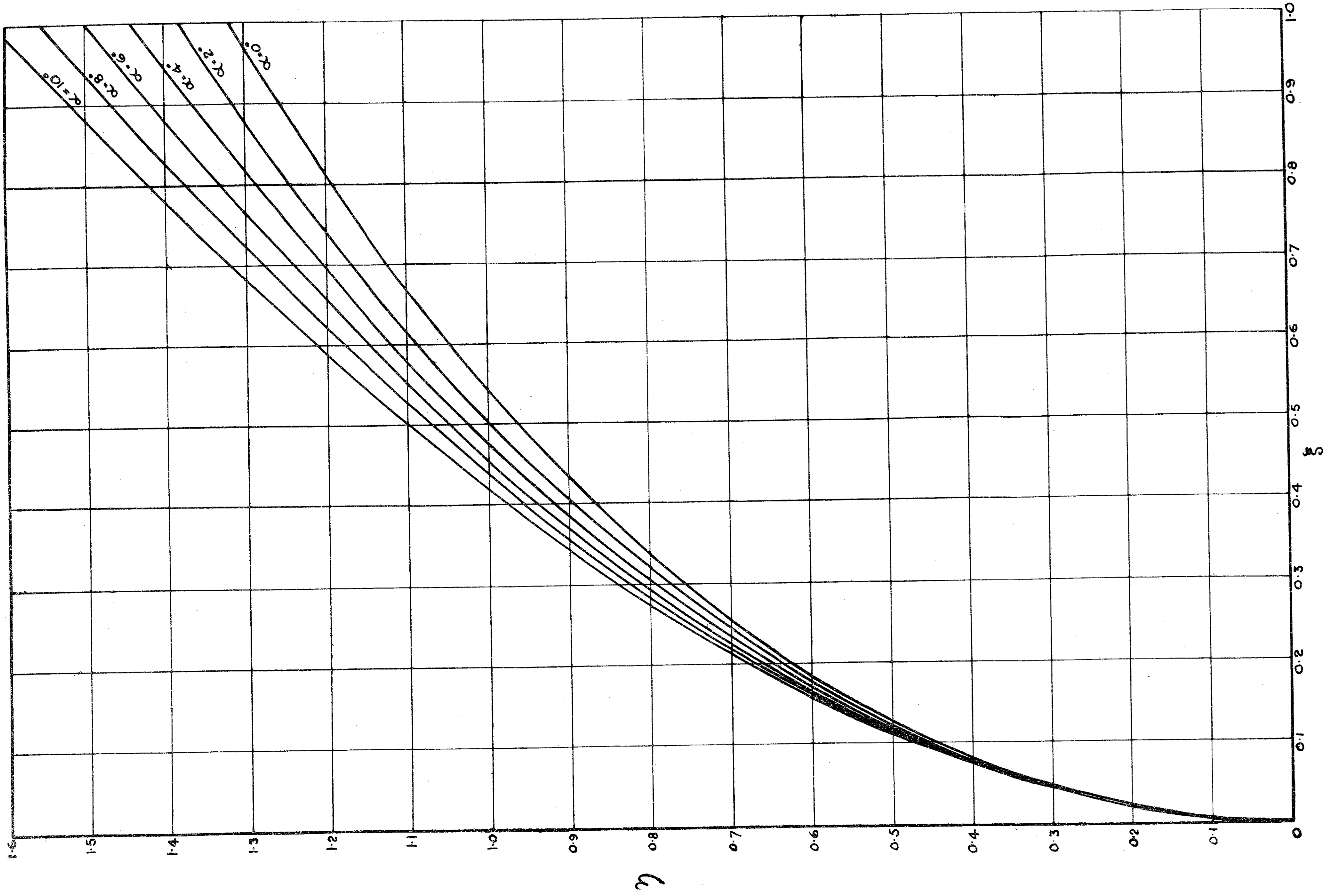


FIG 5.

HEIGHT AND LENGTH OF CABLE.

$\mu = 2 \tan\alpha = \frac{w}{R}$

LENGTH OF CABLE, $s = \sigma \frac{T_0}{R}$

VERTICAL DISTANCE, $y = \eta \frac{T_0}{R}$

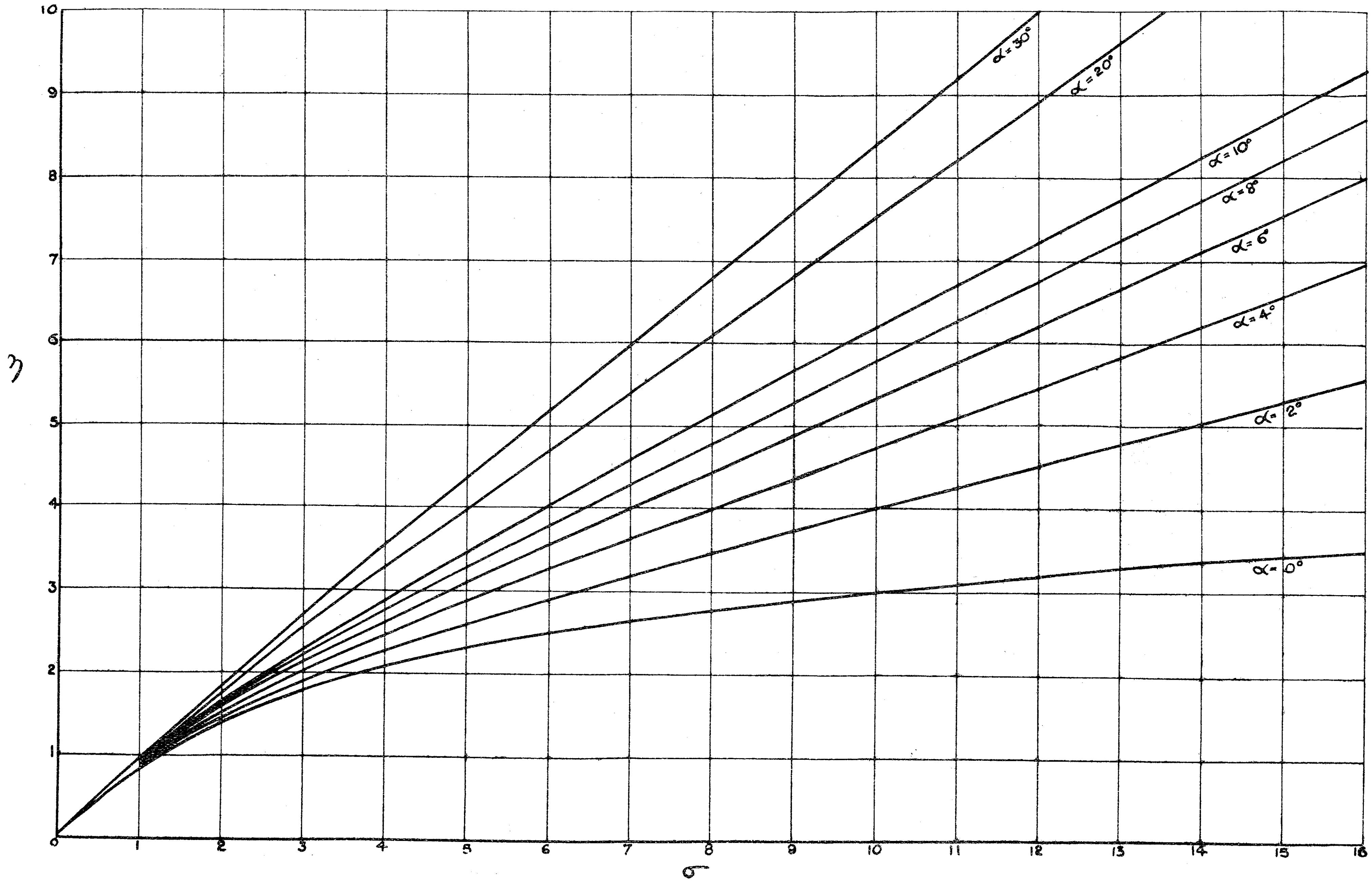


FIG. 6.

ANGLE AND LENGTH OF CABLE.

$\mu = 2 \tan \alpha = \frac{w}{R}$

ϕ = ANGLE OF CABLE TO HORIZONTAL

LENGTH OF CABLE, $s = \sigma \frac{T_0}{R}$

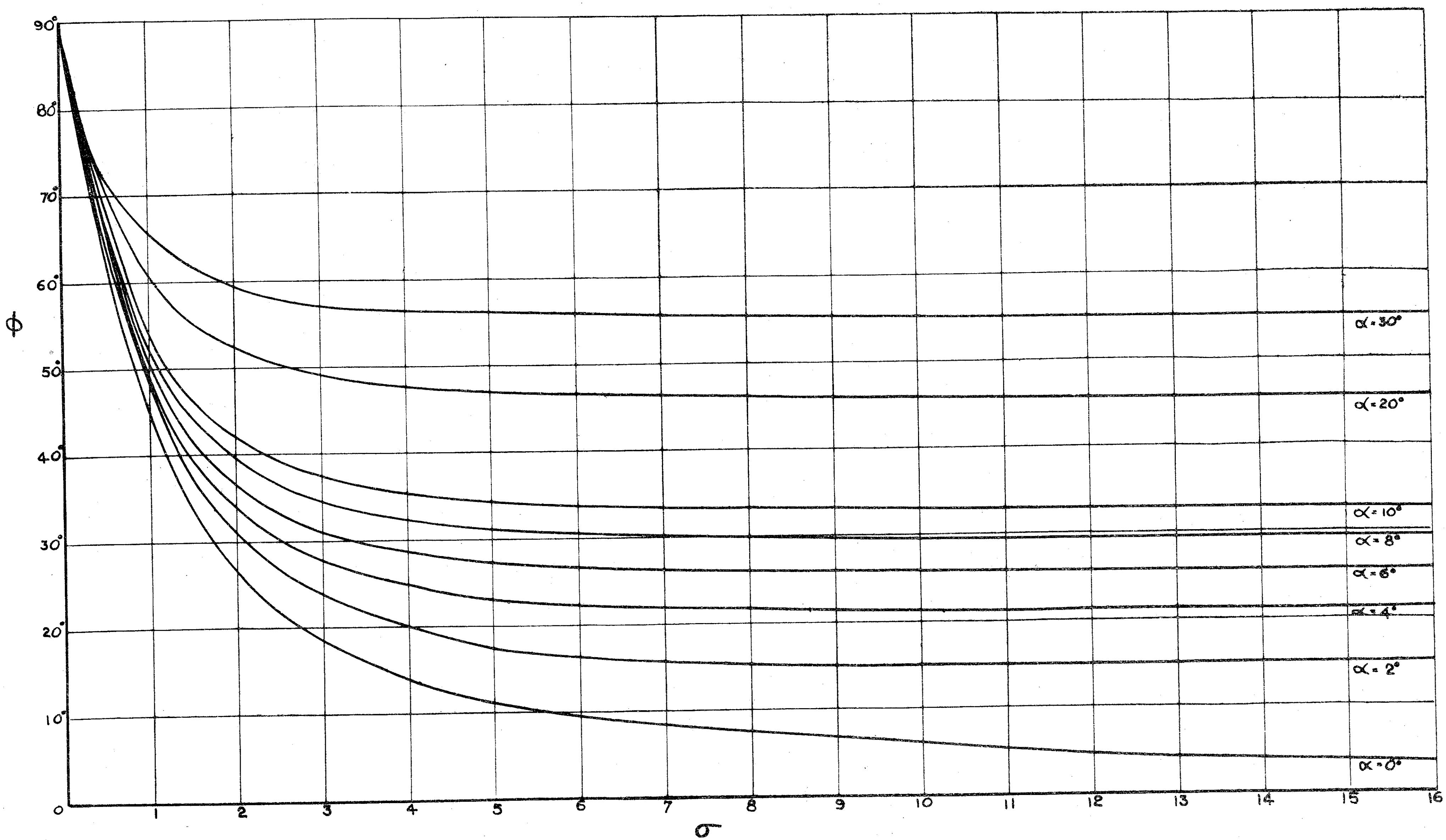


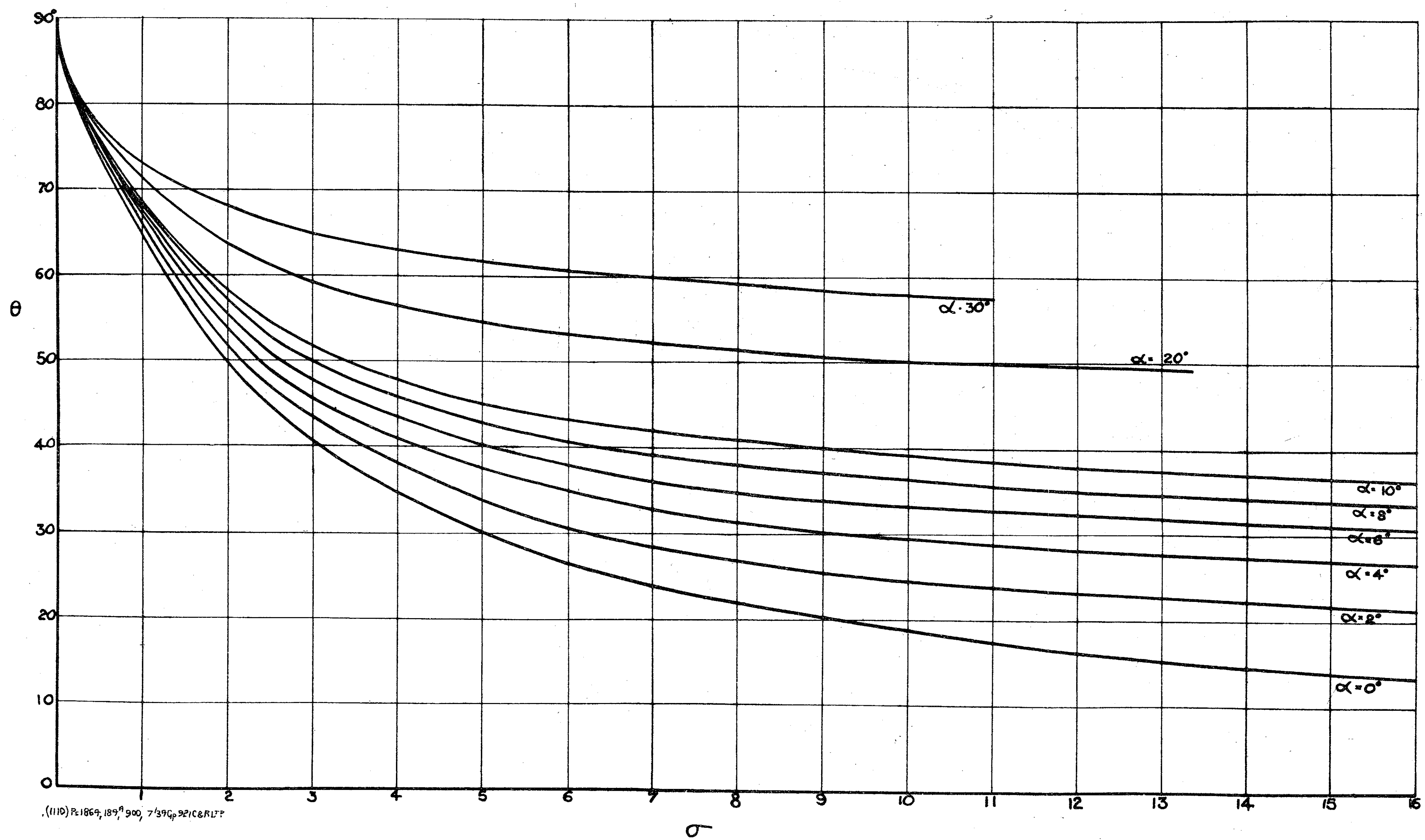
FIG. 7.

ANGLE OF ELEVATION AND LENGTH OF CABLE.

$\mu = 2 \tan \alpha = \frac{w}{R}$

θ = ANGLE OF ELEVATION

LENGTH OF CABLE, $s = \sigma \frac{T_0}{R}$



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R&M.1592. **FIG. 8.**

LIMITING ANGLE OF CABLE.

$\mu = \frac{w}{R}$

ϕ = ANGLE OF CABLE TO HORIZONTAL

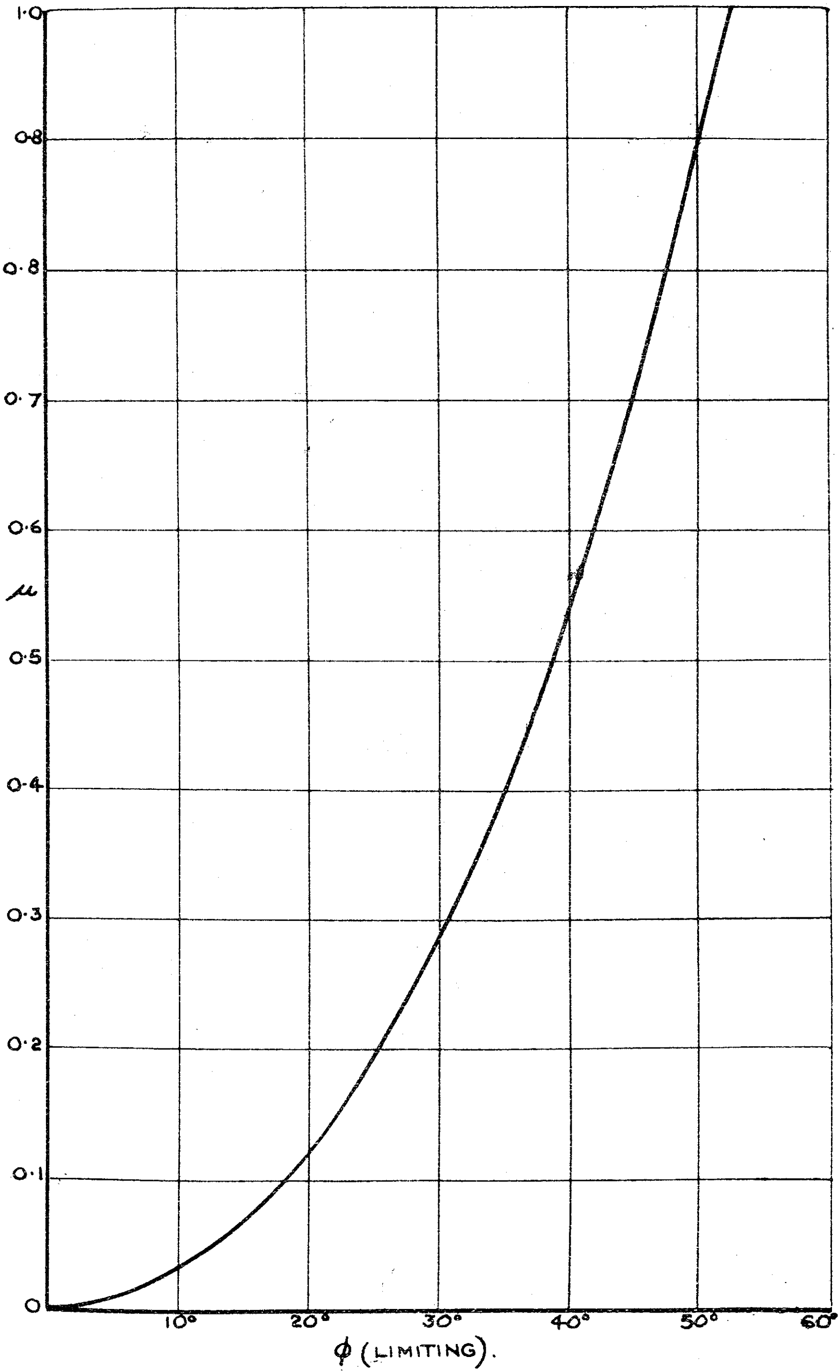

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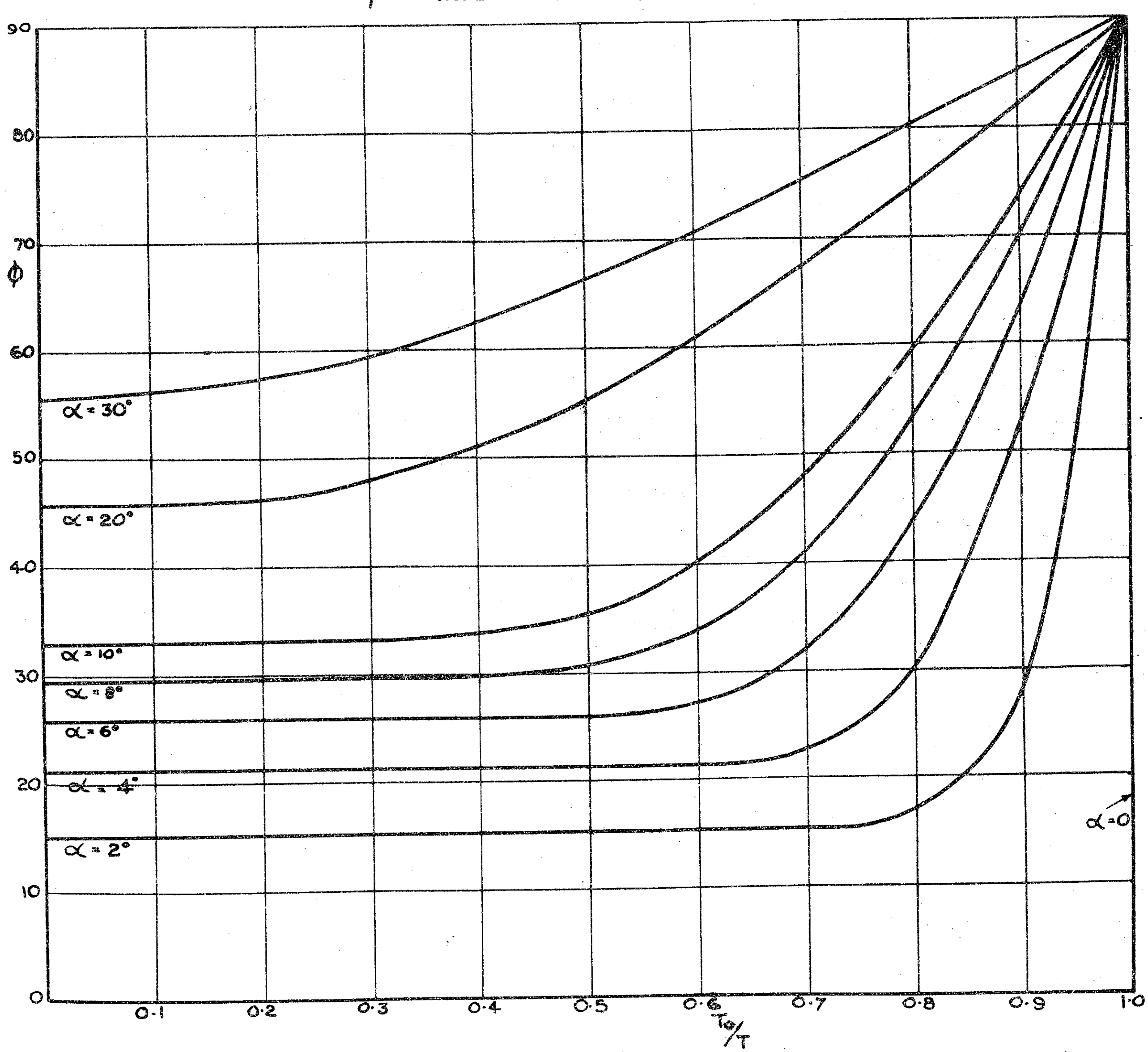
FIG. 9.

ANGLE AND TENSION OF CABLE.

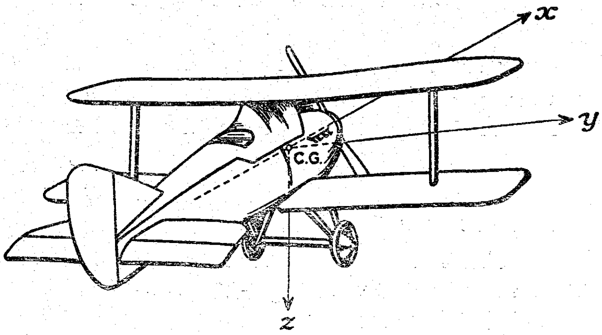
$\mu = 2 \tan\alpha = \frac{w}{R}$

$\frac{T_0}{T}$ = TENSION RATIO

ϕ = ANGLE OF CABLE TO HORIZONTAL.



SYSTEM OF AXES.



Axes	Symbol Designation Positive direction	x longitudinal forward	y lateral starboard	z normal downward
Force	Symbol	X	Y	Z
Moment	Symbol Designation	L rolling	M pitching	N yawing
Angle of Rotation	Symbol	ϕ	θ	ψ
Velocity	Linear Angular	u p	v q	w r
Moment of Inertia		A	B	C

Components of linear velocity and force are positive in the positive direction of the corresponding axis. Components of angular velocity and moment are positive in the cyclic order y to z about the axis of x, z to x about the axis of y, and x to y about the axis of z.

The angular movement of a control surface (elevator or rudder) is governed by the same convention, the elevator angle being positive downwards and the rudder angle positive to port. The aileron angle is positive when the starboard aileron is down and the port aileron is up. A positive control angle normally gives rise to a negative moment about the corresponding axis. The symbols for the control angles are :—

ξ aileron angle
η elevator angle
η_T tail setting angle
ζ rudder angle