C P No. 129
(15,253)
A R C Technical Report



MINISTRY OF SUPPLY



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# Surface Slopes and Curvatures of the RAE 100 - 104 and Other Rooftop-type Aerofoil Sections

By

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1953

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13th October, 1952

Summary

Formulae and tables have been obtained for the accurate and rapid calculation of the surface slopes and curvatures of the RAE 100-104 aerofoil sections, and of more general "rooftop-type" sections. In particular, the surface slopes of the RAE 102 and 104 shapes have been evaluated for application with a "tangent-plane" method of model constructic

## 1. Introduction

The aerofoil sections now designated RAE 100-104 were suggested by Squire[1] (1945) and their ordinates have already been listed at closely-spaced intervals[2]. More recently, their surface slopes and curvatures have been required in connection with a "tangent-plane" method of model construction, and the results may be of wider interest and usefulness, e.g., for the calculation of the simple-wave flow round aerofoils moving at high speeds[4].

The sections were designed to have a "rooftop-type" of velocity distribution, but the trailing-edge regions were modified to provide wedge-shaped tails, as discussed later. Algebraic formulae have been derived for the values of $dy/dx$ and $d^2y/dx^2$ relating to rooftop sections, in terms of transcendental functions which have been tabulated. The values of $dy/dx$ for the 10 per cent thick RAE 102 and 104 sections have been evaluated (see Table 1), and can be linearly scaled for other thickness-chord ratios.

## 2. General Characteristics of RAE 100-104 Section Shapes

Forward of the trailing-edge modification the ordinates of the RAE 100-104 shapes are given by the relation[2,*]

$$y = a\,\{f_0(x) + f_1(x)\} + c\,f_2(x) \qquad \ldots(1)$$

where $x\ [= \frac{1}{2}(1 - \cos\theta)]$ denotes the chordwise distance behind the

loading/

*More general rooftop sections are given by[3]

$$y = a\,f_0(x) + b\,f_1(x) + c\,f_2(x)\ .$$

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leading edge. The constants $a$ and $c$, together with the position of maximum velocity $X_1$ [ $= \frac{1}{2}(1 - \cos\theta_1)$], prescribe the velocity distribution on the basis of Goldstein's Approximation I. The auxiliary functions $f_r(x)$ are of the generic form

$$f_r(x) = A_r(\cos\theta - \cos\theta_1)^2 \log_e\{\sin\tfrac{1}{2}|\theta - \theta_1| / \sin\tfrac{1}{2}(\theta + \theta_1)\} + B_r \sin\theta + C_r \sin 2\theta \quad \ldots(2)$$

and have been tabulated at closely-spaced intervals of $x$ by Goldstein and Richards[3].

It is important to note that the rooftop sections from which the RAE series is derived have a point of inflection near the trailing-edge and a non-zero trailing-edge radius. Squire selected the value of $c$ (relative to $a$) so as to make the tangent at the inflection point pass through the trailing-edge, and the rooftop section was modified to have this wedge-shaped tail. Our calculations would enable more accurate values of $a$ and $c$ to be determined than those given in Ref.2, but in order to avoid confusion we have used the latter values throughout, and have merely indicated the difference between the surface slope at the corresponding inflection point and the slope given by the tail ordinates quoted in Ref.2.

## 3. Formulae for Surface Slopes and Curvatures

The functions $df_r/dx$ and $d^2f_r/dx^2$, required for the determination of $dy/dx$ and $d^2y/dx^2$ respectively from equation (1), can be expressed in the forms

$$\frac{df_r}{dx} = A'_r(\cos\theta - \cos\theta_1)\log_e\{\sin\tfrac{1}{2}|\theta - \theta_1| / \sin\tfrac{1}{2}(\theta + \theta_1)\}$$

$$+ B'_r \operatorname{cosec}\theta + C'_r \cot\theta + D'_r \sin\theta . \quad \ldots(3)$$

$$\frac{d^2f_r}{dx^2} = A''_r \log_e\{\sin\tfrac{1}{2}|\theta - \theta_1| / \sin\tfrac{1}{2}(\theta + \theta_1)\}$$

$$+ B''_r \operatorname{cosec}\theta + C''_r \cos\theta \operatorname{cosec}^3\theta + D''_r \operatorname{cosec}^3\theta + E''_r \cot\theta . \quad \ldots(4)$$

The constants $A'_r$, etc., depend only on the value of $X_1$ and are given by the formulae in the Appendix.

The transcendental functions arising in equations (3) and (4) are tabulated against $x$ in Table 3*, so as to expedite the calculation of $dy/dx$ and $d^2y/dx^2$ for a wide range of rooftop sections. Table 2 contains the values of $df_r/dx$, for $X_1 = 0.4$ and $0.6$, which were evaluated in the determination of $dy/dx$ for the RAE 102 and 104 sections (see Table 1).

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*It should be noted that the short list of values of cosec θ given in Ref.3 contains some inaccuracies in the last two places of decimals.

As an alternative to the foregoing procedure, numerical differencing of the section ordinates could be used to provide somewhat less accurate values of $dy/dx$ and $d^2y/dx^2$. In order to avoid difficulties near the nose, it is then best to write $\psi = 2y \operatorname{cosec} \theta$, to evaluate $d\psi/dx$ rather than $dy/dx$, and to use the relations

$$\frac{dy}{dx} = \frac{1}{2}\frac{d\psi}{dx}\sin\theta + \psi\cot\theta, \qquad \ldots(5)$$

$$\frac{d^2y}{dx^2} = \frac{1}{2}\frac{d^2\psi}{dx^2}\sin\theta + 2\frac{d\psi}{dx}\cot\theta - 2\psi\operatorname{cosec}^3\theta. \qquad \ldots(6)$$

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## References


| No. | Author(s) | Title, etc. |
|---|---|---|
| 1 | H. B. Squire | Tables of aerofoil sections for high speed aircraft. R.A.E. Aero. Memo. 27. (1945). |
| 2 | R. C. Pankhurst and H. B. Squire | Calculated pressure distributions for the R.A.E. 100-104 aerofoil sections. R.A.E. Technical Note No.Aero.2039. Current Paper No. 80. March, 1950. |
| 3 | S. Goldstein and E. J. Richards | Approximate two-dimensional aerofoil theory, Part III. C.P.70. (1942). |
| 4 | D. W. Holder, A. Chinneck and R. J. North | Pressure measurements in a supersonic tunnel on a two-dimensional aerofoil of RAE 104 section. C.P.62. (1951). |


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## Acknowledgement

The writers are indebted to Miss L. M. Esson for assistance with the computational work for this paper.


Appendix/

## Appendix

### Expressions for Coefficients $A'_r$ etc., in Equations (3) and (4)

$$\pi(1 - \cos\theta_1)\,\{A'_0;\ B'_0;\ C'_0;\ D'_0\}$$

$$= \{1;\ \tfrac{1}{2}(\theta_1 - \sin\theta_1\cos\theta_1);\ (\sin\theta_1 - \theta_1\cos\theta_1);\ -\theta_1\}. \qquad \ldots(A.1)$$

$$-\tfrac{1}{2}\pi\,(1 - \cos\theta_1)\,\{A''_0;\ B''_0;\ C''_0;\ D''_0;\ E''_0\}$$

$$= \{1;\ \sin\theta_1;\ \tfrac{1}{2}(\theta_1 - \sin\theta_1\cos\theta_1);\ (\sin\theta_1 - \theta_1\cos\theta_1);\ \ \theta_1\}. \qquad \ldots(A.2)$$

$$\{A'_1\ ;\ B'_1;\ C'_1;\ D'_1\}$$

$$= \left\{-\frac{2}{\pi\sin^2\theta_1};\ \frac{\cot\theta_1}{\pi} - \frac{\theta_1}{\pi\sin^2\theta_1} + \frac{1}{2(1+\cos\theta_1)};\ \frac{1}{1+\cos\theta_1} - \frac{2(\sin\theta_1 - \theta_1\cos\theta_1)}{\pi\sin^2\theta_1};\ -\frac{1}{1+\cos\theta_1} + \frac{2\theta_1}{\pi\sin^2\theta_1}\right\} \qquad \ldots(A.3)$$

$$\tfrac{1}{2}\,\{A''_1;\ B''_1;\ C''_1;\ D''_1;\ E''_1\}$$

$$= \left\{\frac{2}{\pi\sin^2\theta_1};\ \frac{2}{\pi\sin\theta_1};\ -\frac{\cot\theta_1}{\pi} + \frac{\theta_1}{\pi\sin^2\theta_1} - \frac{1}{2(1+\cos\theta_1)};\ -\frac{1}{1+\cos\theta_1} + \frac{2(\sin\theta_1 - \theta_1\cos\theta_1)}{\pi\sin^2\theta_1};\ \frac{1}{1+\cos\theta_1} + \frac{2\theta_1}{\pi\sin^2\theta_1}\right\} \qquad \ldots(A.4)$$

$$\pi(1 + \cos\theta_1)\,\{A'_2;\ B'_2;\ C'_2;\ D'_2\}$$

$$= \{1;\ -\tfrac{1}{2}\,(\overline{\pi - \theta_1} + \sin\theta_1\cos\theta_1);\ \sin\theta_1 + \overline{\pi - \theta_1}\cos\theta_1;\ \overline{\pi - \theta_1}\} \qquad \ldots(A.5)$$

$$-\tfrac{1}{2}\pi(1 + \cos\theta_1)\,\{A''_2;\ B''_2;\ C''_2;\ D''_2;\ E''_2\}$$

$$= \{1;\ \sin\theta_1;\ -\tfrac{1}{2}\,(\overline{\pi - \theta_1} + \sin\theta_1\cos\theta_1);\ (\sin\theta_1 + \overline{\pi - \theta_1}\cos\theta_1);\ -\overline{\pi - \theta_1}\} \qquad \ldots(A.6)$$

TABLE 1

Surface Slopes for RAE 102 and 104 Sections. (10% Thick).

| x | RAE 102 $\frac{dy}{dx}$ | RAE 102 $\tan^{-1}\frac{dy}{dx}$ (degs.) | RAE 104 $\frac{dy}{dx}$ | RAE 104 $\tan^{-1}\frac{dy}{dx}$ (degs.) |
|---|---|---|---|---|
| 0 | ∞ | 90.000 | ∞ | 90.000 |
| 0.001 | 1.848201 | 61.584 | 1.718481 | 59.804 |
| 0.002 | 1.304228 | 52.521 | 1.212994 | 50.498 |
| 0.003 | 1.062733 | 46.742 | 0.988643 | 44.673 |
| 0.004 | 0.918477 | 42.567 | 0.854663 | 40.519 |
| 0.005 | 0.819830 | 39.346 | 0.763066 | 37.346 |
| 0.006 | 0.746863 | 36.755 | 0.695331 | 34.812 |
| 0.007 | 0.690037 | 34.607 | 0.642593 | 32.725 |
| 0.0075 | 0.665951 | 33.662 | 0.620244 | 31.809 |
| 0.008 | 0.644137 | 32.787 | 0.600006 | 30.964 |
| 0.009 | 0.606039 | 31.217 | 0.564668 | 29.452 |
| 0.01 | 0.573744 | 29.845 | 0.534719 | 28.134 |
| 0.012 | 0.521567 | 27.545 | 0.486352 | 25.936 |
| 0.0125 | 0.510493 | 27.044 | 0.476090 | 25.459 |
| 0.014 | 0.480848 | 25.680 | 0.443626 | 24.162 |
| 0.016 | 0.447889 | 24.127 | 0.418106 | 22.690 |
| 0.018 | 0.420475 | 22.806 | 0.392734 | 21.442 |
| 0.02 | 0.397186 | 21.662 | 0.371192 | 20.365 |
| 0.025 | 0.351412 | 19.362 | 0.328889 | 18.205 |
| 0.03 | 0.317264 | 17.602 | 0.297375 | 16.561 |
| 0.035 | 0.290441 | 16.195 | 0.272655 | 15.251 |
| 0.04 | 0.268586 | 15.034 | 0.252544 | 14.173 |
| 0.05 | 0.234638 | 13.205 | 0.221370 | 12.482 |
| 0.06 | 0.209020 | 11.806 | 0.197919 | 11.195 |
| 0.07 | 0.188658 | 10.684 | 0.179340 | 10.167 |
| 0.075 | 0.179891 | 10.198 | 0.171360 | 9.724 |
| 0.08 | 0.171867 | 9.752 | 0.164070 | 9.318 |
| 0.09 | 0.157632 | 8.958 | 0.151170 | 8.596 |
| 0.1 | 0.145302 | 8.267 | 0.140037 | 7.972 |
| 0.12 | 0.124719 | 7.109 | 0.121562 | 6.931 |
| 0.14 | 0.107897 | 6.158 | 0.106590 | 6.084 |
| 0.15 | 0.100489 | 5.738 | 0.100045 | 5.713 |
| 0.16 | 0.093604 | 5.348 | 0.093992 | 5.370 |
| 0.18 | 0.081094 | 4.636 | 0.083083 | 4.749 |
| 0.2 | 0.069880 | 3.997 | 0.073422 | 4.199 |
| 0.22 | 0.059622 | 3.412 | 0.064707 | 3.702 |
| 0.24 | 0.050073 | 2.867 | 0.056723 | 3.247 |
| 0.25 | 0.045502 | 2.605 | 0.052954 | 3.031 |
| 0.26 | 0.041040 | 2.350 | 0.049311 | 2.823 |
| 0.28 | 0.032364 | 1.854 | 0.042348 | 2.425 |
| 0.3 | 0.023904 | 1.369 | 0.035740 | 2.047 |
| 0.32 | 0.015517 | 0.889 | 0.029407 | 1.684 |
| 0.34 | 0.007043 | 0.404 | 0.023283 | 1.334 |

TABLE 1 (Continued)/

TABLE 1 (Continued).

| x | RAE 102 | | RAE 104 | |
|---|---|---|---|---|
| | $\frac{dy}{dx}$ | $\tan^{-1}\frac{dy}{dx}$ (degs.) | $\frac{dy}{dx}$ | $\tan^{-1}\frac{dy}{dx}$ (degs.) |
| 0.35 | 0.002709 | 0.155 | 0.020281 | 1.162 |
| 0.36 | -0.001740 | -0.100 | 0.017311 | 0.992 |
| 0.38 | -0.011240 | -0.644 | 0.011438 | 0.655 |
| 0.4 | -0.023232 | -1.331 | 0.005615 | 0.322 |
| 0.42 | -0.034938 | -2.001 | -0.000206 | -0.012 |
| 0.44 | -0.043576 | -2.495 | -0.006076 | -0.348 |
| 0.45 | -0.047374 | -2.712 | -0.009047 | -0.518 |
| 0.46 | -0.050908 | -2.914 | -0.012052 | -0.691 |
| 0.48 | -0.057318 | -3.281 | -0.018199 | -1.043 |
| 0.5 | -0.062993 | -3.604 | -0.024597 | -1.409 |
| 0.52 | -0.068046 | -3.893 | -0.031357 | -1.796 |
| 0.54 | -0.072554 | -4.150 | -0.038638 | -2.213 |
| 0.55 | -0.074620 | -4.268 | -0.042551 | -2.437 |
| 0.56 | -0.076569 | -4.379 | -0.046708 | -2.674 |
| 0.58 | -0.080134 | -4.582 | -0.056123 | -3.212 |
| 0.6 | -0.083279 | -4.761 | -0.069503 | -3.976 |
| 0.62 | -0.086029 | -4.917 | -0.082639 | -4.724 |
| 0.64 | -0.088403 | -5.052 | -0.091320 | -5.218 |
| 0.65 | -0.089454 | -5.112 | -0.094925 | -5.423 |
| 0.66 | -0.090416 | -5.166 | -0.098162 | -5.606 |
| 0.68 | -0.092080 | -5.261 | -0.103714 | -5.921 |
| 0.7 | -0.093403 | -5.336 | -0.108235 | -6.177 |
| 0.72 | -0.094393 | -5.392 | -0.111870 | -6.383 |
| 0.74 | -0.095053 | -5.430 | -0.114713 | -6.544 |
| 0.75 | -0.095261 | -5.442 | -0.115857 | -6.609 |
| 0.76 | -0.095387 | -5.449 | -0.116825 | -6.663 |
| 0.78 | | | -0.118246 | -6.744 |
| 0.8 | | | -0.119004 | -6.786 |
| Wedge Tail | -0.095507 | -5.456 | -0.119096 | -6.792 |
| At Inflection Point | -0.095432 | -5.451 | -0.119151 | -6.795 |

TABLE 2/

Table 2

Values of Auxiliary Functions $df_r/dx$ for $X_1$ = 0.4 and 0.6.

| x | $X_1$ = 0.4 | | | $X_1$ = 0.6 | | |
|---|---|---|---|---|---|---|
| | $\frac{df_0}{dx}$ | $\frac{df_1}{dx}$ | $\frac{df_2}{dx}$ | $\frac{df_0}{dx}$ | $\frac{df_1}{dx}$ | $\frac{df_2}{dx}$ |
| 0 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| 0.001 | 8.068362$_5$ | 6.247809$_5$ | 1.471488$_5$ | 9.670645$_5$ | 5.366008 | 0.751007$_5$ |
| 0.002 | 5.660098$_5$ | 4.444158 | 1.042514$_5$ | 6.801117$_5$ | 3.813688$_5$ | 0.531964$_5$ |
| 0.003 | 4.584692 | 3.650026 | 0.852860$_5$ | 5.522861$_5$ | 3.129615 | 0.435102 |
| 0.004 | 3.938678$_5$ | 3.179469 | 0.740033$_5$ | 4.756789 | 2.723926$_5$ | 0.377465 |
| 0.005 | 3.494482$_5$ | 2.860248 | 0.663193$_5$ | 4.231238$_5$ | 2.448482 | 0.338203$_5$ |
| 0.006 | 3.164149 | 2.625993$_5$ | 0.606589 | 3.841270$_5$ | 2.246186$_5$ | 0.309274$_5$ |
| 0.007 | 2.905529 | 2.444993 | 0.562688 | 3.536623$_5$ | 2.089753$_5$ | 0.286832$_5$ |
| 0.0075 | 2.795475$_5$ | 2.368732$_5$ | 0.544138$_5$ | 3.407193$_5$ | 2.023804$_5$ | 0.277349 |
| 0.008 | 2.695551$_5$ | 2.299937$_5$ | 0.527374$_5$ | 3.289798$_5$ | 1.964287$_5$ | 0.268776$_5$ |
| 0.009 | 2.520380$_5$ | 2.180476$_5$ | 0.498186 | 3.084314 | 1.860880 | 0.253848$_5$ |
| 0.01 | 2.371145 | 2.079994 | 0.473546$_5$ | 2.909608 | 1.773833$_5$ | 0.241244 |
| 0.012 | 2.128314 | 1.919482 | 0.433986 | 2.626150 | 1.634633 | 0.220999 |
| 0.0125 | 2.076456$_5$ | 1.885751 | 0.425635$_5$ | 2.565765$_5$ | 1.605353 | 0.216724 |
| 0.014 | 1.937003$_5$ | 1.796127$_5$ | 0.403376$_5$ | 2.403678$_5$ | 1.527502$_5$ | 0.205326 |
| 0.016 | 1.780735$_5$ | 1.697788 | 0.378816 | 2.222610$_5$ | 1.441979$_5$ | 0.192743$_5$ |
| 0.018 | 1.649612$_5$ | 1.617213 | 0.358567 | 2.071214$_5$ | 1.371814$_5$ | 0.182363$_5$ |
| 0.02 | 1.537279 | 1.549773 | 0.341519$_5$ | 1.941936$_5$ | 1.313015$_5$ | 0.173619$_5$ |
| 0.025 | 1.313469$_5$ | 1.420440 | 0.308525$_5$ | 1.685717 | 1.200040$_5$ | 0.156677$_5$ |
| 0.03 | 1.143374 | 1.327327 | 0.284488 | 1.492366 | 1.118511 | 0.144312 |
| 0.035 | 1.007501 | 1.256637 | 0.266064 | 1.338881$_5$ | 1.056505 | 0.134816 |
| 0.04 | 0.895112 | 1.200884 | 0.251431$_5$ | 1.212624 | 1.007547 | 0.127257 |
| 0.05 | 0.717164 | 1.118002$_5$ | 0.229575$_5$ | 1.014063 | 0.934755 | 0.115923$_5$ |
| 0.06 | 0.579978 | 1.058745 | 0.214011 | 0.862087$_5$ | 0.882844$_5$ | 0.107802$_5$ |
| 0.07 | 0.469190 | 1.013713$_5$ | 0.202399$_5$ | 0.739959$_5$ | 0.843647 | 0.101697 |
| 0.075 | 0.421059 | 0.994855 | 0.197654$_5$ | 0.687030 | 0.827354$_5$ | 0.099184 |
| 0.08 | 0.376825 | 0.977846$_5$ | 0.193468$_5$ | 0.638431$_5$ | 0.812753$_5$ | 0.096955 |
| 0.09 | 0.298037 | 0.948158 | 0.186461 | 0.551907$_5$ | 0.787559$_5$ | 0.093188$_5$ |
| 0.1 | 0.229676 | 0.922762$_5$ | 0.180895 | 0.476767 | 0.766416$_5$ | 0.090150 |
| 0.12 | 0.116250 | 0.880223 | 0.172896 | 0.351489$_5$ | 0.732236 | 0.085643$_5$ |
| 0.14 | 0.025586 | 0.844037$_5$ | 0.167879$_5$ | 0.250085$_5$ | 0.704807 | 0.082610 |
| 0.15 | -0.013238 | 0.827225 | 0.166209$_5$ | 0.206029$_5$ | 0.692667$_5$ | 0.081499 |
| 0.16 | -0.048453$_5$ | 0.810876 | 0.165003 | 0.165575 | 0.681243$_5$ | 0.080607$_5$ |
| 0.18 | -0.109608$_5$ | 0.778743$_5$ | 0.163791$_5$ | 0.093696$_5$ | 0.659853 | 0.079377 |
| 0.2 | -0.160293$_5$ | 0.746322$_5$ | 0.163971 | 0.031673$_5$ | 0.639566$_5$ | 0.078760 |
| 0.22 | -0.202124$_5$ | 0.712657$_5$ | 0.165393$_5$ | -0.022396$_5$ | 0.619666 | 0.078657 |
| 0.24 | -0.236196 | 0.676977 | 0.167999$_5$ | -0.069867 | 0.599642$_5$ | 0.079005 |
| 0.25 | -0.250560$_5$ | 0.658164 | 0.169747 | -0.091447$_5$ | 0.589462 | 0.079335$_5$ |
| 0.26 | -0.263235 | 0.638587$_5$ | 0.171800 | -0.111728$_5$ | 0.579115 | 0.079766 |
| 0.28 | -0.283683$_5$ | 0.596785$_5$ | 0.176877 | -0.148725$_5$ | 0.557782$_5$ | 0.080921$_5$ |
| 0.3 | -0.297730 | 0.550771 | 0.183394$_5$ | -0.181424$_5$ | 0.535393 | 0.082467 |
| 0.32 | -0.305292 | 0.499530$_5$ | 0.191633$_5$ | -0.210263 | 0.511724 | 0.084411 |
| 0.34 | -0.305925 | 0.441619$_5$ | 0.202066 | -0.235579$_5$ | 0.486565$_5$ | 0.086774$_5$ |

TABLE 2 (Continued)/

TABLE 2 (Continued)

| x | $X_1 = 0.4$ | | | $X_1 = 0.6$ | | |
|---|---|---|---|---|---|---|
| | $\frac{df_0}{dx}$ | $\frac{df_1}{dx}$ | $\frac{df_2}{dx}$ | $\frac{df_0}{dx}$ | $\frac{df_1}{dx}$ | $\frac{df_2}{dx}$ |
| 0.35 | -0.303347 | 0.409486 | 0.208347 | -0.247001 | 0.473364 | 0.088123 |
| 0.36 | -0.298550$_5$ | 0.374668 | 0.215549$_5$ | -0.257634$_5$ | 0.459711$_5$ | 0.089589$_5$ |
| 0.38 | -0.280505$_5$ | 0.293764 | 0.233967 | -0.276625 | 0.430945$_5$ | 0.092905 |
| 0.4 | -0.238270$_5$ | 0.175982$_5$ | 0.266412 | -0.292693$_5$ | 0.400031$_5$ | 0.096786 |
| 0.42 | -0.194096 | 0.058074$_5$ | 0.298110 | -0.305933 | 0.366703 | 0.101318 |
| 0.44 | -0.170206 | -0.023207$_5$ | 0.314287 | -0.316388$_5$ | 0.330645 | 0.106617 |
| 0.45 | -0.161001$_5$ | -0.058302 | 0.319807 | -0.320572$_5$ | 0.311475$_5$ | 0.109601 |
| 0.46 | -0.153005 | -0.090767 | 0.324029 | -0.324053$_5$ | 0.291472 | 0.112839 |
| 0.48 | -0.139695$_5$ | -0.149480 | 0.329207$_5$ | -0.328861$_5$ | 0.248695 | 0.120198$_5$ |
| 0.5 | -0.129000 | -0.201666$_5$ | 0.330666$_5$ | -0.330666$_5$ | 0.201666$_5$ | 0.129000 |
| 0.52 | -0.120198$_5$ | -0.248695 | 0.328861$_5$ | -0.329207$_5$ | 0.149480 | 0.139695$_5$ |
| 0.54 | -0.112839 | -0.291472 | 0.324053$_5$ | -0.324029 | 0.090767 | 0.153005 |
| 0.55 | -0.109601 | -0.311475$_5$ | 0.320572$_5$ | -0.319807 | 0.058302 | 0.161001$_5$ |
| 0.56 | -0.106617 | -0.330645 | 0.316388$_5$ | -0.314287 | 0.023207$_5$ | 0.170206 |
| 0.58 | -0.101318 | -0.366703 | 0.305933 | -0.298110 | -0.058074$_5$ | 0.194096 |
| 0.6 | -0.096786 | -0.400031$_5$ | 0.292693$_5$ | -0.266412 | -0.175982$_5$ | 0.238270$_5$ |
| 0.62 | -0.092905 | -0.430945$_5$ | 0.276625 | -0.233967 | -0.293764 | 0.280505$_5$ |
| 0.64 | -0.089589$_5$ | -0.459711$_5$ | 0.257634$_5$ | -0.215549$_5$ | -0.374668 | 0.298550$_5$ |
| 0.65 | -0.088123 | -0.473364 | 0.247001 | -0.208347 | -0.409486 | 0.303347 |
| 0.66 | -0.086774$_5$ | -0.486565$_5$ | 0.235579$_5$ | -0.202066 | -0.441619$_5$ | 0.305925 |
| 0.68 | -0.084411 | -0.511724 | 0.210263 | -0.191633$_5$ | -0.499530$_5$ | 0.305292 |
| 0.7 | -0.082467 | -0.535393 | 0.181424$_5$ | -0.183394$_5$ | -0.550771 | 0.297730 |
| 0.72 | -0.080921$_5$ | -0.557782$_5$ | 0.148725$_5$ | -0.176877 | -0.596785$_5$ | 0.283683$_5$ |
| 0.74 | -0.079766 | -0.579115 | 0.111728$_5$ | -0.171800 | -0.638587$_5$ | 0.263235 |
| 0.75 | -0.079335$_5$ | -0.589462 | 0.091447$_5$ | -0.169747 | -0.658164 | 0.250560$_5$ |
| 0.76 | -0.079005 | -0.599642$_5$ | 0.069867 | -0.167999$_5$ | -0.676977 | 0.236196 |
| 0.78 | -0.078657 | -0.619666 | 0.022396$_5$ | -0.165393$_5$ | -0.712657$_5$ | 0.202124$_5$ |
| 0.8 | -0.078760 | -0.639566$_5$ | -0.031673$_5$ | -0.163971 | -0.746322$_5$ | 0.160293$_5$ |
| 0.82 | -0.079377 | -0.659853 | -0.093696$_5$ | -0.163791$_5$ | -0.778743$_5$ | 0.109608$_5$ |
| 0.84 | -0.080607$_5$ | -0.681243$_5$ | -0.165575 | -0.165003 | -0.810876 | 0.048453$_5$ |
| 0.85 | -0.081499 | -0.692667$_5$ | -0.206029$_5$ | -0.166209$_5$ | -0.827225 | 0.013238 |
| 0.86 | -0.082610 | -0.704807 | -0.250085$_5$ | -0.167879$_5$ | -0.844037$_5$ | -0.025586 |
| 0.88 | -0.085643$_5$ | -0.732236 | -0.351489$_5$ | -0.172896 | -0.880223 | -0.116250 |
| 0.9 | -0.090150 | -0.766416$_5$ | -0.476767 | -0.180895 | -0.922762$_5$ | -0.229676 |
| 0.92 | -0.096955 | -0.812753$_5$ | -0.638431$_5$ | -0.193468$_5$ | -0.977846$_5$ | -0.376825 |
| 0.925 | -0.099184 | -0.827354$_5$ | -0.687030 | -0.197654$_5$ | -0.994855 | -0.421059 |
| 0.94 | -0.107802$_5$ | -0.882844$_5$ | -0.862087$_5$ | -0.214011 | -1.058745 | -0.579978 |
| 0.95 | -0.115923$_5$ | -0.934755 | -1.014063 | -0.229575$_5$ | -1.118002$_5$ | -0.717163$_5$ |
| 0.96 | -0.127257 | -1.007547 | -1.212624 | -0.251431$_5$ | -1.200884 | -0.895112 |
| 0.975 | -0.156677$_5$ | -1.200040$_5$ | -1.685717 | -0.308525$_5$ | -1.420440 | -1.313469$_5$ |
| 0.98 | -0.173619$_5$ | -1.313015$_5$ | -1.941936$_5$ | -0.341519$_5$ | -1.549773 | -1.537279 |
| 0.9875 | -0.216724 | -1.605353 | -2.565765$_5$ | -0.425635$_5$ | -1.885751 | -2.076456$_5$ |
| 1.0 | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ |

TABLE 3/

TABLE 3a

Values of Transcendental Functions for the Determination of $df_r/dx$ and $d^2 f_r/dx^2$.

| x | cos θ | sin θ | cosec θ | cot θ | $cosec^3$ θ |
|---|---|---|---|---|---|
| 0 | 1.0 | 0 | ∞ | ∞ | ∞ |
| 0.001 | 0.998 | 0.0632139 | 15.819300 | 15.787661 | 3958.7837 |
| 0.002 | 0.996 | 0.0893532 | 11.191537 | 11.146771 | 1401.7456 |
| 0.003 | 0.994 | 0.1093801 | 9.1424333 | 9.0875787 | 764.16192 |
| 0.004 | 0.992 | 0.1262379 | 7.9215531 | 7.8581807 | 497.08541 |
| 0.005 | 0.99 | 0.1410674 | 7.0888120 | 7.0179239 | 356.22171 |
| 0.006 | 0.988 | 0.1544539 | 6.4744247 | 6.3967316 | 271.39607 |
| 0.007 | 0.986 | 0.1667453 | 5.9971700 | 5.9132096 | 215.69450 |
| 0.0075 | 0.985 | 0.1725543 | 5.7952759 | 5.7083467 | 194.63563 |
| 0.008 | 0.984 | 0.1781685 | 5.6126657 | 5.5228630 | 176.81028 |
| 0.009 | 0.982 | 0.1888809 | 5.2943411 | 5.1990430 | 148.40064 |
| 0.01 | 0.98 | 0.1989975 | 5.0251891 | 4.9246853 | 126.89871 |
| 0.012 | 0.976 | 0.2177705 | 4.5919897 | 4.4817820 | 96.828394 |
| 0.0125 | 0.975 | 0.2222049 | 4.5003516 | 4.3878428 | 91.146362 |
| 0.014 | 0.972 | 0.2349809 | 4.2556659 | 4.1365073 | 77.073057 |
| 0.016 | 0.968 | 0.2509502 | 3.9848545 | 3.8573391 | 63.275763 |
| 0.018 | 0.964 | 0.2659022 | 3.7607807 | 3.6253926 | 53.190494 |
| 0.02 | 0.96 | 0.2800000 | 3.5714286 | 3.4285714 | 45.553936 |
| 0.025 | 0.95 | 0.3122499 | 3.2025631 | 3.0424349 | 32.846801 |
| 0.03 | 0.94 | 0.3411744 | 2.9310519 | 2.7551888 | 25.180858 |
| 0.035 | 0.93 | 0.3675595 | 2.7206478 | 2.5302025 | 20.138030 |
| 0.04 | 0.92 | 0.3919184 | 2.5515518 | 2.3474277 | 16.611665 |
| 0.05 | 0.9 | 0.4358899 | 2.2941573 | 2.0647416 | 12.074512 |
| 0.06 | 0.88 | 0.4749737 | 2.1053798 | 1.8527342 | 9.3323573 |
| 0.07 | 0.86 | 0.5102940 | 1.9596545 | 1.6853029 | 7.5255549 |
| 0.075 | 0.85 | 0.5267827 | 1.8983160 | 1.6135686 | 6.8407783 |
| 0.08 | 0.84 | 0.5425864 | 1.8430245 | 1.5481405 | 6.2602733 |
| 0.09 | 0.82 | 0.5723635 | 1.7471414 | 1.4326559 | 5.3331544 |
| 0.1 | 0.8 | 0.6000000 | 1.6666667 | 1.3333333 | 4.6296296 |
| 0.12 | 0.76 | 0.6499231 | 1.5386436 | 1.1693692 | 3.6426222 |
| 0.14 | 0.72 | 0.6939741 | 1.4409760 | 1.0375028 | 2.9920599 |
| 0.15 | 0.7 | 0.7141428 | 1.4002801 | 0.9801961 | 2.7456472 |
| 0.16 | 0.68 | 0.7332121 | 1.3638618 | 0.9274260 | 2.5369453 |
| 0.18 | 0.64 | 0.7683749 | 1.3014480 | 0.8329267 | 2.2043496 |
| 0.2 | 0.6 | 0.8000000 | 1.2500000 | 0.7500000 | 1.9531250 |
| 0.22 | 0.56 | 0.8284926 | 1.2070114 | 0.6759264 | 1.7584665 |
| 0.24 | 0.52 | 0.8541663 | 1.1707323 | 0.6087808 | 1.6046221 |
| 0.25 | 0.5 | 0.8660254 | 1.1547005 | 0.5773503 | 1.5396007 |
| 0.26 | 0.48 | 0.8772685 | 1.1399019 | 0.5471529 | 1.4811615 |
| 0.28 | 0.44 | 0.8979978 | 1.1135885 | 0.4899789 | 1.3809381 |
| 0.3 | 0.4 | 0.9165151 | 1.0910895 | 0.4364358 | 1.2989160 |
| 0.32 | 0.36 | 0.9329523 | 1.0718662 | 0.3858718 | 1.2314639 |
| 0.34 | 0.32 | 0.9474175 | 1.0555008 | 0.3377603 | 1.1759145 |
| 0.35 | 0.3 | 0.9539392 | 1.0482848 | 0.3144855 | 1.1519614 |
| 0.36 | 0.28 | 0.9600000 | 1.0416667 | 0.2916667 | 1.1302807 |
| 0.38 | 0.24 | 0.9707729 | 1.0301071 | 0.2472257 | 1.0930678 |
| 0.4 | 0.2 | 0.9797959 | 1.0206207 | 0.2041241 | 1.0631466 |
| 0.42 | 0.16 | 0.9871170 | 1.0130511 | 0.1620882 | 1.0396666 |
| 0.44 | 0.12 | 0.9927739 | 1.0072787 | 0.1208734 | 1.0219954 |
| 0.45 | 0.1 | 0.9949874 | 1.0050378 | 0.1005038 | 1.0151897 |
| 0.46 | 0.08 | 0.9967949 | 1.0032154 | 0.0802572 | 1.0096774 |
| 0.48 | 0.04 | 0.9991997 | 1.0008010 | 0.0400320 | 1.0024048 |
| 0.5 | 0 | 1.0 | 1.0 | 0 | 1.0 |
| 1 - x | -cos θ | sin θ | cosec θ | -cot θ | $cosec^3$ θ |

TABLE 3b/

## TABLE 3b*

Values of $-\frac{1}{\pi}\log_e \frac{\sin\frac{1}{2}|\theta-\theta_1|}{\sin\frac{1}{2}(\theta+\theta_1)}$

| x \ $X_1$, $\theta_1$ | 0.3<br>1.1592795 | 0.4<br>1.3694384 | 0.5<br>1.5707963 | 0.6<br>1.7721542 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0.001 | 0.0307910 | 0.0246809 | 0.0201485 | 0.0164493 |
| 0.002 | 0.0436010 | 0.0349392 | 0.0285180 | 0.0232797 |
| 0.003 | 0.0534689 | 0.0428344 | 0.0349566 | 0.0285324 |
| 0.004 | 0.0618203 | 0.0495108 | 0.0403983 | 0.0329704 |
| 0.005 | 0.0692067 | 0.0554107 | 0.0452046 | 0.0368887 |
| 0.006 | 0.0759106 | 0.0607609 | 0.0495608 | 0.0404390 |
| 0.007 | 0.0820996 | 0.0656959 | 0.0535770 | 0.0437110 |
| 0.0075 | 0.0850367 | 0.0680362 | 0.0554808 | 0.0452616 |
| 0.008 | 0.0878829 | 0.0703033 | 0.0573246 | 0.0467631 |
| 0.009 | 0.0933360 | 0.0746438 | 0.0608534 | 0.0496362 |
| 0.01 | 0.0985141 | 0.0787619 | 0.0641995 | 0.0523594 |
| 0.012 | 0.1082018 | 0.0864562 | 0.0704465 | 0.0574411 |
| 0.0125 | 0.1105062 | 0.0882844 | 0.0719298 | 0.0586472 |
| 0.014 | 0.1171820 | 0.0935758 | 0.0762209 | 0.0621350 |
| 0.016 | 0.1256076 | 0.1002437 | 0.0816232 | 0.0665233 |
| 0.018 | 0.1335852 | 0.1065455 | 0.0867232 | 0.0706633 |
| 0.02 | 0.1411922 | 0.1125431 | 0.0915720 | 0.0745964 |
| 0.025 | 0.1589390 | 0.1264900 | 0.1028257 | 0.0837135 |
| 0.03 | 0.1753203 | 0.1393014 | 0.1131340 | 0.0920495 |
| 0.035 | 0.1907060 | 0.1512739 | 0.1227398 | 0.0998027 |
| 0.04 | 0.2053376 | 0.1626009 | 0.1318008 | 0.1071024 |
| 0.05 | 0.2329667 | 0.1838183 | 0.1486970 | 0.1206747 |
| 0.06 | 0.2591043 | 0.2036616 | 0.1643987 | 0.1332365 |
| 0.07 | 0.2842980 | 0.2225550 | 0.1792490 | 0.1450672 |
| 0.075 | 0.2966626 | 0.2317372 | 0.1864283 | 0.1507679 |
| 0.08 | 0.3089233 | 0.2407800 | 0.1934732 | 0.1563494 |
| 0.09 | 0.3332602 | 0.2585374 | 0.2072296 | 0.1672108 |
| 0.1 | 0.3575355 | 0.2759794 | 0.2206356 | 0.1777450 |
| 0.12 | 0.4066673 | 0.3103818 | 0.2467428 | 0.1981042 |
| 0.14 | 0.4577565 | 0.3447543 | 0.2723400 | 0.2178474 |
| 0.15 | 0.4844927 | 0.3621252 | 0.2850746 | 0.2275823 |
| 0.16 | 0.5123000 | 0.3797152 | 0.2978231 | 0.2372661 |
| 0.18 | 0.5721481 | 0.4158389 | 0.3235137 | 0.2565824 |
| 0.2 | 0.6399199 | 0.4537243 | 0.3496992 | 0.2759794 |
| 0.22 | 0.7198018 | 0.4940597 | 0.3766594 | 0.2956192 |
| 0.24 | 0.8194671 | 0.5377006 | 0.4046870 | 0.3156554 |
| 0.25 | 0.8812915 | 0.5610998 | 0.4192007 | 0.3258695 |
| 0.26 | 0.9559566 | 0.5857831 | 0.4341069 | 0.3362406 |
| 0.28 | 1.1834367 | 0.6399202 | 0.4652991 | 0.3575357 |
| 0.3 | ∞ | 0.7025652 | 0.4987277 | 0.3797153 |
| 0.32 | 1.1955687 | 0.7777866 | 0.5349833 | 0.4029780 |
| 0.34 | 0.9803269 | 0.8731657 | 0.5748478 | 0.4275551 |

*This Table was prepared sometime ago under the supervision of Dr. Goldstein, but has not been published hitherto.

TABLE 3b (Continued)/

TABLE 3b (Continued)

| x | $X_1$ = 0.3 | $X_1$ = 0.4 | $X_1$ = 0.5 | $X_1$ = 0.6 |
|---|---|---|---|---|
| 0.35 | 0.9118480 | 0.9329715 | 0.5964555 | 0.4404211 |
| 0.36 | 0.8562632 | 1.0056833 | 0.6194024 | 0.4537243 |
| 0.38 | 0.7693164 | 1.2294431 | 0.6702150 | 0.4818281 |
| 0.4 | 0.7025653 | ∞ | 0.7297037 | 0.5123000 |
| 0.42 | 0.6484818 | 1.2347423 | 0.8019070 | 0.5457056 |
| 0.44 | 0.6030569 | 1.0163328 | 0.8943846 | 0.5828058 |
| 0.45 | 0.5828055 | 0.9463090 | 0.9527724 | 0.6030569 |
| 0.46 | 0.5639006 | 0.8892138 | 1.0240931 | 0.6246624 |
| 0.48 | 0.5294736 | 0.7993177 | 1.2451163 | 0.6728272 |
| 0.5 | 0.4987276 | 0.7297037 | ∞ | 0.7297037 |
| 0.52 | 0.4709151 | 0.6728272 | 1.2451163 | 0.7993177 |
| 0.54 | 0.4454837 | 0.6246624 | 1.0240931 | 0.8892138 |
| 0.55 | 0.4335256 | 0.6030569 | 0.9527724 | 0.9463090 |
| 0.56 | 0.4220137 | 0.5828058 | 0.8943846 | 1.0163328 |
| 0.58 | 0.4001779 | 0.5457056 | 0.8019070 | 1.2347423 |
| 0.6 | 0.3797153 | 0.5123000 | 0.7297037 | ∞ |
| 0.62 | 0.3604136 | 0.4818281 | 0.6702150 | 1.2294431 |
| 0.64 | 0.3420967 | 0.4537243 | 0.6194024 | 1.0056833 |
| 0.65 | 0.3332602 | 0.4404211 | 0.5964555 | 0.9329715 |
| 0.66 | 0.3246159 | 0.4275551 | 0.5748478 | 0.8731657 |
| 0.68 | 0.3078426 | 0.4029780 | 0.5349833 | 0.7777866 |
| 0.7 | 0.2916644 | 0.3797153 | 0.4987277 | 0.7025652 |
| 0.72 | 0.2759794 | 0.3575357 | 0.4652991 | 0.6399202 |
| 0.74 | 0.2606938 | 0.3362406 | 0.4341069 | 0.5857831 |
| 0.75 | 0.2531727 | 0.3258695 | 0.4192007 | 0.5610998 |
| 0.76 | 0.2457182 | 0.3156554 | 0.4046870 | 0.5377006 |
| 0.78 | 0.2309647 | 0.2956192 | 0.3766594 | 0.4940597 |
| 0.8 | 0.2163434 | 0.2759794 | 0.3496992 | 0.4537243 |
| 0.82 | 0.2017575 | 0.2565824 | 0.3235137 | 0.4158389 |
| 0.84 | 0.1870982 | 0.2372661 | 0.2978231 | 0.3797152 |
| 0.85 | 0.1797020 | 0.2275823 | 0.2850746 | 0.3621252 |
| 0.86 | 0.1722366 | 0.2178474 | 0.2723400 | 0.3447543 |
| 0.88 | 0.1570087 | 0.1981042 | 0.2467428 | 0.3103818 |
| 0.9 | 0.1411922 | 0.1777450 | 0.2206356 | 0.2759794 |
| 0.92 | 0.1244591 | 0.1563494 | 0.1934732 | 0.2407800 |
| 0.925 | 0.1200769 | 0.1507679 | 0.1864283 | 0.2317372 |
| 0.94 | 0.1062702 | 0.1332365 | 0.1643987 | 0.2036616 |
| 0.95 | 0.0963413 | 0.1206747 | 0.1486970 | 0.1838183 |
| 0.96 | 0.0855838 | 0.1071024 | 0.1318008 | 0.1626009 |
| 0.975 | 0.0669820 | 0.0837135 | 0.1028257 | 0.1264900 |
| 0.98 | 0.0597124 | 0.0745964 | 0.0915720 | 0.1125431 |
| 0.9875 | 0.0469749 | 0.0586472 | 0.0719298 | 0.0882844 |
| 1.0 | 0 | 0 | 0 | 0 |

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