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DRAG OF CIRCULAR CYLINDERS AND SPHERES

By A. FAGE
A.R.C.Sc.

MAY 1930

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AERODYNAMIC SYMBOLS.

1. GENERAL.

m	mass
t	time
V	resultant linear velocity
Ω	resultant angular velocity
ρ	density, σ relative density
ν	kinematic coefficient of viscosity
R	Reynolds number, $R = lV/\nu$ (where l is a suitable linear dimension), to be expressed as a numerical coefficient x 10^6

Normal temperature and pressure for aeronautical work are 15° C. and 760 mm.

For air under these conditions $\begin{cases} \rho = 0{\cdot}002378 \text{ slug/cu. ft.} \\ \nu = 1{\cdot}59 \times 10^{-4} \text{ sq. ft./sec.} \end{cases}$

The slug is taken to be 32·2 lb.-mass.

α	angle of incidence
ϵ	angle of downwash
S	area
c	chord
s	semi-span
A	aspect ratio, $A = 4s^2/S$
L	lift, with coefficient $k_L = L/S\rho V^2$
D	drag, with coefficient $k_D = D/S\rho V^2$
γ	gliding angle, $\tan \gamma = D/L$
L	rolling moment, with coefficient $k_l = L/sS\rho V^2$
M	pitching moment, with coefficient $k_m = M/cS\rho V^2$
N	yawing moment, with coefficient $k_n = N/sS\rho V^2$

2. AIRSCREWS.

	volutions per second
	ameter
	D
	ver
	ust, with coefficient $k_T = T/\rho n^2 D^4$
	que, with coefficient $k_Q = Q/\rho n^2 D^5$
	ciency, $\eta = TV/P = Jk_T/2\pi k_Q$

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THE DRAG OF CIRCULAR CYLINDERS AND SPHERES AT HIGH VALUES OF REYNOLDS NUMBER.

By A. Fage, A.R.C.Sc.

Reports and Memoranda No. 1370.

(Ae. 497.)

May, 1930.

Summary.—The paper gives the results of experiments made recently to measure the drag of a circular cylinder of large diameter (23 in.). The more important measurements made in this country and abroad of the drags of circular cylinders and spheres at high values of Reynolds number are also included. An analysis of these measurements leads to the conclusion that the flow in an open-jet tunnel of the Göttingen type, with a contracting mouth and with the honeycomb at the larger end, is steadier than that in an N.P.L. type of tunnel.

The drag coefficients of a circular cylinder and of a sphere appear to be slowly increasing, at the highest values of Reynolds number attained.

Experiments on long circular cylinders and on spheres immersed in a fluid stream have shown that for each type of body there is a sensitive range of high values of Reynolds number (VD/ν) over which the drag coefficient (K_D) experiences a large drop; and also that the value of (VD/ν) at which this drop occurs is influenced by the turbulence in the general stream, in such a manner that an increase of turbulence causes the drop to occur at an earlier value of (VD/ν). Experimenters in America and Germany have made use of this characteristic of cylinders and spheres to compare the steadiness of flow in wind tunnels of different design.*

Experiments have recently been made to measure in the Duplex Tunnel the drag of a circular cylinder of large diameter (23 in.) at values of (VD/ν) beyond the sensitive range of Reynolds number. These results are given in the present note. The opportunity has also been taken to collect together the more important measurements, made both in this country and abroad, of the drag of both cylinders and spheres at high values of Reynolds number; and also to obtain from a general survey of these data further information on the comparative steadiness of different types of wind tunnel.

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* *See* Prandtl, Göttingen Nachrichten Math. Phys., 1914. Pannell, R. & M. 190. Bacon and Reid, N.A.C.A. Report, 185. Jacobs, N.A.C.A. Technical Note 312. Flachsbart, Phys. Zeit., July, 1927. Dryden & Kuethe, N.A.C.A. Report No. 342.

Drag of Cylinders.—Measurement in the Duplex Tunnel of the drag of a cylinder of diameter 23 *in.*—A hollow cylinder with thin stiff walls made of an electrical insulating material was used. The outer surface was varnished. The length of the cylinder was just sufficient to allow it to extend between the roof and floor of the tunnel. The drag of the cylinder was not measured directly, but was estimated from observations of pressure taken at a point in the median section, as the cylinder was rotated by steps of 5° through 360°. To lessen any inaccuracy arising from slight irregularities in the shape of the model two series of pressure observations were taken at two holes spaced 90° apart. The drag was estimated from the mean of these pressure distributions. The total drag of the cylinder in an infinite stream was predicted from the experimental results given in Fig. 7 of R. & M. 1223. The values of the drag coefficient estimated from the pressure distributions are given in Column (*a*) of Table 2 (appended); and the values of the total drag coefficient in the tunnel, estimated on the assumption that the frictional drag coefficient can be taken as 0·005 (*see* R. & M. 1369*), are given in Column (*b*). The final values of the drag coefficient obtained after correction for the interference of the tunnel walls, by the method described in R. & M. 1223†, are tabulated in Column (*c*).

Values of the drag coefficient for a 9 in. cylinder tested over the sensitive range of (VD/ν) in the 7 ft. No. 2 and the 4 ft. No. 1 tunnels, and for an 8·9 in. cylinder tested in the 4 ft. No. 2 tunnel are also given in Table 1. These values were obtained in the same manner as those for the 23 in. cylinder.

The results given in Table 2 are plotted in the curves B and D of Fig. 1. The agreement between these two curves in the region where they overlap is not very satisfactory; and it is not improbable that the lowest value of K_D for the 23 in. cylinder is in error, for at this speed of 25 ft./sec. the two series of pressure observations taken were not in very close agreement. The interesting features revealed are that the drag coefficient appears to be rising at a very slow rate at a highest value of (VD/ν) (Curve D), and also that the results for the sensitive range of (VD/ν) obtained from the measurements in the three N.P.L. tunnels are in fairly good agreement (Curve B).

Included in Fig. 1 are the results obtained for cylinders of diameters 42, 80 and 300 mms. tested in the open-jet tunnel at Göttingen (Curve A); and also the results for cylinders of diameters 6 in. and 8·9 in. tested 36 in. behind a rope netting in the No. 2 4-ft. tunnel at the N.P.L. (Curve C). The diameter of the rope used was 0·25 inches and the mesh 1·5 inches. The turbulence artificially created

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* Further experiments on the Flow around a Circular Cylinder.—Fage and Falkner.

† On the two-dimensional flow past a body of symmetrical cross-section mounted in a channel of finite breadth.—Fage.

in the tunnel stream by the rope netting is seen to cause the sensitive range of (VD/ν) to occur much earlier. It was also found that this effect progressively increased as the rope netting was moved toward the cylinder.*

The Curve A measured at Göttingen is observed to lie well to the right of the Curve B measured at the N.P.L. Thus whilst the sensitive range of (VD/ν) for the N.P.L. measurements is from 10^5 to 2×10^5, that for the Göttingen measurements is from 2×10^5 to 5×10^5. The conclusion may therefore be drawn from the curves in Fig. 1, that the wind in the Göttingen open-jet tunnel is steadier than that in an N.P.L. closed-jet tunnel.

Drag of Spheres.—Further information on the relative steadiness of flow in the N.P.L. and Göttingen types of tunnel can be obtained from experiments on spheres. The experimental curves selected for examination are given in Fig. 2. Of these curves those for the wind-tunnel experiments (except the dotted curves D_1 and G) were obtained when a sphere was supported on a back spindle entirely enclosed in the wake ; a method of support which is known to have a minimum interference on the flow. The dotted curves D_1 and G are included to show that the interference of other methods of support on the flow affects appreciably the measured drag. The curve D_1 was obtained at Göttingen in 1923 when a sphere was supported on two inclined wires attached to points in the great circle facing the wind (in addition to a back spindle). In later experiments the back spindle only was used and the curve D was obtained. Further, in the first experiments made at the N.P.L. to measure the resistance of a sphere (Pannell, R. & M. 190), the sphere was supported on a crosswind spindle and the curve G was obtained. This curve is seen to differ appreciably in shape from the curve H which was measured when the sphere was held on a back spindle. To obtain therefore a reliable comparison of the characteristics of flow in different tunnels the same method of support must be used. Included in Fig. 2 are the results obtained when spheres were dropped in calm air, and also when towed through air and water. No details of the method of towing are available, but if a wire at the nose were used, the method of support will differ from that used in the wind tunnels, and the two series of results will not be strictly comparable. Excluding curve A, and for the reason given the dotted curves D_1 and G, there are marked resemblances in the shapes of the curves of Fig. 2.

The principal features of the experiments from which the results shown in Fig. 2 were obtained are given in Table 1. The experiments are there referred to in the order in which the drop of K_D occurs, beginning with the experiment for which the drop occurs at the

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* *See* R. & M. 1283. The effects of Turbulence and Surface Roughness on the Drag of a Circular Cylinder.—Fage and Warsap.

(5526) Wt. 95/6003/2114 375 5/31 Hw. G.7/1

highest value of (VD/ν). It is seen that the arrangement of the experiments is such that they fall naturally into four groups :—

First Group (A, B, C), spheres tested either in calm air or water.

Second Group (D, E), spheres tested in wind tunnels which have contracting mouths with the honeycombs at the larger end.

Third Group (H, *see also* G), spheres tested in N.P.L. tunnel.

Fourth Group (J, K, L), spheres tested close behind a honeycomb, a screen or a rope netting.

Further, the conditions of flow in the first group are the most steady, whereas those in the fourth group are the most disturbed. It would appear then from the sequence of the groups that the flow in a Göttingen type of open-jet tunnel with a contracting mouth and with a honeycomb at the larger end is steadier than that in an N.P.L. type of tunnel. In other words, the disturbances introduced into the flow by a honeycomb can be appreciably damped out if subsequently the stream be made to contract by passing it through a contracting mouth of good shape. That the contraction of the stream has a beneficial effect can hardly be doubted for the relative positions of the curves F and L of Fig. 2, indicate that the flow in the rebuilt N.A.C.A. Variable Density Tunnel with its short contracting mouth is appreciably steadier than that in the original tunnel, which had no contracting mouth and within which a model was mounted close behind the honeycomb.*

The curves A, L and F of Fig. 2 indicate that the drag coefficient of a sphere tends to a constant value at the highest values of Reynolds number at which the measurements were made.

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* *See* Jacobs loc. cit.

TABLE 1.

Features of the Experiments from which the curves given in Fig. 2 were obtained.

Group No.	Curve.	Source of Information.	General features of Experiment.
1	A	N.A.C.A. No. 185, Bacon and Reid.	Spheres dropped in calm air.
	B	N.A.C.A. No. 185, Bacon and Reid.	Spheres towed in calm air.
	C	Costanzi (*see* R. & M. 190)	Spheres towed in calm water.
2	D	Flachsbart Göttingen Phys. Zeit. July, 1927.	Open-jet tunnel with a contracting mouth, and honeycomb at the large end.
	E	Bacon & Reid, N.A.C.A. No. 185.	Closed-jet tunnel, N.A.C.A. No. 1 (with fine honeycomb in front of working section removed). This tunnel has a contracting mouth with a honeycomb at the large end.
Query 2 or 3	F	Jacobs, N.A.C.A. No. 312.	N.A.C.A. variable density tunnel as rebuilt, 1928. Open-jet with a *short* contracting mouth and honeycomb at the large end.
3	H*	Fage.	5 in. sphere in 4 ft. No. 2 Tunnel (N.P.L.). (Back Spindle).
4	J	N.A.C.A. No. 185, Bacon and Reid.	N.A.C.A. (No. 1) tunnel. Sphere close behind a screen.
	K*	Fage.	5 in. sphere 36 in. behind a rope netting. N.P.L. 4 ft. No. 2 Tunnel.
	L	Bacon and Reid, N.A.C.A. No. 185.	N.A.C.A. variable Density Tunnel as first built (closed jet). Sphere close behind honeycomb.

* These results are uncorrected for the small drag of the spindle at the back of the sphere.

TABLE 2.

Cylinder Results (N.P.L.).

K_D = (Drag of a unit length of a cylinder)/$\rho V^2 D$.

n = ratio of drag of cylinder in tunnel to the drag in an infinite stream.

The values of n were obtained from Fig. 7 of R. & M. 1223.

Tunnel.	D ins.	V Ft./Sec.	Log$_{10}$ (VD/ν)	K_D (Tunnel).		K_D (Infinite stream).	Remarks.
				From Pressure Distribution.	Total.		
				(*a*)	(*b*)	(*c*)	
Duplex	23·0	25·0	5·479	0·097	0·102	0·095	n = 1·07, Fage. (First publication of results.)
				0·131	0·136	0·128	
		40·0	5·682	0·130	0·135	0·126	
		60·0	5·858	0·142	0·147	0·137	
		80·0	5·983	0·162	0·167	0·156	
7 Ft. No. 2.	9·0	30	5·150	0·445	0·450	0·433	n = 1·04, Relf. (First publication.)
		35	5·217	0·350	0·355	0·341	
		40	5·275	0·200	0·205	0·197	
		45	5·327	0·160	0·166	0·159	
		50	5·372	0·180	0·185	0·178	
		55	5·414	0·165	0·170	0·163	
		60	5·452	0·173	0·178	0·171	
4 Ft. No. 1.	9·0	25	5·071	0·568	0·573	0·503	n = 1·14, Relf. (First publication.)
		30	5·150	0·422	0·427	0·375	
		45	5·327	0·183	0·188	0·165	
		50	5·372	0·182	0·187	0·164	
		60	5·452	0·182	0·187	0·164	
4 Ft. No. 2.	8·9	22·0	5·011	0·634	0·639	0·561	n = 1·14, Fage. (From R. & M. 1179.)
		26·9	5·098	0·511	0·516	0·453	
		39·2	5·262	0·227	0·232	0·203	
		57·9	5·431	0·192	0·197	0·173	
		71·4	5·522	0·218	0·223	0·196	
		30·5	5·154	0·400	0·405	0·355	Additional Values.
		35·0	5·213	0·306	0·311	0·272	
		35·5	5·219	0·314	0·319	0·280	
		50·7	5·373	0·166	0·171	0·150	

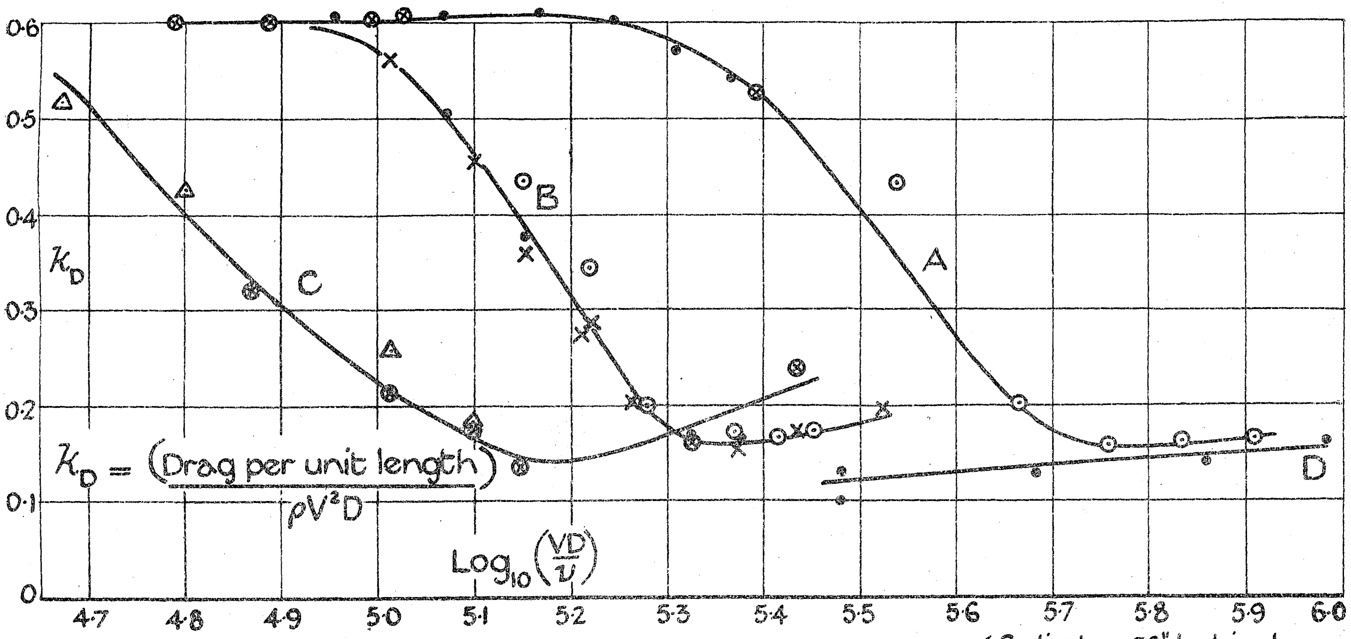
P.4110, 55-6003, 214, 775, 5/31.

R.&M.1370.

THE DRAG OF A LONG CIRCULAR CYLINDER AT HIGH REYNOLDS' NUMBER.

N.P.L. results corrected for interference of tunnel walls.

FIG.1.



Curve D, 23" cylinder N.P.L.

Curve B, 8·9" cylinder N.P.L.
- ● 4 ft No1 Tunnel
- × 4 ft No2 "
- ⊙ 7 ft No3 "

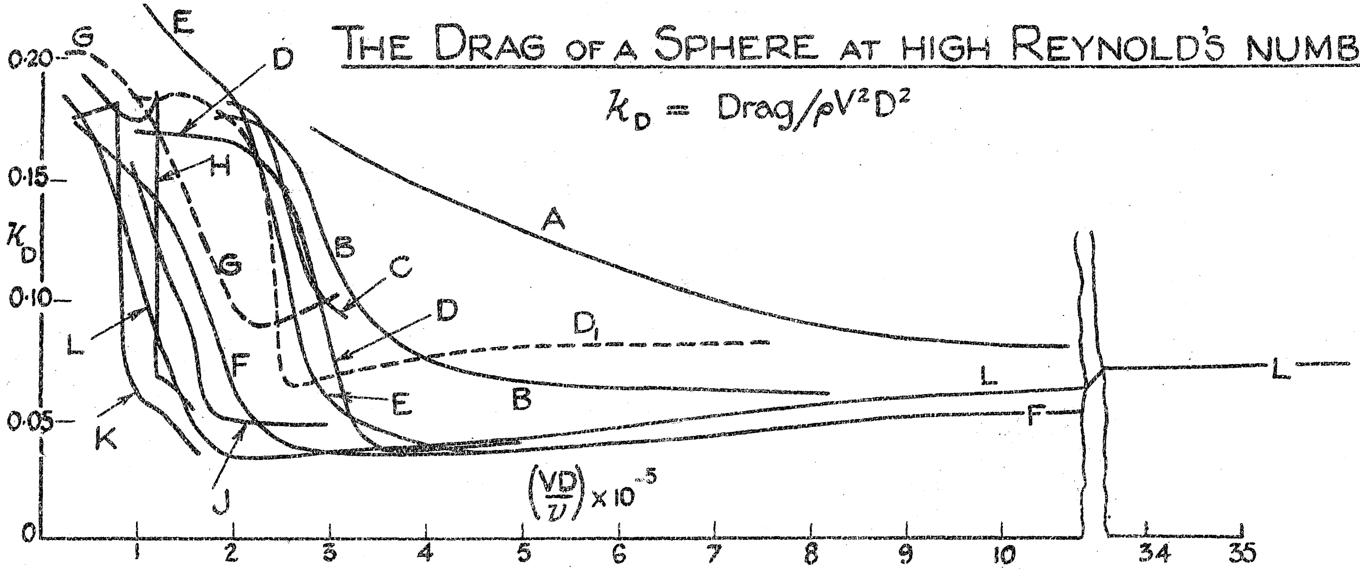
Curve C: Cylinders 36" behind rope netting. 4 ft No 2.
- △ 6" cylinder
- ⊗ 8·9" cylinder

Curve A: Göttingen open-jet Tunnel,
- ⊗ 42 mm. cylinder
- ● 80 mm. "
- ⊙ 300 mm. "

Malby & Sons, Photo Lith.

THE DRAG OF A SPHERE AT HIGH REYNOLD'S NUMBER.

$k_D = \text{Drag}/\rho V^2 D^2$

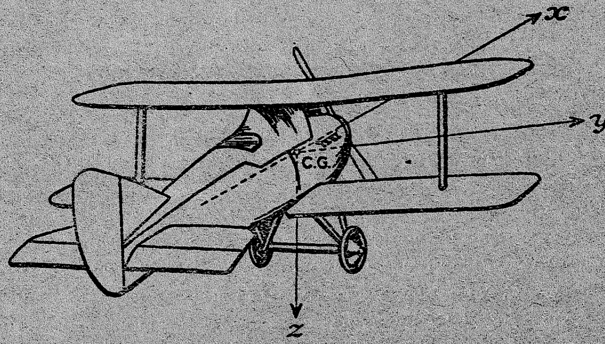


See Table 2 for further details of Curves.

Curve A, Calm air (dropping)
B, Calm air (towing)
C, Calm water (towing)
D, New curve 1926 } Göttingen open jet Tunnel with contracting mouth.
D_1, Old curve 1923 }
E, N.A.C.A.(No1) atmospheric Tunnel [contracting mouth].
F, N.A.C.A (Variable density Tunnel 1928) [open-jet with short contracting mouth.
G, 3 ft, 4 ft, 7 ft Tunnels (N.P.L) [cross-wind spindle]
H, 4 ft Tunnel ,(N.P.L) [back spindle]
J, N.A.C.A.(No 1) Tunnel [sphere behind a screen]
K, 4 ft Tunnel (N.P.L) [sphere behind rope netting]
L, N.A.C.A [Variable density Tunnel 1923] [sphere close behind honeycomb]

FIG. 2.

SYSTEM OF AXES.



Axes	Symbol Designation. Positive direction	x longitudinal forward	y lateral starboard	z normal downward
Force	Symbol	X	Y	Z
Moment	Symbol Designation	L rolling	M pitching	N yawing
Angle of Rotation	Symbol	ϕ	θ	ψ
Velocity	Linear Angular	u p	v q	w r
Moment of Inertia		A	B	C

Components of linear velocity and force are positive in the positive direction of the corresponding axis. Components of angular velocity and moment are positive in the cyclic order y to z about the axis of x, z to x about the axis of y, and x to y about the axis of z.

The angular movement of a control surface (elevator or rudder) is governed by the same convention, the elevator angle being positive downwards and the rudder angle positive to port. The aileron angle is positive when the starboard aileron is down and the port aileron is up. A positive control angle normally gives rise to a negative moment about the corresponding axis. The symbols for the control angles are :—

ξ aileron angle
η elevator angle
η_T tail setting angle
ζ rudder angle

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