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by

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SUMMARY

The theory of partial retraction of alinearly elastic ropes has been developed and illustrated with data on dynamic behaviour derived from slow-speed extension/recovery curves of nylon yarn and ropes. It shows that the retraction velocities are less than those calculated by converting all the strain energy released into kinetic energy of motion but that they can still reach 100 m/s when the tension is relaxed by about 80%. The analysis illustrates the need for direct measurements of dynamic modulus and sonic velocity in ropes.

1. Introduction

It is well known that the speed of retraction at breakage of steel, polyester and nylon ropes is dangerously high; for nylon, because of the material's high specific strain energy and low density, it is about 100 m/s. In a partial retraction much energy is initially released at a high rate of working and this property has been used to study the drawing of polymers at speed[1], the inflation of small parachutes[2] and the performance of ply-tear webbing at speed[3]. The principle of partial retraction lies behind the use of ply-tear webbing as a device to absorb the energy released by a breaking cable[4].

In the accounts of these applications the theory presented assumed a perfectly elastic, uniaxial material whereas in practice polyester and nylon ropes have marked alinear elasticity and also display non-elastic creep. It is, therefore, desirable to establish a more representative theory of partial retraction, or relaxation**, that applies to alinearly elastic materials.

This note presents a theory and discusses it using the load extension/recovery data presented in the earlier work[1,2,5]. The theory implicitly assumes fractional extension and would not be appropriate to a very extensible material like rubber for which the theory of retraction has been studied by Mason[6] treating the material as virtually incompressible. Attention is drawn to the apparent physical anomalies that arise in using the various approaches to estimate retraction velocities.

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*Replaces A.R.C.35 795.

**The phenomenon is called 'retraction' in this paper rather than 'relaxation' as in previous papers[1,2] because it appears that other authors use the former term. 'Relaxation' is sometimes associated with other plastic behaviour.

2. Simple Theory of Strain Recovery in a Perfectly Elastic Material

2.1 Definition of modulus

In basic mechanics it is usual to express the elastic modulus of a material in terms of change of stress, force per unit area, across an elementary section divided by the corresponding change in fractional extension (strain). In a system of uniaxial stress the modulus is known as Young's Modulus which, for metal rods whose area of cross-section can be satisfactorily measured, can be evaluated as defined. However, in the case of textile ropes the area of cross-section cannot be measured precisely so it is the practice to define an elastic modulus in terms of change of tension divided by the change in strain. Because of the difference in physical dimensions from Young's Modulus, force instead of stress, it has been called the Stretch Modulus. This quantity must be distinguished from 'Spring Constant' which is used with reference to a member of specific length and has the dimensions of force per unit length.

In textile technology, when dealing with fibres and yarns, it is the practice first to divide the force by the unstrained mass per unit length of the material and to call the resultant quantity the specific stress. From this a modulus is obtained by dividing the change in specific stress by the corresponding change in strain. The modulus so derived is called the specific modulus and it has the physical dimensions of velocity squared.

It is possible to put these various quantities in perspective when the velocity of propagation of changes in strain is considered. It can be shown[7] that, by the interaction of the elastic properties with the inertia of the material, cyclic variations of strain will be propagated along the rope at a finite velocity, c, which is defined by

$$c^2 = \frac{\text{Young's Modulus}}{\text{Density}} = \frac{\text{Stretch Modulus}}{\text{Mass per unit length}} = \text{Specific Modulus} \qquad ..(1)$$

The above velocity, c, strictly relates to infinitesimal changes of strain at a finite frequency in a perfectly elastic uniaxial system.

The concept of specific stress and modulus, as applied to fibres and yarns is, in this paper, extended to ropes because it shows up the efficiency with which yarns are used in the various constructions and because specific modulus has a direct dynamic significance.

2.2 Geometric continuity in the propagation

The equation of motion,[8] uniaxially, in a heavy elastic rod is

$$m \frac{\partial^2 u}{\partial t^2} = E \frac{\partial^2 u}{\partial x^2} \qquad ..(2)$$

where u, m and E are respectively the particle displacement, the mass per unit length and the Elastic (Stretch) Modulus. The distance, x, is measured from an origin at the end of the rod. This same relationship will apply to a rope so long as it is always in positive tension.

The general solution of equation (2) is of the form

$$u = f_1(ct - x) + f_2(ct + x) \qquad ..(3)$$

where f_1 and f_2 are arbitrary functions and $c^2 = E/m$. If the rope is initially at rest and a displacement suddenly applied to one end, then it is sufficient to

retain/

retain only one function to represent the propagation of the motion away from the end of the rope, that is,

$$u = f(ct - x) \qquad ..(4)$$

showing that the disturbance at $x = 0$ travels away with a velocity, c. By partial differentiation of the function with respect to the variables

$$\frac{\partial u}{\partial t} = cf'(ct - x) = -c\frac{\partial u}{\partial x} \qquad ..(5)$$

Now $\partial u/\partial t$ is the particle velocity, v, and, by definition, $\partial u/\partial x$ is the strain, e. If the distance, x, is measured positively along the rope the strain and the particle velocity will be of opposite sign and, therefore,

$$v = ce \qquad ..(6)$$

This equation holds at all points behind the wave front, irrespective of its form, and relates the particle velocity at any point with the change of strain at that point. If v and e are replaced by differential quantities dv and de, then the equation can be extended to an alinearly elastic material so long as there is negligible creep, that is, $v.\partial e/\partial x$ is much greater than $\partial e/\partial t$.

2.3 Rate of change of momentum

As the retracted rope moves it is gaining momentum towards the wave front. If the rope were perfectly elastic, then the relation $E = T/(\partial u/\partial x)$ could be used from which it follows that the tension, T, is given by

$$T = mvc \qquad ..(7)$$

This equation is the basic one in the earlier theories[1,2].

2.4 Equation of energy

It follows from equations (6) and (7), by eliminating c, that

$$\tfrac{1}{2}Te(\text{Strain energy}) = \tfrac{1}{2}mv^2(\text{Kinetic energy}) \qquad ..(8)$$

This equation has been used in the earlier work to estimate the retraction velocity of an alinearly elastic rope although one could only make equations (7) and (8) compatible by assuming that a different value of c from that given by equation (1) satisfied equation (7). Thus, whereas equation (1) defined the phase velocity a finite wave front travelled along the rope with a group velocity[8].

2.5 Partial retraction in a perfectly elastic rope

A partial retraction will require a resisting force on the end of the rope. The initial strain energy in the rope will be apportioned between residual strain energy, rope kinetic energy and work done against the resistance. Let the tension be reduced by a fraction, r. The fall in tension across the strain front will be $(1 - r)T$ and this will be equal to the rate of change of momentum of the retracting rope. Thus the velocity of retraction, v_r, will be given by

$$v_r = (1 - r)T/mc \qquad ..(9)$$

In unit time the length of rope whose state of strain and motion is changed will be of length c. The strain energy retained will be $\tfrac{1}{2}r^2Tec$, the kinetic energy gained

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by the rope will be $\frac{1}{2}(1 - r)^2Tec$ and the work done will be rTv_r which can be rewritten as $r(1 - r)Tec$. Thus it is seen that

$$\tfrac{1}{2}r^2Tec + \tfrac{1}{2}(1 - r)^2Tec + r(1 - r)Tec = \tfrac{1}{2}Tec. \qquad ..(10)$$

Equation (7) can be rewritten in a form for a partial retraction. If T_o is the initial tension in the rope and it is allowed to fall to a tension, T_p, then the partial retraction velocity will be v_p given by

$$T_c - T_p = mcv_p \qquad ..(11)$$

and the rate of working (power) of the system will be $T_p v_p$.

3. Alinear Elastic Theory

3.1 Definition of modulus

When the stress-strain curve of a rope is no longer straight and proportionality between T and e no longer holds, it is necessary to redefine the modulus as a differential dT/de. Its value must be related to the appropriate tension, T. In a real rope which shows plastic creep the stress-strain curve for increasing load will differ from the unloading curve and their slopes at the same level of tension will differ. In a slow-speed load-extension curve the creep taking place during the change in strain could be significant and it is, therefore, desirable to express the modulus as a partial differential

$$dT = \left(\frac{\partial T}{\partial e}\right)_t de + \left(\frac{\partial T}{\partial t}\right)_e dt \qquad ..(12)$$

In imperfect elastic materials it has been the practice in past work[1,2] to define the elastic (stretch) modulus at a given tension, T, as the slope of the tension-strain recovery curve immediately below this value. However, it could be too high because, if the rope is held at the constant strain, the tension in the rope will be observed to fall. Load-extension curves are usually performed on test machines at a constant rate, say a, and, therefore, the apparent modulus dT/de can be related to the true modulus by

$$\frac{dT}{de} = \left(\frac{\partial T}{\partial e}\right)_t + \frac{1}{a}\left(\frac{\partial T}{\partial t}\right)_e \qquad ..(13)$$

$\frac{\partial T}{\partial e}$ will be different for increasing and decreasing strain and it is also known that it is dependent upon, from the general knowledge of polymeric materials, on the preceding tension-time integral.

In the interpretation of the modulus $\partial T/\partial e$ the constant time relates to the moment of stress reversal. For the purposes of the further analysis the modulus will be considered as a complete differential which is a function of T.

3.2 Determination of retraction velocity

In place of equation (6) the following equation is used

$$dv = c.de \qquad ..(14)$$

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and c^2 is given by $(dT/de)/m$. Equation (14) is assumed to hold for infinitesimal changes of strain.

If the concept of specific stress, T_s, of textile technology is introduced the modulus can be written dT_s/de and is the same as the specific modulus M_s. Thus one can write

$$c^2 = dT_s/de(T_s) = M_s(T_s) \qquad ..(15)$$

indicating that the modulus and the velocity of propagation of strain disturbances, the sonic velocity, is a function of T_s. Therefore, the retraction velocity can be expressed as an integral

$$v = \int_e^{e_o} \left(\frac{dT_s}{de}\right)^{\frac{1}{2}} de \qquad ..(16)$$

Because c now depends upon the tension, equation (7) no longer holds and the strain energy given up no longer equals the kinetic energy acquired by the rope. A thermodynamic equilibrium has to be satisfied.

3.3 Determination of a consistent stress-strain relationship

In order to perform the integration of equation (16) it is necessary to express dT_s/de as a function of the strain, e, whereas it has been determined for particular stress levels from which a plot of dT_s/de against T_s can be constructed. With respect to a particular initial tension T_o the drop in the strain level corresponding to a lower stress level, T_s, can be determined from the following integration,

$$e_o - e = \int_{T_s}^{T_o} 1/(dT_s/de).dT_s \,, \qquad ..(17)$$

from which a purely elastic recovery curve can be constructed. Whilst this conversion has not been used in this paper it has elsewhere[9] for data on nylon fibre where it indicates that the estimated elastic recovery departs more steeply from the experimental strain recovery curve at low stresses. For a relief of stress not more than half there is very little difference between the experimental and derived curves. In this paper the transposition of dT_s/de from a function of stress to one of strain has been made with reference to the experimental stress-strain recovery curves for the evaluations in Figure 7 onwards. This procedure will give overestimates of the retraction velocity particularly when the relief of stress is substantially complete.

3.4 Energy in an alinear material

The specific strain energy, E_s, released by the rope can be expressed by

$$E_s = \int_e^{e_o} T_s.de \qquad ..(18)$$

and the specific kinetic energy, K_s, is given by

K_s/

$$K_s = \tfrac{1}{2}v^2 = \tfrac{1}{2}\left(\int_{e}^{e_0} \left(\frac{dT_s}{de}\right)^{\frac{1}{2}} de\right)^2. \qquad ..(19)$$

Although it is possibly more convenient to evaluate the retraction velocity and the energies by graphical integrations of the appropriate variables plotted against the strain, further analysis is possible if T_s is expressed as a polynomial function of e. (See Appendix A).

4. Extension/Recovery Data for Ropes

4.1 Representation of data

The load extension/recovery data for the ropes referred to in earlier work are represented in Figs. 1-3 using S.I. Units. It is interesting to note the difference in the characteristics of the two nylon ropes (Figs. 2-3) because the latter shows much more hysteresis than the former. This difference probably depends upon the history of each rope: the former was a much used rope and treated with a proofing agent to reduce abrasion; the latter was a new, untreated rope as tested by the manufacturers. The difference may be an indication of the degree of hysteresis that can be worked out of ropes of nylon by use.

It is interesting to compare the same data for the ropes with that on fibres and yarns when all is presented on a specific stress basis. This is done in Fig. 4 which shows how efficiently the intrinsic strength of the nylon yarn is achieved in the constructions. The tenacity (specific stress to break) drops and the ultimate extension increases as the construction becomes more complex and bulky. The information for a tubular woven rope, nylon tubular webbing to Specification IAC S1116, is included for comparison because this construction is representative of the cord described in Fig. 1.

4.2 Efficiency of construction

The efficiency of the various constructions can be compared on the basis of specific energy and stress. This has to be done in Table 1 below for Nylon Type 242. It is seen that, as the fibres become compounded into larger woven structures, the

Table 1

Comparison of Specific Energies Recovered and Expended

	Specific Stress	% Ult.	Work to Stretch x 10^4	Energy Recovered x 10^4	Work Expended	Scaled to 70% Ult. Recovered x 10^4	Scaled to 70% Ult. Expended x 10^4
Fibre	8.2×10^5	89%	4.63	2.46	2.17×10^4	2.07	1.83
Yarn	6.2	82%	2.70	1.55	1.15	1.42	1.05
S1116	3.6	86%	2.96	2.26	0.70	1.97	0.81
Rope A Fig.2	1.5	66%	1.30	1.01	0.29	1.15	0.33
Rope B Fig.3	1.8	68%	2.13	1.07	1.06	1.18	1.17

S.I. Units

specific stress that can be supported falls. The specific work to stretch and the specific energy recovered also fall but not so markedly and this comparison

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has been based upon the measured values being scaled to 70% ultimate tenacity assuming that these small corrections can be made on the basis that specific energy is proportional to the square of the specific stress as if the material were perfectly elastic. Although S1116 nylon tubular webbing is an efficient construction, because the tension-bearing yarns can straighten under load without pressing on one another, the values for it, quoted in Table 1, appear to be rather too high as one would hardly expect the webbing to be better than the yarn. On the other hand, Rope A is about 30% heavier than its unproofed counterpart and this would make the efficiency of the unproofed rope comparable with the yarn.

The data in Table 1 has been collected from miscellaneous sources and, as it has been impossible to check the original measurements, it is to be expected that some small discrepancies may be presented. However, the value of the basis for comparison has been illustrated and it is recommended that a broader range of constructions are studied and compared in this way.

5. Evaluation of Retraction Velocity

5.1 The elastic moduli

The moduli of the ropes have been determined from the slopes of the stress recovery curves. Although this estimate may be affected by plastic creep during the time of unloading, evidence from tests on fibres[9] shows that it is not significant and within the error within which one can assess the slope of a curve. Creep will be neglected at this stage to simplify the analysis. Plots of the specific modulus against specific stress for the ropes are presented in Fig. 5. In this figure the determination of this modulus for one of the ropes (S1116) by an oscillation method[10] is plotted for comparison; it shows that the effective modulus for vibrations increases more with specific tension than that determined from the slope of the recovery curve. However, the moduli for the fibre and yarn are much higher than for the ropes and it is necessary to plot these basic elements against a more closed-up scale of specific modulus in Fig. 6 including the stiffest of the ropes (S1116) for comparison. The measurement of these moduli from steep-sloped recovery curves can only be made approximately and it may be fortuitous that the yarn appears to be slightly stiffer.

5.2 The sonic velocity

In the absence of direct measurements of the sonic velocity in the ropes, this quantity has been determined from the square root of the specific modulus. Plots of this velocity against strain are required and these are given in Fig.7. In the same figure the retraction velocity is also plotted against strain, obtained by integration under the sonic velocity curves, assuming that the strain has been relieved suddenly from the level at which the tension was reversed in the recovery test.

The slope of the recovery curve for a fibre is extremely steep and the estimates obtained therefrom of the modulus and sonic velocity at the high tensions are much higher than that to be expected.[11]

5.3 Retraction velocity

The retraction velocity has been calculated for particular initial conditions by using equation (16). A plot of the retraction velocity against specific tension will give the partial retraction velocity at that tension for the given initial tension. Plots for the examples evaluated are given in Fig. 8.

It is strictly necessary to make the above calculations for each initial tension selected. However, from a small number of selected cases it is possible to interpolate for any initial tension.

It is noted that the alinear trend of the retraction velocity with specific tension is to make the curves slightly concave upwards. There is not a great difference between the two ropes of earlier work although the older rope has a steeper curve. It is useful to compare these estimates of retraction velocity with those based upon the direct equation of strain energy to kinetic energy. This has been done for ropes A and B in Fig. 9. It is seen that the equation of energy substantially overestimates the retraction velocity in a partial retraction; in fact, only a half, or slightly less, of the strain energy goes into kinetic energy for the remainder must be dissipated as heat. It is noted, however, that the disparity is least, or may even show a gain of energy, in a complete retraction but it is expected that, for such a large change in strain level, the sonic group velocity is just that much less than the sonic phase velocity, as estimated from the modulus, so that the estimate of the retraction velocity in such an extreme case is lowered so that the kinetic energy gained does not exceed the strain energy given up.

From the ratio of velocities in Fig. 9 it is possible to calculate the efficiency of the conversion. The square of these velocities is the energy ratio and the percentage of energy converted into retraction energy plotted against the percentage relief of tension in Fig. 10. It appears that a better conversion occurs for the greater relief of tension. This is a little surprising but it must be remembered that any marked disparity between phase and group velocities has not been taken into account and it is the group velocity that must be used in equation (16) if the continuity of the solid medium of the rope is to be satisfied.

The two ropes show much the same trend and in view of the uncertainties in the data and of the approximate methods of calculation, no account should be taken of the small differences.

6. Propagation of Finite Strain-Recovery Fronts

The data in the paper points to the fact that a small disturbance of strain will travel faster in a rope at a higher tension than at a lower tension. Thus, if a finite drop in tension occurs causing initially a finite relief of strain, the strain front will not remain sharp as it travels away; it transforms into a wedge front becoming flatter the further it travels along the rope. This is illustrated in Fig. 11.

Although one may initiate a sharp recovery front with the breakage of a rope it must be remembered that, when dealing with mechanisms like catapults and safety devices, releases and links cannot be designed without introducing small terminal inertias. These can be very important in suppressing the sharpness of any initial recovery phenomena.

7. The Measurement of Dynamic Modulus and Sonic Velocity

Whereas the analysis of this paper has used estimates of the elastic modulus based upon the slopes of slow-speed stress curves, it is apparent that direct measurement of the elastic modulus by a dynamic method will give different values. This has been revealed in Fig. 5 for a particular material. The method of allowing a weight to spring on a length of rope[10] is a simple experiment that can be arranged to cover a range of tensions and frequencies. This method would tend to give an estimate of the phase velocity. The experiment that is particularly desirable, although not so simple to perform, is to study the travel of finite stress/strain pulse along a length of rope, its change of form and effective velocity of travel. Field experiments have been conducted with ropes[12] but it would be preferable if one could devise some laboratory experiments on a modest scale. Such experiments should reveal whether differences between phase and group velocities are significant.

8. Conclusions

By using a method of analysis applicable to an alinearly elastic material the retraction velocity of a nylon rope following a partial relief of tension is shown to be less than that calculated from a complete conversion of the strain energy given up. On the revised basis of calculation it appears that ropes of the sizes used and initially tensioned to about two-thirds of the ultimate strength will retract at about 60 m/s when the tension is relaxed to half; when relaxed to 20% of the initial tension the retraction velocity will reach 100 m/s.

Whilst the analysis has been illustrated using estimates of moduli obtained from the slopes of slow-speed extension/recovery curves, doubts have been cast regarding the viability of these estimates because they appear to be too high at the high tensions, in particular with respect to the fibre. Thus there is a clear case for reliable dynamic measurements.

A comparison has been made of the energy-absorbing capacities and the specific strength of the ropes in relation to the fibre and yarns from which they are made. As ropes are made larger it appears that, although the extensibility is increased, the specific tension and specific energy that can be realised is less.

9. Recommendations

Although there is a doubt about the use of slow-speed tests to estimate dynamic characteristics it is felt that there is a need to determine the slow-speed extension/recovery curves on a range of construction of ropes. All the major variants should be covered including braid over braid, hollow tubular webbing and conventional webbing. In addition, creep tests at tension should be made to discover whether corrections are necessary to the apparent modulus.

The dynamic modulus should be measured by methods of longitudinal resonance such as by oscillating a weight on a length of rope or by resonating a diaphram inserted in a tensioned rope. Direct measurements of the velocity of propagation of strain pulses along ropes, together with a study of the distortion of the strain pulses, should be made. This should be done first for incremental strain pulses at various tensions and later for larger changes of strain. Whilst the sonic velocity can be estimated from the dynamic modulus, it may not agree with that measured directly and the comparison may throw some light on the phenomena of phase and group velocities.

References/

References

No.	Title, Author, etc.
1	A technique for studying the high-velocity drawing of polymers.- G.W.H. Stevens and F.C. Bluett. ARC CP No.1061. 1966.
2	The principle of a nylon rope catapult for testing parachutes.- G.W.H. Stevens. RAE TR No.65179. ARC 27 773 - EP 1015. August 1965.
3	Tests on ply-tear webbing.- J.C.H. Longrigg. RAE TR No.69012. ARC 31 183 - Strut 3087 - EP 1267. February 1969.
4	Ply-tear webbing energy absorber.- G.W.H. Stevens. 7th Aerospace Mechanisms Symposium MSC 1972, NASA TM X-58106. November 1972.
5	The RAE nylon rope catapult.- F.C. Bluett and J.C.H. Longrigg. RAE TR No.69108. ARC 31 977 - EP 1347. May 1969.
6	Finite elastic wave propagation in rubber.- P. Mason. Proc.Roy.Soc. Vol.272, pp 319-321. March 1963.
7	The mathematical theory of elasticity.- A.E.H. Love. Cambridge Univ. Press. 1927.
8	Stress waves in solids.- H. Kolsky. Oxford Clarendon Press. 1953.
9	The estimation of elastic modulus and sonic and retraction velocities of nylon fibres and yarns from the slow-speed extension and recovery characteristics.- G.W.H. Stevens. Journal Textile Institute, Vol.66, No.7, pp 255-267. July 1975.
10	A theory of vibrations in parachutes.- G.W.H. Stevens. Journal of Aircraft, Vol.9, No.1, pp 74-78. January 1972.
11	An investigation of the dynamic properties of some high polymers.- K.W. Hillier and H. Kolsky. Proc.Phys.Soc.(B), Vol.62, pp 111-120. February 1949.
12	The propagation of strain in textile cables under longitudinal impact.- B.J. Brinkworth. Proceedings of Conference on the Properties of Materials at High Rates of Strain, 1957, Institute of Mechanical Engineers - London.

APPENDIX/

APPENDIX A

Analytical Determination of the Retraction Velocity and the Energy Balance

The experiments[10] to determine the dynamic modulus of a rope indicate that the modulus is of the form

$$dT_s/de \;=\; a + bT_s \;=\; c^2. \qquad ..(A1)$$

Experiments[9] on fibres indicate that c is of the form

$$c \;=\; c_o + c_1 T_s. \qquad ..(A2)$$

From either of these forms it is possible to construct a representative stress/strain recovery curve and express T_s and c as functions of the strain e. The relationship (A1) is more practical to manipulate. The solution of equation (A1) for the condition $e = 0$ for $T_s = 0$ is

$$\log_e\left(1 + \frac{bT_s}{a}\right) \;=\; be. \qquad ..(A3)$$

By rearrangement and substitution from equation (A3)

$$T_s \;=\; a/b.(\exp.be - 1) \qquad ..(A4)$$

$$dT_s/de \;=\; a.\exp.be \qquad ..(A5)$$

$$c \;=\; a^{\frac{1}{2}}\exp.\tfrac{1}{2}be. \qquad ..(A6)$$

In a partial retraction the initial strain energy is partitioned into

(a) strain energy retained,
(b) kinetic energy gained,
(c) work done against resistance to retraction,
(d) heat exchange.

Let the initial strain be e_o, which is allowed to fall to a value e, then

$$\int_o^e T_s de + \tfrac{1}{2}v^2 + T_s v/c_E + H\ (\text{heat}) \;=\; \int_o^{e_o} T_s de \qquad ..(A7)$$

The retraction velocity is given by

$$v \;=\; \int_e^{e_o} c.de \qquad ..(A8)$$

and the effective mean value of the sonic velocity, c_E, by

$$c_E \;=\; \int_e^{e_o} c.de \Big/ \int_e^{e_o} de. \qquad ..(A9)$$

It is thus possible to write

$$\int_o^e T_s de + \tfrac{1}{2}\left(\int_e^{e_o} c.de\right)^2 + T_s(e_o - e) + H \;=\; \int_o^{e_o} T_s de. \qquad ..(A10)$$

Substitutions for T_s and c as functions of e can now be made from equations (A4) and (A6) respectively and the integrals evaluated.

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$$\tfrac{1}{2}\left(\int_{e}^{e_o} a^{\frac{1}{2}}\exp.\tfrac{1}{2}be.de\right)^2 + a/b.(\exp.be - 1)(e_o - e) + H = \int_{e}^{e_o} a/b.(\exp.be - 1).de. \qquad ..(A11)$$

The above equation (A11) can be written more compactly if x is written for be

$$\tfrac{1}{2}\left(\int_{x}^{x_o} a^{\frac{1}{2}}/b.\exp.\tfrac{1}{2}x.dx\right)^2 + a/b^2.(\exp.x - 1)(x_o - x) + H = \int_{x}^{x_o} a/b^2(\exp.x - 1).dx. \qquad ..(A12)$$

On integration of the particular terms

$$2a/b^2(\exp.\tfrac{1}{2}x_o - \exp.\tfrac{1}{2}x)^2 + a/b^2(\exp.x - 1)(x_o - x) + H = a/b^2(\exp.x_o - \exp.x) - a/b^2(x_o - x) \qquad ..(A13)$$

or

$$2(\exp.\tfrac{1}{2}x_o - \exp.\tfrac{1}{2}x)^2 + (\exp.x)(x_o - x) + b^2H/a = \exp.x_o - \exp.x.$$

The above equation (A13) can be simplified further to

$$\exp.x_o - 4\exp.\tfrac{1}{2}(x + x_o) + 3\exp.x + (x_o - x)\exp.x + b^2H/a = 0. \qquad ..(A14)$$

The numerical results for a number of light ropes[10] show that x_o will be slightly larger than unity so that the use of expansions of the exponentials will not be very helpful. However, it is important to note that, in equating all the terms of a particular order in the expansion, all orders below the third vanish by the terms cancelling out. The third order term is

$$x_o^3/6 + x_o^2x/4 + x_ox^2/3 + 5x^3/12 + b^2H/a = 0 \qquad ..(A15)$$

which would require a negative value for H, that is, extraction of heat. This is not the result obtained by graphical calculation which shows that not all strain energy is converted to kinetic energy over the more important range of values. However, it is noted that, in equation (A14), the first three terms can be expressed as a pair of factors thus,

$$(\exp.\tfrac{1}{2}x_o - 3\exp.\tfrac{1}{2}x)(\exp.\tfrac{1}{2}x_o - \exp.\tfrac{1}{2}x) + (x_o - x)\exp.x + b^2H/a = 0. \qquad ..(A16)$$

In a practical partial retraction a drop in strain level to one half is typical; therefore,

$$\exp.\tfrac{1}{2}x_o\left(\exp.\frac{x_o}{4} - 3\right)\left(\exp.\frac{x_o}{4} - 1\right) + \tfrac{1}{2}x_o + b^2H/a = 0. \qquad ..(A17)$$

If negative values of H are possible they can only arise when $\exp.\frac{x_o}{4}$ has a value between 1 and 3 and, then, only in a restricted range near the value 2.

It should be remembered that, in equation (A3), the boundary condition is that $T_s = 0$ when $e = 0$, instead of the residual strain e_r. However, equation (A12) is unchanged whether $x = be$ or $x = b(e - e_r)$. This is because the effective origin for equating the energy changes is e_o, or x_o.

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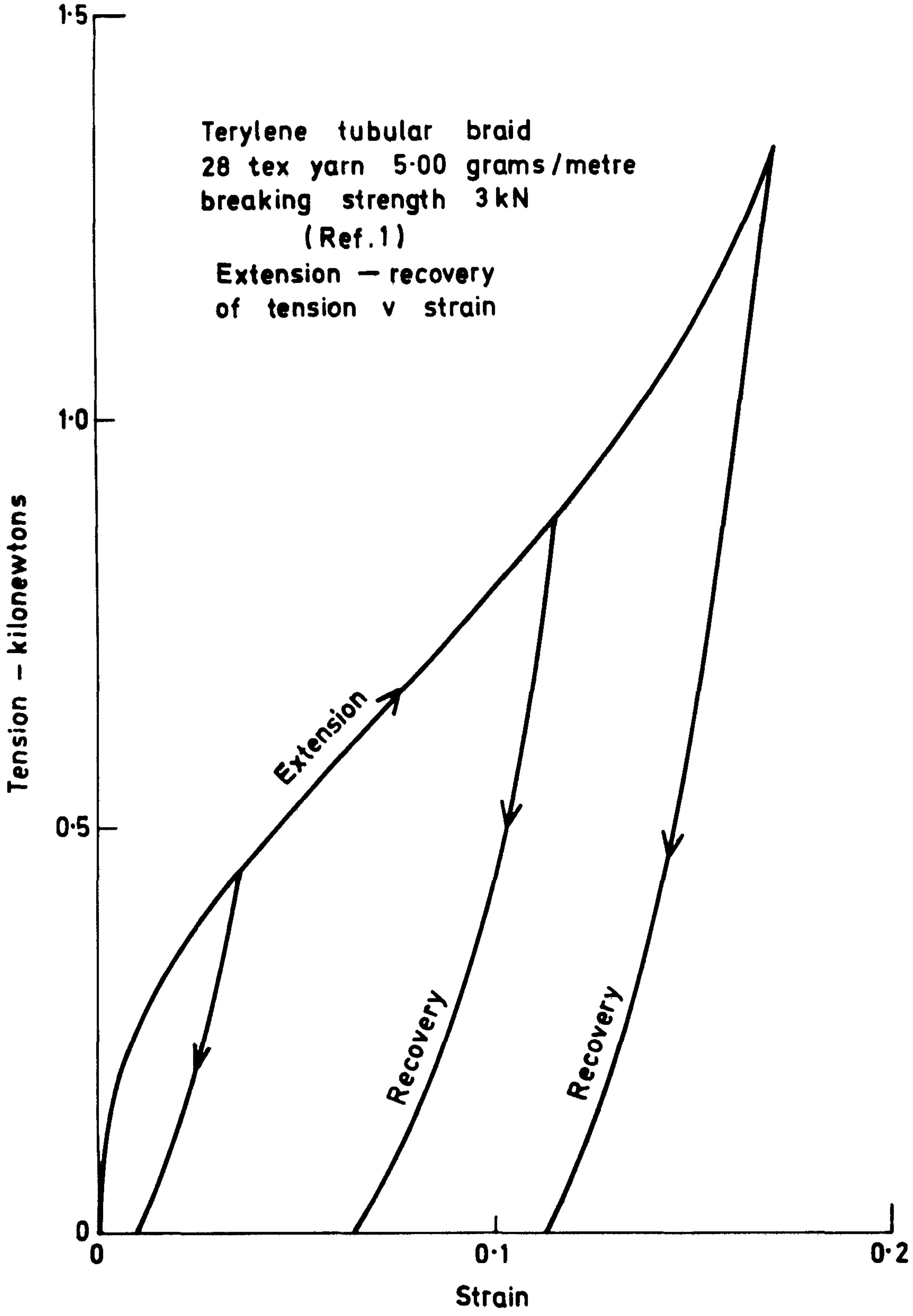

Terylene tubular braid
28 tex yarn 5·00 grams/metre
breaking strength 3 kN
(Ref.1)
Extension — recovery
of tension v strain
Extension
Recovery
Recovery
Tension – kilonewtons
1·5
1·0
0·5
0
0
0·1
0·2
Strain

FIGURE 1

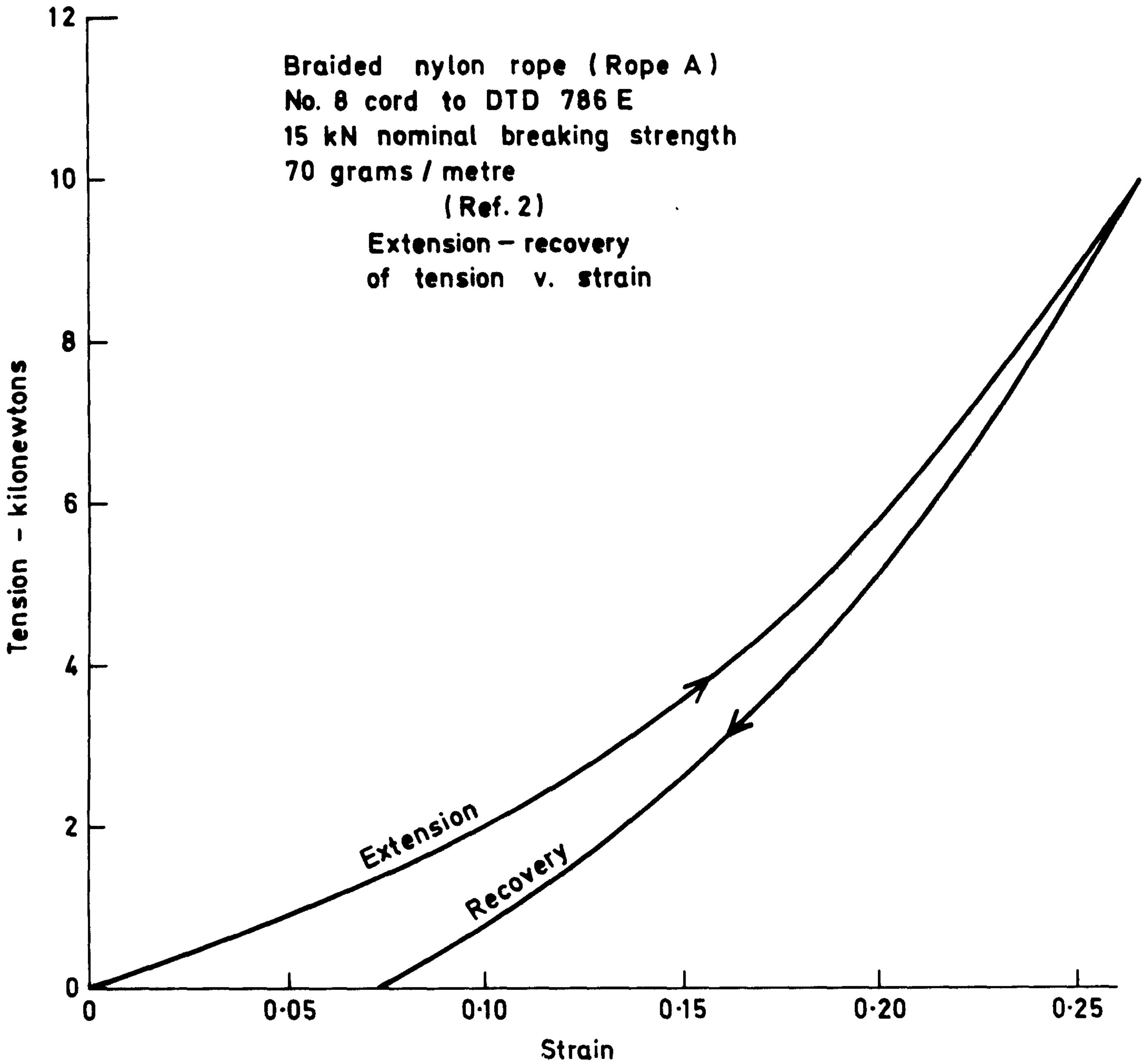

Braided nylon rope (Rope A)
No. 8 cord to DTD 786 E
15 kN nominal breaking strength
70 grams / metre
(Ref. 2)
Extension – recovery
of tension v. strain
Tension - kilonewtons
Strain
0
2
4
6
8
10
12
0
0·05
0·10
0·15
0·20
0·25
Extension
Recovery

FIGURE 2

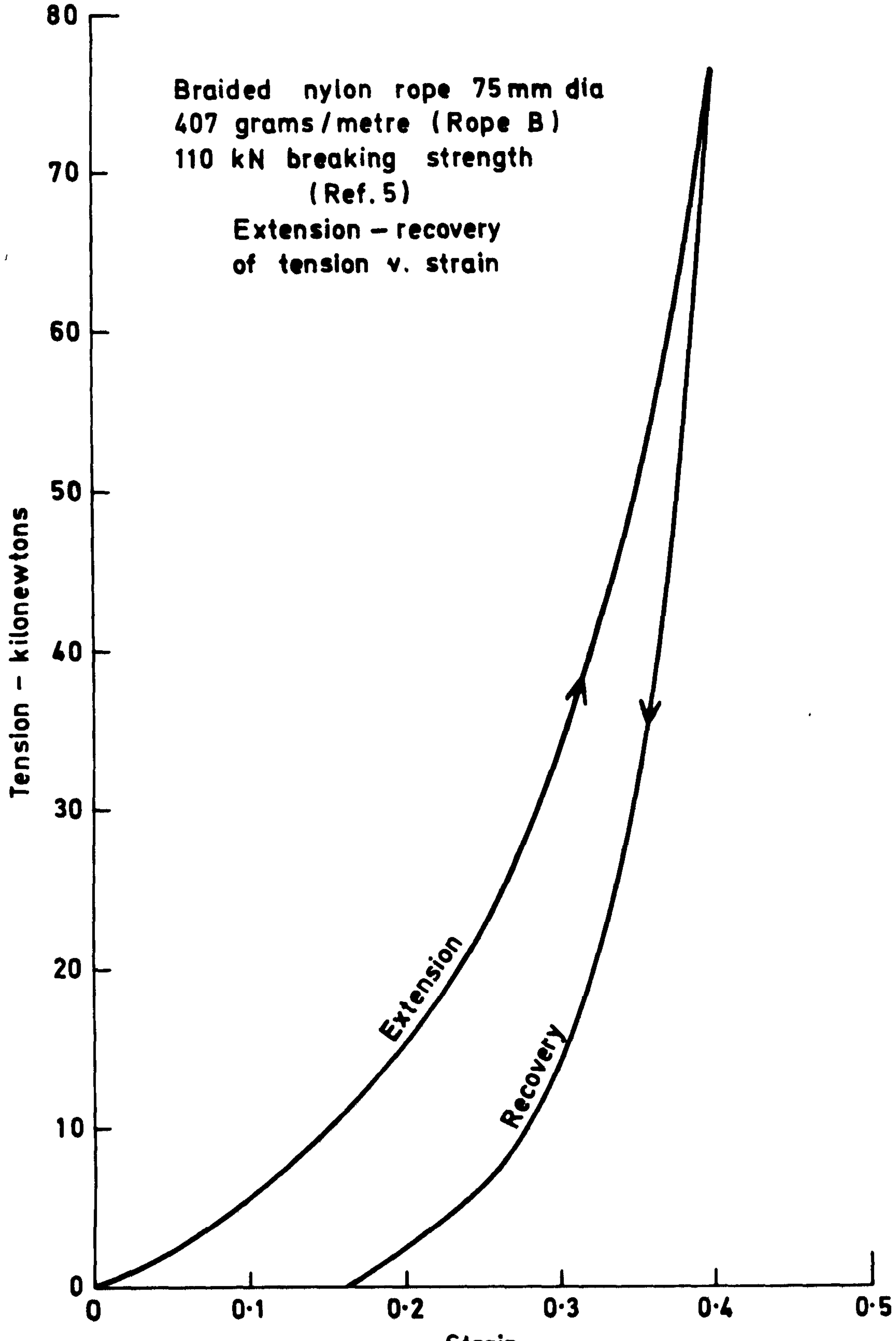

Braided nylon rope 75 mm dia
407 grams/metre (Rope B)
110 kN breaking strength
(Ref. 5)
Extension – recovery
of tension v. strain
Tension – kilonewtons
Strain
Extension
Recovery
0
10
20
30
40
50
60
70
80
0·1
0·2
0·3
0·4
0·5

FIGURE 3

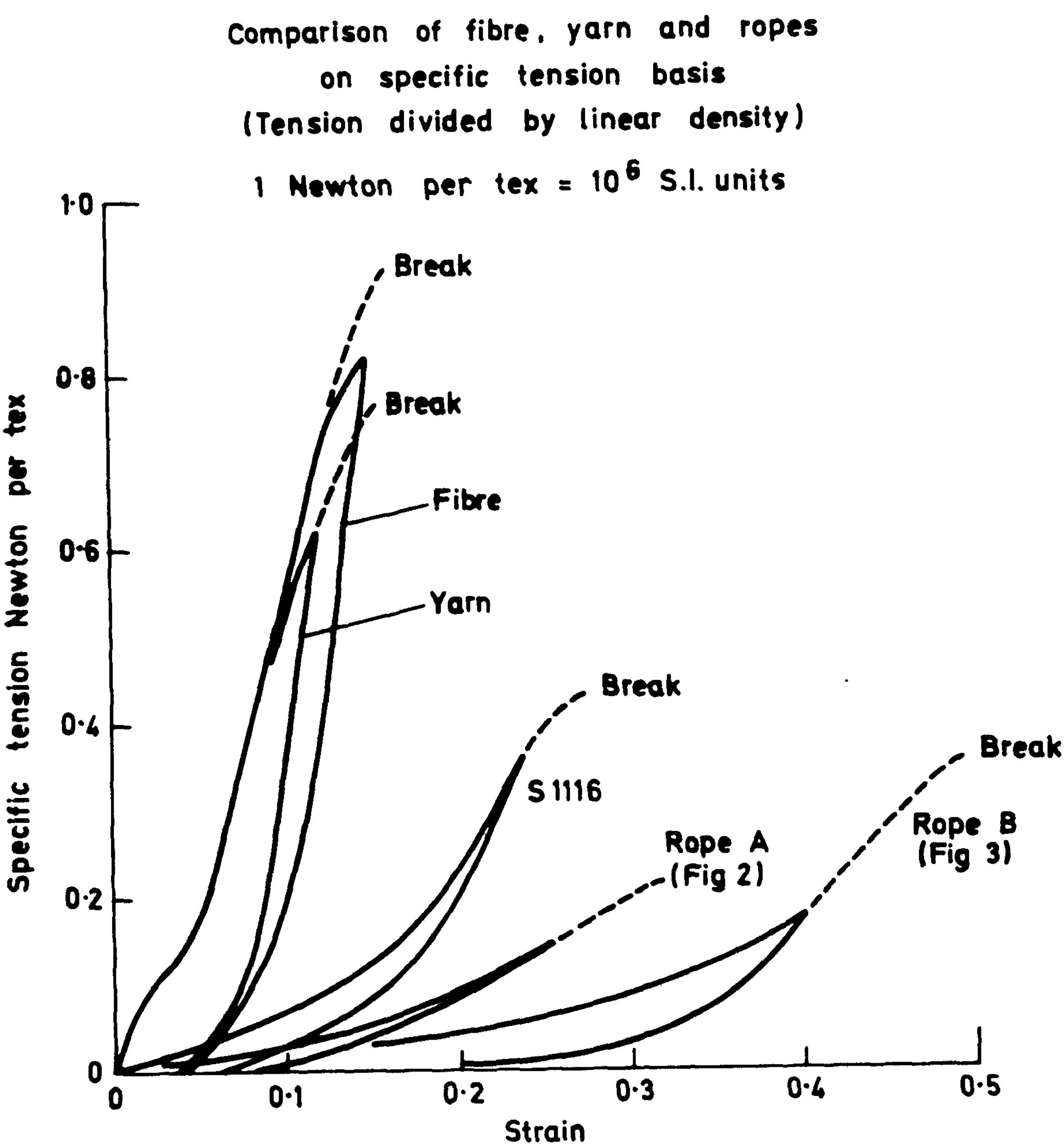

Comparison of fibre, yarn and ropes
on specific tension basis
(Tension divided by linear density)
1 Newton per tex = 10^6 S.I. units
Break
Break
Fibre
Yarn
Break
S 1116
Break
Rope A
(Fig 2)
Rope B
(Fig 3)
Specific tension Newton per tex
1·0
0·8
0·6
0·4
0·2
0
0
0·1
0·2
0·3
0·4
0·5
Strain

FIGURE 4

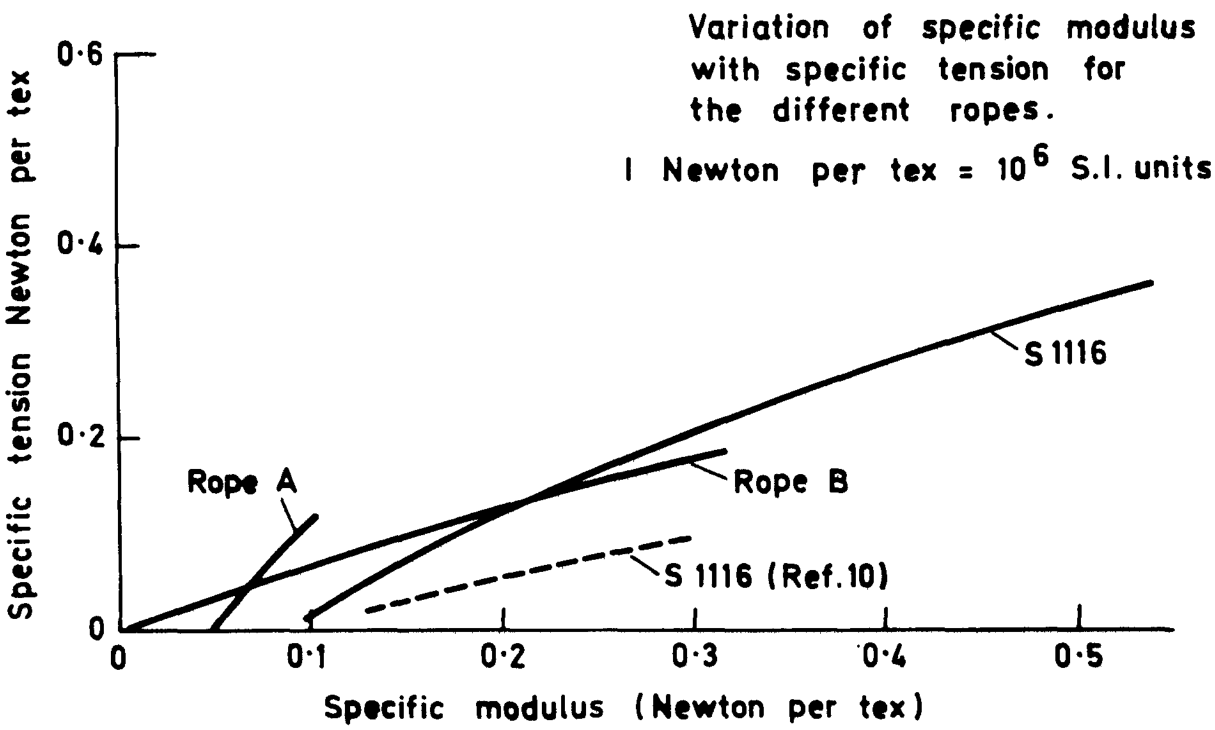


FIGURE 5

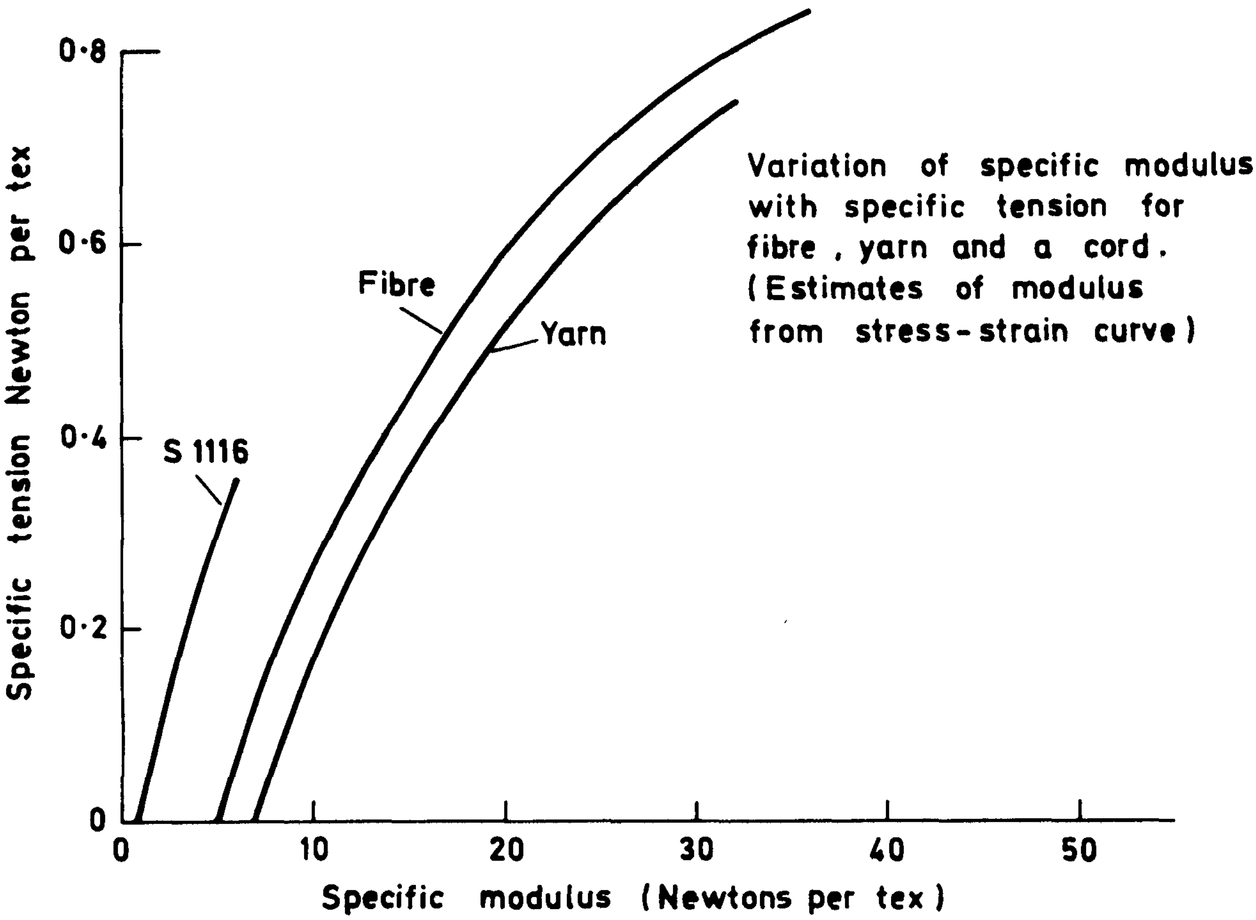
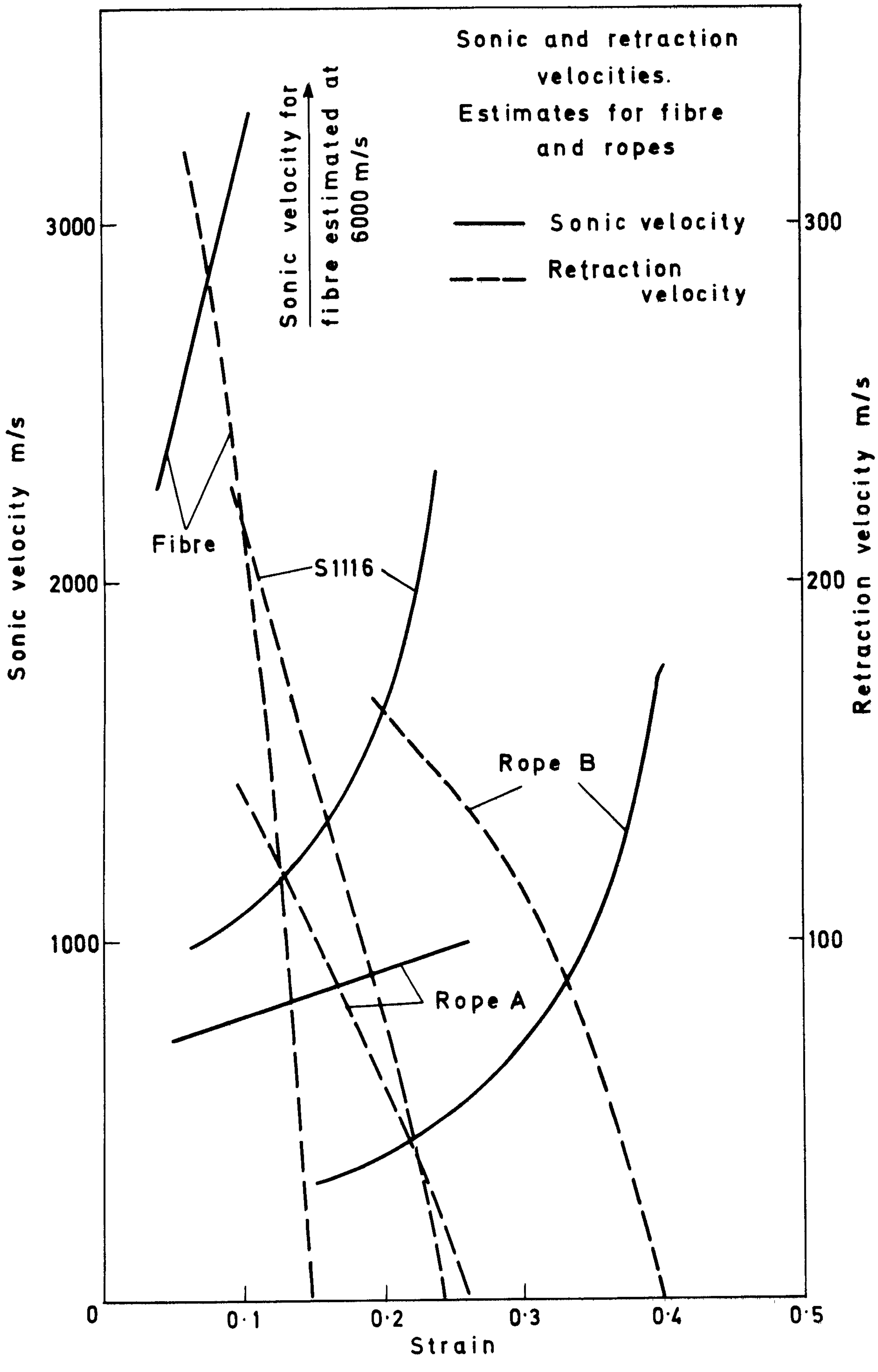


FIGURE 6


Sonic and retraction
velocities.
Estimates for fibre
and ropes
Sonic velocity for fibre estimated at 6000 m/s
Sonic velocity
Retraction velocity
Fibre
S1116
Rope B
Rope A
Sonic velocity m/s
Retraction velocity m/s
3000
2000
1000
300
200
100
0
0·1
0·2
0·3
0·4
0·5
Strain

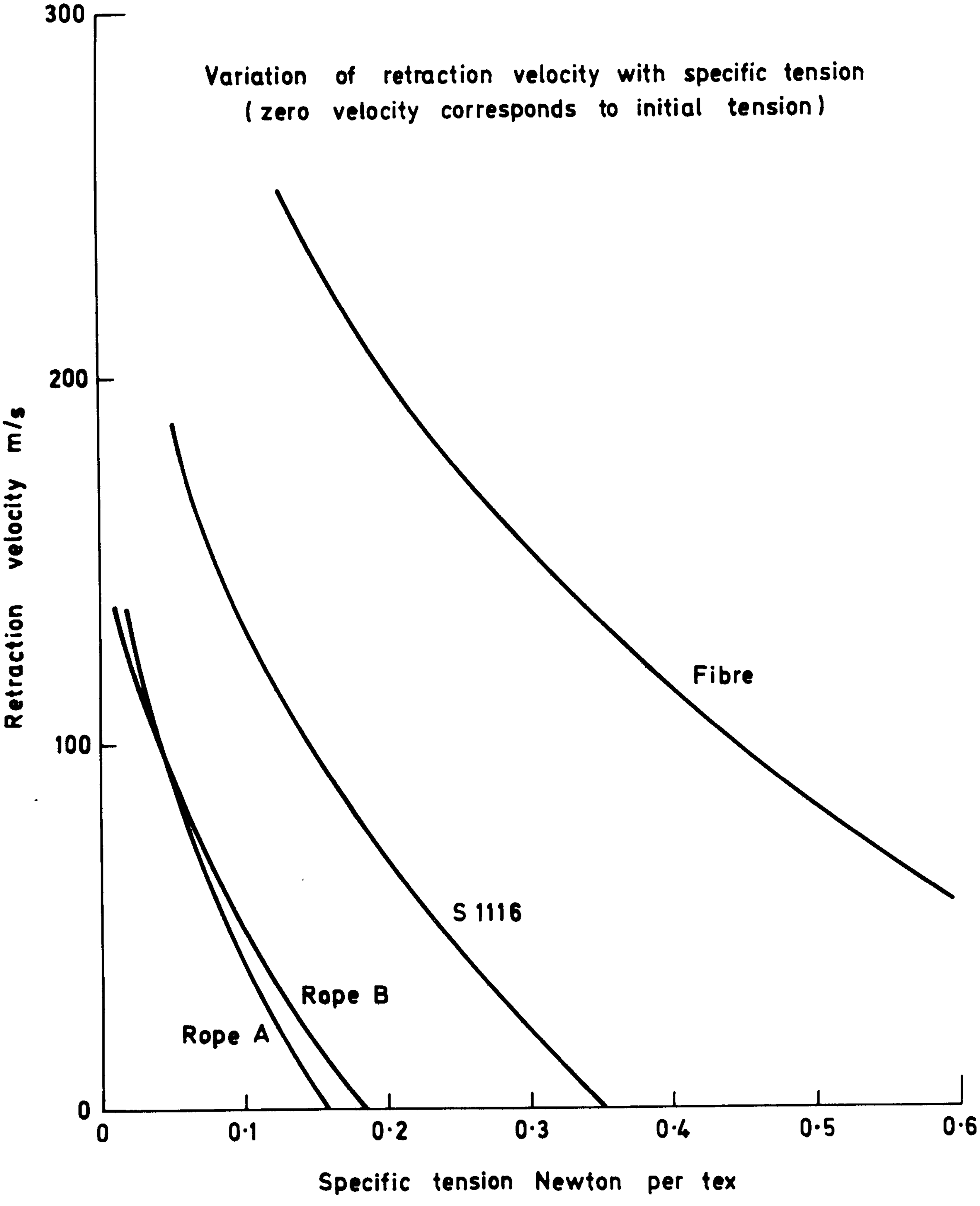

300
200
100
0
Variation of retraction velocity with specific tension
(zero velocity corresponds to initial tension)
Retraction velocity m/s
Fibre
S 1116
Rope B
Rope A
0
0·1
0·2
0·3
0·4
0·5
0·6
Specific tension Newton per tex

FIGURE 8

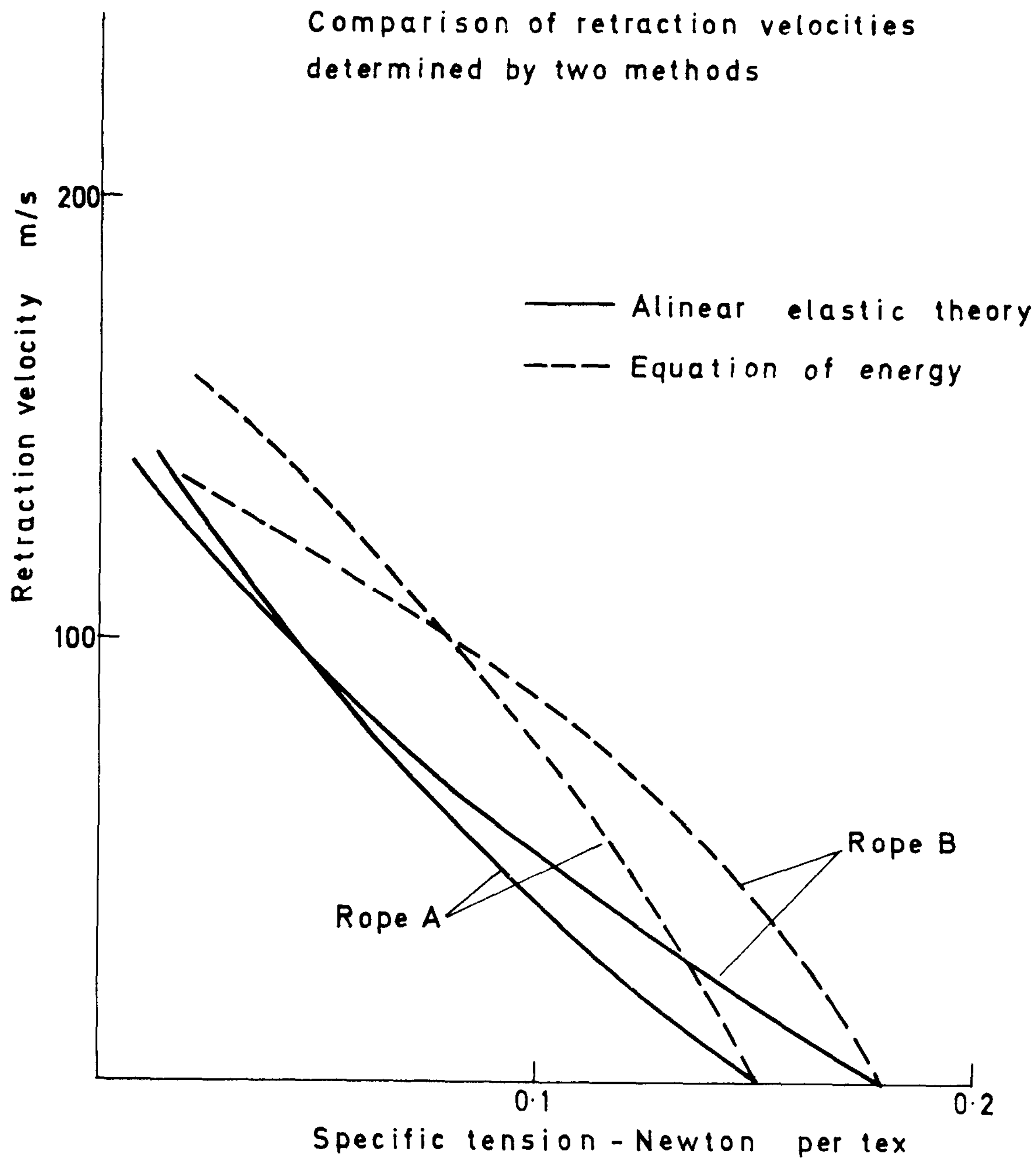

Comparison of retraction velocities
determined by two methods
Retraction velocity m/s
200
100
Alinear elastic theory
Equation of energy
Rope A
Rope B
0·1
0·2
Specific tension - Newton per tex

FIGURE 9

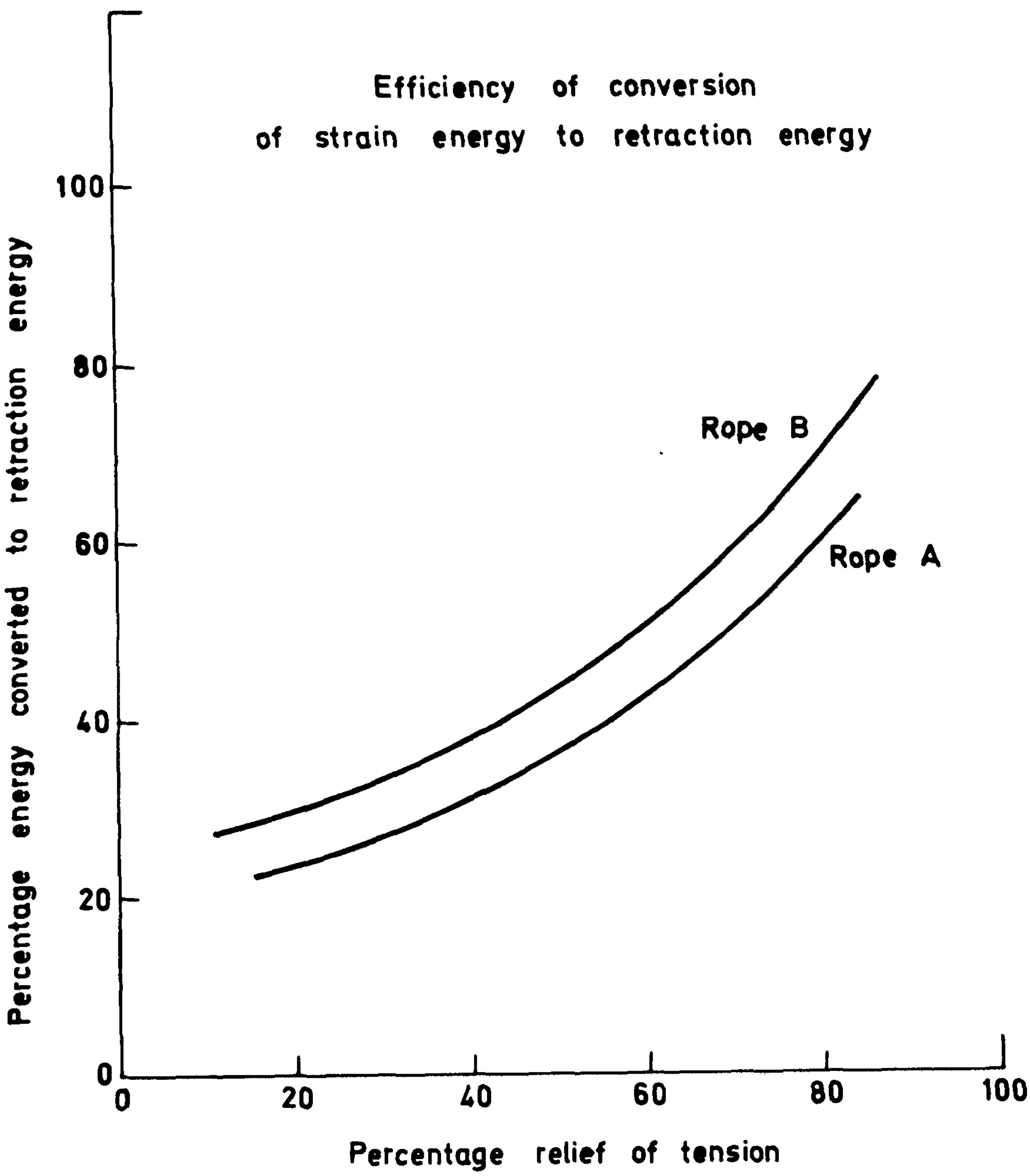

Efficiency of conversion
of strain energy to retraction energy
Percentage energy converted to retraction energy
100
80
60
40
20
0
0
20
40
60
80
100
Percentage relief of tension
Rope B
Rope A

FIGURE 10

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December 1974

Stevens, G.W.H.

THE THEORY OF PARTIAL RETRACTION
OF IMPERFECTLY ELASTIC ROPES

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DETACHABLE ABSTRACT CARDS

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