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A Finite-Difference Scheme for Computing Supercritical Flows in Arbitrary Co-ordinate Systems

by

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A FINITE-DIFFERENCE SCHEME FOR COMPUTING SUPERCRITICAL FLOWS IN ARBITRARY COORDINATE SYSTEMS

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C. M. Albone

SUMMARY

In the computation of flows with local Mach numbers slightly in excess of unity, difficulty is experienced in obtaining numerical stability, where the coordinate system is not aligned with the local velocity vector. A new scheme is proposed for overcoming this difficulty. Its effectiveness is checked on a test case, where it is found to be stable, while alternative schemes fail.

* Replaces RAE Technical Report 74090 - ARC 35564.

CONTENTS

1 INTRODUCTION

The mixed finite-difference scheme introduced by Murman and Cole[1] has revolutionised the computation of transonic flows. Many researchers have adopted it, and methods for computing transonic flow fields of increasing complexity continue to appear in the literature. In the scheme, derivatives are approximated by central differences at grid points where the flow is subsonic. For points at which the flow is supersonic certain derivatives are approximated by differences that are backward with respect to the stream direction. The first successful applications[2,3] have been to cases in which the coordinate system is aligned (at least approximately) with the local velocity vector. However, if the departure of a coordinate direction from the local velocity vector is considerable at a point where the flow is supersonic, the use of simple backward difference approximations may lead to numerical instability. The domain of dependence of the finite-difference equation at each point must, for stability, contain the domain of dependence of the differential equation for that point.

Attempts to solve problems in which the velocity vector departs significantly from the coordinate directions exposed the limitations of Murman's scheme. The problem which prompted the investigation reported here is that of solving the three-dimensional transonic small-perturbation equation for flow past a swept wing, using a sheared non-orthogonal coordinate system which is swept with the wing. The departure of coordinate lines from the direction normal to the velocity vector occurs over most of the field and depends upon the angle of sweep of the wing. For sufficiently large angles, stability problems are encountered with Murman's scheme wherever the flow is supersonic.

It was discovered during the course of this investigation that Jameson of the Courant Institute, New York had encountered similar problems in the solution of the exact potential equation with exact boundary conditions, for supersonic axisymmetric flow past a body. He had used a coordinate system, based upon the body shape, which is generally considered to be most suitable for this problem. However, for such a coordinate system, the departure of coordinate lines from the local velocity vector can be appreciable especially in the far field. Since the velocity is almost everywhere supersonic, problems of numerical stability again arise with Murman's scheme.

Correspondence revealed that solutions to the problem employed at RAE and at the Courant Institute were essentially the same. Jameson stressed the importance of retaining dissipative terms of the correct type in addition to ensuring an adequate domain of dependence for the difference equations. He went further to perform a linear stability analysis of the new scheme. The check of the new scheme reported here took the form of a two-dimensional test case specifically constructed to simulate as closely as possible the case of the transonic swept wing.

In the following sections of this Report a more precise statement of the problem is made, together with a summary of how it has limited existing methods. The new scheme is described in detail and this is followed by a description of the test case and the conclusions drawn from it.

2 STATEMENT OF THE PROBLEM

Consider solving the hyperbolic equation

$$A\Phi_{xx} + 2B\Phi_{xz} + C\Phi_{zz} = 0 \quad , \qquad B^2 - AC > 0 \quad , \tag{1}$$

where Φ is the velocity potential and x and z are fixed cartesian coordinates. Fig.1 shows a situation where the velocity vector is in the x-direction. The values of Φ at the points labelled 0, 1, 2, ..., 6 are those which would normally be used in a finite difference approximation to equation (1) when a central-difference approximation is adopted for Φ_{zz} but backward-difference approximations are adopted for Φ_{xx} and Φ_{xz}. The dotted lines (in Fig.1) are characteristics, which in this case are Mach lines, enclosing the domain of dependence of the differential equation at the point P. The Mach angle, μ, equals $\sin^{-1}\{1/M_{local}\}$. The domain of dependence of the difference equation at P covers the whole region $x < x_p$, and so includes, as required, the domain of dependence of the differential equation. In Fig.2, the velocity vector is not in the x-direction, and so the Mach lines enclose a region which extends outside of $x < x_p$, the domain of dependence of the difference equation. Disturbances reaching P from the region PQR are not taken into account by the difference equation, and the numerical solution will become unstable.

To identify the cause of this difficulty more precisely, consider Fig.3, where we have non-orthogonal axes $\bar{x}$ and $\bar{z}$, with $\bar{x}$ in the direction of the velocity vector. The form taken by equation (1) is similar in the $\bar{x}$, $\bar{z}$ system and again values of Φ at the points 0, 1, 2, ..., 6 are those normally used in a scheme employing a central-difference approximation for $\Phi_{\bar{z}\bar{z}}$ and backward-difference approximations for $\Phi_{\bar{x}\bar{x}}$ and $\Phi_{\bar{x}\bar{z}}$. Here, as in Fig.2, the domain of dependence of the difference equation does not include that of the differential equation for values of μ sufficiently near $\pi/2$, despite the fact that the velocity vector is in the $\bar{x}$-direction. Thus it is the limiting direction of the characteristics as the local Mach number tends to unity that is relevant, and so, unless one of the coordinate lines is normal to the velocity vector, the domain of dependence of the difference equation will fail to include that of the differential equation at some Mach number.

There is no doubt, however, that some flow-field calculations do converge satisfactorily for flows with locally supersonic regions when none of the coordinate axes lies normal to the velocity vector. This can be explained. The problem regarding the domain of dependence occurs with the first appearance of supersonic flow. It vanishes, however, when the local Mach number rises sufficiently for both Mach lines to fall within the domain of dependence of the difference equations (see Figs.2 and 3). The problem is thus confined to near-sonic flows. The range of local Mach numbers over which the problem occurs is related to β, the angle between the z-direction and the normal to the velocity vector, and is given by

$$1 < M_{local} < \sec \beta \quad .$$

With $\beta = 15^{\circ}$, the problem is restricted to fields in which the local supersonic Mach numbers are less than 1.035, and, for $\beta = 25^{\circ}$, this bounding Mach number is 1.10. For larger angles, the local Mach number range increases more rapidly and, for $\beta = 45^{\circ}$, the problem can occur for values of local Mach number up to 1.414. Thus, if β is fairly small everywhere, and the flow accelerates rapidly through the low supersonic range, any numerical instabilities will be slight, and highly localized, and they may be sufficiently damped for the computation to converge successfully. The main aim here is to ensure that the domain of dependence of the difference equation includes that of the corresponding differential equation, even when β is not small.

3 LIMITATIONS OF EXISTING METHODS

A method of solving the exact potential equation in two dimensions,

$$\left(a^2 - \Phi_x^2\right)\Phi_{xx} - 2\Phi_x\Phi_z\Phi_{xz} + \left(a^2 - \Phi_z^2\right)\Phi_{zz} = 0 \quad , \tag{2}$$

with exact boundary conditions for the flow past an aerofoil has been given by Garabedian and Korn[3]. In this method, the coordinate lines can depart significantly from the direction of the velocity vector and its normal. Numerically stable solutions have, however, been obtained for cases where supersonic flow is confined to a region where the departure is small, close to the aerofoil surface and away from the leading and trailing edges. Attempts to compute flow fields with supersonic regions which extend far from the aerofoil or towards the trailing edge have encountered stability problems. The method of Garabedian and Korn cannot be used to obtain solutions to equation (2) at free stream Mach numbers of unity or above. In practice, it is also not possible to obtain solutions for certain important classes of aerofoils at off-design conditions[4], even if the free stream Mach number is well below unity.

The numerical solution of the transonic small-perturbation equation,

$$\left\{K - (\gamma + 1)\phi_x\right\}\phi_{xx} + \phi_{\bar{z}\bar{z}} = 0 \quad , \tag{3}$$

first given by Murman and Cole[1], does not suffer from this limitation although it has the shortcoming of being a near sonic thin aerofoil approximation. It is solved by use of a cartesian coordinate system $(x,\bar{z})$ with axes normal and parallel to the undisturbed stream direction (x). Since $\phi_{\bar{z}}$ does not appear in the coefficients of equation (3), the characteristics of the equation are symmetric with respect to the x-direction throughout the field, and so the domain of dependence of the difference equation always contains that of the differential equation.

Attempts to compute the flow past swept wings have brought out more serious consequences of the numerical difficulty. Ballhaus and Bailey[5] have developed a method for solving the three-dimensional transonic small-perturbation equation

$$\left\{1 - M_\infty^2 - M_\infty^2(\gamma + 1)\phi_x\right\}\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \quad , \tag{4}$$

using a coordinate system which, for economy of grid points, is swept with the wing. Fig.4 shows how, with such a swept system (X,Y) the domain of dependence of the difference equation, for some supersonic Mach numbers, does not contain that of the differential equation. In general, numerical instability results. The severity of the numerical instability depends upon the sweep of the wing, the fineness of the finite-difference grid, and the local Mach number. Ballhaus and Bailey have presented results for a 24° swept, untapered wing. They delay the switching of X derivatives from central to backward differences until the local Mach number is such that $\mu < \pi/2 - \beta$, where β is the sweep angle and μ is the Mach angle. Under this condition, the domain of dependence, of the differential equation, using Murman's scheme, becomes sufficiently small to be contained within that of the difference equation. For this case, their scheme is equivalent to switching from central to backward differences when the local Mach number normal to the leading edge becomes sonic, which, for a 24° swept wing, corresponds to a real local Mach number of 1.095. In other words, a slightly supersonic part of the flow is treated as if it were subsonic. Despite their success in computing the flow past this wing, serious stability problems must be expected for wings with significantly greater sweep. Bailey[6] has suggested a scheme in which the swept coordinate system (X,Y,Z) defines the grid points, which it does with considerable efficiency, but in which the differential equation is solved in a cartesian system, (x,y,z). The problem associated with swept coordinate systems is thus avoided, but the scheme involves considerable interpolation.

4 A NEW SCHEME

Consider the problem of calculating the flow about an aerofoil by solving numerically the exact two-dimensional potential equation

$$\left(a^2 - \Phi_x^2\right)\Phi_{xx} - 2\Phi_x\Phi_z\Phi_{xz} + \left(a^2 - \Phi_z^2\right)\Phi_{zz} = 0 \ , \tag{5}$$

where $a^2 + \left(\frac{\gamma - 1}{2}\right)\left(\Phi_x^2 + \Phi_z^2\right) = \text{constant}$, and Φ is the full velocity potential.

Let x be the direction of the undisturbed stream and z be normal to x . This coordinate system is unsuitable for the solution of the potential equation by use of Murman's scheme, since the direction of the velocity vector may depart greatly from the x-direction.

In order to determine what form a new scheme should take for this equation, it is convenient to consider the equation in a new coordinate system. Firstly, a transformation is made from the x, z system to some coordinate system for which Murman's scheme will be numerically stable. This means finding a coordinate system in which the taking of backward differences in a single coordinate direction will for locally supersonic flow, give a domain of dependence for the difference equations that includes that for the differential equation, and will introduce dissipative terms of the appropriate type. An orthogonal system, (s,n) such that s is aligned with the local velocity vector, is a natural choice, since it satisfies both these requirements. The transformation is thus a function of the direction of the local velocity vector. Under this transformation, equation (5) reduces to the canonical form

$$\left(a^2 - \Phi_s^2\right)\Phi_{ss} + a^2\Phi_{nn} = 0 \ . \tag{6}$$

Murman's scheme for this equation would involve the use of central-difference approximations everywhere, except for locally supersonic flow, where backward-difference approximations would be used for derivatives in the s-direction.

It is here proposed that each derivative in the s, n system is resolved into its components in the x, z system, and that the individual components be differenced in the same way as the terms in the s, n system from which they originated. Thus, the difference scheme which should be employed for each term of equation (5) is dictated by the difference scheme used for the terms in the s, n system, from which it arose by transformation. To illustrate this, suppose that $\theta \left(= \tan^{-1} (\Phi_z/\Phi_x)\right)$ is the angle between the s and x directions. Resolution of the terms in the canonical equation gives

$$\Phi_s = \cos\theta\,\Phi_x + \sin\theta\,\Phi_z \quad , \tag{7}$$

$$\Phi_{ss} = \Phi_{xx}\cos^2\theta + 2\Phi_{xz}\sin\theta\cos\theta + \Phi_{zz}\sin^2\theta \quad , \tag{8}$$

$$\Phi_{nn} = \Phi_{xx}\sin^2\theta - 2\Phi_{xz}\sin\theta\cos\theta + \Phi_{zz}\cos^2\theta \quad . \tag{9}$$

All terms on the right-hand side of equation (8) should be approximated by backward differences when the flow is locally supersonic, whereas those on the right-hand side of equation (9) should always be approximated by central differences. Hence, equation (6) may be rewritten in the following form:

$$\underline{\left(a^2 - \Phi_s^2\right)\cos^2\theta\,\Phi_{xx}} + a^2\sin^2\theta\,\Phi_{xx} + \underline{2\left(a^2 - \Phi_s^2\right)\sin\theta\cos\theta\,\Phi_{xz}} - 2a^2\sin\theta\cos\theta\,\Phi_{xz} + \underline{\left(a^2 - \Phi_s^2\right)\sin^2\theta\,\Phi_{zz}} + a^2\cos^2\theta\,\Phi_{zz} = 0 \quad , \tag{10}$$

where each term is split into a part to be approximated by central differences throughout, and a part (underlined) to be approximated by backward differences in locally supersonic flow. The part of each term which has its difference scheme switched at sonic conditions has $\left(a^2 - \Phi_s^2\right)$ as a factor, which means that a smooth change from central to backward differences takes place. This would not have been possible with Murman's scheme applied to equation (5) in the x, y system. The fact that part of the term Φ_{xx} is represented by a central-difference approximation everywhere ensures that the domain of dependence of the difference equation contains that of the differential equation. If $\theta = 0$, the x and z axes are aligned with, and normal to the velocity vector, and the new scheme reduces to that of Murman.

A question which should be asked of the new scheme is whether it will be stable in a supersonic region. Experience indicates that the exclusive use of central differences in a supersonic region leads to numerical instability. With the new scheme, however, there will always be part of the term ϕ'_{xx} which will be approximated by backward differences in the supersonic region. In fact, the amount of dissipation introduced by the new difference scheme in the x, z system is precisely the same as that used in the s, n system. In the discussion with Jameson, it was learned that he had studied the stability of the new scheme under an iterative method of solution by performing a linear analysis of a 'time-dependent' equation obtained by considering iterations as successive levels in artificial time. The results of his studies appear in the concluding section. We did not attempt a linear stability analysis, but decided, before using the new scheme in the three-dimensional problem, to set up a simplified test case. Of course, no single test case can show that the scheme is stable in general. Our aim was to provide some evidence that the scheme would be stable for the transonic swept-wing problem, by considering a very similar problem in two dimensions. A description of the test case follows in the next section.

5 A TEST CASE

A solution of the transonic small-perturbation equation in two dimensions is considered. The equation is written in the form (3) adopted by Murman,

$$\left\{K - (\gamma + 1)\phi_x\right\}\phi_{xx} + \phi_{\bar{z}\bar{z}} = 0 \quad , \tag{11}$$

where the undisturbed stream is in the x direction, $\bar{z}$ is a scaled coordinate normal to x , ϕ is a scaled disturbance potential and K is a transonic similarity parameter. We choose to solve this equation, not in the orthogonal system, but in a sheared, non-orthogonal system, X, Z defined by

$$X = x - h(\bar{z}) \; ; \qquad Z = \bar{z} \quad . \tag{12}$$

The coordinate lines X = const have a slope that depends on the form of the function $h(\bar{z})$, and the use here of Murman's finite difference scheme would be expected to lead to numerical instability, just as with a sheared coordinate system for a swept wing. In the X, Z system equation (11) becomes

$$\left\{\underline{K - (\gamma + 1)\phi_x} + \{h'\}^2\right\}\phi_{xx} - 2h'\phi_{xz} + \phi_{zz} - h''\phi_x = 0 \ . \qquad (13)$$

A computer program[7] written earlier to solve equation (11), subject to the usual linearised aerofoil boundary conditions, was modified in order to solve equation (13). Murman's difference scheme for equation (13) involves the use of backward-difference approximations for each term containing an X-derivative wherever the flow is supersonic. The new scheme employs backward-difference approximations for the underlined term only in equation (13). Both schemes were programmed for the test case. Here, hyperbolae are chosen for the lines X = constant, because it is then unnecessary to change the treatment of aerofoil boundary conditions used for equation (11). Also, hyperbolae have finite sweep at infinity. The grid shown in Fig.5 is such that the sweep of the hyperbolae is 40° at a distance of about 30% chord above the aerofoil and reaches $45\frac{1}{2}^{\circ}$ at infinity.

A 12% thick circular-arc section at zero lift was chosen for the test. This section has a critical Mach number of 0.75. The reformulated program was initially tested by considering a sub-critical case and checking that results from the original program agreed with those obtained by use of the non-orthogonal system.

The calculation of a supercritical flow was made for a free stream Mach number of 0.85. The flow past the circular-arc section was first computed with the orthogonal coordinate system, then with the non-orthogonal coordinate system using the new scheme, and finally with the non-orthogonal coordinate system using Murman's scheme. Fig.6 shows the pressure distribution obtained using the orthogonal coordinate system with 40 points in the X-direction and 80 points in the Z-direction. The embedded supersonic region extends to a distance of 74% chord above the aerofoil. The lines X = constant in the non-orthogonal grid of Fig.5 are thus swept back at about 44° to the normal to the stream direction at the extremity of the supersonic region, where the local Mach number is near unity. The computation was then repeated, using the non-orthogonal system, and with the new finite-difference scheme. It was stable and converged satisfactorily with a 40 × 80 grid. The resulting pressure distribution is included in Fig.6, to provide a comparison with that obtained (using the same number of grid points) with the orthogonal system. The shock-wave location differs by about 6% of the chord, but, away from the shock, the agreement is good. This discrepancy may be accounted for by the fact that with the non-orthogonal system, finite-difference approximations are carried out across the shock wave, whereas this does not occur (except at isolated points) with the orthogonal system. To investigate the effects of grid refinement the calculation was repeated with an 80 × 160 grid. Convergence was again achieved satisfactorily. A comparison of the result with that using the orthogonal system is shown in Fig.7.

The calculation was repeated with the sweep of the hyperbolae near the tip of the shock wave increased to 60°. Again, the computation remained stable with a 40 × 80 grid, and convergence was achieved satisfactorily. The shock wave was now spread over several grid intervals even at the aerofoil surface.

The effectiveness of Murman's scheme was tested by applying it to the earlier case, for which the sweep of the hyperbolae near the tip of the shock wave was 44°. It failed to converge, even on a coarse 20 × 40 grid. The coordinate system was modified so that the sweep of the hyperbolae near the tip of the shock wave was about 15°. For this case, it converged on the 20 × 40 grid, but became unstable when the grid was refined to 40 × 80. This failure to converge was thought to be possibly due to the lack of diagonal-dominance of the tri-diagonal matrices formed by the coefficients of the difference equations, rather than an inadequate domain of dependence. In order to remedy this, the term $\{h'\}^2\phi_{xx}$ in equation (13) was evaluated in terms of ϕ calculated at the previous iteration. With this modification, the computation converged successfully. The angle of sweep of the coordinate lines was then increased to 24° at the tip of the shock wave, and again, a converged solution was obtained, although the first signs of instability were just noticeable with the 40 × 80 grid near the extremity of the supersonic region. When the angle of sweep was increased to 29°, the calculation with a 40 × 80 grid became unstable. Central difference approximations for ϕ_Z and $\phi_{X,Z}$ in the supersonic region were tried but with no success. Thus, no solutions for angles of sweep greater than 24° near the tip of the shock wave were obtained with Murman's scheme.

Finally, an independent test of the new finite-difference scheme was carried out by Gilbert[8] of the University of Lancaster. Gilbert used a non-orthogonal system in which the sweep of the coordinate lines was constant over the field. He took the NACA 0012 aerofoil as his test section, and performed all his calculations at a free stream Mach number of 0.8, with zero lift. The angle of sweep of the coordinate lines was taken as 0, 0.5, 0.75 and 1.0 radians. He tested Murman's scheme and the new scheme. Computations with the new scheme remained stable for all the angles of sweep. These results are shown in Fig.8, where it is seen that the thickness of the shock wave increases with the angle of sweep of the coordinate lines. Computations using Murman's scheme failed to converge for angles of sweep greater than half a radian.

6 CONCLUDING REMARKS

The finite-difference scheme proposed in this Report overcomes the problem associated with an inadequate domain of dependence. It treats terms in the differential equations in a manner consistent with the correct treatment of the corresponding terms in the equation expressed in flow-orientated or canonical form. This results in the use of central-difference approximations, where the flow is supersonic for terms which are usually approximated by backward differences. In the test case, the new scheme is numerically stable under conditions which cause Murman's scheme to become unstable.

The conclusion drawn from Jameson's linear stability analysis of the new scheme is that it is stable, for an equation with constant coefficients, provided that a certain ratio of 'old' and 'new' values is used for central-difference approximations. The specific combination of 'old' and 'new' values recommended by Jameson was not used in the test case. The relaxation scheme employed 'new' values wherever they were available, and no stability problems were encountered, despite the use of a very fine grid (80 × 160) and large angles of sweep of the coordinate lines. It is possible that the relaxation scheme would have become unstable with further grid refinement; however, such excessively fine grids are not contemplated for three-dimensional problems. It is also possible that the non-linearity of the coefficients of the differential equation or the presence of boundary constraints causes instabilities to become damped.

For practical purposes, the differences in the details of the two versions of the new scheme appear, so far, to be of little consequence. Jameson has now incorporated his version of the scheme in a solution of the full potential equation for supersonic flow past aerofoils, and has extended it to a treatment of finite yawed wings. At RAE, the version of the new scheme presented in this Report is incorporated into a program for solving the transonic small-perturbation equation for subsonic or supersonic flow past finite swept wings of arbitrary planform. These programs have been used for many calculations, and so far no problems of numerical instability have been encountered.

An important practical consequence is that the new scheme gives the researcher a free hand in his choice of coordinate system. Being free of constraints placed upon his coordinate system by considerations of numerical stability, he can tailor his system to suit other needs, such as that for the efficiency spacing of grid points or the treatment of complex geometries.

REFERENCES

No.	Author	Title, etc.
1	E.M. Murman J.D. Cole	Calculation of plane steady transonic flows. AIAA Journal 9, 114-121 (1971)
2	J.A. Krupp	The numerical calculation of plane steady transonic flow past thin lifting airfoils. Ph.D. Dissertation, Univ. of Washington (1971)
3	P.R. Garabedian D.G. Korn	Analysis of transonic aerofoils. Comm. P & Appl. Math., Vol XXIV (1971)
4	R. O'Mahoney R.C. Lock Gaynor Joyce	Notes on a computer program of Garabedian and Korn for computing steady inviscid two-dimensional transonic flow past an aerofoil. RAE Technical Memorandum Aero 1482 (ARC 35159) (1973)
5	W.F. Ballhaus F.R. Bailey	Numerical calculation of transonic flow about swept wings. AIAA 5th Fluid and Plasma Dynamics Conference, Paper No. 72-677, June 1972
6	F.R. Bailey	Private communication. (1972)
7	C.M. Albone D. Catherall M.G. Hall Gaynor Joyce	An improved numerical method for solving the transonic small-perturbation equation for the flow past a lifting aerofoil. RAE Technical Report 74056 (ARC 35563) (1974)
8	J. Gilbert	Private communication. (1973)

SYMBOLS

a	local speed of sound
h	function defining shape of coordinate lines X = const
K	transonic similarity parameter
M_{local}	local Mach number
M_∞	Mach number in the undisturbed stream
n	coordinate normal to the direction of the velocity vector
s	coordinate in the direction of the velocity vector
x	coordinate in the undisturbed stream direction
X	coordinate in the undisturbed stream direction
z	coordinate normal to the undisturbed stream direction
Z	swept coordinate direction
β	angle between the normal to the velocity vector and the swept coordinate direction
γ	ratio of specific heats
θ	angle between directions of velocity vector and free stream
μ	Mach angle = $\sin^{-1} \{1/M_{local}\}$
ϕ	scaled disturbances potential
Φ	full velocity potential

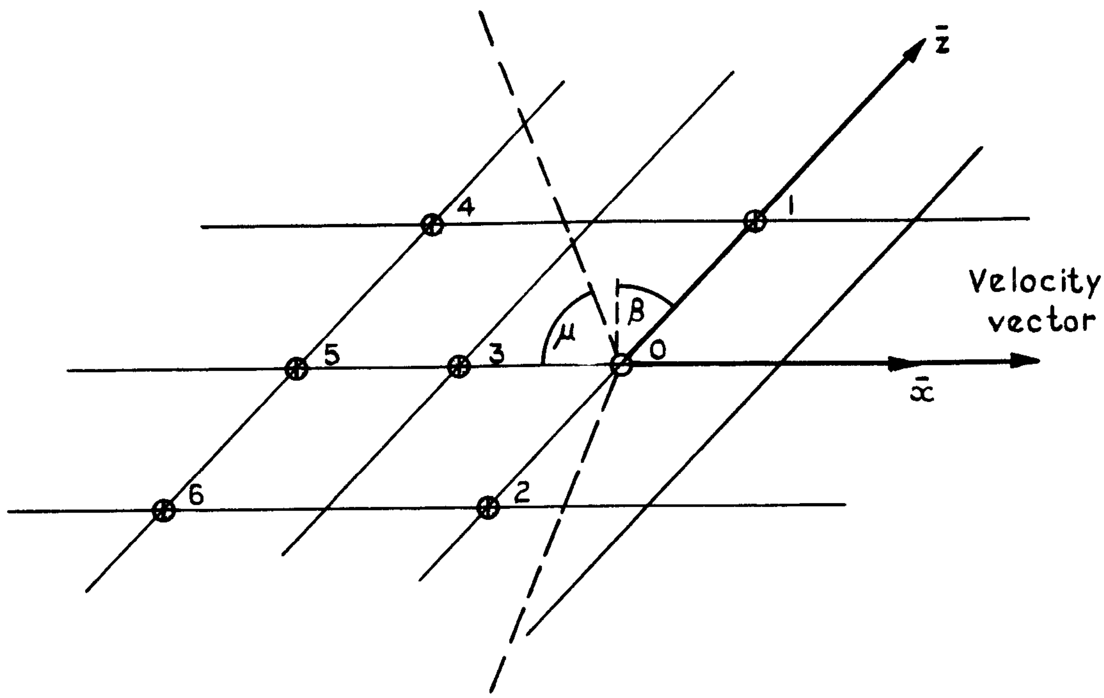

Fig. 3 Non-orthogonal coordinate system

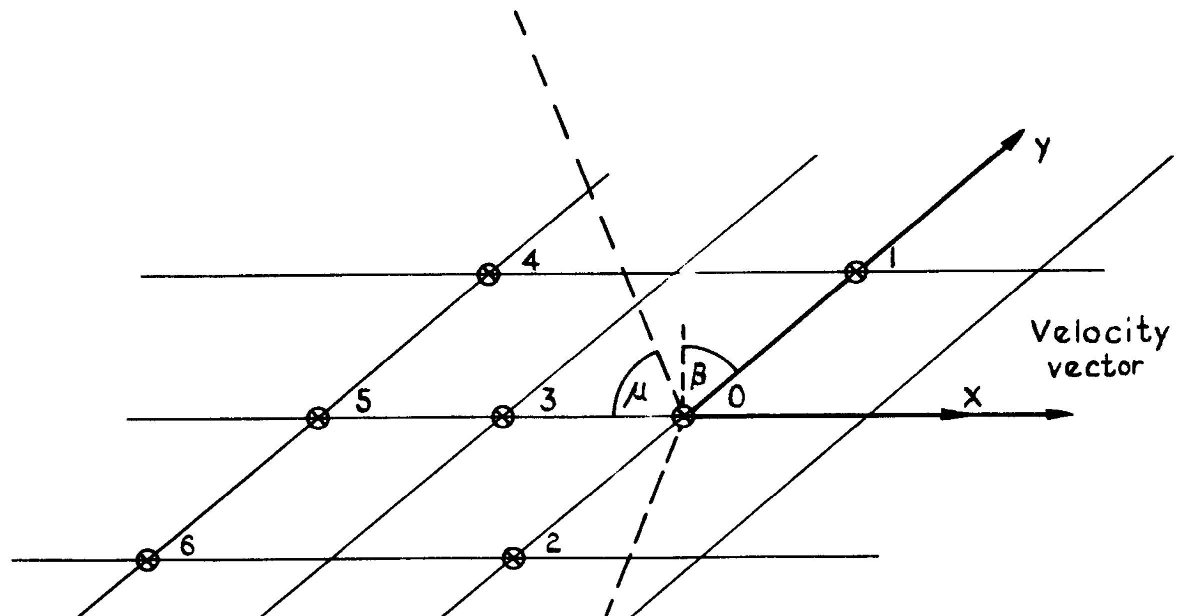

Fig. 4 Swept wing coordinate system

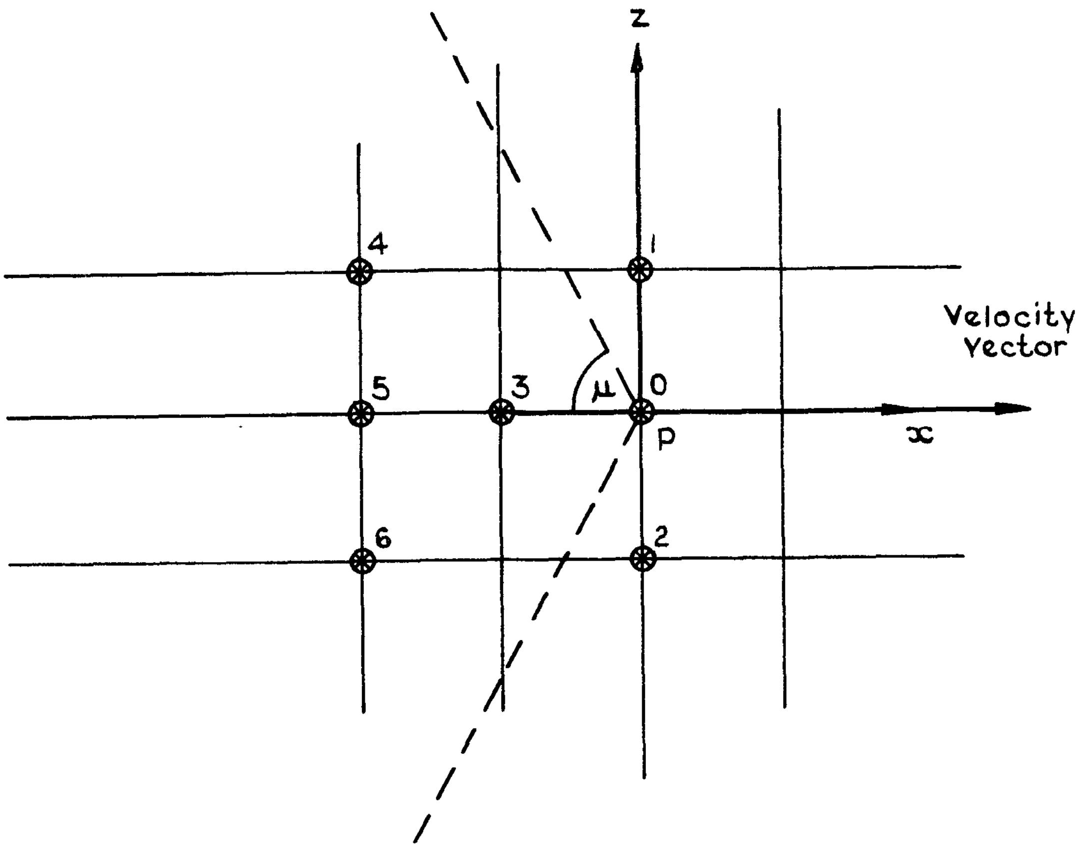

Fig. 1 Orthogonal coordinate system aligned with the flow

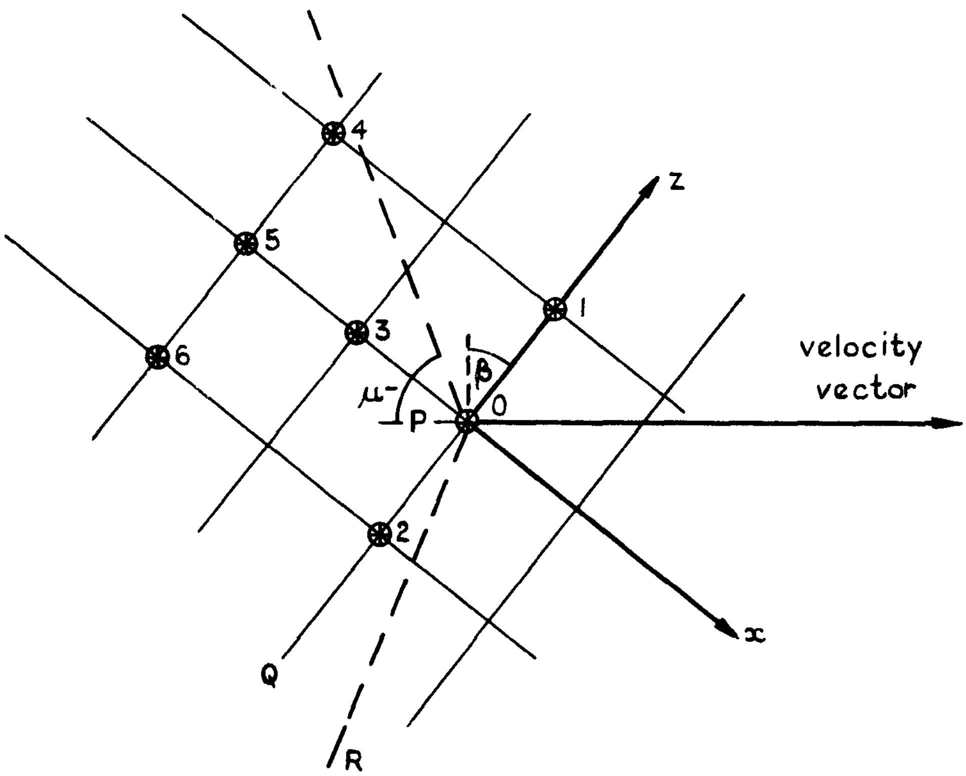

Fig. 2 Orthogonal coordinate system not aligned with the flow

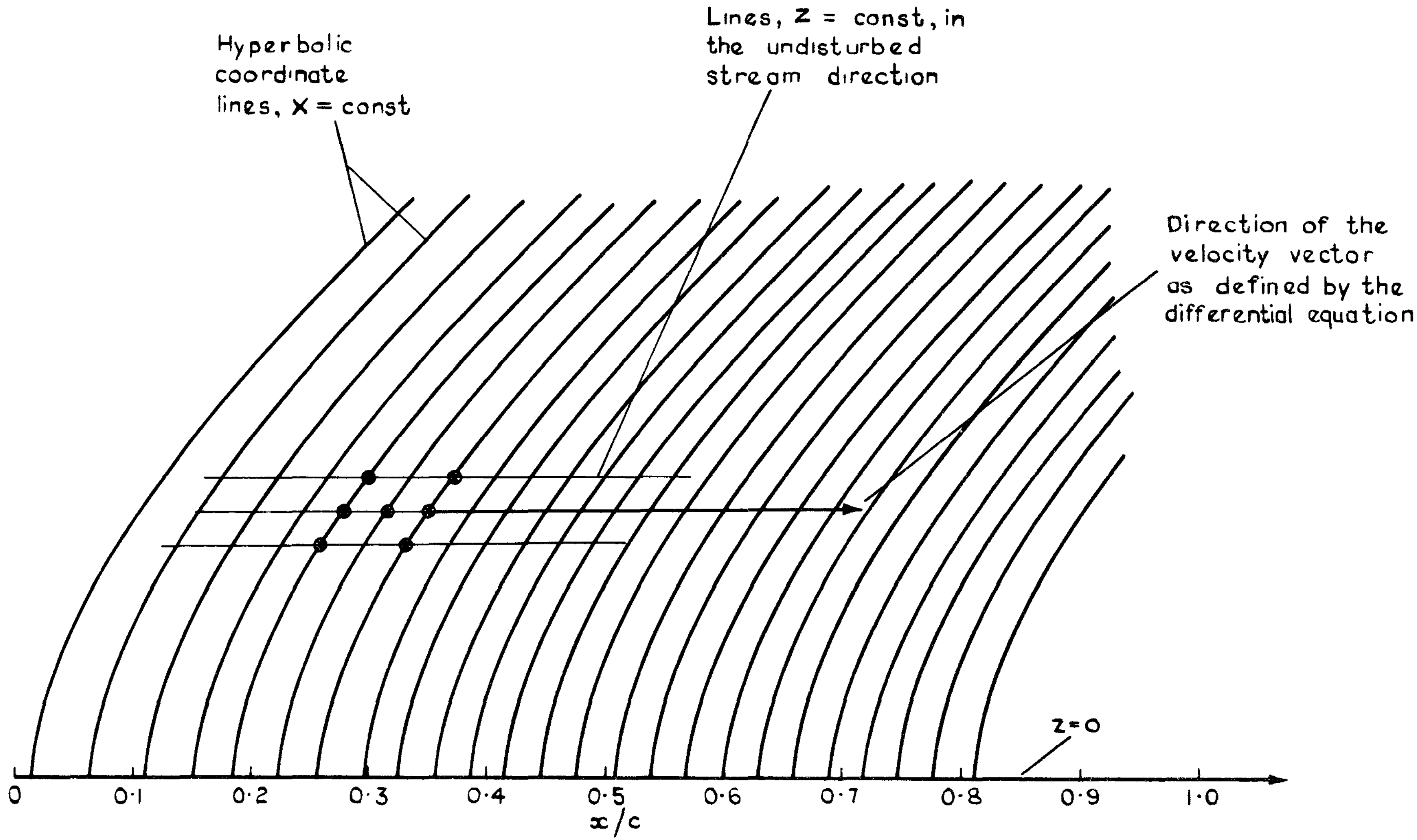

Fig. 5 The non-orthogonal coordinate system (X,Z).
Coordinate lines in the physical plane

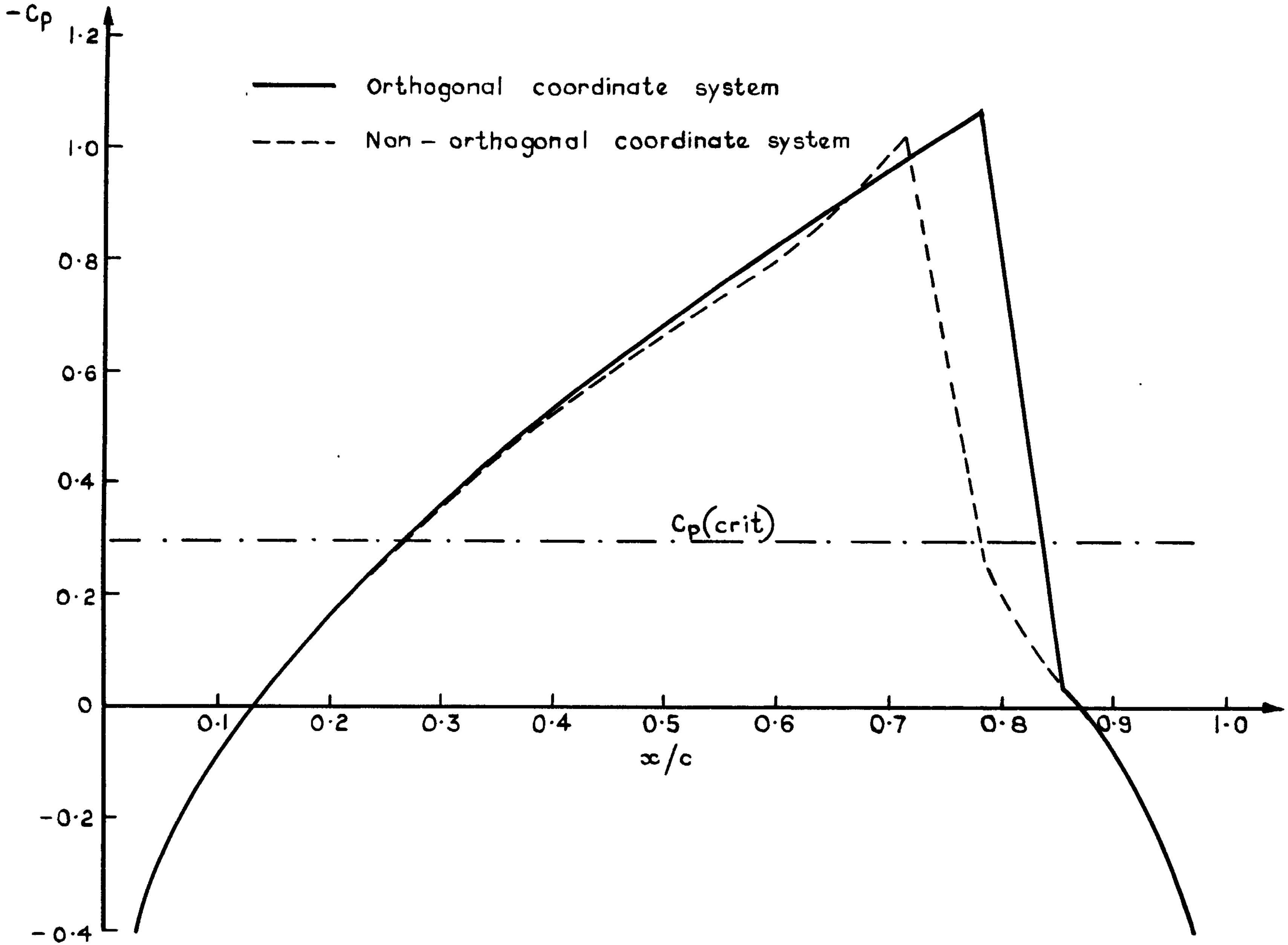

Fig. 6 12% thick circular arc-section at zero lift, $M_\infty = 0{\cdot}85$ computed pressure distributions using a 40x80 grid

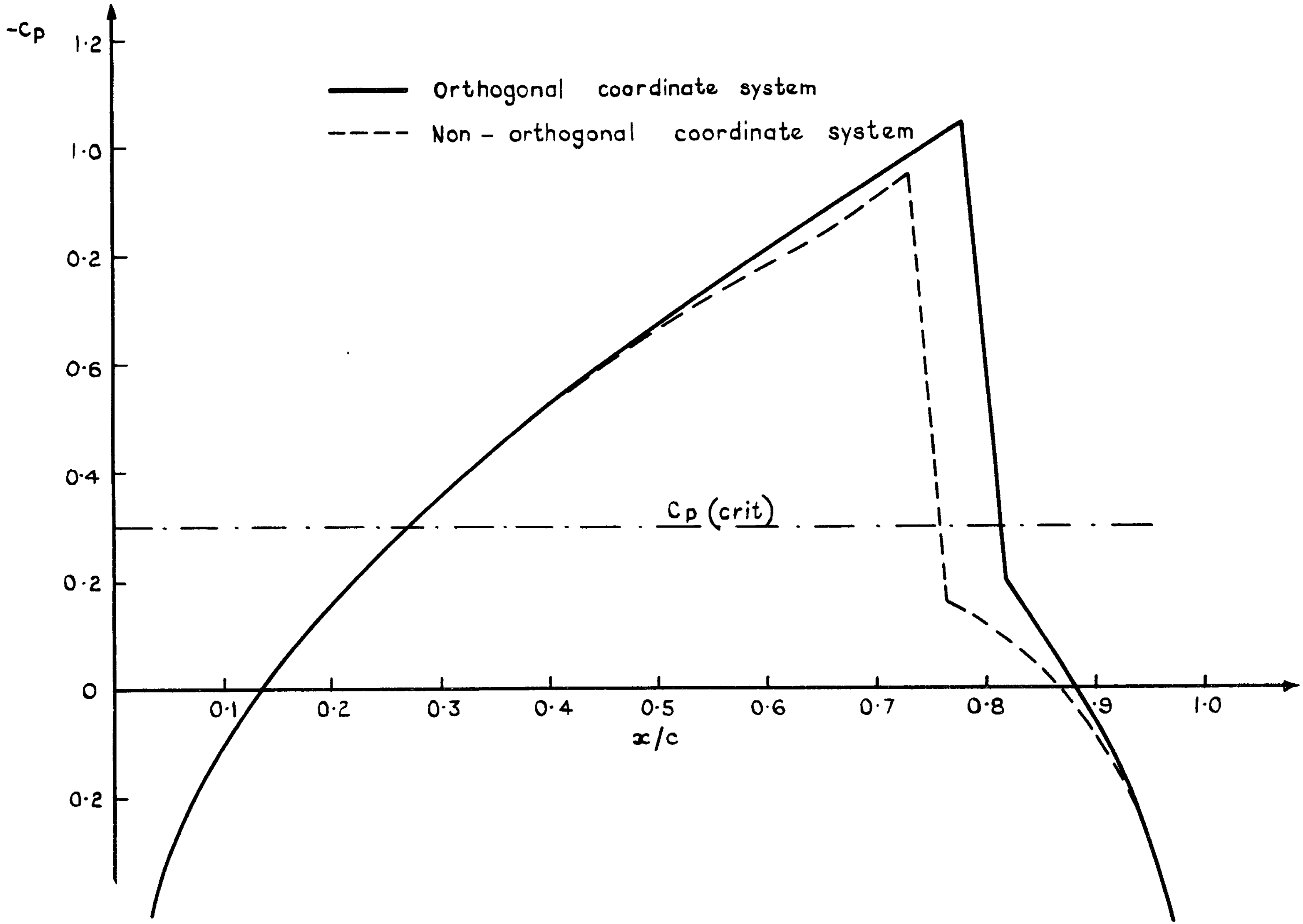

Fig. 7 12% thick circular-arc section at zero lift, $M_\infty = 0{\cdot}85$ computed pressure distributions using a 80 x 160 grid

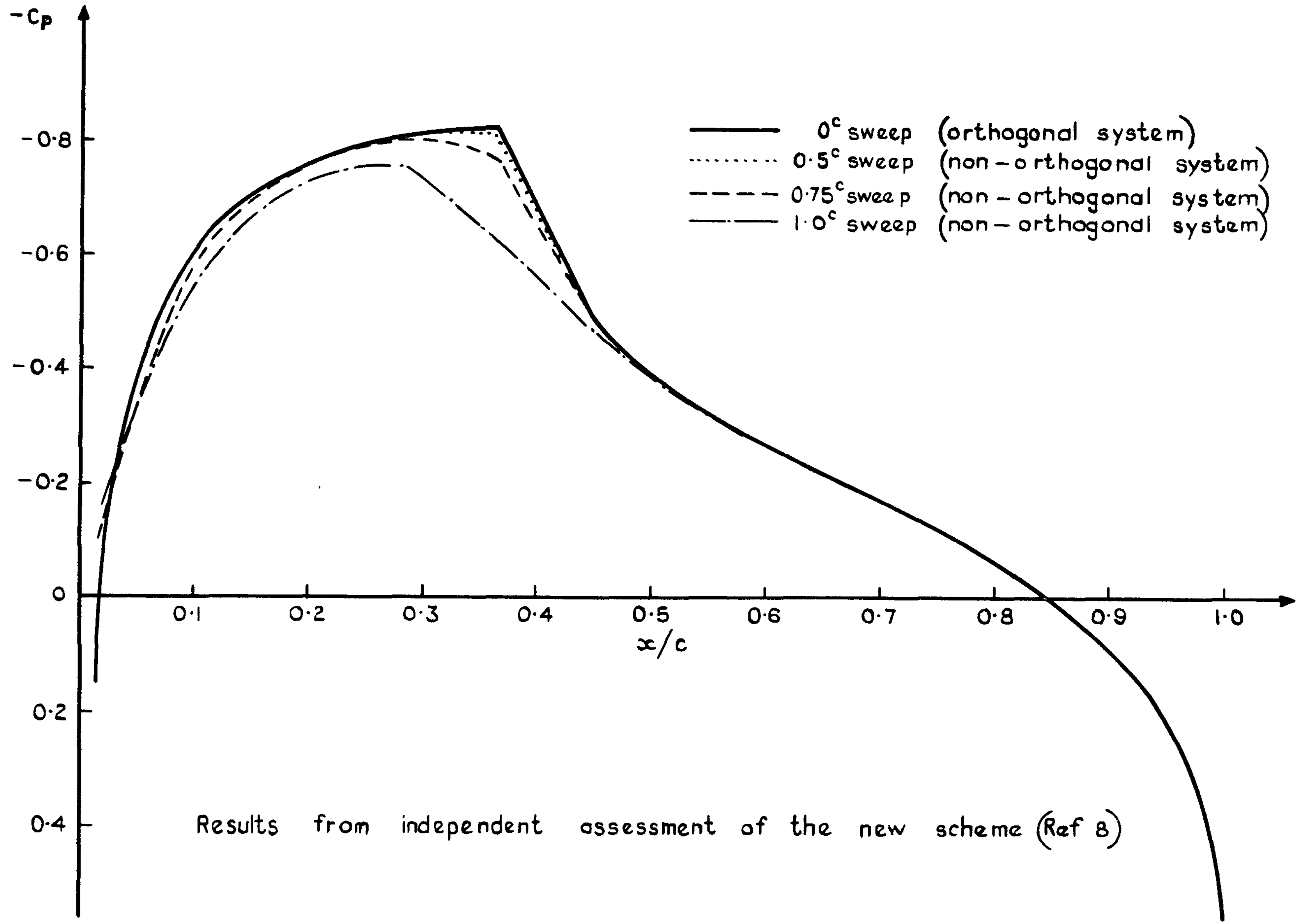

Fig. 8 NACA 0012 section at zero lift, $M_\infty = 0{\cdot}80$ computed pressure distributions from Ref 8

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