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The Variability of Fatigue Damage from Flight to Flight

by

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1974

PRICE 55*p* NET

UDC 539.431 : 629.13.012 : 62.004.6

CP No.1297 *
February 1973

THE VARIABILITY OF FATIGUE DAMAGE FROM FLIGHT TO FLIGHT

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SUMMARY

The variability of fatigue damage from flight to flight is examined for a fleet of aircraft, both within aircraft and between aircraft. A method is described of modifying the standard analysis of variance technique to take account of correlation between consecutive flights and an example is given. It is then shown how to use the estimates of variance so obtained in the derivation of factors for damage prediction, both on a short term basis for a few unmetered flights and also for complete life estimation.

* Replaces RAE Technical Report 73046 - ARC 34723

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1 INTRODUCTION

In many cases, whole fleets of aircraft are fitted with fatigue load meters in order to monitor fatigue damage. These instruments are basically counting accelerometers designed to count at convenient levels for the particular aircraft.

A common procedure is then to convert the numbers of accelerations to fatigue damage by means of a standard formula derived for the aircraft type. The result of this is to obtain a number, on some arbitrary scale, representing the fatigue damage, on a flight by flight basis, for each aircraft of a fleet.

The question of variability arises when it is necessary to use these observed values for the purpose of prediction, either on a long term basis for life prediction, or on a short term basis, when perhaps an instrument has become unserviceable for a few flights, and it is required to allow for this. It is then necessary to know whether there are significant differences between aircraft and if so to what extent, and also to estimate the variability within the experience of a single aircraft due to differences of environment and operation.

Superficially this is a classical case where the analysis of variance can be used, but there are certain qualifications to be made. In the first place, the distributions with which we are dealing are often far from normal and, strictly speaking, the analysis of variance is only valid in the normal case. However, it has been found that, in most cases encountered in practice, the departure from normality has little effect of the significance test[1]. Departures from normality do however, have a larger effect on the estimates of confidence limits for the variances and for this reason no such confidence limits are quoted below.

A further difficulty of a fundamental nature occurs because successive values of the variate under consideration are often highly correlated, whereas the analysis of variance assumes independence. This point was considered brieflv in an earlier paper[2] but the full implications were not then realised. The present report makes a detailed study of the effects of correlation between flights and shows how allowance can be made for it in the analysis.

Finally, having established whether significant differences between aircraft exist, having estimated their magnitude and having examined the variation within the experience of a single aircraft, it is shown how this information can be applied in fatigue life estimation problems and illustrative examples are given.

2 OUTLINE OF METHOD OF ANALYSIS

Consider a series of values of a variate, $x_1, x_2, \ldots x_N$, and let the mean of n successive values be distributed about the true mean with variance σ_n^2. When the values of x are independent, then $\sigma_n^2 = \sigma_1^2/n$, where σ_1^2 is the variance of the individual values. When, however, there is correlation between successive values this is no longer true. For example, when there is a very high correlation, successive values tend to be alike, and several consecutive values contain little more information than a single value. The correlation thus has the effect of suppressing a number of degrees of freedom, so that estimating the mean from n correlated values is equivalent to estimating from fewer than n, say n', independent values, the relationship between n and n' depending on the degree of correlation.

The behaviour of σ_n^2, considered as a function of n, is no longer inversely proportional to n. The relationship can be found in two ways. It can be found directly, first of all by calculating the variance of single estimates σ_1^2, then finding the means of consecutive pairs and estimating σ_2^2, and so on for a selection of higher values of n. (In the example considered below σ_n^2 is found for n = 1,2,4,8 and 16.)

Alternatively, the serial correlations for the sequence can be calculated and the values of σ_n^2 deduced from these. The way in which this can be done is given in the Appendix. In the subsequent analysis a combination of these methods is used. A smoothed form for the correlogram is assumed which gives a good fit to the estimated values of σ_n^2. This facilitates extrapolation when n takes on values equal to the number of flights per aircraft, and avoids difficulties of estimation, including sampling errors and bias, of the serial correlations themselves.

We now define n' as the number of independent values equivalent to n correlated values, that is to say the number of independent values giving the same variance σ_n^2 as the n correlated values, and hence

$$n' = \sigma_1^2 / \sigma_n^2 \quad . \tag{1}$$

Now let N be the number of flights made by a particular aircraft. In order to find the equivalent number, N', of uncorrelated flights, we set n = N and n' = N' in (1). The analysis of variance then proceeds with the values of N replaced by N'.

For small samples there is however, an adjustment to be made. The values of σ_1^2 were originally calculated using $N - 1$ as the number of degrees of freedom for each aircraft whereas, as we have seen, the true value is $N' - 1$. A small second order error can thus be removed by weighting the estimated values of σ_1^2 for each aircraft by $N' - 1$ rather than $N - 1$ in determining the overall pooled estimate of σ_1^2.

For each aircraft then we have

$$\sigma_1^2 = \frac{\Sigma(x - \bar{x})^2}{N} \frac{N'}{N' - 1} \qquad (2)$$

determined from $N' - 1$ degrees of freedom. Similar adjustments are necessary to $\sigma_2^2, \sigma_4^2, \sigma_8^2$ and σ_{16}^2.

Thus the precise determination of the functional dependence of σ_n^2 upon n is an iterative process, which is however, rapidly convergent and it is seldom that it is necessary to repeat the calculation a second time.

In this way the necessary modifications are made to the standard analysis of variance technique, and the test of significance for differences between aircraft made. Having done this it is then necessary to estimate the variance between aircraft, if any, and the variance within aircraft, and use these estimates to assess 'unmetered factors' in various circumstances.

In order to do this, use is made of the method described in an earlier paper[2] and the distribution of the damage-per-flight parameter assumed to be of gamma-function form. The appropriate factor is then derived by considering the distribution of the ratio of two samples drawn from such gamma distributions. Detailed examples are given of this procedure.

3 ANALYSIS OF DATA

A fleet of twenty trainer aircraft are considered, and from these all flights classified as 'general handling' are taken as the basic data. For each flight the damage is estimated on some arbitrary scale from a standard formula, and it is these numbers that are used in the analysis. Table 1 gives the number of flights, N, the mean $\bar{x}$, and the values of $\sqrt{\{\Sigma(x - \bar{x})^2/N\}}$ for each aircraft, x being the value of the damage parameter for a single flight. This information is used to obtain an initial estimate of σ_1^2 and since at this stage there is no information regarding correlation effects, the N values for each aircraft are assumed independent to give a first approximation. Later the accuracy is improved by substituting N' for N as shown below. It is also necessary at this stage to investigate how σ_n^2 varies with n. For this purpose calculations are made to determine $\sigma_2^2, \sigma_4^2, \sigma_8^2$ and σ_{16}^2. Table 2 gives the number of pairs of consecutive flights for each aircraft, N_2, and also the means and root mean square deviations as in the case of single flights. Tables 3 to 5 give similar information for groups of four, eight and sixteen flights respectively. For all these cases the calculations based on Table 1 are exactly paralleled and the resulting first approximations to the variances are

$$\sigma_1^2 = 16.389$$

$$\sigma_2^2 = 8.916$$

$$\sigma_4^2 = 4.860$$

$$\sigma_8^2 = 2.677$$

$$\sigma_{16}^2 = 1.386 \ .$$

These values are shown plotted in Fig.1 and confirm that successive flights do not give altogether independent values, as in this case the values of σ_n^2 would be inversely proportional to n and lie along the dashed line of the figure.

Now the relationship between σ_n^2 and n is required in order to estimate the equivalent number of flights N' for each aircraft by substituting N for n. We can extrapolate from Fig.1 to values of the order of 100 for n, (not quite so much of a guess as it seems at first sight since the scales are logarithmic and the experimental values lie close to a straight line) or we can consider the approach through the autocorrelation function. In the Appendix a relationship is derived relating σ_n^2 to the serial correlations. Fig.2 gives the first twenty values of these and shows that because of sampling errors they fluctuate wildly. In view of this what amounts to a combination of methods is used. A mathematical form is chosen for the correlogram which gives a good fit to the values for σ_n^2, the primary concern, and lies reasonably well through the points representing the serial correlations calculated from the observations.

A simple autoregressive scheme of the form

$$x_n = kx_{n-1} + \varepsilon$$

where k is a constant and ε a random variable with zero mean, leads to

$$r_n = k^n$$

(see Kendall and Stuart[3])

The values of r_n obtained here indicate an underlying relationship of this kind with an additional large random element superimposed. The form chosen for the correlogram is therefore

$$r_0 = 1 , \quad r_s = \alpha k^{s-1} , \quad s \geqslant 1 \tag{3}$$

where α and k are parameters defining the values of r_s, and the curve in Fig.2 is based on this with the final estimates of k and α.*

With the formula for the serial correlations given in (3) and the relationship (A-5) in the Appendix, the value of n' becomes

$$n' = n \Bigg/ \left\{ 1 + \frac{2\alpha}{n} \left(\frac{n-1}{1-k} - \frac{k - k^n}{(1-k)^2} \right) \right\} . \tag{4}$$

Equating n to N for each aircraft then gives the equivalent number of independent observations N' .

In a similar way, for groups of two or more, the equivalent numbers of independent observations are found from the ratios of variances, for pairs:-

$$N_2' = \sigma_2^2 \Big/ \sigma_2^2 N_2 \tag{5}$$

where N_2 is the number of pairs and N_2' the corresponding equivalent number of independent pairs. Similar relationships apply for the larger groups. An iterative process is then employed to refine the estimates of σ_1^2 to σ_{16}^2 and to fit the parameters of equation (3). The final estimates of α and k are

$$\alpha = 0.090$$

$$k = 0.58$$

and the final estimates of σ_1^2 to σ_{16}^2 are given in the following table where they are compared with the values estimated from the formula using the above values for α and k and fitting at σ_1^2.

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* In addition to the scatter there appears to be a bias in the calculated values of r. Since the main object at this stage is to derive a satisfactory method for estimating σ_n^2 this question of bias is not pursued. The theory of bias in estimates of serial correlations is discussed by Kendall and Stuart[3].

	From observations	From formula
σ_1^2	16.470	16.470
σ_2^2	8.980	8.976
σ_4^2	4.903	4.950
σ_8^2	2.737	2.682
σ_{16}^2	1.398	1.405

Tables 1 to 5 give for each aircraft the values of N, N_2, N_4, N_8 and N_{16}; the final estimates of N', N_2', N_4', N_8' and N_{16}' and the initial and final estimates of σ_1^2, σ_2^2, σ_4^2, σ_8^2 and σ_{16}^2. It will be seen that successive approximations have produced only very small changes in the estimates of the variances.

We are now in a position to make a variance ratio test to see whether the differences between aircraft are significant. To do this we proceed exactly as if the calculated values of N' were the true number of observations. The first step is to compute the population mean on the basis of the null hypothesis, that is to say, assuming no significant differences between aircraft. This is calculated by weighting with respect to N', that is, inversely as the true variance of each aircraft mean. The overall mean is found to be 4.2750 and this is used to determine the sum of squares between aircraft. With the above value of the mean, and with the values of N' already determined, an analysis of variance following the standard procedure (see for example Bennett and Franklin[1], p.327) is made on the data in Table 1. The following variance estimates are so obtained.

$$\sigma_1^2 = 16.470 \quad \text{with 1216.4 degrees of freedom} \qquad (6)$$

$$61.564\rho^2 + \sigma_1^2 = 25.542 \quad \text{with 19 degrees of freedom} \qquad (7)$$

where ρ^2 is the variance between aircraft.

The variance ratio is 1.551 and this, with the numbers of degrees of freedom given above, indicates a level of significance of between 6% and 7%. In other words, if there were no differences between aircraft, the variance ratio would exceed that observed about once in fifteen times, so the result is hardly conclusive. However, in order to illustrate subsequent steps in the analysis it will be deemed a sufficient refutation of the null hypothesis.

Solving equations (6) and (7) for ρ^2 gives

$$\rho^2 = 0.14737 \quad .$$

Now the population mean on which this estimate is based has been estimated assuming the null hypothesis, and this is not necessarily the same value as would be estimated on another hypothesis, in this case, the hypothesis that there is a variation between aircraft given by the above value of ρ^2. This implies weighting the mean for each aircraft inversely as $\rho^2 + \sigma_1^2/N'$ $\left(\text{rather than inversely as } \sigma_1^2/N'\right)$. The population mean so found is 4.2764 compared with the value 4.2750 derived earlier so that in this case the change in hypothesis has made little difference to the mean and does not significantly affect the estimate of ρ^2. (In extreme cases, iteration might be necessary.) The variance of the final estimate of the mean is given by $1/\Sigma\left\{1/\left(\rho^2 + \sigma_1^2/N'\right)\right\}$ and is 0.02108. The information so far derived is now available for the estimation of factors for use in damage calculations but before considering this problem a short digression will be made in order to compare the procedure described above with that in which correlation effects are neglected.

4 ANALYSIS WITH CORRELATION EFFECTS NEGLECTED

If in the analysis of variance correlation effects are neglected, the values of N are used directly in setting up the estimates of variance. The equations so derived are:-

$$\sigma_1^2 = 16.389 \quad \text{with 1732 degrees of freedom} \tag{8}$$

$$87.228\rho^2 + \sigma_1^2 = 36.192 \quad \text{with 19 degrees of freedom} \quad . \tag{9}$$

These are to be compared with equations (6) and (7). The equation giving σ_1^2 is practically unchanged, the difference resulting from the use of factors $N/(N - 1)$ in place of $N'/(N' - 1)$. The number of degrees of freedom changes due to the replacement of N by N'. In the second equation the coefficient of ρ^2 and the right-hand side change from the same cause. Although the change in the number of degrees of freedom from 1216.43 to 1732 has little effect, the overall effect is to give a spuriously high value to the variance ratio and the probability based on this ratio is found to be about 1%. The estimate of ρ^2 based on equations (8) and (9) is 0.22703, so that the incorrect procedure leads not only to the rejection of the null hypothesis more often than is justified, but also to incorrect estimates of the variance between classes.

5 PREDICTION AND ESTIMATION PROBLEMS

We now consider how the analysis so far made can be applied in the estimation of factors for use in fatigue damage estimations. The general situation is the common one of being given one sample from a population, and from it, it is required to estimate the limit outside which a second sample will lie with a given small probability. Specifically, we are given the data from the fleet of aircraft, from which it is necessary to predict the probability that the average damage per flight for a further aircraft exceeds a certain specified value. A similar situation arises when attention is concentrated on a single aircraft which has made a number of unmetered flights for which it is required to make an estimate.

A convenient way of attacking the problem is to assume that each of the samples has a gamma distribution, the two distributions having the same mean but differing shape parameters. As shown in Ref.2 it is then a simple matter to derive a theoretical expression comparing the two samples. For completeness the mathematical analysis involved is repeated here and with a slightly different notation.

Let the element of the frequency distribution of y be given by

$$\frac{1}{(p-1)!}\left(\frac{py}{m}\right)^{p-1} \exp\left(-\frac{py}{m}\right)\frac{pdy}{m} \qquad (10)$$

and that of z by

$$\frac{1}{(q-1)!}\left(\frac{qz}{m}\right)^{q-1} \exp\left(-\frac{qz}{m}\right)\frac{qdz}{m} \quad . \qquad (11)$$

The mean is common to both distributions and equal to m and the variances of y and z are m^2/p and m^2/q respectively. In the examples to be considered y and z are average damage rates for given numbers of flights and the values of p and q are determined from the estimated mean and the variances.

The analysis leads to a comparison of y and z and the estimate of the mean is used only to determine p and q and does not appear explicitly in the result.

Make the substitution

$$u = (py + qz)/m \tag{12}$$

$$v = py/(py + qz) \tag{13}$$

and

$$pqdydz = m^2 ududv \quad . \tag{14}$$

The element of the frequency distribution of u and v is then

$$\frac{1}{(p - 1)!(q - 1)!} (uv)^{p-1}\{u(1 - v)\}^{q-1}e^{-u}ududv \quad . \tag{15}$$

This can be written as the product of two functions, one of u and one of v, showing that the variables are independent, *viz*

$$\left\{\frac{1}{(p + q - 1)!} u^{p+q-1}e^{-u}du\right\} \left\{\frac{(p + q - 1)!}{(p - 1)!(q - 1)!} v^{p-1}(1 - v)^{q-1}dv\right\} \quad . \tag{16}$$

The constants are such that the integral of the expression in each of the curly brackets is unity; u has a gamma-distribution and v a beta-distribution. Here we are concerned with the distribution of v, leading to a comparison of y and z. Some worked examples illustrate the application of this result.

5.1 Example (i)

Suppose we have to make allowance for 50 consecutive unmetered flights on a new aircraft of the fleet analysed above. The first step is to estimate the values of p and q. Using values of the fleet mean and its variance found above gives:-

$$p = 4.2764^2/0.02108$$
$$= 867.53 \quad .$$

The variance of the mean of 50 unmetered flights is $\rho^2 + \sigma_1^2/N'$ where N' is the equivalent number of independent observations corresponding to 50 flights.

From (4), $N' = 35.507$, and putting $\rho^2 = 0.14737$ and $\sigma_1^2 = 16.470$ gives for the required variance 0.61122. Hence

$$q = 4.2764^2/0.61122$$
$$= 29.92 .$$

The next stage in the analysis involves the probability level at which it is decided to work. Without prejudging the issue as to what are necessary factors, the level chosen here for illustrative purposes is one chance in forty, that is, $P = 0.025$, In this example we now let y be the mean of the fleet and z the mean of the unmetered flights. Remembering that $v = py/(py + qz)$, we have to solve

$$\frac{(p + q - 1)!}{(p - 1)!(q - 1)!} \int_0^v v^{p-1}(1 - v)^{q-1} dv = P \qquad (17)$$

for $P = 0.025$ and the values of p and q derived above. When z becomes large, v becomes small, and therefore the tail of the distribution which is of interest is from 0 to v, which are therefore the limits of integration in (17).

The values of p and q take us outside the tabulation of Pearson and Hartley[4], but the method due to Carter[5] for evaluating the integral, described therein[4], gives adequate accuracy. The value of v so found is 0.95396. The required factor F is the corresponding value of z/y and thus

$$F = p(1 - v)/(qv) \qquad (18)$$

so that finally $F = 1.399$.

(It is worth noting that when using Carter's method as described by Pearson and Hartley[4], our F is given directly by their e^{2z}.)

The damage is therefore assumed to be 1.399 × 50 × 4.2764, i.e. 299.1 units of damage.

5.2 Example (ii)

Assume that aircraft number 2 in Table 1 has made four unmetered flights and it is required to determine the appropriate factor to apply. As before, we first estimate p and q. Since the assumption is that the variance within aircraft is constant, the pooled estimate of 16.470 is taken for σ_1^2. We have now to estimate σ_{78}^2 and σ_4^2 from equations (1) and (4). There is no contribution from ρ^2 since we are making a 'within aircraft' estimation. These give $\sigma_{78}^2 = 0.29891$ and $\sigma_4^2 = 4.9506$; the mean for aircraft number 2 is 4.3242 so that $p = 62.56$ and $q = 3.777$. Using Table 16 of Pearson and Hartley[4] to solve equation (17) for the above values of p and q gives $v = 0.8765$ and from equation (18), $F = 2.334$. The total damage to be assumed is thus $2.334 \times 4 \times 4.3242$, i.e. 40.37 units of damage.

5.3 Chart for the determination of unmetered factors

A clearer idea of how the unmetered factors vary with p and q can be gained from a diagram, which can also prove a quick method for the approximate determination of such factors. For a given probability level the required factor depends only on p and q and it is thus possible to prepare a chart showing curves of constant values of F on a grid of values of p and q. This is done in Fig.3 for the value $P = 0.025$. On the chart points are shown corresponding to examples (i) and (ii). For practical use curves for more values of F would be required. It is to be noted that the scales are logarithmic and that the curves are asymmetric with respect to p and q.

6 CONCLUSIONS

In certain branches of fatigue damage estimation it is necessary to examine variability within the experience of a single aircraft and also between the many aircraft of a fleet. The usual techniques of analysis of variance are not directly applicable for a number of reasons, the most important being the existence of correlation between successive flights. The usual procedure can, however, be modified to take account of this, and 'unmetered factors' based on the resulting estimates of variance, and assuming a certain distribution for the damage parameter, can be found.

The procedure described leads to a consistent approach to such factors. It remains to establish the precise probability level that is acceptable in practice.

Appendix

THE RELATIONSHIP BETWEEN THE VARIANCE WITHIN AIRCRAFT AND THE SERIAL CORRELATIONS

Let the sum of squares from a sample of n flights for a given aircraft be S, so that

$$S = \sum_{m=1}^{n} \left\{ \frac{1}{n} (x_1 + x_2 \ldots + x_n) - x_m \right\}^2 \qquad \text{(A-1)}$$

where x is referred to the population mean as origin.

(If the origin is shifted then the shift disappears in the above expression.)

Expanding the above expression for S gives

$$S = \frac{n-1}{n} \sum_{m=1}^{n} x_m^2 - \frac{2}{n} (x_1 x_2 + x_2 x_3 \ldots x_{n-1} x_n)$$

$$- \frac{2}{n} (x_1 x_3 + x_2 x_4 \ldots x_{n-2} x_n)$$

$$- \frac{2}{n} (x_1 x_4 + x_2 x_5 \ldots x_{n-3} x_n)$$

etc. .

The sample value of S thus provides an estimate of the mean value, that is, the expectation, of the expression of the right of the equation. Replacing, then, the terms on the right by their expectations gives

$$S = (n-1)\sigma_1^2 - 2\sigma_1^2 \left(\frac{n-1}{n} r_1 + \frac{n-2}{n} r_2 \ldots \frac{1}{n} r_{n-1} \right)$$

i.e.

$$S = \sigma_1^2 \left(n - 1 - 2 \sum_{m=1}^{n-1} \frac{n-m}{n} r_m \right) . \qquad \text{(A-2)}$$

Writing

$$n' = n \Bigg/ \left(1 + 2 \sum_{m=1}^{n-1} \frac{n-m}{n} r_m \right) \qquad \text{(A-3)}$$

equation (A-2) can be rearranged to give

$$\sigma_1^2 = \frac{S}{n} \frac{n'}{(n' - 1)} \tag{A-4}$$

showing that the effective number of observations is reduced from n to n'.

In a similar way the variance of the mean of a sample of n consecutive values can be found, as follows. The square of the mean of a sample of n consecutive values is

$$\frac{1}{n^2}\left(\sum_{m=1}^{n} x_m\right)^2 = \frac{1}{n^2}\left\{\sum_{m=1}^{n} x_m^2 + 2(x_1x_2 + x_2x_3 \ldots x_{n-1}x_n) + 2(x_1x_3 + x_2x_4 \ldots x_{n-2}x_n) \ldots \text{etc.}\right\} .$$

The expectation of this expression is then the required variance, σ_n^2. Replacing by expectations gives

$$\sigma_n^2 = \frac{1}{n^2}\left\{n\sigma_1^2 + 2(n - 1)r_1\sigma_1^2 + 2(n - 2)r_2\sigma_1^2 \ldots\right\}$$

i.e.

$$\sigma_n^2 = \frac{\sigma_1^2}{n}\left(1 + 2\sum_{m=1}^{n-1} \frac{n - m}{n} r_m\right) \tag{A-5}$$

and therefore

$$\sigma_n^2 = \frac{\sigma_1^2}{n'} . \tag{A-6}$$

Table 1

SINGLE FLIGHTS

Aircraft number	N	$\bar{x}$	$\sqrt{\frac{\Sigma(x - \bar{x})^2}{N}}$	N'
1	113	4.2377	3.9458	79.60
2	78	4.3242	4.9081	55.10
3	119	5.4309	4.2637	83.80
4	76	3.6150	3.1673	53.70
5	112	4.4239	3.7842	78.90
6	65	4.0524	3.6497	46.01
7	53	5.2544	3.9501	37.61
8	68	4.8842	4.0499	48.11
9	43	4.8832	5.4392	30.61
10	92	3.6617	3.3239	64.90
11	128	4.2570	3.6599	90.10
12	119	4.3389	4.5192	83.80
13	76	3.9591	3.9008	53.70
14	123	3.5291	3.7848	86.56
15	102	3.2758	3.5891	71.90
16	79	4.3379	4.4116	55.80
17	63	3.3120	4.2163	44.61
18	104	5.2122	4.8691	73.30
19	69	3.9207	3.2913	48.81
20	70	4.8819	3.5311	49.51

Mean weighted with respect to N' = 4.2750

Initial estimate of σ_1^2 (based on N) = 16.389

Final estimate of σ_1^2 (based on N') = 16.470

Table 2

PAIRS OF FLIGHTS

Aircraft number	N_2	$\bar{x}$	$\sqrt{\frac{\Sigma(x - \bar{x})^2}{N_2}}$	N'_2
1	56	4.2555	3.1872	43.00
2	39	4.3242	3.2861	30.03
3	59	5.4769	2.9485	45.29
4	38	3.6150	2.0727	29.27
5	56	4.4239	2.7916	43.00
6	32	4.0957	2.8081	24.69
7	26	5.2063	2.8632	20.11
8	34	4.8842	2.9534	26.22
9	21	4.9585	3.8347	16.30
10	46	3.6617	2.4311	35.37
11	64	4.2570	2.7066	49.11
12	59	4.3452	3.3182	45.29
13	38	3.9591	2.7494	29.27
14	61	3.4815	2.8397	46.82
15	51	3.2758	2.4505	39.19
16	39	4.3412	3.3245	30.03
17	31	3.3526	3.0943	23.93
18	52	5.2122	3.6840	39.95
19	34	3.9784	2.6527	26.22
20	35	4.8819	2.7531	26.98

Initial estimate of σ_2^2 (based on N_2) = 8.9157

Final estimate of σ_2^2 (based on N'_2) = 8.9797

Table 3

GROUPS OF FOUR FLIGHTS

Aircraft number	N_4	$\bar{x}$	$\sqrt{\frac{\Sigma(x-\bar{x})^2}{N_4}}$	N_4'
1	28	4.2555	2.5258	23.72
2	19	4.4155	2.0257	16.14
3	29	5.5078	2.1847	24.56
4	19	3.6150	1.4195	16.14
5	28	4.4239	2.1440	23.72
6	16	4.0957	1.9430	13.62
7	13	5.2063	1.4954	11.09
8	17	4.8842	2.0805	14.46
9	10	5.1687	3.3845	8.57
10	23	3.6617	1.8021	19.51
11	32	4.2570	1.7271	27.08
12	29	4.2987	2.2181	24.56
13	19	3.9591	1.6254	16.14
14	30	3.4623	2.0150	25.40
15	25	3.2217	1.9469	21.19
16	19	4.3209	2.9390	16.14
17	15	3.3801	2.2723	12.78
18	26	5.2122	2.9180	22.03
19	17	3.9784	1.9570	14.46
20	17	4.8635	1.7156	14.46

Initial estimate of σ_4^2 (based on N_4) = 4.8596

Final estimate of σ_4^2 (based on N_4') = 4.9029

Table 4

GROUPS OF EIGHT FLIGHTS

Aircraft number	N_8	$\bar{x}$	$\sqrt{\frac{\Sigma(x - \bar{x})^2}{N_8}}$	N'_8
1	14	4.2555	1.7140	12.85
2	9	4.4905	1.7961	8.29
3	14	5.3966	1.6896	12.85
4	9	3.6240	1.0418	8.29
5	14	4.4239	1.3817	12.85
6	8	4.0957	1.5621	7.38
7	6	5.0889	0.9895	5.55
8	8	4.7581	1.7448	7.38
9	5	5.1687	2.3330	4.64
10	11	3.7454	1.4659	10.11
11	16	4.2570	1.3576	14.67
12	14	4.2768	1.8637	12.85
13	9	3.9438	1.1330	8.29
14	15	3.4623	1.4075	13.76
15	12	3.1146	1.1241	11.02
16	9	4.3560	1.9736	8.29
17	7	3.2353	1.2253	6.47
18	13	5.2122	2.1125	11.94
19	8	4.1643	1.2405	7.38
20	8	4.8365	1.3653	7.38

Initial estimate of σ_8^2 (based on N_8) = 2.6767

Final estimate of σ_8^2 (based on N'_8) = 2.7374

Table 5

GROUPS OF SIXTEEN FLIGHTS

Aircraft number	N_{16}	$\bar{x}$	$\sqrt{\frac{\Sigma(x - \bar{x})^2}{N_{16}}}$	N'_{16}
1	7	4.2555	0.9528	6.730
2	4	4.3928	1.1414	3.865
3	7	5.3966	1.2971	6.730
4	4	3.6143	1.0326	3.865
5	7	4.4239	1.0398	6.730
6	4	4.0957	1.0169	3.865
7	3	5.0889	0.3571	2.909
8	4	4.7581	0.7797	3.865
9	2	5.4187	0.4141	1.954
10	5	3.8408	1.2107	4.820
11	8	4.2570	1.1147	7.686
12	7	4.2768	1.5300	6.730
13	4	4.2414	0.2779	3.865
14	7	3.5547	0.9734	6.730
15	6	3.1146	0.8163	5.775
16	4	4.2729	1.0238	3.865
17	3	3.1542	0.5444	2.909
18	6	5.3484	1.6417	5.775
19	4	4.1643	0.5155	3.865
20	4	4.8365	0.6165	3.865

Initial estimate of σ^2_{16} (based on N_{16}) = 1.3863

Final estimate of σ^2_{16} (based on N'_{16}) = 1.3977

SYMBOLS

F	the required unmetered factor
k	parameter used in defining the correlogram
m	the mean damage per flight
n	the number of consecutive flights considered as a group
N	the number of flights made by a given aircraft
N_n	the number of groups considered of size n
	The dashed values n', N' and N'_n refer to estimates of the equivalent numbers of uncorrelated flights.
p	the shape parameter of the distribution of y
P	the probability level to which the factor F refers
q	the shape parameter of the distribution of z
r_s	the serial correlation with lag s
u, v	transformed variates depending on y and z
y, z	variates defining the average damage-per-flight distributions for two samples
α	parameter used in defining the correlogram
ρ^2	the variance of the mean damage per flight between aircraft
σ_n^2	the variance of the mean of n consecutive flights for a given aircraft

REFERENCES

No.	Author	Title, etc.
1	C.A. Bennett N.L. Franklin	Statistical analysis in chemistry and the chemical industry. Wiley (1954)
2	N.I. Bullen	The analysis of normal accelerations experienced by fleets of transport aircraft. R.Ae.S., CASI, AIAA International Conference on Atmospheric Turbulence, May 1971
3	M.G. Kendall A. Stuart	The advanced theory of statistics. Vol.3, Second Edition, Griffin (1968)
4	E.S. Pearson H.O. Hartley, (Eds.)	Biometrika tables for statisticians. Vol.1, Third Edition, Cambridge University Press (1966)
5	A.H. Carter	Biometrika. Vol.34, pp.352-8

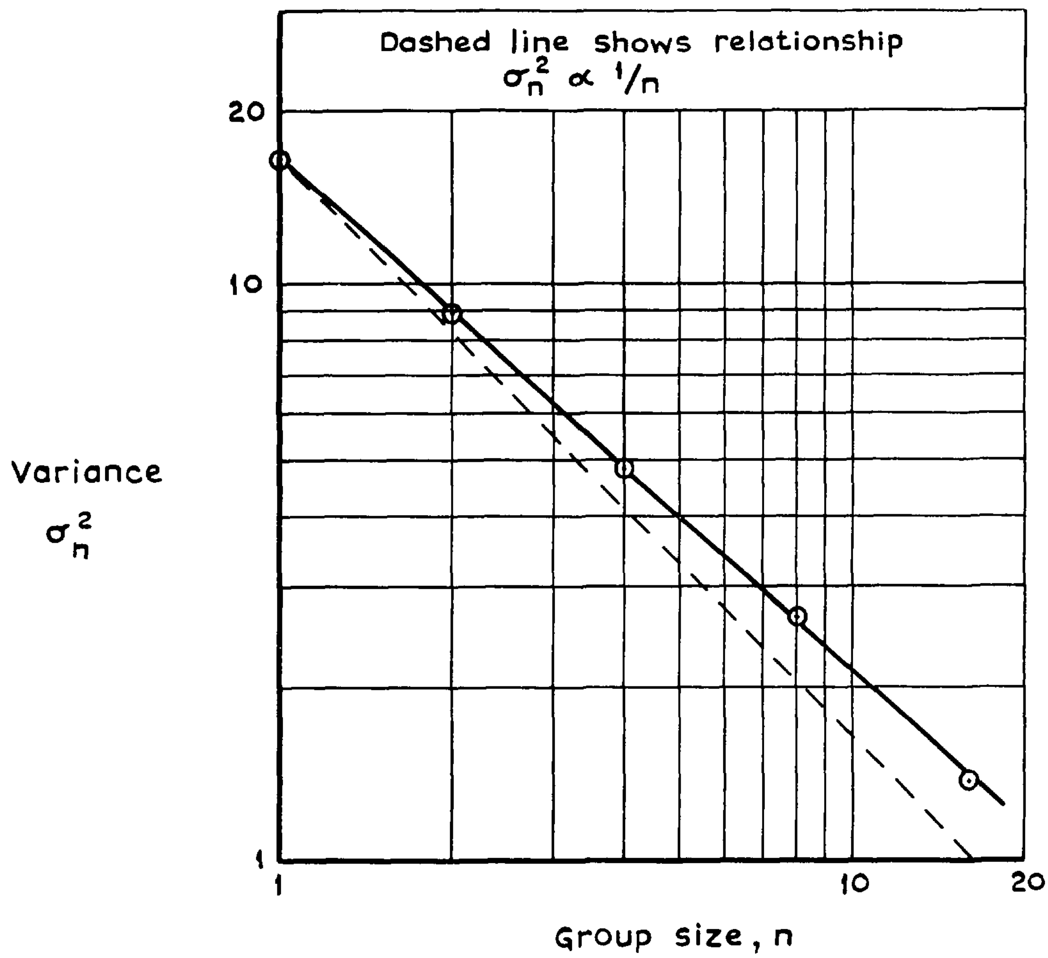

Fig.1 Relationship between variance within aircraft and number of flights per group

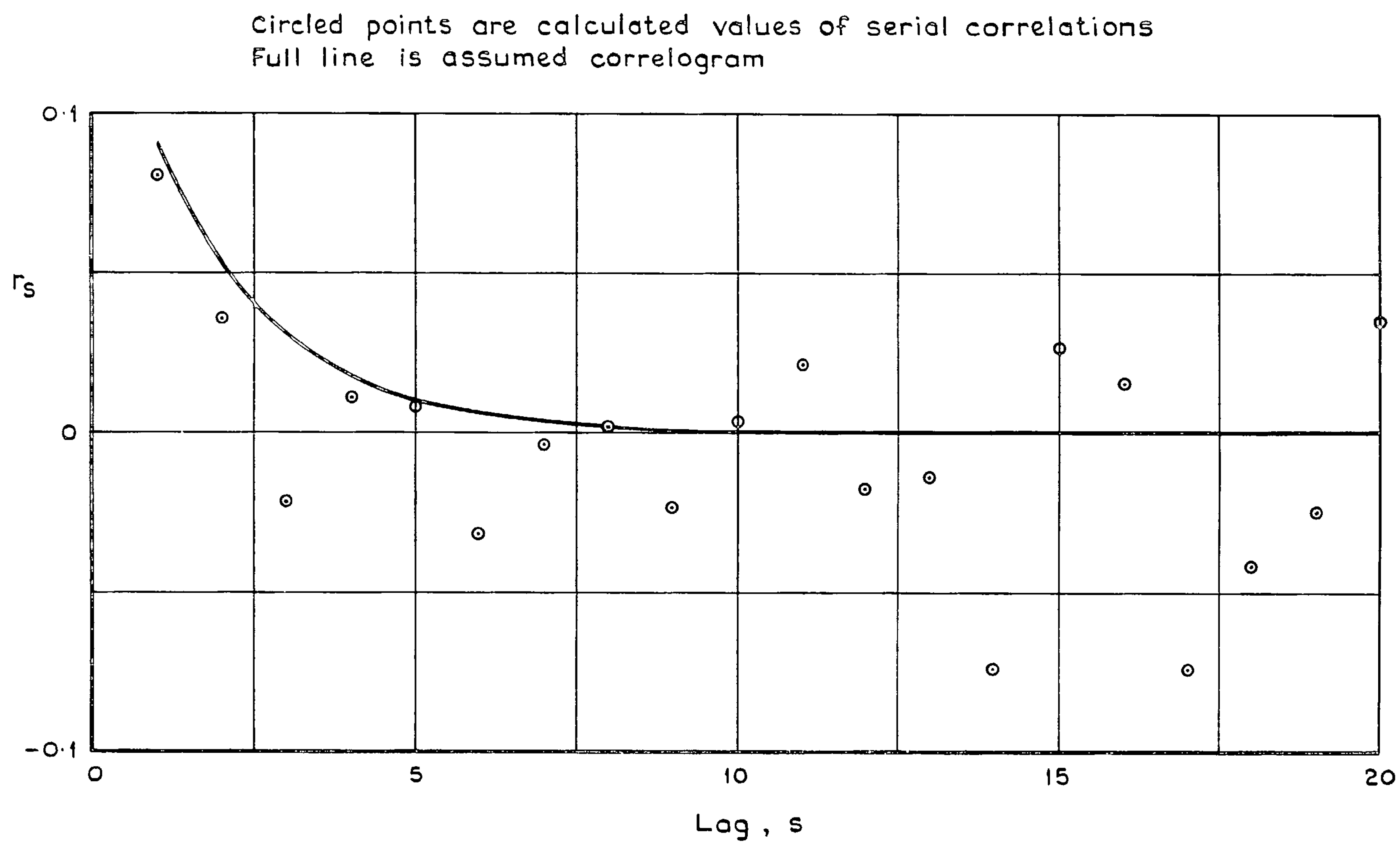

Fig.2 Calculated serial correlations and assumed correlogram

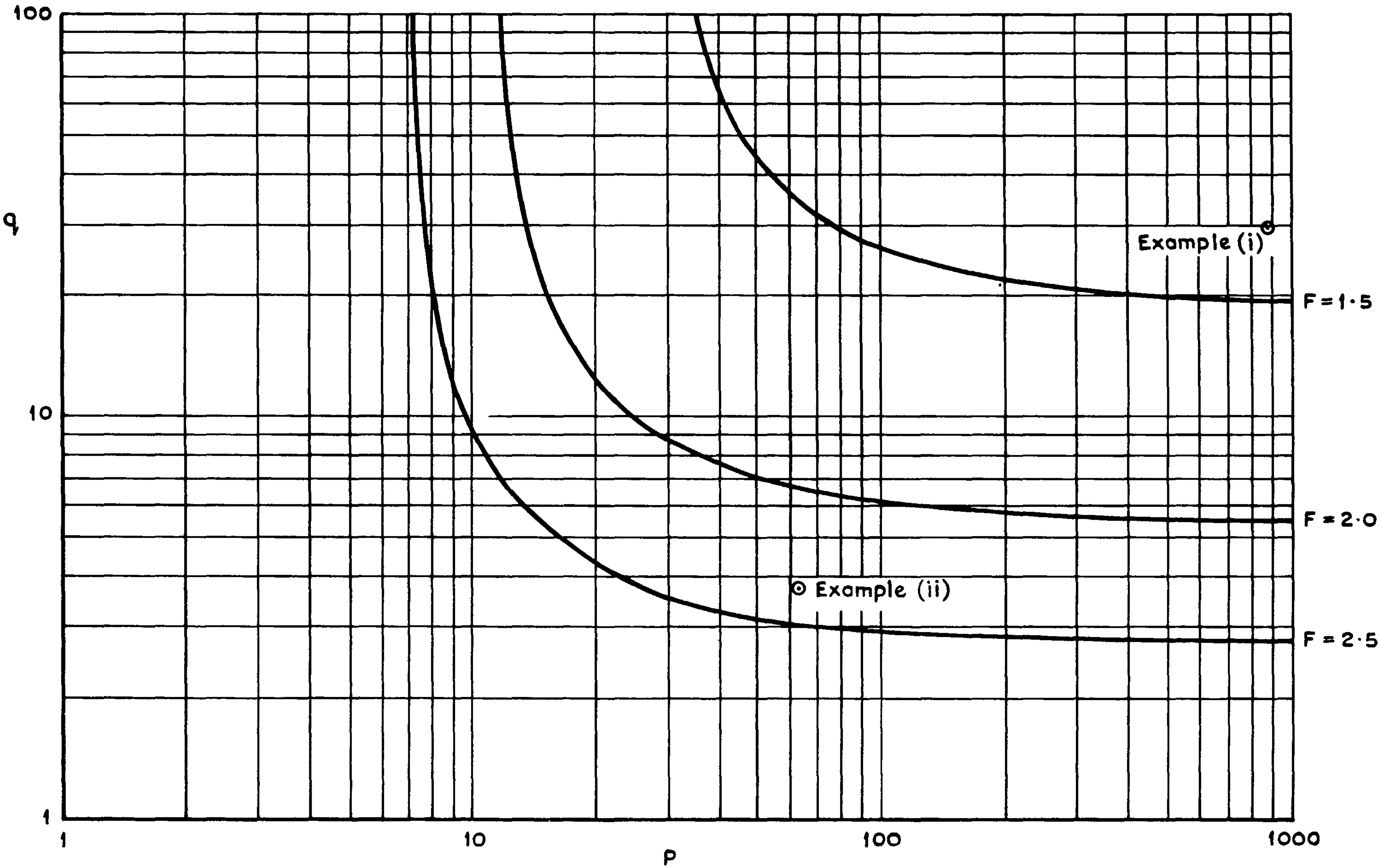

Fig.3 Chart of required factors

Printed in England for Her Majesty's Stationery Office by the Royal Aircraft Establishment, Farnborough. Dd. 505376 K4 6/74

ARC CP No.1297
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ISBN 011 470831 9