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Numerical Studies on Hypersonic Delta Wings with Detached Shock Waves

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NUMERICAL STUDIES ON HYPERSONIC DELTA WINGS WITH DETACHED SHOCK WAVES

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SUMMARY

In this report a numerical procedure is described for calculating the inviscid hypersonic flow about the lower surface of a conical wing of general cross-section. The method is based on thin-shock-layer theory and the cross-section of the wing may be either described by a polynomial (up to fourth degree) or given as tabulated data. The actual numerical scheme is an improvement on that used by earlier workers and the computation time is much shorter. This reduction in computation time has been exploited to produce a complete iterative procedure for the calculation of the pressure distribution and shock shape on a given wing at given flight conditions. (In earlier work graphical interpolation was used.)

The report includes a complete set of tabulated non-dimensional pressures and shock shapes for flat wings with detached shocks for reduced aspect ratios from 0.1 to 1.99, and some sample results for wings with caret and bi-convex cross-sections.

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*Replaces A.R.C.34 617

Introduction

It has been shown by Messiter[1], Squire[2.3], Hillier[4] and others that thin shocklayer theory gives pressure distributions and shock shapes on delta wings with simple cross-sections which are in very close agreement with experiments. The use of this theory involves the solution of a complex integral equation for the cross flow velocity (w) with boundary conditions at the centre line and at the leading edge. Once this cross flow velocity is found the pressure distribution and shock shape follow by direct integration. Most of the calculated results for the detached shock case have been obtained by Squire and Hillier for wings with simple cross-sections (flat wings, diamond cross-section wings, and some circular arc sections). They converted the integral equation into differential equation and marched out from the centre line using the first derivative of w at the centre line as a parameter (a_1). This method was very lengthy but by obtaining results for a number of values of a_1, they could use graphical interpolations from these results to obtain results for a particular wing at given flight conditions. However, the direct application of this numerical scheme to iterate to find the actual solutions corresponding to given flight conditions would require a very large computer time. Also this method can only be used for the simple sections mentioned above.

In the present report a direct method of solution of the integral equation is described which produces a considerable reduction in computer time and therefore it is possible to combine this method with a direct iterative scheme for the calculation of pressure distribution and shock shape on a given wing at given flight conditions.

2. Derivation of Equations

For steady flow of an ideal, inviscid gas the continuity, momentum and entropy equations can be written as

Continuity

$$\frac{\partial}{\partial \bar{x}}(\bar{\rho}\bar{u}) + \frac{\partial}{\partial \bar{y}}(\bar{\rho}\bar{v}) + \frac{\partial}{\partial \bar{z}}(\bar{\rho}\bar{w}) = 0$$

Momentum

$\bar{x}$
$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial \bar{x}} = 0$$

$\bar{y}$
$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{v}}{\partial \bar{z}} + \frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial \bar{y}} = 0$$

$\bar{z}$
$$\bar{u}\frac{\partial \bar{w}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{w}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{w}}{\partial \bar{z}} + \frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial \bar{z}} = 0$$

Entropy

$$\bar{u}\frac{\partial}{\partial \bar{x}}\left(\frac{\bar{p}}{\bar{\rho}^{\gamma}}\right) + \bar{v}\frac{\partial}{\partial \bar{y}}\left(\frac{\bar{p}}{\bar{\rho}^{\gamma}}\right) + \bar{w}\frac{\partial}{\partial \bar{z}}\left(\frac{\bar{p}}{\bar{\rho}^{\gamma}}\right) = 0$$

These equations must be solved subject to Rankine-Hugoniot jump conditions at the shock. These are:

Continuity:-
$$\left[\bar{\rho}\cdot(\vec{\bar{q}}\cdot\vec{n}_s)\right] = 0$$

Momentum:-
$$\left[\bar{p} + \bar{\rho}(\vec{\bar{q}}\cdot\vec{n}_s)^2\right] = 0$$

Energy:-
$$\left[\frac{1}{2}(\vec{\bar{q}}\cdot\vec{n}_s)^2 + \frac{\gamma}{\gamma-1}\frac{\bar{p}}{\bar{\rho}}\right] = 0$$

Tangential velocity:-
$$\left[\vec{\bar{q}}\times\vec{n}_s\right] = 0$$

where the square brackets denote the change in the enclosed quantity across the shock discontinuity and $\vec{n}_s$ denotes a unit vector normal to the shock surface and directed away from the body. The body boundary conditions require the streamlines to become tangential to the surface i.e.

$$\vec{\bar{q}}_B \cdot \vec{n}_B = 0 \qquad \text{on } \bar{y} = \bar{y}_B \tag{3}$$

In thin-shock layer theory for conical wings the co-ordinate system is first stretched to

$$\begin{aligned} x &= \bar{x} \\ y &= \bar{y} / x \epsilon \tan\alpha \\ z &= \bar{z} / x \epsilon^{1/2} \tan\alpha \end{aligned} \tag{4}$$

Where the barred symbols refer to physical co-ordinate system and unbarred quantities refer to transformed (or stretched) co-ordinates. In this transformed co-ordinate system the wing semi-span and thickness become

$$\begin{aligned} \Omega &= b / x \epsilon^{1/2} \tan\alpha \\ t_o &= h / x \epsilon \tan\alpha \end{aligned} \tag{5}$$

respectively.

For a shock which differs only slightly from a plane shock Messiter suggested an expansion of flow properties in terms of ϵ which is the inverse of the density ratio across a basic shock, lying in the plane of the leading edges of the wing. In the limit $\epsilon \rightarrow 0$ the expressions tend to basic Newtonian solutions. The basic density ratio across the shock is given by

$$\epsilon = \frac{\gamma - 1}{\gamma + 1} + \frac{2}{\gamma + 1} \frac{1}{M_\infty^2 \sin^2\alpha} \tag{6}$$

where α is the incidence of the plane of the leading edges. The suggested expansions for flow properties are

$$c_p = \frac{\bar{p} - \bar{p}_\infty}{\frac{1}{2}\bar{\rho}\,U_\infty^2} = 2\,\sin^2\alpha\left(1 + \epsilon\,p(y,z)\right) + O(\epsilon^2)$$

$$\frac{\bar{u}}{U_\infty} = \cos\alpha + \epsilon\,\sin\alpha\,\tan\alpha\;u(y,z) + O(\epsilon^2)$$

$$\frac{\bar{v}}{U_\infty} = \epsilon\;\sin\alpha\;v(y,z) + O(\epsilon^2) \qquad (7)$$

$$\frac{\bar{w}}{U_\infty} = \epsilon^{1/2}\,\sin\alpha\;w(y,z) + O(\epsilon^{3/2})$$

Substitution of these quantities into the equations of motion lead to a consistent system of equations and boundary conditions which are

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$(v-y)\frac{\partial v}{\partial y} + (w-z)\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} \qquad (8)$$

$$(v-y)\frac{\partial w}{\partial y} + (w-z)\frac{\partial w}{\partial z} = 0$$

with shock boundary conditions

$$v_s = \left(y_s - z\,\frac{dy_s}{dz}\right) - 1 - \left(\frac{dy_s}{dz}\right)^2$$

$$w_s = -\frac{dy_s}{dz} \qquad (9)$$

$$p_s = -1 - \left(\frac{dy_s}{dz}\right)^2 + 2\left(y_s - z\frac{dy_s}{dz}\right)$$

where $y_s(z)$ is the equation for the shock, and v_s, w_s and p_s denote components of velocities and pressure immediately downstream of the shock. The equation (8) have two sets of real characteristics given by Z = const. and ξ = const. where

$$(v-y)\frac{\partial \xi}{\partial y} + (w-z)\frac{\partial \xi}{\partial z} = 0 \qquad (10)$$

Since the operator $(v-y)\frac{\partial}{\partial y} + (w-z)\frac{\partial}{\partial z}$ is the total derivative along a streamline to this approximation, the ξ = const. characteristic coincides with the projected streamlines in the conical plane. Equations (8) also show w to be constant along a streamline and therefore it is a function of ξ only.

Messiter fixed the constant on ξ characteristics by putting $\xi = Z$ on the shock. He also showed that solution of equations (8) depend on one parameter $w(\xi)$ and by considering body boundary conditions he showed that

$$w(\xi) = Z_b \qquad (11)$$

for the detached shock case.

The solution of equation (8) leads after much algebra to

$$\left(\frac{dy}{dz}\right)_{body} = -W(z_b) - \frac{1}{W(z_b)-z_b} + \int_{\xi}^{z_b} \frac{ds}{[W(s)-s]^2} \qquad (12)$$

This is the fundamental equation for the determination of $W(\xi)$.

The pressure on the body and the shock shape are given by

$$p(z,\xi) = -1 - W^2(z) + 2\left[\Delta_0 + t_0 - \int_0^z W(s)\,ds\right]$$

$$+ 2z\,W(z) + \left[-1 + \frac{1}{\{W(s)-s\}^2}\right] \frac{dW(z)}{dz} \int_{\xi}^{z} \frac{[W(s)-z]^3}{[W(s)-s]^2}\,ds$$

$$p^* = p(z,\xi) - 2t_0 \qquad (14)$$

$$y_s(z) = \Delta_0 + t_0 - \int_0^z W(s)\,ds \qquad (15)$$

Messiter also showed that the appropriate boundary conditions for $W(\xi)$ are $W(0) = 0$ and $W(\Omega) = 1 + \Omega$. The first condition corresponds to zero cross-flow on the centre line. The second condition was chosen to give a singularity in the shock curvature at the leading edge since a similar singularity occurs in certain two-dimensional blunt body flows. By equation (9) there is a similar singularity in the span-wise derivative of $W(Z)$ at the edge and this leads to some difficulty in the numerical solution of equation (12).

3. Numerical Solution of the Integral Equation

The analysis up to this section was similar to that of Messiter and Squire. Squire solved the integral equation by differentiating once again and then using Runge-Kutta procedure for integration of the resultant differential equation. This procedure takes a long time for computation of $w(\xi)$ for a single value of the parameter a_1. Again the step size for Runge-Kutta type of integration must be extremely small (0.001) so that large storage was required. The Runge-Kutta procedure was used up to a certain point (i.e. $w(t) > 1 + 0.75t$) and for the remaining part manual graphical extrapolation was used. By a suitable choice of a_1 it was thus possible to get a set of results for a range of Ω and ϵ where ϵ is thickness ratio of the wings with diamond cross-sections. These results were then used to produce a set of charts which could be used to find the pressure distributions and shock shapes for any given wing with diamond cross-section at given flight conditions. A similar method was used for caret wings, and for wings with biconvex cross-sections. In general this method cannot be used for general cross-sections.

In the present evaluation of the integral equation (12), a different approach was used*. This approach is as follows.

Let us assume that the solution has been obtained up to the i^{th} station. Then at the i^{th} station

$$\left(\frac{dy_B}{dz}\right)_i = -W(Z_b)_i - \frac{1.0}{W(Z_b)_i - Z_{b_i}} + \int_{\xi_i}^{Z_{b_i}} \frac{ds}{[W(s) - s]^2} \qquad (16)$$

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* This method was originally suggested by Dr. R. Hillier, but he only applied the method to the case $w(t) > t$.

Similarly at i + 1th station

$$\left(\frac{dy_B}{dz}\right)_{i+1} = -W(z_b)_{i+1} - \frac{1.0}{W(z_b)_{i+1} - z_{b_{i+1}}} + \int_{\xi_{i+1}}^{z_{b_{i+1}}} \frac{ds}{[W(s) - s]^2} \qquad (17)$$

$$\therefore \left(\frac{dy_B}{dz}\right)_{i+1} - \left(\frac{dy_B}{dz}\right)_i = W(z_b)_i - W(z_b)_{i+1}$$

$$+ \frac{1.0}{W(z_b)_i - z_{b_i}} - \frac{1.0}{W(z_b)_{i+1} - z_{b_{i+1}}}$$

$$- \int_{\xi_i}^{z_{b_i}} \frac{ds}{[W(s) - s]^2} + \int_{\xi_{i+1}}^{z_{b_{i+1}}} \frac{ds}{[W(s) - s]^2} \qquad (18)$$

$$= W(z_b)_i - W(z_b)_{i+1} + \frac{1.0}{W(z_b)_i - z_{b_i}} - \frac{1.0}{W(z_b)_{i+1} - z_{b_{i+1}}}$$

$$+ \int_{z_{b_i}}^{z_{b_{i+1}}} \frac{ds}{[W(s) - s]^2} - \int_{\xi_i}^{\xi_{i+1}} \frac{ds}{[W(s) - s]^2} \qquad (19)$$

If we take the step length to be sufficiently small the integrals can be evaluated using the trapezoidal rule and the equation become

$$\left(\frac{dy_B}{dZ}\right)_{i+1} - \left(\frac{dy_B}{dZ}\right)_i = W(Z_b)_i - W(Z_b)_{i+1}$$

$$+ \frac{1.0}{W(Z_b)_i - Z_{b_i}} \qquad \frac{1.0}{W(Z_b)_{i+1} - Z_{b_{i+1}}}$$

$$+ \frac{\Delta g}{2}\left[\frac{1.0}{\{W(Z_b)_{i+1} - Z_{b_{i+1}}\}^2} + \frac{1.0}{\{W(Z_b)_i - Z_{b_i}\}^2}\right]$$

$$- \frac{\Delta \zeta}{2}\left[\frac{1.0}{\{W(\zeta)_{i+1} - \zeta_{i+1}\}^2} + \frac{1.0}{\{W(\zeta)_i - \zeta_i\}^2}\right] \qquad (20)$$

where

$$\Delta g = Z_{b_{i+1}} - Z_{b_i} \qquad (21)$$

$$\Delta \zeta = \zeta_{i+1} - \zeta_i$$

Equation (20) was solved by a marching process for a given starting parameter a_1, until $W(t) > 1 + 0.75t$ and then a parabolic type of extrapolation was used to find the value of Ω which satisfies $W(\Omega) = 1 + \Omega$. The whole process was iterated to get correct value of a_1 (i.e. correct $W(\zeta)$ function) for given boundary conditions of cross-sectional profile, Mach number and incidence.

Equation (20) was solved subject to boundary conditions $W(0) = 0$ and $W(\Omega) = 1 + \Omega$. Near the origin (i.e. $Z_b = \zeta = 0$) a fifth power series solution was assumed for $W(\zeta)$ i.e.

$$W(\zeta) = a_1\zeta + a_2\zeta^2 + a_3\zeta^3 + a_4\zeta^4 + a_5\zeta^5 + O(\zeta^6) \qquad (22)$$

This is similar to Squire's treatment for analytic cross-section case. Here a_2, a_3, a_4 and a_5 are related to the cross-sectional shape of the body by the following expression

$$\left(\frac{dy}{dz}\right)_{body} = \mp\left[\frac{a_1-1}{a_1(a_1-1)^2} - \frac{2\log a_1}{(a_1-1)^3}\right] a_2$$

$$- \left[a_2^2 - a_2a_3 + a_1^4\right]\frac{Z}{a_1^3}$$

$$+ \left[\frac{(1+a_1)}{a_1^5}\left(2a_1^3 - 3a_1a_2a_3 + a_1^2a_4\right) - \frac{a_2}{a_4}\left(a_1^4 - a_1a_3 + a_2^2\right)\right] Z^2$$

$$+ \left[-a_3 - \frac{a_3^2}{a_1^5}\left(a_1^2 + 2a_1 + 2\right) + \frac{a_5}{3a_1^4}\left(3a_1^2 + 4a_1 + 3\right)\right.$$

$$- \frac{2a_2a_4}{a_1^5}\left(a_1^2 + 2a_1 + 2\right) + \frac{a_2^2a_3}{a_1^6}\left(3a_1^2 + 8a_1 + 10\right)$$

$$\left. - \frac{a_2^2}{3a_1^7}\left(3a_1^2 + 10a_1 + 15\right)\right] Z^3 \qquad (23)$$

In the case of an analytic cross-section in the form of a polynomial up to fourth degree it is easy to calculate the slope $\left(\frac{dy}{dz}\right)_{body}$ and to equate coefficients of powers of Z to calculate a_2, a_3, a_4, a_5, in terms of a_1. (This is the parameter which is to be determined by the iterative procedure mentioned above.) This procedure was used by Squire and Hillier for delta wings with diamond and circular arc cross-sections.

But the real problem in the general case arises as follows; first, if the given profile was a polynomial of more than fourth degree and secondly, if the profile was given in the form of a table at finite number of discrete points. To overcome this problem, we approximate the cross-sectional shape by a five point Lagrangian formula then by differentiating this formula with respect to Z, an expression for $\left(\frac{dy}{dz}\right)_{body}$ can be obtained at any Z. This expression for $\left(\frac{dy}{dz}\right)_{body}$ can be expressed as a third degree polynomial in Z as follows.

$$\left(\frac{dy}{dz}\right)_{body} = K_4 Z^3 + K_3 Z^2 + K_2 Z + K_1$$

where

$$K_1 = -\sum_{i=1}^{5} y_i \frac{\sum\limits_{\substack{j=1,\, k=1,\, l=1 \\ j \neq k \neq l \neq i}}^{5} Z_j Z_k Z_l}{\prod\limits_{\substack{j=1 \\ j \neq i}}^{5} (Z_i - Z_j)}$$

$$K_2 = 2\sum_{i=1}^{5} y_i \frac{\sum\limits_{\substack{j=1,\, k=1 \\ j \neq k \neq i}}^{5} Z_j Z_k}{\prod\limits_{\substack{j=1 \\ j \neq i}}^{5} (Z_i - Z_j)}$$

$$K_3 = -3\sum_{i=1}^{5} y_i \frac{(Z_1 + Z_2 + Z_3 + Z_4 + Z_5 - Z_i)}{\prod\limits_{\substack{j=1 \\ j \neq i}}^{5} (Z_i - Z_j)}$$

$$K_4 = 4\sum_{i=1}^{5} \frac{y_i}{\prod\limits_{\substack{j=1 \\ j \neq i}}^{5} (Z_i - Z_j)}$$

These coefficients were used to calculate a_2, a_3, a_4, a_5 at the origin for any given a_1.

After the initial polynomial expansion for $W(t)$ (t being the running variable) has been found, the direct solution of the equation can be undertaken. The actual step by step procedure is best understood by noting that $W(z_b)$ is the same function of z_b as $W(\xi)$ is that of ξ. So if a solution has been obtained up to a particular value of the independent variable (say t_f) then $W(z_b)$, z_b, $W(\xi)$ and ξ are known for all values of z_b and ξ less than, or equal to t_f.

Now suppose, at t_f; $W(t_f) < t_f$, in this case we can identify t with ξ and since $z_b = W(\xi) < \xi$, $W(z_b)$ is known. Therefore equation (20) can be written as,

$$\frac{\Delta\xi}{2}\left[\frac{1.0}{(W(\xi)_{i+1} - \xi_{i+1})^2}\right] = \left(\frac{dy_B}{dz}\right)_i - \left(\frac{dy_B}{dz}\right)_{i+1}$$

$$+ W(z_b)_i - W(z_b)_{i+1}$$

$$+ \frac{1.0}{W(z_b)_i - z_{b_i}} - \frac{1.0}{W(z_b)_{i+1} - z_{b_{i+1}}}$$

$$+ \frac{\Delta g}{2}\left[\frac{1.0}{\{W(z_b)_{i+1} - z_{b_{i+1}}\}^2} + \frac{1.0}{\{W(z_b)_i - z_{b_i}\}^2}\right]$$

$$- \frac{\Delta\xi}{2}\left[\frac{1.0}{\{W(\xi)_i - \xi_i\}^2}\right] \qquad (24)$$

In this equation $W(\xi)_{i+1}$ and $W(z_b)_{i+1}$ are unknowns. But if we know $W(\xi)_{i+1}$ which is equal to $(z_b)_{i+1}$, and as $W(\xi)_{i+1} < \xi_{i+1}$, $W(z_b)_{i+1}$ will be less than $z_{b\,i+1}$ and can be interpolated from previous values. The method of bisections was used to evaluate the correct value of $W(\xi)_{i+1}$ from a first approximation (which was the linear extrapolated value from the previous step), so that equation (24) was satisfied. Once the correct value of $W(\xi)_{i+1}$ was obtained, the solution was carried for the next step.

On the other hand, if $W(t_f) > t_f$, then t is identified with Z_b and since in this case $\xi < Z_b$ it can be interpolated from already computed solutions at previous t values. Re-arranging the equation (20) we get a cubic equation in $W(Z_b)_{i+1}$ i.e.

$$W(z_b)^3_{i+1} + W(z_b)^2_{i+1}\left\{-2 z_{b\,i+1} + RHS\right\}$$

$$+ W(Z_b)_{i+1}\left[z^2_{b_{i+1}} + 1 - 2 z_{b\,i+1}(RHS)\right]$$

$$+ \left[(RHS)\, z^2_{b_{i+1}} - z_{b\,i+1} - \frac{\Delta g}{2}\right] = 0 \qquad (25)$$

where

$$RHS = \left(\frac{dy_B}{dz}\right)_{i+1} - \left(\frac{dy_B}{dz}\right)_i - W(z_b)_i - \frac{1.0}{W(z_b)_i - z_{b\,i}}$$

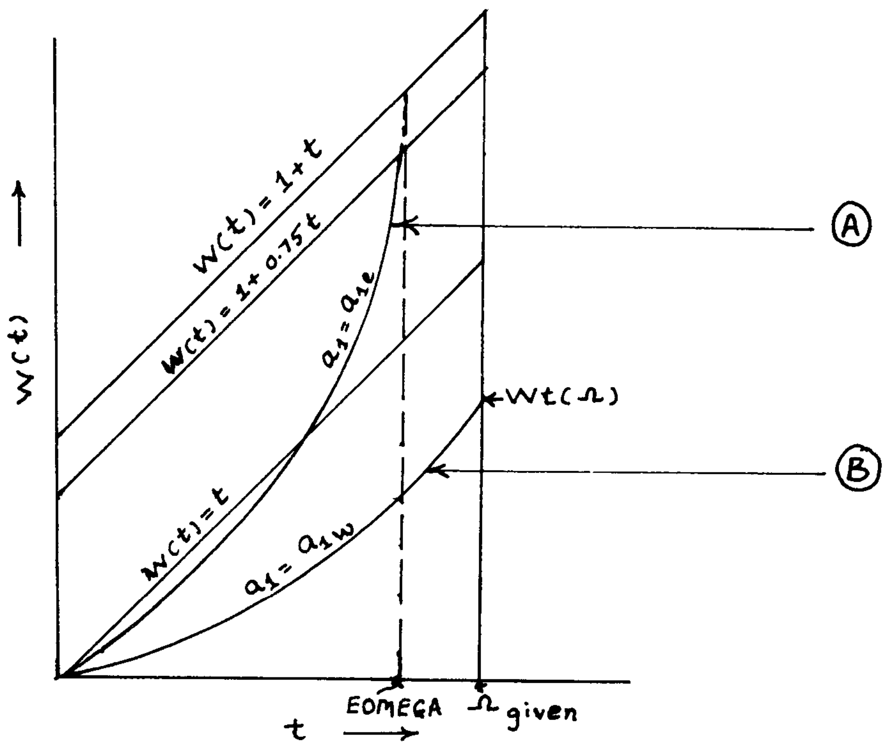
$$- \frac{\Delta g}{2}\left[\frac{1.0}{\{W(z_b)_i - z_{b\,i}\}^2}\right]$$

$$+ \frac{\Delta \xi}{2}\left[\frac{1.0}{\{W(\xi)_{i+1} - \xi_{i+1}\}^2} + \frac{1.0}{\{W(\xi)_i - \xi_i\}^2}\right] \qquad (26)$$

Equation (25) was solved by Newton-Raphson method for $W(z_b)_{i+1}$

So, to summarise, if $W(t) > t$ equation (25) was solved whereas if $W(t) < t$ equation (24) was solved.

Solving the above equations step by step, cross-flow velocity distributions of one of the following two cases (i.e. case A or B) are obtained.



If the cross-flow velocity distribution was as in the case A the whole set of calculations were repeated with a new value of a_1 equal to half of a_{ie} and conversely if the distribution was as in the case B , the new value of a_1 was taken as twice the value of a_{1_w} . This process was repeated till we get both cases A and B . The correct value of a_1 and the corresponding $W(\xi)$ distribution lies in between these two cases. After obtaining this upper and lower bounds of the cross-flow velocity distribution, the iterative process was continued with new a_1 parameter such that

$$a_{1_{new}} = a_{1_e} + \frac{(a_{1_e} - a_{1_w})[Wt(\Omega)]}{[Wt(\Omega) - \Omega_{given} - EOMEGA]}$$

This process was repeated till we get the $W(\xi)$ distribution such that $EOMEGA$ or $Wt(\Omega)$ is within one percent of the correct value of Ω given or 1 + Ω given respectively. After obtaining the correct cross-flow velocity distribution, the non-dimensional pressure coefficient, c_p, shock shape, C_L, C_D, etc., were calculated.

A different approach, namely

$$a_{1_{new}} = \frac{a_{1_e} + a_{1_w}}{2}$$ was also tried

but it was found that the first procedure converges slightly faster than this second procedure.

There are two main difficulties in the integration procedure. One concerns the outer boundary condition given by $W(\Omega) = 1 + \Omega$. This boundary condition was chosen by Messiter to coincide with the singularity in the shock curvature. Squire (2) has found that near the point where $W(\Omega) = 1 + \Omega$, $W(t) \propto (\Omega - t)^{1/2}$ + ... So the solution of the equation was stopped when $W(t) > 1 + 0.75t$ and then remaining portion of the curve was obtained by a parabolic type of extrapolation with the vertex of the parabola having co-ordinates (Ω extrapolated, 1 + Ω extrapolated) consistent with the $W(t)$ values calculated so far.

The other difficulty arises when $w(t) = t$ since at $w(t) = t$ the equation becomes indeterminate. A trap was therefore included such that when $w(t)$ curve crosses the line $w(t) = t$, as found during the solution of equation (24) the value for that step was obtained by 6 point Nevil type of extrapolation from previous solutions. This is best explained in fig. 2 curve (a).

If $w(t)$ as extrapolated above falls below the $w(t) = t$ line as in the case of case (B) fig. 2, this indicates that $w(t)$ is increasing rapidly and the $w(t)$ value is influenced by the square root singularity at the leading edge. So $w(t)$ value was re-calculated using a parabolic type of extrapolation (stipulating similar type of singularity as that at the leading edge).

4. Result

Using the above programme, the pressure p^*, pressure coefficient c_p and shock shapes were calculated on flat delta wings. Sample calculations are also given for a caret wing and for a circular arc cross-section wing when the cross-section was given in the analytic form as well as in the form of a table ($\bar{z}_i$, $\bar{y}_i$) at 51 points. For flat wing the functions, p^* in the pressure coefficient and the non dimensional shock shape are functions of z/Ω and Ω . These functions have been calculated for the range $0.1 \leq \Omega \leq 1.99$ and are tabulated in tables Ia and Ib and plotted in Figs. For these calculations the programme was modified to read Ω directly, together with number of steps into which wing span has to be divided, which determines step length, and the starting value of a_1. The number of steps used when $\Omega \leq 1.0$ was 200 and $\Omega \geq 1.0$ was 400. The accuracy of the result was tested by doubling the number of steps in the same case and it was found that there is no variation of results up to four figures. A typical solution for flat delta wing takes about 5 to 8 seconds on Cambridge University IBM 370/165 computer with FORTRAN G1 compiler.

An interesting result shows up if we plot the correct a_1 parameter against Ω for the flat delta wing, fig. (4). In the region between $\Omega = 0.5$ and 0.51 a_1 jumps from $a_1 > 1.0$ to $a_1 < 1.0$. This can be explained by the sketches of the flow field (fig. 3). If $a_1 < 1.0$ we get $W(t) < t$ and the flow field is as shown in fig. (3a) and if $W(t) > t$ the flow field will look like fig. 3(b) and so at certain the flow field will jump from (a) to (b) or vice versa, depending on whether Ω is increased or decreased.

Table II gives p^*, c_p and shock shapes for a caret wing.

Table III compares results for circular arc (biconvex) cross-section when the cross-section was given in the analytic form with that when the cross section was given in the form of a table at 51 points. Both the results compare very well. The results of calculations were compared with experimental results of Squire (ref. 5) in fig. 5, which shows a good agreement.

Although the programme converged successfully from any starting value of a_1 for a variety of shapes, such as flat wings, caret wings, biconcave wings and thin biconvex wings and also a wavy type of cross-section(sketch a), some difficulties were experienced on more extreme shapes. In particular it was very difficult to get converged solutions for the shape shown in sketch (b) and for very thick biconvex wings. The difficulties appeared to be caused by the fact that if the initial value of a_1 was too far from the correct value then the computed cross-flow, $W(\xi)$, was completely unrealistic and the iterative procedure did not converge. To overcome these difficulties it was necessary to do a preliminary series of computations using the basic programme (i.e. without iteration) for a range of values of a_1 .

By plotting these results, it was usually possible to find values of a_1 which appeared to be in the correct range. The iterative procedure could then be used to complete the solution. However, it should be pointed out that on caret wings a_1 is usually small particularly near design, whereas for thick wings a_1 can be large. On complicated shapes such as that shown in sketch (b) it was found that possible values of a_1 lay in a very narrow range and that with values of a_1 outside this range the computed curves of $W(\xi)$ were completely unrealistic. Thus it may require a few preliminary runs to find appropriate range of a_1 .

(a) (b)

Acknowledgements

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References

No.	Author(s)	Title, etc.
1	A. F. Messiter	Lift of slender delta wings according to Newtonian theory. AIAA J.3, pp.427-433. 1965.
2	L. C. Squire	Calculated pressure distributions and shock shapes on thick conical wings. Aeronautical Quarterly, Vol.18, p.185. May, 1967.
3	L. C. Squire	Calculation of pressure distribution on lifting conical wings with application to off design behaviour of wave-riders. AGARD Conference Proceedings, 30. 1968.
4	R. Hillier	Some application of thin shock layer theory to hypersonic wings. Ph.D. Thesis. Engineering Department, Cambridge University. 1970.
5	L. C. Squire	Pressure distributions and flow patterns on some conical shapes with sharp edges and symmetrical cross-sections at M = 4.0. ARC R & M 3340. 1963.

TABLE Ia

$\frac{z}{\Omega}$	P^* distributions on flat delta Wings for Ω =									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.51
0.00	- 0.1081	0.0257	0.1095	0.1733	0.2275	0.3224	0.3389	0.3512	0.3669	0.3140
0.05	- 0.1085	0.0255	0.1094	0.1734	0.2278	0.3226	0.3393	0.3516	0.3694	0.3141
0.10	- 0.1096	0.0248	0.1091	0.1735	0.2283	0.3234	0.3404	0.3530	0.3711	0.3148
0.15	- 0.1114	0.0237	0.1087	0.1736	0.2271	0.3249	0.3423	0.3553	0.3739	0.3159
0.20	- 0.1139	0.0221	0.1080	0.1738	0.2282	0.3271	0.3451	0.3586	0.3779	0.3180
0.25	- 0.1171	0.0200	0.1072	0.1740	0.2302	0.3299	0.3486	0.3629	0.3831	0.3203
0.30	- 0.1213	0.0173	0.1060	0.1744	0.2327	0.3332	0.3530	0.3682	0.3897	0.3233
0.35	- 0.1265	0.0142	0.1045	0.1747	0.2358	0.3376	0.3582	0.3747	0.3975	0.3263
0.40	- 0.1328	0.0099	0.1026	0.1750	0.2391	0.3421	0.3642	0.3823	0.4070	0.3306
0.45	- 0.1403	0.0048	0.1000	0.1749	0.2426	0.3477	0.3716	0.3913	0.4180	0.3350
0.50	- 0.1495	- 0.0012	0.0966	0.1748	0.2466	0.3538	0.3796	0.4017	0.4309	0.3400
0:55	- 0.1603	- 0.0091	0.0923	0.1739	0.2508	0.3608	0.3890	0.4137	0.4459	0.3456
0.60	- 0.1735	- 0.0185	0.0867	0.1725	0.2554	0.3689	0.3997	0.4274	0.4632	0.3518
0.65	- 0.1898	- 0.0309	0.0791	0.1700	0.2600	0.3776	0.4119	0.4431	0.4832	0.3586
0.70	- 0.2102	- 0.0464	0.0691	0.1656	0.2639	0.3878	0.4255	0.4611	0.5064	0.3657
0.75	- 0.2361	- 0.0669	0.0540	0.1589	0.2669	0.3984	0.4409	0.4817	0.5334	0.3744
0.80	- 0.2702	- 0.0952	0.0330	0.1461	0.2672	0.4111	0.4585	0.5054	0.5651	0.3833
0.85	- 0.3172	- 0.1363	- 0.0004	0.1256	0.2606	0.4244	0.4782	0.5329	0.6026	0.3935
0.90	- 0.3877	- 0.2008	- 0.0581	0.0848	0.2385	0.4394	0.5004	0.5646	0.6474	0.4043
0.95	- 0.5140	- 0.3287	- 0.1852	- 0.0135	0.1663	0.4564	0.5261	0.6002	0.6991	0.4163
1.00	- 1.5584	- 1.1435	- 1.3181	- 1.2360	- 1.1402	-0 .8440	- 0.7799	- 0.7263	- 0.6288	- 0.8756
Integral of F^* i.e. $\frac{1}{l}\int_0^l F^* dz$ =										
	- 0.2181	- 0.0650	0.0396	0.1293	0.2160	0.3462	0.3786	0.4083	0.4480	0.3284

TABLE II

P* , Cp , and Non-dimensional Shock shape for Caret-Wing.

Mach Number = 3.97 ; Incidence = 23.8 degrees ;

b = 0.1918 , h = - 0.10023 , Ω = 0.62039 , C = - 0.745537

Z/Ω	P*	Cp	Shock Shape
0.00	0.6041	0.2743	0.2869
0.05	0.6075	0.2749	0.2868
0.10	0.6116	0.2755	0.2865
0.15	0.6163	0.2763	0.2859
0.20	0.6214	0.2771	0.2852
0.25	0.6275	0.2781	0.2842
0.30	0.6345	0.2792	0.2828
0.35	0.6427	0.2805	0.2812
0.40	0.6517	0.2820	0.2792
0.45	0.6632	0.2838	0.2769
0.50	0.6753	0.2857	0.2741
0.55	0.6897	0.2880	0.2707
0.60	0.7079	0.2909	0.2669
0.65	0.7281	0.2942	0.2623
0.70	0.7536	0.2983	0.2570
0.75	0.7846	0.3032	0.2508
0.80	0.8227	0.3093	0.2434
0.85	0.8716	0.3171	0.2347
0.90	0.9365	0.3275	0.2240
0.95	1.0210	0.3410	0.2106
1.00	- 0.0999	0.1286	0.1916

T A B L E   III

Comparision of P* distributions and Cp distribu'ions on a Biconvex wing when the profile is given in the analytic form as well as in the form of a table at 51 descrete points.

Equation of the cross-sectional profile :-

$$\frac{\bar{Y}}{\bar{X}} = 0.047856 \left(1 - \frac{\bar{Z}^2}{b^2 \bar{X}^2} \right)$$

Mach-Number = 3.97 ; Incidence = 23.8 ; Aspect-ratio = 2/3

Omega = 0.533096 ; C = 0.409627

Iterated value of omega in the calculations is,

Case I   Analytic cross-section   = 0.5339587

Case II   Tabular cross-section   = 0.5337565

$\frac{\bar{Z}}{\Omega}$	P* distribution		Cp distribution	
	Case I	Case II	CAse I	Case II
0.00	0.118011	0.118757	0.415261	0.415381
0.05	0.116998	0.117773	0.415099	0.415223
0.10	0.113987	0.114793	0.414617	0.414746
0.15	0.108964	0.109857	0.413814	0.413956
0.20	0.101867	0.102897	0.412678	0.412843
0.25	0.092665	0.093889	0.411205	0.411401
0.30	0.081219	0.082648	0.409373	0.409602
0.35	0.067383	0.068912	0.407159	0.407404
0.40	0.051077	0.052853	0.404550	0.404834
0.45	0.031861	0.033797	0.401475	0.401785
0.50	0.009804	0.012035	0.397945	0.398302
0.55	- 0.016041	- 0.013811	0.393809	0.394166
0.60	- 0.045689	- 0.043045	0.389063	0.389487
0.65	- 0.08022	- 0.077004	0.383538	0.383909
0.70	- 0.120895	- 0.118479	0.377029	0.377416
0.75	- 0.169089	- 0.165721	0.369317	0.369856
0.80	- 0.228076	- 0.224650	0.359877	0.360425
0.85	- 0.303636	- 0.300503	0.347785	0.438236
0.90	- 0.408206	- 0.405276	0.331051	0.331408
0.95	- 0.583507	- 0.583965	0.302997	0.302924
1.00	- 1.460570	- 1.459090	0.162641	0.163839

APPENDIX

Computer Programme (FORTRAN)

```
      CALCULATION OF PRESSURE DISTRIBUTION ON DELTA WING OF GENERAL
       CROSS-SECTION AT HYPERSONIC SPEEDS

      REAL MACH,INCRT
      DIMENSION ZBAR(105),YBAR(105),Z(105),Y(105),DERFD(1),STOREE(2)
      DIMENSION AK(4),STOREA(2),WAPZYI(2),AKZ(2),COFP(4)
      DIMENSION PSTARL(21),PSTARD(21)
      DIMENSION  WT(405),SUMWT(405),EITA(405),DER1(405)
      DOUBLE PRECISION ZBAR,YBAR,Z,Y,WT,EITA,SUMWT,DER1,AK,DERFD
      DOUBLE PRECISION RHS,EOMEGA,APPWI1,APPWI2,CC,CCD,BB2,BB1,BBO
      DOUBLE PRECISION WZYI1,WZYI,WWZBI1,WZBI,DABS,DSQRT,EFG
      DOUBLE PRECISION    A2,A3,A4,A5,B1,B2,B3,B4,B5,DELTA
      DOUBLE PRECISION WAPZYI,AKZ,AINCRT,AA,AB,AC,OMX,OMX1,PSTARL,PSTARD
       DOUBLE PRECISION PRESUR,SHOCKS,DYBYDZ,RHS1,CL,CD,CLBYCD
      COMMON  WT,EITA,SUMWT,II,INCRT,T,TT,ZBP,ZYIP
      EXTERNAL FCTZB ,FCTZYI,SINT,F
   10 CONTINUE
      READ (5,20,END=1000)ANALTC
   20 FORMAT(F15.7)


      IF ONE IS INTERESTED IN CALCULATION OF COEFFICIENT OF LIFT
      COEFFICIENT OF DRAG,CL/CD THEN GIVE FOR NOCLCD AVALUE OTHER
      THAN 2 AND INCLUDE  CORRECT EXPRESSION FOR DY/DX

      READ (5,30) NOCLCD
      READ (5,30)NSC
   30 FORMAT (I3)


       ABOVE NSC REPRESENTS NUMBER OF POINTS UP-TO WHICH POWER  SIRIES
       SOLUTION   IS USED FOR W(T) NEAR ORIGIN

      IF (ANALTC.EQ.1.) GO TO 40
       GO TO 60
   40  READ (5,20) COFP(4),COFP(3),COFP(2),COFP(1)
       WRITE (6,50) COFP(4),COFP(3),COFP(2),COFP(1)
   50  FORMAT ('1',' COEFFICIENTS, A=',F15.7, ' B=',F15.7, ' C=',F15.7,
     1  ' D=',F15.7)

     COFP(4),COFP(3),COFP(2),COFP(1) REPRESENT COEFFICIENTS A,B,C,D
       RESPECTIVELY OF THE EQUATION OF THE CROSS SECTIONAL PROFILE
       YBAR= A*ZBAR**4 + B*ZBAR**3 + C*ZBAR**2 + D*ZBAR + F

      GO TO 100
   60 READ (5,30) NP
      WRITE (6,70) NP
   70 FORMAT ('1',' NUMBER OF POINTS IN CROSS-SECTIONAL PROFILE=',I4)
      READ (5,80) (ZBAR(I),I=1,NP)
      READ (5,80) (YBAR(I),I=1,NP)
   80 FORMAT (4D15.0)
      WRITE (6,90)(ZBAR(I),I=1,NP)
      WRITE (6,90)(YBAR(I),I=1,NP)
   90 FORMAT (4F20.7)
  100 READ (5,20) MACH,ALPHA,GAMA
       WRITE (6,110) MACH,ALPHA,GAMA
  110 FORMAT(' MACH NUMBER=',F10.4, ' INCIDENCE=',F10.4, ' GAMA=',F10.4)
      READ (5,20) XBAR,BSPAN,HMOR
      WRITE (6,120)XBAR,BSPAN,HMOR
  120 FORMAT (' XBAR=',F15.6,' SEMI-SPAN=',F15.6,' MAXIMUM ORDINATE=',
     1  F15.6)
      ALP1=(ALPHA/180)*3.141593
      EPS=((GAMA-1.0)/(GAMA+1.0))+2/((GAMA+1.0)*((MACH*SIN(ALP1))**2))
      SQEPS=SQRT(EPS)
      WRITE (6,130) EPS,SQEPS
  130 FORMAT ('EPSYLON=',E16.7,'SQURE ROOT EPSYLON=',E16.7)
      TALP=TAN(ALP1)
      SNALPH=SIN(ALP1)
      IF (ANALTC.EQ.1.) GO TO 150
      DO 140 I=1,NP
      Z(I)=ZBAR(I)/(SQEPS*TALP*XBAR)
      Y(I)=YBAR(I)/(EPS*TALP*XBAR)
  140 CONTINUE
  150 OMEGA=BSPAN/(SQEPS*TALP*XBAR)
      CONIC=HMOR/(XBAR*BSPAN*SQEPS)
      TOOO=HMOR/(XBAR*EPS*TALP)
       READ (5,30)NNXU
      INCRT=OMEGA/NNXU
      READ (5,20)COEA1
      ILIMIT=OMEGA/INCRT
    . WRITE(6,160)INCRT,COEA1,NSC,OMEGA,ILIMIT,CONIC
  160 FORMAT(' INCRIMENT DT=',F12.6,' COEFFICIENT A1=',F12.6,' NSC=',I4,
     1 ' OMEGA=',F12.6,'     ILIMIT=',I6,' PARAMETER C=',F12.6)
      IL=1+ILIMIT
      DO 190 IM=1,IL
      T=(IM-1)*INCRT
      IF (IM.EQ.IL)T=OMEGA
      IF (ANALTC.EQ.1.) GO TO 170
      CALL LGR (Z,Y,T,NP,AK,DERFD)
      GO TO  180
  170 CALL ANSLOP (T,AK,DERFD,COFP,SQEPS,TALP,XBAR)
  180 DER1(IM)=DERFD(1)
  190  CONTINUE
      KKKKK=0
      KKKK=1
      KKKI=1
      KKIK=1
      COEA2=0.0
      CALL  SCLOCK
C
      READ (5,20) ACONTY
       IF (ACONTY.EQ.2.) GO TO 200
      READ (5,20) STOREA(1),STOREE(1),STOREA(2),STOREE(2)
      KKKK=2
      KKKI=2
      KKIK=2
  200 DELTA=(COEA1*ALOG(COEA1)+1.0-COEA1)/(1.0-COEA1)**2
      IF (COEA2.EQ.COEA1) GO TO  710
      COEA2=COEA1
      CALL RCLOCK (ITIME)
      IF (ITIME.GT.1000) GO TO 730
      WRITE (6,210)COEA1 ,DELTA
  210 FORMAT (' VALUE OF A1 =',E20.7,' DELTA=',E18.7)
      II=1
      WT(1)=0.0
```

```
      SUMWT(1)=0.0
C
C
C     CALCULATION OF COEFFICIENTS OF POWER SIRIES SOLUTION NEAR ORIGIN
C
C
       T=0.0
      IF (ANALTC.EQ.1.) GO TO 220
       CALL LGR (Z,Y,T,NP,AK,DERFD)
      GO TO 230
  220 CALL ANSLOP (T,AK,DERFD,COFP,SQEPS,TALP,XBAR)
  230  CONTINUE
  240  A1=COEA1
      A2=+AK(1)/((A1+1.0)/(A1*(A1-1)**2)-((2*ALOG(A1))/(A1-1.0)**3))
      A3=(AK(2)*A1**2)+A1**3+(A2**2/A1)
      A4=A2*(A1**4-A1*A3+A2**2)/(A1*(1+A1))+(A1**3)*(AK(3))/(1+A1)
     1-(2*A2**3)/(A1**2)+(3*A2*A3)/A1
      A5=3*((A1**4)*(AK(4)+A3)+(A1**2+2*A1+2.0)*(A3**2+2*A2*A4)/A1-(A3
     1*A2**2)*(3*A1**2+8*A1+10.0)/A1**2+(A2**4)*(3*A1**2+10*A1+15)/(3*
     2A1**3))/(3*A1**2+4*A1+3)
      B1=1.0/A1
       B2=-A2/(A1**3)
      B3=2*(A2**2)/(A1**5)-A3/(A1**4)
      B4=-5*(A2**3)/(A1**7)+5*A2*A3/(A1**6)-A4/(A1**5)
      B5=14*(A2**4)/(A1**9)-21*(A2**2)*A3/(A1**8)+3*(A3**2)/(A1**7)
     1 +6*A2*A4/(A1**7)-A5/(A1**6)
  250  IF (II.GE.NSC) GO TO 270
      II=II+1
      T=(II-1)*INCRT
      WT(II)=A1*T+A2*T**2+A3*T**3+A4*T**4+A5*T**5
      SUMWT(II)=SUMWT(II-1)+INCRT*((WT(II)+WT(II-1))/2)
      EITA(II)=B1*T+B2*(T**2)+B3*(T**3)+B4*(T**4)+B5*(T**5)
C
C     IF ONE IS INTERESTED IN THE PRINT OUT OF ALL THE W(T) PRINT
C     OUT, REMOVE THE  C FROM FIRST COLUMN  IN THE FOLLOWING TWO CARDS
C     WRITE (6,260)T,WT(II),SUMWT(II),EITA(II),DER1(II)
C 260 FORMAT (F10.6,4E15.6)
C
C
        GO TO 250
  270 II=II+1
      T   =(II-2)*INCRT
      TT  =(II-1)*INCRT
      IF (WT(II-1).GT.(1.0+0.75*T)) GO TO 630
      IF (II.GT.(1+ILIMIT)) GO TO 660
      IF (WT(II-1).LT.T) GO TO 350
      IF (WT(II-1).EQ.T)GO TO  430
      JJ=0
  280 JJ=JJ+1
      IF (JJ.GT.(II-1)) GO TO 430
      IF (WT(JJ).EQ.TT) GO TO 290
      IF (WT(JJ).GT.TT) GO TO 300
      GO TO 280
  290 EITA(II)=(JJ-1)*INCRT
       GO TO 310
  300 EITA(II)=(JJ-2)*INCRT+INCRT*(TT-WT(JJ-1))/(WT(JJ)-WT(JJ-1))
  310 RHS=DER1(II)-DER1(II-1)-WT(II-1)-1.0/(WT(II-1)-T)-(0.5*INCRT)
     1 *(1.0/(WT(II-1)-T)**2)+0.5*(EITA(II)-EITA(II-1))*((1.0/(TT
     2 -EITA(II))**2)+(1.0/(T-EITA(II-1))**2))
      BB2=-2.*TT+RHS
      BB1=TT**2+1.0-2.*TT*RHS
      BB0=(TT**2)*RHS-TT-(INCRT/2)
      APPWI1=2.*WT(II-1)-WT(II-2)
      IF (APPWI1.GE.(1.0+0.75*TT)) GO TO 630
      JK=1
      APPWI2=1.0+OMEGA
  320 JK=JK+1
      IF (JK.GT.20) GO TO 430
      CC=APPWI1**3+BB2*APPWI1**2+BB1*APPWI1+BB0
      CCD=3*APPWI1**2+2.*BB2*APPWI1+BB1
       IF (DABS(APPWI2-APPWI1).LE.0.000001) GO TO 520
      APPWI2=APPWI1
      APPWI1=APPWI2-(CC/CCD)
      IF (APPWI1.GE.(1.0+TT)) GO TO 330
      IF (APPWI1.LE.WT(II-1)) GO TO 430
      GO TO 320
  330  IF (WT(II-1).GT.(1.0+0.5*(TT-INCRT))) GO TO 630
      IF (ABS((TT-OMEGA)/OMEGA).LE.0.05) GO TO 630
      EOMEGA=T
      WRITE (6,340) COEA1,EOMEGA
  340  FORMAT(' COEFFICIENT A1=',F15.7,' CURVE RISES STEEPLY,CASEB,EOMEG
     1A=TI=',F16.7)
      KKKK=KKKK+1
      GO TO 650
  350 WZBI=WT(II-1)
      IK=II-1
      CALL NEVIL (6,WZBI,0.0,INCRT,IK,WT,IER)
      EITA(II)=TT
      WZYI1=WT(II-1)
      AINCRT= (WT(II-1)-WT(II-2))/2.
      JK=0
      RHS1=DER1(II-1)-DER1(II)+WZBI+1.0/(WZBI-WT(II-1))-0.5*INCRT*
     1  (1.0/(WT(II-1)-T)**2)
      IF ((WT(II-1)-WT(II-2)).LT.0.0) AINCRT=DABS(AINCRT)
  360 JK=JK+1
      KZ=1
      AKZ(1)=WT(II-1)
      WAPZYI(1)=(0.5*INCRT)*((1.0/(WT(II-1)-TT)**2)+(1.0/(WT(II-1)-T **
     1  2))-DER1(II-1)+DER1(II)
      IPP=2
      KIQ=1
  370 WZYI1=WZYI1+AINCRT
      IF (WZYI1.EQ.TT) GO TO 420
      IF (WZYI1.GT.TT) GO TO 490
      WWZBI1=WZYI1
      CALL NEVIL (6,WWZBI1,0.0,INCRT,IK,WT,IER)
      RHS=RHS1-WWZBI1-1.0/(WWZBI1-WZYI1)+0.5*(WZYI1-WT(II-1))*(1.0/
     1  (WWZBI1-WZYI1)**2+1.0/(WZBI-WT(II-1))**2)
      WAPZYI(IPP)=(0.5*INCRT)/((WZYI1-TT)**2)-RHS
  380 IPP=2
      IF (WAPZYI(1)*WAPZYI(2).LT.0.0) GO TO 390
      WAPZYI(1)=WAPZYI(2)
       AKZ(1)=WZYI1
      IF (KIQ.EQ.2) GO TO 400
      GO TO 370
  390 KZ=KZ+1
      AKZ(KZ)=WZYI1
```

```
      IF(DABS(AKZ(2)-AKZ(1)).LE.0.000001) GO TO 410
      KZ=1
      KIQ=2
  400 AINCRT=AINCRT/2.
      WZYI1=AKZ(1)
      GO TO 370
  410 WZYI1=WZYI1-(AINCRT/2.)
  420 WT(II)=WZYI1
      GO TO  530
  430 JK=0
      EFG=TT
      IK=II-1
      CALL NEVIL (6,EFG,0.0,INCRT,IK,WT,IER)
      WT(II)=EFG
      SUMWT(II)=SUMWT(II-1)+INCRT*(WT(II)+WT(II-1))/2.0
      IF (TT.GT.WT(II)) GO TO 450
      JKK=0
  440   JKK=JKK+1
       IF (WT(JKK).GT.TT) GO TO  460
       IF (WT(JKK).EQ.TT) GO TO  470
      GO TO 440
  450 EITA(II)=TT
      GO TO 530
  460  EITA(II)=(JKK-2)*INCRT+INCRT*(TT-WT(JKK-1))/(WT(JKK)-WT(JKK-1))
      IF (WT(II).LT.(TT+INCRT)) GO TO 480
      GO TO 530
  470  EITA(II)=(JKK-1)*INCRT
      IF (WT(II).LT.(TT+INCRT)) GO TO 480
      GO TO 530
  480  II=II+1
       T=(II-2)*INCRT
      TT=(II-1)*INCRT
      IF (WT(II-1).GT.(1.0+0.75*T)) GO TO 630
       IF (II.GT.(1+ILIMIT)) GO TO 660
  490 JK=0
      EFG=TT
      IK=II-1
      CALL NEVIL (6,EFG,0.0,INCRT,IK,WT,IER)
      WT(II)=EFG
      SUMWT(II)=SUMWT(II-1)+INCRT*(WT(II)+WT(II-1))/2.0
      IF (EFG.GT.TT) GO TO 510
      WT(II)=(SQRT(2.0)*WT(II-1)-WT(II-2))/(SQRT(2.0)-1.0)
      SUMWT(II)=SUMWT(II-1)+INCRT*(WT(II)+WT(II-1))/2.0
      WRITE (6,500) TT,WT(II)
  500  FORMAT (' PARABOLIC EXTRAPOLATION IN INTERVAL AT WT=T,=',F12.6,
     1E15.6)
        EOMEGA=TT
        WRITE (6,640) COEA1,EOMEGA
       KKKK=KKKK+1
       GO TO 650
  510  JKK=0
      GO TO 440
  520 CONTINUE
      WT(II)=APPWI1
  530 SUMWT(II)=SUMWT(II-1)+INCRT*(WT(II)+WT(II-1))/2
C
C     IF ONE IS INTERESTED IN ALL THE CALCULATED SLOPES  AT  EACH OF
C     POINTS FOR  REFERANCE REMOVE THE  C  FROM FIRST COLUMN
C      IN THE FOLLOWING  CARDS
C
C     IF (WT(II).EQ.TT) GO TO 590
C     IF (WT(II).GT.TT) GO TO 570
C     ZYIP=TT
C     ZBP=WT(II)
C     GO TO 580
C 570 ZBP=TT
C     ZYIP=EITA(II)
C     CALL QG6 (ZYIP,ZBP,SINT,SLOPE)
C     GO TO 600
C 580 CALL QG6 (ZBP,ZYIP,SINT,SLOPE)
C     SLOPE=-SLOPE
C     EFG=ZBP
C     CALL NEVIL (6,EFG,0.0,INCRT,II,WT,IER)
C      DYBYDZ=-EFG-1.0/(EFG-ZBP)+SLOPE
C      GO TO 610
C 590  WRITE (6,595) II,JK,TT,WT(II),SUMWT(II),EITA(II),DER1(II)
C 595  FORMAT(I4,I7,F12.7,3E15.7,E30.7)
C        GO TO 270
C 600 DYBYDZ=-WT(II)-1.0/(WT(II)-TT)+SLOPE
C 610  CONTINUE
C     WRITE (6,615) II,JK,TT,WT(II),SUMWT(II),EITA(II),DYBYDZ,DER1(II),
C    1  SLOPE
C 615  FORMAT (I4,I7,F12.7,6E15.7)
C
C
C
C
C
C
C
C
C
C
C
C
C     IF ONE IS INTERESTED IN THE PRINT OUT OF ALL THE W(T) PRINT
C     OUT, REMOVE THE  C FROM FIRST COLUMN  IN THE FOLLOWING TWO CARDS
C
C     WRITE (6,620) II,JK,TT,WT(II),SUMWT(II),EITA(II),DER1(II)
C 620  FORMAT (I4,I7,F12.7,4E15.7)
C
C
      GO TO 270
  630 KKKK=KKKK+1
      AA=2*(WT(II-1)-WT(II-2))-INCRT
      AB=WT(II-2)*(WT(II-2)-2.0+2*(II-2)*INCRT) -WT(II-1)*(WT(II-1)-
     1 2.0+2*(II-3)*INCRT)  -2*INCRT
      AC=(II-3)*INCRT*WT(II-1)*(WT(II-1)-2.0)  -(II-2)*INCRT*WT(II-2)*
     1 (WT(II-2)-2.0)  -INCRT
      EOMEGA= (-AB+DSQRT(AB**2-4*AA*AC))/(2.0*AA)
      WRITE (6,640) COEA1,EOMEGA
  640 FORMAT('            COEFFICIENT A1=',F16.7,'    EXTRAPOLATED OMEGA
     1=',E20.7)
```

```
C
650   CONTINUE
      WT(II)=1.0+EOMEGA
      SUMWT(II)=SUMWT(II-1)+(EOMEGA-(II-2)*INCRT)*(WT(II)+WT(II-1))/2
       IF (DABS ((EOMEGA-OMEGA)/OMEGA).LE.0.01) GO TO 750
      STOREA(1)=COEA1
      STOREE(1)=EOMEGA
      IF (KKKI.GE.2) GO TO 690
       IF (KKIK.EQ.2) GO TO 690
      COEA1=COEA1/2.
       GO TO 200
660 CONTINUE
       OMX1=WT(II-1)
      STOREA(2)=COEA1
      STOREE(2)=1.0+OMEGA-OMX1
      KKIK=2
      WRITE (6,670)STOREA(2),OMX1
670 FORMAT ('              COEFFICIENT A1=',F15.7, ' EXTRAPOLATED WT(OMEGA
     1)=',E15.7)
      IF (DABS((OMX1-(1.0+OMEGA))/(1.0+OMEGA)).LE.0.01) GO TO 750
      IF (KKKI.GE.2) GO TO 690
      IF (KKKK.GE.2) GO TO 680
      COEA1=2*COEA1
       GO TO 200
680 KKKI=KKKI+1
      IF (KKKKK.GT.6) GO TO 700
      KKKKK=KKKKK+1
       COEA1=STOREA(2)+(STOREA(1)-STOREA(2))*(STOREE(2))/ (STOREE(2)+
     1(OMEGA-STOREE(1)))
       GO TO 200
690 IF (KKKKK.GT.6) GO TO 700
      KKKKK=KKKKK+1
       COEA1=STOREA(2)+(STOREA(1)-STOREA(2))*(STOREE(2))/ (STOREE(2)+
     1(OMEGA-STOREE(1)))
       GO TO 200
700 COEA1=(STOREA(1)+STOREA(2))/2.
      GO TO 200
710  WRITE (6,720) COEA1
720 FORMAT (' A1 CONVERGES TO SAME VALUE BUT  OMEGA NOT SATISFIED A1=
     1 ',F15.7)
      GO TO 730
730 WRITE (6,740)
740 FORMAT (' ITERATION DID NOT CONVERGE, NEXT DATA TAKEN')
      GO TO 10
750  WT(1+ILIMIT)=1.0+OMEGA
      SUMWT(1+ILIMIT)=SUMWT(II-1)+(1+ILIMIT-(II-1))*INCRT*(1.0+OMEGA-
     1  WT(II-1))
       ISTEP=ILIMIT/20
      JK=1
      KKK=1
      WRITE (6,760)
760 FORMAT(' Z      W(Z)        INTEGRAL O - Z W(S) DS        P*
     1 CP(Z)     SHOCK SHAPE    DY/DZ(CAL)     DY/DZ(BODY)')
      ZBP=0.0
       XKB=1.0/(COEA1-1.0)
      PRESUR=-(((3.0*XKB+4.5)*XKB+3.0)*XKB+0.5)  + ((((3.0*XKB+6.0)*XKB
     1 +5.0)*XKB+2.0)*XKB)*(ALOG(COEA1)) +2.*TOOO-2.*OMEGA*CONIC
       ILLL=1
```

```
      SHOCKS=DELTA+TOOO
      CPPP=2*(SNALPH**2)*(1.0+(EPS*(2.*CONIC*OMEGA+PRESUR)))
      IF (NOCLCD.EQ.2) GO TO 785
      IF (ANALTC.EQ.1.) GO TO 770
      AZL=ZBP
      CALL LGR (Z,Y,AZL,NP,AK,DERFD)
      DYBYDX=(HMOR/XBAR)-(EPS*TALP)*((3.*AK(4)*ZBP+2.*AK(3))*ZBP+AK(2))
     1  *ZBP*ZBP
      GO TO 780
770  AZBAR=ZBP*SQEPS*TALP
      DYBYDX=HMOR-(((3.*COFP(4)*AZBAR)+2.*COFP(3))*AZBAR+COFP(2))*AZBAR
     1 *AZBAR
780 BT=ATAN(DYBYDX)
      BT1=ALP1+BT
      PSTARL(1)=CPPP*COS(BT1)
      PSTARD(1)=CPPP*SIN(BT1)
785 WRITE(6,890)ZBP,WT(1 ),SUMWT(1 ),PRESUR,CPPP,SHOCKS
790 JK=JK+ISTEP
      ILLL=ILLL+1
       IF (JK.GT.(1+ILIMIT)) GO TO 910
      ZBP=(JK-1)*INCRT
       IF (JK.GE.ILIMIT) ZBP=OMEGA
      M=1
      GO TO 800
800 M=M+1
      IF(WT(M).GT.ZBP) GO TO 810
       IF (WT(M).EQ.ZBP) GO TO  820
      GO TO 800
810 ZYIP=(M-2)*INCRT+INCRT*((  ZBP  -WT(M-1))/(WT(M)-WT(M-1)))
      GO TO 830
820 ZYIP=(M-1)*INCRT
830 IF (ZYIP.GT.ZBP) GO TO  840
       IF (ZYIP.EQ.ZBP) GO TO 790
      CALL QG6 ( ZYIP,ZBP,FCTZB,YINT)
      CALL QG6 ( ZYIP,ZBP,SINT,SLOPE)
      GO TO 850
840  CALL QG6 (ZBP,ZYIP,FCTZYI,YINT)
      CALL QG6 (ZBP,ZYIP,SINT,SLOPE)
      SLOPE=-SLOPE
850 PRESUR=-1.0-WT(JK)**2-2*SUMWT(JK)+2*ZBP*WT(JK)+2*DELTA+YINT*
     1  ((WT(JK)-WT(JK-1))/INCRT)*(-1.0+1.0/(WT(JK)-ZBP)**2) +2.*TOOO
     2 -2.*CONIC*OMEGA
      CPPP=2*(SNALPH**2)*(1.0+(EPS*(2.*CONIC*OMEGA+PRESUR)))
      DYBYDZ=-WT(JK)-1.0/(WT(JK)-ZBP)+SLOPE
      SHOCKS=DELTA+TOOO-SUMWT(JK)
      IF (NOCLCD.EQ.2) GO TO 860
      IF (ANALTC.EQ.1.) GO TO 860
      AZL=ZBP
      CALL LGR (Z,Y,AZL,NP,AK,DERFD)
      DYBYDX=(HMOR/XBAR)-(EPS*TALP)*((3.*AK(4)*ZBP+2.*AK(3))*ZBP+AK(2))
     1  *ZBP*ZBP
      GO TO 870
860  AZBAR=ZBP*SQEPS*TALP
      DYBYDX=HMOR-(((3.*COFP(4)*AZBAR)+2.*COFP(3))*AZBAR+COFP(2))*AZBAR
     1 *AZBAR
870 BT=ATAN(DYBYDX)
      BT1=ALP1+BT
      PSTARL(ILLL)=CPPP*COS(BT1)
```

```
      PSTARD(ILLL)=CPPP*SIN(BT1)
  880 WRITE(6,890)ZBP,WT(JK),SUMWT(JK),PRESUR,CPPP,SHOCKS,DYBYDZ,DER1(JK
     1)
  890 FORMAT (F10.5,7E15.6)
       IF (NOCLCD.EQ.2) GO TO 790
      WRITE (6,900)DYBYDX,BT
  900 FORMAT(' DYBYDX=',E16.7,' SLOPE IN RADIANS=',E15.6)
      GO TO 790
  910  IF (NOCLCD.EQ.2) GO TO 10
       CL=PSTARL(1)-PSTARL(21)
      CD=PSTARD(1)-PSTARD(21)
      DO 920 IMM=2,20,2
      CL=CL+4.0*PSTARL(IMM)+2.0*PSTARL(1+IMM)
      CD=CD+4.0*PSTARD(IMM)+2.0*PSTARD(1+IMM)
  920  CONTINUE
      CL=CL/60.0
      CD=CD/60.
      CLBYCD=CL/CD
      WRITE (6,930)CL,CD,CLBYCD
  930 FORMAT(' LIFT COEFF. CL=',E15.6,' DRAG COEFF. CD=',E15.7, ' LIFT/D
     1RAG RATIO L/D=',E15.7)
      GO TO 10
 1000  CONTINUE
       RETURN
      END
C
C
      FUNCTION FCTZB(X)
      REAL INCRT
      DIMENSION WT(405),SUMWT(405),EITA(405)
      DOUBLE PRECISION WT,SUMWT,EITA
      DOUBLE PRECISION WS
      COMMON  WT,EITA,SUMWT,II,INCRT,T,TT,ZBP,ZYIP
      XINT=INCRT
      WS=X
      CALL NEVIL (6,WS,0.0,XINT,II,WT,IER)
      FCTZB=(WS-ZBP)**3/(WS-X)**2
      RETURN
      END
C
C
C
       FUNCTION FCTZYI(X)
      REAL INCRT
      DIMENSION WT(405),SUMWT(405),EITA(405)
      DOUBLE PRECISION WT,SUMWT,EITA
      DOUBLE PRECISION WS
      COMMON  WT,EITA,SUMWT,II,INCRT,T,TT,ZBP,ZYIP
      XINT=INCRT
      WS=X
      CALL NEVIL (6,WS,0.0,XINT,II,WT,IER)
      FCTZYI=-(WS-ZBP)**3/(WS-X)**2
      RETURN
      END
C
C
C
C
      SUBROUTINE NEVIL (N,X,X1,RINT,M,W,IER)
      DIMENSION F(18),W(M)
      DOUBLE PRECISION W,X
      IER=0
   10 IF (N-2)20,40,40
   20 WRITE (6,30)
   30 FORMAT ( 'NEVIL ERROR.N 2 OR 18')
      IER=1
      GO TO 180
   40 IF (N-18)50,50,20
   50 U=(X-X1)/RINT
      J=IFIX(U+0.00001)
      I=J+N/2.+0.1
      K=0
      IF (M-N)160,60,60
   60 IF (I-M+1)80,80,70
   70 K=M-N
      GO TO 100
   80 KK=J-N/2.+1.1
      IF (KK)100,90,90
   90 K=KK
  100 UU=U-K
      DO 110 L=1,N
      L1=K+L
      F(L)=W(L1)
  110 CONTINUE
      LL=1
      J=N-1-LL
  120 JJ=J+1
      U=UU
      DO 130 L=1,JJ
      L2=L+1
      F(L)=((U+1-L)*F(L2)-(U+1-L-LL)*F(L))/LL
  130 CONTINUE
      LL=LL+1
      IF (J)150,150,140
  140 J=J-1
      GO TO 120
  150 X=F(1)
      GO TO 180
  160 WRITE (6,170) M
  170  FORMAT ( 'NEVIL ERROR.M N.CONTINUE WITH N=M=',I2)
      N=M
      IER=2
      GO TO 10
  180 CONTINUE
      RETURN
      END
```

```
C
C
C
C
      SUBROUTINE LGR (A,B,C,IP,D,E)
      DIMENSION A(IP),B(IP),D(4),E(1),AZ(6),AY(5)
      DOUBLE PRECISION A,B,D,E,AZ,AY,DIM,DIM1,ANUM1,ANUM2,ANUM3
      D(1)=0.0
      D(2)=0.0
      D(3)=0.0
      D(4)=0.0
      DO 10 I=1,IP
      K=I
      IF (C    -A(K))20,100,10
   10 CONTINUE
   20 IF ((K+2).GT.IP) GO TO 80
      IF (K.LE.3)GO TO 60
      DI1=A(K)-C
      DI2=C    -A(K-1)
      IF (DI2.GT.DI1) GO TO 40
      DO 30 L=1,5
      M=K+L-4
      AZ(L)=A(M)
   30 AY(L)=B(M)
      GO TO 120
   40 DO 50 L=1,5
      M=K+L-3
      AZ(L)=A(M)
   50 AY(L)=B(M)
      GO TO 120
   60 DO 70 L=1,5
      AZ(L)=A(L)
   70 AY(L)=B(L)
      GO TO 120
   80 DO 90 L=1,5
      M=IP+L-5
      AZ(L)=A(M)
   90 AY(L)=B(M)
      GO TO 120
  100 IF (K.LE.3) GO TO 60
      IF((K+2).GE.IP) GO TO 80
      DO 110 L=1,5
      M=K+L-3
      AZ(L)=A(M)
  110 AY(L)=B(M)
  120 AZ(6)=AZ(1)
      DO 130 I=1,5
      DIM=(AZ(1)-AZ(2))*(AZ(1)-AZ(3))*(AZ(1)-AZ(4))*(AZ(1)-AZ(5))
      DIM1=AY(I)/DIM
      ANUM1=AZ(2)+AZ(3)+AZ(4)+AZ(5)
      ANUM2=(AZ(2)*AZ(3))+(AZ(2)*AZ(4))+(AZ(2)*AZ(5))+(AZ(3)*AZ(4))
     1+(AZ(3)*AZ(5))+(AZ(4)*AZ(5))
      ANUM3=(AZ(2)*AZ(3)*AZ(4))+(AZ(2)*AZ(3)*AZ(5))+(AZ(2)*AZ(4)*AZ(5))+
     1(AZ(3)*AZ(4)*AZ(5))
      D(1)=D(1)-(ANUM3*DIM1)
      D(2)=D(2)+(ANUM2*DIM1)
      D(3)=D(3)-(ANUM1*DIM1)
      D(4)=D(4)+DIM1
      AZ(1)=AZ(2)
      AZ(2)=AZ(3)
      AZ(3)=AZ(4)
      AZ(4)=AZ(5)
      AZ(5)=AZ(6)
      AZ(6)=AZ(1)
  130 CONTINUE
      E(1)=((4.0*D(4)*C+3.0*D(3))*C+2.0*D(2))*C+D(1)
       D(2)=2.0*D(2)
      D(3)=3.0*D(3)
      D(4)=4.0*D(4)
      RETURN
      END
C
C
C
      SUBROUTINE QG6 ( XL,XU,FCTT,Y)
      A=0.5*(XL+XU)
      B=XU-XL
      C=.4662348*B
      Y=.08566225*(FCTT(A+C)+FCTT(A-C))
      C=.3306047*B
      Y=Y+.1803808*(FCTT(A+C)+FCTT(A-C))
      C=.1193096*B
      Y=B*(Y+.2339570*(FCTT(A+C)+FCTT(A-C)))
      RETURN
      END
C
C
      FUNCTION SINT (X)
      REAL INCRT
      DIMENSION WT(405),SUMWT(405),EITA(405)
      DOUBLE PRECISION WT,SUMWT,EITA
      DOUBLE PRECISION WK
      COMMON  WT,EITA,SUMWT,II,INCRT,T,TT,ZBP,ZYIP
      XINT=INCRT
      WK=X
      CALL NEVIL (6,WK,0.0,XINT,II,WT,IER)
      SINT=1.0/((WK-X)**2)
      RETURN
      END
C
C
C
      SUBROUTINE ANSLOP (A,B,C,D,E,F,G)
     DIMENSION B(4),C(1),D(4)
     DOUBLE PRECISION  B,C
      E1=1.0/E
     E2=E*F*G
      B(1)=E1*D(1)
      B(2)=2.0*E1*D(2)*E2
     B(3)=3.0*E1*D(3)*E2*E2
      B(4)=4.0*E1*D(4)*E2*E2*E2
      C(1)=((B(4)*A+B(3))*A+B(2))*A+B(1)
     RETURN
     END
```

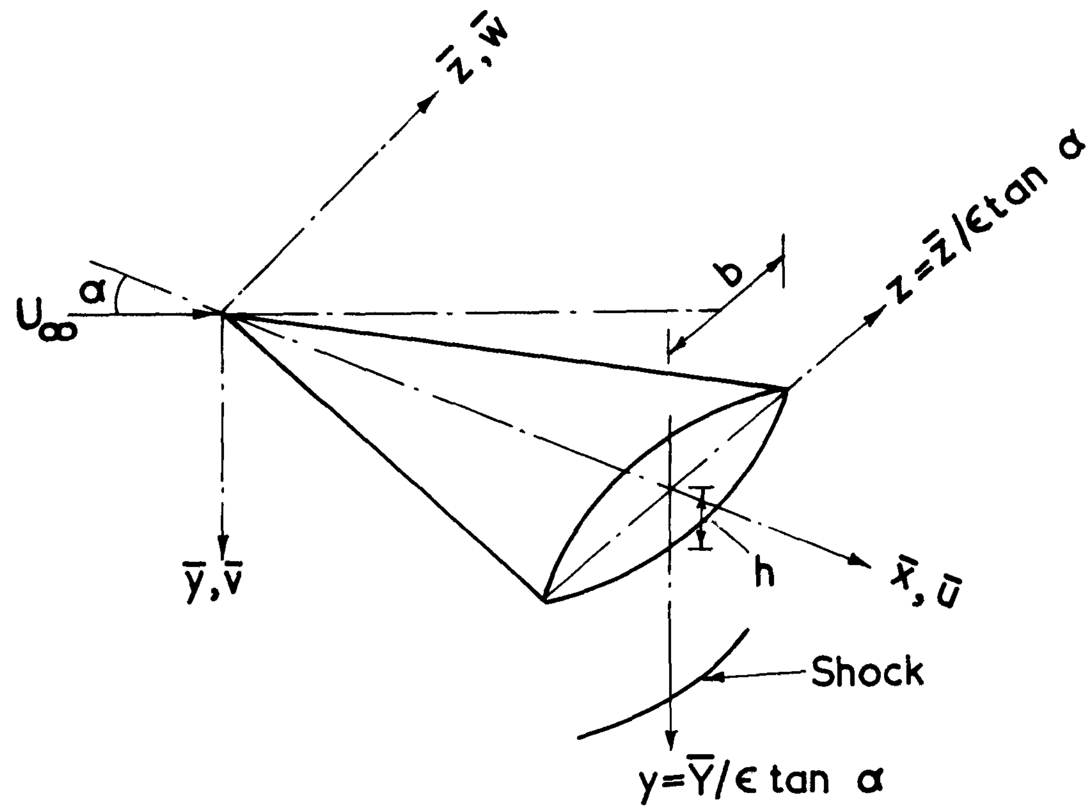

FIG. 1 Notation

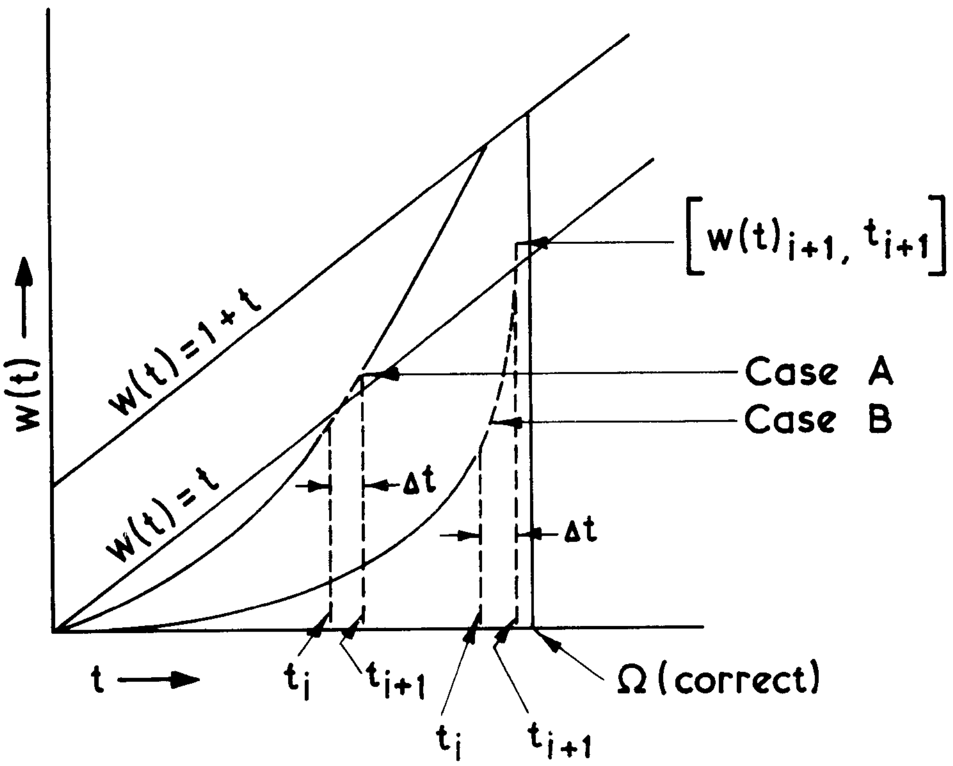

FIG.2. Variation of w(t) Vs t (not to scale)

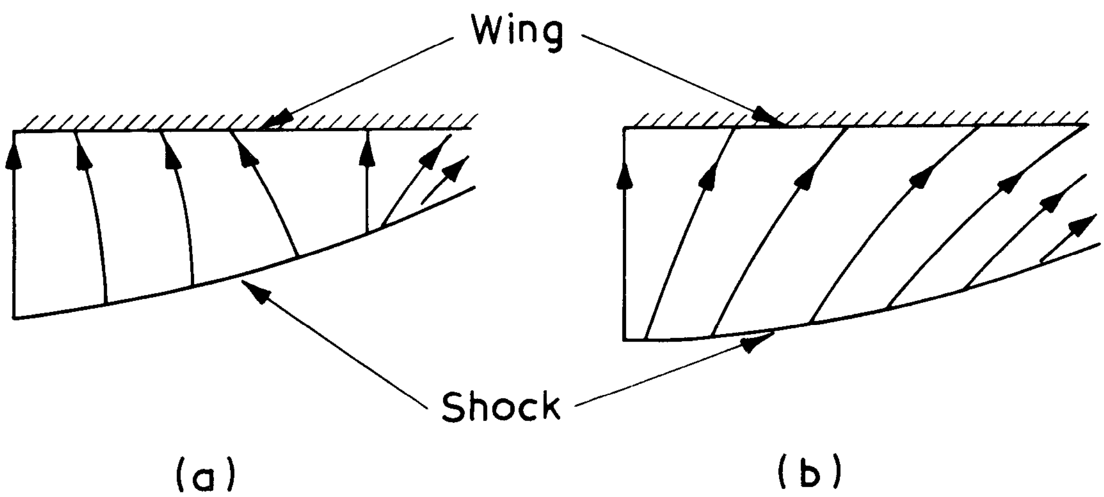

FIG.3. Streamline Patterns

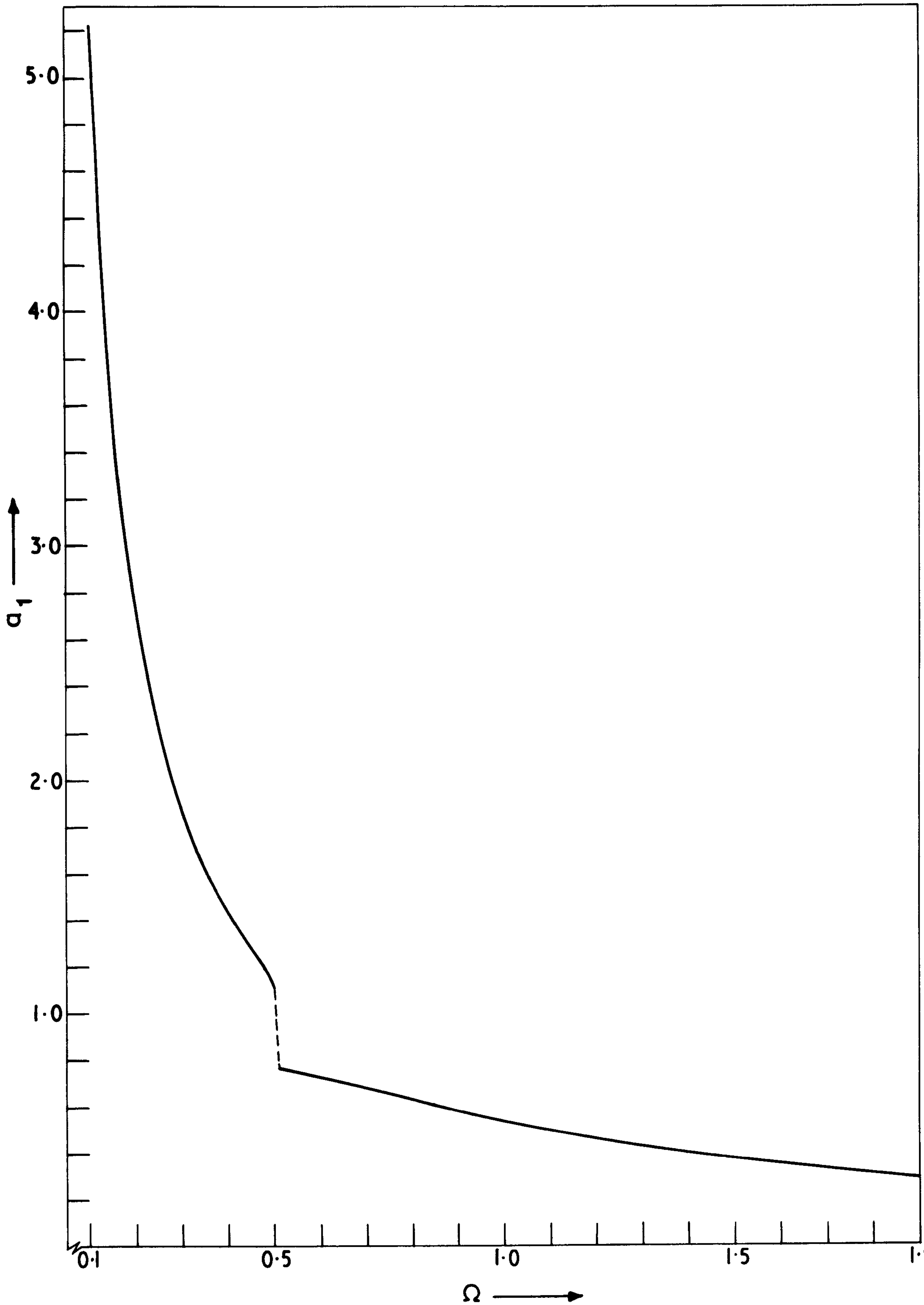

FIG.4 Parameter a_1 Vs Ω for flat delta wing

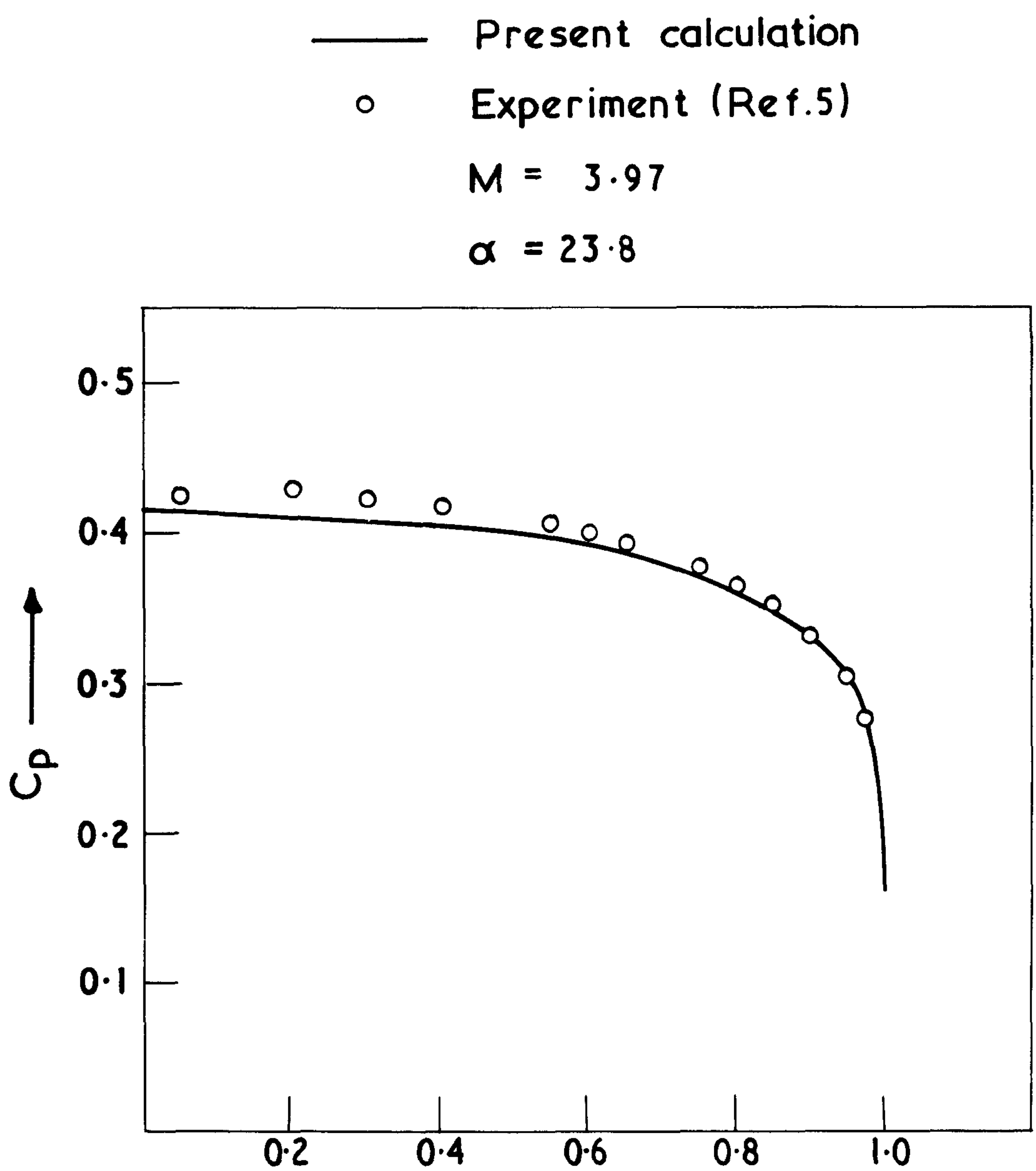

FIG.5 Spanwise pressure distribution on circular-arc cross-section delta wing Æ = 2/3

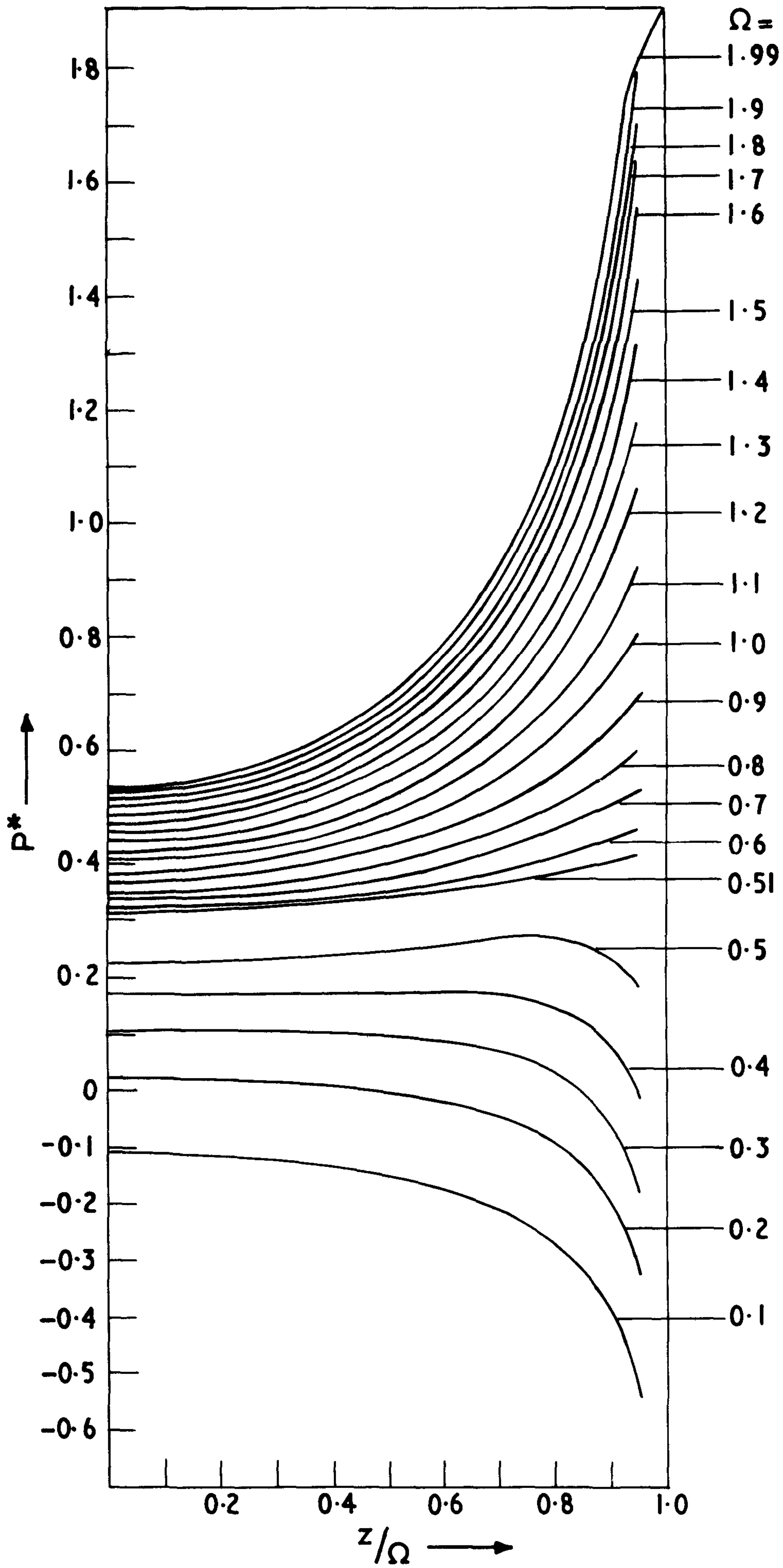

FIG.6 Spanwise P^* distribution on flat – delta wing

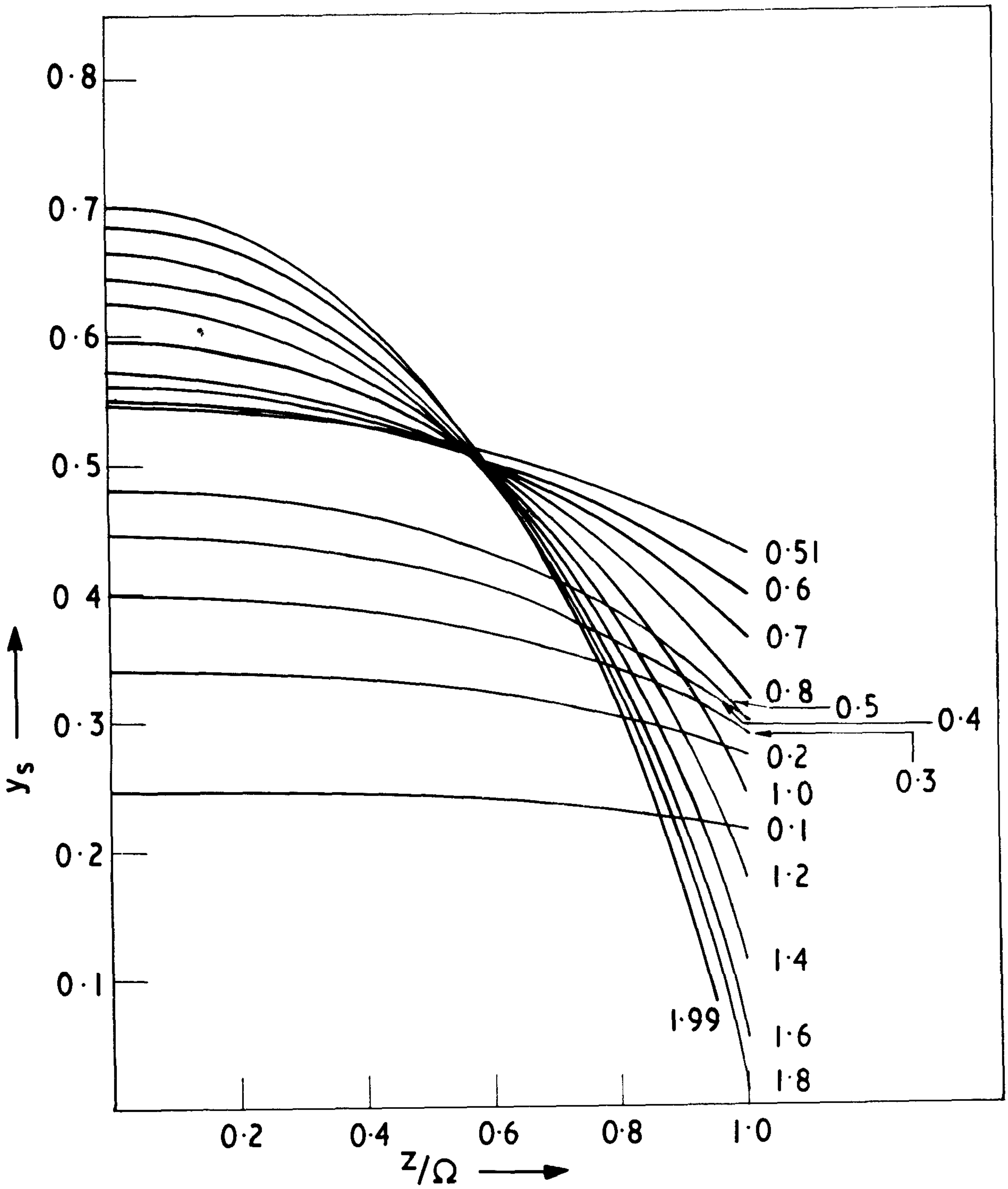

FIG. 7 Theoritical non-dimensional shock shapes on flat delta wings

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The report includes a complete set of tabulated non-dimensional pressures and shock shapes for flat wings with detached shocks for reduced aspect ratios from 0.1 to 1.99, and some sample results for wings with caret and bi-convex cross-sections.

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