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A Two - Dimensional Mathematical Model of a Parachute in Steady Descent

by

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A TWO DIMENSIONAL MATHEMATICAL MODEL OF A PARACHUTE IN STEADY DESCENT

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R. G. Hume **

SUMMARY

A two dimensional parachute model has been developed to compute various characteristics of the steady descent of a parachute system.

The model demonstrates similar characteristics to those shown from both qualitative and quantitative measurements on full-scale parachutes.

The model is in a form suitable for investigating the effect of parachute system parameters and experimentally measured aerodynamic characteristics on the stability of a descent - in particular the relationship between oscillation and gliding.

The model is also suitable for investigating the effect of wind on parachute oscillation using measured wind profiles.

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1 INTRODUCTION

Various authors[1,2,3,4] have developed nonlinear equations to model the behaviour of parachutes during descent. These equations have been used chiefly to establish stability criteria for parachute parameters. Ludwig and Heins[4] solve their particular model for a personnel parachute.

Parachutes often undergo a pendulum type oscillation. From this observation, a useful approach in studying parachute behaviour would be to bring out the pendulum type motion in the modelling equations. This procedure does not appear to have been adopted by previous authors.

Furthermore to examine the characteristics of a parachute during descent it would appear logical to confine the complexity of the model parameters within the bounds that these parameters are known.

With these points in view a parachute model has now been developed to examine:

(1) The influence of wind on the parachute with wind as:

  (a) gust (step function)

  (b) gradient (ramp function with time and height).

(2) The influence of rigging line length and store mass on:

  (a) stability

  (b) oscillation frequency.

(3) The influence of store drag on:

  (a) stability (damping)

  (b) oscillation frequency.

(4) Gliding and oscillation.

(5) The effect of gliding and oscillation on the mean descent velocity.

(6) Vertical velocity fluctuations as a result of oscillation.

(7) The linearised oscillation theory in relation to the nonlinear theory.

(8) The stability and oscillation of a gliding parachute in comparison to a nongliding parachute.

The model could be extended to examine coning and wind effects in three dimensions.

2 TWO DIMENSIONAL PARACHUTE MODEL

2.1 Initial assumptions

The parachute model is sketched in Fig.1. The assumptions made in deriving this model were:-

(1) The parachute store mass, M_s, the canopy mass, M_c, and the aerodynamic added mass, M_a, can be represented by point masses.

(2) The added mass depends only on the scale of the canopy.

(3) The added mass and the canopy mass points are coincident.

(4) The canopy mass and store mass lie on the parachute axis a fixed distance apart, L.

(5) An aerodynamic force acts on the canopy through the centre of the canopy mass. This force, with axial component, F_A and normal component, F_N, is dependent only on the velocity vector.

Implicit in this assumption, aerodynamic moments are neglected.

(6) Aerodynamic forces on the store are neglected.

2.2 Derivation of the system equations

The parachute model has been restricted to two dimensions for the present study. The variables shown in Fig.1 are used in the analysis below.

The motion may be divided into motion about the centre of mass and motion of the centre of mass.

The motion about the centre of mass is:

$$I_{cm} \frac{d^2\theta}{dt^2} + M_s gLs \sin\theta - M_c gL_c \sin\theta - F_N L_c = 0 \quad , \qquad (1)$$

where $I_{cm} = M_s L_s^2 + (M_c + M_a)L_c^2$.

L_c and L_s may be eliminated knowing the position of the centre of mass:

$$L_c = LM_s/(M_s + M_c + M_a) \quad ,$$

and

$$L_s = L(M_c + M_a)/(M_s + M_c + M_a) \quad .$$

Hence I_{cm} becomes:

$$I_{cm} = M_s(M_a + M_c)L^2/(M_s + M_c + M_a)$$

and equation (1) reduces to:

$$\frac{d^2\theta}{dt^2} + \left[\frac{g}{L(1 + M_c/M_a)}\right] \sin\theta - \frac{F_N}{L(M_a + M_c)} = 0 \quad . \tag{2}$$

The equations tracing the motion of the centre of mass are:

$$M' \frac{d^2 z}{dt^2} = Mg - F_A \cos\theta + F_N \sin\theta \quad , \tag{3}$$

and

$$M' \frac{d^2 y}{dt^2} = - F_A \sin\theta - F_N \cos\theta \quad . \tag{4}$$

where $M = M_s + M_c$
$M' = M + M_a$.

The aerodynamic forces on the canopy are usually written in the form:

$$\text{the axial force, } F_A = C_A \tfrac{1}{2}\rho A U^2 \tag{5}$$

and

$$\text{the normal force, } F_N = C_N \tfrac{1}{2}\rho A U^2 \tag{6}$$

where C_A and C_N are functions of the flow incidence, α
A is the area appropriate to the coefficients C_A, C_N
ρ is the air density, and
U the velocity of the canopy relative to the air flow.

The complete model equations to be solved are from above:

$$\frac{d^2\theta}{dt^2} = -\left[\frac{g}{L(1 + M_c/M_a)}\right] \sin\theta - C_N \left[\frac{\rho A}{2L(M_a + M_c)}\right] U^2 \tag{7}$$

$$\frac{d^2 z}{dt^2} = \left[\frac{Mg}{M'}\right] - C_A \left[\frac{\rho A}{2M'}\right] U^2 \cos\theta + C_N \left[\frac{\rho A}{2M'}\right] U^2 \sin\theta \tag{8}$$

$$\frac{d^2y}{dt^2} = - C_A \left[\frac{\rho A}{2M'}\right] U^2 \sin\theta - C_N \left[\frac{\rho A}{2M'}\right] U^2 \cos\theta \tag{9}$$

$$U_y = \left[\frac{dy}{dt}\right] - \left[\frac{LM_s}{M'}\right] \left[\frac{d\theta}{dt}\right] \cos\theta - V_y \tag{10}$$

$$U_z = \left[\frac{dz}{dt}\right] + \left[\frac{LM_s}{M'}\right] \left[\frac{d\theta}{dt}\right] \sin\theta - V_z \tag{11}$$

$$\alpha + \theta = \tan^{-1} \left[\frac{U_z}{U_y}\right] . \tag{12}$$

Equations (10) and (11) give the y,z components of the flow velocity, U. The terms V_y, V_z are the y,z components of a wind vector.

2.3 Solution of the system equations

2.3.1 Parameter values

Equations (7) to (12), while nonlinear appear quite stable under the Runge Kutta numerical solution method.

To test the modelling equations, parameters akin to a personnel parachute were selected as some experimental data on oscillations was available for comparison.

The values of the parameters chosen were:

$$M_a, M_s: 91 \text{ kg} ,$$

$$M_c: 7 \text{ kg} ,$$

$$L: 8.4 \text{ m} ,$$

and

$$\rho A: 91.5 \text{ kg/m} .$$

The aerodynamic added mass, M_a, is difficult to determine. Lester[1] has shown theoretically that the added mass is not necessarily independent of flow direction. However, due to difficulties in measuring, the added mass of the canopy is generally[2,3,4] assumed constant.

Heinrich[6] gave the added mass of a flat parachute as:

$$\left(0.25 + \frac{2}{3}\right)\pi R^3 \rho$$

where R is the mouth radius of the inflated canopy.

For the present example, M_a calculated from the above formula is 122 kg.

Ludwig and Heins[4] assumed the apparent mass of the personnel parachute to be of the order of the mass of the load. This latter assumption which follows an earlier suggestion by Von Karman, does not differ markedly from the Heinrich result and was adopted for the example calculations.

Axial and normal force coefficients were taken from work by Heinrich and Haak[5] and are reproduced in Fig.2. (These results are also available in Ref.6.) While these coefficients were measured on small scale parachutes they were thought representative of man carrying parachutes and have been used by other authors for stability calculations[2,3,4].

No wind function was inserted into the example calculations.

An approximate estimate of the descent velocity was used as a starting value. To initiate oscillations two starting angles of θ were used; $0.25(14^\circ)$ and $0.60(34^\circ)$.

2.3.2 Results

The results of the two example calculations are presented in Fig.3. The limited number of cycles shown illustrate the trend of the results.

Points noted from these trial calculations were:

(1) The horizontal velocity component of the canopy was an order of amplitude less than that of the store. This result is borne out from observation of personnel or larger parachutes where the parachute system appears to rotate about a point in the canopy.

(2) The frequency of oscillation is close to that given by a linearised version of equation (7), (derived in Appendix A):

$$\frac{d^2\theta}{dt^2} + \left[\frac{g}{L(1 + M_c/M_a)}\right] \sin\theta = 0 \quad . \tag{13}$$

Ludwig and Heins[4] give a formula for the frequency of oscillation derived empirically from their calculations. Their formula was also of the simple pendulum type.

The C_N term, equation (7), is in general small compared to the $\sin\theta$ term. Further, $\sin\theta$ may be approximated to θ with little effect on the frequency of oscillation - a second order adjustment on this linearisation increases the period by a factor $(1 + \theta^2/16)$. For a 30^o amplitude oscillation, quite usual in parachute descents, this second order adjustment is 1.7%.

For the two example calculations the frequencies of oscillation were:

$\theta(t=0) = 0.6(34^o)$
frequency 0.165 Hz (period 6.05 s)

$\theta(t=0) = 0.25(14^o)$
frequency 0.170 Hz (period 5.9 s) .

The frequency of oscillation from equation (13) is:

$$\left(\frac{g}{L}\left(1 + \frac{M_c}{M_a}\right)\right)^{\frac{1}{2}} \Big/ 2\pi \quad . \tag{14}$$

Substituting the values used in the example calculations in the expression (14) gives a frequency of 0.166 Hz.

(3) The frequency of oscillation was very close to that of the personnel parachute which the model was chosen to represent, Fig.4. However, this result only really implies a correct choice of L. The error in this choice of L is not critical as the frequency is proportional to $L^{-\frac{1}{2}}$.

(4) The two example calculations show that there is little damping in the modelling equations though the large amplitude oscillation does show a greater proportional reduction in amplitude with time. The form of the equations, in particular the C_N curve with the statically unstable zero oscillation position, leads one to suspect that a limit cycle may exist.

(5) Some glide was associated with each calculated descent. The glide velocity was 0.7 m for the 0.6 starting angle but continued to increase for the small starting angle.

A statically stable glide position occurs at 20^{o} (C_N curve, Fig.2). With the zero position statically unstable, there is probably some relationship between glide angle and oscillation - parachutes with large oscillation showing little tendency to glide. This may account for the large range in descent velocities of parachutes near impact.

(6) The amplitude and phase of the descent velocity oscillations with parachute oscillations agree well with those measured in actual parachute descents[7], Fig.4. The parachute oscillation induces the descent velocity fluctuation at twice its own frequency in a similar fashion to the vertical velocity fluctuations of a pendulum.

(7) The mean descent velocity differs slightly between the two example calculations even though the systems are similar. The higher oscillation causes an apparently lower drag coefficient.

3 DISCUSSION

The two dimensional analysis appears quite realistic in a comparison with a man carrying parachute.

The problem with using this or any parachute model lies in choosing suitable parameters to model the actual situation. In particular the parameters M_a and C_N have had little experimental examination.

The equations involving the oscillation are dominated by the simple pendulum type equation:

$$\frac{d^2\theta}{dt^2} + \left[\frac{g}{L(1 + M_c/M_a)}\right] \sin\theta = 0 \quad .$$

In any comparison with a parachute system, this motion and the normal variation in parachute characteristics masks the effect of parametric changes; M_c/M_a is generally small and L appears only as the half power in the frequency equation.

Nevertheless the only way of introducing a wind function is through the nonlinear terms involving the uncertain parameters.

It is shown in Appendix A that the canopy movement is dependent on the ratio M_c/M_a. This ratio increases with decrease in scale. In practice with large scale parachutes, the canopy often does appear to have a centre of rotation

/

near the canopy. However for small scale, where M_c/M_a is no longer small, the mode of oscillation changes.

The model results in Fig.3 bear out the linear analysis. The oscillations in the horizontal velocity component of the canopy and store are in phase with the derivative of the angle θ. Further, for the case with the smaller starting angle, the ratio of the amplitudes of the oscillations in the horizontal velocity of the canopy and store lie within 15% of that estimated from the linearised equations of Appendix A.

Ludwig and Heins[4] neglect the canopy mass and therefore get little horizontal oscillatory motion in the canopy.

The assumed position of the centre of pressure can be relinquished by using the measured aerodynamic moments as well as the normal and axial forces. The moment term is added into equation (1). The normal force in that equation is now multiplied by the distance the centre of mass and the point about which the aerodynamic forces were measured rather than the assumed centre of pressure. The equations relating the motion of the centre of mass of the system remain unchanged. However in view of both the small size of the moment term and the assumptions made regarding the aerodynamic added mass it was not felt necessary to include the moment term in the initial study.

4 CONCLUSIONS

(1) A parachute model has been developed to exhibit the dominant pendulum type motion of parachutes. It demonstrates similar characteristics to those shown for both qualitative and quantitative measurement on full scale parachutes.

(2) The model is in a form suitable for investigating the effect of parachute system parameters and experimentally measured aerodynamic characteristics on the stability of a descent - in particular the relationship between oscillation and gliding.

(3) The model is suitable for investigating the effect of wind on parachute oscillation using measured wind profiles.

Appendix A

LINEARISED ANALYSIS

The influence of the canopy mass to added mass ratio, M_c/M_a, can be seen in the following linearised analysis.

We assume:

(1) The angle of oscillation is small and U constant so that $\sin\theta \triangleq \theta$, and $Mg = C_A A \frac{1}{2}\rho U^2$,

(2) C_A is constant, and

(3) $C_N = 0$.

Then equations (7) and (9) become:

$$\frac{d^2\theta}{dt^2} = -\left[\frac{g}{L(1 + M_c/M_a)}\right] \sin\theta \quad ,$$

and

$$\frac{d^2y}{dt^2} = -\left[\frac{Mg}{M'}\right] \sin\theta \quad .$$

Combining these equations to eliminate $-g \sin\theta$ we get:

$$\frac{d^2y}{dt^2} = \frac{d^2\theta}{dt^2}\left[\frac{M}{M'}\left(1 + \frac{M_c}{M_a}\right)L\right] \quad .$$

For vertical descent this may be integrated to give:

$$y = \theta\left[\frac{M}{M'}\left(1 + \frac{M_c}{M_a}\right)L\right] \quad .$$

Horizontal canopy movement equals:

$$y - L_c \sin\theta \triangleq y - L_c\theta$$

and

$$y - L_c\theta = \theta\left[\frac{M}{M'}\left(1 + \frac{M_c}{M_a}\right)L\right] - \left[L\frac{M_s}{M'}\right]\theta = \theta L\frac{M_c}{M_a} \quad .$$

Similarly the store movement equals:

$$\theta L \left(1 + \frac{M_c}{M_a}\right) .$$

These equations imply that the horizontal motion of the canopy and store is heavily dependent on the canopy mass to added mass ratio.

SYMBOLS

A	area appropriate to the coefficients C_A and C_N
C_A	aerodynamic force coefficient associated with F_A
C_N	aerodynamic force coefficient associated with F_N
F_A	aerodynamic force component along the parachute axis
F_N	aerodynamic force component normal to the parachute axis
g	acceleration due to gravity
I_{cm}	inertia of the parachute system about the centre of mass (including the effect of the aerodynamic added mass)
M	$M_c + M_s$, parachute canopy plus store mass
M'	$M_a + M_c + M_s$
M_a	aerodynamic added mass
M_c	canopy mass
M_s	store mass
L	canopy centroid to store distance
L_c	canopy centroid to centre of mass distance
L_s	store to centre of mass distance
R	inflated canopy mouth radius
t	time
U	velocity of the canopy relative to the air
U_y, U_z	velocity components of U in the y and z directions
V_y, V_z	wind velocity components in the y and z directions
y, z	the stationary horizontal and vertical axes respectively with z positive downwards. (The parachute system is assumed to move only in the y, z plane.)
α	inclination of the air flow to the canopy measured from the parachute axis
θ	inclination of the parachute axis to the vertical
ρ	density of air flow

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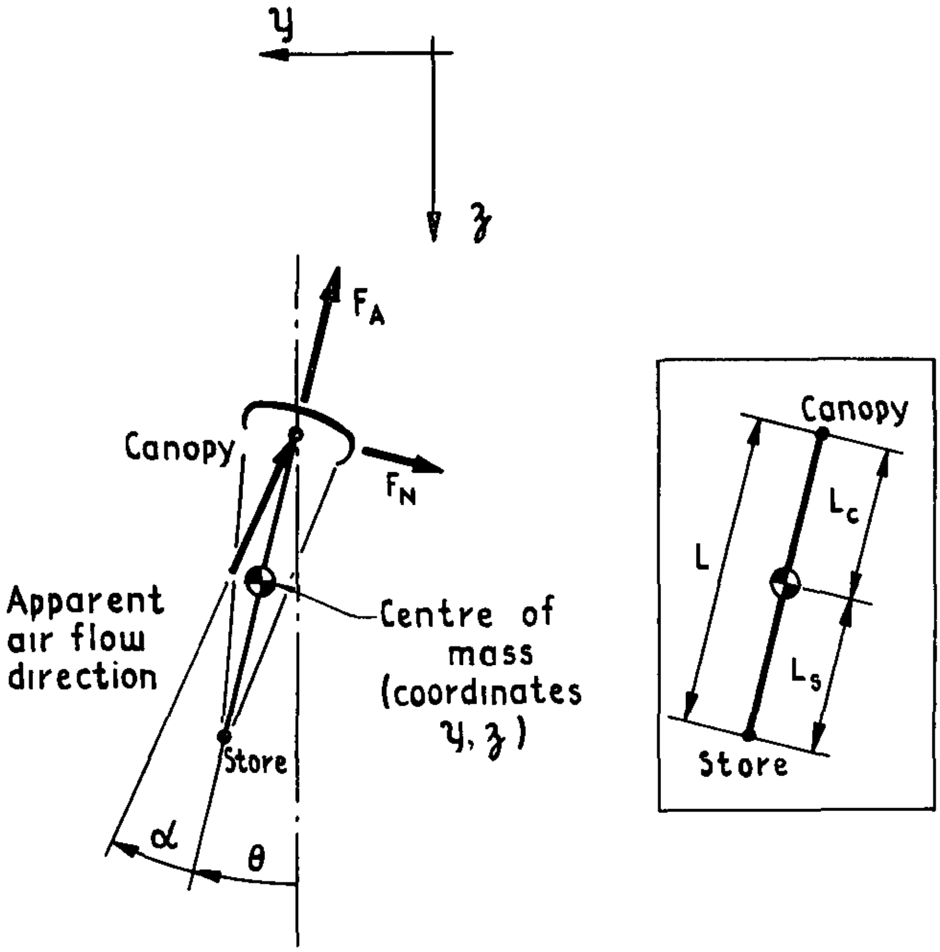

Fig. 1 Parachute model (two dimensional)

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Fig. 2

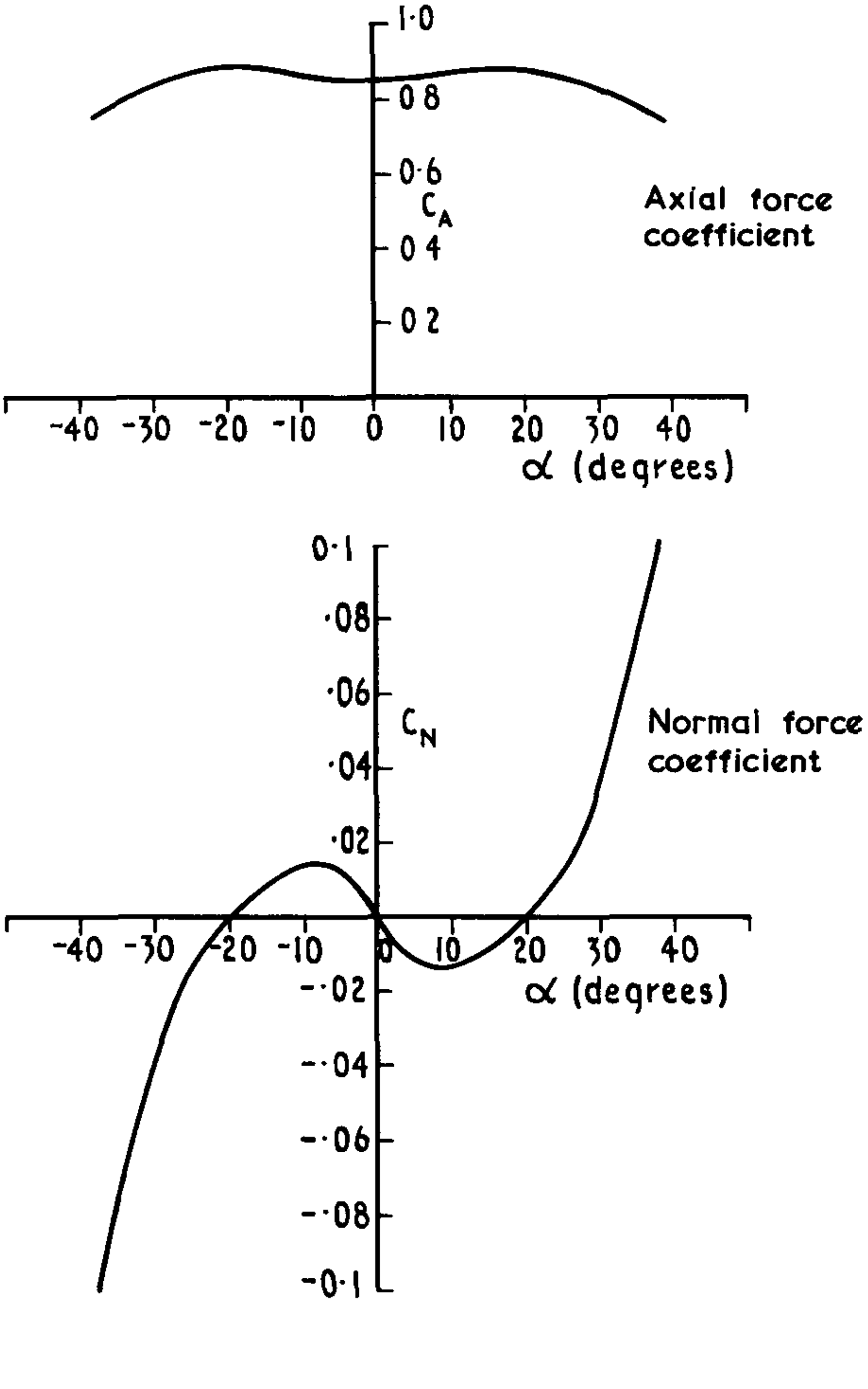

Fig. 2 Normal and axial force coefficients variation with flow angle to the parachute axis

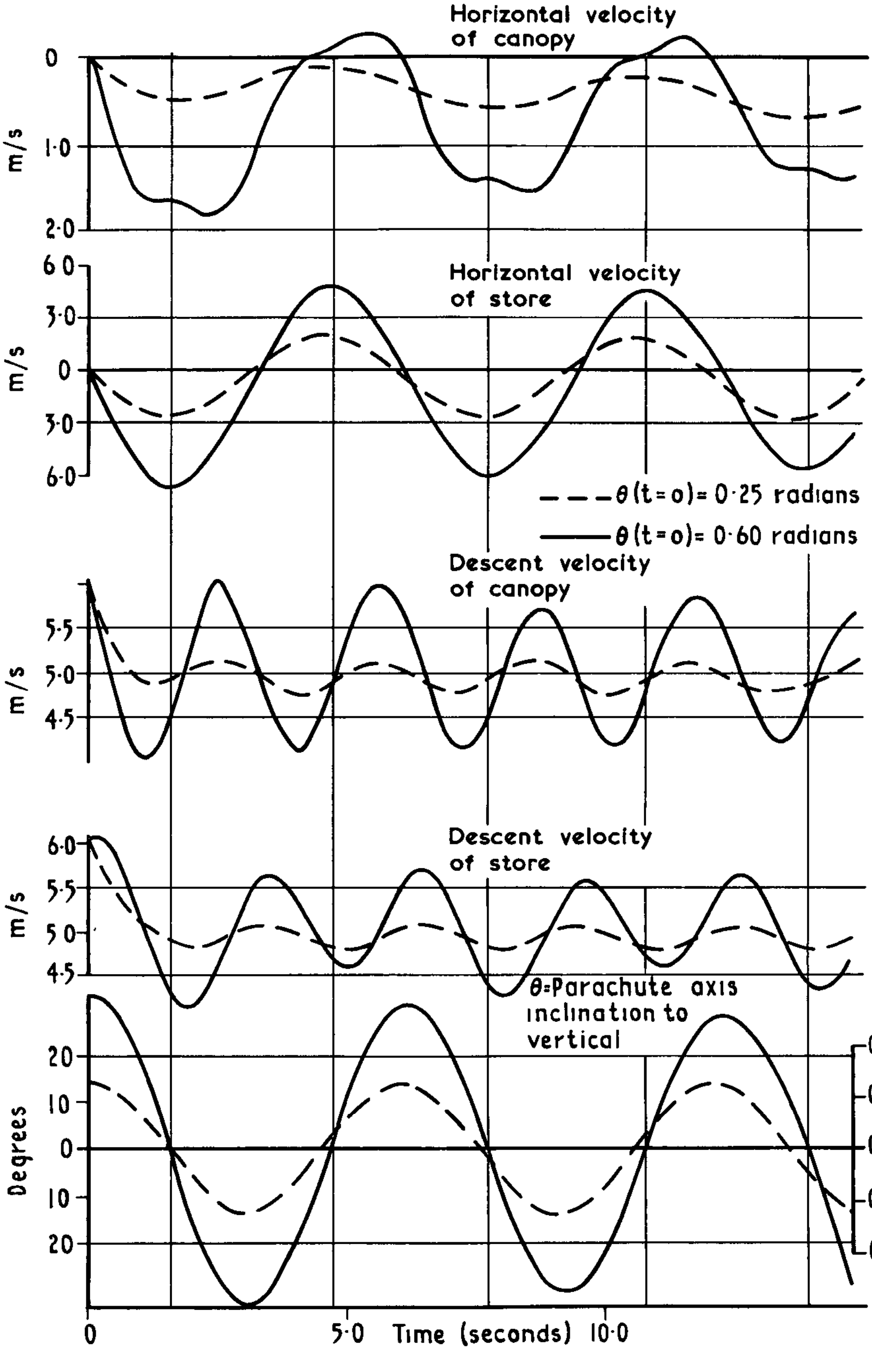

Fig. 3 Calculated store / canopy movement during a descent

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Fig. 4

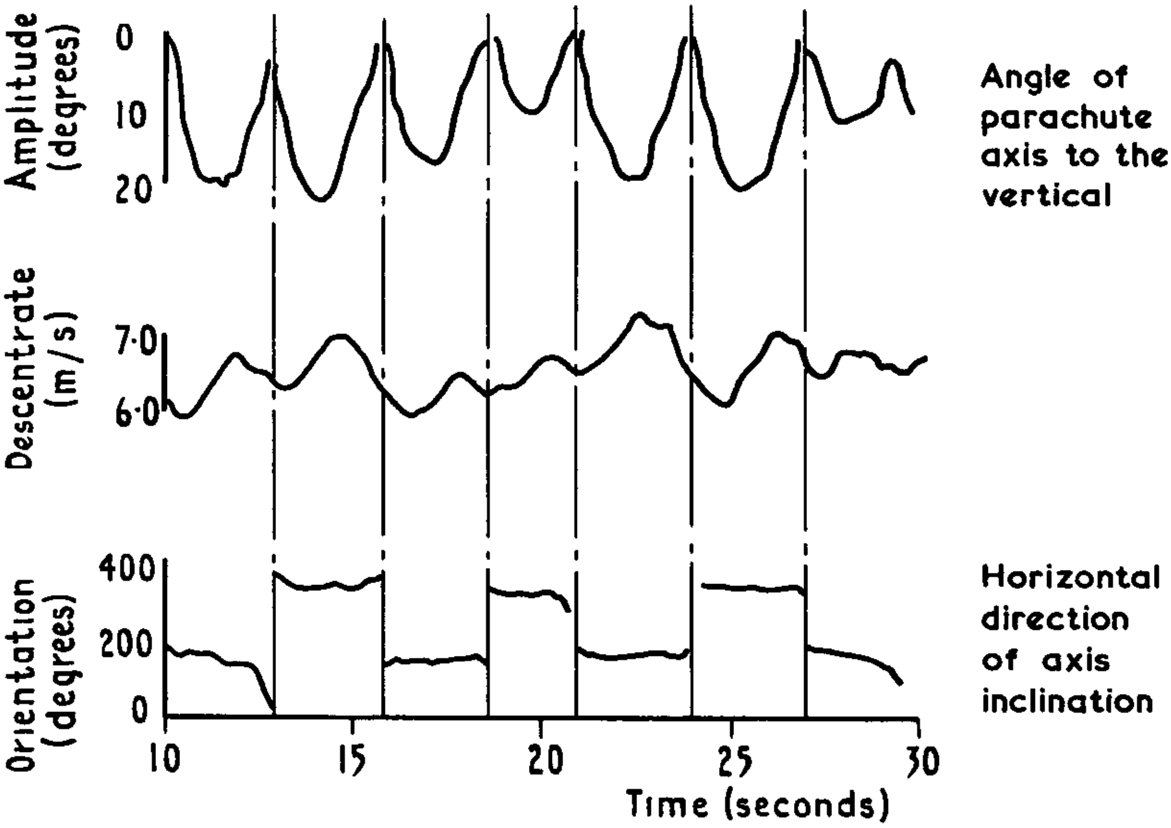

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Fig. 4 Parachute pendulum oscillations and descent rate
PX1 parachute, 136kg store

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