C.P. No. 114
(14,790)
A.R.C. Technical Report

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A Method for Calculating Pressure Distributions on Circular Arc Ogives at Zero Incidence at Supersonic Speeds, using the Prandtl-Meyer Flow Relations

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1953

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A Method for Calculating Pressure Distributions on Circular Arc Ogives at Zero Incidence at Supersonic Speeds, Using the Prandtl-Meyer Flow Relations.

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Report No. LA.t.024

21st January, 1952

Summary

A method is given for a rapid determination of pressure distributions on circular arc ogives at zero incidence at supersonic speeds. The method is based on the characteristics pressure distributions of Ref. 1 and involves the use of the two-dimensional Prandtl-Meyer flow relations. It gives very good agreement with characteristics results from other sources and with experimental results.

The method can probably be extended to ogives with profiles other than the circular arc.

List of Symbols

$C_p = \dfrac{p - p_o}{0.7\, p_o M_o^2}$, the pressure coefficient.

D = maximum diameter of tangent ogive.

$K = \dfrac{M_o}{L/D}$, the hypersonic similarity parameter.

L = length of the tangent ogive head.

M = Mach number.

p = static pressure

p_s = stagnation (reservoir) pressure.

x = distance from the nose of ogive, measured along the axis of symmetry.

Δp = increment of pressure in axi-symmetric flow.

δp = corresponding increment of pressure in two-dimensional simple-wave flow.

λ = function defined by $p = \lambda \delta p$

ϕ = change of direction of flow (see fig. 1.)

Suffix o refers to the free stream conditions.
All other symbols are defined in the text.

1. Introduction.

There are a number of methods available for calculating pressure distributions on bodies of revolution at supersonic speeds, each of them requiring a considerable amount of computing. Of these, the linearised theories are of little use as the accuracy one can expect in calculation of the wave drag coefficient C_D is of the order of ± 30%. Approximate solutions of any desired accuracy can be obtained by the method of characteristics, but the method is very laborious. With the 2nd. order theory of Van Dyke (Ref. 2.) accuracy of about ± 2% in C_D can be obtained for a wide range of the free stream Mach number and body fineness ratio, but computing work is still rather heavy.

The present note gives an alternative method of calculating the pressure distribution on circular arc ogives. The method is based mainly on the numerical characteristics results of Ref. 1. and involves the use of the two-dimensional Prandtl-Meyer flow relations. The time required to pressure plot a given head at one Mach number is about 30-45 mins. by this method, as compared with about 25 hrs by van Dyke's 2nd order theory and 80-100 hrs by the method of characteristics.

2. Method.

Consider an aerofoil profile APQB (Fig. 1.) in a uniform supersonic stream of Mach number M. and static pressure p_o. If the effects of vorticity produced by the nose shock wave are neglected, the change of flow conditions between points P and Q on the surface depends only on the conditions at the point P and on the change of direction, ϕ, which the flow undergoes between P and Q. When we consider a body of revolution with the same profile APQB, the conditions at Q are also affected by the change of cross-sectional area between P and Q of the stream tube adjacent to the body, and the flow can no longer be described by the simple Prandtl- Meyer relations. However, it may not be unreasonable to assume that it is possible to separate these two effects (i.e. the effect of the change of direction and that of the change of streamtube cross-sectional area) and to relate the conditions at the point Q in axi-symmetric flow, to the conditions that would exist at Q if the flow were two-dimensional, starting from the same conditions at the point P. In particular, one may expect that if Δp is the change of pressure between P and Q in axi-symmetric flow and δp is the corresponding quantity in the two-dimensional flow, then

$$\Delta p = \lambda \delta p \qquad (1).$$

where λ is some function of the conditions at P and of the shape of the profile between P and Q.

In the particular case of the circular arc ogives, it was found most convenient to take the point P to be just downstream of the nose shock and to define λ by

$$(p_n - p) = \lambda(p_n - p_t) \qquad (2).$$

where p_n = pressure at the nose just downstream of the shock wave in axi-symmetric flow.

p = pressure at a point P on the surface in axi-symmetric flow.

p_t = pressure at P if the flow expanded two-dimensionally from p_n at the nose.

Characteristics pressure distributions of Ref.1. were used to calculate λ and it was found that λ was, within the accuracy of the distributions, constant along the profile and varied only with the free stream Mach number, M_o, and the ogive fineness ratio, L/D.

3. Results and Comments.

Fig. 3. shows the variation of λ with M_o and L/D. For convenience, the lines of constant $K = \frac{M_o}{L/D}$ are also shown. K is the hypersonic similarity parameter; its significance is fully explained in Ref.1. where it is shown that for a wide range of M_o and L/D the pressure distribution on a family of geometrically similar ogives is a function of K only. The dotted parts of the curves correspond to regions in which λ varies along the surface by more than about ± 2-3% of its average value. The values of λ were not calculated for $K < 0.5$ because of the lack of characteristics pressure distributions. The method can probably be extended to $M_o > 3.5$, $0.5 \leq K \leq 1$ and to $L/D > 2$, $K > 1$; however, no calculations were made, since these regions are of less interest from the point of view of practical applications.

Fig. 4(a) to (d) shows the comparison of pressure distributions on circular arc ogives obtained by using equation (2) and Fig.3. with some theoretical and one experimental pressure distributions, which were not used in the determination of λ; to illustrate the method, one calculation of pressure distribution is shown in detail in the Appendix. It is seen that the agreement in cases (a), (b) and (c) is excellent. Case (d) is at a low Mach number and falls within the region (shown dotted in Fig. 3) where the applicability of the λ relation is doubtful. Even here, the agreement with the characteristics pressure distributions is good, the maximum error in C_p being about 2% of C_p at the nose.

It may be possible to extend the present method to profiles other than the circular arc; for a non-circular arc profile, λ would probably depend also on the local radius of curvature of the profile, or some related variable. It is hoped to investigate this further when the work (at present in progress) is completed on calculating the pressure distribution on some parabolic ogives by the 2nd. order theory of van Dyke (Ref. 2.).

4. Conclusions.

Numerical analysis of results of Ref.1. shows that pressure distributions on circular arc ogives at zero incidence can be very simply related to the pressure distribution on the same profile in two-dimensional simple-wave flow, calculated by the Prandtl-Meyer relation.

It is found that the relation $\Delta p = \lambda \, \delta p$ (see equs. (1) and (2)) holds with very good accuracy for $M_o \geq 1.6$, $0.5 \leq K = \frac{M_o}{L/D} \leq 1.0$.

Variation of λ with L/D and M_o is determined from characteristics results in the region $0.5 \leq K \leq 1.0$, $L/D > 2$, $1.6 \leq M_o \leq 3.5$, and is given in Fig. 3.

It is hoped to investigate the possibility of extending this relation to profiles other than the circular arc.

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Appendix.

The following numerical example is given to illustrate the method of this report.

Free stream Mach number $M_o = 2.0$

Ogive fineness ratio, $L/D = 3.12$; nose semi-angle, $\theta_s = 18.20^\circ$

Let p_o and p_{s_o} denote the free stream static pressure and stagnation pressure, respectively;

p_n and p_s the static and stagnation pressures, respectively, at the nose of the ogive, just downstream of the nose shock wave.

p the static pressure at a point P on the surface in axi-symmetric flow.

p_t the static pressure at P if the flow expanded two-dimensionally from p_n at the nose.

From Kopal's cone tables (Ref. 6.) it is found that the pressure coefficient at the nose is $C_{p_n} = \dfrac{p_n - p_o}{0.7\, p_o M_o^2} = 0.277_5$, hence $\dfrac{p_n}{p_o} = 1.776$.

From Ref. 7. the stagnation pressure ratio across the nose shock is found to be $\dfrac{p_s}{p_{s_o}} = 0.994$ and from Table I of Ref. 8. $\dfrac{p_s}{p_{s_o}} = 0.1278$,

so tho

$$\frac{p_n}{p_s} = \frac{p_n}{p_o}\,\frac{p_o}{p_{s_o}}\,\frac{p_{s_o}}{p_s} = 0.2283.$$

Let ν denote the angle through which the flow would have to expand in a Prandtl-Meyer expansion from the sonic velocity, to reach the pressure ratio p/p_s. Then, from Table I of Ref. 8. one finds that at the nose

$$\nu_n = 15.46^\circ, \text{ (corresponding to } \frac{p_n}{p_s} = 0.2283).$$

From Fig. 3., the value of λ at $M_o = 2.0$ and $L/D = 3.12$ is 0.814. Taking values of θ (see Fig. 2.) at intervals of $0.1\,\theta_s$ and using table I of ref. 8. the following table can be drawn up:

TABLE I.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
θ (degrees)	$\frac{x}{L} = 1 - \frac{\sin\theta}{\sin\theta_s}$	$\nu = y_n + (\theta_s - \theta)$	$\frac{p_t}{p_o}$ (Ref. 8. Table I.)	$\frac{p_n - p}{p_s} = 0.2283 - (4)$	$\frac{p_n - p}{p_s} = \lambda.(5)$	$\frac{p_n}{p_s} = 0.2283 - (6)$	$C_p = \frac{p - p_o}{0.7 p_o M_o^2}$
18.20	0	15.46	0.2283	0	0	0.2283	0.277
16.38	0.097	17.28	0.2083	0.0200	0.0163	0.2120	0.232.
14.56	0.196	19.10	0.1895	0.0388	0.0316	0.1967	0.190
12.74	0.295	20.92	0.1722	0.0561	0.0457	0.1826	0.151
10.92	0.394	22.74	0.1563	0.0720	0.0586	0.1697	0.114
9.10	0.494	24.56	0.1415	0.0868	0.0707	0.1576	0.080
7.28	0.595	26.38	0.1278	0.1005	0.0818	0.1465	0.050
5.46	0.696	28.20	0.1152	0.1131	0.0920	0.1363	0.021
3.64	0.797	30.02	0.1036	0.1247	0.1015	0.1268	- 0.005
1.82	0.898	31.84	0.0929	0.1354	0.1102	0.1181	- 0.029
0	1	33.66	0.0831	0.1452	0.1182	0.1101	- 0.051

These values of C_p are shown plotted in Fig.4(a).

Continuation to: IA.t.024. Sheet No.6

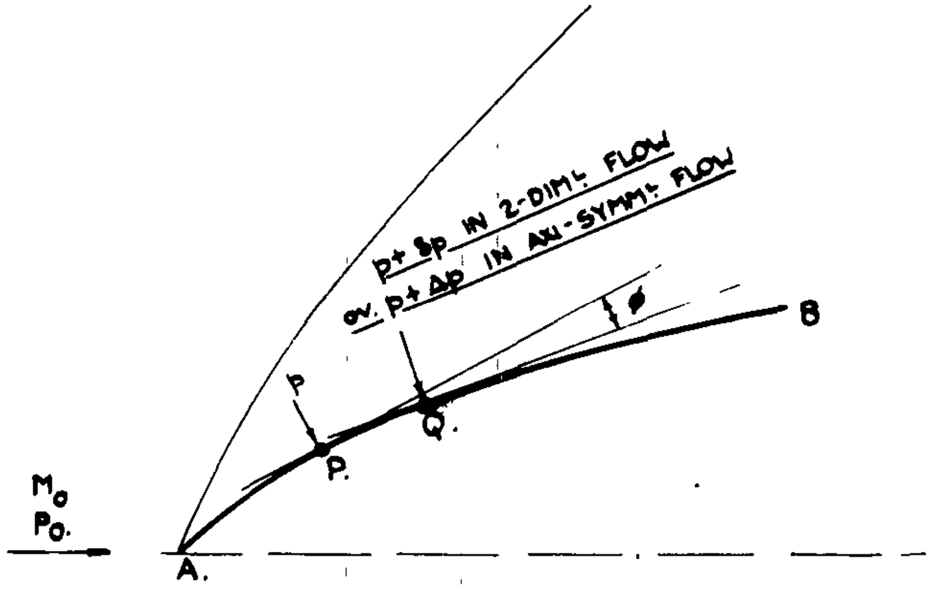

FIG. 1. SUPERSONIC FLOW PAST A CURVED PROFILE.

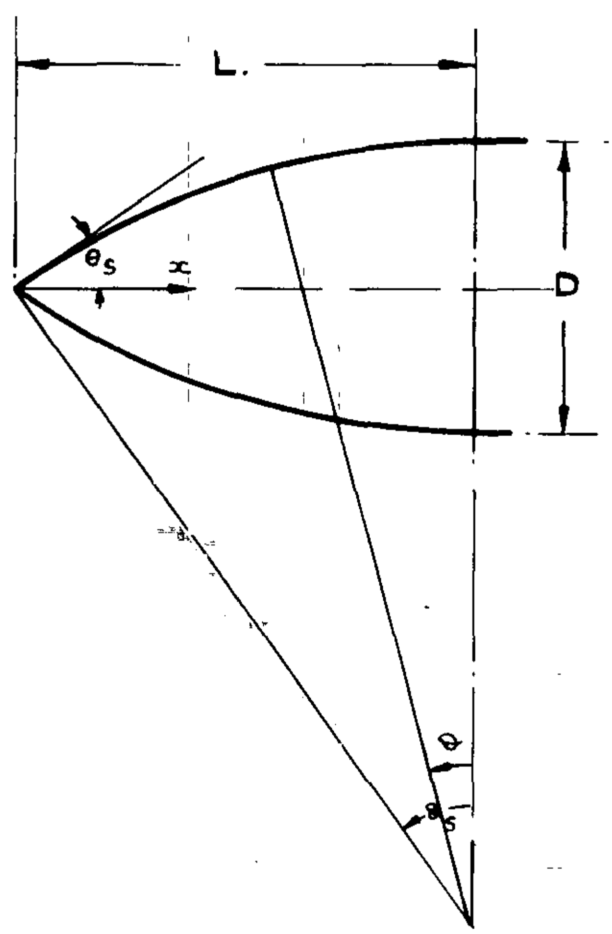

FIG 2 GEOMETRY OF TANGENT CIRCULAR ARC OGIVES

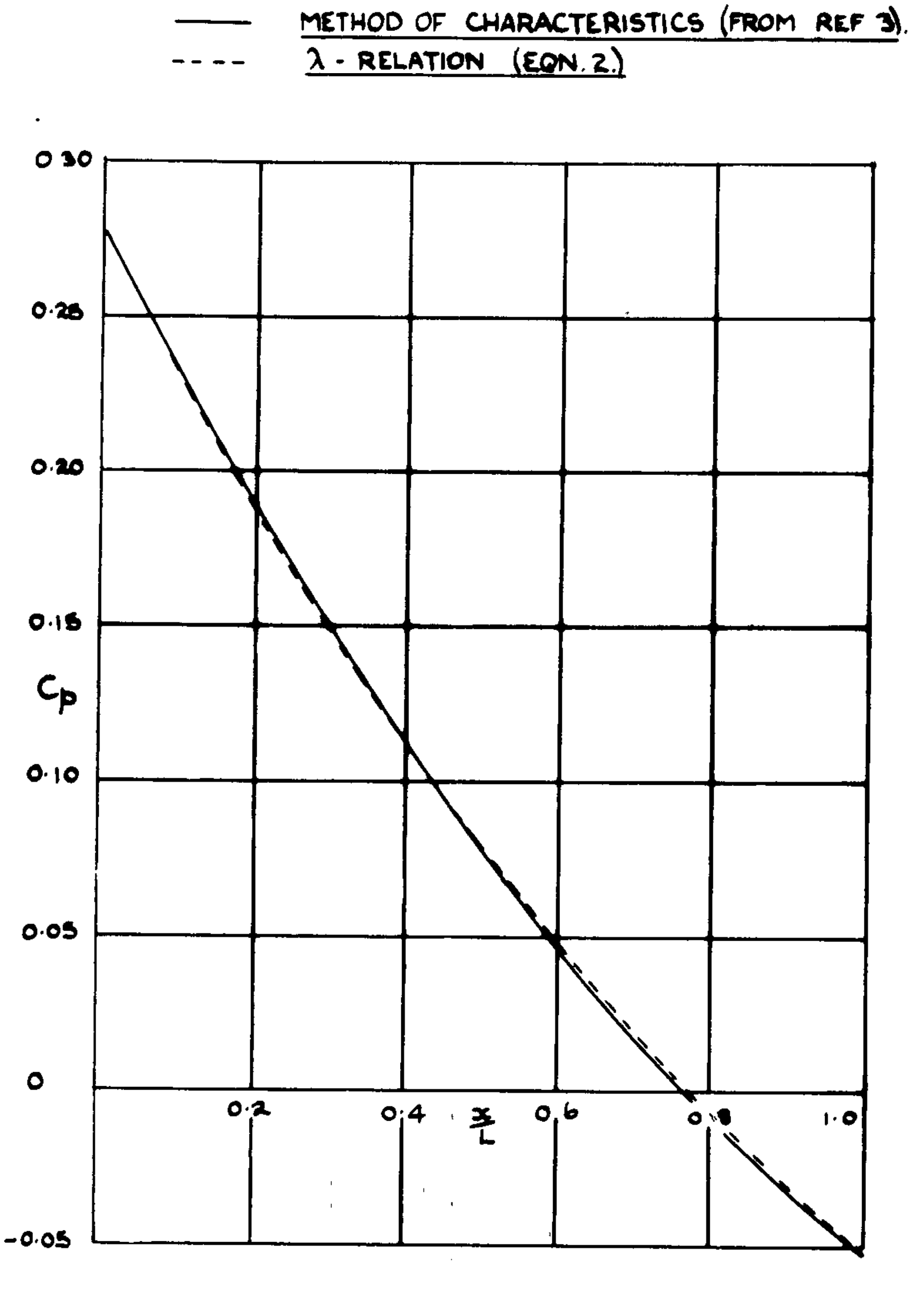

FIG. 4. PRESSURE DISTRIBUTION ON CIRCULAR ARC OGIVES AT ZERO INCIDENCE

(a) $L/D = 3.12$ $M_0 = 2.0$.

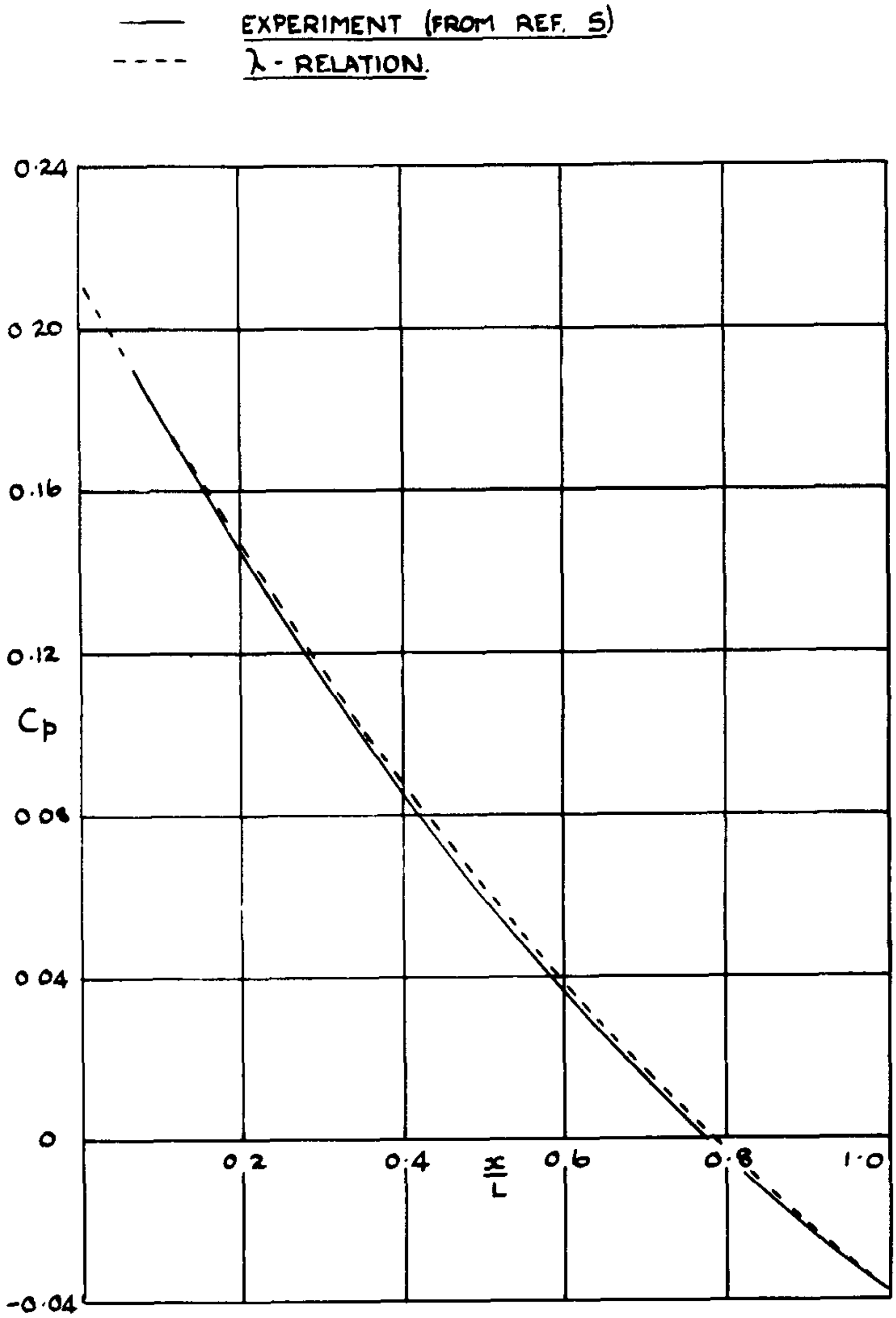

FIG 4 CONTINUED

(b) L/D = 3 50. M_0 = 2·47.

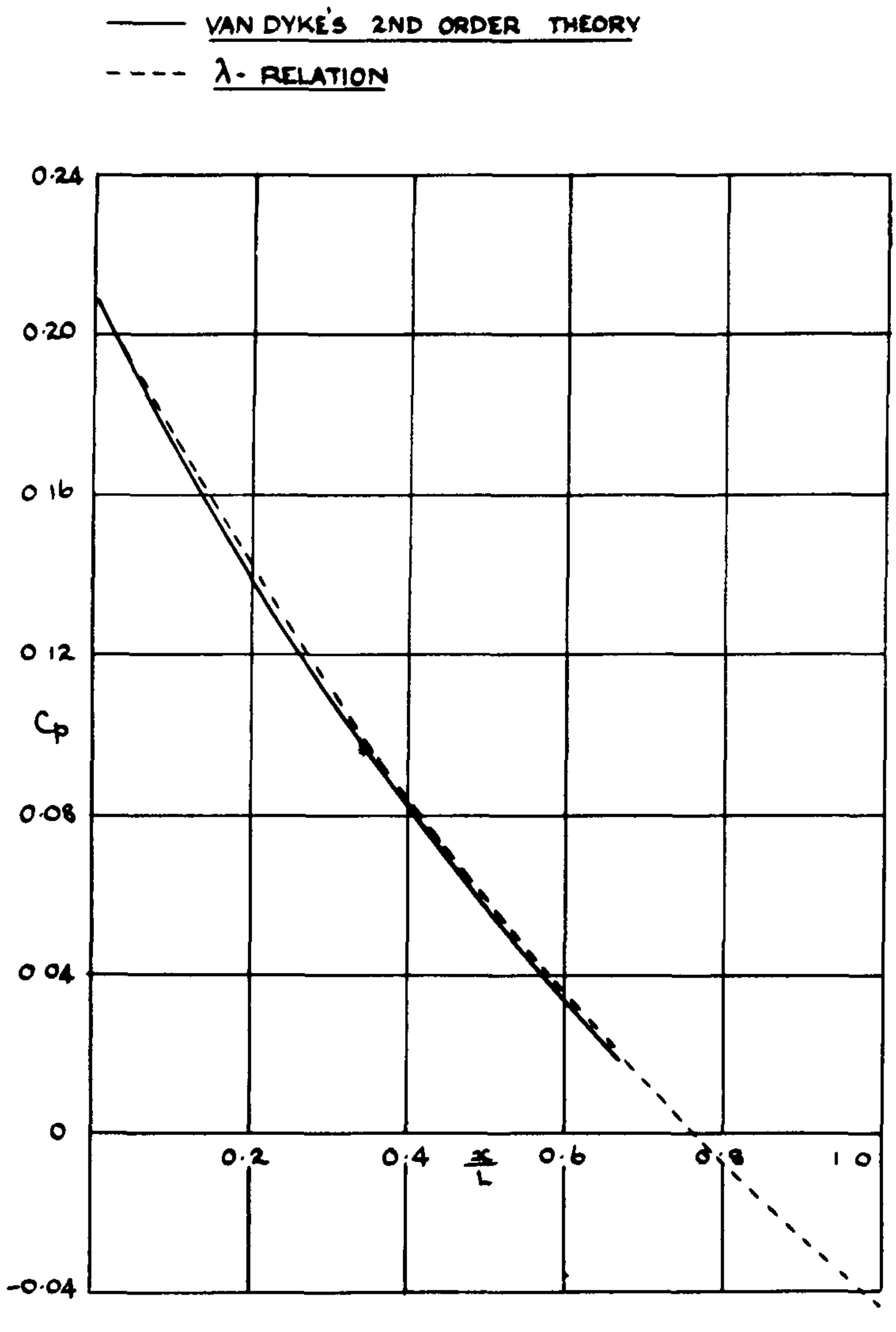

FIG. 4 CONTINUED

(c) $L/D = 3.70$ $M_0 = 2.0$

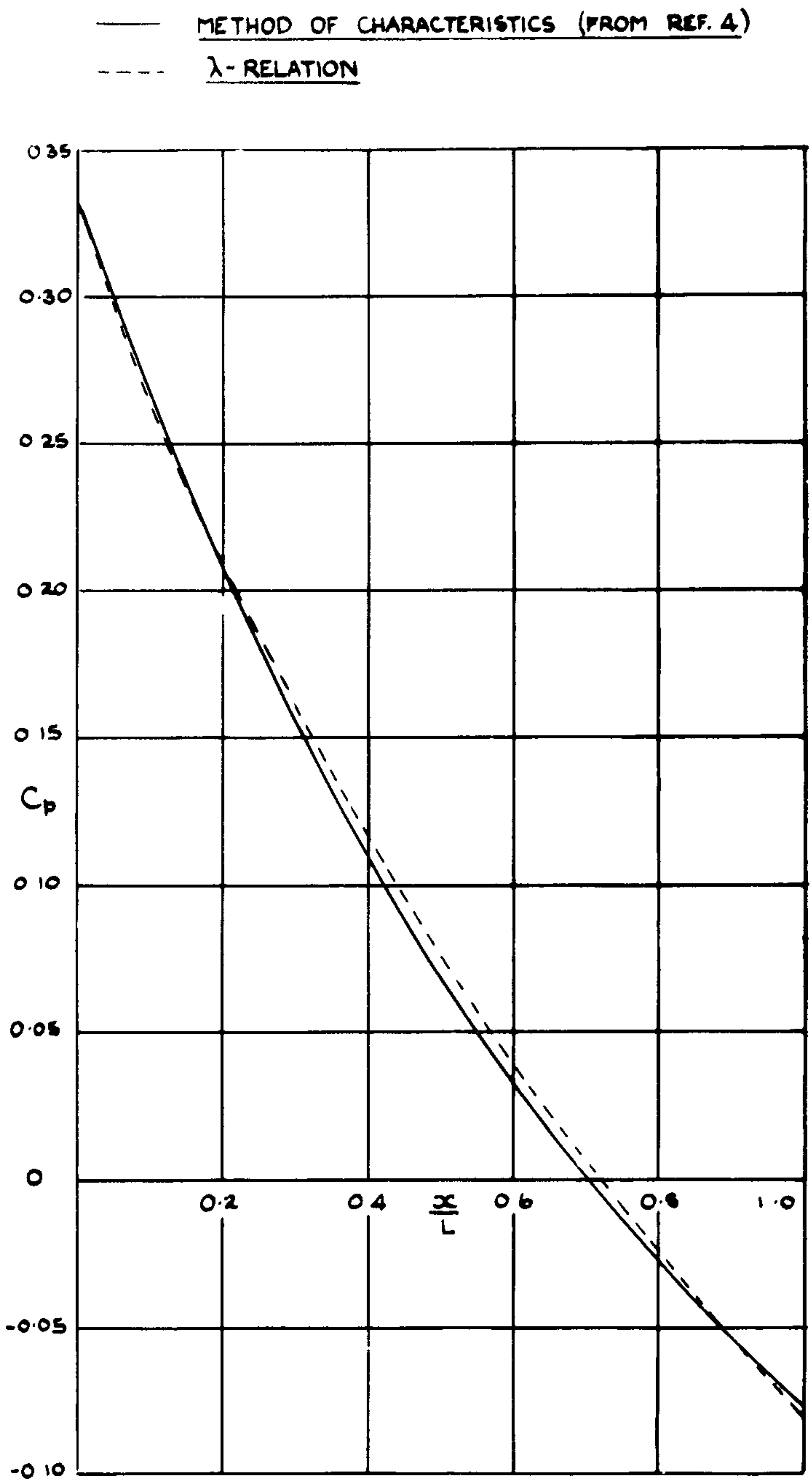

FIG. 4 CONTINUED

(d) $L/D = 3.12$, $M_0 = 1.5$

C.P. No. 114
(14,790)
A R C Technical Report

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