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**Boundary-Layer Pressure Fluctuations
at High Reynolds Numbers on a
Free-Flight Test Vehicle**

by

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BOUNDARY-LAYER PRESSURE FLUCTUATIONS AT HIGH REYNOLDS NUMBERS ON A
FREE-FLIGHT TEST VEHICLE

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SUMMARY

Measurements have been made of the boundary-layer pressure fluctuations on the body of a free-flight aerodynamic test vehicle powered by a solid-fuel rocket motor. The vehicle reached a maximum Mach number of 2.2 with a maximum Reynolds number of about 215 millions.

Pressure spectra have been deduced, and have been found to compare reasonably with a theoretical spectrum for homogeneous isotropic turbulence.

The scale of the boundary-layer turbulence was found to fluctuate between 47% and 76% of the turbulence boundary-layer thickness over a range of Mach numbers from 1.5 to 2.2, while being essentially equal to 50% of this thickness over the range $Ma = 2.0$ to $Ma = 2.2$.

At $Ma = 2.2$ the root mean square boundary-layer pressure was equal to 0.0045 of the free stream dynamic pressure.

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1 INTRODUCTION

A series of three free-flight aerodynamic test vehicles, designated the 'Shark'* series, was envisaged for the investigation of boundary-layer characteristics at high Reynolds number.

On Shark 1, measurements were made of skin friction drag by means of surface pitot tubes, and boundary-layer pressure fluctuations using a piezo-electric pressure transducer. On Shark 2, it was planned to make direct measurements of skin friction drag by means of a modified force-balance accelerometer, for comparison with the results obtained from Shark 1. Difficulties with the instrument resulted in the abandonment of the experiment. On Shark 3, measurements were made of boundary-layer pressure fluctuations employing two piezo-electric pressure transducers, one at a rear station similar to that used on Shark 1, and the other at a station positioned to give a Reynolds number half that at the rear station. This Report presents the pressure-fluctuation results from Shark 1.

2 DESCRIPTION OF TEST EXPERIMENT

2.1 Description of test vehicle

2.1.1 General

The vehicle consisted of a 4 calibre tangent circular ogival nose fitted to a cylindrical body 8 calibre long followed by a 2 calibre tail section, to the tail section were fitted three stabilising fins, radially equally distributed. The body and tail tube were constructed of resin bonded paper tube, and the nose was a glass fibre moulding. The vehicle was propelled by a non-separating solid-fuel rocket motor mounted inside the body. The overall length was about 233 inches (5.92 m) and the launch weight was about 1204 pounds (546 kg). A general arrangement drawing is given in Fig.1a.

2.1.2 Instrumentation

Transducers were installed to measure:

(a) Boundary-layer pressure fluctuations (special piezo-electric lead-zirconate bimorph transducer, $\pm 0.2 \text{ lbf/in}^2$ [$\pm 1379 \text{ N/m}^2$]).

*These vehicles were designed and constructed by Aerodynamics Department, RAE, and launched by them at the RAE Aberporth firing range.

(b) Pitot and static pressure differential at each of five surface pitot tubes (variable inductance pressure transducers).

(c) Acceleration normal to the vehicle body surface near the boundary-layer pressure measuring station (piezo-electric accelerometer).

(d) Accelerations normal and perpendicular to a given vehicle-body radius (variable inductance accelerometers).

The output signals of these transducers were translated through suitable sequence-switches and modulator units to frequency-modulate the transmission of three 465 MHz oscillator units. The signals from these oscillators were received and translated at the ground receiving station, and recorded on magnetic tape and oscilloscope record films.

One transmitter was used to telemeter the data from (b) and (d), to give measurements of vehicle accelerations and skin-friction. The second transmitter telemetered the output of (c) to give measurements of the environmental vibration local to the boundary-layer pressure-fluctuation transducer. The third transmitter telemetered the output from (a); the information derived from this transducer is considered in this paper.

2.2 Aerodynamic environment

The maximum Mach number achieved was about 2.2 [$q_{\max} \approx 6870 \text{ lbf/ft}^2$ (330 kN/m^2)].

The maximum local Reynolds number was about 215 millions.

Maximum altitude was about 1456 ft (444 m).

2.3 Range facilities

In addition to the telemetry receiving and recording station, the firing range provided the following:-

(a) a number of observation posts suitably sited, equipped with high-speed cine cameras and kinetheodolites, which tracked the vehicle during flight, and afforded data from which was deduced velocity and altitude information.

(b) Radar cover to provide additional velocity and altitude data, particularly when the vehicle was beyond optical range of the observation posts, or obscured by cloud.

(c) A central source of timing pulses which were transmitted to all the data recording systems, and also initiated a firing sequence which resulted in the electrical pulse sent to the rocket motor igniter circuit. Measurement of the roll of the vehicle was also made by the telemetry receiving station.

The transmission of data from the boundary-layer pressure fluctuation transducer ceased after about 5 seconds of flight. This record deals with the data received during this time.

3 METHOD OF DATA ANALYSIS

The signal representing the output of the boundary-layer pressure fluctuation transducer was frequency-translated after reception and recorded on $\frac{1}{2}$ inch magnetic tape at 60 inches per second. It was later played back at 7.5 inches per second, and re-recorded at 30 inches per second. Thus one second of vehicle flight time was represented by 240 inches of magnetic tape. This operation was carried out by Instrumentation and Ranges Department at the Royal Aircraft Establishment, Farnborough, using the equipment known as BRAMBLE¹. This analogue record was digitised by means of a high-speed digital recorder made to an RAE design by English-Electric/Leo/Marconi Ltd., the output from which was taken to a five-hole paper tape punch. For ease of handling, the record of the five seconds of flight time was divided into fifty punched paper tapes, each representing approximately 0.1 second of flight time, and designated runs 00 to 49, the designation representing the approximate time of the beginning of that section of data. Thus run 23, for instance, refers to the data received during the section beginning at about 2.3 seconds after the commencement of the flight.

Each of these fifty runs consisted of approximately 6170 discrete readings of pressure amplitude, expressed in units of analogue/digital converter output. For convenience in the computing process, these output units were retained throughout the analysis. The relationship between these units and standard physical units is given below:-

$$\begin{aligned} 1 \text{ pressure unit} &= 0.0031163 \text{ lbf/in}^2 &= 0.02149 \text{ kN/m}^2 \\ 1(\text{pressure})^2 \text{ unit} &= 9.7113 \times 10^{-6} (\text{lbf/in}^2)^2 &= 0.4618 \times 10^{-6} (\text{kN/m}^2)^2. \end{aligned}$$

The readings were spaced at intervals of 15.5 microseconds of flight time. Figs.2a, 2b and 2c show plots of pressure-readings for the first 5 milliseconds of each of the three runs, 32, 33 and 34.

Using the RAE Mathematics Department ICL 1907 computer, the data for each run was treated separately, the following operations being performed.

(a) The arithmetic mean of all the values in the run was found, and subtracted from each value in turn:-

$$y_t = (\text{recorded value}) - (\text{arithmetic mean}) .$$

(b) The root mean square of the pressure fluctuations was evaluated using the expression

$$\sigma_p = \left[\frac{1}{(N+1)} \sum_{t=1}^{N+1} y_t^2 \right]^{\frac{1}{2}}$$

where $(N+1)$ is the number of discrete values contained in the run.

(c) The autocorrelation function is defined as

$$R_\tau = \frac{1}{(N+2-\tau)} \sum_{t=1}^{N+2-\tau} y_t y_{(t+\tau-1)}$$

where $\tau = 1, 2, 3, \dots, (M+1)$

where M is the number of correlated intervals used in the spectral density analysis.

The expression was evaluated for $\tau = 1, 2, 3, \dots, (100)$.

(d) The unsmoothed spectral density estimates were calculated from²:-

$$V_k = \Delta t \left\{ R_1 + R_{(M+1)} \cos [(k-1)\pi] + 2 \sum_{\tau=2}^M R_\tau \cos \left[\frac{[(k-1)(\tau-1)\pi]}{M} \right] \right\}$$

where $k = 1, 2, 3, \dots, (M+1)$

and Δt is the time interval between each two consecutive discrete values.

Tabulated values of unsmoothed spectral density are presented in Table 1.

(e) The frequency to which this estimate refers is given by:-

$$f_k = \frac{(k-1)}{2\Delta t M}$$

Values of k with the corresponding values of frequency are presented in Table 2.

The expression in (d) and (e) were evaluated for $k = 1, 2, 3, \dots, 100$.

(f) The estimates of hanning-smoothed and hamming-smoothed spectral densities are given by:-

hanning smoothing

$$\begin{aligned}
 W_k &= 0.5 V_1 + 0.5 V_2 & k &= 1 \\
 &= 0.25 (V_{k-1} + V_{k+1}) + 0.5 V_k & 1 < k < M \\
 &= 0.5 V_M + 0.5 V_{M+1} & k &= M + 1
 \end{aligned}$$

hamming smoothing

$$\begin{aligned}
 Z_k &= 0.54 V_1 + 0.46 V_2 & k &= 1 \\
 &= 0.23 (V_{k-1} + V_{k+1}) + 0.54 V_k & 1 < k < M \\
 &= 0.46 V_M + 0.54 V_{M+1} & k &= M + 1
 \end{aligned}$$

These expressions were also evaluated for $k = 1, 2, 3 \dots 100$.

Autocorrelation functions plotted against τ , and smoothed and unsmoothed spectral density estimates plotted against k , are presented for a number of representative runs in Figs.6a to 6e. The maximum value of unsmoothed spectral density and the frequency f_m at which it occurred, were determined, for each run. It should be noted that f_m was taken as the frequency of the peak value of the unsmoothed spectral density, regardless of fluctuations about the mean path through the experimental points. In a few runs, two peaks of similar magnitudes occurred at fairly close frequencies, causing some doubt as to which represented the proper value of f_m . In such a case, reference was made to the plots of smoothed spectral density, which indicated one sensible peak value. The frequency of this value was taken as the value of f_m .

(g) For very high Reynolds numbers (non-dimensional viscosity function $A = 0$) it was shown³ that the maximum (pressure)² spectral density occurred at

$$X = 2.111$$

where X , the non-dimensional frequency, was defined as

$$X = a\omega = \frac{2\pi fa}{U}$$

where ω was the wave-number, f the measured frequency, U the velocity of the stream relative to the measuring station, and a was a length to make wave-numbers non-dimensional.

The frequency at which the spectral density is a maximum is defined as f_m so that

$$2.111 = \frac{2\pi f_m a}{U}$$

or

$$a = \frac{2.111U}{2\pi f_m} .$$

The scale of the turbulence was defined as

$$\lambda_u = LUa$$

and

$$LU = 0.74677 \quad \text{when } A = 0 .$$

If the frequency position of the maximum value of the experimental spectrum is made to coincide with that of the theoretical spectrum, the scale of the turbulence is therefore given by -

$$\begin{aligned} \lambda_u &= \frac{2.111 (0.74677U)}{2\pi f_m} \\ &= \frac{0.25088U}{f_m} . \end{aligned}$$

This expression was used to calculate the value of λ_u for each run.

(h) The local Reynolds number R_x was calculated for each run, assuming that the effective start of the turbulence was at a distance x upstream of the measuring station, equal to the axial distance of the station from the nose of the test vehicle, and using the data of velocity and altitude.

(i) Taylor⁴ has concluded that a reasonable estimate of the turbulence thickness δ_T is equal to about 80% of the total boundary-layer thickness δ predicted by Spalding⁵, so that

$$R_{\delta_T} = 0.8 R_{\delta} .$$

The relationship between R_{δ} and R_x has also been defined by Spalding, and using this relationship together with the expression

$$\delta_T = \left(\frac{0.8 R_{\delta}}{R_x} \right) x$$

the values of δ_T were calculated for each run.

(j) The ratio λ_u / ϕ_T was then calculated for each run.

3.1 For runs 00 to 20, ($Ma = 0$ to $Ma \sim 1.2$) the signal/noise ratio of the recorded data was too small for the effective determination of a peak in the plot of spectral density against frequency factor. Under these conditions, a spurious peak imposed by the reading frequency was often apparent, at the lowest frequency used. This would lead, of course, to a false value of λ_u being obtained if this frequency was used in the calculation of the scale of the turbulence in the expression given in 3(g) above. For this reason, the values of the computed spectral density estimates for these runs have not been tabulated, or used in analysis.

The data for runs 21 to 24 inclusive have not been fully analysed because the signal/noise ratios, though higher than those for earlier runs, were still too low for really adequate identification of a significant peak. The data have been included in Table 1 merely for possible use in transonic studies.

The spectral density estimates for runs 21 to 49 are tabulated for a range of $k = 1$ to $k = 100$, in Table 1, and for runs 25 to 49 ($Ma \approx 1.5$ to $Ma \approx 2.2$) the values of the scale of the turbulence λ_u and of the ratio λ_u / ϕ_T were calculated. For runs 33 and 49, however, the spectral density plots show very low mean values and low signal/noise ratios, and in these cases the results have not been tabulated. It is suspected that these two runs were affected by transient telemetry faults, and immediately after run 49, the transmission of data effectively ceased.

The values of λ_u obtained are plotted against time in Fig.7a and against Mach number in Fig.7b. The positions of runs 33 and 49 are indicated, but no values plotted. Similarly, positions are indicated, but no values plotted, for runs 33 and 49 in the plots of λ_u / ϕ_T against time (Fig.10) and against Mach number (Fig.11).

Values of all the environmental experimental data and derived quantities are presented in Table 3, and the vehicle trajectory data in Figs.3, 4 and 5.

3.2 To reinforce the conclusions derived from the analyses described above, the computing technique was modified to accept the pressure readings for five consecutive runs and use them as the data for one run.

The readings for runs 41 to 45 were used as the data for this collective run, which thus contained about 30800 discrete readings, separated by intervals $\Delta t = 15.55$ microseconds. The mean velocity and mean altitude over the

considered time span were used to derive Reynolds numbers and hence σ_T , and the autocorrelation function, unsmoothed and smoothed spectral density estimates, scale ℓ_u , ratio ℓ_u/σ_T and root mean square pressure σ_p were obtained as for the individual runs.

The unsmoothed spectral density estimates are shown in graphical form in Figs.9 and 12c, and the experimental environmental data are given in Table 4.

4 DISCUSSION OF RESULTS

The pressure-transducer used to measure the wall pressure was carefully mounted and inspected to ensure that the measuring diaphragm was flush with the exterior skin of the vehicle. The diaphragm diameter was 0.125 inch (3.175 mm) and the maximum projection above the surrounding surface was 0.003 inch (0.076 mm).

It was arbitrarily assumed that pressure-fluctuations having a wavelength of less than ten times the diameter of the transducer diaphragm would be attenuated to such an extent that they should be regarded as being unreliable, i.e. the highest usable frequency of pressure-fluctuation is given by

$$f = \frac{v}{10d}$$

where v is the vehicle velocity and d is the diaphragm diameter. For the range of vehicle velocities considered (i.e. 1692 ft/s to 2444 ft/s) and the transducer diaphragm diameter of 0.01042 ft, the value of f lies in the range 16.24 kHz to 23.45 kHz, corresponding to values of $k = 51$ to $k = 73$. Since all the significant peaks in the plots of spectral density against k occur well below $k = 40$, it is assumed that errors due to spatial resolution of the transducer may be ignored.

The data obtained from the piezo-electric accelerometer (c) showed that although the vibration local to the piezo-electric pressure transducer was of considerable amplitude in the frequency-range 100 to 300 Hz, the amplitude at higher frequencies was very much smaller, and would have negligible effect on the accuracy of the pressure data.

4.1 Comparison of experimentally determined spectra with a theoretical spectrum for homogeneous isotropic turbulence

In Ref.3, Taylor tabulated a theoretical (pressure)² spectrum for very high Reynolds numbers, ($A = 0$) in a non-dimensional form, and this was used as a basis for comparison with the experimental spectra.

Figs.12a, b and c show respectively the experimental spectra determined for run 35 ($Ma = 2.060$), run 45 ($Ma = 2.188$) and the collective run described in section 3.2 (mean $Ma = 2.193$). The strong resemblance between these three spectra confirms a reasonably consistent state of turbulence throughout this portion of the available data.

The theoretical spectrum has been superimposed on each of these three figures. The full line indicates the spectrum so placed as to have its maximum value at the same frequency as that of the maximum experimental value, and to represent the same value of $(\sigma_p)^2$ as the experimental data. The broken line indicates the spectrum placed so as to obtain a good fit with the experimental points over the high frequency range, whilst retaining the same value of $(\sigma_p)^2$.

Although the test vehicle was subject to longitudinal acceleration ranging, over the time span considered, from $+22 g$ to $-4 g$, and could not therefore be regarded as being in a steady state and thus not directly comparable with the theoretical case, nevertheless a surprisingly good agreement was apparent. It may be noted that the collective run considered in Fig.12c was derived from data obtained during the time span centred on the 'zero g' point of the flight.

4.2 The scale of the turbulence, ℓ_u

The values of ℓ_u given in Tables 3 and 4 are based on the values of f_m indicated by the coincident peaks of the experimental points, and the full-line theoretical spectrum. If the values of ℓ_u were to be based on values of f_m given by the peak of the broken-line spectrum, they would be greater by a factor of about 2.6. The values of ℓ_u obtained from the two values of f_m for the three runs illustrated in Figs.12a, b and c are given below:

Run	(Full-line) $\ell_{u(f)}$	(Broken-line) $\ell_{u(b)}$	$\frac{\ell_{u(b)}}{\ell_{u(f)}}$
35	0.09849 ft (0.03002 m)	0.2616 ft (0.0797 m)	2.66
45	0.08953 ft (0.02729 m)	0.2266 ft (0.0691 m)	2.53
Collective	0.08970 ft (0.02734 m)	0.2266 ft (0.0691 m)	2.53

The true value of ℓ_u associated with the pressure fluctuations due entirely to turbulence probably lies somewhere between these two extremes.

The values of ℓ_u presented throughout this paper are obtained from calculations based on the frequency of the experimental peak values. However, since the flight of the test vehicle was of such short duration, it is possible that the turbulence was not fully established and that the values of ℓ_u given are somewhat low.

The plot of ℓ_u against elapsed time Fig.7a indicates a mean value of about 0.09 ft (27.4 mm), the individual points showing very little deviation in level after 3.3 seconds. Fig.7b, a plot of ℓ_u against Mach number, again shows very little deviation from the mean above a value of $Ma = 2$. As the value of $\$T$ is sensibly constant over the considered range of Mach number, the values of $\ell_u/\$T$ shown in Figs.10 and 11 also exhibit this constancy.

The mean value of ℓ_u for the runs 41 \rightarrow 45 is 0.09245 ft (28.2 mm) and ℓ_u for the collective run is 0.0897 ft (27.3 mm). The mean value of $\ell_u/\$T$ for the runs 41 \rightarrow 45 is 0.5275, and the value for the collective run is 0.5128. Therefore, the results obtained from the collective run may be assumed to be representative of those from the individual runs.

Taylor⁴ cites an example of experimental results derived from an investigation of turbulence at subsonic velocities, which are compared below with the results from the present investigation.

	Ma	$R_{\$T}$	$\$T$	$\$$	ℓ_u	$\frac{\ell_u}{\$T}$	$\frac{\sigma_p}{q}$
Taylor	0.176	3.8×10^4	0.328 ft (100 mm)	0.410 ft (125 mm)	0.0975 ft (29.7 mm)	0.297	0.0056
Shark	2.2	2×10^6	0.175 ft (53.3 mm)	0.219 ft (66.8 mm)	0.0897 ft (27.3 mm)	0.513	0.0045

It is seen that although the Mach number has increased from the first case to the second by a factor of 12.5, and the turbulence Reynolds number by a factor of 53, the scale of the turbulence has remained almost the same, and the ratio $\ell_u/\$T$ has increased only by a factor of 1.73.

5 CONCLUSIONS

Measurements of boundary-layer pressure fluctuations have been made for a short flight-time on a free-flight aerodynamic test vehicle at Mach numbers in the range 1.5 to 2.2, and local Reynolds numbers of 1.51 to 2.15×10^8 .

Autocorrelation functions and (pressure)² spectral densities have been determined and are presented in tabular and graphical form. Representative experimental spectra are compared with a theoretical spectrum for homogeneous isotropic turbulence and are found to be in reasonable agreement.

Values have been obtained of the scale of the turbulence, and the ratio of scale of the turbulence to turbulence boundary-layer thickness, and these are similarly presented. Over the considered range of $Ma = 1.5$ to $Ma = 2.2$, the scale of the turbulence was found to fluctuate between 0.47 and 0.76 of the turbulence boundary-layer thickness. At $Ma = 2.2$ the scale of the turbulence was about 0.51 of the turbulence boundary-layer thickness, and the root mean square pressure was 0.0045 of the free-stream dynamic pressure.

Table.1 page 1 of 8 pages

DENSITY OF (PRESSURE SQUARED X 10000) SPECTRUM WITH RESPECT TO FREQUENCY FACTOR "k". (100 PRESSURE UNITS = 0.3116 LB/SQ INCH)							
k	RUN NUMBER						
	21	22	23	24	25	26	27
1	499	366	404	623	269	245	257
2	490	454	338	207	265	331	482
3	662	551	469	164	311	482	710
4	539	399	387	176	187	262	614
5	431	535	525	184	418	464	677
6	394	529	430	217	353	324	676
7	352	317	436	202	381	294	252
8	425	345	432	340	336	449	636
9	412	400	351	241	608	465	1027
10	222	430	411	399	414	497	650
11	457	362	352	423	573	580	547
12	415	411	418	243	480	510	957
13	263	386	296	298	498	586	622
14	319	212	236	228	634	465	915
15	258	252	270	183	400	445	539
16	312	387	332	234	488	524	429
17	215	252	269	223	416	680	763
18	175	286	299	164	432	569	503
19	265	301	336	303	352	473	493
20	191	157	304	247	343	432	376
21	195	157	206	141	207	337	368
22	200	115	247	188	181	288	350
23	164	178	137	153	232	165	410
24	139	148	200	161	282	286	291
25	137	112	147	110	210	276	185
26	141	118	232	90	213	180	290
27	116	130	178	81	196	244	253
28	142	91	177	63	249	265	176
29	84	122	179	150	166	284	256
30	106	131	122	120	215	202	279
31	82	113	97	144	204	228	187
32	107	157	136	101	174	146	281
33	59	94	98	87	200	144	203
34	100	86	120	96	196	166	117
35	94	81	129	103	110	207	210
36	77	89	134	85	147	128	197
37	87	100	84	76	135	219	178
38	53	95	80	55	140	190	191
39	59	63	104	71	165	125	158
40	74	76	115	73	144	129	170
41	51	72	58	84	81	119	127
42	55	98	87	80	99	187	133
43	45	69	85	38	99	167	136
44	59	46	75	64	135	112	156
45	50	46	51	69	89	96	144
46	57	57	74	43	83	168	134
47	33	45	68	43	88	133	165
48	49	52	56	45	83	72	134
49	39	53	55	54	61	85	147
50	51	37	58	41	74	98	99

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DENSITY OF (PRESSURE SQUARED X 10000) SPECTRUM WITH RESPECT TO FREQUENCY FACTOR *k*. (100 PRESSURE UNITS = 0.3116 LB/SQ INCH)							
k	RUN NUMBER						
	21	22	23	24	25	26	27
51	38	44	41	41	75	87	99
52	39	31	46	37	59	67	86
53	40	33	52	33	54	72	110
54	29	25	34	42	62	56	64
55	22	29	57	28	67	52	109
56	32	51	50	28	72	86	63
57	19	23	29	48	43	40	72
58	32	27	45	36	62	80	74
59	21	22	44	35	52	73	71
60	28	33	46	20	67	53	78
61	15	30	31	23	59	51	95
62	25	25	28	21	50	48	71
63	24	38	34	28	45	43	69
64	14	17	28	18	40	31	58
65	18	14	25	23	49	38	51
66	17	13	28	31	41	32	46
67	14	14	19	20	51	51	76
68	17	19	25	24	31	31	73
69	17	20	18	23	36	38	51
70	15	19	20	20	51	34	59
71	17	19	22	27	35	27	42
72	14	16	26	16	28	40	56
73	8	15	18	16	26	37	41
74	15	13	21	16	28	19	46
75	13	12	18	21	25	27	63
76	13	9	23	13	26	25	38
77	10	13	19	16	39	34	46
78	13	18	19	17	40	28	60
79	12	11	13	20	30	23	44
80	14	16	17	18	33	25	30
81	8	9	10	18	23	21	36
82	14	20	21	21	39	30	38
83	8	5	9	17	27	20	52
84	86	69	104	63	107	119	117
85	19	21	27	27	50	56	91
86	12	9	25	23	47	74	101
87	17	32	45	37	64	44	37
88	30	21	33	21	34	23	34
89	22	16	18	15	29	20	32
90	15	13	17	8	27	19	37
91	15	17	17	16	26	27	27
92	10	9	19	11	28	18	23
93	17	14	12	13	16	19	29
94	12	8	8	16	19	21	23
95	8	5	10	15	22	20	27
96	9	6	7	7	17	14	24
97	7	8	9	10	24	12	28
98	7	8	9	8	27	15	33
99	6	10	11	11	28	20	22
100	5	10	12	7	30	15	26

Table.1. Page 3 of 8 pages

DENSITY OF (PRESSURE SQUARED X 10000) SPECTRUM WITH RESPECT TO FREQUENCY FACTOR "K". (100 PRESSURE UNITS = 0.3116 LB/SQ INCH)							
k	RUN NUMBER						
	28	29	30	31	32	33	34
1	397	101	905	957	1530	498	436
2	1158	1693	2579	3036	2984	481	1131
3	1104	1829	1562	4379	4914	640	1282
4	1603	1761	2080	3574	2651	533	1608
5	1063	1655	2665	3650	4159	438	1275
6	866	1942	2198	2209	2526	398	857
7	971	1156	1762	2877	2592	332	1396
8	887	725	1685	2880	2450	411	1149
9	970	1216	1636	1860	2059	406	1447
10	820	1164	1468	2141	3183	219	1431
11	1225	1267	1880	1997	2214	438	1730
12	1020	1748	3627	2353	3039	398	2953
13	1264	1611	4036	4051	2787	255	2602
14	700	2026	2042	3147	3739	302	2031
15	780	1020	1595	2386	2321	252	3005
16	596	1269	1379	2379	2637	311	1248
17	1035	1337	2166	3945	3878	227	4176
18	525	1039	1742	2148	4427	177	4351
19	569	716	1247	2009	5747	254	4124
20	541	535	645	2313	3993	175	5093
21	425	493	670	1060	2837	188	3471
22	391	527	623	842	1443	192	3716
23	259	334	523	778	1029	165	2531
24	343	483	516	733	982	138	1524
25	403	400	703	728	941	137	1102
26	295	539	630	459	678	139	1172
27	493	414	432	795	851	114	996
28	271	395	442	518	663	143	867
29	312	325	446	587	488	84	441
30	440	449	298	500	490	100	711
31	325	365	379	700	523	80	679
32	233	270	384	589	472	101	730
33	192	269	421	456	588	61	518
34	254	311	531	402	434	93	577
35	237	335	409	419	451	97	548
36	221	220	311	476	695	71	546
37	232	237	373	516	477	81	536
38	235	350	259	416	472	52	419
39	173	282	375	415	418	56	424
40	241	154	337	350	456	75	494
41	183	244	162	336	290	50	355
42	191	111	250	341	410	56	326
43	148	198	320	262	268	43	361
44	141	127	261	307	451	57	321
45	75	159	238	275	344	44	385
46	191	196	157	206	220	59	350
47	122	147	219	219	362	38	303
48	121	167	265	293	250	47	318
49	115	174	198	244	346	38	360
50	129	151	167	275	303	47	297

Table 1. Page 4 of 8 pages

DENSITY OF (PRESSURE SQUARED X 10000) SPECTRUM WITH RESPECT TO FREQUENCY FACTOR "k". (100 PRESSURE UNITS = 0.3116 LB/SQ INCH)							
k	RUN NUMBER						
	28	29	30	31	32	33	34
51	120	123	184	210	196	39	265
52	141	164	140	195	230	38	239
53	119	115	174	171	244	39	172
54	117	142	113	159	221	30	224
55	123	93	163	166	162	22	203
56	87	90	122	121	175	30	182
57	76	109	100	165	275	21	215
58	86	112	140	134	213	31	218
59	91	106	107	174	153	23	213
60	61	104	93	82	217	29	172
61	84	98	94	145	173	14	245
62	79	72	136	184	122	21	278
63	50	81	78	108	136	25	119
64	57	60	120	125	110	13	199
65	66	53	116	86	151	17	154
66	54	64	108	109	135	18	160
67	61	53	86	90	125	14	171
68	71	58	87	136	129	17	276
69	64	48	90	125	118	16	193
70	33	65	87	118	131	14	169
71	51	57	87	116	128	17	168
72	56	54	64	75	136	16	215
73	42	57	63	96	117	8	157
74	40	60	59	66	80	15	165
75	50	61	67	107	121	9	131
76	35	38	60	117	85	14	170
77	31	44	62	81	148	11	95
78	46	52	62	119	112	16	107
79	43	44	66	109	135	9	112
80	24	55	62	87	83	14	128
81	38	51	74	101	107	7	66
82	47	66	95	86	107	15	80
83	39	60	68	88	80	8	85
84	134	180	124	135	172	78	178
85	110	89	64	80	82	19	98
86	43	25	57	49	72	11	104
87	35	43	43	80	103	18	79
88	31	22	43	60	88	26	87
89	36	44	41	66	88	16	75
90	23	28	43	51	70	14	97
91	30	34	48	70	73	11	67
92	28	31	37	51	69	11	68
93	29	30	49	47	69	15	89
94	20	23	41	51	69	9	60
95	34	36	32	58	62	7	62
96	18	30	30	50	71	6	74
97	28	22	45	61	76	7	74
98	19	21	47	57	72	7	86
99	29	42	34	54	73	8	69
100	13	27	34	62	78	7	57

Table 1. Page 5 of 8 pages

DENSITY OF (PRESSURE SQUARED X 10000) SPECTRUM WITH RESPECT TO FREQUENCY FACTOR "k". (100 PRESSURE UNITS = 0.3116 LB/SQ INCH)							
k	RUN NUMBER						
	35	36	37	38	39	40	41
1	660	825	680	907	864	1078	1103
2	944	819	770	1010	775	1643	988
3	1911	876	826	997	1022	1251	1649
4	1559	884	1011	985	999	1336	1334
5	1002	1579	895	748	983	1289	1104
6	1150	1225	897	922	1154	1258	1307
7	1345	1239	1003	900	1111	1615	1358
8	1606	1435	1459	1159	1141	941	916
9	1292	1448	1199	1565	1194	1200	1432
10	1792	1864	1654	1169	1090	991	1424
11	1383	2313	1924	1206	1211	1598	1172
12	2452	3188	2289	1353	1611	1304	1414
13	1833	1244	1923	2091	2294	1936	1614
14	1832	2154	1756	1684	1927	1855	1101
15	1324	1594	1568	1883	1433	1523	1320
16	1896	2375	2080	1847	1879	1361	987
17	3424	2230	2320	3054	2085	2360	2748
18	4458	4492	3578	2569	2852	2537	2012
19	5602	7948	3125	3860	3182	4125	3320
20	4344	4831	4231	4145	3211	4379	2931
21	4829	5198	3901	3905	3938	4021	3771
22	3659	2731	4252	3466	3818	3242	3992
23	3321	2671	3878	2847	2951	3252	1953
24	2026	2235	2466	1648	2880	1814	2083
25	1197	1695	1427	1661	1703	1929	2113
26	1067	1018	1503	1807	944	1162	1406
27	1104	1126	1390	1052	1358	1356	1051
28	939	755	784	844	853	699	1118
29	797	855	989	709	941	725	1075
30	620	882	664	879	666	800	901
31	803	829	873	840	911	716	1091
32	689	468	619	901	775	687	905
33	734	631	847	564	560	593	769
34	436	535	459	943	550	782	483
35	638	743	851	673	802	328	573
36	664	394	499	539	655	711	674
37	553	583	467	482	576	571	577
38	415	578	464	405	414	499	539
39	585	710	498	546	480	596	364
40	423	507	466	573	428	417	628
41	566	519	586	320	345	541	348
42	393	710	435	444	447	492	435
43	390	645	424	559	353	446	645
44	385	363	466	408	422	493	580
45	485	489	526	483	424	379	511
46	363	294	396	395	287	428	368
47	352	299	473	406	397	366	513
48	364	275	492	301	417	288	307
49	376	273	459	383	425	387	301
50	372	284	288	342	289	413	471

Table 1. Page 6 of 8 pages

DENSITY OF (PRESSURE SQUARED X 10000) SPECTRUM WITH RESPECT TO FREQUENCY FACTOR "K". (100 PRESSURE UNITS = 0.3116 LB/SQ INCH)							
K	RUN NUMBER						
	35	36	37	38	39	40	41
51	238	357	403	347	269	284	372
52	357	295	296	288	308	224	241
53	247	278	346	321	256	226	276
54	348	275	401	231	314	177	261
55	232	270	274	225	255	256	240
56	252	227	233	280	270	226	281
57	263	284	211	295	243	125	256
58	178	191	174	227	198	202	293
59	302	348	304	364	263	270	341
60	177	262	263	215	182	207	245
61	183	308	270	213	191	221	215
62	268	405	174	193	193	181	250
63	191	292	257	181	186	188	254
64	269	243	218	180	218	239	193
65	174	221	213	201	216	132	211
66	172	252	158	263	178	175	191
67	280	239	173	170	183	160	247
68	166	204	202	140	176	185	236
69	184	109	142	148	181	170	207
70	217	162	180	80	164	150	145
71	164	135	99	170	208	174	201
72	181	112	109	167	154	151	198
73	155	124	158	147	177	138	197
74	154	183	130	155	132	119	197
75	155	108	96	140	182	182	123
76	159	141	155	123	147	110	129
77	133	103	102	118	122	152	112
78	115	78	137	137	164	130	141
79	100	66	121	122	108	87	137
80	101	97	107	110	140	112	108
81	129	124	111	111	117	99	70
82	125	84	109	138	90	158	109
83	87	108	73	127	99	96	104
84	170	173	184	142	190	170	197
85	119	102	92	74	134	97	149
86	67	86	106	111	89	92	136
87	97	110	111	86	89	112	142
88	89	71	105	69	107	117	98
89	118	87	86	96	93	89	110
90	80	69	92	81	95	104	110
91	83	85	98	98	85	61	97
92	69	51	95	85	74	150	118
93	54	69	120	103	79	106	113
94	92	80	73	76	94	96	82
95	87	98	93	72	81	89	71
96	83	84	80	56	88	127	83
97	77	101	98	84	105	119	85
98	46	65	89	65	110	123	112
99	95	79	92	80	103	88	102
100	78	107	104	97	64	79	127

Table.1. Page 7 of 8 pages

DENSITY OF (PRESSURE SQUARED X 10000) SPECTRUM WITH RESPECT TO FREQUENCY FACTOR "k". (100 PRESSURE UNITS = 0.3116 LB/SQ INCH)								
k	RUN NUMBER							
	42	43	44	45	46	47	48	49
1	1457	1003	1019	935	4251	2266	41520	5252
2	1495	1180	1308	2115	1696	1647	7619	1921
3	1582	1887	1903	2018	1559	2393	1092	1828
4	1660	1154	1487	1319	1598	1151	1902	1613
5	1675	1194	1623	1571	1751	1751	1482	1365
6	1610	1745	1223	1608	1715	1622	2034	1375
7	1495	1322	1717	1434	1538	1322	1563	1319
8	1392	1206	1624	1635	1875	1669	1157	1548
9	1353	1898	1211	1340	1183	1617	1711	1066
10	1390	1570	1371	1690	1639	1558	1500	1851
11	1472	1491	1407	1354	1404	2166	1424	1852
12	1552	1577	2621	1401	1884	1737	2463	1705
13	1602	1543	1755	1680	1485	1395	1394	1128
14	1640	1301	1408	1940	1668	1511	1464	913
15	1724	1566	1599	948	977	1594	1203	1166
16	1918	1465	1353	1487	1460	1730	955	1175
17	2242	2634	1762	2193	1776	2355	1489	1212
18	2652	2768	2254	2338	1882	2438	2033	1216
19	3041	2480	3266	1579	2966	1967	1821	1487
20	3284	2577	3479	2847	3294	3443	1939	1214
21	3289	3653	4188	2884	3930	2845	3104	597
22	3043	3342	3693	3799	3268	2646	2272	837
23	2610	1936	2262	1938	2194	1584	1202	1062
24	2108	2416	2111	1899	1174	1442	1360	891
25	1652	1528	1626	1610	1431	1345	1237	1145
26	1319	1415	1164	1152	1333	1211	1026	984
27	1122	1389	917	997	1021	801	1172	608
28	1029	883	1036	812	738	690	480	799
29	983	775	662	918	712	710	601	921
30	938	634	857	708	604	815	516	722
31	874	713	530	722	607	853	788	889
32	796	1002	624	444	760	534	436	989
33	722	589	619	536	540	713	568	504
34	670	689	668	290	605	673	565	1110
35	649	478	720	711	642	577	613	616
36	652	557	639	740	367	508	545	799
37	667	559	424	643	443	676	554	589
38	679	649	453	710	406	619	710	713
39	675	646	444	495	401	513	559	693
40	649	620	519	415	513	664	555	599
41	603	424	486	466	710	444	633	690
42	546	496	343	504	333	443	505	938
43	494	512	535	598	385	528	662	586
44	459	449	633	538	594	684	604	676
45	449	491	534	288	409	695	549	1063
46	459	453	364	441	454	514	458	537
47	479	400	665	359	556	734	441	745
48	492	490	515	489	372	503	504	752
49	492	379	458	474	389	407	658	634
50	479	512	450	541	504	620	421	630

Table.1. Page 8 of 8 pages

DENSITY OF (PRESSURE SQUARED X 10000) SPECTRUM WITH RESPECT TO FREQUENCY FACTOR "k". (100 PRESSURE UNITS = 0.3116 LB/SQ INCH)								
k	RUN NUMBER							
	42	43	44	45	46	47	48	49
51	464	410	485	774	685	641	590	671
52	459	539	513	563	346	459	570	880
53	466	350	451	423	450	719	628	619
54	477	410	463	511	568	774	583	651
55	479	408	464	556	745	863	548	806
56	460	254	417	437	403	711	601	655
57	422	339	551	559	519	398	541	556
58	376	323	333	360	558	613	508	705
59	345	298	381	472	430	551	413	563
60	340	292	290	289	326	504	352	586
61	362	242	224	282	346	600	447	580
62	393	254	235	338	272	555	277	493
63	412	264	273	334	282	421	313	934
64	404	271	282	233	258	458	315	536
65	371	275	217	191	204	327	347	483
66	328	279	155	176	212	329	324	510
67	294	174	198	196	233	341	250	348
68	281	228	176	193	275	359	316	364
69	287	223	184	155	148	257	231	630
70	298	134	165	203	183	372	286	420
71	299	124	203	173	168	285	291	407
72	285	139	163	129	174	310	217	414
73	263	164	214	148	207	333	317	574
74	247	142	196	121	138	295	244	539
75	245	144	111	131	158	351	166	629
76	258	106	83	115	175	243	132	711
77	275	139	153	136	136	299	281	439
78	283	151	173	129	111	264	380	523
79	277	150	133	129	108	247	241	455
80	264	188	83	96	121	397	265	463
81	255	117	170	116	135	288	297	390
82	262	119	207	120	153	259	131	567
83	285	168	81	101	97	239	291	455
84	310	206	248	203	165	293	305	471
85	322	143	122	106	120	226	227	428
86	311	130	93	80	110	260	180	422
87	282	129	101	99	111	199	292	493
88	249	145	116	73	123	263	231	467
89	230	126	119	121	86	246	244	270
90	233	116	150	86	72	247	242	622
91	252	123	110	120	93	204	293	484
92	270	80	162	99	105	245	247	452
93	274	113	112	94	112	261	262	403
94	259	85	75	73	110	351	297	607
95	235	103	115	89	98	215	278	458
96	217	111	143	79	110	169	221	327
97	215	129	67	116	107	312	180	344
98	229	105	109	86	91	261	231	468
99	247	106	92	110	105	271	268	352
100	255	79	111	131	148	293	149	488

Table.2
Frequency and corresponding frequency factor 'k'.

k	FREQUENCY (Hz)	k	FREQUENCY (Hz)	k	FREQUENCY (Hz)	k	FREQUENCY (Hz)
1	0.00	26	8119.77	51	16239.55	76	24359.32
2	324.79	27	8444.57	52	16564.34	77	24684.12
3	649.58	28	8769.36	53	16889.13	78	25008.91
4	974.37	29	9094.15	54	17213.92	79	25333.70
5	1299.16	30	9418.94	55	17538.71	80	25658.49
6	1623.95	31	9743.73	56	17863.50	81	25983.28
7	1948.75	32	10068.52	57	18188.30	82	26308.07
8	2273.54	33	10393.31	58	18513.09	83	26632.86
9	2598.33	34	10718.10	59	18837.88	84	26957.65
10	2923.12	35	11042.89	60	19162.67	85	27282.44
11	3247.91	36	11367.68	61	19487.46	86	27607.23
12	3572.70	37	11692.48	62	19812.25	87	27932.03
13	3897.49	38	12017.27	63	20137.04	88	28256.82
14	4222.28	39	12342.06	64	20461.83	89	28581.61
15	4547.07	40	12666.85	65	20786.62	90	28906.40
16	4871.86	41	12991.64	66	21111.41	91	29231.19
17	5196.66	42	13316.43	67	21436.21	92	29555.98
18	5521.45	43	13641.22	68	21761.00	93	29880.77
19	5846.24	44	13966.01	69	22085.79	94	30205.56
20	6171.03	45	14290.80	70	22410.58	95	30530.35
21	6495.82	46	14615.59	71	22735.37	96	30855.14
22	6820.61	47	14940.39	72	23060.16	97	31179.94
23	7145.40	48	15265.18	73	23384.95	98	31504.73
24	7470.19	49	15589.97	74	23709.74	99	31829.52
25	7794.98	50	15914.76	75	24034.53	100	32154.31

Table 3

RUN NO.	VELOCITY (ft/s)	HEIGHT (ft)	MACH NO.	R _X (X10 ⁻⁶)	R _T (X10 ⁻⁶)
25	1692	470	1.5168	151.478	1.934
26	1762	504	1.5797	157.613	2.005
27	1830	538	1.6409	163.559	2.074
28	1899	572	1.7030	169.584	2.143
29	1965	606	1.7624	175.331	2.209
30	2030	640	1.8209	180.980	2.274
31	2091	682	1.8759	186.226	2.334
32	2149	726	1.9282	191.185	2.390
33	2202	766	1.9761	195.709	2.442
34	2241	800	2.0113	199.010	2.479
35	2295	850	2.0601	203.556	2.531
36	2333	868	2.0944	206.836	2.568
37	2365	944	2.1237	209.284	2.595
38	2392	976	2.1482	211.509	2.620
39	2413	1018	2.1674	213.148	2.639
40	2429	1060	2.1821	214.342	2.652
41	2439	1104	2.1914	214.995	2.660
42	2444	1148	2.1962	215.206	2.662
43	2444	1192	2.1966	214.976	2.659
44	2440	1236	2.1933	214.396	2.653
45	2434	1280	2.1883	213.642	2.644
46	2426	1324	2.1814	212.713	2.634
47	2418	1368	2.1746	211.787	2.623
48	2409	1412	2.1668	210.775	2.612
49	2397	1456	2.1564	209.503	2.598

RUN NO.	R.M.S. PRESSURE (lbf/in ²)	SCALE/\$ _T	SCALE _L (ft)	S.D. MAX	k _m	\$ _T (ft)	RUN NO.
25	0.0964	0.5559	0.10054	0.06335	14	0.181	25
26	0.1015	0.4720	0.08506	0.06800	17	0.180	26
27	0.1148	0.6056	0.10873	0.09150	14	0.180	27
28	0.1332	0.6825	0.12223	0.12635	13	0.179	28
29	0.1544	0.6541	0.11676	0.20259	14	0.178	29
30	0.1842	0.7342	0.13067	0.40361	13	0.178	30
31	0.2152	0.7578	0.13459	0.40512	13	0.178	31
32	0.2355	0.5207	0.09222	0.57465	19	0.177	32
33	0.0835	*	*	*	*	0.177	33
34	0.2184	0.5163	0.09111	0.50926	20	0.176	34
35	0.2232	0.5592	0.09849	0.56015	19	0.176	35
36	0.2269	0.5693	0.10012	0.79481	19	0.176	36
37	0.2171	0.4952	0.08699	0.42521	22	0.176	37
38	0.2112	0.5541	0.09725	0.41451	20	0.175	38
39	0.2094	0.5314	0.09319	0.39385	21	0.175	39
40	0.2126	0.5634	0.09875	0.43791	20	0.175	40
41	0.2100	0.5119	0.08971	0.39923	22	0.175	41
42	0.2261	0.5387	0.09439	0.32889	21	0.175	42
43	0.2130	0.5386	0.09439	0.36527	21	0.175	43
44	0.2167	0.5376	0.09424	0.41882	21	0.175	44
45	0.2111	0.5106	0.08953	0.37992	22	0.175	45
46	0.2146	0.5342	0.09370	0.39302	21	0.175	46
47	0.2321	0.5602	0.09830	0.34427	20	0.175	47
48	0.2475	0.5307	0.09303	0.31045	21	0.176	48
49	0.2229	*	*	*	*	0.176	49

* REFER TO SECTION 3.1

Table.4
Environmental data and derived quantities for collective run.

	IMPERIAL UNITS	S. I. UNITS
MEAN VELOCITY	2440 ft/s	743.7 m/s
MEAN HEIGHT	1192 ft	363.3 m
MEAN MACH NUMBER	2.193	
$R_X (X10^{-6})$	214.64	
$R_{\$T} (X10^{-6})$	2.656	
$\$T$	0.175 ft	0.053 m
k_m	22	
S.D. _{MAX}	0.3562	
SCALE	0.0897 ft	0.027 m
SCALE/ $\$T$	0.5128	
R.M.S. PRESSURE	0.2153 lbf/in ²	1.4844 kN/m ²

SYMBOLS

A	non-dimensional viscosity function
LU	ℓ_u/a
Ma	Mach number
M	number of correlated intervals used
N	number of intervals in one run
R_δ	boundary-layer Reynolds number
R_{δ_T}	turbulence Reynolds number
R_x	local Reynolds number at distance x from the effective start of turbulence
R_τ	autocorrelation function for interval τ
U	stream velocity relative to measuring station
V_k	} unsmoothed spectral density estimate at frequency f_k
SD_k	
SD_{max}	maximum value of unsmoothed spectral density estimate for one run
W_k	hanning-smoothed spectral density estimate at frequency f_k
X	ωa
Z_k	hamming-smoothed spectral density estimate at frequency f_k
a	a length to make wave numbers non-dimensional
f_k	frequency corresponding to frequency factor k (see Table 2)
f_m	frequency at which the maximum spectral density occurs
k	integer included between 1 and $(M + 1)$; the frequency factor corresponding to frequency f_k
k_m	frequency factor corresponding to f_m
ℓ_u	the scale of the turbulence
Δ_t	time interval separating each discrete value y_t from the next
y_t	recorded discrete pressure value at time t , referred to the mean value for that run
δ	total boundary-layer thickness
δ_T	turbulence thickness

SYMBOLS (Contd)

τ	integer included between 1 and (M + 1)
ω	wave number = $2\pi/\text{wavelength}$
σ_p	root mean square pressure
q	free stream dynamic pressure

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|------------|---|--|
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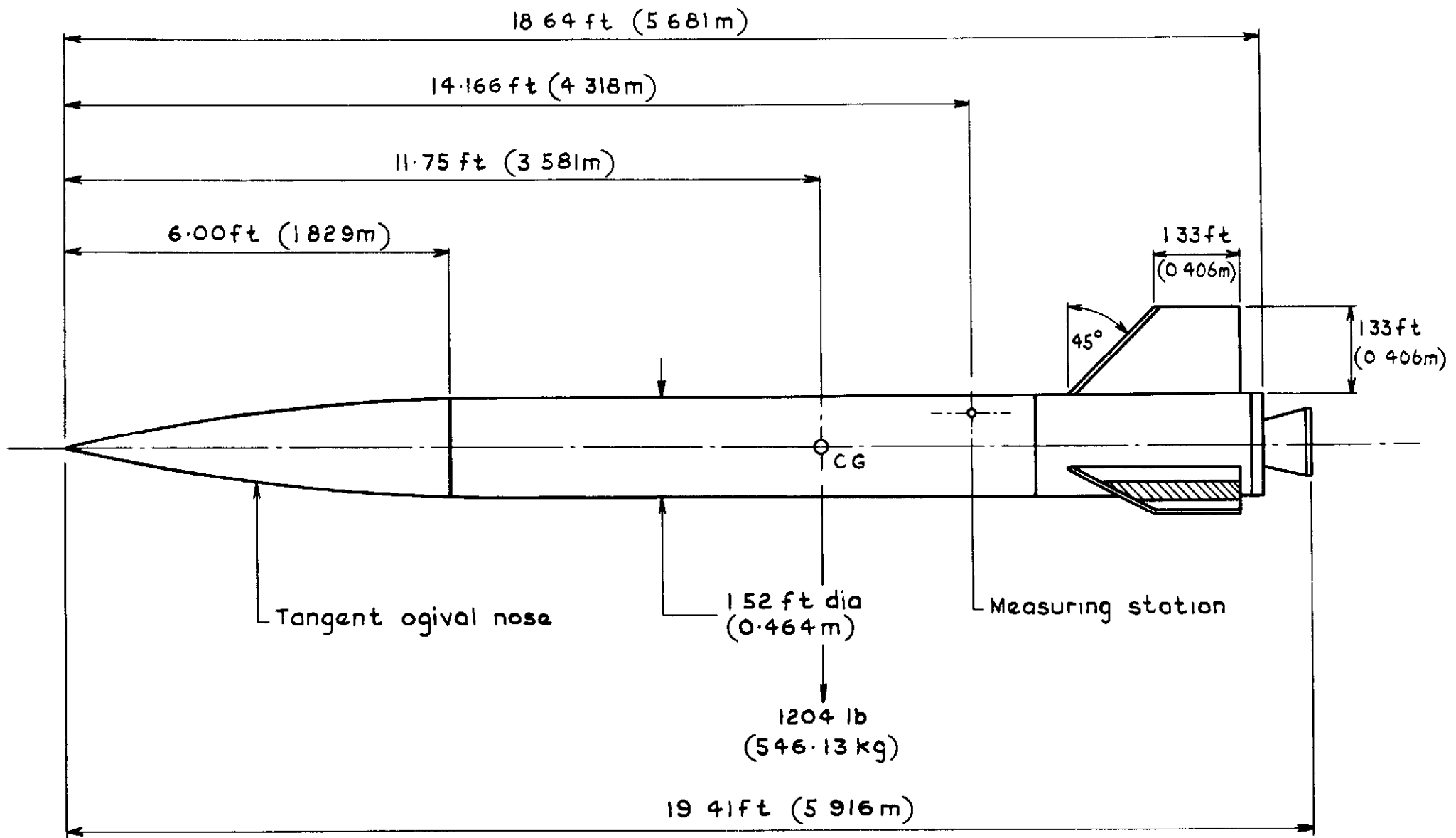


Fig. 1a General arrangement of 'Shark 1'

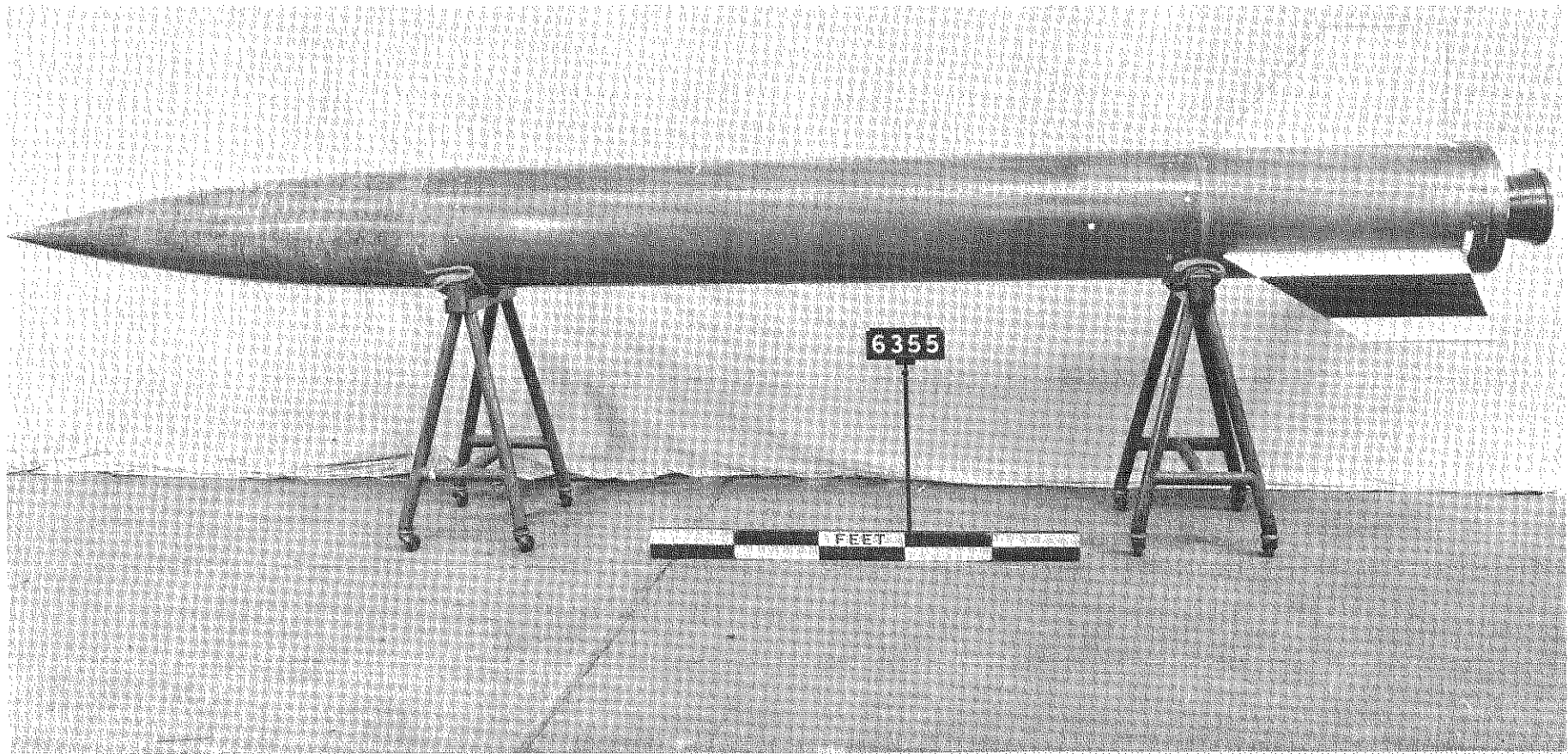


Fig1(b) Side view photograph of 'Shark 1'

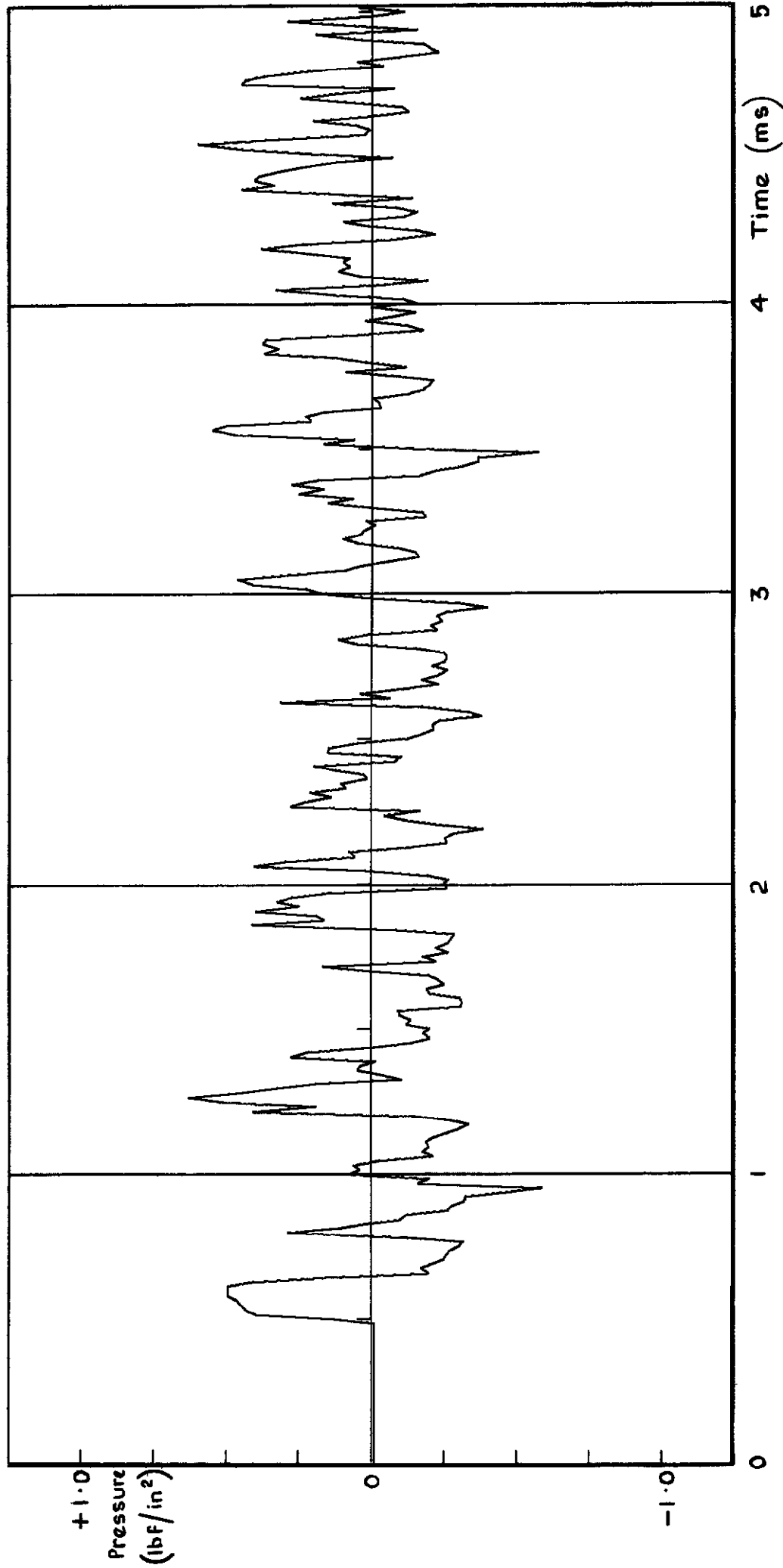


Fig 2a Plot of pressure against time for the first 5 milliseconds of run 32

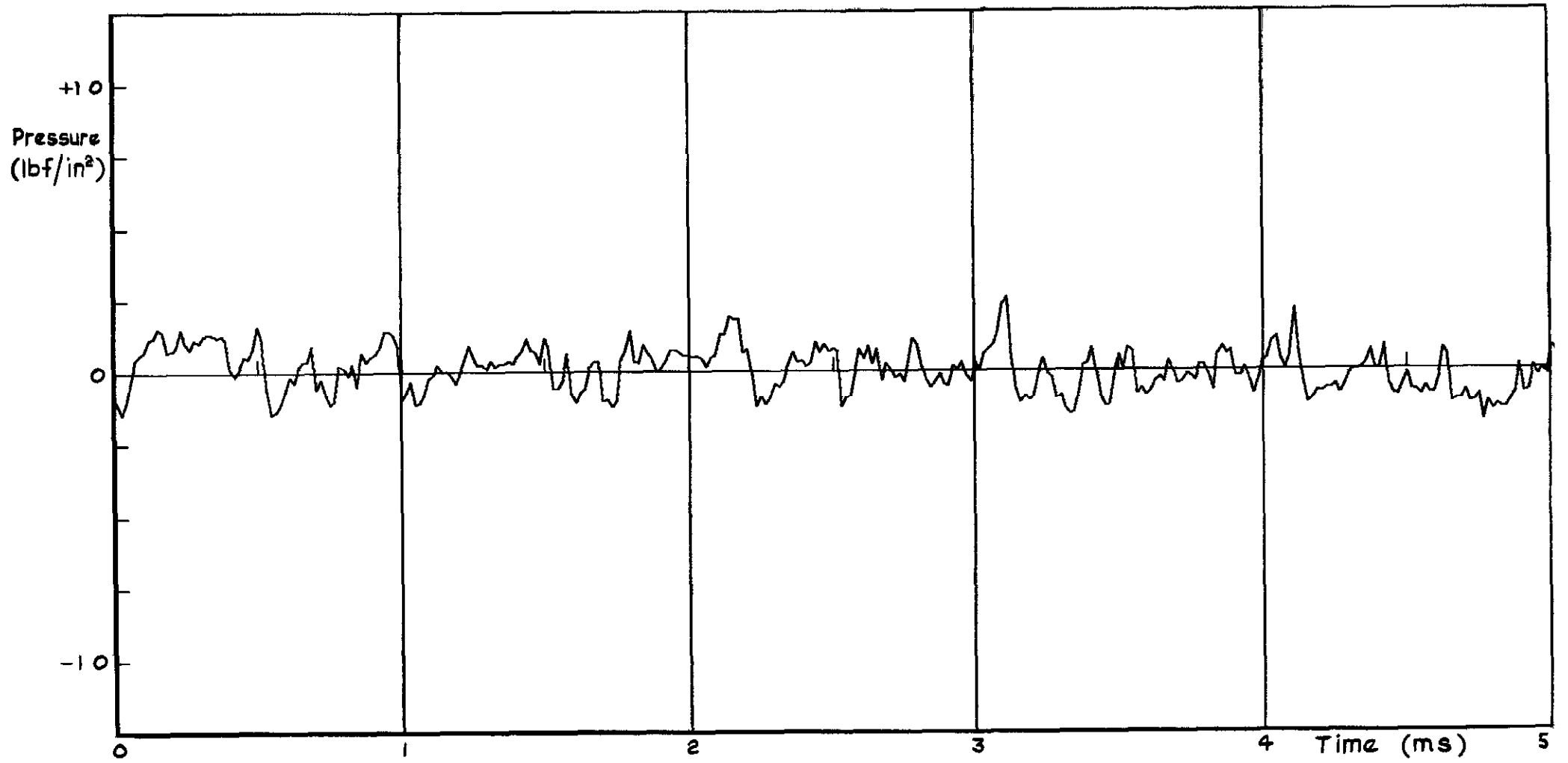


Fig.2b Plot of pressure against time for the first 5 milliseconds of run 33

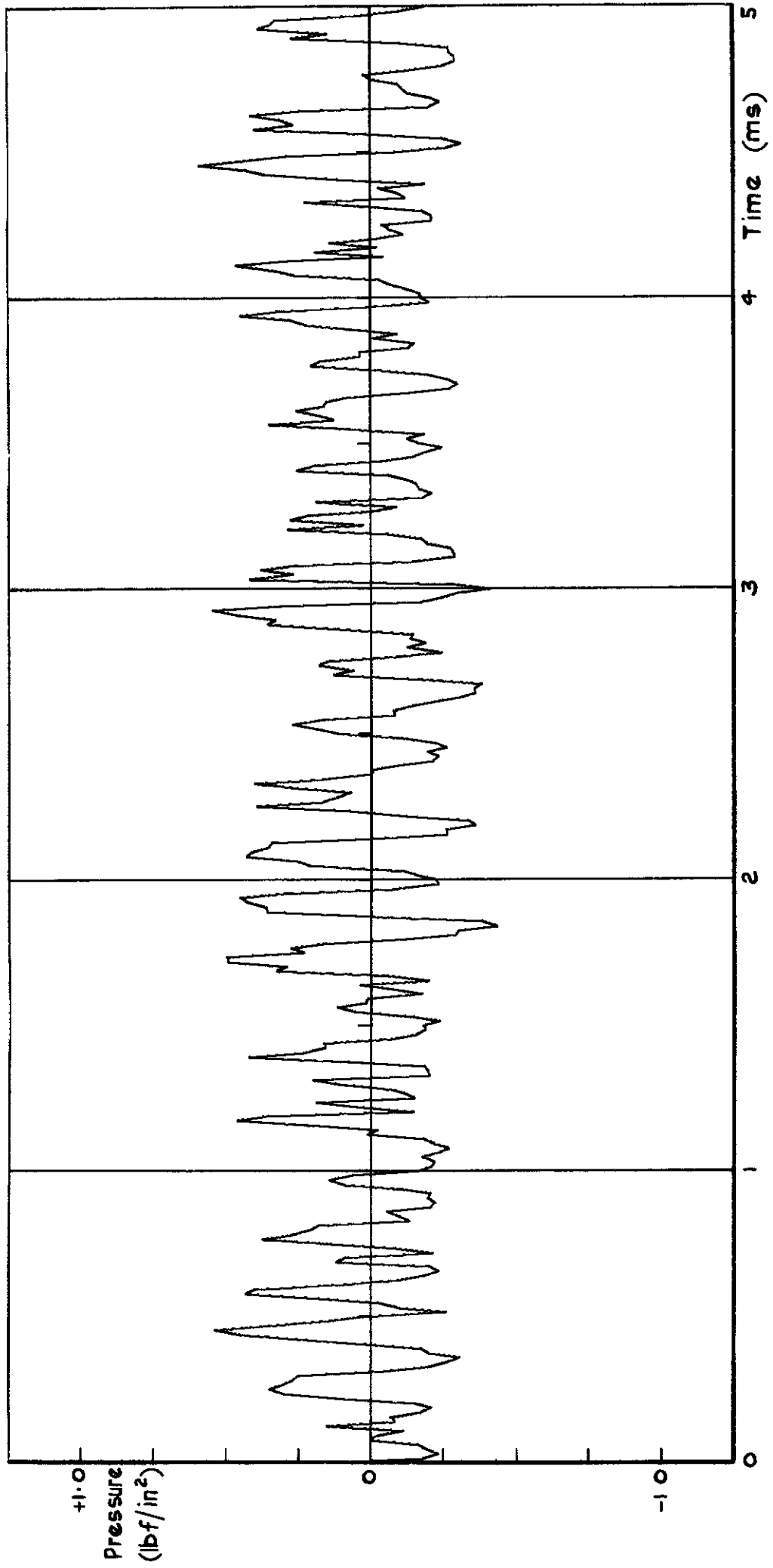


Fig.2c Plot of pressure against time for the first 5 milliseconds of run 34

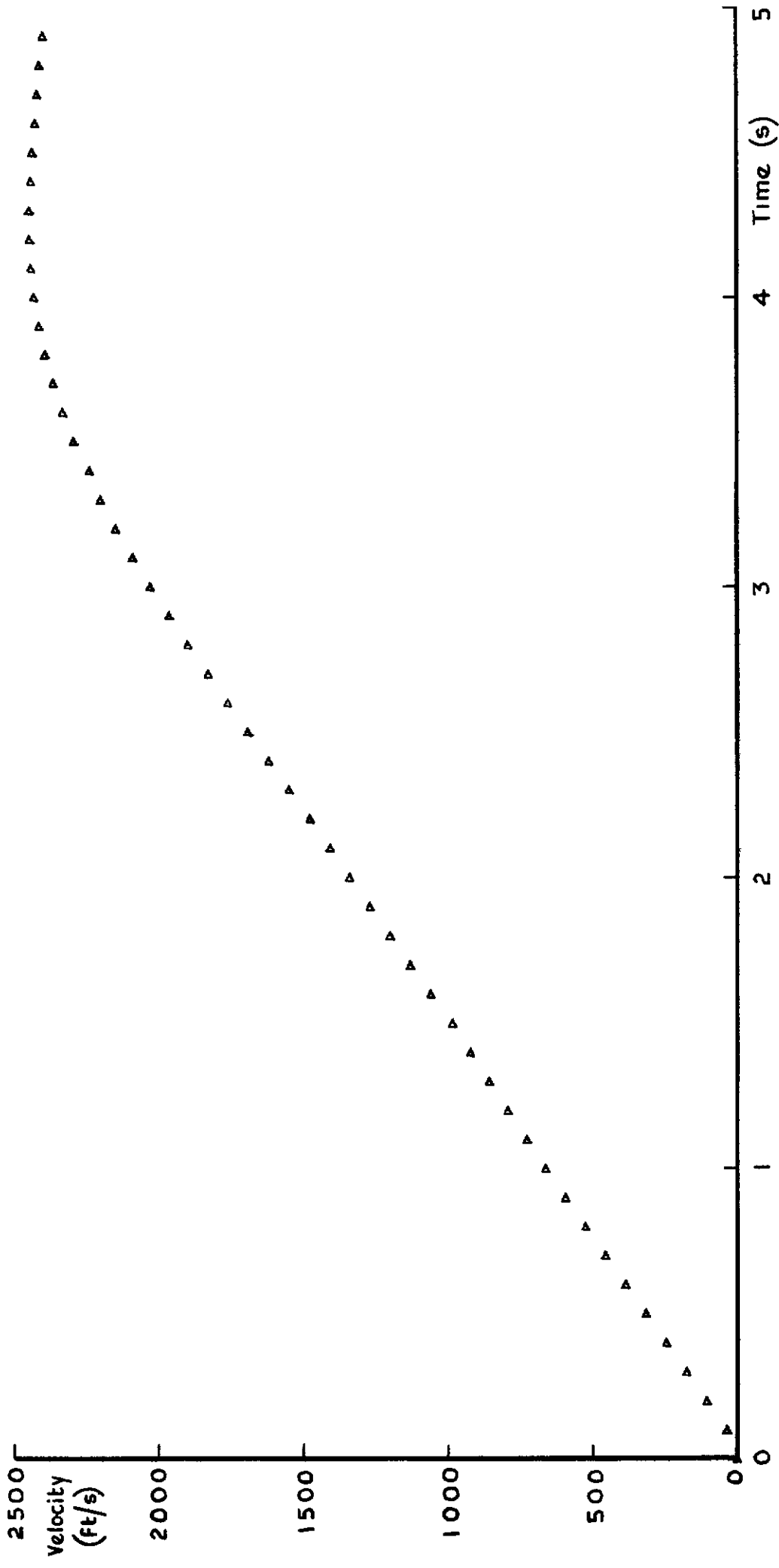


Fig.3 Plot of vehicle velocity/elapsed time

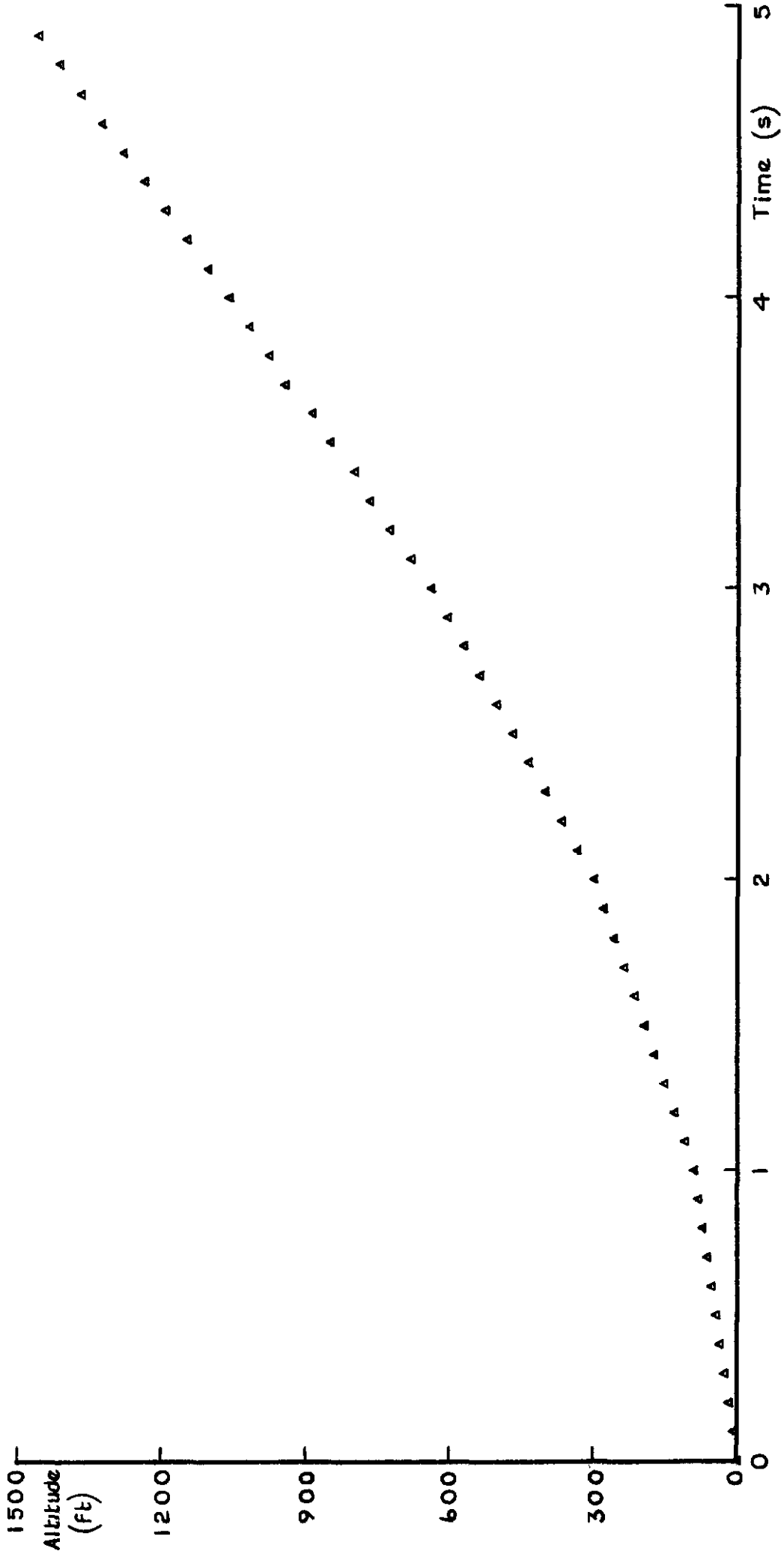


Fig.4 Vehicle altitude/elapsed time

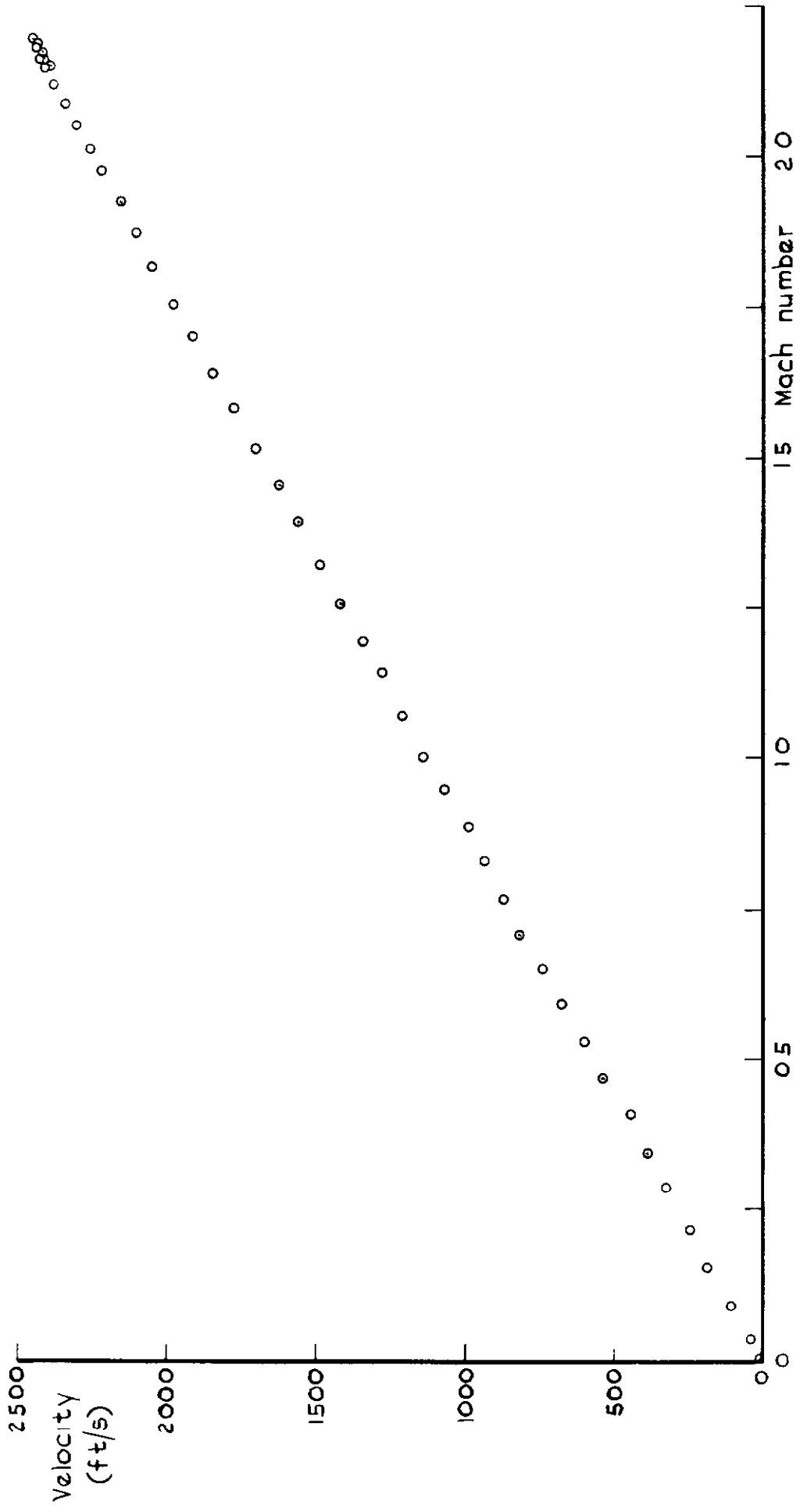


Fig 5 Plot of vehicle velocity/Mach number

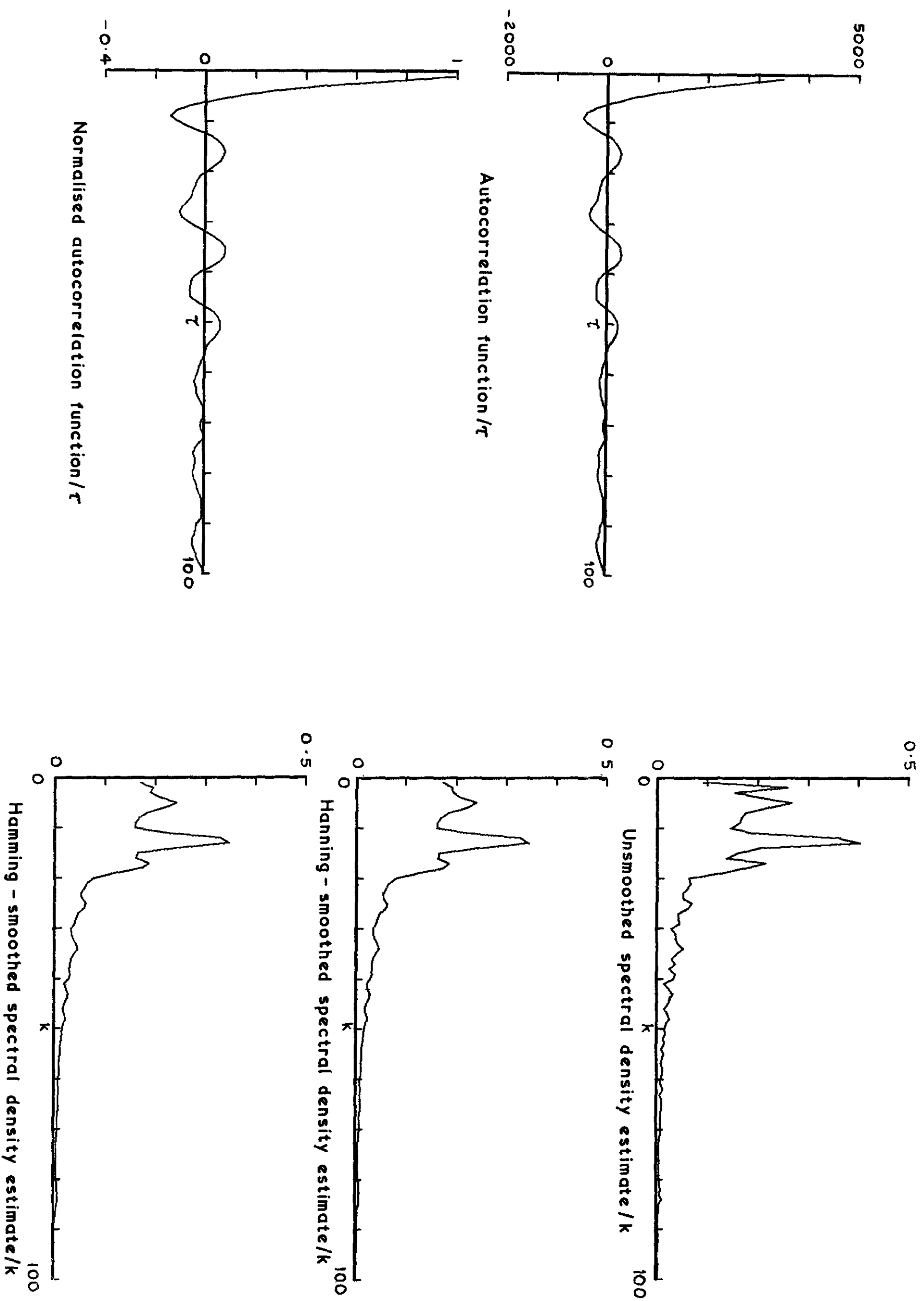
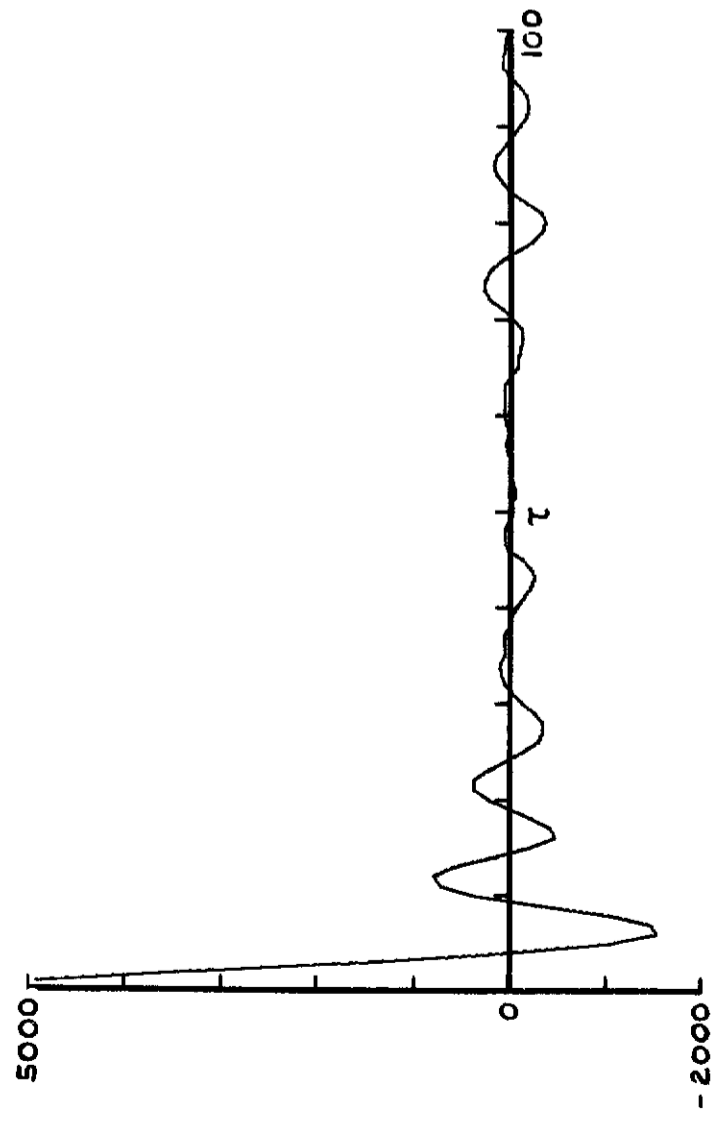
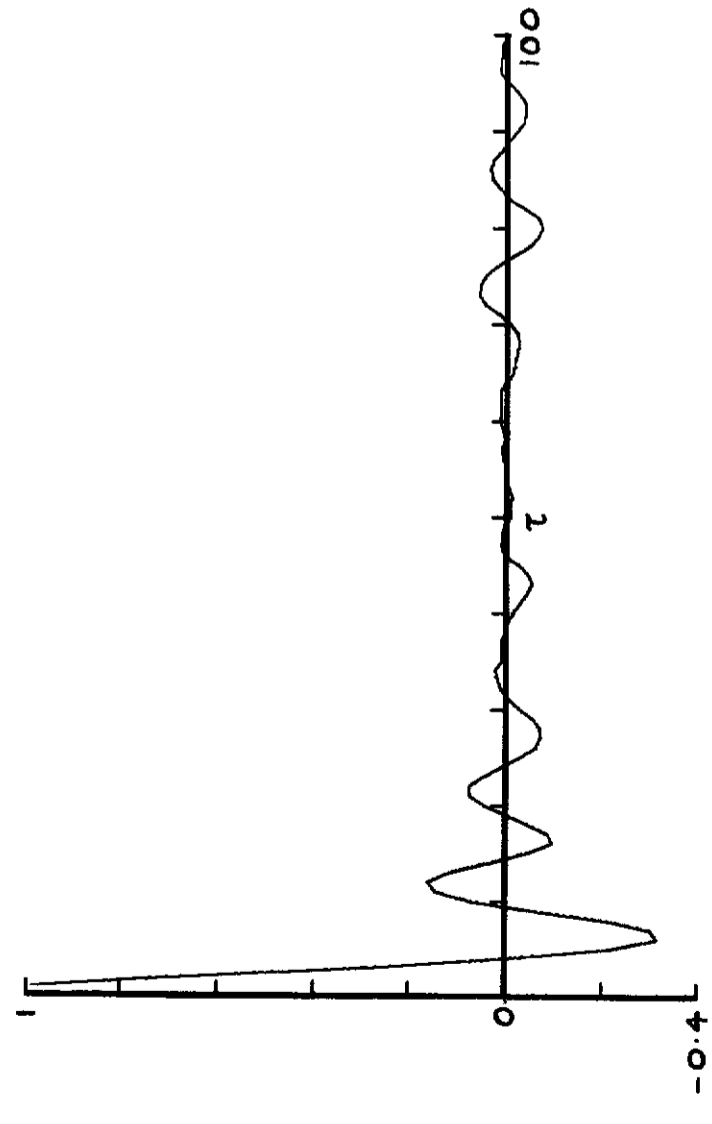


Fig.6a Plots of derived quantities for run 30



Autocorrelation function / τ



Normalised autocorrelation function / τ

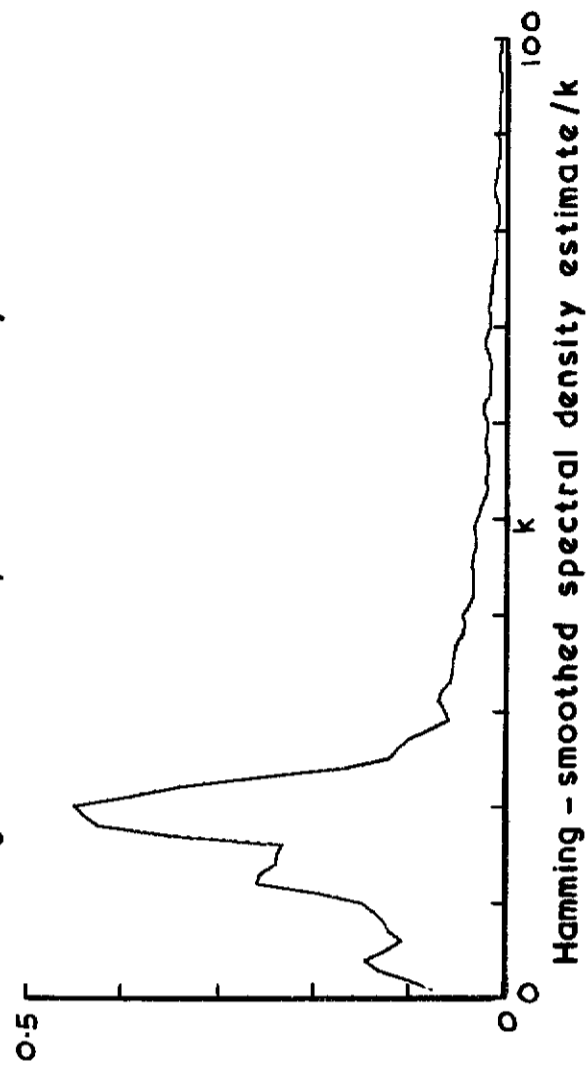
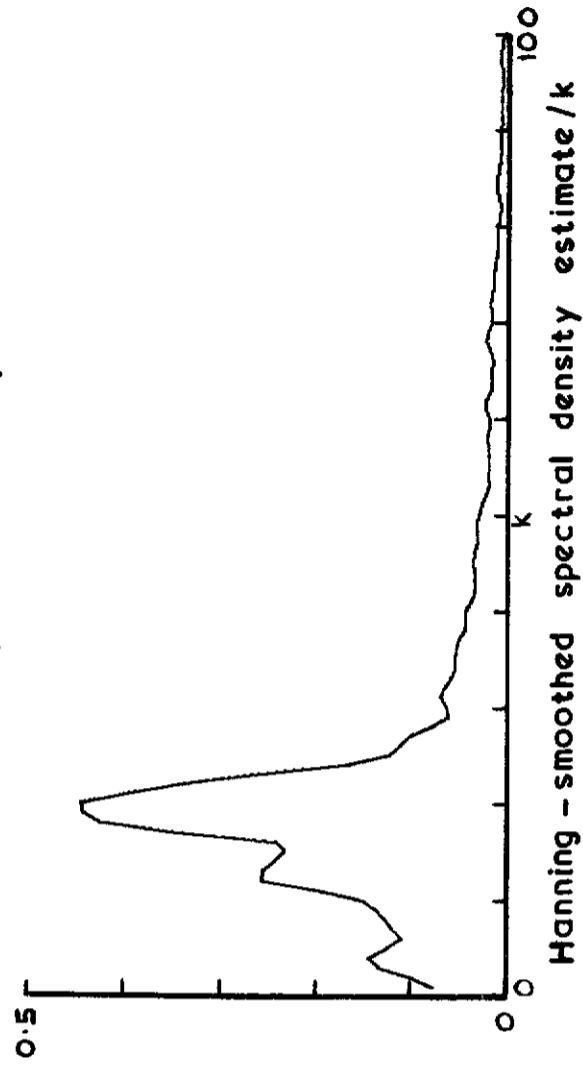
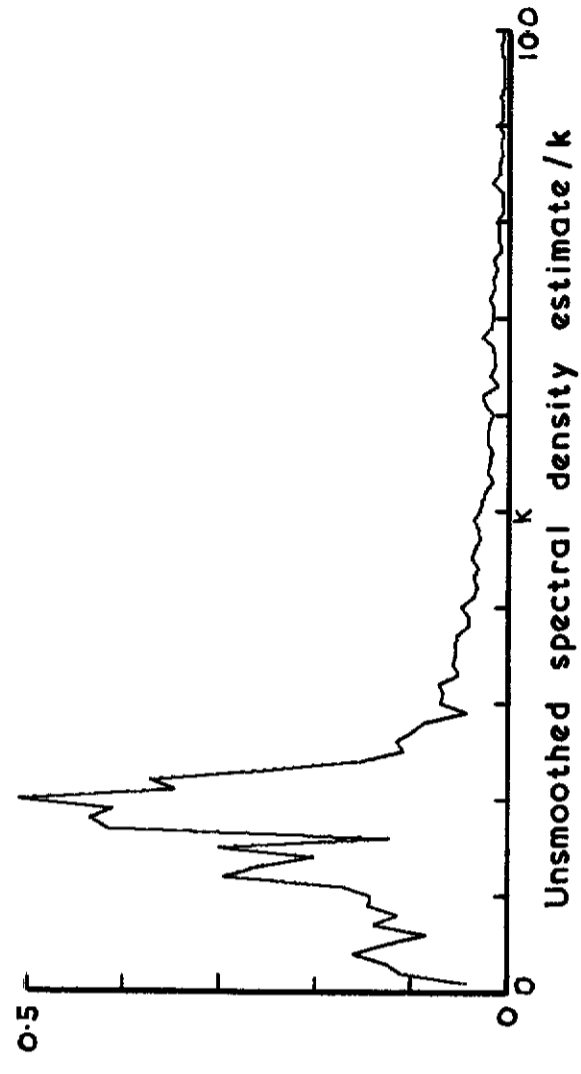


Fig.6b Plots of derived quantities for run 34

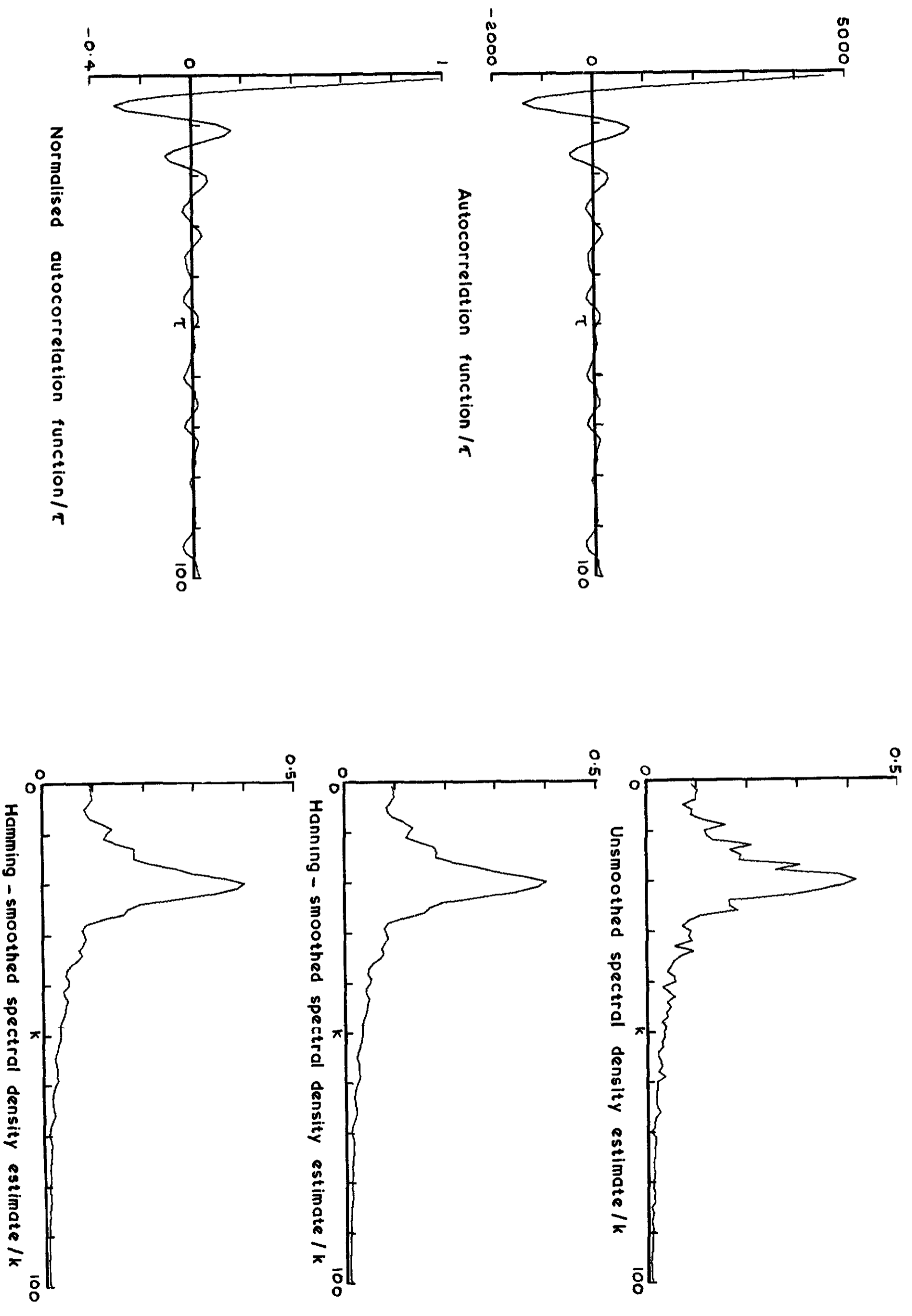


Fig.6c Plots of derived quantities for run 38

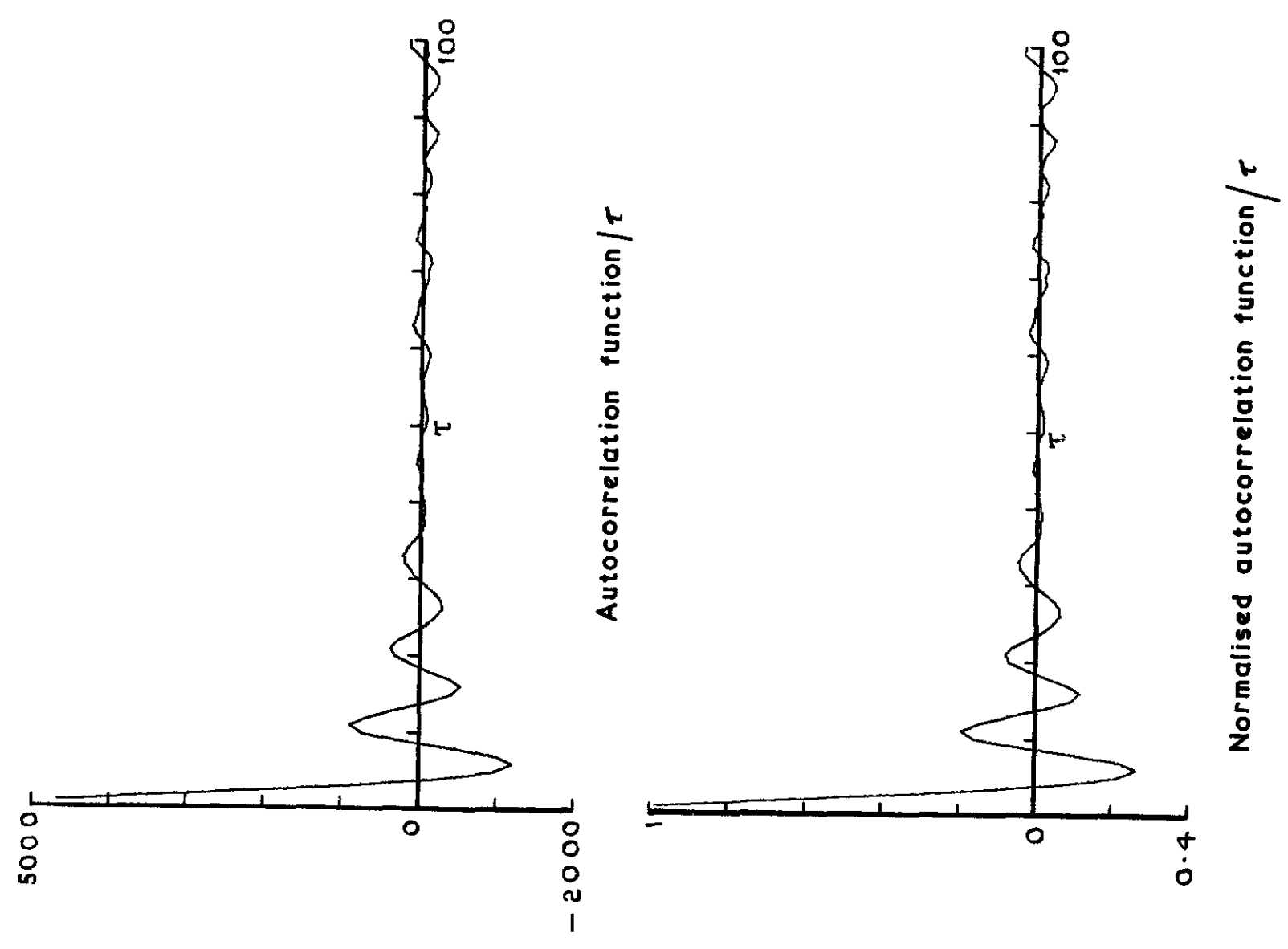
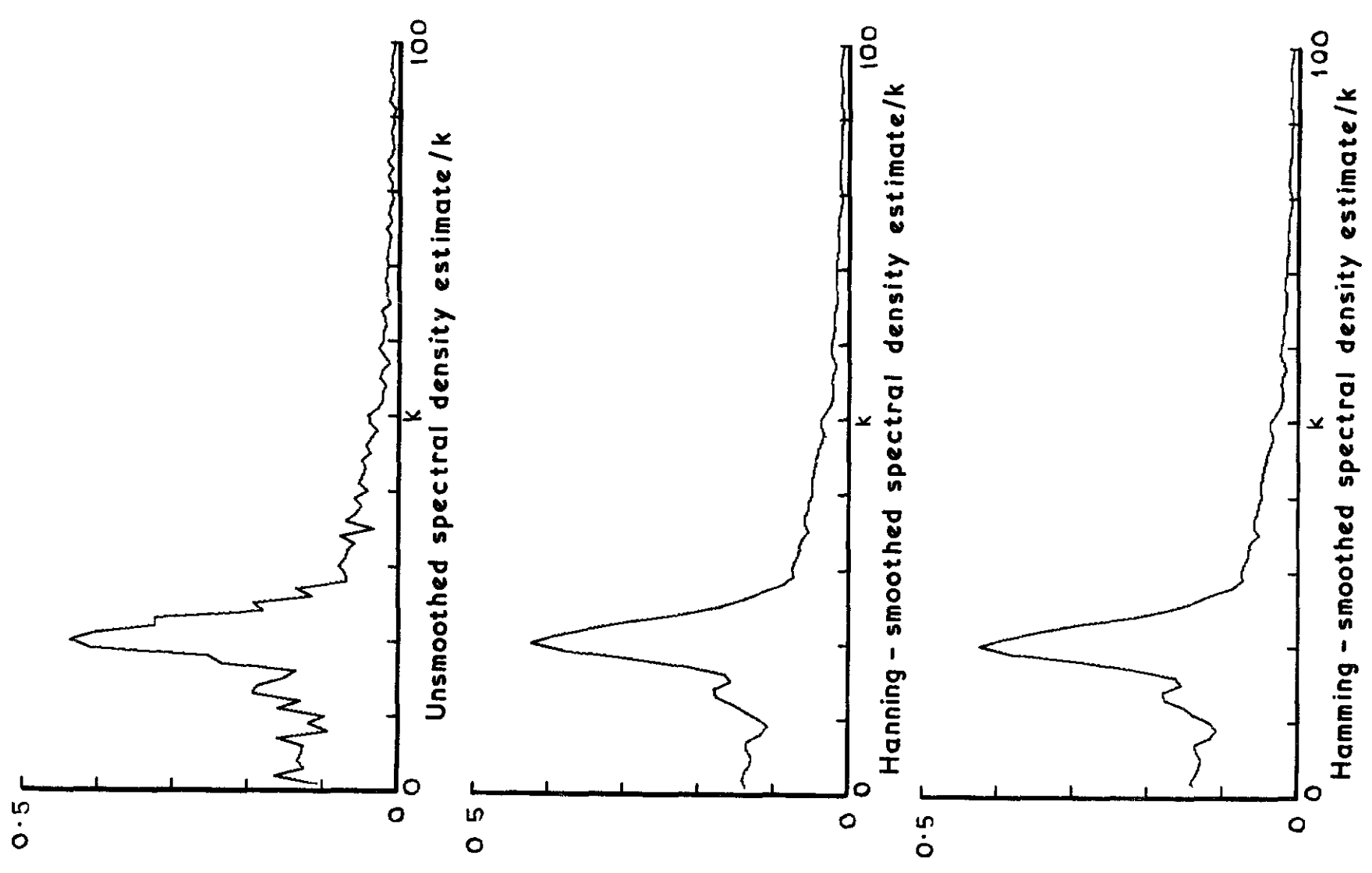


Fig.6d Plots of derived quantities for run 40

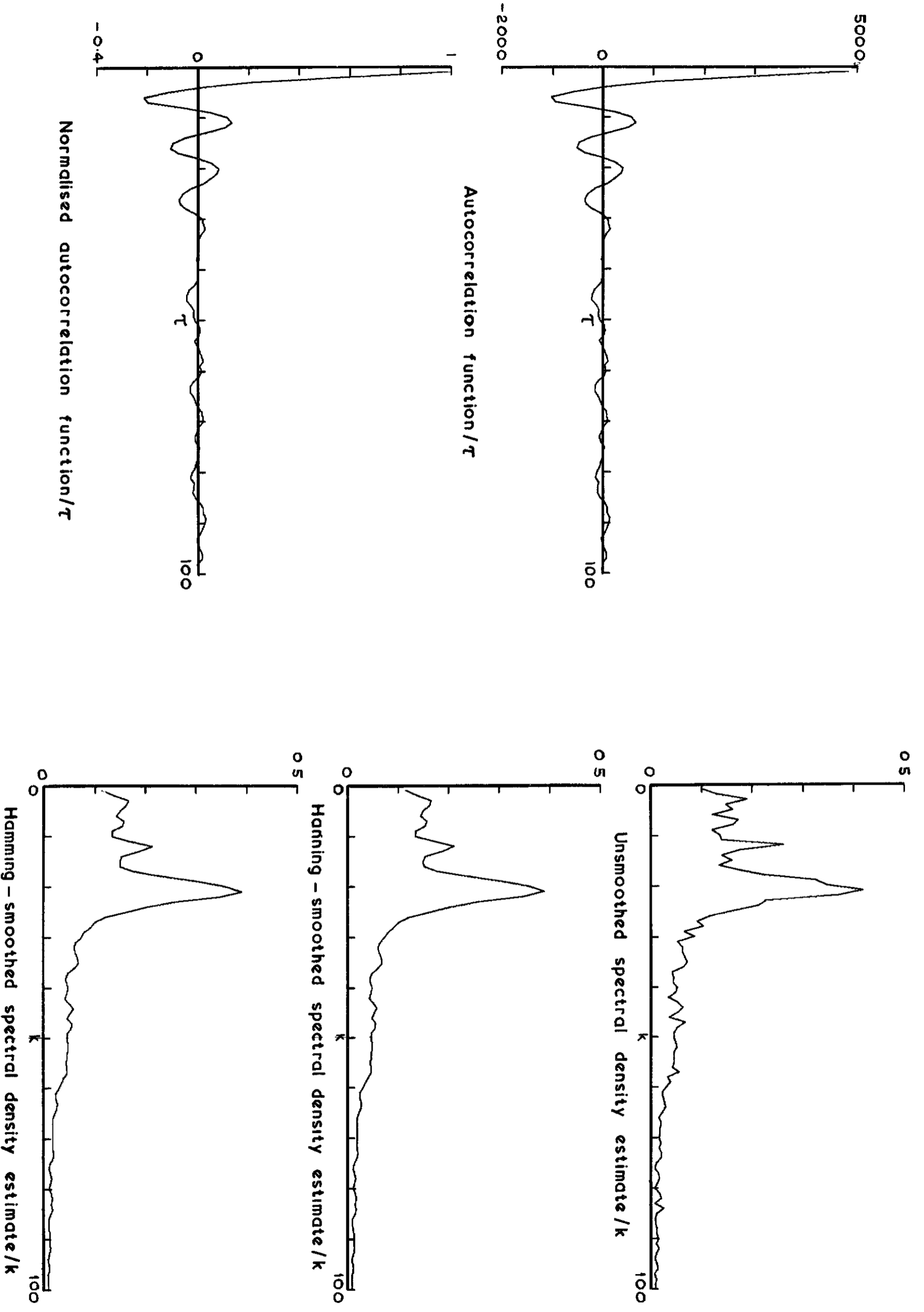
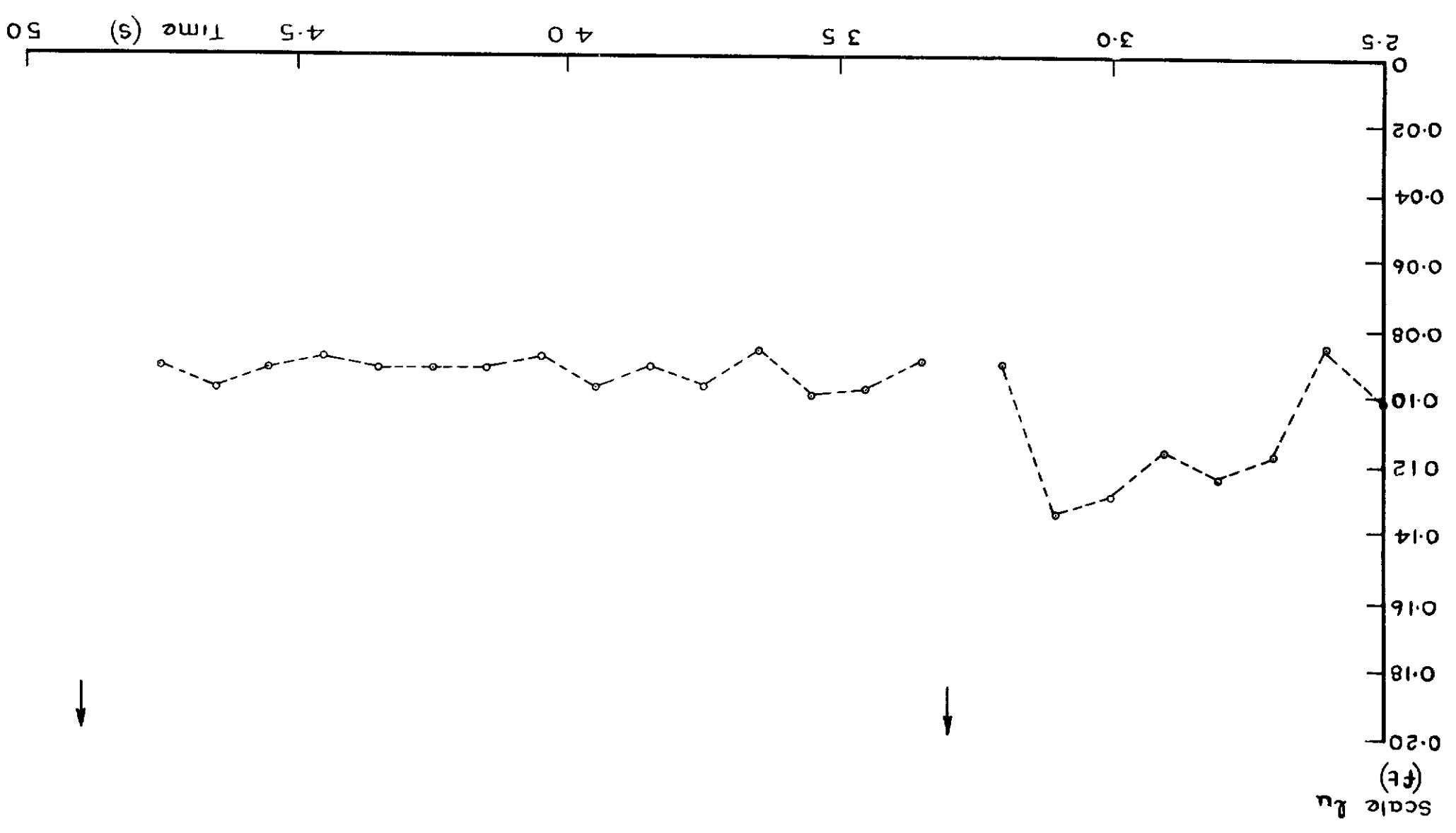


Fig.6e Plots of derived quantities for run 44

Fig. 7a Scale/elapsed time



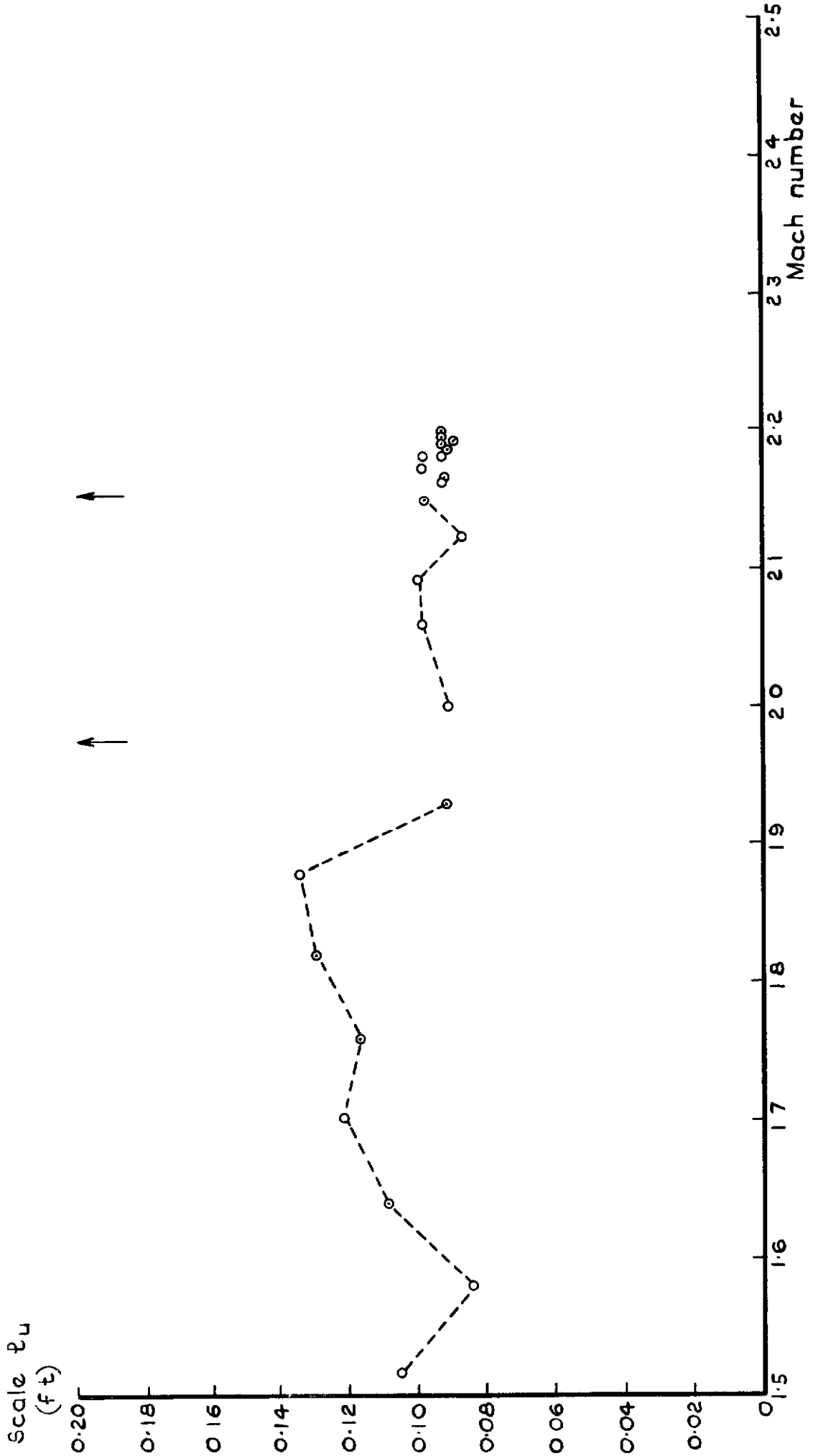


Fig 7b Scale / Mach number

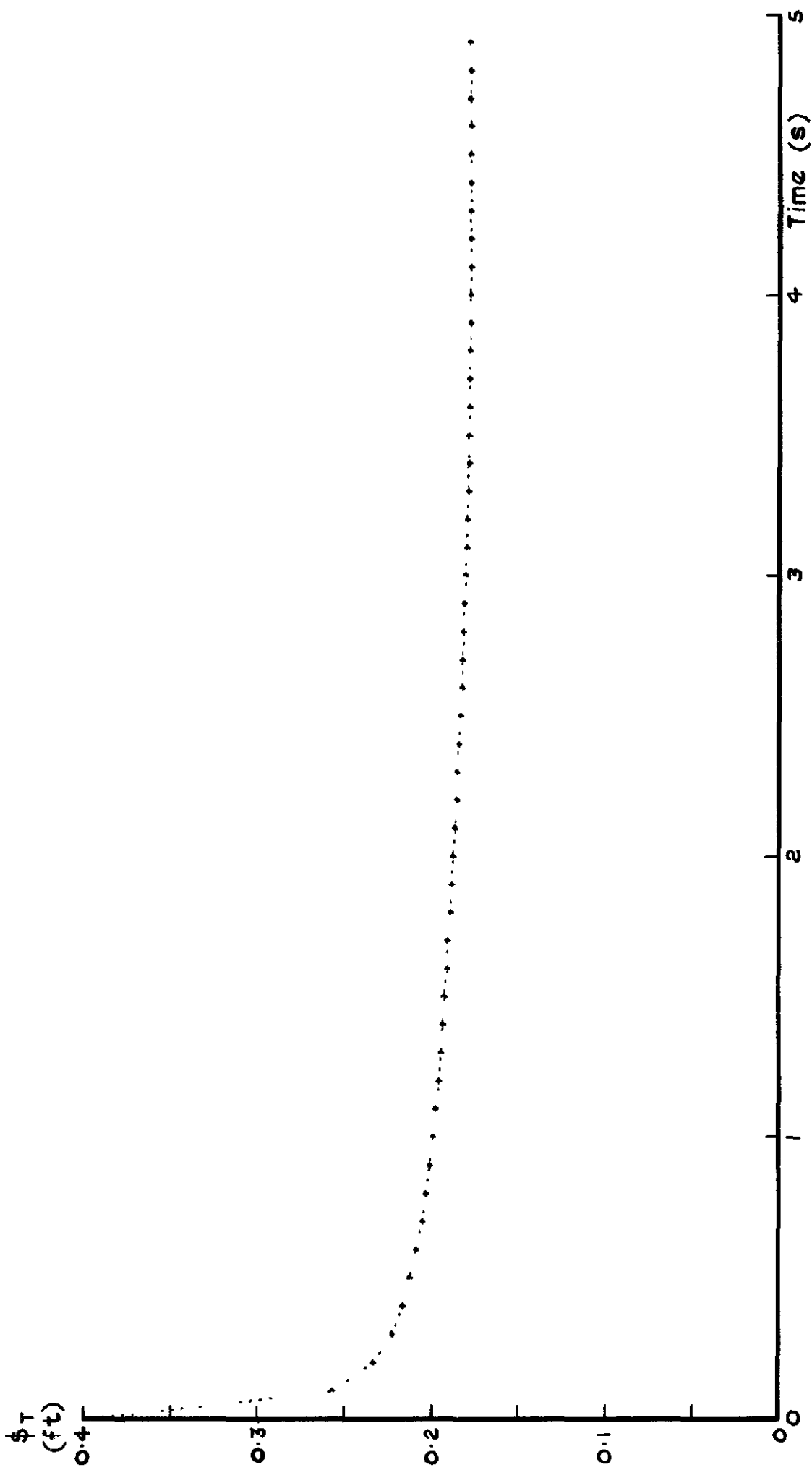


Fig.8a $\$T$ / elapsed time

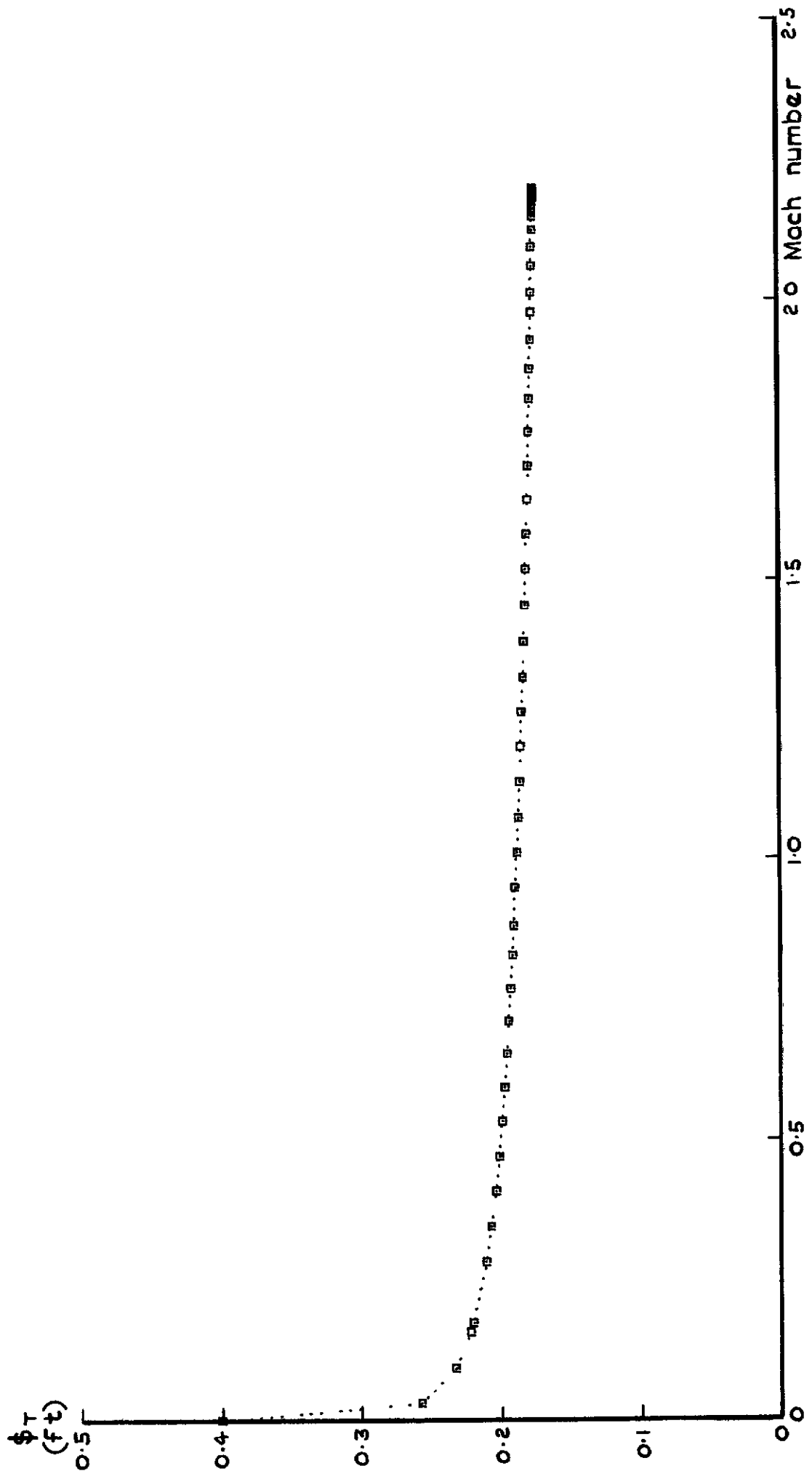


Fig.8 b \$T / Mach number

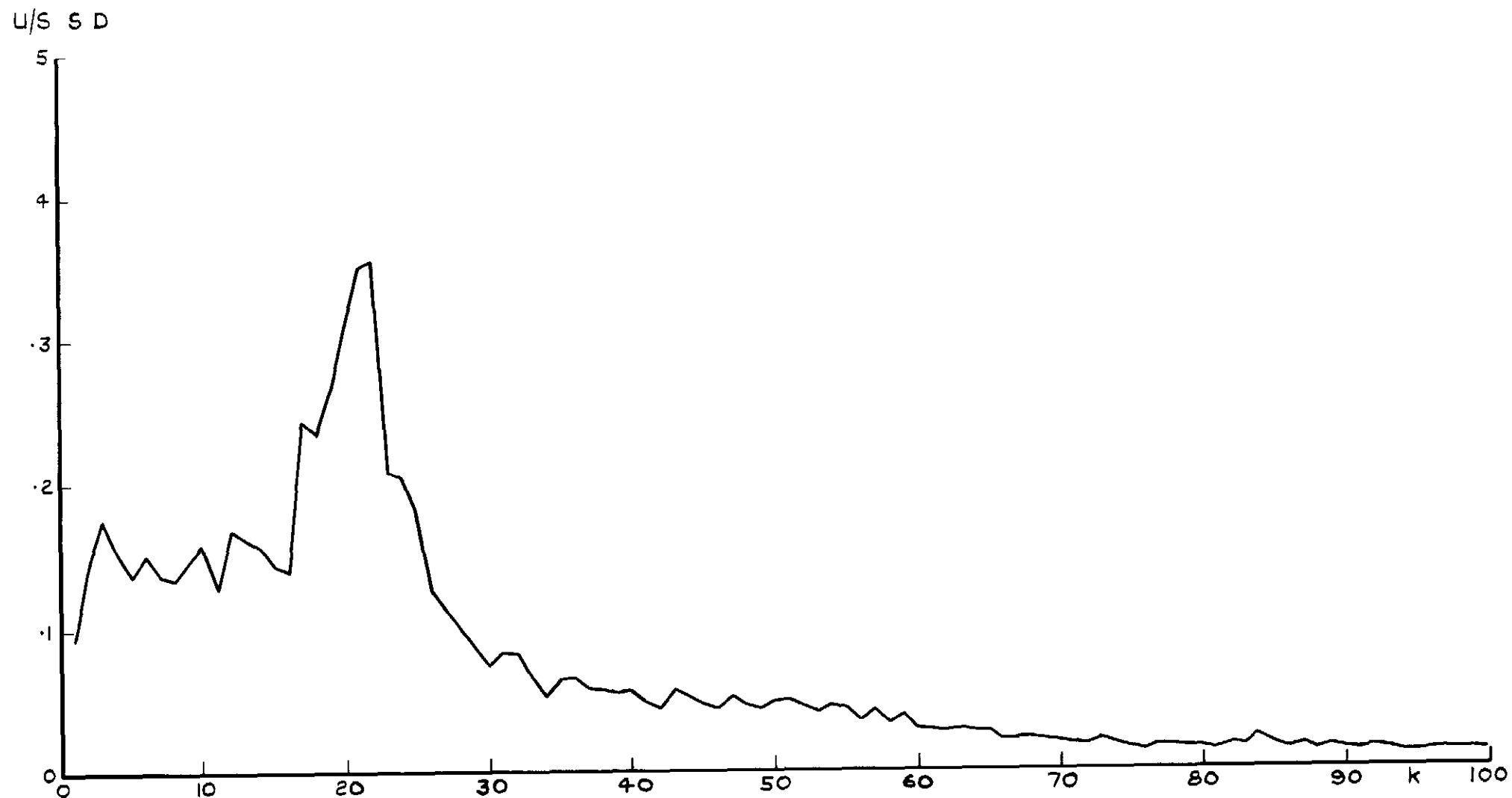


Fig 9 Collective run-unsmoothed spectral density/k

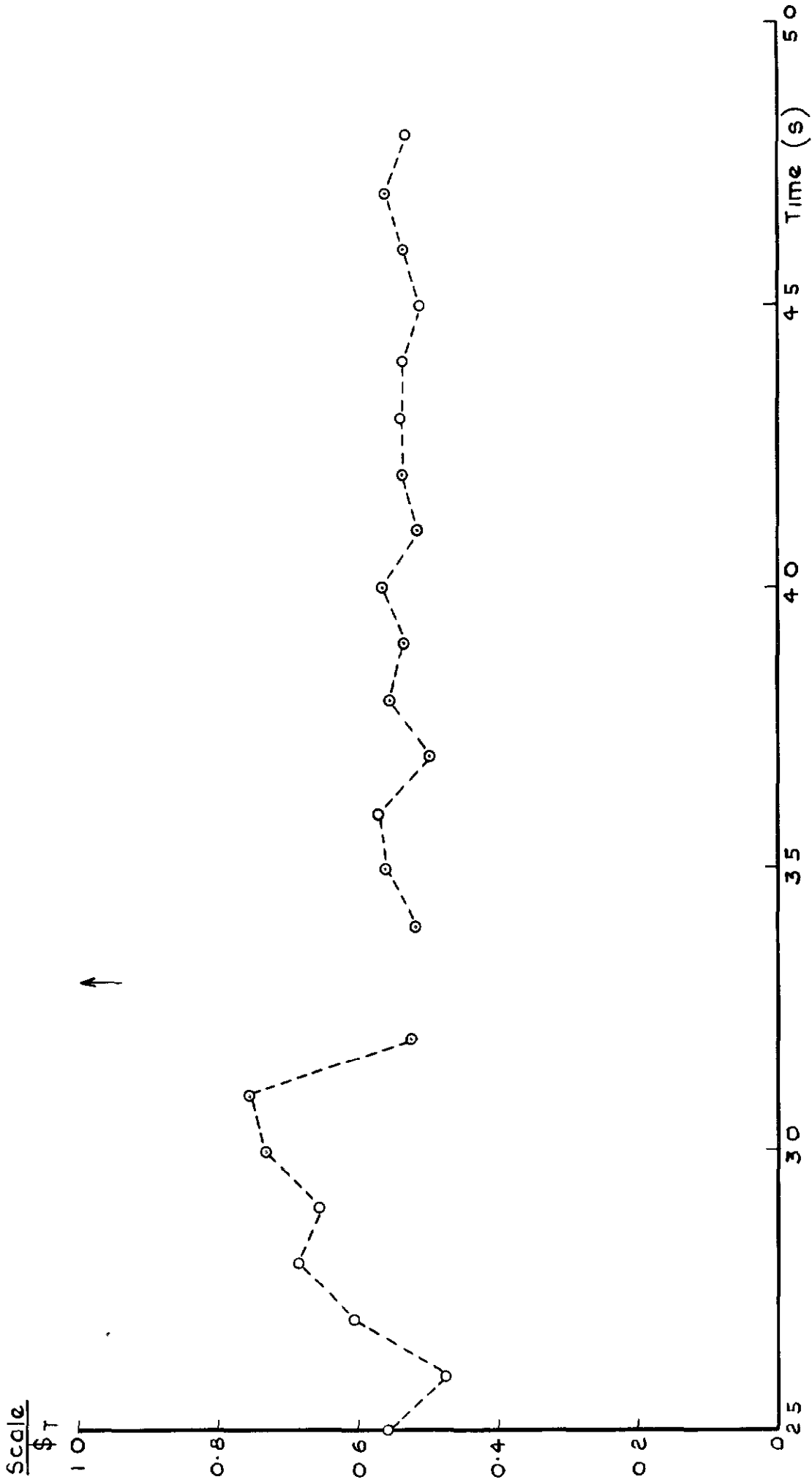


Fig 10 $\frac{\text{Scale}}{\$T}$ / elapsed time

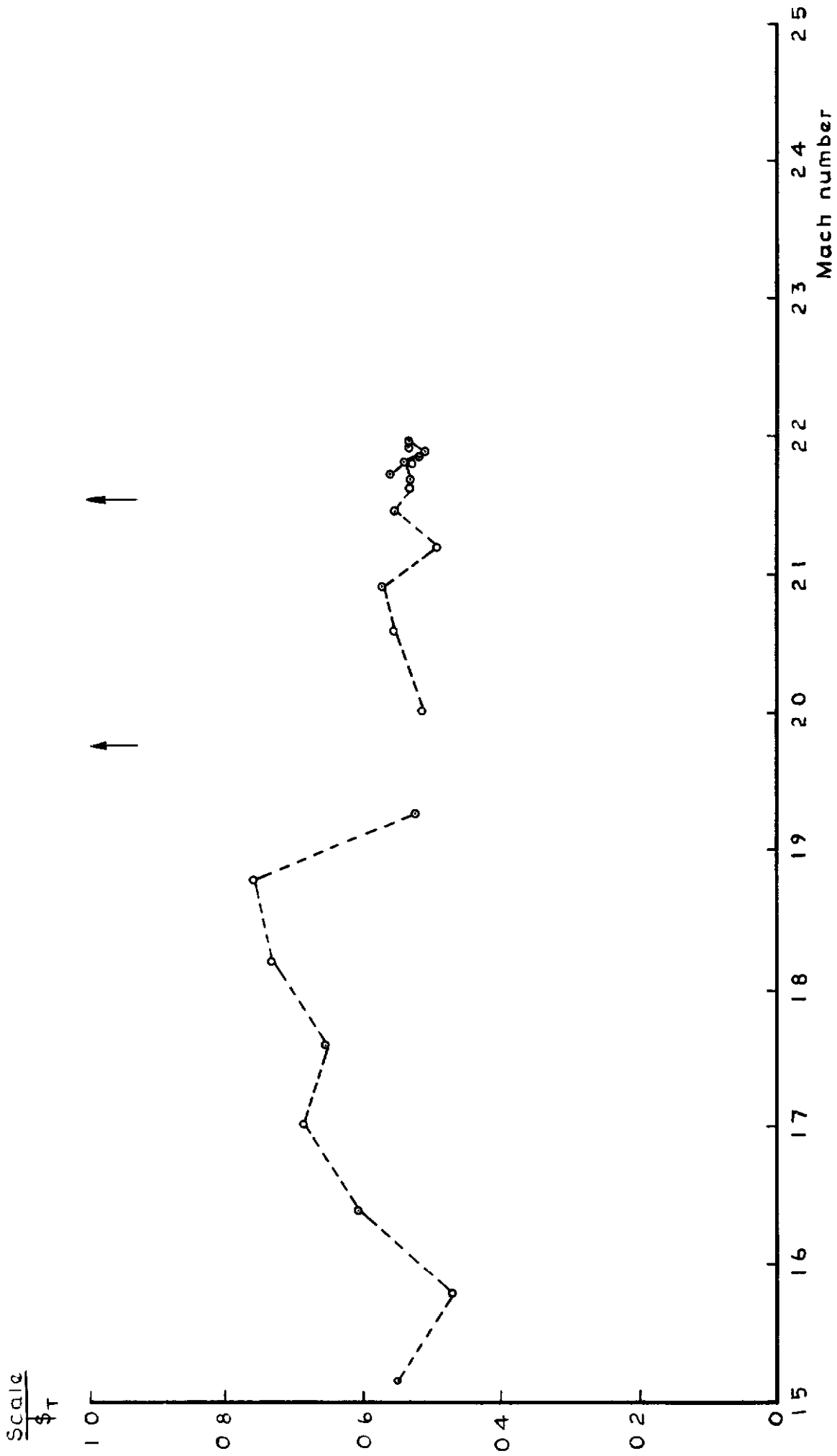


Fig. 11 $\frac{\text{Scale}}{\$T}$ / Mach number

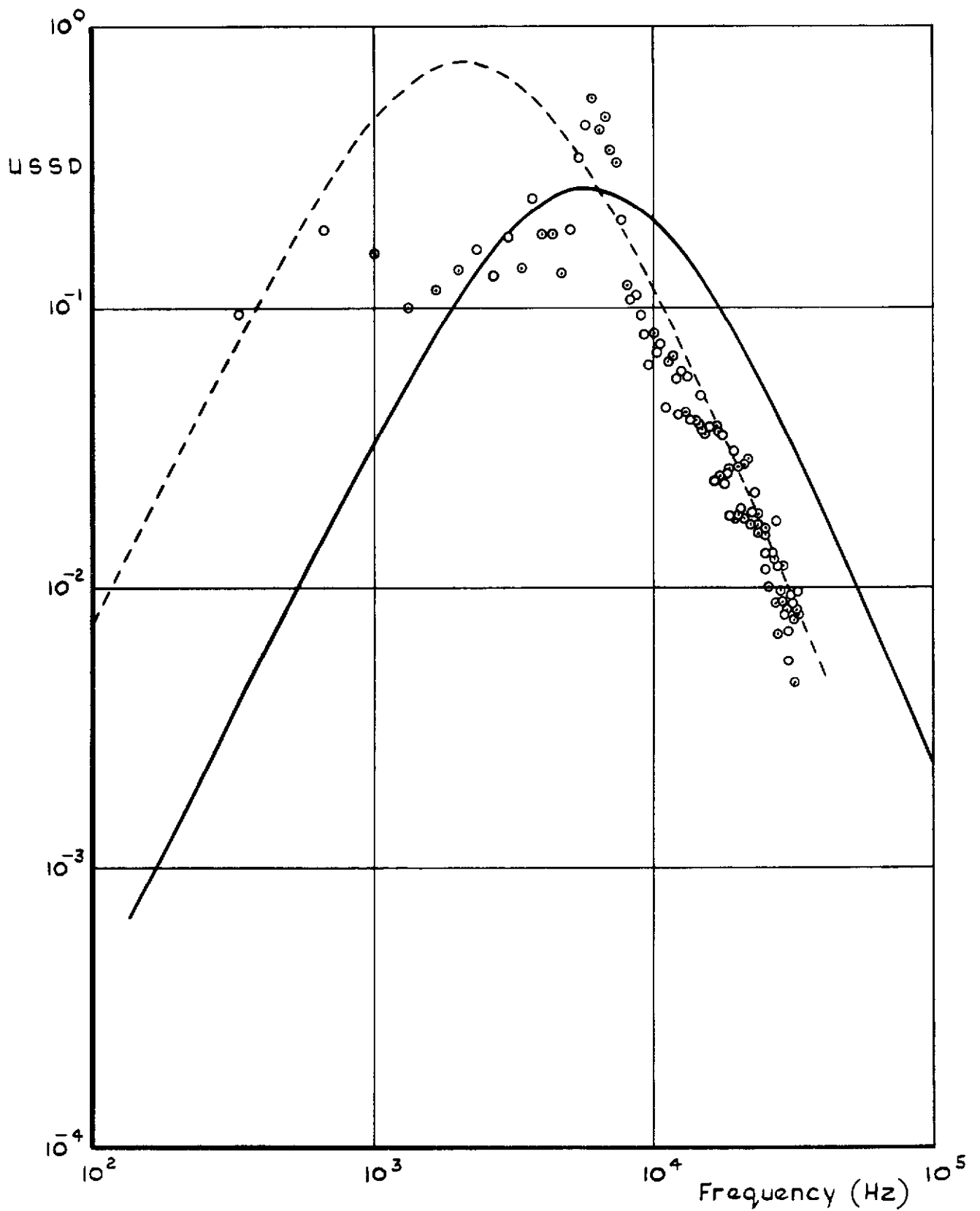


Fig.12a Plot of unsmoothed spectral density (USSD) against frequency, for run 35

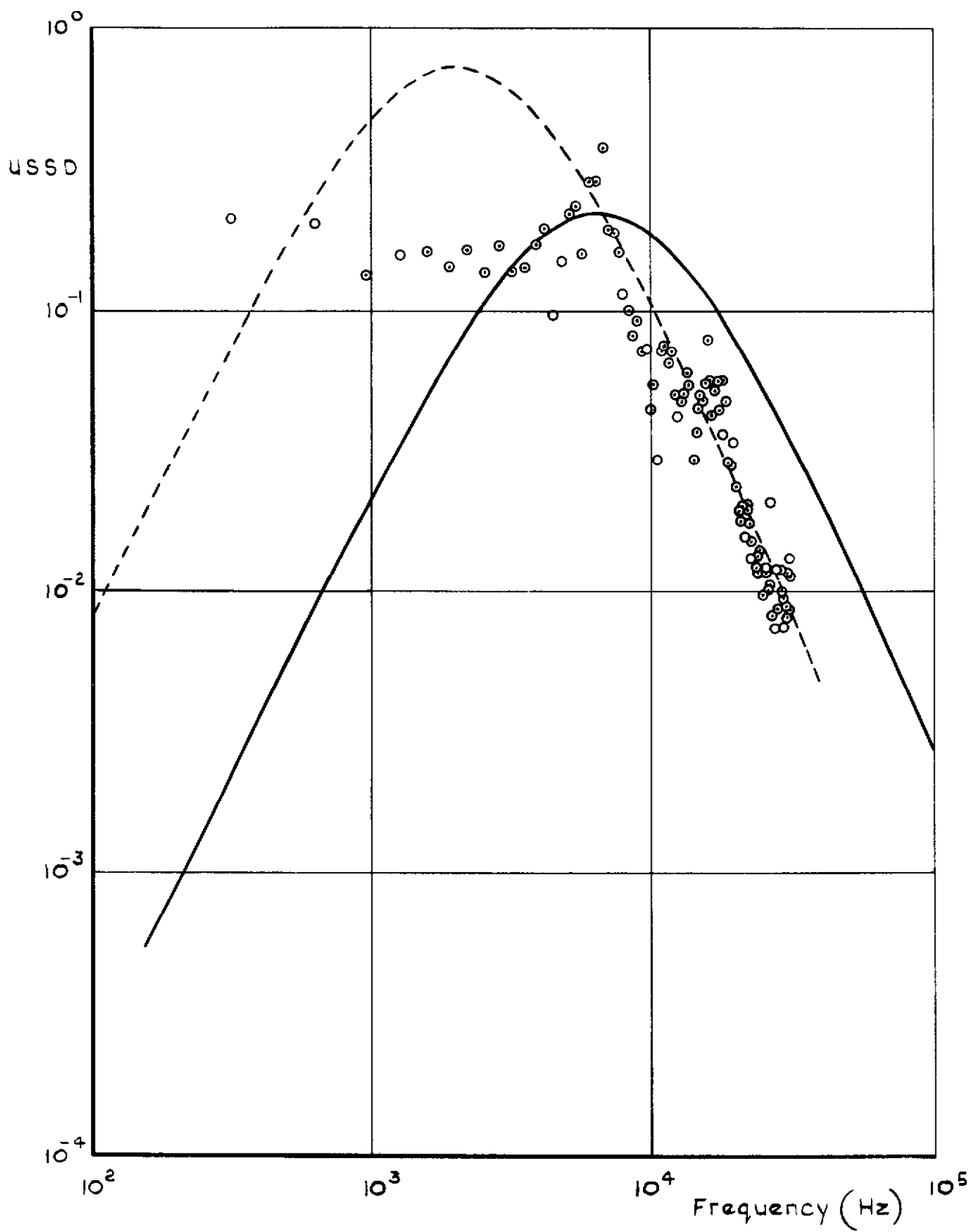


Fig 12b Plot of unsmoothed spectral density (USSD) against frequency, for run 45

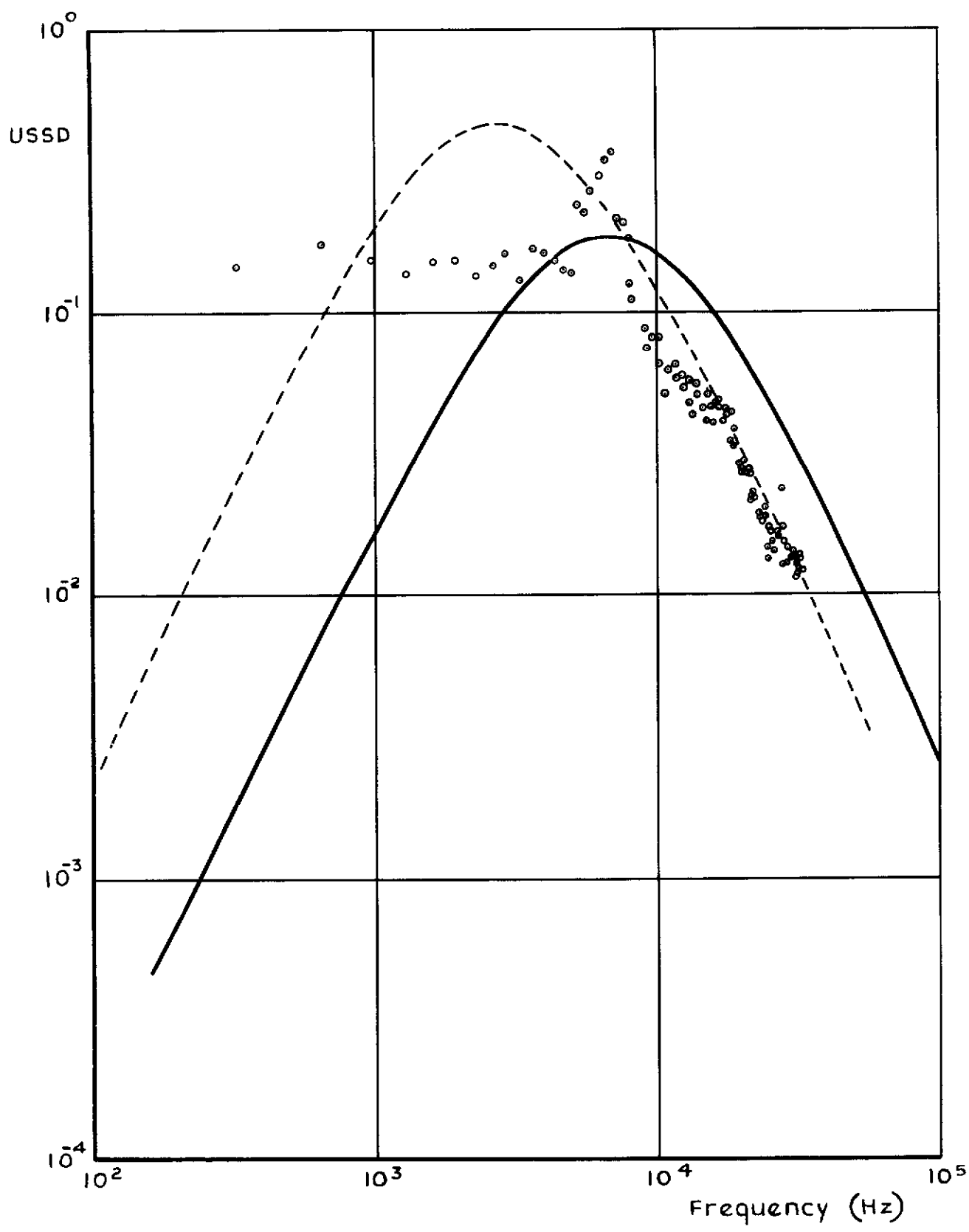


Fig.12c Plot of unsmoothed spectral density (USSD) against frequency, for collective run

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AT HIGH REYNOLDS NUMBERS ON A FREE-
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Measurements have been made of the boundary-layer pressure fluctuations on the body of a free-flight aerodynamic test vehicle powered by a solid-fuel rocket motor. The vehicle reached a maximum Mach number of 2.2 with a maximum Reynolds number of about 215 millions.

Pressure spectra have been deduced, and have been found to compare reasonably with a theoretical spectrum for homogeneous isotropic turbulence.

The scale of the boundary-layer turbulence was found to fluctuate between 47% and 76% of the turbulence boundary-layer thickness over a range of Mach numbers from 1.5 to 2.2, while being essentially equal to 50% of this thickness over the range $Ma = 2.0$ to $Ma = 2.2$.

At $Ma = 2.2$ the root mean square boundary-layer pressure was equal to 0.0045 of the free stream dynamic pressure.

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