C.P. No. 1201

MINISTRY OF DEFENCE (PROCUREMENT EXECUTIVE)

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

Vibrational Equilibrium Calculations of Properties Behind Reflected Shock Waves with Tables for CO_2 and N_2O

By

R. J. Hine

Department of the Mechanics of Fluids

University of Manchester

LONDON· HER MAJESTY'S STATIONERY OFFICE

1971

20p net

C.P. No. 1201*
September, 1969

Vibrational Equilibrium Calculations of Properties Behind Reflected Shock Waves with Tables for CO_2 and N_2O

by

R. J. Hine
Department of the Mechanics of Fluids,
University of Manchester

SUMMARY

Vibrational equilibrium conditions behind a reflected shock wave may be calculated for a real gas by considering the gas as ideal with constant specific heats with heat extraction. Use of a computer allows the equilibrium properties to be determined and tables for CO_2 and N_2O are given as an example.

In the Rayleigh-line method developed by Johannesen[1](1961) vibrational relaxation regions in shock waves are analysed by treating the flow as that of an ideal gas with constant specific heats with heat extraction. Rees[2](1965) adapted this method for the use on a computer and compiled tables of equilibrium conditions behind normal shock waves in CO_2 and N_2O. In the present paper the properties behind a shock wave reflected from a plane wall are found by using a modified version of the computer program of Rees. Johannesen, Bird and Zienkiewicz[3] applied the method of characteristics together with the Rayleigh-line and shock wave problem and determined the distribution of properties in the non-uniform region immediately after reflection, but their method is not particularly suitable for an accurate determination of equilibrium properties.

Fig.1 shows an x,t-diagram of a shock wave reflection with particle paths (whose curvature in the relaxation regions is greatly exaggerated).

Using an approach similar to that of Rees the equilibrium conditions behind the incident shock are found. These conditions become the initial conditions for the reflected shock wave. The gas behind the reflected shock comes to rest at equilibrium. With reference to Fig.2, the frozen Mach number m_R of the gas in region 2 relative to the reflected shock is given by:-

$$m_R = \frac{|V_R| + U_2}{a_{\alpha_2}}$$

where V_R = velocity of reflected shock relative to the wall

U_2 = velocity of the gas in region 2 relative to the wall

a_{α_2} = frozen velocity of sound in region 2

The velocity of the gas in region 5 relative to the reflected shock is equal to the velocity of the reflected shock relative to the wall.

$$|V_R| = m_5 . a_{\alpha_5}$$

where/

---

* Replaces A.R.C.31 421

where m_5 = frozen Mach number in region 5 relative to the shock.

a_{α_5} = frozen velocity of sound in region 5.

Thus
$$m_R = \frac{m_5 \cdot a_{\alpha_5} + U_2}{a_{\alpha_2}}$$

A flow diagram of the computer program is given. Using the identical vibrational energy approximation to that of Rees, the Mach number of the α gas (m) is decreased from its value at the shock front (m_a) in steps of 0.1 until the energy extracted from the α gas (η) is greater than the energy absorbed in vibration ($\bar{\eta}$) i.e., until ($\bar{\eta} - \eta$) becomes negative. The previous value of m is taken and m is decreased in steps of 0.01. This process is repeated until equilibrium is reached (i.e., until ($\bar{\eta} - \eta$) < 0.0001 $\bar{\eta}$). The equilibrium properties behind the primary shock are calculated and used as initial conditions for the reflected shock. m_R is calculated using trial values of m_5 and a_{α_5}. For the first trial it is sufficient to use the frozen Mach number of the flow in region 2 relative to the incident shock and the frozen speed of sound in region 2. The whole process is repeated yielding better values of m_5 and a_{α_5}. A better value of m_R is calculated and the cycle is repeated until m_R does not change by 0.0001 m_R. The equilibrium conditions behind the reflected shock are then calculated using the Rayleigh-line equations. Examples of tables calculated in this way are given for CO_2 and N_2O. The frozen/frozen values at the reflecting wall are also included.

M_I Mach number of the incident shock in the real gas

m_R Frozen Mach number of the α gas in front of the reflected shock relative to the reflected shock

V_R Velocity of the reflected shock relative to the wall

U_2 Velocity of the gas in front of the reflected shock relative to the wall

a_{α_2} Frozen velocity of sound in the α gas in front of the reflected shock

m_5 Frozen Mach number in the α gas behind the reflected shock relative to the shock

a_{α_5} Frozen velocity of sound in the α gas behind the reflected shock

m Mach number of the α gas

m_a Mach number of the α gas at the shock front

η Energy extracted from α gas

$\bar{\eta}$ Energy absorbed by the real gas in vibration

T_F Frozen/frozen temperature at the reflecting wall

P_F/P_1/

P_F/P_1 The ratio of frozen/frozen pressure at the reflecting wall to the initial pressure in front of the incident shock

ρ_F/ρ_1 The ratio of frozen/frozen density at the reflecting wall to the initial density in front of the incident shock

T_a Temperature at the reflected shock front

P_a/P_1 The ratio of the pressure at the reflected shock front to the initial pressure in front of the incident shock

ρ_a/ρ_1 The ratio of the density at the reflected shock front to the initial density in front of the incident shock

T_5 Equilibrium temperature of the gas behind the reflected shock

P_5/P_1 The ratio of the equilibrium pressure behind the reflected shock to the initial pressure in front of the incident shock

ρ_5/ρ_1 The ratio of the equilibrium density behind the reflected shock to the initial density in front of the incident shock

V_R/a_1 The Mach number of reflected shock relative to the initial gas in front of the incident shock

---

REFERENCES

No.	Author(s)	Title, etc.
1	N. H. Johannesen	Analysis of vibrational relaxation regions by means of the Rayleigh-line method. J. Fluid Mech. Vol.10, pp.25-32. 1961
2	T. Rees	Computer calculations of relaxation regions and equilibrium conditions for shock waves with tables for CO_2 and N_2O. ARC R. & M.3472, H.M.S.O. 1965
3	N. H. Johannesen, G. A. Bird and H. K. Zienkiewicz	Theoretical and experimental investigations of the reflexion of normal shock waves with vibrational relaxation. J. Fluid Mech. Vol.30, pp.5-64. 1967

BW

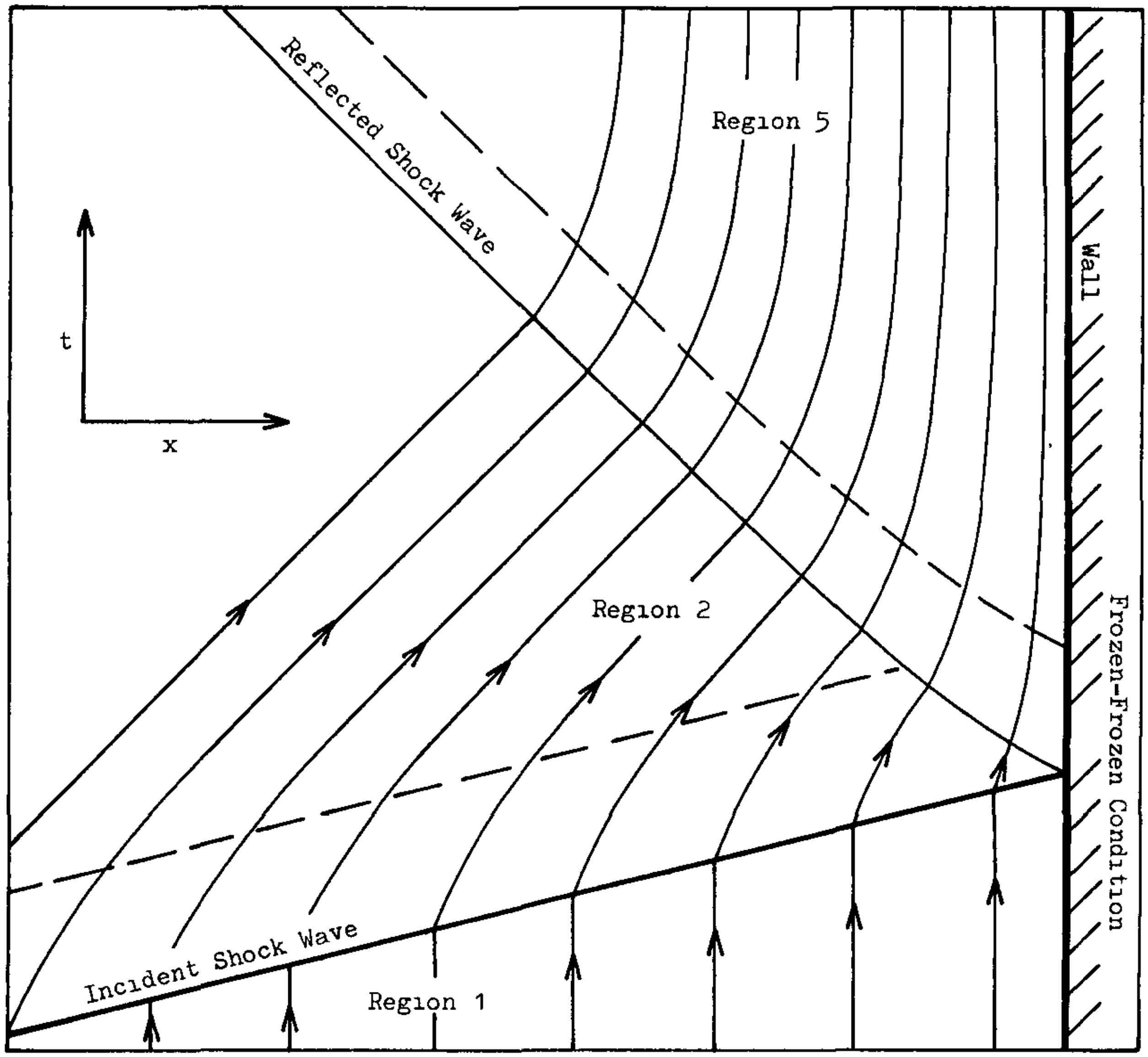


Figure 1

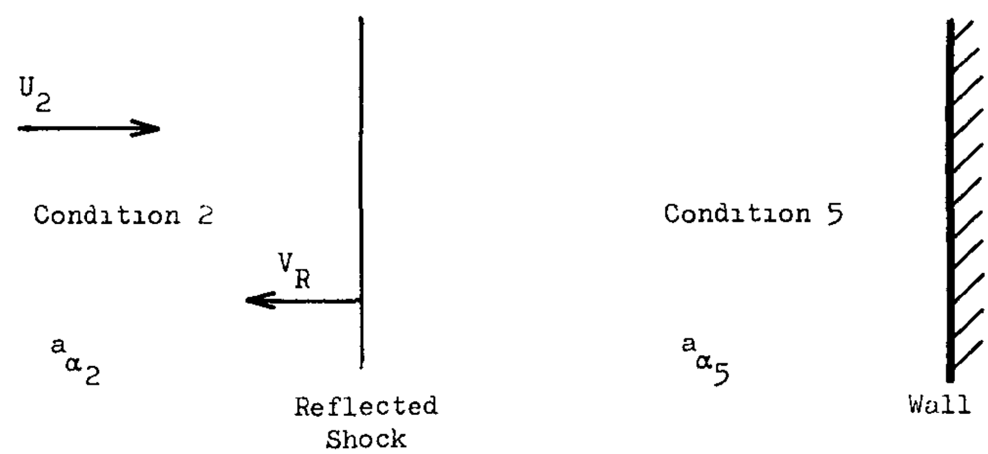
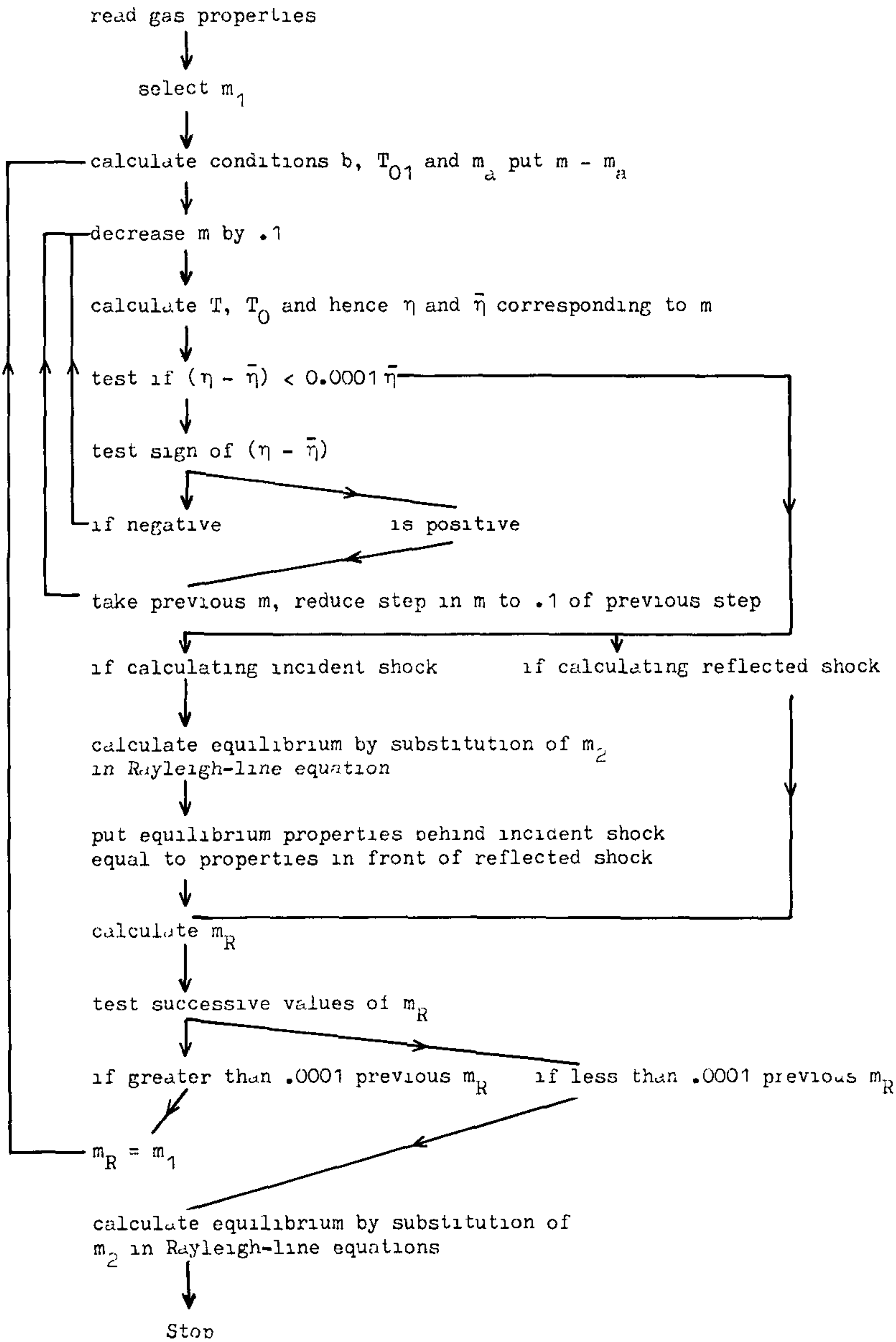


Figure 2

Flow Diagram for Calculation of Equilibrium Values



Equilibrium Values for Reflected Shocks in Carbon Dioxide for T_1 = 295°K

M_I	T_F	P_F/P_1	ρ_F/ρ_1	T_a	P_a/P_1	ρ_a/ρ_1	T_5	P_5/P_1	ρ_5/ρ_1	V_R/a_1
1.1	316.8	1.28	1.195	319.9	1.39	1.283	324.2	1.53	1.391	0.9142
1.2	354.0	1.88	1.567	352.0	2.02	1.691	351.8	2.21	1.857	0.8789
1.3	391.5	2.63	1.983	384.0	2.80	2.155	378.8	3.08	2.397	0.8515
1.4	429.9	3.55	2.435	416.5	3.77	2.671	405.7	4.14	3.009	0.8302
1.5	469.7	4.64	2.917	449.7	4.93	3.234	432.7	5.41	3.690	0.8137
1.6	510.9	5.93	3.422	483.9	6.30	3.839	460.0	6.92	4.436	0.8010
1.7	554.0	7.41	3.944	519.3	7.89	4.481	487.9	8.67	5.241	0.7914
1.8	598.9	9.09	4.477	556.0	9.71	5.154	516.4	10.68	6.103	0.7844
1.9	645.9	10.98	5.014	594.0	11.79	5.853	545.5	12.97	7.015	0.7794
2.0	694.9	13.08	5.553	633.5	14.12	6.574	575.3	15.55	7.972	0.7762
2.1	746.1	15.40	6.088	674.3	16.71	7.311	605.8	18.42	8.971	0.7745
2.2	799.5	17.93	6.616	716.7	19.59	8.062	637.1	21.61	10.005	0.7740
2.3	855.1	20.68	7.135	760.5	22.74	8.822	669.2	25.12	11.072	0.7746
2.4	913.0	23.65	7.642	805.9	26.20	9.589	702.0	28.95	12.166	0.7762
2.5	973.1	26.84	8.136	852.7	29.95	10.360	735.7	33.13	13.284	0.7785
2.6	1035.6	30.25	8.616	901.1	34.01	11.133	770.1	37.65	14.422	0.7815
2.7	1100.4	33.87	9.080	951.1	38.38	11.905	805.4	42.53	15.577	0.7851
2.8	1167.5	37.71	9.527	1002.6	43.08	12.675	841.5	47.77	16.746	0.7893
2.9	1237.0	41.76	9.959	1055.6	48.10	13.440	878.4	53.38	17.926	0.7940
3.0	1308.8	46.03	10.374	1110.2	53.45	14.201	916.1	59.36	19.115	0.7991
3.1	1383.0	50.50	10.772	1166.4	59.13	14.956	954.6	65.72	20.309	0.8046
3.2	1459.6	55.19	11.154	1224.1	65.16	15.704	994.0	72.47	21.507	0.8104
3.3	1538.6	60.08	11.520	1283.3	71.54	16.445	1034.3	79.61	22.708	0.8166
3.4	1619.9	65.19	11.871	1344.2	78.26	17.177	1075.3	87.15	23.908	0.8230
3.5	1703.6	70.49	12.206	1406.6	85.35	17.900	1117.3	95.09	25.107	0.8298
3.6	1789.7	76.00	12.527	1470.5	92.78	18.614	1160.1	103.44	26.303	0.8368
3.7	1878.2	81.71	12.834	1536.0	100.59	19.318	1203.8	112.19	27.495	0.8440
3.8	1969.0	87.62	13.127	1603.1	108.75	20.012	1248.3	121.37	28.681	0.8515
3.9	2062.3	93.73	13.407	1671.8	117.29	20.697	1293.7	130.95	29.860	0.8591
4.0	2158.0	100.03	13.675	1742.0	126.20	21.371	1340.0	140.96	31.032	0.8670

Equilibrium Values for Reflected Shocks in Carbon Dioxide for T_1 = 295°K.

M_I	T_F	P_F/P_1	ρ_F/ρ_1	T_a	P_a/P_1	ρ_a/ρ_1	T_5	P_5/P_1	ρ_5/ρ_1	V_R/a_1
4.1	2256.1	106.53	13.930	1813.9	135.49	22.035	1387.2	151.40	32.195	0.8750
4.2	2356.5	113.23	14.175	1887.2	145.15	22.689	1435.4	162.26	33.349	0.8832
4.3	2459.4	120.12	14.408	1962.2	155.19	23.332	1484.4	173.55	34.492	0.8916
4.4	2564.7	127.20	14.631	2038.7	165.62	23.965	1534.3	185.28	35.624	0.9001
4.5	2672.4	134.47	14.844	2116.9	176.43	24.587	1585.1	197.44	36.745	0.9087
4.6	2782.5	141.93	15.048	2196.6	187.63	25.199	1636.8	210.04	37.854	0.9175
4.7	2895.0	149.58	15.243	2277.9	199.22	25.801	1689.5	223.08	38.951	0.9264
4.8	3009.9	157.42	15.429	2360.7	211.20	26.392	1743.1	236.56	40.035	0.9355
4.9	3127.2	165.45	15.607	2445.2	223.57	26.973	1797.6	250.48	41.105	0.9447
5.0	3246.9	173.66	15.778	2531.2	236.34	27.544	1853.1	264.85	42.162	0.9539
5.1	3369.0	182.06	15.941	2618.9	249.50	28.104	1909.4	279.66	43.206	0.9633
5.2	3493.6	190.64	16.098	2708.1	263.05	28.655	1966.8	294.92	44.235	0.9728
5.3	3620.5	199.41	16.248	2798.9	277.00	29.196	2025.0	310.62	45.251	0.9824
5.4	3749.9	208.35	16.391	2891.3	291.35	29.727	2084.3	326.78	46.252	0.9920
5.5	3881.7	217.49	16.529	2985.3	306.10	30.248	2144.4	343.39	47.239	1.0018
5.6	4015.8	226.80	16.661	3081.0	321.25	30.760	2205.5	360.44	48.211	1.0117
5.7	4152.4	236.30	16.787	3178.2	336.80	31.262	2267.6	377.95	49.169	1.0216
5.8	4291.4	245.98	16.909	3277.0	352.74	31.755	2330.6	395.91	50.113	1.0316
5.9	4432.9	255.84	17.025	3377.4	369.09	32.239	2394.6	414.31	51.042	1.0417
6.0	4576.7	265.87	17.137	3479.4	385.84	32.713	2459.5	433.17	51.957	1.0518
6.1	4722.9	276.09	17.245	3583.0	402.98	33.179	2525.4	452.49	52.857	1.0620
6.2	4871.6	286.49	17.349	3688.2	420.53	33.636	2592.2	472.25	53.743	1.0723
6.3	5022.7	297.07	17.448	3795.0	438.48	34.085	2660.0	492.47	54.616	1.0827
6.4	5176.2	307.83	17.544	3903.4	456.83	34.525	2728.8	513.14	55.474	1.0931
6.5	5332.1	318.76	17.636	4013.5	475.58	34.956	2798.5	534.26	56.318	1.1035
6.6	5490.4	329.87	17.724	4125.1	494.73	35.380	2869.2	555.83	57.149	1.1141
6.7	5651.1	341.16	17.809	4238.3	514.28	35.795	2940.8	577.85	57.965	1.1246
6.8	5814.3	352.63	17.892	4353.2	534.22	36.202	3013.4	600.32	58.769	1.1353
6.9	5979.8	364.28	17.971	4469.7	554.57	36.602	3087.0	623.25	59.559	1.1459
7.0	6147.8	376.10	18.047	4587.7	575.32	36.994	3161.5	646.62	60.336	1.1566
7.1	6318.2	388.10	18.121	4707.4	596.46	37.379	3237.0	670.45	61.100	1.1674
7.2	6491.0	400.28	18.192	4828.7	618.01	37.756	3313.5	694.72	61.851	1.1782
7.3	6666.2	412.63	18.260	4951.6	639.95	38.126	3390.9	719.44	62.590	1.1890
7.4	6843.8	425.16	18.326	5076.1	662.29	38.489	3469.3	744.61	63.316	1.1999
7.5	7023.9	437.86	18.390	5202.3	685.02	38.845	3548.6	770.23	64.030	1.2108

Equilibrium Values for Reflected Shocks in Nitrous Oxide for T_1 = 295°K.

M_I	T_F	P_F/P_1	ρ_F/ρ_1	T_a	P_a/P_1	ρ_a/ρ_1	T_5	P_5/P_1	ρ_5/ρ_1	V_R/a_1
1.1	314.6	1.25	1.174	318.2	1.37	1.273	322.9	1.53	1.394	0.9078
1.2	351.5	1.84	1.541	349.6	1.99	1.681	349.4	2.21	1.864	0.8716
1.3	388.8	2.57	1.952	380.9	2.77	2.147	375.3	3.07	2.410	0.8434
1.4	426.9	3.47	2.399	412.7	3.73	2.666	401.1	4.12	3.030	0.8214
1.5	466.3	4.55	2.876	445.2	4.88	3.234	427.1	5.39	3.722	0.8043
1.6	507.2	5.81	3.377	478.7	6.24	3.846	453.5	6.89	4.481	0.7912
1.7	549.9	7.26	3.895	513.4	7.82	4.495	480.5	8.64	5.303	0.7811
1.8	594.4	8.91	4.423	549.3	9.64	5.178	508.0	10.65	6.183	0.7737
1.9	640.9	10.77	4.958	586.5	11.71	5.888	536.2	12.93	7.115	0.7684
2.0	689.4	12.84	5.494	625.2	14.03	6.621	565.1	15.51	8.096	0.7649
2.1	740.1	15.12	6.026	665.2	16.62	7.372	594.7	18.38	9.118	0.7629
2.2	792.9	17.61	6.553	706.7	19.49	8.137	625.1	21.57	10.179	0.7622
2.3	847.9	20.32	7.070	749.7	22.65	8.912	656.3	25.08	11.273	0.7626
2.4	905.2	23.25	7.576	794.2	26.10	9.695	688.3	28.92	12.396	0.7639
2.5	964.7	26.39	8.069	840.2	29.85	10.481	721.0	33.10	13.544	0.7660
2.6	1026.5	29.74	8.548	887.7	33.91	11.270	754.6	37.63	14.712	0.7688
2.7	1090.6	33.32	9.012	936.8	38.29	12.058	788.9	42.52	15.899	0.7723
2.8	1157.0	37.10	9.459	987.3	42.99	12.844	824.1	47.77	17.100	0.7763
2.9	1225.7	41.10	9.891	1039.4	48.01	13.627	860.0	53.39	18.312	0.7807
3.0	1296.8	45.31	10.306	1093.1	53.37	14.404	896.8	59.38	19.533	0.7857
3.1	1370.2	49.72	10.705	1148.2	59.06	15.175	934.4	65.76	20.760	0.7910
3.2	1445.9	54.35	11.088	1204.9	65.10	15.938	972.8	72.52	21.991	0.7967
3.3	1524.0	59.18	11.455	1263.2	71.48	16.694	1012.1	79.68	23.225	0.8027
3.4	1604.4	64.21	11.806	1323.0	78.22	17.441	1052.2	87.23	24.458	0.8090
3.5	1687.2	69.45	12.143	1384.3	85.31	18.179	1093.1	95.19	25.690	0.8156
3.6	1772.4	74.89	12.465	1447.2	92.76	18.908	1134.9	103.56	26.918	0.8224
3.7	1859.9	80.53	12.772	1511.7	100.57	19.626	1177.5	112.33	28.142	0.8295
3.8	1949.8	86.36	13.067	1577.6	108.75	20.334	1221.0	121.52	29.360	0.8368
3.9	2042.1	92.40	13.348	1645.2	117.29	21.032	1265.4	131.13	30.571	0.8443
4.0	2136.7	98.63	13.617	1714.3	126.21	21.720	1310.6	141.16	31.775	0.8520

Equilibrium Values for Reflected Shocks

in Nitrous Oxide for T_1 = 295°K.

M_I	T_F	P_F/P_1	ρ_F/ρ_1	T_a	P_a/P_1	ρ_a/ρ_1	T_5	P_5/P_1	ρ_5/ρ_1	V_R/a_1
4.1	2233.7	105.05	13.874	1784.9	135.51	22.396	1356.6	151.62	32.969	0.8598
4.2	2333.0	111.66	14.119	1857.1	145.18	23.062	1403.6	162.50	34.153	0.8678
4.3	2434.8	118.47	14.354	1930.8	155.23	23.717	1451.4	173.81	35.327	0.8760
4.4	2538.9	125.46	14.578	2006.1	165.67	24.362	1500.1	185.56	36.490	0.8843
4.5	2645.4	132.65	14.792	2083.0	176.49	24.995	1549.7	197.74	37.642	0.8928
4.6	2754.3	140.02	14.997	2161.4	187.70	25.618	1600.2	210.36	38.781	0.9014
4.7	2865.6	147.58	15.193	2241.4	199.29	26.230	1651.6	223.42	39.907	0.9101
4.8	2979.2	155.33	15.381	2322.9	211.28	26.831	1703.8	236.92	41.021	0.9189
4.9	3095.2	163.26	15.560	2406.0	223.65	27.422	1757.0	250.86	42.121	0.9279
5.0	3213.6	171.38	15.732	2490.6	236.42	28.003	1811.0	265.25	43.207	0.9369
5.1	3334.4	179.68	15.896	2576.8	249.58	28.572	1866.0	280.08	44.280	0.9461
5.2	3457.6	188.16	16.054	2664.6	263.14	29.132	1921.8	295.36	45.338	0.9553
5.3	3583.2	196.82	16.204	2754.0	277.09	29.682	1978.6	311.09	46.383	0.9646
5.4	3711.1	205.67	16.349	2844.9	291.44	30.221	2036.2	327.27	47.413	0.9740
5.5	3841.4	214.70	16.488	2937.3	306.19	30.751	2094.8	343.89	48.429	0.9835
5.6	3974.1	223.91	16.621	3031.4	321.33	31.271	2154.3	360.97	49.431	0.9931
5.7	4109.2	233.29	16.748	3127.0	336.88	31.781	2214.6	378.50	50.418	1.0027
5.8	4246.7	242.86	16.870	3224.1	352.82	32.282	2275.9	396.47	51.391	1.0125
5.9	4386.6	252.61	16.988	3322.9	369.16	32.773	2338.1	414.90	52.349	1.0222
6.0	4528.8	262.53	17.101	3423.2	385.90	33.255	2401.2	433.78	53.294	1.0321
6.1	4673.5	272.64	17.209	3525.1	403.04	33.729	2465.2	453.12	54.224	1.0420
6.2	4820.5	282.92	17.314	3628.5	420.57	34.193	2530.0	472.90	55.140	1.0519
6.3	4969 9	293.37	17.414	3733.5	438.51	34.649	2595.9	493.14	56.042	1.0619
6.4	5121.7	304.01	17.510	3840.1	456.85	35.096	2662.6	513.83	56.931	1.0720
6.5	5275.9	314.82	17.603	3948.3	475.59	35.534	2730.2	534.98	57.805	1.0821
6.6	5432.5	325.81	17.692	4058.0	494.72	35.965	2798.7	556.57	58.667	1.0922
6 7	5591.4	336.97	17.778	4169.3	514.26	36.387	2868.1	578.62	59.515	1.1024
6.8	5752.8	348.31	17.861	4282.1	534.20	36.801	2938.4	601.12	60.350	1.1126
6.9	5916.5	359.83	17.941	4396.6	554.53	37.207	3009.6	624.08	61.171	1.1229
7.0	6082.7	371.52	18.018	4512.6	575.26	37.606	3081.7	647.48	61.981	1.1332
7.1	6251.2	383.38	18.092	4630.2	596.39	37.998	3154.7	671.34	62.777	1.1435
7.2	6422.1	395.42	18.164	4749.3	617.92	38.382	3228.6	695.65	63.561	1.1539
7.3	6595.4	407.63	18.233	4870.0	639.85	38.758	3303.4	720.41	64.333	1.1643
7.4	6771.0	420.02	18.300	4992.3	662.17	39.128	3379.1	745.62	65.094	1.1747
7.5	6949.1	432.58	18.364	5116.2	684.89	39.491	3455.7	771.28	65.842	1.1851

C.P. No. 1201

HER MAJESTY'S STATIONERY OFFICE

Government Bookshops

49 High Holborn, London WC1V 6HB
13a Castle Street, Edinburgh EH2 3AR
109 St Mary Street, Cardiff CF1 1JW
Brazennose Street, Manchester M60 8AS
50 Fairfax Street, Bristol BS1 3DE
258 Broad Street, Birmingham B1 2HE
80 Chichester Street, Belfast BT1 4JY

Government publications are also available through booksellers

CP. No. 1201

SBN 11 470469 4