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Layers in Axial Flow Turbomachines

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CALCULATION OF THE ANNULUS WALL BOUNDARY  
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SUMMARY

An integral method for calculating the turbulent wall boundary layers in axial flow turbomachines is described. The method is applied to flow through annular cascades and single and multistage machines. Agreement between prediction and experiment is good provided lift coefficients and flow deflections of the blade rows are small.

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Nomenclature/

A1

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\* Replaces A.R.C. 31 955

Nomenclature

c	velocity in boundary layer
C	velocity at edge of boundary layer
$c_f$	wall shear stress coefficient, $\tau_w / \frac{1}{2}\rho C^2$
f	blade force
p	static pressure
$\alpha$	flow angle relative to axial direction
$\delta$	absolute thickness of boundary layer
$\delta^*$	displacement thickness
$\theta$	momentum thickness
$\epsilon_w$	difference between $\alpha$ at the wall and $\alpha$ at the edge of the boundary layer
$\pi$	3.1416
$\Pi$	Coles' wake factor
$\rho$	density
$\tau$	shear stress
$\tau_w$	shear stress at wall
$\omega$	wall shear stress parameter, $\sqrt{c_f/2}$
x,y,z	co-ordinates, x axial, y tangential, z perpendicular to wall
s,n,z	co-ordinates, s streamline, n normal to streamline, z perpendicular to wall

## 1. Introduction

Several attempts have been made to calculate the development of the annulus wall boundary layer in axial flow turbomachines. Three approaches may be followed:-

- (i) An inviscid approach, following an entry shear flow through the machine, calculating the angle variations by secondary flow analysis and using these angles in a three-dimensional flow calculation, (e.g., Horlock (1963)).
- (ii) A boundary layer approach, in which integral momentum equations are written for the boundary layer, the blade force being eliminated by subtraction of the free stream momentum equation from the boundary layer momentum equation before integration, (e.g., Raily and Howard (1962) and Stratford (1967)).
- (iii) Use of empirical data for the growth of the boundary layer across a blade row, (e.g., Hanley (1968) and Smith (1970)).

Gregory-Smith (1970) has shown that the first approach gives accurate estimates of the axial velocity profiles for flow through an isolated rotor, if the exit boundary layer distribution is known (from which the secondary flow angle distribution and loss distribution may be determined). However, the problem of making a first estimate of this exit profile remains.

The approach followed here is essentially similar to that followed by Raily and Howard. The analysis, leading to two momentum integral equations (written in  $s, n$  co-ordinates, along and normal to the streamline outside the boundary layer) was outlined by Horlock (1970), and is reproduced briefly in the Appendix. Raily and Howard used axial ( $x$ ) and tangential ( $y$ ) co-ordinates, as did Hansen and Herzig (1956); Stratford simply writes the  $x$  momentum equation assuming collateral flow. In all these cases it is assumed that the blade force may be eliminated in the formation of the momentum integral equations.

It should be explained that the validity of these equations, and indeed of the form of the "difference" momentum equation (A5) (see Appendix) before integration,

$$\frac{1}{\rho} \frac{d\bar{r}}{dz} = (\bar{c} \cdot \nabla) \bar{c} - (\bar{c} \cdot \nabla) \bar{c} \quad \dots (A5)$$

where  $\bar{C}$  and  $\bar{c}$  are mean velocities across the blade pitch, is limited by the assumption that various terms are neglected in the momentum equations averaged across the pitch (A1, A2). Essentially, this amounts to assuming

- (i) That variations in flow across the blade pitch are small, which may be shown to imply that the local blade lift coefficient is small ( $C_L/4 \ll 1$ ).
- (ii) That variations in flow through the boundary layer are also small. This implies that the change of the flow angle from free stream to wall is small.
- (iii) That tip clearance effects may be ignored.

Thus/

Thus not only is the boundary layer assumption made (the pressure distribution is determined by the main stream and transmitted through the boundary layer) but also the idea of a small flow perturbation from the mainstream flow is implied, which is essentially the basic assumption of secondary flow analysis.

These are quite severe restrictions on the program that has been developed. But several important points result from the calculations that have been made, and these are discussed in detail below.

## 2. Analysis

The method of analysis is essentially a variation of a three-dimensional boundary layer analysis developed by Hoadley (1970) for swirling flow in a conical diffuser. (This in turn was an extension of a two-dimensional boundary layer analysis described by Lewkowicz et al. (1970).)

Hoadley wrote the momentum integral equations along the  $s$  (streamline) and  $n$  (normal) directions, for axisymmetric flow. By (justifiably) assuming axisymmetric flow he could express each of these two equations in terms of one independent variable  $x$ , with the flow angle outside the boundary layer ( $\alpha$ ) known as a function of  $x$ . Using also the entrainment equation derived by Cumpsty and Head (1967) and Coles' (1956) expression for the wall shear stress, he obtained four differential equations. By assuming the streamwise flow could be represented by Coles' velocity profile, and that the cross flow could be represented by the Mager (1952) profile, all four differential equations could be expressed in terms of four dependent variables,

- $\delta$  (boundary layer thickness)
- $\Pi$  (Coles' wake factor)
- $c_f$  (skin friction coefficient)
- $\epsilon_w$  (the difference between flow angle at the wall and flow angle in the mainstream)

Simultaneous solution of the equations by a Runge-Kutta method gave fair predictions of the swirling flow observed by Hoadley in the conical diffuser.

A similar approach has been followed in tackling the problem of the flow through a blade row. Equation (A5) is valid for the averaged boundary layer flow in the blade passage within the assumptions listed above. Integration of this equation yields the momentum integral equations (A6, A7) of the Appendix, but it is important to note that since the mean tangential velocity ( $\bar{C}_y$ ) changes in the flow through a blade row, an extra term (due to

the blade forces and represented by the "smeared" vorticity  $\zeta = \frac{d\bar{C}_y}{dx}$ )

appears in the equation, compared with Hoadley's original form. A similar term arises in the entrainment equation, but the wall shear stress law is assumed to be unchanged from Hoadley's formulation.

The differential equations for solution are thus a simple modification of Hoadley's original equations. The input has been simplified considerably, so that the only data required are starting values for  $\delta$ ,  $\Pi$ ,  $c_f$  and  $\epsilon_w$ , together with free stream data for  $\bar{C}_x(x)$  (mean velocity) and  $\bar{\alpha}(x)$  (mean flow angle). Values of the dependent parameters are calculated at downstream stations, and from these the streamwise and cross flow profiles are determined, together with the axial displacement and momentum thicknesses,

$$\delta_x^* = \int_0^\delta \left(1 - \frac{\bar{c}_x}{\bar{C}_x}\right) dz, \quad \theta_x = \int_0^\delta \frac{\bar{c}_x}{\bar{C}_x} \left(1 - \frac{\bar{c}_x}{\bar{C}_x}\right) dz$$

One important point is that by specifying  $\bar{C}_x$  and  $\bar{\alpha}(x)$  (mean axial velocity and mean absolute flow angle) there is no requirement to say whether the blade row through which the fluid flows is stationary or rotating. Equation (A4) is written in absolute co-ordinates, as are the momentum integral equations, and all are valid within the limits of the stated assumptions for the averaged flow through rotors of stators. Stagnation pressure changes do take place in rotors where the dot product of blade force and velocity ( $\bar{f} \cdot \bar{C}$ ) is non-zero, but this does not change equation (A5). Thus the boundary layer flow through a turbine stator row with given  $\bar{C}_x(x)$  and  $\bar{\alpha}(x)$  is the same as that through a compressor rotor row with the same  $\bar{C}_x(x)$  and  $\bar{\alpha}(x)$ .

### 3. Calculations

Attempts have been made to calculate three separate boundary layer flows:-

- (a) the flow through an isolated rotating cascade (described by Gregory-Smith (1970)). The annulus wall boundary layer on the casing is studied.
- (b) The casing boundary layer flow through a complete compressor stage of three rows (experiments described by Horlock (1963)).
- (c) the flow through a multi-stage Rolls-Royce axial compressor.

#### 3(a) Flow through an isolated rotating cascade

Fig.1 shows the axial velocity and angle variations through the boundary layer at exit from Gregory-Smith's isolated rotor. The entry boundary layer was assumed to be a flat plate boundary layer ( $\Pi = 0.55$ ) and from the measured displacement thickness  $\delta^*$ , the skin friction  $c_f$  and the "thickness"  $\delta$  were derived from the Coles profiles using the equations

$$\delta^* = \delta \sqrt{\frac{c_f}{2}} (1 + \Pi)/0.41$$

and

$$\sqrt{\frac{2}{c_f}} = \frac{1}{0.41} \log_e \left( \frac{0.41 R_{\delta^*}}{1 + \Pi} \right) + 5.0 + \frac{2\Pi}{0.41}$$

where  $R_{\delta^*}$  is the Reynolds number based on displacement thickness.

Gregory-Smith had calculated the angle and axial velocity variations at the casing from the Wu/Marsh program (Marsh (1968)) assuming that the annulus was running full, and his values of  $\bar{C}_x$ ,  $\bar{\alpha}(x)$  were used as input to the program.

Agreement between theory and experiment on axial velocity profiles is good, and the cross-flow is predicted quite well, although the assumption that the value of  $\bar{\alpha}(x)$  at the casing is the angle outside the boundary layer mars the cross flow prediction. The prediction of displacement and momentum thickness growth is shown in Fig.2 and a small improvement over Stratford's method can be seen.

It should be emphasised that this flow is one of small overall deflection, and that the limiting assumptions listed in the introduction are not exceeded in this calculation.

### 3(b) Flow through an axial compressor stage

The boundary layer flow through a single stage - guide vane, rotor, stator - was measured by Horlock (1963). Results for a calculation of this flow are given in Figs.3 and 4 (axial velocity profiles and angle variations). Although  $\Pi$  becomes negative it does not drop below -1 in the calculations and the Coles profile still has validity. Many "aerodynamic" boundary layer experiments achieve negative values of  $\Pi$  in accelerating flow, such as those of Herring and Norbury as shown by Coles and Hirst (1968).

The axial velocity profiles are quite well predicted, but the cross flow predictions are not as good. Clearly the Mager cross flow profile is not adequate to describe cross flows in turbomachine stages with large deflection. (Note that for the purpose of these calculations, free stream values of  $\bar{\alpha}(x)$  and  $\bar{C}_x(x)$  were taken from the experimental data, the edge of the boundary layer being assumed to be at the point of maximum stagnation pressure.)

It is of interest to note that guide vane and rotor produce effectively a double "acceleration", dropping  $\Pi$  progressively. The stator diffuses the flow, increasing  $\Pi$  back to about zero, (i.e., the boundary layer is virtually logarithmic). In view of the large changes in  $\Pi$  produced by the rapid changes in free stream conditions it is remarkable that the axial velocity profiles are so well predicted. The large changes in  $\epsilon_w$  and  $\bar{\alpha}$  produce the peak and subsequent decrease in the calculated axial displacement thickness shown in Fig.5.

### 3(c) Flow through a multi-stage compressor

Rolls-Royce provided a streamline curvature calculation of the flow through a four-stage Avon compressor. The values of  $\bar{\alpha}(x)$  and  $\bar{C}_x(x)$  at the casing were used to calculate the values of  $\delta_x^*$ ,  $\Pi$ ,  $c_f$  and  $\epsilon_w$  through the four stages.  $\delta_x^*$  and  $\Pi$  are shown in Fig.6, for the first three rows.

Again the pattern of "double acceleration" in guide vane and first rotor was apparent, but the calculation then loses validity because  $\Pi$  drops below -1. Values of  $\delta_x^*$  for a Stratford calculation are also shown.



#### 4. Discussion

Within the limitations initially stated - small variations of flow across the pitch (small lift coefficient), small variations through the boundary layer (small overall deflection), and negligible effects of tip clearance - the present method for calculating boundary layers through blade rows gives reasonable results. For example, the flow through Gregory-Smith's rotor is well predicted. However in conditions of large overall deflection and lift coefficient, (as exist in turbine blade rows or large deflection guide vane rows), the validity of the method is open to doubt.

The most striking feature of the compressor calculations described in Sections 3(b) and 3(c) is that the boundary layers never appear to be subjected to conditions which produce large values of the Coles wake parameter  $\Pi$ . This is in conflict with the conditions observed by Hanley (1968) in cascades (rapidly increasing  $\delta_x^*$ , giving a wall stall at large  $\Pi$ ).

However, the cross flow model used in the present calculation method does not allow for the under turning near the outer edge of the boundary layer which was present in Hanley's experiments.

There is room for several improvements in the analysis described here:-

- (i) A better model for the cross flow should be used. It is known that the cross flow profile depends on the blade aspect ratio and pitch-chord ratio (see Hawthorne (1955)). Use of a semi-analytical form for the cross flow profile in line with secondary flow predictions was suggested by Mellor and Wood (1970).
- (ii) The limitations of small lift coefficient and small deflection should be removed. Marsh (1970) suggests that equation (A5) is valid on a mean stream surface but that the averaged momentum integral equations (A6, A7) should contain an extra body force term which was overlooked into the present analysis. The significance of this term is currently being assessed.
- (iii) The effects of tip clearance should be allowed for.
- (iv) Alternative entrainment assumptions should be tested, especially under conditions of rapid acceleration.

#### 5. Conclusions

A boundary layer calculation method, for determining the end wall flow through blade rows of an axial turbomachine, has been compared with a range of experiments. For small deflection flows through blades of low lift coefficient, the method works satisfactorily. But for flows through blades of large deflection, the cross flow is poorly predicted and this leads to incorrect prediction of displacement thickness. It is expected that the calculation method would be improved if a more realistic cross flow model is used.

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APPENDIX/

APPENDIX

Horlock (1970) has derived momentum integral equations for the averaged boundary layer flow.

If the local velocities within the layer in an  $x, y$ , co-ordinate system are  $c_x, c_y$ , then the equations of motion averaged across the pitch  $S$  of the blade are given by

$$\frac{\partial \bar{\tau}_x}{\partial z} - \frac{1}{\rho} \frac{d\bar{p}}{dx} - \frac{(p_p - p_s)}{\rho S} \tan \alpha_b = \bar{c}_x \frac{\partial \bar{c}_x}{\partial x} + \bar{c}_z \frac{\partial \bar{c}_x}{\partial z} \quad \dots (A1)$$

$$\frac{\partial \bar{\tau}_y}{\partial z} + \frac{(p_p - p_s) \tan \alpha_b}{\rho S} = \bar{c}_x \frac{\partial \bar{c}_y}{\partial x} + \bar{c}_z \frac{\partial \bar{c}_y}{\partial z} \quad \dots (A2)$$

where  $\tau_x, \tau_z$  are the shear stress,  $p$  the pressure, subscripts  $p$  and  $s$  indicate pressure and suction surfaces and a superscript  $\bar{\quad}$  indicates an average across the pitch. (The blades are assumed to be thin and defined by the angle  $\alpha_b$ ).

Neglect of second order terms in the above equations in comparison with those retained involves assuming that ratios of terms such as

$$\chi = \frac{\frac{d}{dx} (\overline{c'_x c'_y})}{\bar{c}_x \frac{d\bar{c}_y}{dx}}$$

are small, where  $c'_x, c'_y$  are the maximum variations of the axial and tangential velocities from the mean values (i.e.,  $c_x = \bar{c}_x + c'_x$ ,  $c_y = \bar{c}_y + c'_y$ ).

$\frac{c'_x}{\bar{c}_x}$  may be shown to be of order  $(F_y/2 \rho \bar{c}_x^2 \sec^2 \alpha_b b)$  where  $F_y$  is the tangential force  $\int_0^b (p_p - p_s) dx$  and  $b$  is the blade axial chord.

This implies that  $c'_x/\bar{c}_x \approx C_L/4$  and  $c'_y/\bar{c}_y$  is of similar order of magnitude. It may be shown that  $\chi$  is of order  $C_L/4$ . This is satisfied by lightly loaded compressor blades but not by highly loaded turbine blades.

With/

With  $\chi$  and similar terms small equations (A1) and (A2) may be written

$$\frac{1}{\rho} \frac{d\bar{\tau}}{dz} + \bar{f} - \frac{\nabla \bar{p}}{\rho} = (\bar{c} \cdot \nabla) \bar{c} \quad \dots (A3)$$

where vector quantities are now mean across the pitch, and

$$\bar{f} = \frac{p_p - p_s}{\rho S} \sec \alpha_b \quad \dots (A4)$$

In the main stream

$$\bar{f} - \frac{\nabla \bar{p}}{\rho} = (\bar{c} \cdot \nabla) \bar{c} \quad \dots (A4)$$

and subtraction of (A4) from (A3) yields

$$\frac{1}{\rho} \frac{d\bar{\tau}}{dz} + (\bar{c} \cdot \nabla) \bar{c} = (\bar{c} \cdot \nabla) \bar{c} \quad \dots (A5)$$

A more general discussion of this problem for a mean stream surface, without the assumption of  $\chi$  small, is given by Marsh (1970).

From integration of (A5) between  $z = 0$  and  $z = \delta$ , the momentum equations may be derived in the form

$$\begin{aligned} \frac{d\theta_{11}}{ds} + \frac{d\theta_{12}}{dn} + \frac{1}{C} \left( \frac{dC}{ds} \right)_n (\theta_{11} + \delta_1^*) - K_1 (\theta_{11} - \theta_{22}) \\ - \frac{\zeta}{C} (\theta_{12} + \delta_2^*) = \frac{\tau_w}{\rho C} 2 \cos \epsilon_w \quad \dots (A6) \end{aligned}$$

$$\begin{aligned} \frac{d\theta_{21}}{ds} + \frac{d\theta_{22}}{dn} + \frac{2}{C} \left( \frac{dC}{ds} \right)_n \theta_{21} + \frac{1}{C} \left( \frac{dC}{dn} \right)_s (\theta_{22} + \theta_{11} + \delta_1^*) \\ - 2K_1 \theta_{21} + \frac{\zeta}{C} (\theta_{11} + \delta_1^* - \theta_{22}) = \frac{\tau_w}{\rho C} 2 \sin \epsilon_w \quad \dots (A7) \end{aligned}$$

where momentum and displacement thickness are

$$\begin{aligned} \delta_1^* &= \int_0^\delta \left(1 - \frac{c_s}{C}\right) dz & \delta_2^* &= - \int_0^\delta \frac{c_n}{C} dz \\ \theta_{11} &= \int_0^\delta \left(1 - \frac{c_s}{C}\right) \frac{c_s}{C} dz & \theta_{21} &= - \int_0^\delta \frac{c_s c_n}{C^2} dz \\ \theta_{12} &= \int_0^\delta \left(1 - \frac{c_s}{C}\right) \frac{c_n}{C} dz & \theta_{22} &= - \int_0^\delta \frac{c_n^2}{C^2} dz \end{aligned}$$

and  $C$  is the resultant velocity at the edge of the boundary layer,  $K_1$  is the convergence/divergence of the streamlines at the edge of the layer, and  $\zeta$  is the vorticity at the edge in the  $z$  direction.

For an axisymmetric flow, these equations may be written in terms of the single independent variable  $x$ , the flow angle  $\bar{\alpha}(x)$ , and the axial velocity  $\bar{C}_x(x)$  outside the boundary layer.

The entrainment equation may be written

$$\begin{aligned} \frac{d}{ds} (\delta - \delta_1^*) - \frac{d\delta}{dn} 2^* &= F (H_\delta - \delta^*) - (\delta - \delta_1^*) \left( \frac{1}{C} \frac{dC}{ds} - K_1 \right) \\ &\quad - \frac{\delta_2^* \zeta}{C} \end{aligned} \quad \dots (A8)$$

where  $F$  is Head's (1958) entrainment function and  $H_\delta - \delta^* = \frac{\delta - \delta_1^*}{\theta_{11}}$

The wall shear stress equation in differential form is

$$\frac{1}{\delta} \frac{d\delta}{ds} + \frac{2d\Pi}{ds} + \left( \frac{1}{\omega} + \frac{0.41}{\omega^2} \right) \frac{d\omega}{ds} = - \frac{1}{C} \frac{dC}{ds} \quad \dots (A9)$$

where  $\omega = \sqrt{\frac{c_f}{2}}$

Writing the streamwise velocity profile in Coles' form

$$\frac{C - c_s}{C} = \frac{1}{0.41} \sqrt{\frac{c_f}{2}} \left[ \Pi \left( 1 + \cos \frac{\pi z}{\delta} \right) - \log_e \frac{z}{\delta} \right] \quad \dots (A10)$$

and the cross flow velocity profile in Mager's form

$$\frac{c_n}{c_s} = \left(1 - \frac{z}{\delta}\right)^2 \tan \epsilon_w$$

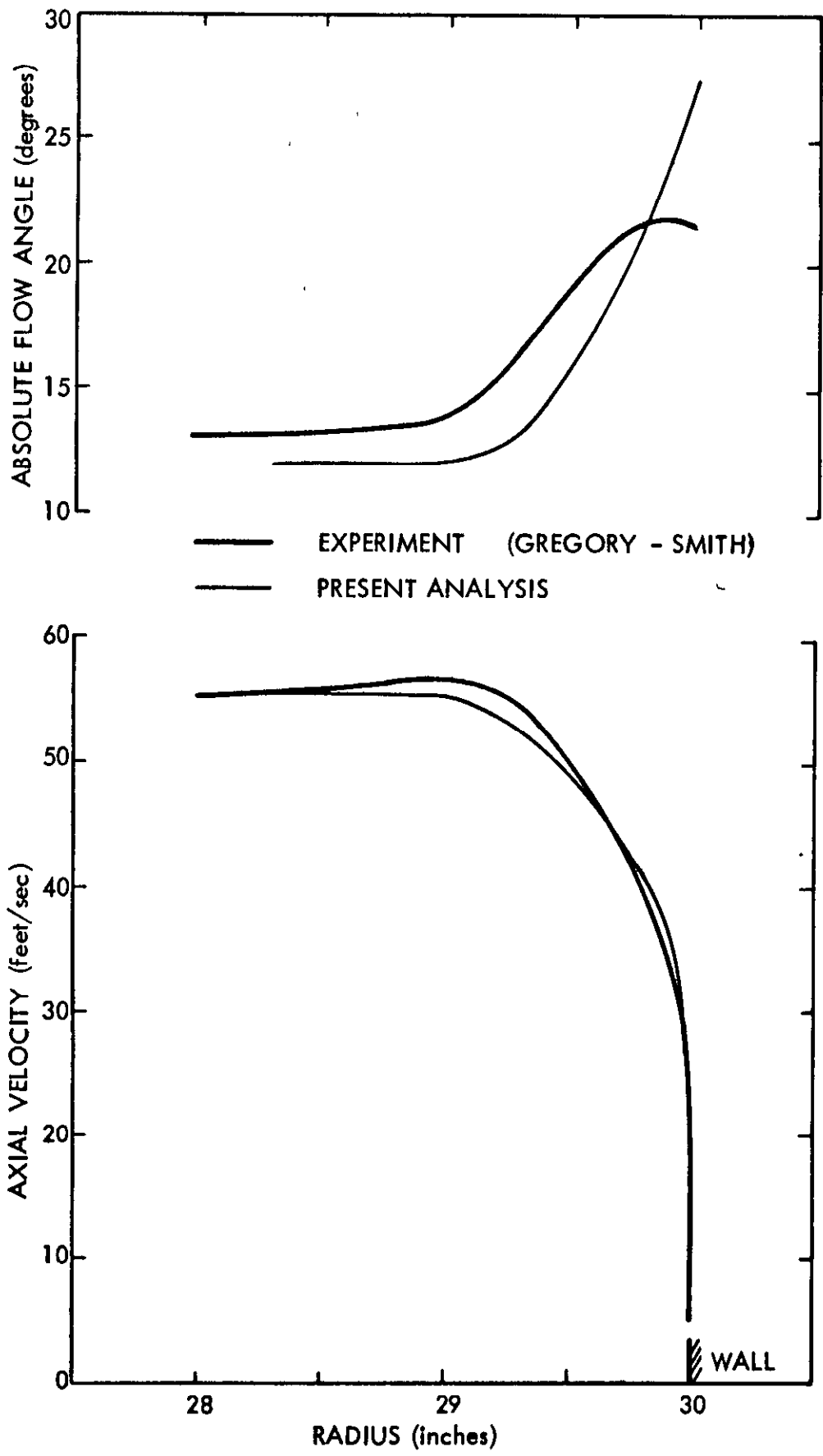
equations (A6 - (A9) may be reduced to the form

$$A_i \frac{d\delta}{dx} + B_i \frac{d\Pi}{dx} + C_i \frac{dc_f}{dx} + D_i \frac{d\epsilon_w}{dx} = E_i$$

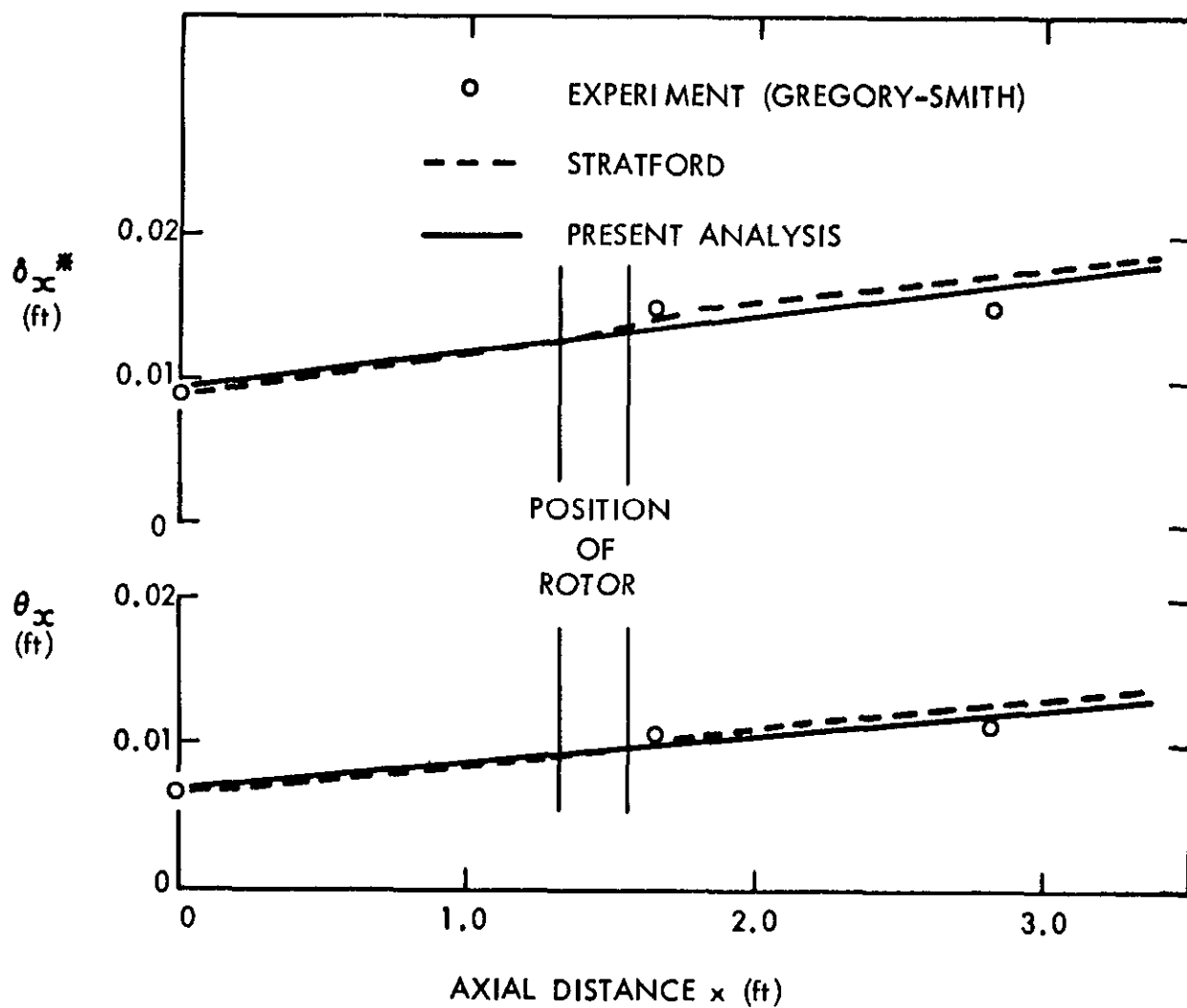
where  $i = 1, 2, 3, 4$  and  $A_i, B_i, C_i, D_i$  and  $E_i$  are functions of  $\delta, \Pi, c_f, \epsilon_w, \bar{a}(x)$  and  $\bar{C}_x(x)$ . (A12) may be solved by Runge-Kutta techniques.

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**FIG.1. OUTER WALL FLOW AT EXIT FROM ISOLATED ROTOR**



**FIG. 2. OUTER WALL BOUNDARY LAYER GROWTH THROUGH ISOLATED ROTOR**

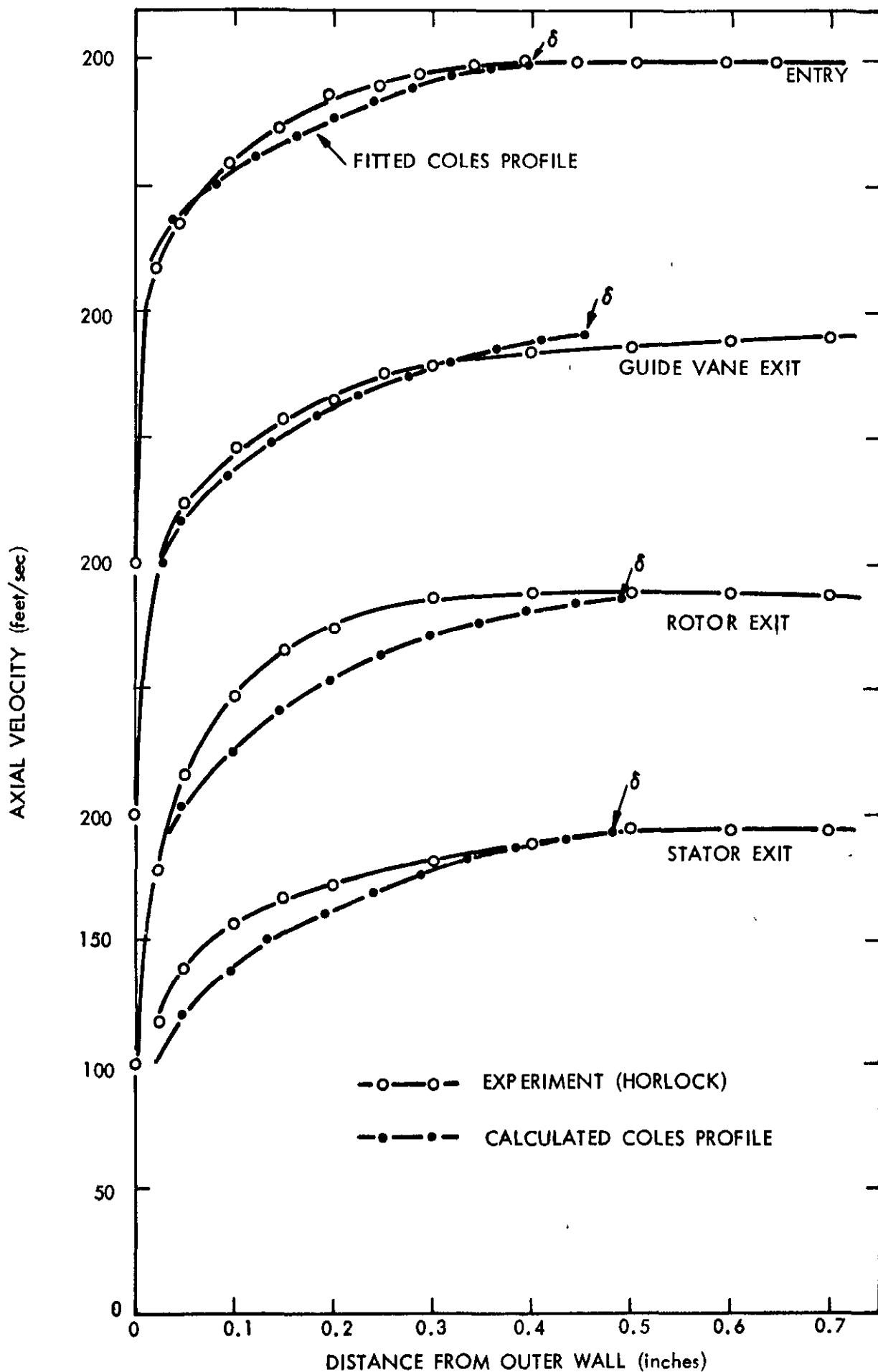


FIG. 3. FLOW THROUGH AXIAL COMPRESSOR STAGE

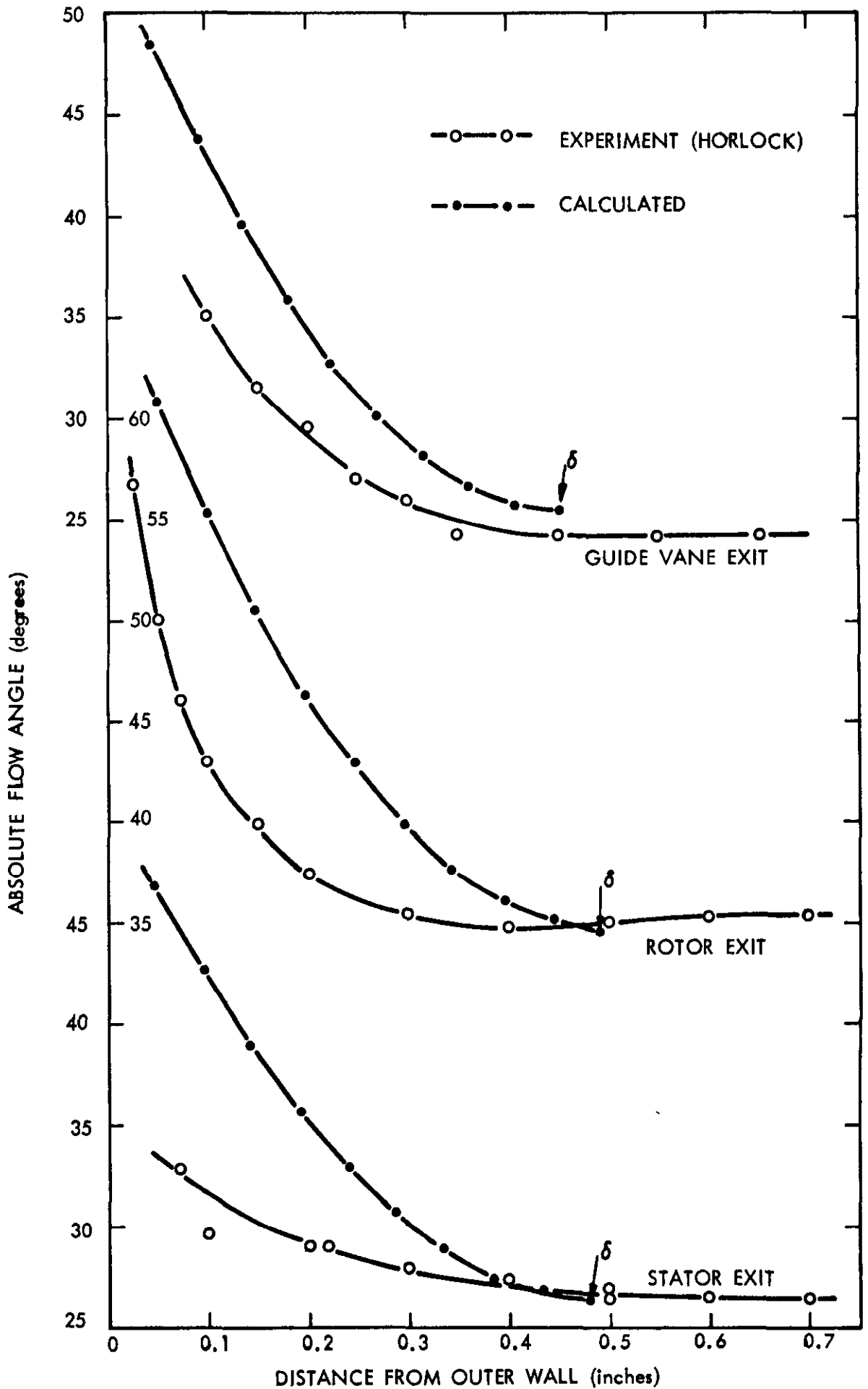
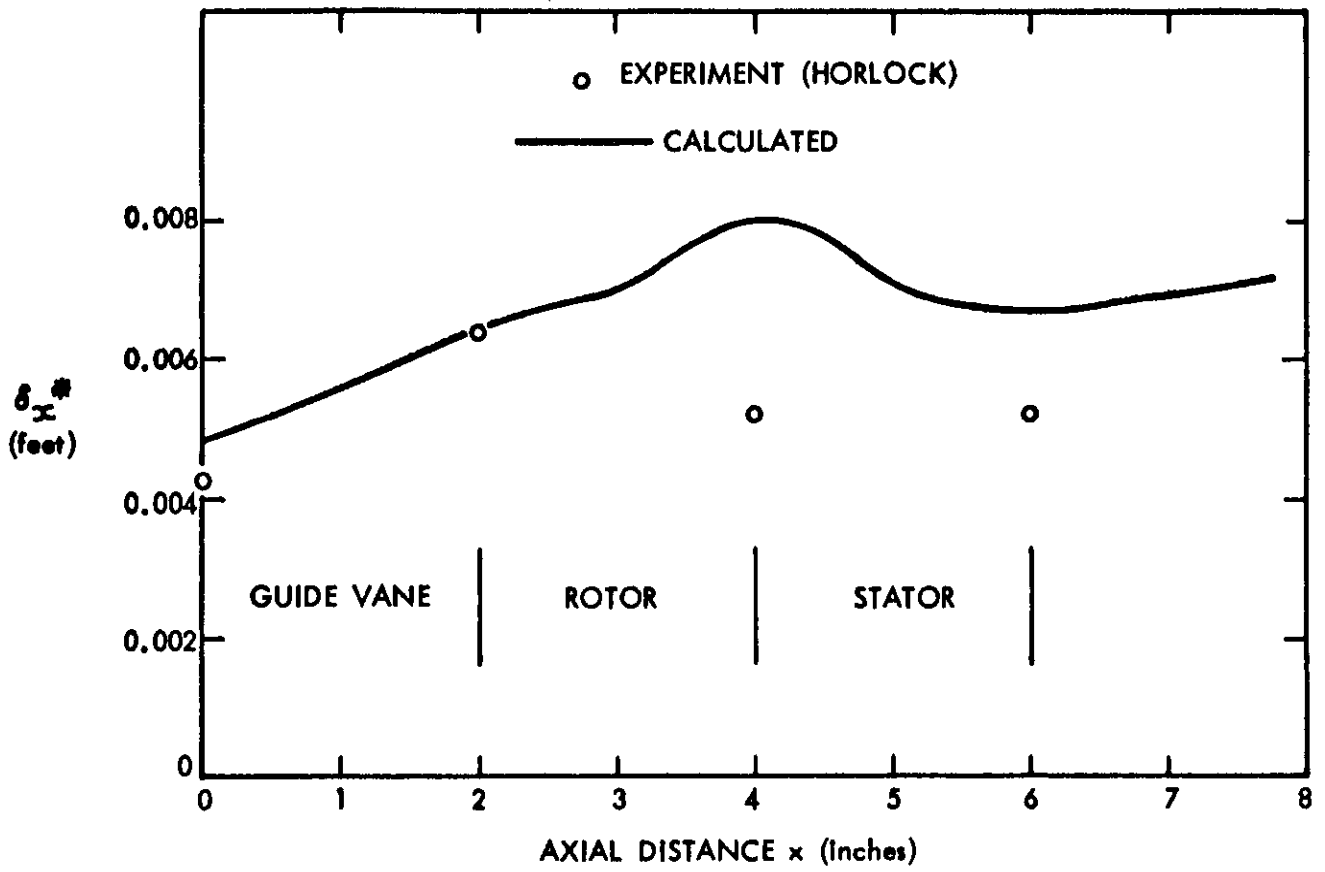


FIG. 4. FLOW THROUGH AXIAL COMPRESSOR STAGE



**FIG. 5. OUTER WALL BOUNDARY LAYER GROWTH THROUGH AXIAL COMPRESSOR STAGE**

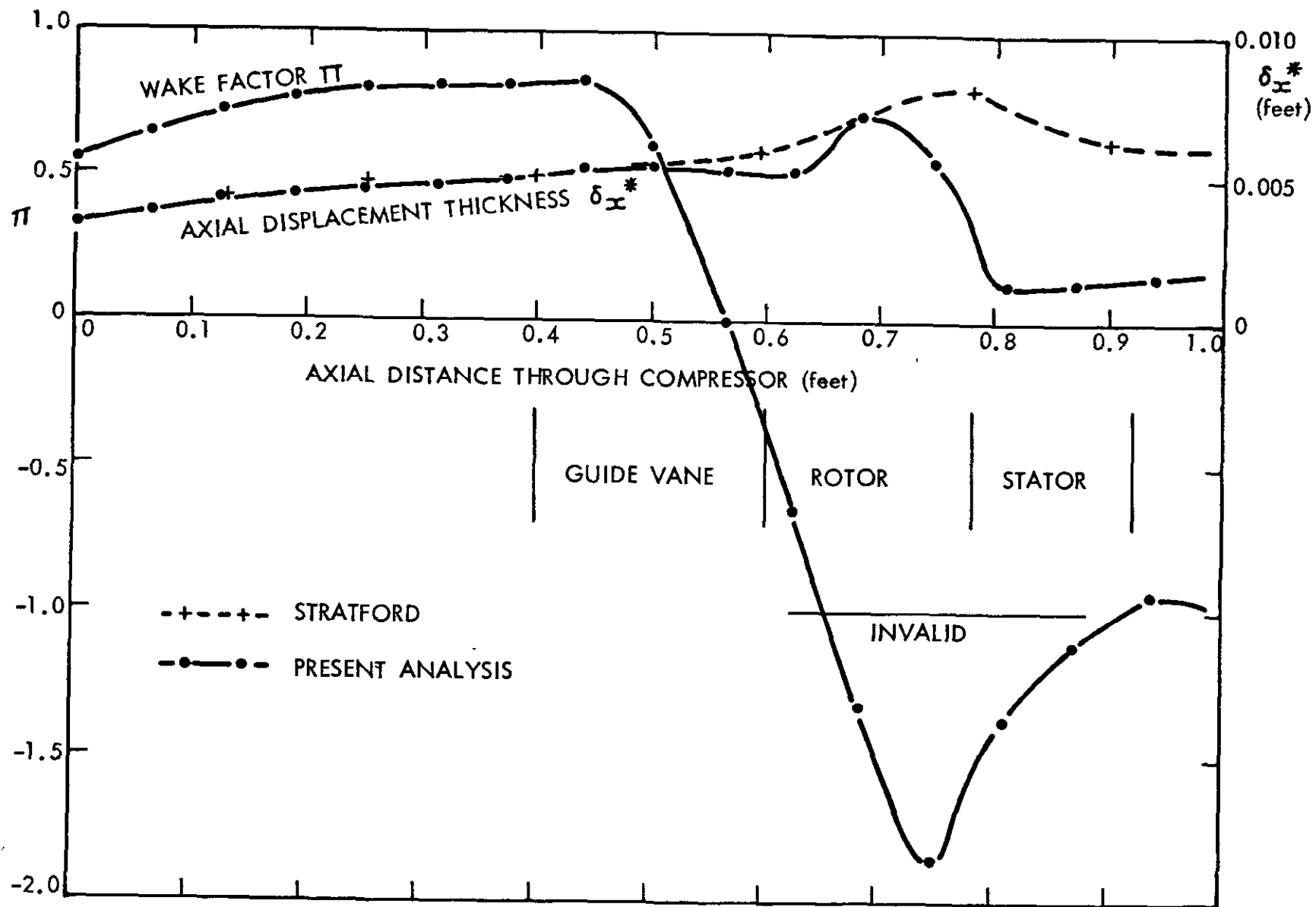


FIG. 6. FLOW THROUGH AVON COMPRESSOR (BASED ON ROLLS ROYCE DATA)



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