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Vertical Cockpit Accelerations
Measured on an Operational
Jet Transport Aircraft

by

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1971

PRICE 40 p NET

C.P. No.1183

October 1969*

VERTICAL COCKPIT ACCELERATIONS MEASURED ON AN OPERATIONAL
JET TRANSPORT AIRCRAFT

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G. B. Hutton

SUMMARY

An analysis has been made of vertical accelerations recorded in the cockpit of a Boeing 707-436 jet transport aircraft during take-off and landing. Twenty-five take-offs and twenty-five landings from twenty airports were studied. The results could be used to prepare a flight simulator input programme representing a generalized flight deck vibration environment of a subsonic jet transport aircraft.

Turbulence response data from 9436 flying hours on the same type of aircraft was used to obtain assessments of the probability of exceeding various levels of cockpit acceleration while airborne for comparison with the high speed taxiing phases of landing and take-off.

* Replaces RAE Technical Report 69214 - ARC 32768

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1 INTRODUCTION

During take-off and landing a pilot is subjected to low frequency vibration in the vertical and lateral planes. The frequency and amplitude of the oscillations depend upon the resonance characteristics of the aircraft, the runway surface profile and the aircraft ground speed. Up to the present time vibration of civil transport aircraft during the landing and taxiing phases of operation under normal conditions (viz. excluding cases of shimmy) has been insufficient to impair the mental or physical capabilities of pilots. However, theoretical work carried out to assess the vertical accelerations produced at the flight deck during take-off on future large flexible aircraft has indicated that high incremental accelerations may be experienced and therefore the performance of the pilot may be impaired.

This aspect of the pilot's environment is being evaluated using a cockpit vibration simulator at the British Aircraft Corporation, Weybridge. It is envisaged that vertical motions typical of those encountered on a large jet aircraft will be fed into a simulator and the resulting pilot performance assessed.

A Boeing 707-436 instrumented to provide selected analogue flight data for the Civil Aircraft Airworthiness Data Recording Programme (CAADRP)¹, was fitted with a special accelerometer supplied by Aerodynamics Department to assess vibrational environments likely to affect crew performance. The accelerometer was attached to a deep-sectioned floor beam 3 feet behind the captain's seat and had a natural frequency of 7 Hz and 70% damping. It is shown² that beyond 7 Hz accelerations at the levels encountered become increasingly unimportant to the pilot.

2 ANALYSIS OF RECORDS

The plan view and elevations of the aircraft instrumented, with the position of the accelerometer indicated, are shown in Fig.1.

Fig.2 presents recordings of one landing and take-off from the sample analysed.

From a total of 94 flights measured, 34 consecutive flights were examined. From these, nine take-off and nine landing runs were rejected as the character of the vibration was too complex to measure, but the severity of these flights was not abnormal. Due to the relatively low recording film speed adopted, i.e. 1.3 inches per minute, oscillations above 2 Hz could not

be resolved; their peaks but not frequency, could be measured. Where there was no dominant frequency of less than 2 Hz the time spent at frequencies in the range 2 to 6 Hz was shown as a uniform distribution as shown in Figs.3 to 5; it was appreciated that a predominant frequency of 5.4 Hz, corresponding to the fundamental fuselage mode, might be present but it was not practicable to evaluate it. Frequencies were measured by counting the number of positive acceleration peaks (upward accelerations were designated positive and downward, negative) and dividing by the time over which they recurred regularly.

From each take-off and landing run in the sample, dominant response frequencies and their duration were measured. The mean positive of the acceleration peaks and of the negative acceleration peaks over each of these periods was also measured. The measurements are presented in Tables 1 and 2 for the take-off and landing runs respectively. Frequencies were grouped in bandwidths of 0.2 Hz and histograms produced of frequency plotted against percentage of the sample time period. Similarly, the mean amplitudes of the accelerations were grouped in bandwidths of 0.05 Δg and plotted against per cent sample time in the form of histograms. The envelope shape of each complete run was noted. The landing and take-off phases, in addition to being treated as a whole, were considered separately in order to determine the differences in aircraft response. Any recordings at speeds below 30 knots ias were disregarded as accelerations below this speed were found to be insignificant.

3 RESULTS

3.1 Take-off and landing severity

The frequency distributions with time for the take-off, landing and the two phases amalgamated are displayed in Figs.3, 4 and 5 respectively. Fig.3 shows that 0.8 Hz was dominant for 53% of the take-off sample time and high frequency oscillations only (i.e. above 2 Hz) were present for 7% of the time. Fig.4 indicates that 82% of the landing sample was distributed fairly evenly between frequencies from 0.6 to 1.0 Hz, while high frequencies only were present for 30% of the time. The histogram of the amalgamated results, Fig.5, also shows 0.8 Hz to be predominant, in this case occurring 34% of the total time. High frequency oscillations dominated for 20% of the total.

Figs.6 and 7 show the distribution in time of the mean amplitudes measured at the dominant frequencies observed during the take-off and landing phases respectively. Fig.8 presents the distribution for the amalgamated results of both phases. On each figure is quoted the calculated mean amplitudes and the

standard deviations. The diagrams show that average values were greater during the landing phase by about 40%; probably caused by high nosewheel loads during braking. This may not be significant, however, owing to the physiological demands upon the crew being less severe than in the take-off case. It is much more important that the intensity should be acceptably low during the take-off run when a high degree of concentration and accurate instrument monitoring is demanded of the crew and errors on their part could be disastrous, particularly during the last 10 seconds. The calculated mean values show that positive accelerations were greater than the negative by 10% in the take-off cases and 20% in the landings. Averaging positive and negative values, the mean amplitude of the cockpit acceleration exceeded $0.25 \Delta g$ during take-off and $0.35 \Delta g$ during landing for 25 seconds during the twenty-five flights, representing 81 flying hours, that have been analysed.

The average take-off duration of the sample was 26 seconds and the average landing duration 32 seconds.

The examples of a landing and take-off shown in Fig.2 have cockpit vibration envelope shapes which were typical of the majority studied. In the take-off case peak acceleration values varied progressively, commencing at a very low level and reaching a maximum at the start of rotation and thereafter falling rapidly to an insignificant level at the moment of lift-off. The accelerations in the landing phase commenced at a high level and remained so for about 50% of the time before steadily decaying to a low level on completion of the landing run. The take-off run illustrated lasted 27 seconds and comprised a dominant frequency of 1.1 Hz, with a superimposed higher frequency, at a mean amplitude of $\pm 0.25 \Delta g$. The duration of the landing run was approximately 37 seconds. The first 11 seconds comprised a high frequency component superimposed upon an oscillation of 0.6 Hz. This was followed by 13 seconds of predominantly high frequency vibration and the remainder at 0.8 Hz with high frequencies superimposed.

At the instant of touchdown on a moderately heavy landing there are high peak accelerations for an extremely short time which are not included in the analysis

3.2 Comparison with severity in turbulence

A study has been made of cg accelerations measured in periods of severe (storm) turbulence during 9436 flying hours on a Boeing 707-436³. In order to compare the time spent above different intensities of normal acceleration in

the cockpit resulting from high speed taxiing with the time spent above the same levels of intensity of approximately the same duration while airborne the results from the storm turbulence study were processed as described below.

The ratio of cockpit to cg acceleration in flight was estimated at 0.75 from the data displayed in Fig.9 (see Appendix B). This is the most severe patch of turbulence in which both cockpit and cg accelerations have been measured on this aircraft and corresponds to a mean peak acceleration at the cockpit of 0.22 g increment.

Appendix C derives the ratio of the mean of the acceleration peaks to the rms of the acceleration time history as 1.25 assuming the acceleration time history is a random stationary Gaussian process. Therefore, in flight $1 \Delta g$ rms at the cg corresponds to cockpit mean peak accelerations of $0.94 \Delta g$ (i.e. $0.75 \times 1.25 \Delta g$).

The airborne phase curve in Fig.10 was derived by using the factor of 0.94 on King's data. Added to the figure for comparison are curves derived from the high speed taxiing records. The positive and negative acceleration values of individually measured patches within each take-off or landing run were combined and a mean level for the whole run thereby obtained for comparison with the turbulence data.

The turbulence encounters included three incidents of relatively very long duration and the median duration is therefore appreciably smaller than the average. There are no landings or take-offs that have either a relatively very long or a very short duration and the median duration is almost identical to the average. The median durations of landing and take-off combined and of the severe turbulence are both $\frac{1}{2}$ minute.

Levels of intensity below $0.2 \Delta g$ are not considered significant and comparisons are confined to levels above this. Comparison between the take-off run curve and the airborne curve is only possible between the intensity levels of 0.2 and $0.25 \Delta g$ as no take-off runs in the sample studied had a mean level greater than $0.25 \Delta g$; the sample represented 81 flying hours and the data cannot reasonably be extrapolated to evaluate the intensities that would occur in one take-off during the 9436 hours of the turbulence records. Between the levels of 0.2 and $0.25 \Delta g$ the proportion of flying time for which the indicated levels due to take-off runs were exceeded was about 10 to 30 times greater than the proportion of flying time spent above the same levels due to the storm turbulence.

The curve derived from the landing run measurements extends to the same intensities as the airborne curve and on comparing these it is seen that the times for which acceleration levels due to the landing runs were exceeded are 50 to 100 times those of the airborne case. Although the cockpit vibration intensity was greater during the landing runs than the take-off it is much more important that the intensity should be acceptably low during take-off as was mentioned in 3.1.

4 CONCLUSIONS

The information presented provides some knowledge of cockpit vibrations on operational aircraft during the landing and take-off runs and can be used to provide a generalized but more realistic cockpit vibration simulation than has been possible in the past.

The acceleration intensity was found to be 40% greater in the landing than the take-off case but the affect of this on the crew is most probably offset by psychological demands upon them being less severe during the landing than the take-off. A comparison was made between the proportion of time spent exceeding different levels of cockpit acceleration intensity attributable to ground vibration and to turbulence. It was found that, within the range of intensities measured, the time spent exceeding different levels due to high speed taxiing is greater than that due to turbulence.

Acknowledgement

The author thanks Mr. N.I. Bullen for providing the calculations presented in Appendices A and C.

Appendix A

FITTING A RAYLEIGH DISTRIBUTION BY MAXIMUM LIKELIHOOD

A.1 Rice⁴ has shown that the peak values of a random variable approximately have a Rayleigh distribution. So let the probability of value between x and $x + dx$ be

$$\frac{x}{a^2} e^{-x^2/2a^2} dx$$

where a = rms of acceleration.

(Integral from 0 to ∞ = 1.)

Maximizing:

$$\prod_{p=1}^N \left(\frac{x_p}{a^2} e^{-x_p^2/2a^2} \right)$$

where N = number of peaks

x_p = peak crossing threshold.

Or

$$\sum_{p=1}^N \left(-\frac{x_p^2}{2a^2} + \log x_p - 2 \log a \right) .$$

Differentiating with respect to a

$$\frac{\sum x_p^2}{a^3} = \frac{2N}{a}$$

$$a^2 = \frac{\sum_{p=1}^N x_p^2}{2N} .$$

A.2 As it is usually uneconomical to count peaks below an arbitrary threshold, c , a Rayleigh distribution truncated at the lower end must be considered. Therefore by maximum likelihood as before let probability of value between x and $x + dx$ be

$$\frac{x}{a^2} e^{c^2 - x^2/2a^2} dx$$

(Integral from c to ∞ = 1)

x in range c to ∞ .

Maximizing:

$$\sum \left(\frac{c^2 - x_p^2}{2a^2} + \log x_p - 2 \log a \right) .$$

Differentiating with respect to a

$$-\frac{N c^2 - \sum x_p^2}{a^3} - \frac{2N}{a} = 0$$

$$a^2 = -\frac{c^2}{2} + \frac{\sum x_p^2}{2N}$$

$$= \frac{1}{2} \left(\frac{\sum x_p^2}{N} - c^2 \right) .$$

Appendix B

COMPARISON OF COCKPIT AND CG ACCELERATION DURING THE PERIOD OF
TURBULENCE DEPICTED IN FIG.9

The cg rms acceleration during turbulence situations in Ref.3 was assessed from the distribution of peaks of acceleration at the aircraft cg. The rms of the cg and cockpit acceleration for the 3 minute duration of turbulence depicted in Fig.9 was determined by the same method. The distributions of peaks of cg and cockpit accelerations were used to determine rms accelerations from the formula:-

$$a^2 = \frac{1}{2} \left(\frac{\sum_{p=1}^{p=N} x_p^2}{N} - c^2 \right) \quad (\text{Ref. Appendix A})$$

where a = rms of acceleration

x_p = $\pm \Delta g$ level

c = lowest Δg threshold

N = number of peaks exceeding c.

Distribution of peaks of acceleration

$\pm \Delta g$ level (x_p)	(N) number of peaks > (x_p)	
	cg	cockpit
0.20		36
0.25	46	15
0.30	14	12
0.35	9	3
0.40	5	0
0.45	4	
0.50	2	
0.55	1	

Substituting the values in the above table into the formula gives a rms value of 0.24 g for the cg and 0.18 g for the cockpit. Thus the ratio of cockpit to cg acceleration during this patch of turbulence was 0.75.

Appendix C

DERIVATION OF THE RELATIONSHIP BETWEEN THE MEAN OF PEAK ACCELERATIONS
AND THE ROOT MEAN SQUARE VALUE OF THE ACCELERATION TIME HISTORY
FOR GAUSSIAN DISTRIBUTIONS

In deriving the above relationship it is assumed that x is a random stationary Gaussian process and that x and \dot{x} are independent.

The distribution of x is given by

$$\frac{1}{a\sqrt{2\pi}} e^{-x^2/2a^2}$$

where $a = \text{rms}$.

Since x and \dot{x} are independent the number of crossings of a level x is proportional to the time spent in the range x to $x + dx$ and is thus proportional to $e^{-x^2/2a^2}$.

If N_0 is the number of positive zero crossings and N_x the number of positive crossings of x , then

$$N_x = N_0 e^{-x^2/2a^2}$$

It is now assumed that the crossing distribution is approximately equal to the cumulative peak distribution so that the number of peaks above x is $N_x = N_0 e^{-x^2/2a^2}$.

The number of peaks between x and $x + dx$ is

$$N_0 \frac{x}{a^2} e^{-x^2/2a^2} dx$$

and the mean value of the positive peaks is

$$\int_0^{\infty} x N_0 \frac{x}{a^2} e^{-x^2/2a^2} dx / N_0$$

Evaluate the integral by substituting $x^2/2a^2 = u$ giving:

$$\begin{aligned}\int_0^{\infty} a \sqrt{2} \sqrt{u} e^{-u} du &= a \sqrt{2} \Gamma\left(1\frac{1}{2}\right) \\ &= a \sqrt{\frac{\pi}{2}} \quad \left(\text{since } \Gamma\left(1\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2}\right) \\ &= 1.2533 a \quad .\end{aligned}$$

Table 1

COCKPIT ACCELERATION FREQUENCY AND INTENSITY DURING TAKE-OFF RUNS

Run No.	Time (sec)	Freq. (Hz)	Mean of peaks (Δg)		Run No.	Time (sec)	Freq. (Hz)	Mean of peaks (Δg)	
			Pos.	Neg.				Pos.	Neg.
1	20	1.10	0.15	0.10	14	38	1.10	0.20	0.20
1	18	0.88	0.20	0.20	14	6	2.0	0.15	0.15
2	16	0.83	0.15	0.10	15	28	0.85	0.10	0.10
3	23	1.70	0.10	0.10	16	31	0.83	0.15	0.15
4	28	0.80	0.20	0.15	16	9	2.0	0.15	0.15
5	21.5	0.75	0.15	0.10	17	22	0.77	0.25	0.20
6	19.5	0.90	0.20	0.20	18	22	0.50	0.20	0.15
7	34	0.90	0.20	0.20	19	27	1.10	0.25	0.25
8	23	0.80	0.20	0.20	20	25	2.0	0.10	0.10
9	17.5	1.20	0.15	0.15	21	25	0.75	0.25	0.20
10	30	0.80	0.10	0.10	22	14	0.55	0.20	0.15
10	10	2.0	0.10	0.10	22	11	0.90	0.20	0.15
11	20	0.83	0.15	0.10	23	25	0.80	0.25	0.20
12	16.5	0.66	0.25	0.20	24	25	1.10	0.30	0.20
13	35	1.10	0.15	0.15	25	16	2.0	0.15	0.10

Table 2

COCKPIT ACCELERATION FREQUENCY AND INTENSITY DURING LANDING RUNS

Run No.	Time (sec)	Freq. (Hz)	Mean of peaks (Δg)		Run No.	Time (sec)	Freq. (Hz)	Mean of peaks (Δg)	
			Pos.	Neg.				Pos.	Neg.
1	26	0.90	0.25	0.20	12	11	>2.0	0.25	0.20
1	5½	0.60	0.25	0.25	12	11	1.00	0.25	0.20
2	13½	0.66	0.3	0.25	13	4	0.55	0.25	0.25
2	15½	>2.0	0.25	0.20	13	30	1.00	0.25	0.20
3	43	0.66	0.20	0.20	14	10	0.60	0.45	0.25
4	5	0.58	0.20	0.20	14	27	1.00	0.25	0.20
4	27	0.94	0.20	0.15	15	8	0.65	0.30	0.25
5	16	1.00	0.20	0.20	15	30	1.10	0.25	0.20
5	7	>2.0	0.25	0.20	16	29	>2.0	0.30	0.25
5	5½	0.57	0.25	0.20	17	36	>2.0	0.30	0.30
6	7	0.63	0.35	0.30	18	8	0.48	0.30	0.20
6	16	>2.0	0.25	0.25	18	35	0.80	0.25	0.20
6	10	0.90	0.40	0.30	19	25	0.60	0.35	0.30
6	8	>2.0	0.15	0.15	20	5	0.62	0.50	0.30
7	4½	0.30	0.30	0.25	20	9½	>2.0	0.35	0.25
7	30	0.88	0.25	0.25	20	11	0.94	0.25	0.20
8	2	0.55	0.30	0.20	21	6	0.64	0.50	0.30
8	23	2.0	0.25	0.20	21	6	>2.0	0.30	0.25
9	5	0.55	0.50	0.35	21	14	0.70	0.30	0.20
9	18	>2.0	0.30	0.25	22	4	0.62	0.25	0.20
9	10	0.85	0.15	0.15	22	24	>2.0	0.30	0.20
10	5½	0.65	0.35	0.25	23	3½	0.78	0.25	0.20
10	4½	0.88	0.25	0.20	23	26	>2.0	0.25	0.20
10	14½	>2.0	0.30	0.25	24	26	1.00	0.25	0.20
11	4	0.55	0.30	0.25	25	9	>2.0	0.30	0.25
11	36	>2.0	0.25	0.20	25	14½	0.88	0.25	0.20

NOTATION

a	root mean square of acceleration
c	lowest threshold of peak counts
N	number of peaks exceeding lowest threshold
N_0	number of positive zero crossings
N_x	number of positive crossings of x
x	acceleration level from datum
x_p	acceleration peak crossing threshold

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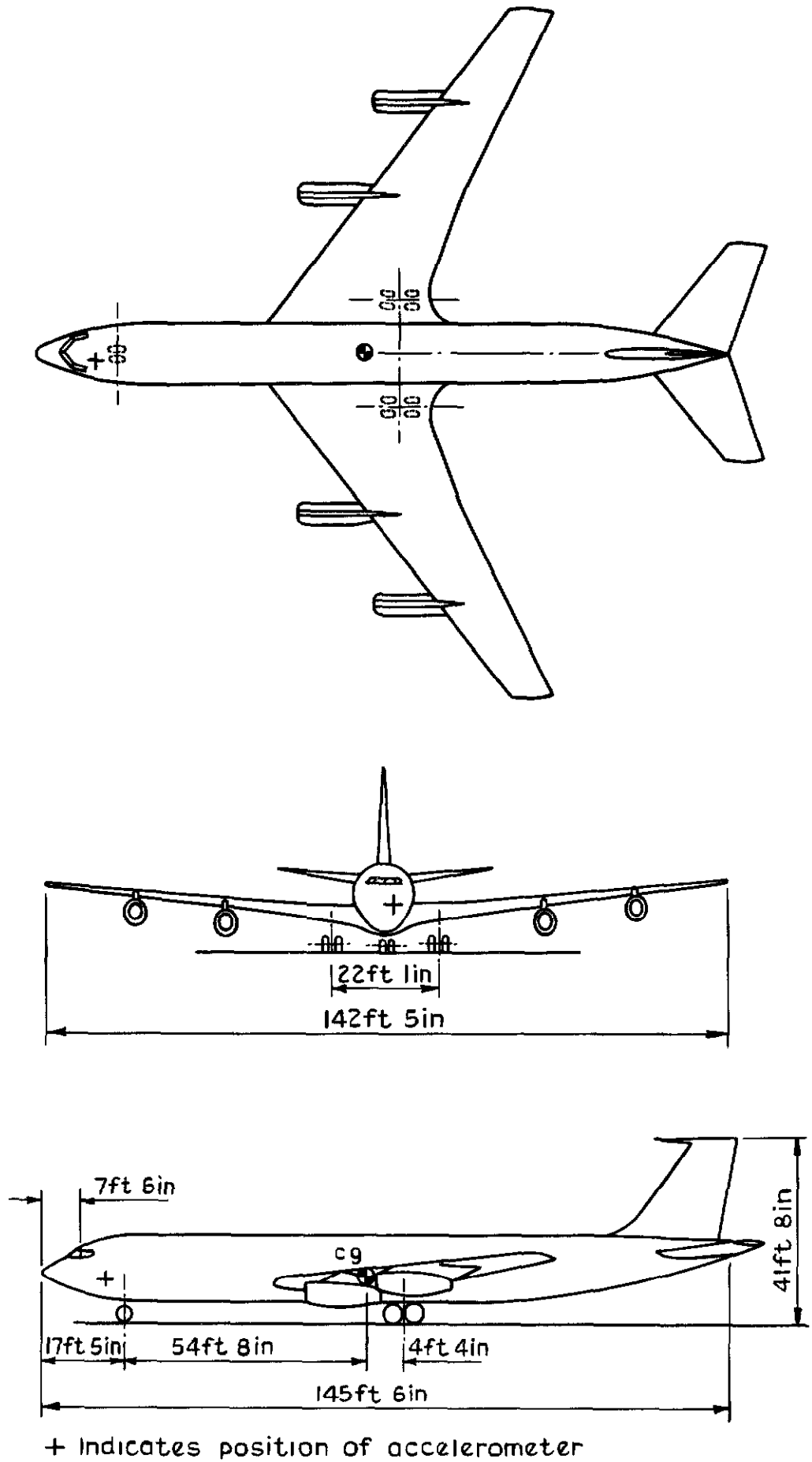


Fig.1 Aircraft general arrangement

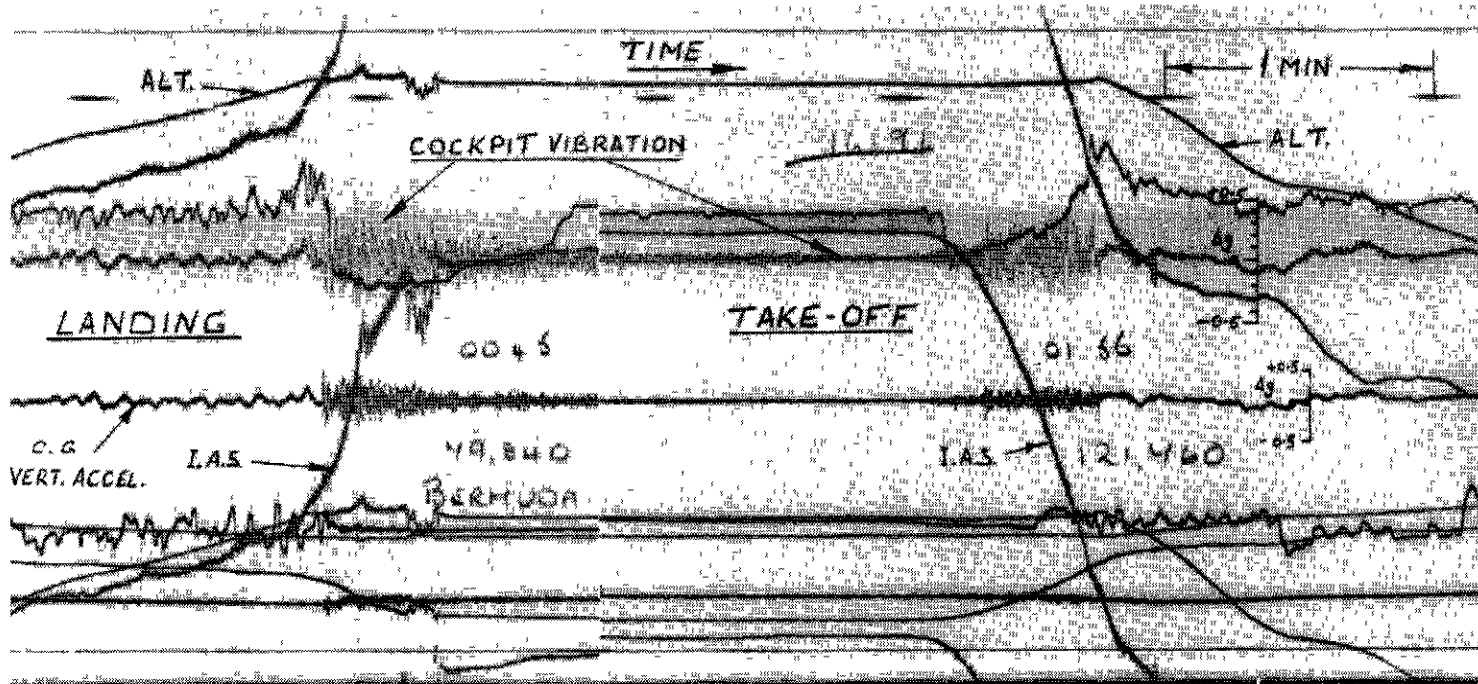


Fig.2. Typical landing and take-off runs

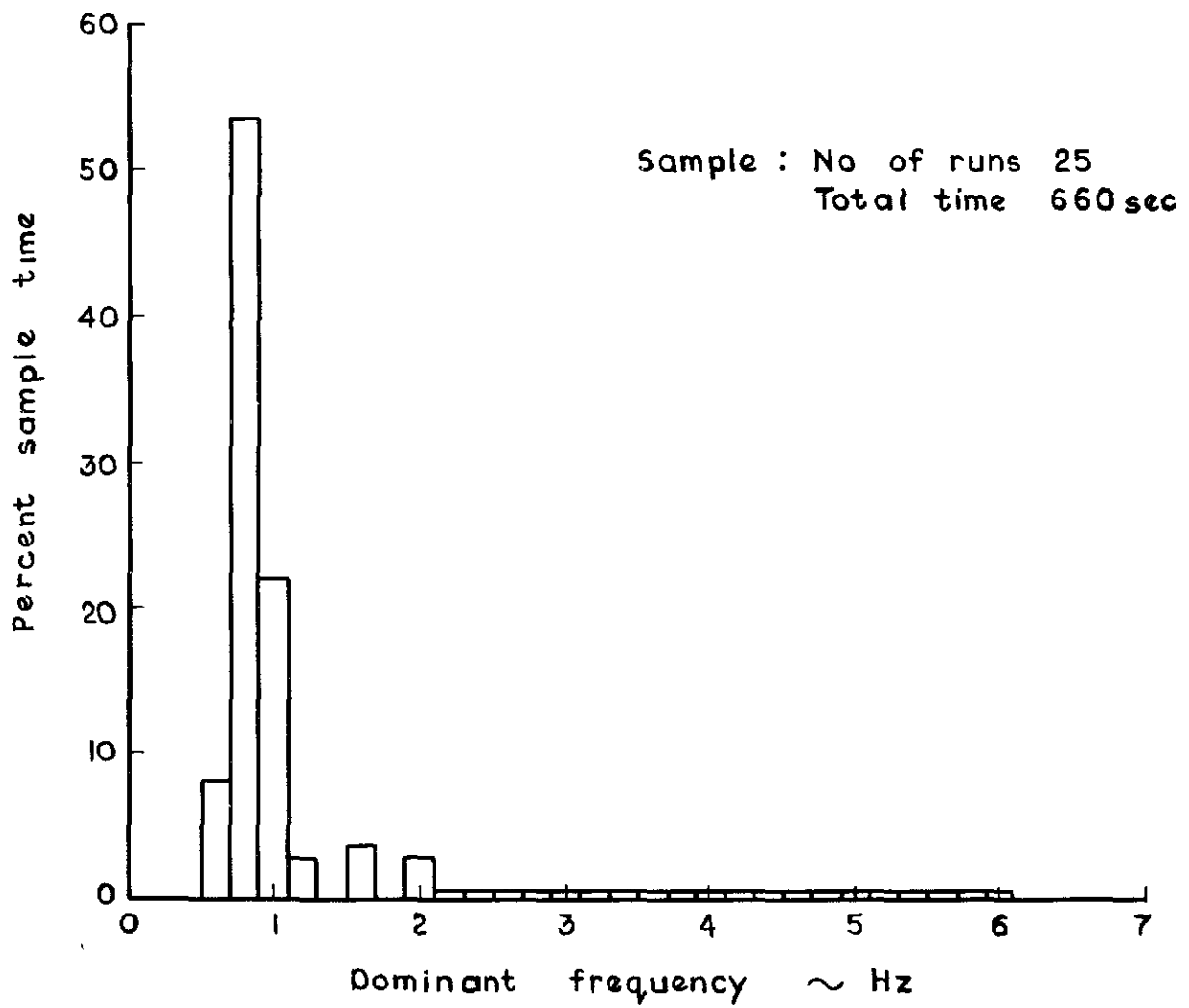


Fig.3 Histogram of dominant frequencies.
Take - off phase

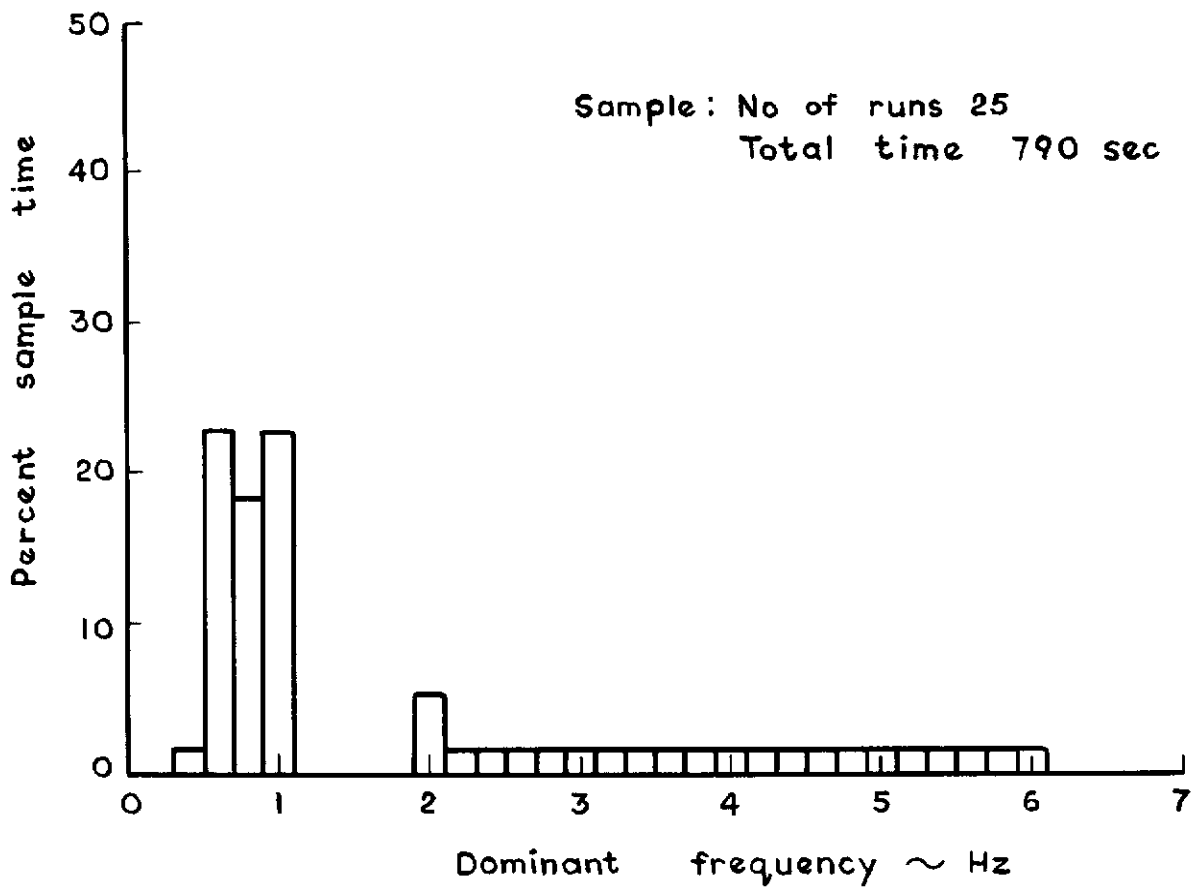


Fig.4 Histogram of dominant frequencies.
Landing phase

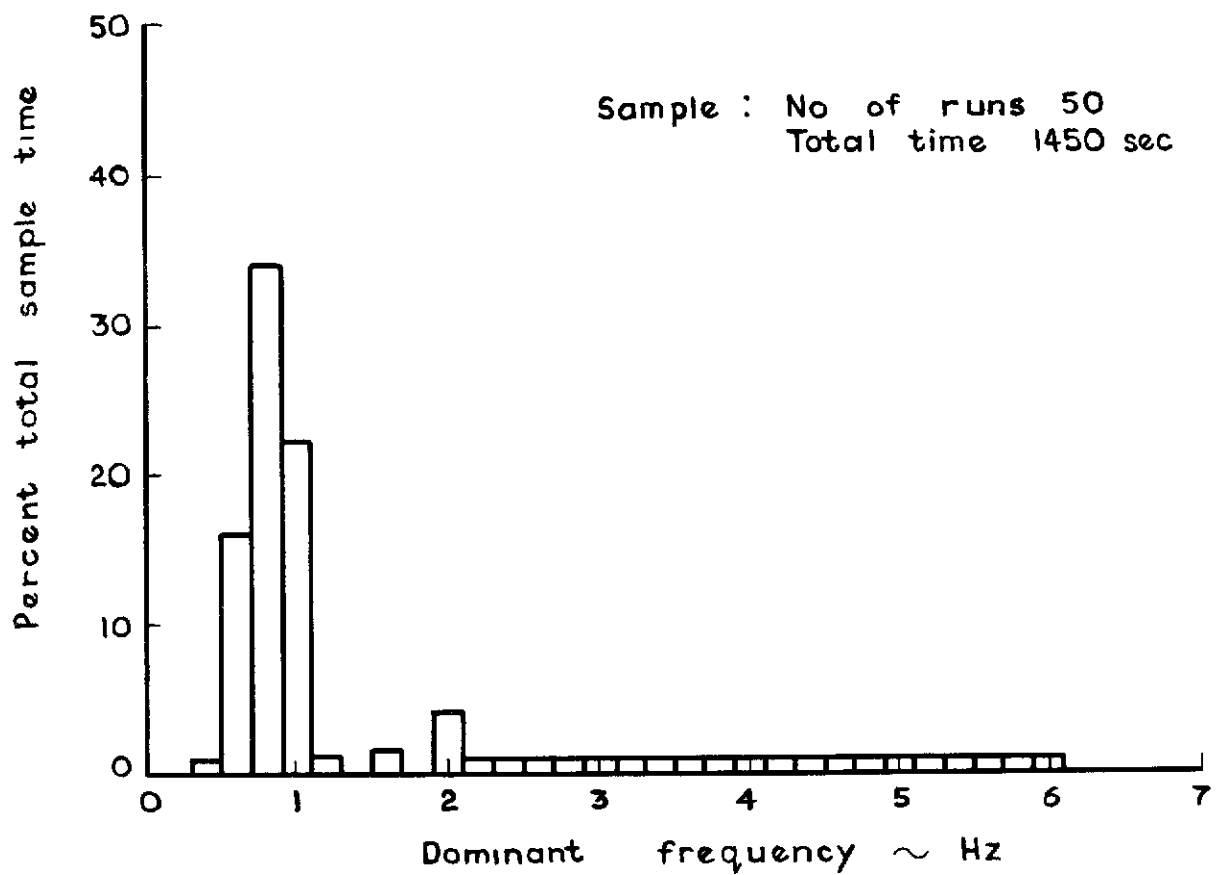


Fig.5 Histogram of dominant frequencies.
Take-off and landing amalgamated

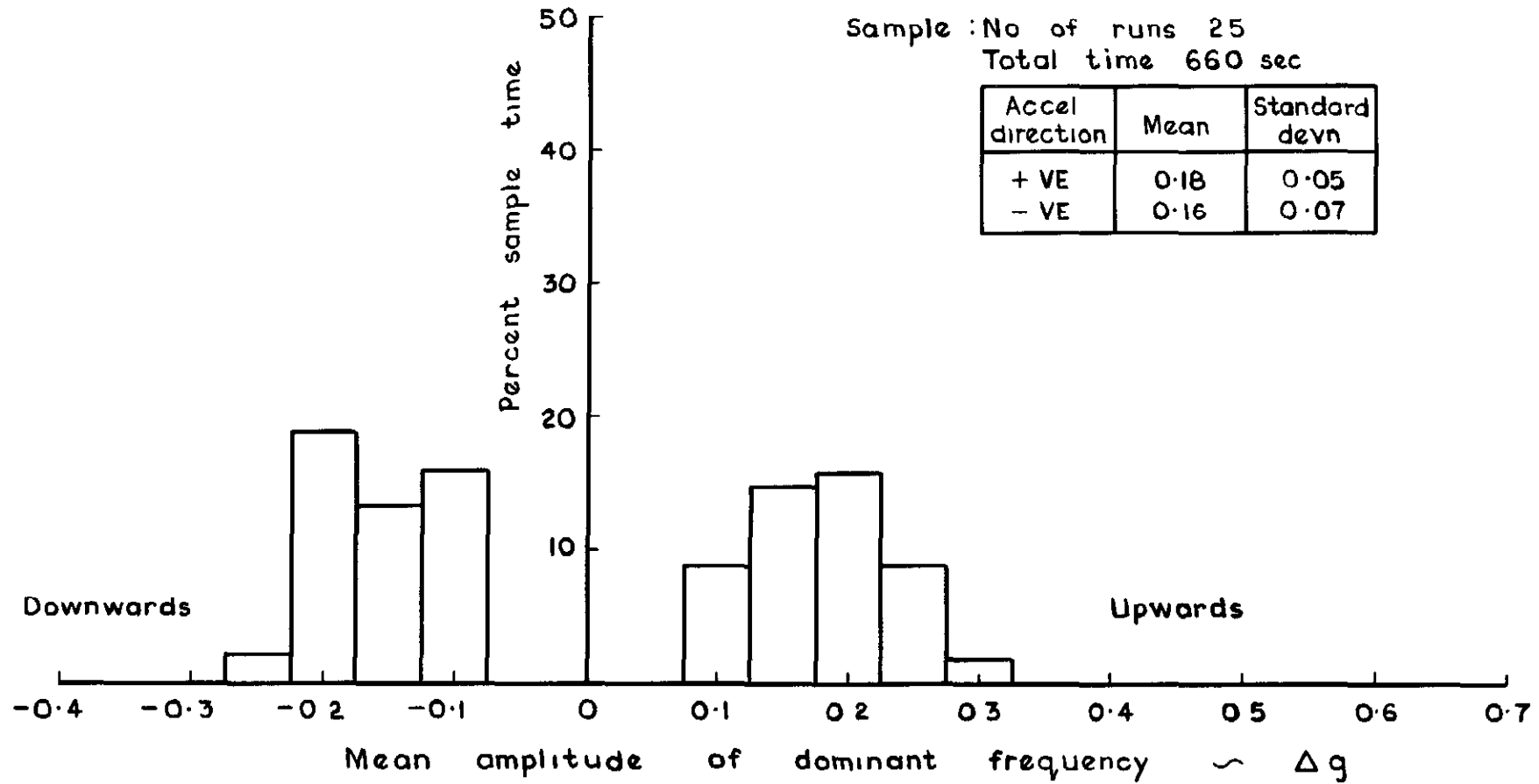


Fig 6 Histogram of amplitudes of dominant frequency of vibration.
Take-off phase

Sample : No of runs 25
 Total time 790 sec

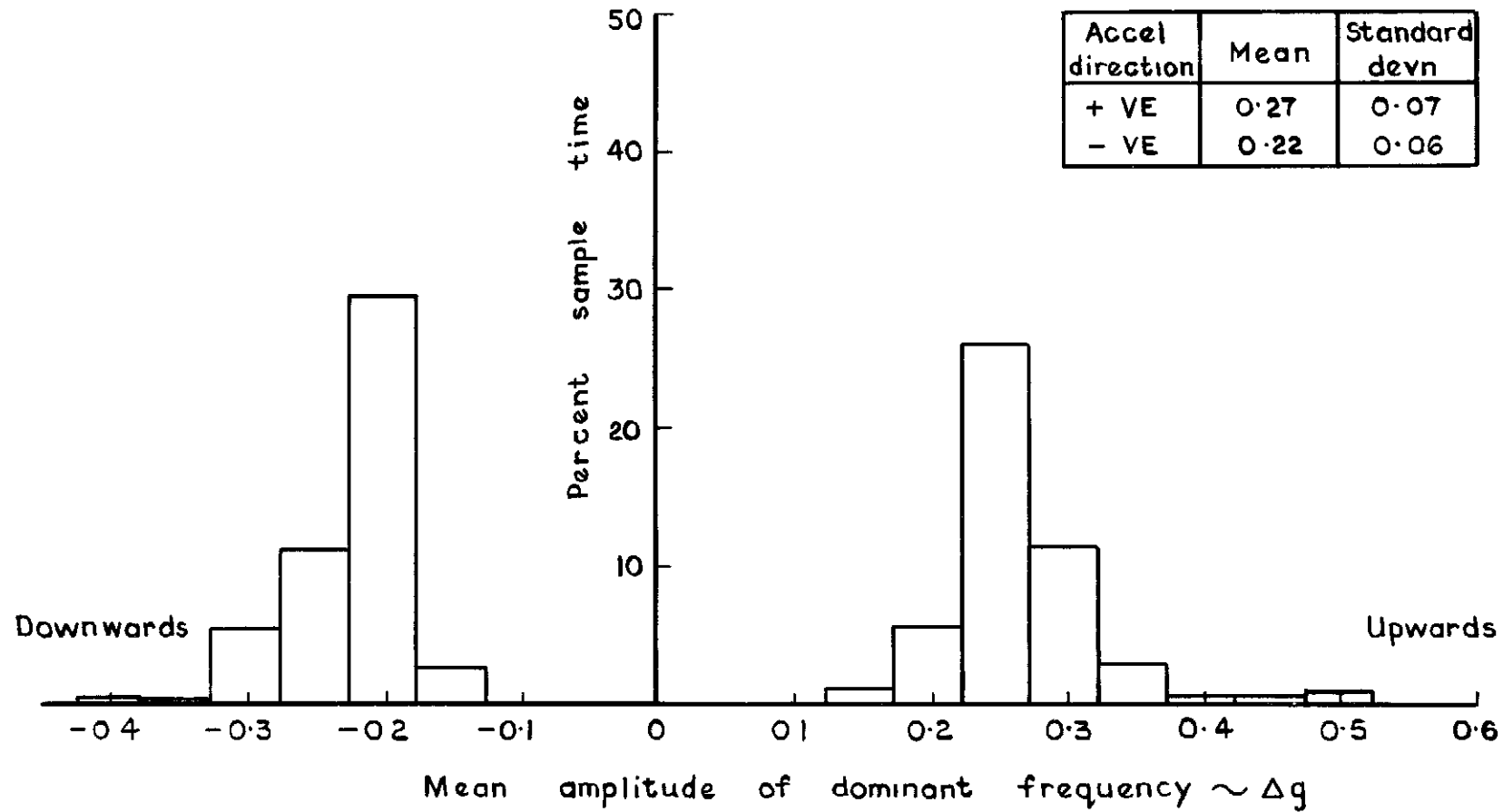


Fig.7 Histogram of amplitudes of dominant frequency of vibration.
 Landing phase

Sample : No of runs 50
 Total time 1450 sec

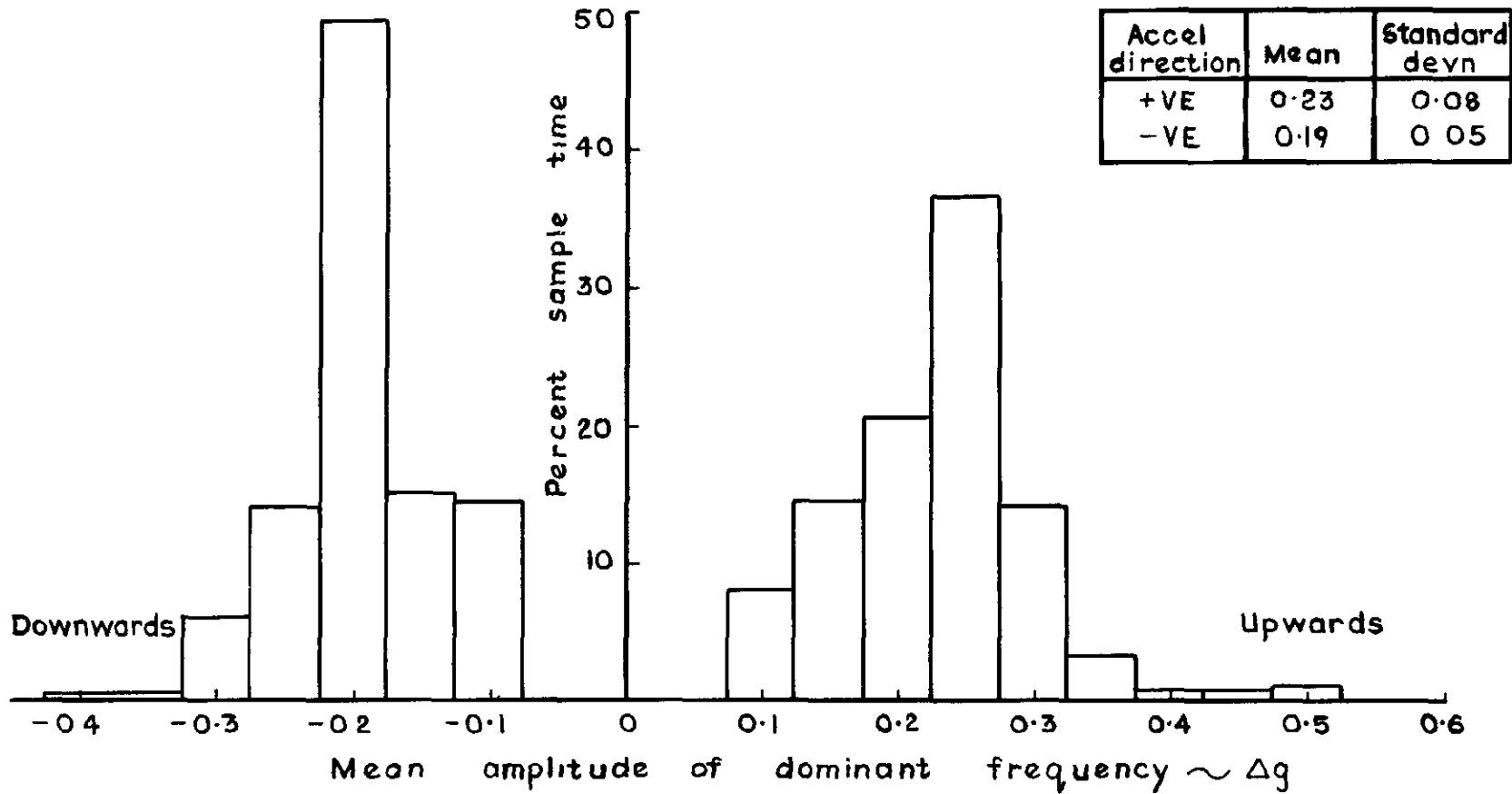


Fig.8 Histogram of amplitudes of dominant frequency of vibration.
 Take-off and landing amalgamated

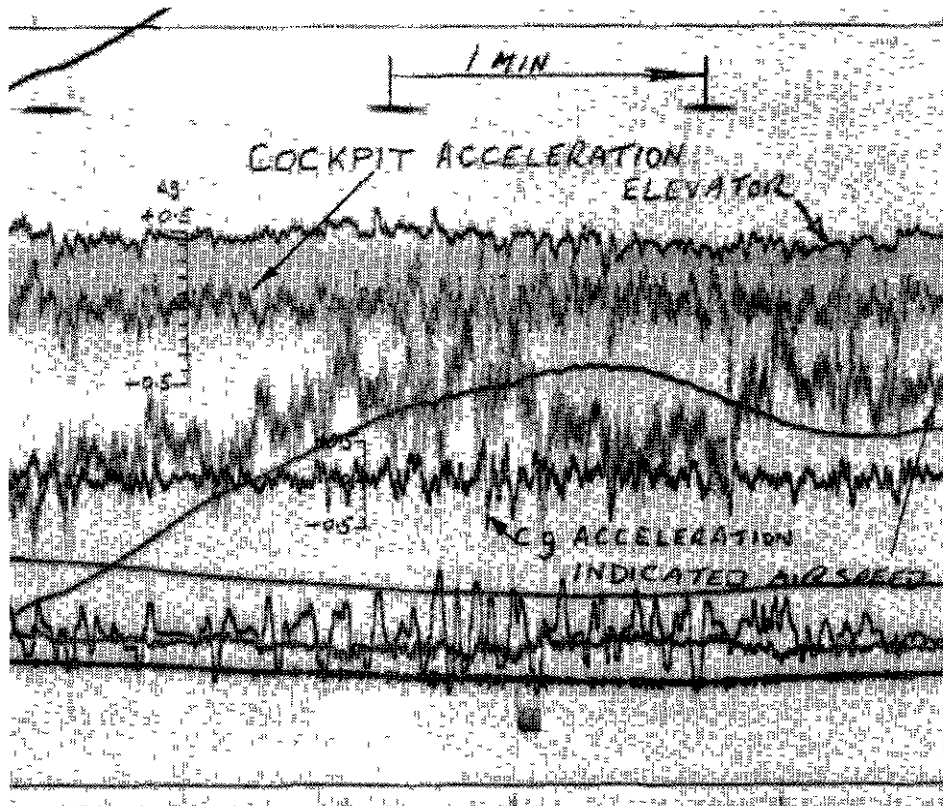


Fig.9. Light turbulence during flight

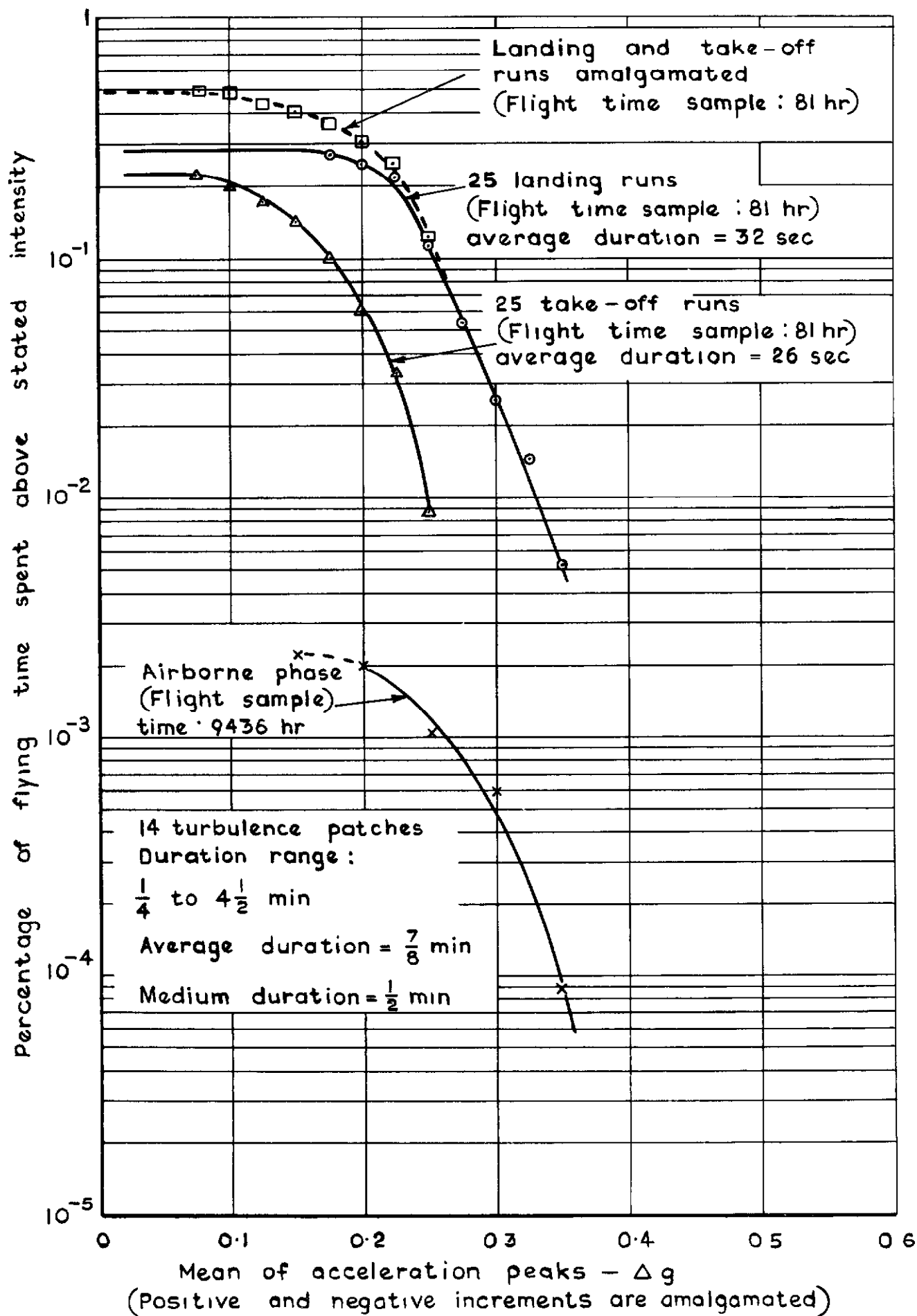


Fig.10 Proportion of time spent at different intensities of normal acceleration in the cockpit

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October 1969

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