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A Systematic Approach to the Design of Radial Inflow and Mixed Flow Turbines

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F. J. Wallace

SUMMARY

The report deals with 3 aspects of the turbine design problem:

1. A simplified one-dimensional steady flow treatment for performance predictions on simple and multiple admission turbines including the case of nozzle or rotor choking.
2. A one-dimensional unsteady flow treatment for the prediction of pulse performance, again for single and multiple admission casings.
3. A brief discussion of pseudo three-dimensional streamline curvature techniques which have been extended to include automatic computer plotting of streamlines, isobars and isotachs, as well as of the rotor geometry itself.

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[*] Replaces A.R.C.32 781.

CONTENTS

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1. INTRODUCTION

Over the last 12 years, intensive work on radial inflow turbines has been proceeding under the writer's direction, arising out of the need to design a turbine for road traction applications operating in association with a free piston gas generator delivering gas at 4 bar (approx) and 450°C. This early work (1958) led to the formulation of a simple, but nevertheless then unique, treatment covering, by one dimensional methods, both design and off design conditions, as well as a rapid prediction technique for blade to blade pressure and velocity gradients in terms of mid-channel values of velocities and geometric properties, and derived values of the tangential acceleration (Reference 1). This treatment was later extended to analyse pulsating flow conditions in radial flow turbines, using the method of characteristics to formulate equations for wave propagation in the supply duct, volute and nozzle ring, on the one hand, and the rotor on the other, the two sets of conditions being joined by continuity, momentum and energy equations in the interspace allowing for entropy gain (References 2, 3 and 4). A largely experimental attack on the problem was reported separately in Reference 5.

Reference 2 dealt exclusively with single entry turbines, whilst References 3 and 4 cover the cases of multi entry casings using either the full unsteady or a simpler quasi steady treatment.

Reference 4 incorporated several new techniques required to deal with exceptionally high pressure ratios leading to either nozzle or rotor choking, and a thermodynamic analysis of interspace conditions with multi entry operation. These aspects, as well as operation with variable nozzles, were treated in a further paper (Reference 6) applying specifically to constant pressure operation of single and multi entry radial inflow turbines.

The work covered by References 2 - 6 has been supported experimentally by an extensive programme, using a range of high speed dynamometers, for both constant pressure and pulse operation.

Finally, the analytical techniques developed for radial inflow turbines have been extended to mixed flow turbines, and combined with throughflow and blade to blade analyses based on the work of Horlock and Hodskinson (Reference 7), as well as with new computer graphics routines for the plotting of streamlines, isobars and isotachs, and of the rotor geometry itself. This integrated computer aided design approach has already been applied to the design, manufacture and testing of a mixed flow rotor.

In the following the various techniques will be briefly described, together with some analytical and experimental results.

2. STEADY FLOW (CONSTANT PRESSURE) ANALYSIS (References 1 and 6)

Under this heading the most important aspect of the one-dimensional treatment will be discussed, particularly the off-design interspace model as applied to subsonic as well as sonic nozzle exit conditions. This work is taken almost in its entirety from Reference 6. It must be emphasised that the only losses explicitly taken into account are nozzle-rotor interspace irreversibilities and rotor exit losses, ie there is no recovery of exit KE. Nozzle and rotor passage losses are explicitly excluded, although the treatment can very readily be modified by the inclusion of loss coefficient as discussed in a recent paper by Benson (8). The chief advantage of the method lies in the fact that it gives a closed solution of the off design problem and that no recourse is had to empirical incidence loss or deviation coefficients. Correlation with experimental results is surprisingly good.

(a) Unchoked Nozzle (Figures 1, 2 and 3)

The thermodynamic model for interspace flow is that of a constant pressure irreversible ('shock') process 22' (Figure 2) with sudden deflection of the nozzle jet leaving velocity c_2 and absolute angle α_2 (Figure 3) to conditions c_2' and α_2' giving a relative velocity vector ω_2', β_2' in conformity with the rotor inlet geometry. Slip is not allowed for, but could readily be incorporated.

The shock' problem is solved by the application of the continuity, energy and momentum equations between conditions 2 and 2', the momentum equation giving the so-called 'shock torque' as distinct from the impeller torque subsequently developed in the rotor itself.

The application of the energy, continuity, and momentum equations across the shock yields:

$$C_pT_2 + \frac{c_2^2}{2g_oJ} = C_pT_2' + \frac{c_2'^2}{2g_oJ}$$

$$+ (c_2 \cos \alpha_2 - c_2 \sin \alpha_2 \frac{T_2'}{T_2} \cot \theta_2 - u_2) \frac{u_2}{g_oJ} \qquad (1)$$

yielding the following solution for the temperature T_2', and hence both the shock temperature rise $\Delta T_{22}'$ and entropy gain $\Delta s_{22}'$ (where $\Delta s_{22}' = C_p \ln T_2'/T_2$)

$$(T_2')^2 \frac{c_2^2 \sin^2 \alpha_2}{\sin^2 \beta_2^2 \; g_oJc_pT_2^2} + T_2'$$

$$- \left\{ T_2 + \frac{c_2^2 - u_2^2}{2g_oJc_p} - \frac{u_2(c_2 \cos \alpha_2 - u_2)}{g_oJc_p} \right\} = 0 \qquad (2)$$

Having determined the shock 'jump' 22' (Figure 2) the rotor end conditions 3 may be evaluated by applying continuity and energy through the rotor, and ultimately ensuring mass flow compatibility between nozzle and rotor by adjusting the initially assumed value of the interspace pressure p_2.

The rotor equations are as follows:

energy:

$$w_3^2 = w_2'^2 + 2g_oJc_pT_2' \left[1 - \left(\frac{p_3}{p_2} \right)^{\frac{\gamma-1}{\gamma}} \right] + u_3^2 - u_2^2 \qquad (3)$$

absolute exit velocity:

$$c_3^2 = w_3^2 + u_3^3 - 2w_3u_3 \cos \theta_3 \qquad (4)$$

absolute rotor exit angle:

$$\sin \alpha_3 = \frac{w_3 \sin \theta_3}{c_3} \tag{5}$$

from which the 'impeller torque' τ_i becomes

$$\tau_i = \frac{\left[c_2' \cos \alpha_2' + \frac{\overline{d_3}}{d_2} \; c_3 \cos \alpha_3 \right] d_2}{2 g_o} \tag{6}$$

whilst the 'shock' torque associated with the transition 22' becomes:

$$\tau_{sh} = \frac{\left[c_2 \cos \alpha_2 - c_2 \sin \alpha_2 \frac{T_2'}{T_2} - u_2 \right] d_2}{g_o} \tag{7}$$

The nozzle and rotor mass flow terms yielding the required interspace pressure p_2 by iteration are:

$$\dot{m}_N = (c_2 \sin \alpha_2) d_2 b_2 \, \rho_{01} \left(\frac{p_2}{p_{01}} \right)^{\frac{1}{\gamma}}$$

$$= \left[\sqrt{2 g_o J c_p T_{01} \left[1 - \left(\frac{p_2}{p_{01}} \right)^{\frac{\gamma - 1}{\gamma}} \right]} \right] \sin \alpha_2 \; \pi \; d_2 b_2 \; \rho_{01} \left(\frac{p_2}{p_{01}} \right)^{\frac{1}{\gamma}} \tag{8}$$

and

$$\dot{m}_R = c_3 \sin \alpha_3 \; \pi \; d_3 b_3 \; \rho_2' \left(\frac{p_3}{p_2} \right)^{\frac{1}{\gamma}} \tag{9}$$

Equations (1) to (9) constitute a closed system yielding the required solutions for interspace pressure p_2, exit velocity c_3 and exit angle α_3 mass flow $\dot{m} = \dot{m}_N = \dot{m}_R$, and torque $\tau = \tau_{sh} + \tau_i$. The efficiency (total to static) may be evaluated from

$$\eta = \frac{(\tau_{sh} + \tau_i)\,\frac{2\pi N}{60}}{2g_o J c_p\, T_{01}\left[1 - \left(\frac{p_3}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \qquad (10)$$

A comparison of calculated and experimental results for a wide range of operating conditions is given in Figures 4 and 5.

It should be noted that the 'shock' model applies equally to rotor speeds above and below the design point.

(b) Choked Nozzle (High Overall Pressure Ratios) (Figures 6a and b)

The thermodynamic model used to describe the flow under these conditions is that of sonic conditions in the nozzle throat followed by a Prandtl Meyer expansion from the nozzle throat pressure p_N to the interspace pressure p_2. This, in turn, is followed by the constant pressure 'shock' described in the previous section. The representation of the flow is similar to that postulated by Jansen,[9] but arrived at quite independently.

The turning angle $\alpha_2 - \alpha_N$ is given by

$$\frac{\sin^2 \alpha_2}{\sin^2 \alpha_N} = \left(\frac{\gamma + 1}{2}\right)^{(\gamma+1)/2(\gamma-1)} \left(\frac{2}{\gamma - 1}\right)^{\frac{1}{2}}$$

$$\times \frac{p_2}{p_{01}} \left[1 - \left(\frac{p_2}{p_{01}}\right)^{\gamma-1/\gamma}\right]^{\frac{1}{2}}$$

where $\left(\frac{p_{01}}{p_2}\right)$ exceeds the critical pressure ratio

$$\frac{p_{01}}{p_N} = \left(\frac{\gamma + 1}{2}\right)^{\gamma/\gamma-1}$$

The supersonic velocity c_2 is obtained by application of the energy equation for isentropic flow. Thereafter the procedure is similar to that described in Section (a).

(c) <u>Two Entry Casing</u> (Figures 7a and b)

This is a form of casing frequently met on turbochargers. This analysis is intended primarily for unsteady flow studies, using the quasi steady approach, ie treating the flow as steady for short time intervals. Under such conditions inlet conditions at the 2 entries can differ widely and may be specified by p_{01}, T_{01} and p_{02}, T_{02}. It is possible by the application of techniques similar to those already given to arrive at the common 'post shock' condition T_2' (Figure 7b) in terms of the entry conditions p_{01}, T_{01} and p_{02}, T_{02}. The general approach is to solve iteratively for the common interspace pressure p_2 until mass flow compatibility between the sum of the nozzle flows $(\dot{m}_N)_1$ and $(\dot{m}_N)_2$ and the rotor flow $\dot{m}_R$ is obtained.

Figures 8a, b and c show typical results obtained from the analysis compared with experimental results taken from Reference 10.

3. PULSE FLOW ANALYSIS (References 2, 3 and 4)

Space precludes a detailed discussion of the methods developed. Basically, the method of characteristics has been applied to solve the mid channel equations for unsteady flow in the rotor passages. This involves detailed analysis of radial and tangential velocity and acceleration components under unsteady conditions, leading to the inclusion of partial derivatives with respect to time as well as to radius.

The resultant solutions for increments of the Riemann variables dλ and dβ within the rotor then become:

$$d\lambda_R = \frac{\gamma-1}{2}\,\omega\sin\theta\,dt\left[\lambda_R - \frac{\gamma-1}{2}\,w\right]\left\{\left(\frac{1}{r}+\frac{1}{b}\frac{\partial b}{\partial r}\right)+\cot\theta\,\frac{\partial\theta}{\partial r}\right\} - \frac{\gamma-1}{2}\,\omega^2 r\sin\theta\,dt \qquad \text{(12}$$

and

$$d\beta_R = \frac{\gamma-1}{2}\,\omega\sin\theta\,dt\left[\beta_R + \frac{\gamma-1}{2}\,w\right]\left\{\left(\frac{1}{r}+\frac{1}{b}\frac{\partial b}{\partial r}\right)+\cot\theta\,\frac{\partial\theta}{\partial r}\right\} + \frac{\gamma-1}{2}\,\omega^2 r\sin\theta\,dt \qquad (13)$$

Thus the leading geometric passage parameters (radius r, channel depth b, inclination θ) and their derivatives, as well as the centrifugal pressure gradient are taken into account.

These characteristic equations are connected with the simpler version appertaining to the fixed passages by the interspace solution for the entropy function S in terms of the λ characteristic incident from the nozzle and the β characteristic incident from the rotor derived from considerations similar to those leading to equation (2) and resulting in:

$$\frac{2}{\gamma-1}\left[\lambda_N - \frac{\gamma-1}{2} C_N\right]^2 (1 - S^2)$$

$$= 2(u_N \cos\alpha_N - u_N \sin\alpha_N S^2 \cot\theta_2 - u_2)u_2 - {C_N}^2$$

$$+ \left\{\frac{2}{\gamma-1}\left[S(\lambda_N - \frac{\gamma-1}{2} u_N) - \beta_R\right]\right\}^2$$

$$+ 2u_2 \cos\theta_2 \left\{\frac{2}{\gamma-1}\left[S(\lambda_N - \frac{\gamma-1}{2} C_N) - \beta_R\right]\right\} \qquad (14)$$

where

$$C_N = \frac{\frac{2}{\gamma-1}(S\lambda_N - \beta_R)}{\left[\frac{F_N}{F_R} S + 1\right] S} \qquad (15)$$

The method has recently been extended to 2 and 3 entry casings and FORTRAN IV programs have been written for these cases (See References 11, Nos 103 and 104).

In addition to the full unsteady flow treatment as outlined above, a simpler analysis has been completed based on the quasi steady approach, and applicable to single and 2 entry casings.

Typical results for single entry casing and comparisons with experiment are shown in Figures 9a and 9b, the former applying to measured and calculated pressures at different stations, and the latter to time averaged mass flow, power and efficiency. 1P refers to the pulse (characteristics) treatment, and 1Q to the quasi steady treatment. It will be observed that discrepancies between the two treatments are slight, and that therefore the much simpler quasi steady flow treatment may safely be used.

4. THREE-DIMENSIONAL AND PLOTTING PROGRAMS

This section should be read in conjunction with Section 2, ie as an extension of the 1-D techniques and intended to form, with these, a complete design or analysis procedure. The work arose, initially, out of a research project intended to lead to the design, manufacture and testing of a small mixed flow turbine having the same rotor diameter and speed as a conventional inward radial flow turbine, but required to give an increase in mass flow over the latter of the order of 30% without significant loss of total to static efficiency. Accordingly parametric studies were first undertaken using the (modified) 1-D treatment of Section 2 to establish the possibility of such an increase in mass flow subject to the above restrictions and without exceeding 'reasonable' blade height.

Details must again be omitted, but the parameter study, cycling systematically over many values of nozzle exit angle α_N, rotor entry angle β_2, 'cone angle' ψ, blade height at entry b_2, meridional velocity ratio j, etc, eventually produced a small number of possible configurations satisfying the above criteria.

The unusual rotor geometry and the exceptionally deep channel passages demanded the subsequent application of hub-shroud and blade to blade analysis. Rather than write completely new programs, it was decided to adapt the streamline curvature treatment as developed by Horlock and Hodskinson (Reference 7), originally for compressors. The major modifications made were:

(i) specification of rotor geometry in analytical form, thus greatly dimplifying data input and giving accurate values of derived geometric quantities such as curvature and length of normals

(ii) inclusion of a graphics subroutine giving automatic plots of streamlines, isobars and isotachs, (lines of constant relative velocity) (see *Figure* 10a)

(iii) development of a completely new graphics package enabling external views (including isometric) and eventually sections in any arbitrary plane to be produced (see Figure 10b)

At the moment the technique is still subject to certain limitations, eg losses are not taken into account, zero exit whirl has to be assumed, and the 'drawing' package is restricted to the particular geometry adopted for the mixed flow design.

It is intended to generalise the method to include losses, to draw sections and to convert these into instructions for numerically controlled machine tools.

Similar procedures, starting with a more complex 1-D treatment making extensive use of loss coefficients will be developed as a design and analysis tool for centrifugal compressors.

Notation

a	acoustic velocity	ft/s
b	blade height	ft
c	absolute velocity	ft/s
d	rotor diameter	ft
F	area	ft^2
g_o	gravitational conversion constant	lb ft/lbf s^2
J	mechanical equivalent of heat	ft lbf/Btu
$\dot{m}$	mass flow rate	lb/s
N	rotational speed	rev/min
p	pressure	lbf/ft^2
r	rotor radius	ft
s	entropy level (= $\frac{\text{acoustic velocity reduced to reference pressure}}{\text{acoustic velocity of reference gas at reference pressure}}$)	
t	time	s
T	temperature (absolute)	$^{\circ}$R
u	peripheral velocity	ft/s
w	relative velocity	ft/s
α	absolute angle	radians
β	Riemann variable	'leftward' wave
γ	ratio of specific heats	
λ	Riemann variable	'rightward' wave
θ	blade angle	radians
ρ	density	lb/ft^3
ω	angular velocity	radians/s

Suffix Notation

01	upstream total head
2	interspace - before shock
2'	interspace - after shock or rotor entry
3	rotor exit
N	nozzle exit
R	rotor entry

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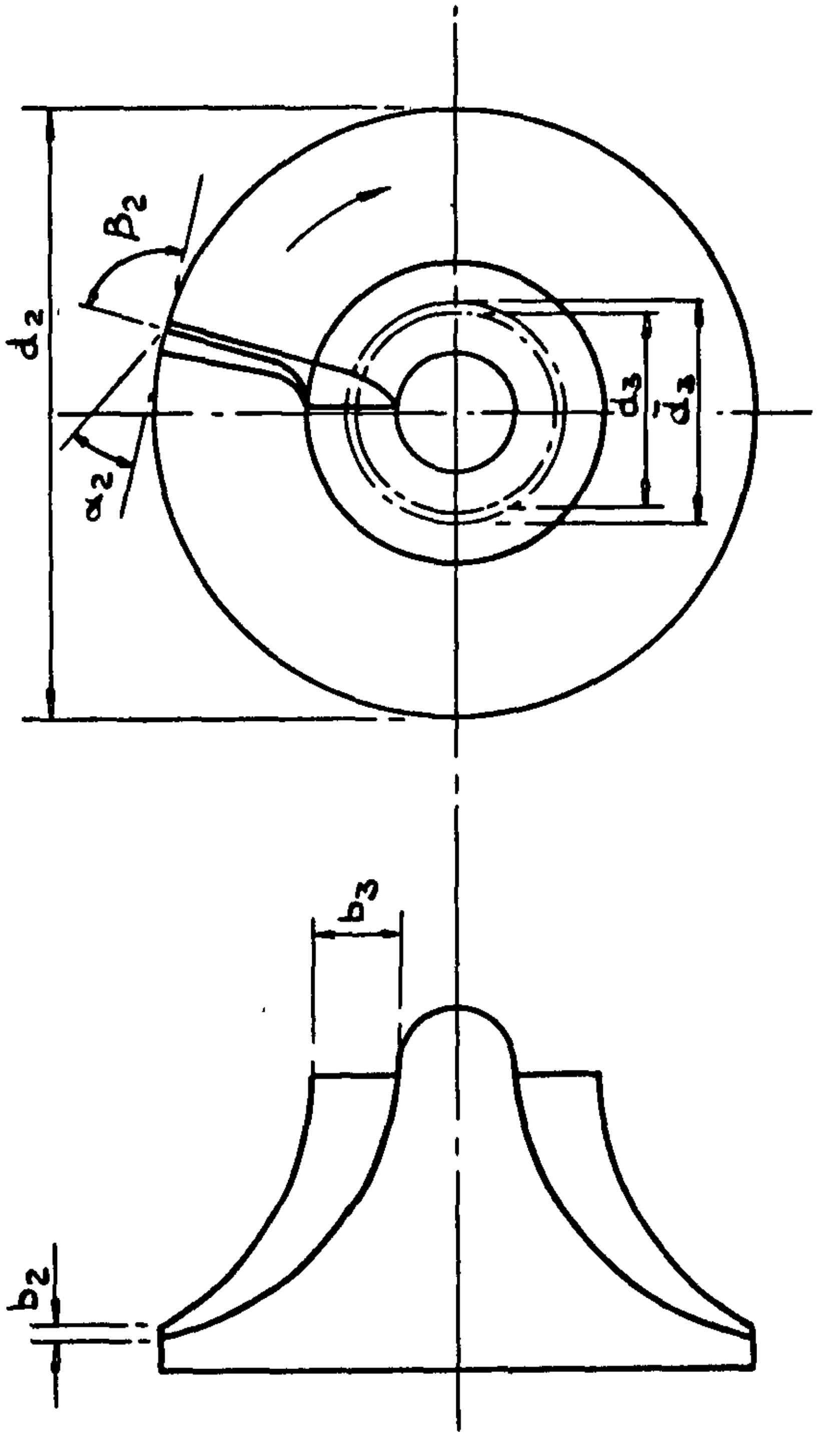
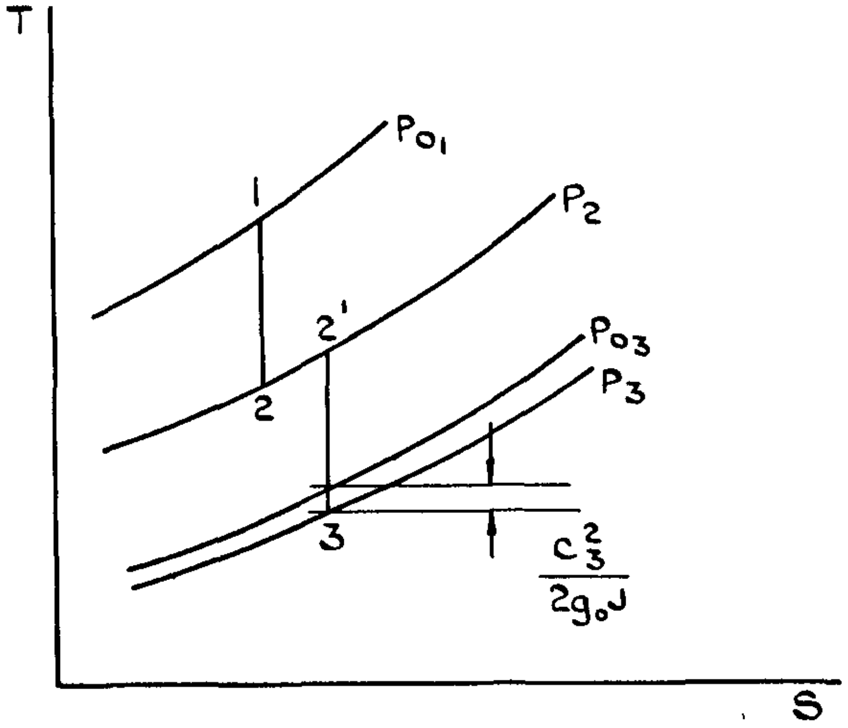


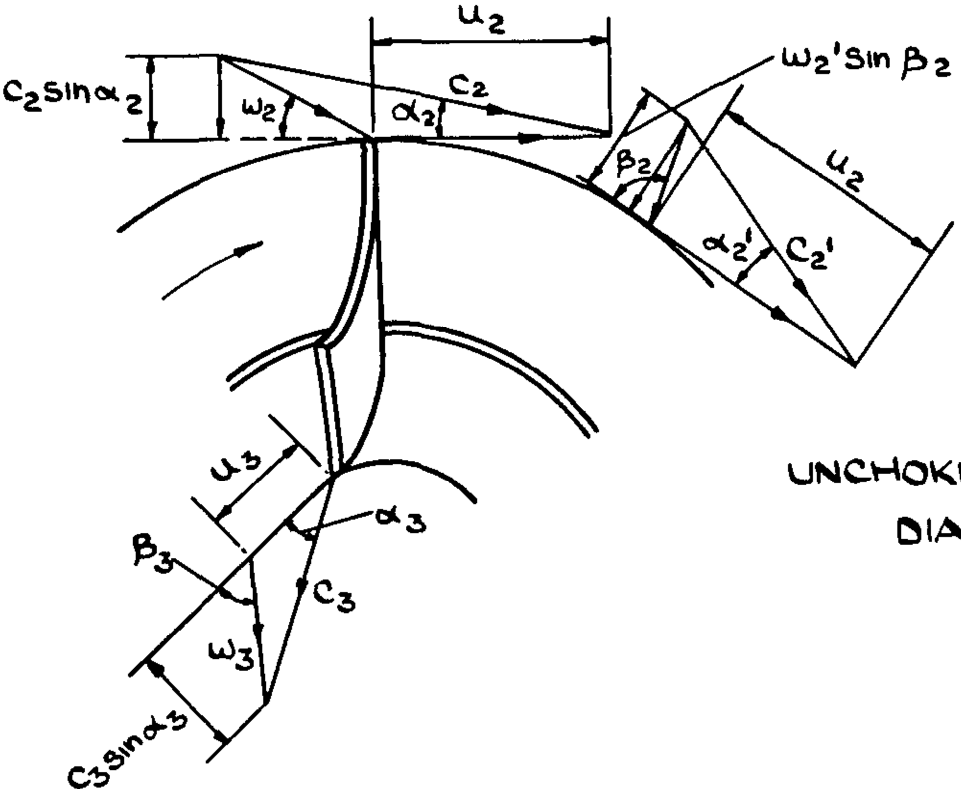
FIG. 1. TYPICAL RADIAL INFLOW ROTOR.

FIG 2



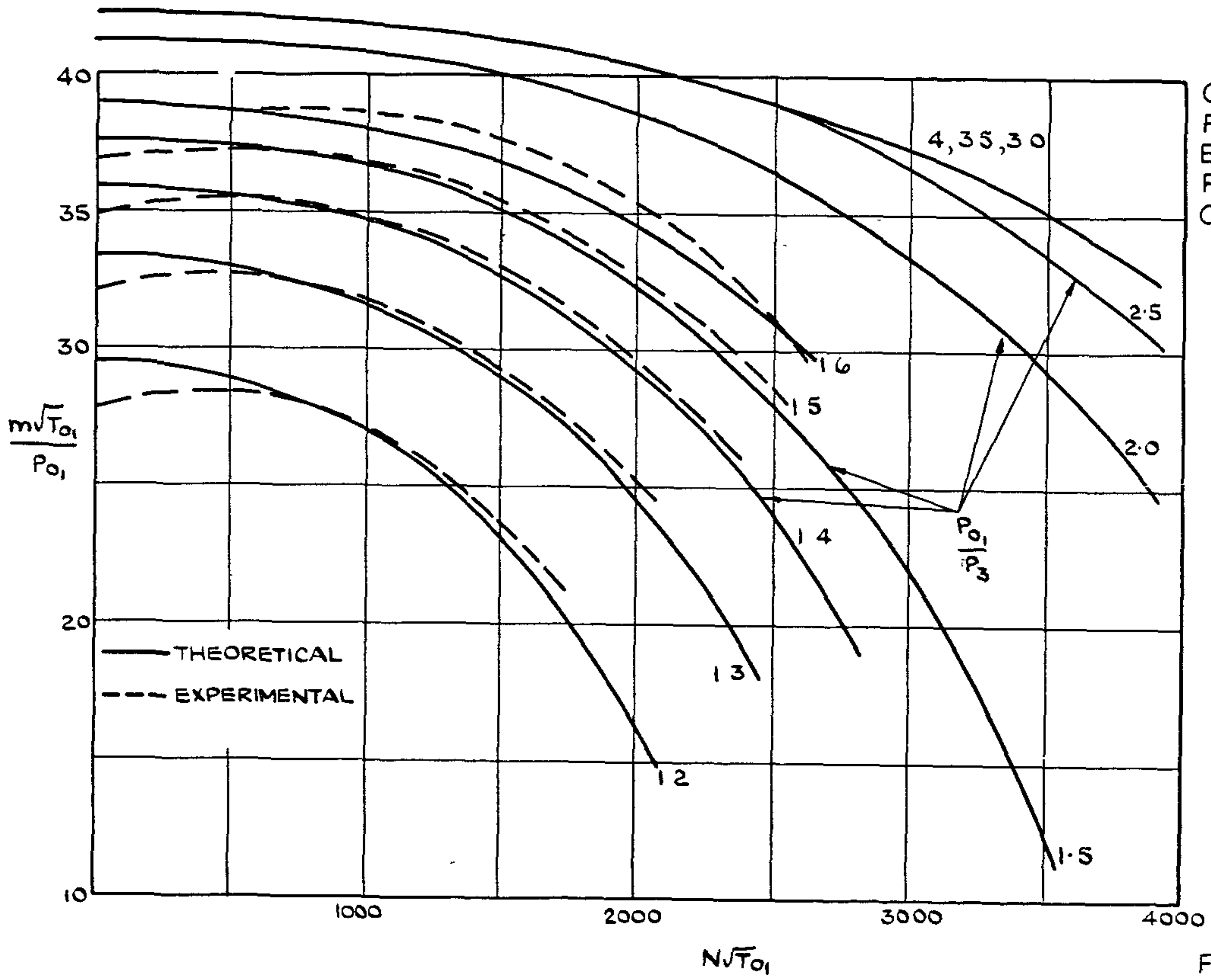
TS DIAGRAM
FOR UNCHOKED
CASE.

FIG 3



UNCHOKED VELOCITY
DIAGRAM.

FIG 4

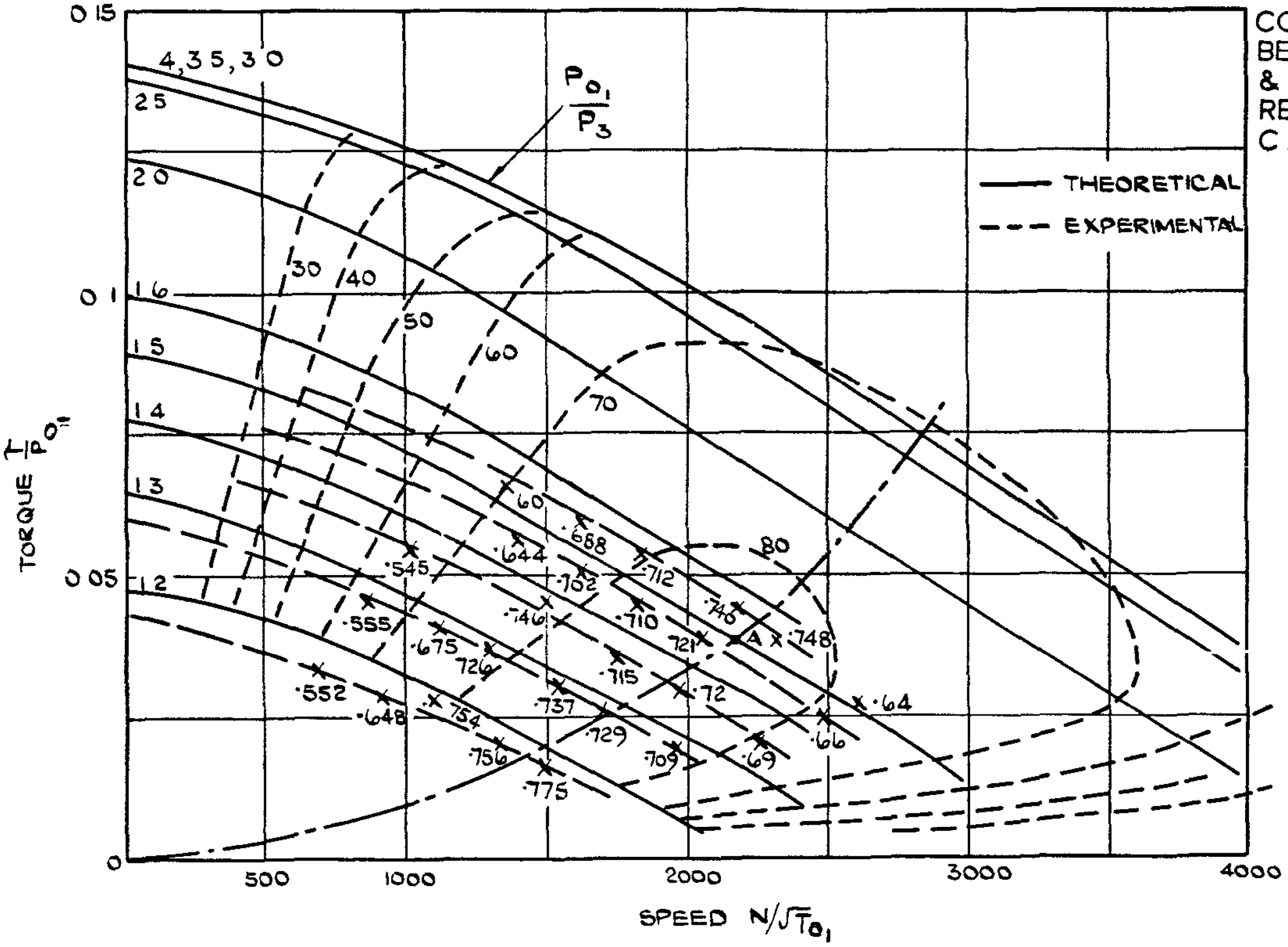


COMPARISON BETWEEN PREDICTED AND EXPERIMENTAL RESULTS FOR CAV TYPE 01

$\dot{m}$ = lb/min
N = rev/min
T_{0_1} = °R
P_{0_1} = lbf/in²

Figure 4

FIG 5

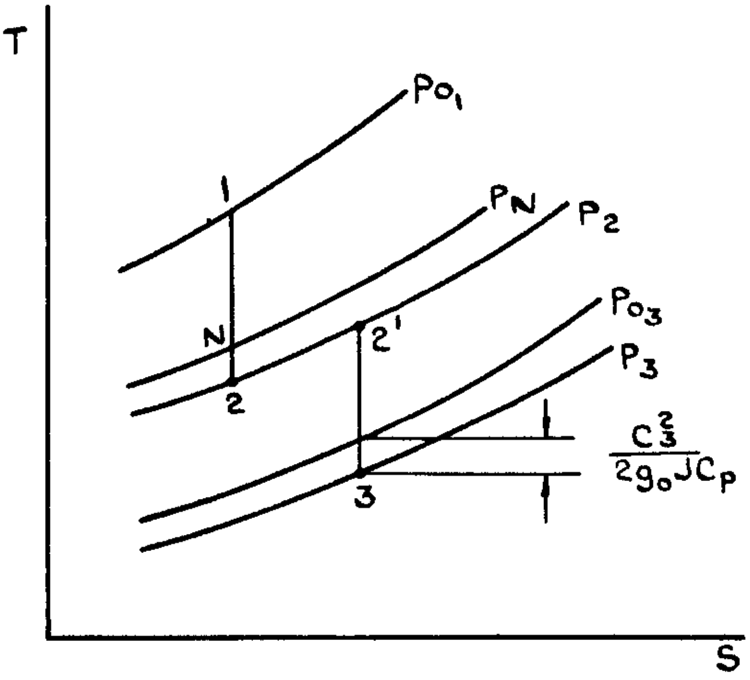


COMPARISON
BETWEEN PREDICTED
& EXPERIMENTAL
RESULTS FOR
CAV TYPE 01.

T = lbf

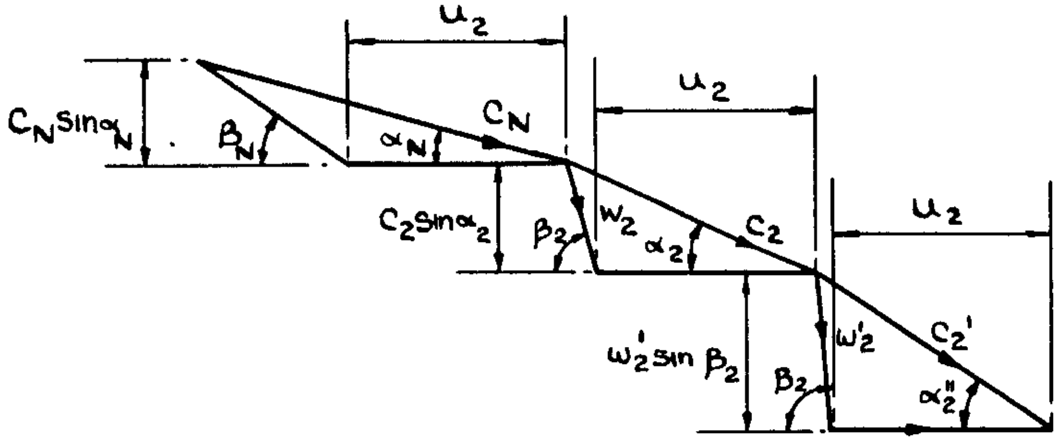
Figure 5

FIG 6a



TS DIAGRAM WITH NOZZLE CHOKING

FIG 6b



VELOCITY DIAGRAM WITH NOZZLE CHOKING

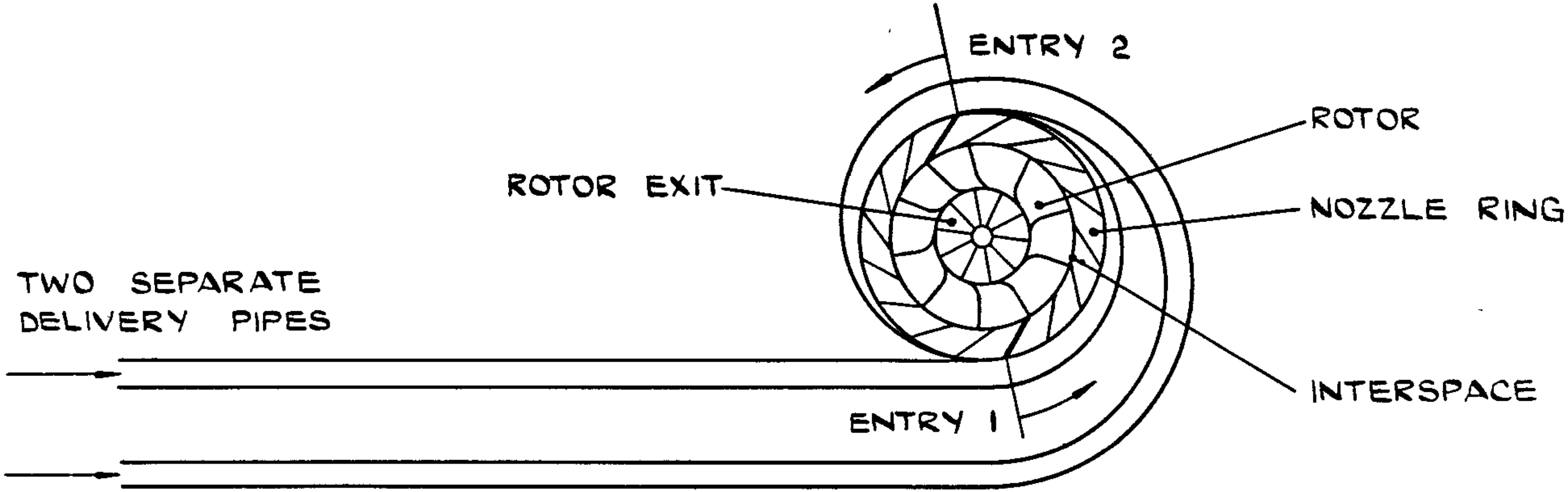


FIG 7a 2 ENTRY TURBINE.

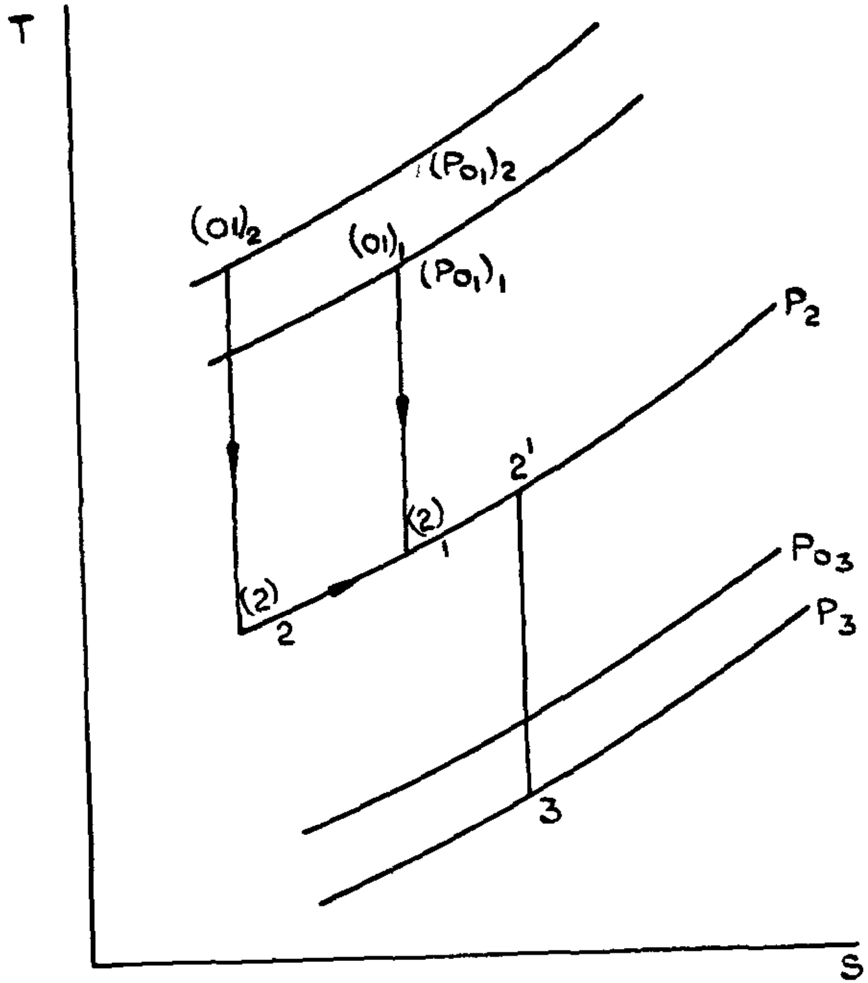
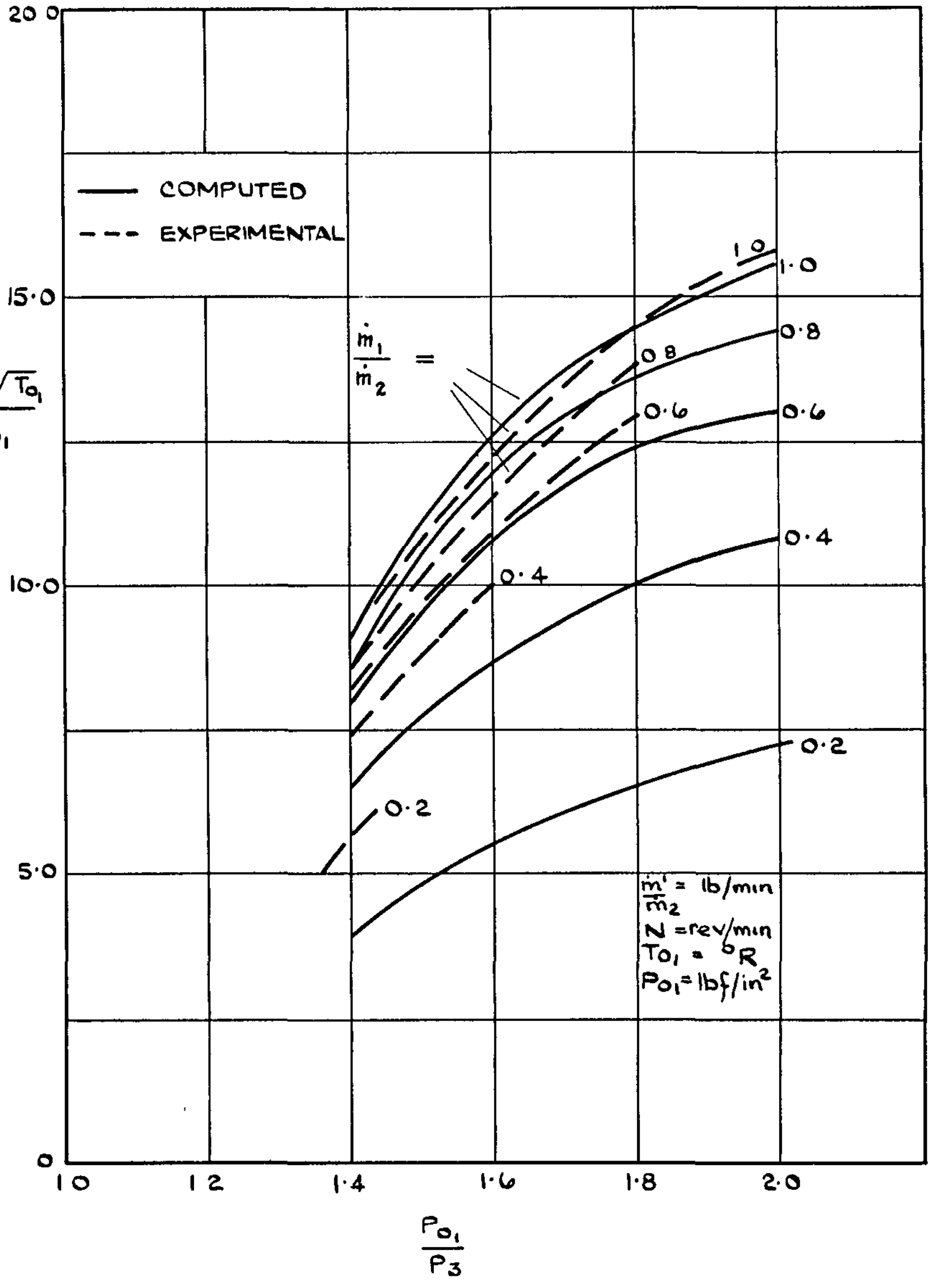


FIG. 7 b 2 ENTRY TURBINE TS DIAGRAM

FIG 8a. PREDICTED AND EXPERIMENTAL RESULTS FOR 2 ENTRY TURBINE.



SINGLE ENTRY CASING (Test 2)

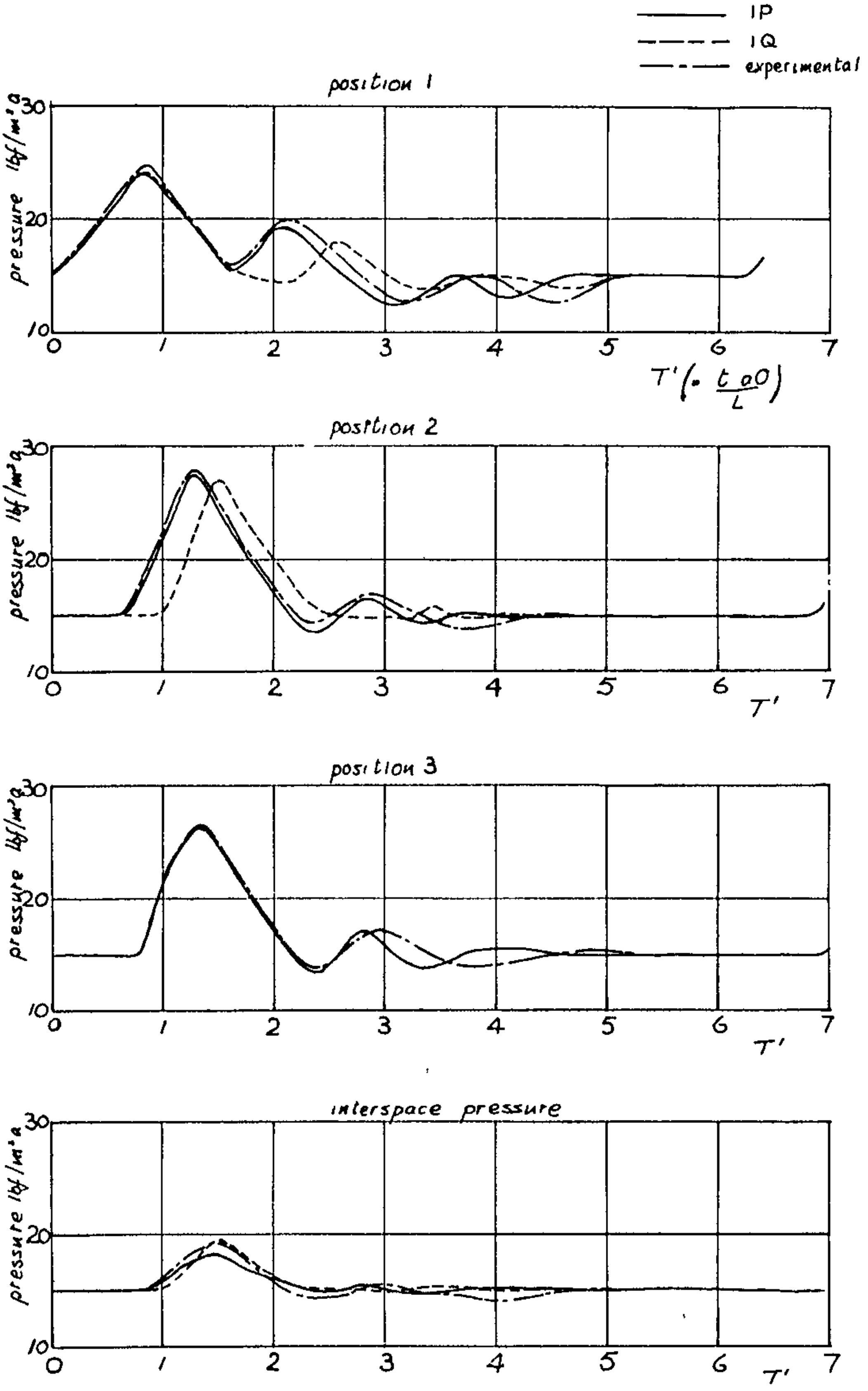


FIG 9a

SINGLE ENTRY CASING (Test 2)

Time averaged values

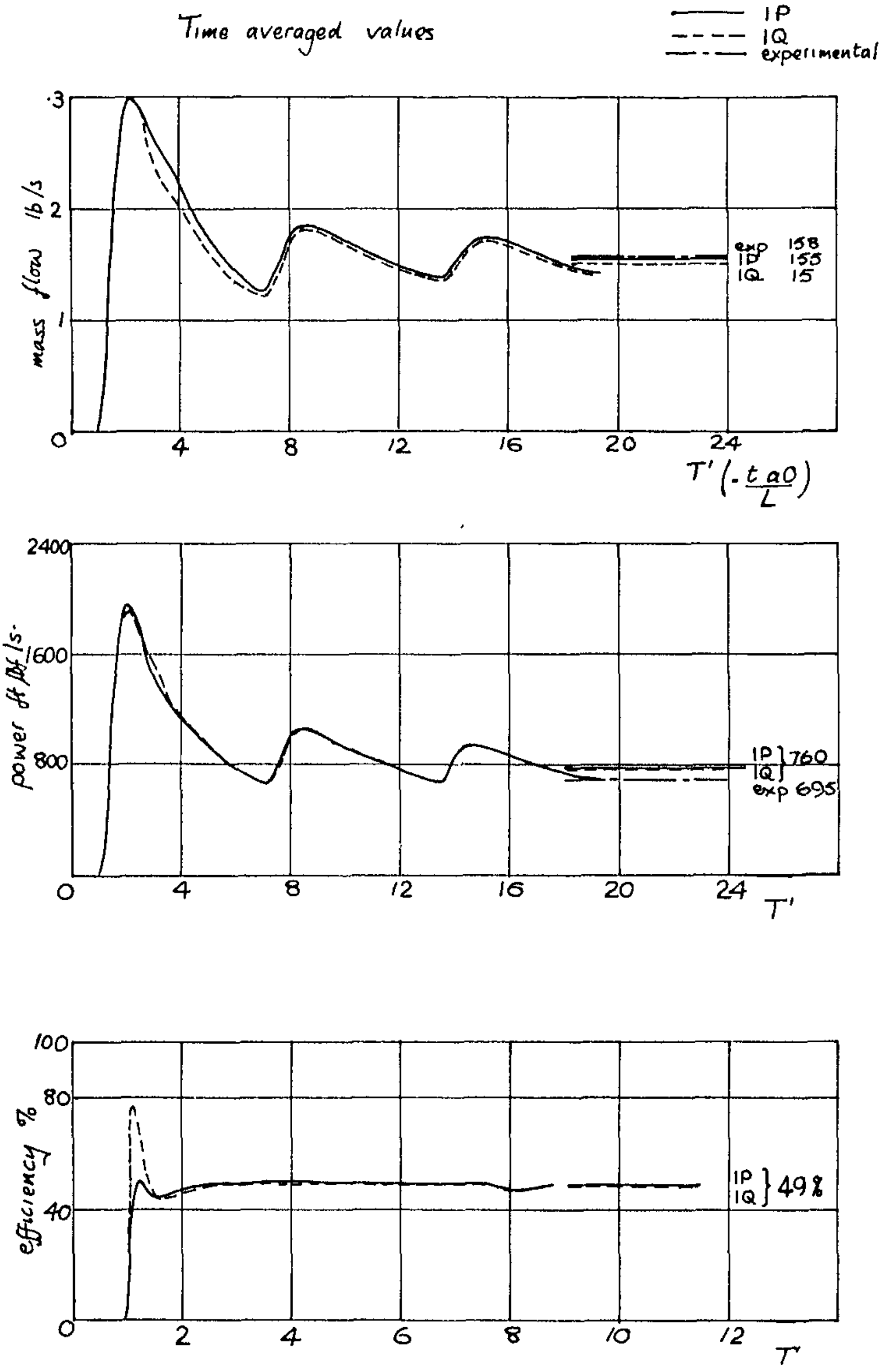
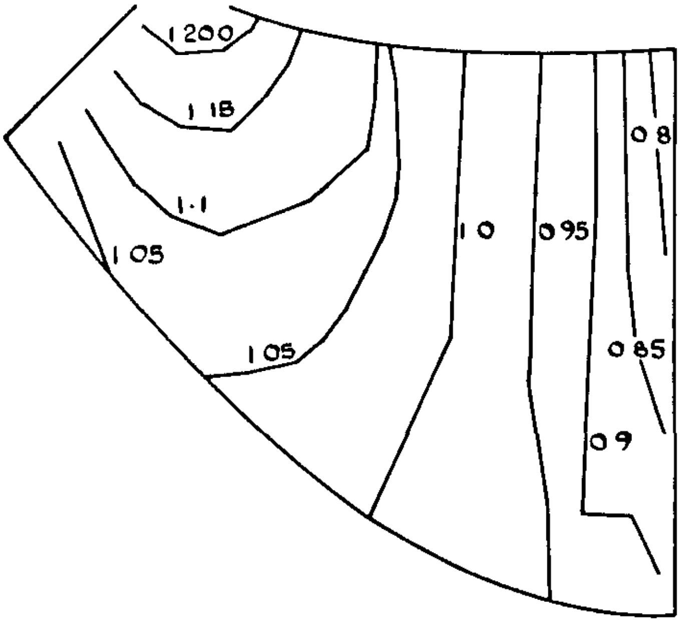
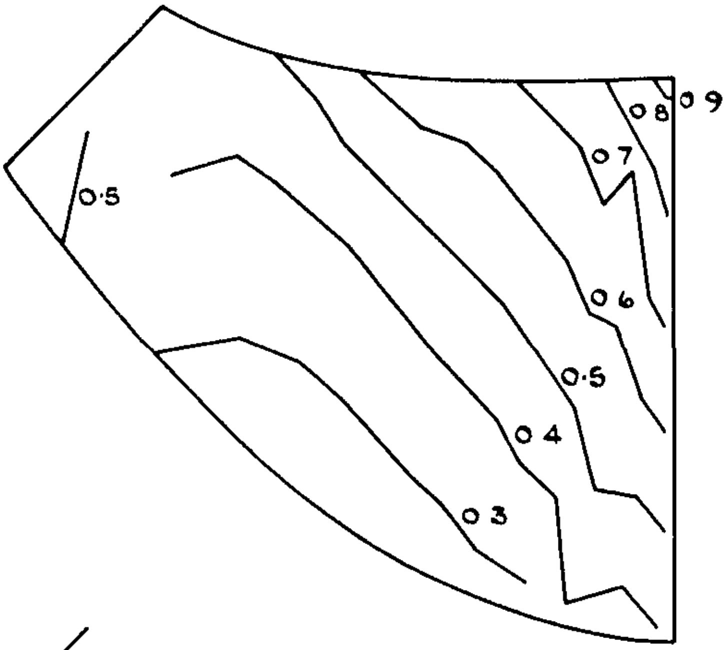


FIG 9b



REL. VEL



STR LINES

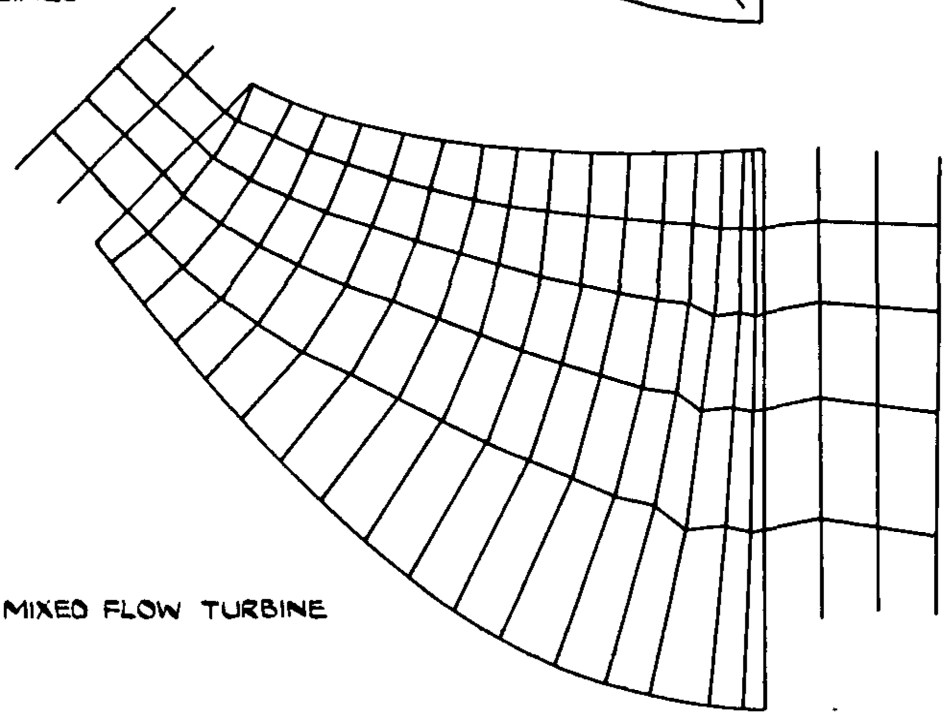
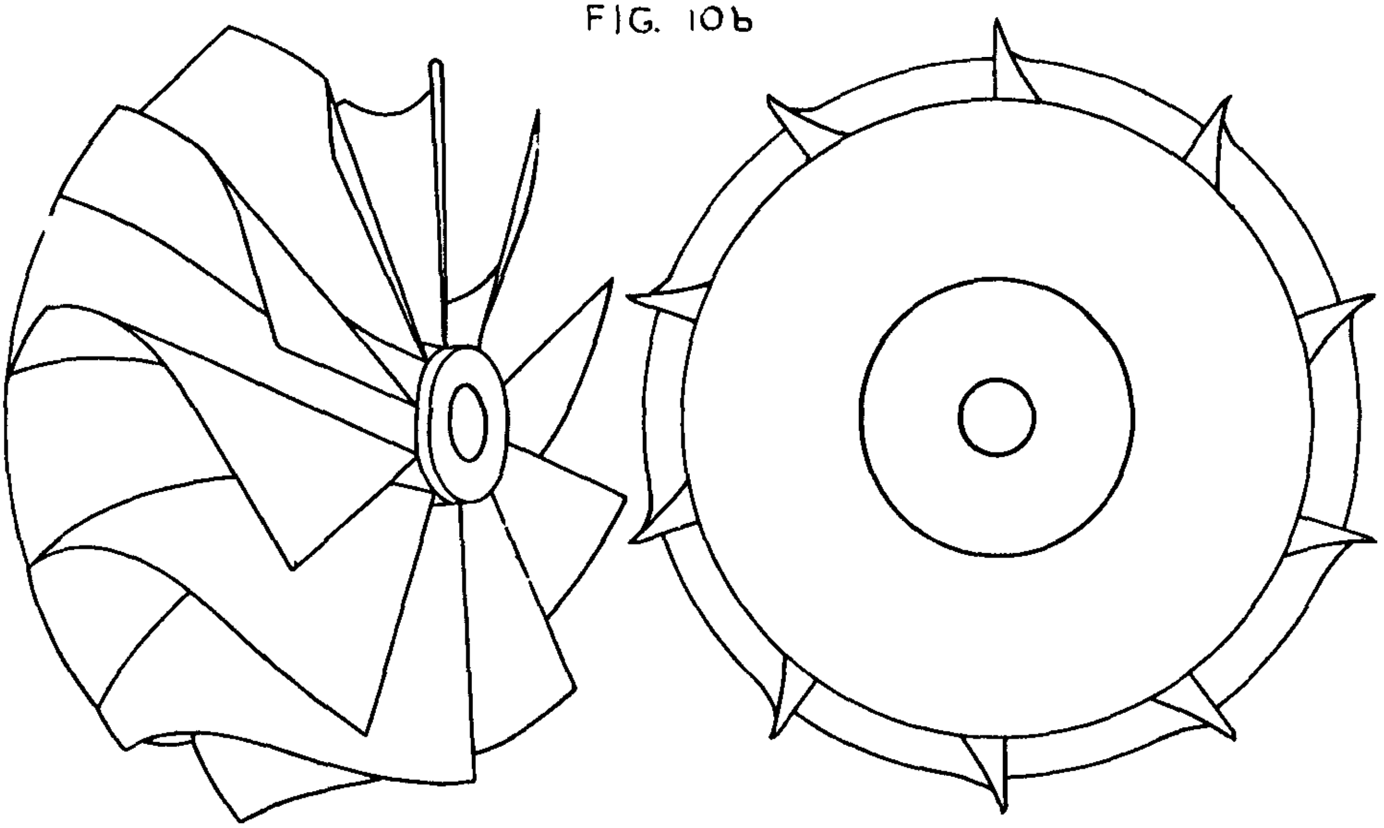


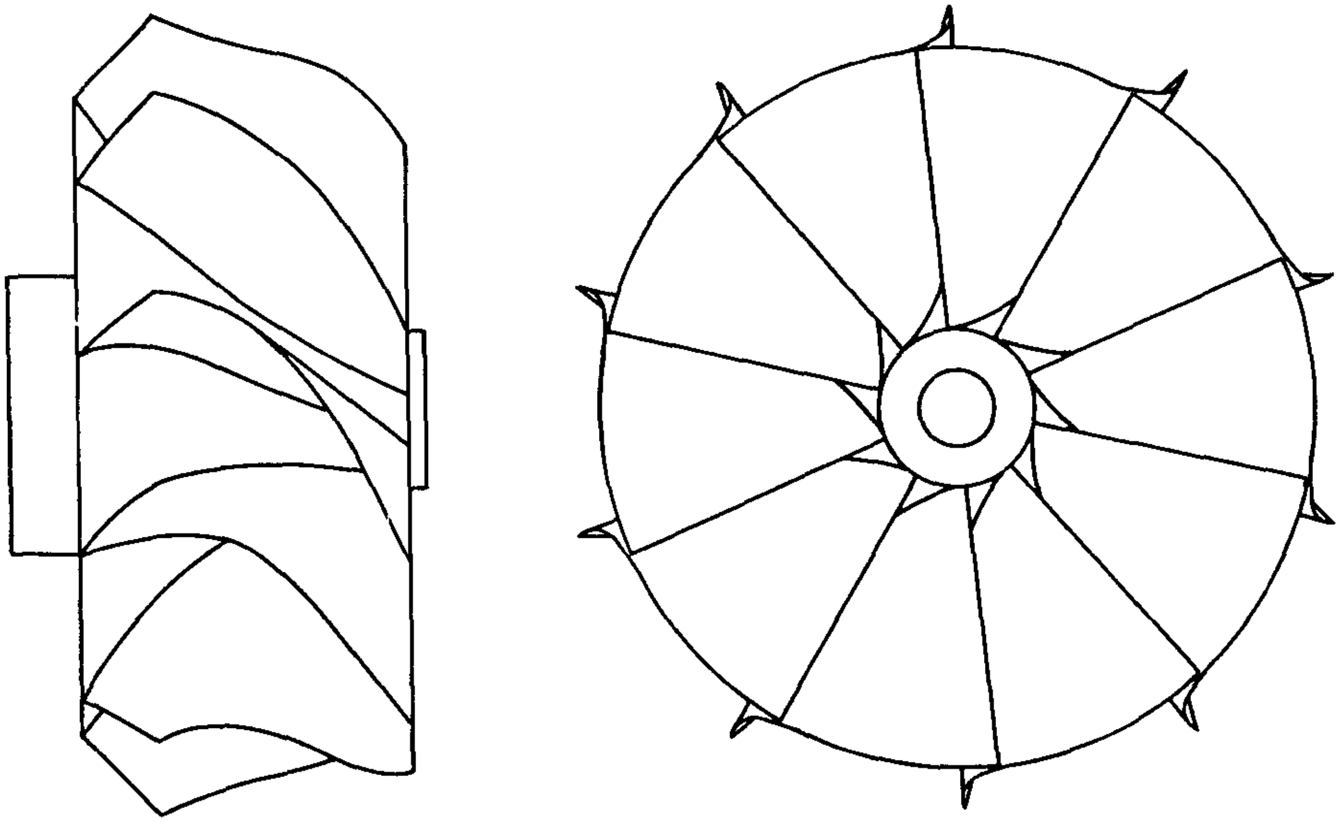
FIG 10a MIXED FLOW TURBINE

FIG. 10b



ISOMETRIC VIEW

BACK ELEVATION



SIDE ELEVATION

FRONT ELEVATION

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