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THE ESTIMATION OF THE LOADING ON SWEPT WINGS WITH EXTENDING CHORD FLAPS AT SUBSONIC SPEEDS

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SUMMARY

A method is given for estimating lift and vortex drag increments due to part-span, extending chord flaps on thin, sweptback, tapered wings of large aspect ratio in inviscid, incompressible flow. It is a linear theory and may be considered as a simple extension of the R.A.E. Standard Method for calculating loadings on such wings and retains similar means of accounting for sweepback, tip and centre effects. Spanwise loadings are obtained by Multhopp's quadrature methods, extended to include discontinuities in wing chord, and examples are given for some typical wing and flap layouts.

* Replaces R.A.E. Technical Report 69034 - A.R.C. 31285

CONTENTS

1 INTRODUCTION

This Report describes a method of estimating lift and vortex drag increments due to part-span flaps. It is applicable to thin, swept, tapered, isolated wings with kinks in the leading and trailing edges occurring only on the line of symmetry. Plain hinged trailing edge flaps that can extend the wing chord are considered. It is assumed that flap deflections are sufficiently small for the flow to remain attached over the flap and for linear aerofoil theory still to be applicable. Further, the flow is assumed to be both inviscid and incompressible (compressibility effects for sub-critical Mach numbers can be included by means of the Prandtl-Glauert analogy).

The deflection of a flap changes the chordwise pressure distribution and for thin aerofoils introduces a second infinite suction peak at the flap hinge. A method which considers the double integral downwash equation as a whole would require a large number of chordwise pivotal points as well as spanwise ones, and thus a large amount of computation. Instead, Küchemann's planar vortex sheet theory[1,2], which forms part of the foundations of the R.A.E. Standard Method, has been extended to include the discontinuities due to part-span flaps. It is assumed that the angle of incidence of the aerofoil and the flap deflection angles are all sufficiently small as to justify neglecting the vertical vortex sheets which in practice are shed from the aerofoil and flap tips. Hence, the horizontal vortex sheets shed from the trailing edge are all assumed to lie in the same plane parallel to the free-stream direction.

Flap deflection is considered to apply camber to the basic flat plate and to increase its angle of incidence by forming a new chordline. A chordwise loading is produced from the same loading equation as used for the basic aerofoil at angle of incidence, i.e. the same relation between downwash and the chordwise load distribution is used. Thus the effects of sweepback, aspect ratio and spanwise position on the loading equation are assumed to be independent of the chordwise shape of the aerofoil section, as is assumed for cambered wings[3].

One of the essential features of Küchemann's method is a way of accounting for the effects of small aspect ratio on the induced angle of incidence through use of the downwash factor. Brebner and Lemaire[4] have considered the effects of part-span flaps on this factor and have given qualitative arguments to account for its modified dependency on sweepback angle and aspect ratio. In order to judge the sensitivity of the spanwise load distribution to changes in the numerical value of the downwash factor, comparisons are made in this Report using a varying spanwise distribution obtained using these ideas, and a constant value as used in Küchemann's method[2].

The discontinuous distributions of angle of incidence and wing chord are dealt with by the methods of Multhopp[5], Weissinger[6] and Weber[7], suitably extended to handle any number of discontinuities.

The effects of part-span flaps on the lift and vortex drag characteristics are shown as examples for some typical wing layouts.

2 THE EQUIVALENT INCIDENCE

Consider the aerofoil to be a flat plate with a simple hinged trailing edge flap of chord ratio c_F. Flap deflection induces an increment in sectional lift, and an 'equivalent angle of incidence' $\Delta\alpha$ can be defined as that angle of incidence the flat plate aerofoil section would have to be given in order to achieve the same increment in lift. For small flap deflections it is supposed that this equivalent angle of incidence is independent of the basic aerofoil incidence. Hence to calculate $\Delta\alpha$ it is sufficient to consider the basic aerofoil at zero incidence to the free-stream direction.

The chordline is defined as the straight line joining the leading edge to the flap trailing edge and forms the x-coordinate axis (see Fig.1). To a first approximation the wing chord is unchanged, but the chordline is inclined at an angle δ to the free-stream direction, where for small flap deflections

$$\delta = c_F \beta \qquad (1)$$

The camber f of the aerofoil, defined as the ratio of the ordinate of the hinge to the chord, is approximately

$$f = c_F(1 - c_F)\ \beta \qquad (2)$$

Using either equation (2) or the exact relation, the camber for a 15° flap deflection, for example, is about 6·5% at the most, which is probably near the limit for linear aerofoil theory; the approximations (1) and (2) are better than 1% for this flap angle.

As in classical aerofoil theory, Küchemann divided the vorticity distribution on the aerofoil and in its wake into two parts[2]:

(i) spanwise system, producing chordwise loading

(ii) streamwise system, producing spanwise loading.

For swept wings of infinite aspect ratio with constant vorticity along lines parallel to the leading edge (and no trailing vorticity), the relation between the downwash and the chordwise loading, both at the centreline and on the sheared part of the wing, can be approximated by[2]

$$\frac{v_{z_1}(x)}{V_o} = \frac{1}{2\pi V_o}\left\{\int_o^1 \gamma_x(\xi)\,\frac{d\xi}{x-\xi} + \sigma(\varphi,y)\,\gamma_x(x)\right\} \tag{3}$$

In the first case σ is a function of sweep angle φ only; in the second case σ is zero. The same type of relation is assumed to hold at any spanwise station and also for wings where the spanwise vorticity is not constant along the span. It has also been used for large aspect ratio cambered wings and will be assumed valid for wings with part-span flaps and aspect ratios not less than about 4.

It is necessary here to solve equation (3) as an integral equation for $\gamma_x(x)$ for prescribed downwash distribution $v_{z_1}(x)$. For this purpose the results of Carleman[1,8] are convenient, and the solution which satisfies the Kutta-Joukowski condition reads:

$$\gamma_x(x) = \frac{2\pi\,\sigma}{\sigma^2+\pi^2}\,v_{z_1}(x) - \frac{2\pi}{\sigma^2+\pi^2}\left[\frac{1-x}{x}\right]^{n_o}\int_o^1 v_{z_1}(\xi)\left[\frac{\xi}{1-\xi}\right]^{n_o}\frac{d\xi}{x-\xi} \quad \ldots (4)$$

where the index n_o is given by

$$n_o = \frac{1}{2\pi}\cos^{-1}\left[\frac{\sigma^2-\pi^2}{\sigma^2+\pi^2}\right]$$

or, more conveniently,

$$\sigma = \pi\cot(\pi\,n_o) \tag{5}$$

On infinite sheared wings, or at mid semi-span of finite aspect ratio wings, σ is zero and thus n_o has the value one half. On the centreline it has been found that provided the sweep angle φ is less than about 50°, then $\sigma = \pi \tan \varphi$ is a good approximation. In this case, therefore, $n_o = \frac{1}{2} - \varphi/\pi$. For wings of finite aspect ratio, the wing tip is considered as the centre section of a wing of opposite sweep, i.e. $n_o = \frac{1}{2} + \varphi/\pi$ there. Küchemann introduced an interpolation function[2] $\lambda(y)$, related to the shift of aerodynamic centre due to the centre effect, so that n_o can be expressed as a function of φ, y:-

$$n_o = \frac{1}{2}\left[1 - \lambda(y)\frac{\varphi}{\pi/2}\right] \qquad (6)$$

Except for very highly swept wings, a suitable expression for $\lambda(y)$ is

$$\lambda(y) = \sqrt{1 + \left(\pi \frac{b}{c} \eta\right)^2} - \sqrt{1 + \left[\pi \frac{b}{c}(1 - |\eta|)\right]^2} + \pi \frac{b}{c}(1 - 2|\eta|) \qquad (7)$$

where b is the wing span, c the local wing chord and η the nondimensional spanwise coordinate. As the aspect ratio becomes infinitely large:-

at the wing tip ($\eta = \pm 1$), $\lambda = -1$

at mid semi-span ($\eta = \pm 0{\cdot}5$), $\lambda = 0$

at the wing centre ($\eta = 0$), $\lambda = +1$

In order to obtain the chordwise load distribution on the flapped aerofoil section, the downwash distribution $v_{z_1}(x)$ must be defined. The total velocity normal to the wing surface must satisfy the stream surface condition which in linear theory reads:

$$\frac{V_z}{V_x} = \frac{\partial z}{\partial x}$$

where V_z = total velocity in z-direction at aerofoil surface

V_x = total velocity in x-direction at aerofoil surface

x,z are nondimensional on local wing chord.

The total perturbation to the downwash is the sum of that due to the spanwise vorticity v_{z_1} and that due to the streamwise vorticity v_{z_2} :-

$$v_{z_1}(x) + v_{z_2}(x) = v_z(x)$$

Thus the stream surface condition becomes

$$\frac{v_{z_1}(x) + v_{z_2}(x) - V_o \sin\delta}{V_o \cos\delta} = \frac{\partial z}{\partial x} \tag{8}$$

The downwash produced by the streamwise vorticity is assumed to be constant over the chord and an induced angle of incidence is defined thus

$$\delta_i = \frac{v_{z_2}}{V_o} \tag{9}$$

For small flap deflection angles, the chordwise distribution of spanwise vorticity $\gamma_x(x)$ must satisfy the relation (3) and induce a downwash distribution $v_{z_1}(x)$ which satisfies the boundary condition (8), i.e.

$$\frac{v_{z_1}(x)}{V_o} = \frac{\partial z}{\partial x} + \delta - \delta_i \tag{10}$$

In this equation, the difference between the geometric angle of incidence δ and the induced angle of incidence δ_i is called the effective angle of incidence. At any spanwise station the slope of the camber line is known, so that for the flapped aerofoil

$$\left.\begin{aligned} \frac{\partial z}{\partial x} &= -c_F\,\beta & \text{for} \quad & 0 \leqslant x < 1 - c_F \\ \frac{\partial z}{\partial x} &= (1 - c_F)\,\beta & \text{for} \quad & 1 - c_F < x \leqslant 1 \end{aligned}\right\} \tag{11}$$

These values may now be used to specify the downwash $v_{z_1}(x)/V_o$ through equation (10), and hence the required distribution of spanwise vorticity $\gamma_x(x)$ can be obtained from equation (4). Thus for small angles β, there results the following equation for $\gamma_x(x)$:-

$$\frac{\gamma_x(x)}{V_o} = (\delta - \delta_i) 2 \sin(\pi n_o) \left[\frac{1-x}{x}\right]^{n_o} - 2 \sin(\pi n_o) \cos(\pi n_o) \beta \begin{bmatrix}1\\0\end{bmatrix} +$$

$$+ 2 \sin(\pi n_o) \left[\frac{1-x}{x}\right]^{n_o} (1 - c_F) \beta + \frac{2}{\pi} \sin^2(\pi n_o) \left[\frac{1-x}{x}\right]^{n_o} \beta B'(x) \quad \ldots (12)$$

In this equation $\begin{bmatrix}1\\0\end{bmatrix}$ means that the upper value holds ahead of the flap hinge, and the lower value over the flap. Also

$$B'(x) = \int_0^{1-c_F} \left[\frac{\xi}{1-\xi}\right]^{n_o} \frac{d\xi}{x - \xi} \quad (13)$$

The index n_o, called the chordwise loading parameter, is defined by equation (6).

If in equation (12) the flap chord ratio c_F is put equal to unity, then δ equals β and

$$\frac{\gamma_x(x)}{V_o} = (\delta - \delta_i) 2 \sin(\pi n_o) \left[\frac{1-x}{x}\right]^{n_o}$$

which represents the chordwise loading on a flat plate aerofoil at an angle of incidence δ (see equation (59) of Ref.2).

To calculate the chordwise load distribution, the following relation is used

$$\Delta C_p(x) = -2 \frac{\gamma_x(x)}{V_o} \frac{\cos\varphi}{\sin(\pi n_o)} \quad (14)$$

where $\cos\varphi/\sin(\pi n_o)$ is a factor introduced by Brebner[3] to account for the fact that in general, the bound vortices on a swept wing are not parallel to the mean sweep direction. If equation (12) is used for $\gamma_x(x)$, then the chordwise load distribution becomes

$$\Delta C_p(x) = -4\cos\varphi\,[(\delta-\delta_i)+\beta(1-c_F)]\left[\frac{1-x}{x}\right]^{n_o} + 4\cos\varphi\,\cos(\pi\,n_o)\,\beta\begin{bmatrix}1\\0\end{bmatrix} -$$

$$-\frac{4}{\pi}\cos\varphi\,\sin(\pi\,n_o)\left[\frac{1-x}{x}\right]^{n_o}\beta\,B'(x) \qquad (15)$$

The sectional lift coefficient is obtained by integrating this equation with respect to x from the leading to trailing edges:-

$$C_L(\eta) = \frac{4\,n_o\,\pi\cos\varphi}{\sin(\pi\,n_o)}\left[(\delta-\delta_i)+\beta(1-c_F)-\beta\,\frac{\sin(\pi\,n_o)}{\pi\,n_o}\,B_{n_o}\right] \qquad (16)$$

where*

$$B_{n_o} = \int_0^{1-c_F}\left[\frac{x}{1-x}\right]^{n_o}dx \qquad (17)$$

If the flap chord ratio is unity, then equation (16) reduces to

$$C_L(\eta) = (\delta-\delta_i)\,\frac{4\pi\,n_o\cos\varphi}{\sin(\pi\,n_o)}$$

which is the familiar form for the sectional lift coefficient of a flat plate aerofoil of large aspect ratio[2]. Since this is a linear theory, the sectional lift slope a is given by

$$a = \frac{C_L}{(\delta-\delta_i)} = \frac{4\pi\,n_o\cos\varphi}{\sin(\pi\,n_o)} \qquad (18)$$

Thus to achieve the increment in lift produced by a plain flap of chord ratio c_F, deflected through a small angle β, the flat plate aerofoil would have to

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* Note that B_{n_o} is a well known integral, being a particular form of the Incomplete Beta Function

$$\beta_x(p,q) = \int_0^x \xi^{p-1}(1-\xi)^{q-1}\,d\xi$$

which is tabulated for certain values of x, p, q.

have an effective angle of incidence α_e (see equation (16)) where

$$\alpha_e = (\delta - \delta_i) + \beta(1 - c_F) - \beta \frac{\sin(\pi n_o)}{\pi n_o} B_{n_o}$$

Operating at the same overall lift coefficient, the flapped aerofoil is assumed to have the same spanwise distribution of induced angle of incidence δ_i as the flat plate aerofoil. Hence the geometric equivalent angle of incidence $\Delta\alpha$ can be written as the sum of α_e and δ_i. Thus in view of equation (1), the equivalent angle of incidence of the flapped aerofoil is directly proportional to the angle of flap deflection β :-

$$\frac{\Delta\alpha}{\beta} = \left[1 - \frac{\sin(\pi n_o)}{\pi n_o} B_{n_o}\right] \qquad (19)$$

and the sectional lift slope is the same as that for the flat plate aerofoil, given by equation (18).

The variation of equivalent angle of incidence with n_o for a range of flap chord ratios is shown on Fig.2. The magnitude of the chordwise loading parameter is determined through wing geometry and spanwise position by equations (6) and (7). n_o is a function of two parameters, φ and λ : if one is fixed then $\Delta\alpha/\beta$ may be plotted against the other. Thus on Fig.2 an alternative abscissa scale is given where n_o has been replaced by sweepback angle φ, and the ordinate shows the equivalent angle of incidence for unit flap deflection angle at the centre-section of a wing of infinite aspect ratio.

The sectional pitching moment referred to the local quarter chord position is given by

$$C_m = - \int_o^1 (\tfrac{1}{4} - x) \, \Delta C_p(x) \, dx$$

The chordwise load distribution $\Delta C_p(x)$ is given by equation (15), thus for the aerofoil at zero angle of incidence

$$C_m = C_L \left\{ \frac{n_o}{2} - \frac{1}{4} - \frac{\sin(\pi\, n_o)\; c_F^{\;1-n_o}\,(1 - c_F)^{1+n_o}}{2[\pi\, n_o - \sin(\pi\, n_o)\; B_{n_o}]} \right\} \tag{20}$$

The local chordwise position of the centre of pressure is obtained from the equation:

$$x_{CP} = \frac{1}{4} - \frac{C_m}{C_L}$$

and hence

$$x_{CP} = \frac{1}{2}\,[1 - n_o] + \frac{\sin(\pi\, n_o)\; c_F^{\;1-n_o}\,(1 - c_F)^{1+n_o}}{2[\pi\, n_o - \sin(\pi\, n_o)\; B_{n_o}]} \tag{21}$$

This function is plotted on Fig.3 against n_o for a range of flap chord ratios. Values at the centre-section of an infinite aspect ratio sweptback wing can be found using the alternative φ scale. For the particular case of an unswept wing, n_o has the value $\frac{1}{2}$ everywhere. Also $B_{n_o} = \frac{1}{2}[\zeta - \sin\zeta]$ where $\zeta = \cos^{-1}(1 - 2c_F)$. Thus in this special case, equation (21) reduces to

$$x_{CP} = \frac{1}{4}\left[\frac{\pi - \zeta + \sin\zeta\,(2 - \cos\zeta)}{\pi - \zeta + \sin\zeta}\right]$$

which is the formula given by Glauert[9] appropriate to such wings.

The values of the chordwise loading parameter and lift slope given by equations (6) and (18) are strictly valid for wings of large aspect ratio only. Indeed, for wings of smaller aspect ratio it is not correct to assume that the downwash produced by the streamwise vorticity $v_{z_2}(x)$ is constant over the chord. Unfortunately, without this approximation it is not possible to easily specify a chordwise distribution of vorticity $v_{z_1}(x)$ which is necessary for the solution of the spanwise vorticity equation (4). Hence in this Report the small aspect ratio corrections introduced by Küchemann[2] are not attempted. It is thought[1] that such corrections are significant only for straight wings of aspect ratio less than about 6, and for wings swept back 45° of aspect ratio smaller than about 3.

For small flap angles, the equivalent angle of incidence $\Delta\alpha$ is a linear function of the flap deflection angle β (equation (19)). Fig.4 shows the variation of the equivalent angle of incidence across the span of a wing of 45° sweepback and constant chord of aspect ratio 4. The full span trailing edge flap has constant flap chord ratio 0·35. The large loss in lift at the centre and gain at the tip is a sweepback effect. This figure also shows some values found on an electrolytic tank analogue computer by Malavard and Duquenne[10], and the results of a calculation made by Brebner and Lemaire[4] based on these values. Over the range $0{\cdot}2 < \eta < 0{\cdot}7$ the agreement is good, but there is considerable scatter in the tank test values at the wing tip, presumably due to experimental difficulties here. This scatter throws some doubt on the accuracy of the wing tip value which was used by Brebner and Lemaire in their calculation. At the wing centre-section the tank test result is much larger than the theoretical: this again may be due to scatter, or it may imply that the equivalent angle of incidence due to flap deflection does not fall off at the centre of sweptback wings as rapidly as is given by the λ-variation of equation (7).

That the effects of sweepback and spanwise position on the equivalent angle of incidence are so large is a result of the fact that the chordwise loading $\Delta C_p(x)$ is completely changed by these effects. To illustrate this, chordwise loadings at the centre-section of wings of large aspect ratio have been calculated by equation (15) for sweepback angles $\varphi^\circ = 0$, 20, 40 and are shown on Fig.5. This figure may also be used to indicate the change in chordwise load distributions with spanwise position for fixed angle of sweepback. For the example of a large aspect ratio wing swept back 40°, the $\varphi = 0$ curve represents the distribution of ΔC_p at mid semi-span, factored by $1/\cos 40^\circ$. The $\varphi = 20^\circ$ curve corresponds to the loading distribution at an intermediate station near to the wing centre section, factored by $\cos 20^\circ/\cos 40^\circ$.

3 SPANWISE LOADING

The induced angle of incidence α_i at the spanwise position η is determined by the whole of the trailing vortex sheet:

$$\alpha_i(\eta) = \frac{\omega}{2\pi} \int_{-1}^{+1} \frac{d}{d\xi}\left[C_L(\xi)\,\frac{c(\xi)}{2b}\right] \frac{d\xi}{\eta - \xi} \tag{22}$$

(Here α_i has been written in place of δ_i to make this section rather more generally suitable: it signifies that the aerofoil itself may have an angle of incidence in addition to the equivalent angle due to flap deflection.

Similarly δ and $(\delta - \delta_i)$ are now replaced by α and α_e, respectively.) The downwash factor ω is defined as the ratio of the mean value of the induced downwash over the chord at a given spanwise station, to one half the downwash at infinity at the same station. Although for most purposes in this Report the downwash factor will be assumed to be constant and equal to unity, it is written into equation (22) to facilitate the possible future application of a non-constant distribution of ω (see Appendix B).

The spanwise loading $\gamma(\eta)$ is related to the spanwise distribution of lift coefficient $C_L(\eta)$ by

$$C_L(\eta) = \frac{2b}{c(\eta)} \gamma(\eta)$$

The boundary condition

$$\alpha = \alpha_e - \alpha_i$$

implies that

$$\alpha_i(\eta) = \alpha(\eta) - \frac{2b \ \gamma(\eta)}{a(\eta) \ c(\eta)} \qquad (23)$$

For physical reasons the spanwise loading $\gamma(\eta)$ must be a continuous function of spanwise position. However, as the wing chord and angle of incidence can both be discontinuous at the flap tips, equation (23) shows that the induced angle of incidence may also be a discontinuous function of spanwise position.

Equations (22) and (23) can be combined to give an integral equation for $\gamma(\eta)$:-

$$\frac{2b}{ac} \gamma(\eta) = \alpha - \frac{\omega}{2\pi} \int_{-1}^{+1} \frac{d}{d\xi} \left[\gamma(\xi) \right] \frac{d\xi}{\eta - \xi} \qquad (24)$$

If $\alpha_i(\eta)$ is a continuous function of η then this equation can conveniently be solved by Multhopp's approximate quadrature method[5]. It is assumed that the spanwise load distribution can be expressed as a Fourier series and equation (24) satisfied exactly at a finite number of given spanwise points. The values of $\gamma(\eta)$ at these points are found from a system of linear, simultaneous, algebraic equations. Obviously the Fourier series technique is invalidated if the spanwise distribution of wing chord or angle of incidence in equation (24) is discontinuous. Multhopp gave a modified scheme[5] for the case of known discontinuities in geometric incidence, and Weissinger[6] has shown how the spanwise loading on an unswept wing of large aspect ratio may be calculated if at one point on the wing there is also a discontinuity in chord. For the general case of a swept wing of any aspect ratio where the lift slope is a function of spanwise position, Weber[7] has given a method for calculating the spanwise loading if there is a discontinuity in geometric incidence, chord and lift slope at one point. In Appendix A Weber's method has been extended to deal approximately with the situation of discontinuities in geometric incidence and chord occurring at any number of spanwise points.

The spanwise load distribution is divided into two parts γ_I and γ^*. The former produces a discontinuous distribution of induced angle of incidence $\alpha_{i_{o_I}}$ depending on the amount and position of the discontinuities and the latter a continuous distribution. It is supposed that γ^* can be represented by a Fourier series and solved exactly at m spanwise Multhopp points (the ν points) and at the k discontinuity points (the s points). It is further assumed that the calculation of γ at the ν points is not seriously affected by the additional points η_s. The loading γ_I and its distribution of induced angle of incidence $\alpha_{i_{o_I}}$, can both be expressed in terms of the values of γ^* at the s points. The (m + k) values of γ^* are calculated from two coupled sets of linear, simultaneous, algebraic equations by an iterative method. With γ^* at the s points known, the loading γ_I may then be calculated and hence the complete spanwise load distribution $\gamma(\eta)$. Details of the method are given in Appendix A.

The spanwise distribution of lift coefficient $C_L(\eta)$ is related to the spanwise loading by

$$C_L(\eta) = \frac{2b}{c}\,\gamma(\eta)$$

and is thus discontinuous at the flap tips if the flaps extend the chord. The total lift is given by

$$\bar{C}_L = A \int_{-1}^{+1} \gamma(\eta)\, d\eta \qquad (25)$$

It is customary to refer $\bar{C}_L$ and A to the wing area with flaps unextended. The spanwise distribution of drag due to lift[1,2] can be approximated by

$$C_D(\eta) = C_{D_v}(\eta) + \lambda(\eta)\ C_L(\eta)^2/a(\eta)$$

where $\lambda(\eta)$ is the spanwise interpolation function of equation (7). The term $\lambda\ C_L^2/a$ arises from the changes in chordwise loading along the span on swept wings of finite aspect ratio. It represents a drag force at the wing centre and a corresponding thrust force at the tips; its sum over the whole wing span is zero. The term $C_{D_v}(\eta)$ is the local vortex drag coefficient given by

$$C_{D_v}(\eta) = \alpha_{i_o}(\eta)\ C_L(\eta)$$

The overall drag due to lift, or vortex drag is the integral of the local vortex drag:-

$$\bar{C}_{D_v} = A \int_{-1}^{+1} \gamma(\eta)\ \alpha_{i_o}(\eta)\ d\eta \qquad (26)$$

where the aspect ratio is the same as that used in equation (25) for the total lift coefficient.

4 EXAMPLES

At present there appear to be no experimental data suitable for making comparisons with the results of this Report, i.e. no test results are available for plain hinged flaps on thin isolated wings. However, in Figs.6 and 7 comparisons are made with some semi-empirical results of Brebner and Lemaire[4], and the electrolytic tank test data[10] upon which they were based. For the case of the unswept wing, the theoretical results derived by using a constant downwash factor, of value unity, agree very well with the tank test results, in fact better than those of Ref.4.

For the swept wing, however, this is not so, the present method apparently underestimates the spanwise loading*(using the electrolytic tank tests as a standard). The first point to note is that the tank value for the equivalent angle of incidence at the centre line (see Fig.4) is much higher than the theoretical, thus giving a higher overall loading. This discrepancy between tank results and those of theories similar to the present one has

*Subsequent to completion of this Report, Garner and Lehrian[14] have published an approximate theoretical method for treating oscillating control surfaces They have considered examples similar to this, and have obtained better agreement with the electrolytic tank results been noted before[4] and has not yet been explained. However, Küchemann's swept wing theory extended to thin cambered wings without flaps[3] gives good agreement with wind-tunnel results, hence it seems possible that in Fig.7 the tank test results have the grosser error.

The second point to note is that this theory assumes the downwash factor to be the same as on a wing without flap (i.e. unity, as an aspect ratio of 4 for 45° of sweepback is considered to be 'large') and thus constant across the whole span. This assumption may well be invalid on a wing with part-span flaps. Brebner and Lemaire used a constant value for the downwash factor, deduced from the tank test results, but introduced a spanwise loading factor which multiplied the loading γ_I due to the flap discontinuities. This factor is a function of spanwise position, is dependent on wing and flap geometry and is unity for straight wings. It cannot be determined theoretically, but its use appears to be an alternative to allowing ω to vary across the span. To investigate the effect of a non-constant downwash factor, Fig.7 also shows the result of a computation using values of ω found after the manner outlined in Appendix B. The value of ω on the wing centreline was 0·85, ω decreased to a minimum of about 0·8 at the flap tip and rose to 1·0 at the wing tip. Outboard of the flap the loading is not changed very much, whereas on the flap itself there is a considerable increase in spanwise loading. Tests using constant values of the downwash factor but not equal to unity have resulted in inferior improvements to the spanwise loading (by increasing the loading inboard of the flap discontinuity but decreasing it outboard). These experiments indicate that the effect on the spanwise loading of using a non-constant downwash factor distribution, such as could be produced by consideration of the downwash due to the flap tip trailing vortices, can be important.

Effects of changes in flap or wing geometry are easily investigated by this method without recourse to any graphical interpolation. Fig.8, for example, indicates the effect of varying the span of an inboard flap on an 'Airbus' type of wing, all other parameters being kept constant. The downwash factor is unity, and the flaps do not extend the wing chord. The ordinate used is the increment of the local lift coefficient made nondimensional on overall lift, and the figure indicates how the peak sectional loading is influenced by flap span.

Part-span factors for the increments in lift due to flap, relevant to the loadings of Fig.8 are shown on Fig.9. This factor is defined as the ratio of the lift increment due to the part-span inboard flap to that due to a similar full-span flap. A comparison is made with factors determined from the Royal Aeronautical Society's Data Sheets[11], and it appears that these can over-estimate K_L by as much as 5%.

The vortex drag factor, defined as the ratio of the vortex drag on the wing with flap to that of the same wing without a flap but twisted to give elliptic loading, varies from 5·434 for the flap of 20% span down to 1·016 for the full-span flap. The vortex drag coefficient can be written

$$\bar{C}_{D_v} = \frac{1}{\pi A}\left[K_1 \bar{C}_L^{\,2} + K_2 \Delta C_L^{\,2} + 2K_3 \bar{C}_L \Delta C_L\right] \qquad (27)$$

where $\bar{C}_L$ is the overall lift coefficient (produced by angle of incidence and flap deflection) and ΔC_L the increment due to flap deflection. The factor K_1 represents the departure from elliptic loading of the basic wing and is unchanged by flap deflection if the wing chord remains unaltered. For the wing described on Fig.8, K_1 has the value 1·015. The method of the Royal Aeronautical Society's Data Sheets is to assume that vortex drag can be expressed as

$$\bar{C}_{D_v} = \frac{1}{\pi A} \bar{C}_L^{\,2} + K_D^{\,2} \Delta C_L^{\,2} \qquad (28)$$

where $\bar{C}_L$ and ΔC_L are defined as for equation (27). From equation (27), vortex drag at zero lift $\bar{C}_{D_{v_o}}$ is given by

$$\bar{C}_{D_{v_o}} = \frac{K_2}{\pi A} \Delta C_L^{\,2}$$

and by equation (28)

$$\bar{C}_{D_{v_o}} = K_D^{\,2} \Delta C_L^{\,2}$$

The part-span factors K_2 and $\pi A\ K_D^{\,2}$ are compared on Fig.10 against relative flap span. Unlike $K_D^{\,2}$, for a full-span flap K_2 is not necessarily zero. Equation (28) assumes that K_1 is unity and that minimum vortex drag occurs at zero lift. From equation (27) this happens when

$$\bar{C}_{L_{MD}} = -\frac{K_3}{K_1} \Delta C_L$$

The factor K_3 is also shown on Fig.10. It is numerically small compared with K_1 and K_2, but as indicated above is not necessarily insignificant.

Fig.11 shows the effect on the spanwise load distribution of having a portion of the flap undeflected at a position typical for an engine installation. The wing is the same as for Fig.8, using a flap span of 80%. Both flaps are deflected equally and again there is no discontinuity in the wing chord. The effect on the spanwise loading is considerable, especially outboard of the cut-out. For comparison purposes, the figure shows the distribution for the continuous flap of the same span, and also that for the unflapped wing twisted to give elliptic loading.

The angle of incidence of the wing is zero and the discontinuity in flap deflection produces a rise in the vortex drag factor from 1·168 to 1·311, i.e. about 12%. However, in a more typical case, for example at take-off with the wing at an angle of incidence of about 11° and a flap deflection of 15°, the loss in lift due to the cut-out requires an increase in angle of incidence of about 1° and the vortex drag factor is raised by only 1%, from 1·026 to 1·039 (see Fig.12). If the flaps extend the wing chord, then the loading due to incidence is also affected by the cut-out. Fig.13 shows the effect of a 20% increase in wing chord, keeping the flap chord ratio constant. The comparison is made for $C_L c$ (or $2b\gamma$) rather than for the sectional lift coefficient, since the latter is discontinuous at the flap tips. The wing area has been increased, so that in order to maintain the same overall lift with the same flap setting of 15°, the wing angle of incidence has been reduced to 10°; the vortex drag factor is increased by approximately 3% to 1·067.

5 CONCLUSIONS

This Report describes a simple extension of the R.A.E. Standard Method for calculating spanwise loadings, to include the effects of part-span trailing edge flaps on wings of large aspect ratio. The sectional lift produced by flap deflection is represented by the lift generated by an equivalent angle of incidence of the basic flat plate aerofoil section. The local lift slope is assumed not to be affected by deflection of a flap, and the chordwise load distribution is modified by sweepback and spanwise position in a similar manner as that due to the angle of incidence of the basic aerofoil. Small aspect ratio and thickness effects are not included. As yet there is no suitable experimental evidence to indicate how better is this theory at predicting lift and vortex drag increments than existing techniques.

Most calculations in this Report have been performed assuming the downwash factor ω to be unchanged from its constant value of unity appropriate to wings of large aspect ratio without flaps. However, a tentative scheme to include part-span effects is put forward, using ideas of Ref.4 to generate a non-constant spanwise distribution for ω, and an example is given which indicates the considerable effect on the spanwise load distribution. To take this matter further would mean revising the theory of section 2 to incorporate a chordwise distribution of downwash due to streamwise vorticity that is not nearly constant over the wing chord. Further, the means of generating ω outlined in Appendix B would have to be considerably refined and put on a firmer basis. Nevertheless, the apparent sensitivity of the spanwise loading to the value of the downwash factor would seem to indicate that the matter merits further attention.

The theory enables the effects of flap or wing geometry changes to be easily investigated using a consistent set of assumptions. The vortex drag increments due to part-span flap deflection include the effects of sweepback and implied camber, which is an advance on the calculation method of the Royal Aeronautical Society's Data Sheets. Systematic investigations into the effects of aspect ratio, sweepback, taper ratio, flap chord ratio, etc., as required for design work can easily be carried out using this theory. As it is an extension of the R.A.E. Standard Method, a number of numerical methods which have been developed for classical aerofoil theory can be applied so that a consistent and complete set of calculations can be carried out. The effects of cranks in the leading and trailing edges may be included by the method of Brebner[12], and the effects of a fuselage by applying the method of Weber, Kirby and Kettle[13].

The theory as developed is linear and numerous approximations have been made (e.g. that in spite of deflecting the flap, linear aerofoil theory still applies; that the vortex sheets from the flap and undeflected parts of the trailing edge all lie in the plane $z = 0$). The test of a mathematical model is how well it agrees with experimental facts. At present such a comparison cannot be made, so it is not possible to predict with any degree of reliability the practical limits of aspect ratio, sweepback, flap deflection angle, etc. for which the theory can be applied.

Appendix A

THE SPANWISE LOAD DISTRIBUTION WITH DISCONTINUITIES IN ANGLE OF INCIDENCE AND WING CHORD

Weber has given a method[7] for the calculation of the spanwise loading at the Multhopp points η_ν and one arbitrary point η_s at which there is a discontinuity in α or c (called a 'discontinuity' point). It was shown that the calculation of γ at the η_ν points is not affected by the additional discontinuity point. However, if there is more than one discontinuity, then it is difficult to produce an analogous method. An approximate scheme is given here whereby it is assumed that the loading at the η_ν points is not affected by the k discontinuity points, and further, that at each of these points, the induced angle of incidence α_{i_o} may be expressed in terms of the loading at the Multhopp points and the loading at that discontinuity point only. Thus the error is likely to be least, if the η_s points are well away from each other or from the η_ν points.

The spanwise load distribution is divided into two parts

$$\gamma(\eta) = \gamma_I(\eta) + \gamma^*(\eta) \qquad \text{(A-1)}$$

where γ_I produces a discontinuous distribution of induced angle of incidence $\alpha_{i_{o_I}}$, and γ^* a continuous distribution $\alpha^*_{i_o}$. Consider the point η_s, where there occurs a jump in the angle of incidence of amount σ_s, i.e.

$$\sigma_s = \alpha(\eta_s + 0) - \alpha(\eta_s - 0) \qquad \text{(A-2)}$$

and also a jump in wing chord. Define

$$\tau_s = \frac{2b}{a}\left[\frac{1}{c(\eta_s - 0)} - \frac{1}{c(\eta_s + 0)}\right] \qquad \text{(A-3)}$$

As the load distribution γ must be continuous at η_s,

$$\gamma(\eta_s + 0) = \gamma(\eta_s - 0) = \gamma_s$$

$$= \frac{a}{2b} c(\eta_s + 0)[\alpha(\eta_s + 0) - \omega\, \alpha_{i_o}(\eta_s + 0)]$$

$$= \frac{a}{2b} c(\eta_s - 0)[\alpha(\eta_s - 0) - \omega\, \alpha_{i_o}(\eta_s - 0)]$$

On equating these two expressions for γ_s, there results for the jump in induced angle of incidence

$$\alpha_{i_o}(\eta_s + 0) - \alpha_{i_o}(\eta_s - 0) = \frac{\tau_s\, \gamma_s + \sigma_s}{\omega} \qquad (A-4)$$

The lift slope a and downwash factor ω can both be functions of spanwise position. They are assumed here to be continous and evaluated at the appropriate point.

If the function $F(\vartheta, \vartheta_s)$ is defined as follows (see Multhopp[5])

$$F(\vartheta, \vartheta_s) = \frac{2}{\pi}\left[(\cos\vartheta - \cos\zeta)\log\left(\frac{\sin\frac{1}{2}(\vartheta + \zeta)}{\sin\frac{1}{2}|\vartheta - \zeta|}\right) + \zeta\sin\vartheta\right]_{\zeta=\vartheta_s}^{\zeta=\pi} \qquad (A-5)$$

then the load distribution γ_I given by

$$\gamma_I(\vartheta) = \sum_{s=1}^{k} \frac{\gamma_s\, \tau_s + \sigma_s}{\omega_s} F(\vartheta, \vartheta_s) \qquad (A-6)$$

produces a discontinuous distribution of induced angle of incidence $\alpha_{i_{o_I}}$ with a jump at each discontinuity of the required amount.

$$\alpha_{i_{o_I}}(\vartheta) \begin{cases} 0 & 0 \leq \vartheta < \vartheta_{s_1} \\ \dfrac{\tau_1\, \gamma_{s_1} + \sigma_1}{\omega_{s_1}} & \vartheta_{s_1} < \vartheta < \vartheta_{s_2} \\ \quad '' \qquad + \dfrac{\tau_2\, \gamma_{s_2} + \sigma_2}{\omega_{s_2}} & \vartheta_{s_2} < \vartheta < \vartheta_{s_3} \\ \text{etc.} & \end{cases} \qquad (A-7)$$

From equation (A-1), the load distribution γ^* can be written:-

$$\gamma^*(\eta) = \frac{ac}{2b}\left[\alpha - \omega\,\alpha_{i_{o_I}} - \gamma_I\,\frac{2b}{ac} - \omega\,\alpha^*_{i_o}\right] \qquad \text{(A-8)}$$

where $\alpha^*_{i_o}$ is the continuous distribution of induced angle of incidence generated by the loading γ^*. $\alpha^*_{i_o}$ can be expressed in terms of γ^* at the Multhopp points and the discontinuity points:-

$$\alpha^*_{i_{o_\nu}} = b_{\nu\nu}\,\gamma^*_\nu - \sum_{n=1}^{m}{}' \, b_{\nu n}\,\gamma^*_n \qquad \text{(A-9)}$$

$$\alpha^*_{i_{o_s}} = b_{ss}\,\gamma^*_s - \sum_{n=1}^{m} b_{sn}\,\gamma^*_n \qquad \text{(A-10)}$$

The second of these two equations in conjunction with equation (A-7) is in accordance with the assumption that at a discontinuity point, α_{i_o} may be written in terms of the loading at the η_ν points and at that point. The coefficients $b_{\nu\nu}$ and $b_{\nu n}$ are the familiar Multhopp coefficients[5]; b_{ss} and b_{sn} are given by Weber[7] as

$$\left.\begin{aligned}
b_{ss} &= \frac{m+1}{2\sin\vartheta_s} \\
b_{sn} &= \frac{a_{sn}\sin\vartheta_n}{(m+1)(\cos\vartheta_s - \cos\vartheta_n)^2} \\
a_{sn} &= \begin{cases} \sin^2\dfrac{(m+1)\,\vartheta_s}{2} & n \text{ even} \\[2ex] \cos^2\dfrac{(m+1)\,\vartheta_s}{2} & n \text{ odd} \end{cases}
\end{aligned}\right\} \qquad \text{(A-11)}$$

In equation (A-8), substitute for $\alpha^*_{i_o}$ from equations (A-9) and (A-10). For the Multhopp points there result m equations:-

$$\left[b_{\nu\nu} + \frac{2b}{a\,c\,\omega}\right]_\nu \gamma^*_\nu = \left[\frac{\alpha}{\omega}\right]_\nu + \sum_{n=1}^{m}{}' b_{\nu n}\,\gamma^*_n - \left[\alpha_{i_{o_I}}\right]_\nu - \left[\gamma_I \frac{2b}{\omega\,a\,c}\right]_\nu \qquad \text{(A-12)}$$

where subscript ν denotes evaluation at the spanwise position η_ν. For the discontinuity points η_s, there is a choice of evaluating equation (A-8) at either side of the discontinuity: the $(\eta_s - 0)$ alternative is chosen here. Thus there are obtained k equations

$$\left[b_{ss} + \frac{2b}{a\,c\,\omega}\right]_{s-} \gamma^*_s = \left[\frac{\alpha}{\omega}\right]_{s-} + \sum_{n=1}^{m} b_{sn}\,\gamma^*_n - \left[\alpha_{i_{o_I}}\right]_{s-} - \left[\gamma_I \frac{2b}{\omega\,a\,c}\right]_{s-} \qquad \text{(A-13)}$$

These two sets of linear, simultaneous equations are coupled through γ^*_ν and $\alpha_{i_{o_I}}$, γ_I which are functions of γ^*_s, and it is convenient to use matrix methods for their solution. Values of γ_I at the Multhopp and discontinuity points are given by equation (A-6) thus:-

$$\gamma_{I\nu} = \mathcal{F}_\nu\,\mathcal{L}_1\,\gamma_s + \mathcal{F}_\nu\,\mathcal{L}_2 \qquad \text{(A-14)}$$

$$\gamma_{Is} = \mathcal{F}_s\,\mathcal{L}_1\,\gamma_s + \mathcal{F}_s\,\mathcal{L}_2 \qquad \text{(A-15)}$$

where

$\gamma_{I\nu}$ is a column vector of the m values $\gamma_{I\nu}$

γ_{Is} " " " " " " k " γ_{Is}

γ_s " " " " " " k " γ_s

$\mathcal{L}_2$ " " " " " " k " σ_s/ω_s

$\mathcal{L}_1$ is a diagonal matrix of the k values τ_s/ω_s

$\mathcal{F}_\nu$ is a m × k matrix with ν^{th} row:

$$F(\vartheta_\nu, \vartheta_{s_1}), \quad F(\vartheta_\nu, \vartheta_{s_2}), \; \ldots., \; F(\vartheta_\nu, \vartheta_{s_k})$$

$\mathcal{F}_s$ is a k × k matrix with s^{th} row:

$$F(\vartheta_s, \vartheta_{s_1}), \quad F(\vartheta_s, \vartheta_{s_2}), \; \ldots., \; F(\vartheta_s, \vartheta_{s_k})$$

In vector notation, equation (A-1) at the discontinuity point can be written

$$\gamma_s = \gamma_{Is} + \gamma^*_s$$

and similarly for the Multhopp points. γ_s can now be expressed in terms of γ^*_s by substitution into (A-15):-

$$\gamma_s = [\mathcal{I} - \mathcal{J}_s \mathcal{L}_1]^{-1} [\gamma^*_s + \mathcal{J}_s \mathcal{L}_2]$$

where $\mathcal{I}$ is the unit matrix of order k, and the inverse operation is assumed to be non-singular. Hence γ_I and γ_{Is} can also be written in terms of γ^*_s and known geometric terms only:-

$$\gamma_{I\nu} = \mathcal{J}_\nu \mathcal{L}_1 [\mathcal{I} - \mathcal{J}_s \mathcal{L}_1]^{-1} [\gamma^*_s + \mathcal{L}_1^{-1} \mathcal{L}_2] \qquad \text{(A-16)}$$

$$\gamma_{Is} = \mathcal{J}_s \mathcal{L}_1 [\mathcal{I} - \mathcal{J}_s \mathcal{L}_1]^{-1} [\gamma^*_s + \mathcal{L}_1^{-1} \mathcal{L}_2] \qquad \text{(A-17)}$$

If there are no discontinuities in chord, then $\mathcal{L}_1^{-1}$ is singular, and in this case $\gamma_{I\nu}$ is simply $\mathcal{J}_\nu \mathcal{L}_2$ and correspondingly for the η_s points.

Equations (A-12), (A-13) and (A-1), (A-7), (A-16), (A-17) constitute a sufficient set of simultaneous, linear equations for the (m + k) unknowns γ^*_ν and γ^*_s. They can be solved by an iterative process. Initially assume that there are no discontinuities in chord (i.e. $\mathcal{L}_1$ zero) so that $\gamma_{I\nu}$ and $\alpha_{1_{oI\nu}}$ are determined solely by geometric quantities. Equation (A-12) can thus be solved for γ^*_ν. Substitute these values into (A-13) and solve for γ^*_s (applying equations (A-17), (A-7) and (A-1)) using the correct values of the discontinuities in chord. The values of γ^*_s thus found may be substituted into (A-16) and (A-7) so that a new estimate of γ^*_ν may be obtained from (A-12), and so on.

Appendix B

THE EFFECT OF PART-SPAN FLAPS ON THE DOWNWASH FACTOR

Küchemann[2] assumed that ω did not vary across the span of swept wings and that it was an adequate approximation to take ω equal to twice the value of n at the mid semi-span position. Thus for wings without flaps, ω is a function of A and φ only. For large aspect ratio wings, $\omega \to 1$ as $A \to \infty$; for small aspect ratios, $\omega \to 2$ as $A \to 0$. However, for wings with part-span flaps, the discontinuous distribution of angle of incidence may produce a spanwise load distribution whose slope is not small at spanwise stations outside the tip regions. In such cases it may no longer be an adequate approximation to take the downwash factor constant across the whole wing span.

In a previous attempt to determine the effects of part-span flaps on swept wings, Brebner and Lemaire[4] considered the influence of the downwash produced by the strong trailing vortices at the flap tip discontinuities. They produced qualitative arguments for the effects of aspect ratio and sweep-back and used them to assist in the analysis of electrolytic tank test data[10]. The spanwise distribution of the downwash factor due to an isolated trailing vortex is a function of the aspect ratio of the wing, its angle of sweepback and the chordwise position of the point of origin of the vortex (the hinge for a flap discontinuity vortex). For unswept wings without flaps, it was argued that there would be little overall change in the distribution of ω and it would still be an adequate approximation to take it constant across the span. Fig.6 compares the spanwise load distribution calculated by the method of this Report using $\omega = 1{\cdot}0$ with electrolytic tank test results for an unswept wing with inboard flap. The agreement is very good, in fact better than that of Brebner and Lemaire which used a constant value for ω slightly greater than unity (small aspect ratio effects are just apparent on unswept wings of aspect ratio 4).

For swept wings with part-span flaps, Brebner and Lemaire considered that there may well be significant departures from ω = constant, but were unable to give a quantitative assessment of $\omega(\eta)$. In order to see what effect such a non-constant distribution of the downwash factor would have on the spanwise loading, a factor has been derived which tends to unity as the angle of deflection, or chord ratio or span of the flap tends to zero. At all spanwise stations, the mean value over the chord of the downwash produced by trailing vortices at the discontinuities springing from the local centres of pressure with the flaps deflected, was divided by the mean downwash produced by trailing vortices of the same strength but springing from the centres of pressure with the flaps undeflected. This factor thus varies with the proportion of the total lift that is due to the flap and is automatically unity when the flap is undeflected. For the purposes of this exercise, this factor has been called the downwash factor, but it is not intended that this should necessarily be the way that ω should be estimated.

SYMBOLS

a	sectional lift slope, C_L/α_e
b	wing span
c	local wing chord
c_F	flap chord
k	number of discontinuities in induced angle of incidence
m	number of spanwise Multhopp points
n_o	chordwise loading parameter
	z-component of induced velocity
x, y, z	rectangular coordinates: x in free stream director, zero at leading edge; y in spanwise direction, positive to starboard; z positive vertically downwards. x, z nondimensional on c
x_v	chordwise point of origin of a trailing vortex
A	wing aspect ratio
B_{n_o}	$\int_0^{1-c_F} \left[\frac{x}{1-x}\right]^{n_o} dx$
$B'(x)$	$\int_0^{1-c_F} \left[\frac{\xi}{1-\xi}\right]^{n_o} \frac{d\xi}{x-\xi}$
C_{D_V}	sectional vortex drag coefficient
$\bar{C}_{D_V}$	overall vortex drag coefficient
$\bar{C}_{D_{V_o}}$	$\bar{C}_{D_V}$ at zero lift
C_L	sectional lift coefficient
$\bar{C}_L$	overall lift coefficient
$\bar{C}_{L_{MD}}$	$\bar{C}_L$ when $\bar{C}_{D_V}$ is a minimum
C_m	sectional pitching moment coefficient about quarter chord position
$K_{1,2,3}$, K_D^2	part-span drag factors
K_L	part-span lift factor
V_o	free-stream velocity
α	local angle of incidence (equals the sum of $\Delta\alpha$ and the angle of incidence of the aerofoil with undeflected flaps)
α_e	effective angle of incidence (equals α less the induced angle of incidence α_i)

SYMBOLS (Contd.)

α_i	induced angle of incidence
α_{i_o}	α_i on wings of large aspect ratio
β	angle of flap deflection (streamwise)
γ	$C_L c/2b$, nondimensional load distribution
γ_I	load distribution that produces a discontinuous distribution of α_{i_o}
γ^*	$\gamma - \gamma_I$
$\gamma_x(x)$	distribution of spanwise vortices along the chord
δ	angle between chordline and aerofoil ahead of the flap hinge
δ_i	value of α_i when the aerofoil with flaps undeflected is at zero angle of incidence
ϑ	spanwise coordinate $\quad \eta = \cos\vartheta$
η	" " $= 2y/b$
λ	spanwise interpolation function
σ	$\pi \cot(\pi n_o)$
φ	angle of sweepback of mid-chord line
ω	downwash factor
$\Delta C_p(x)$	difference between pressure coefficients on upper and lower surfaces of the aerofoil
$\Delta\alpha$	equivalent angle of incidence due to flap deflection

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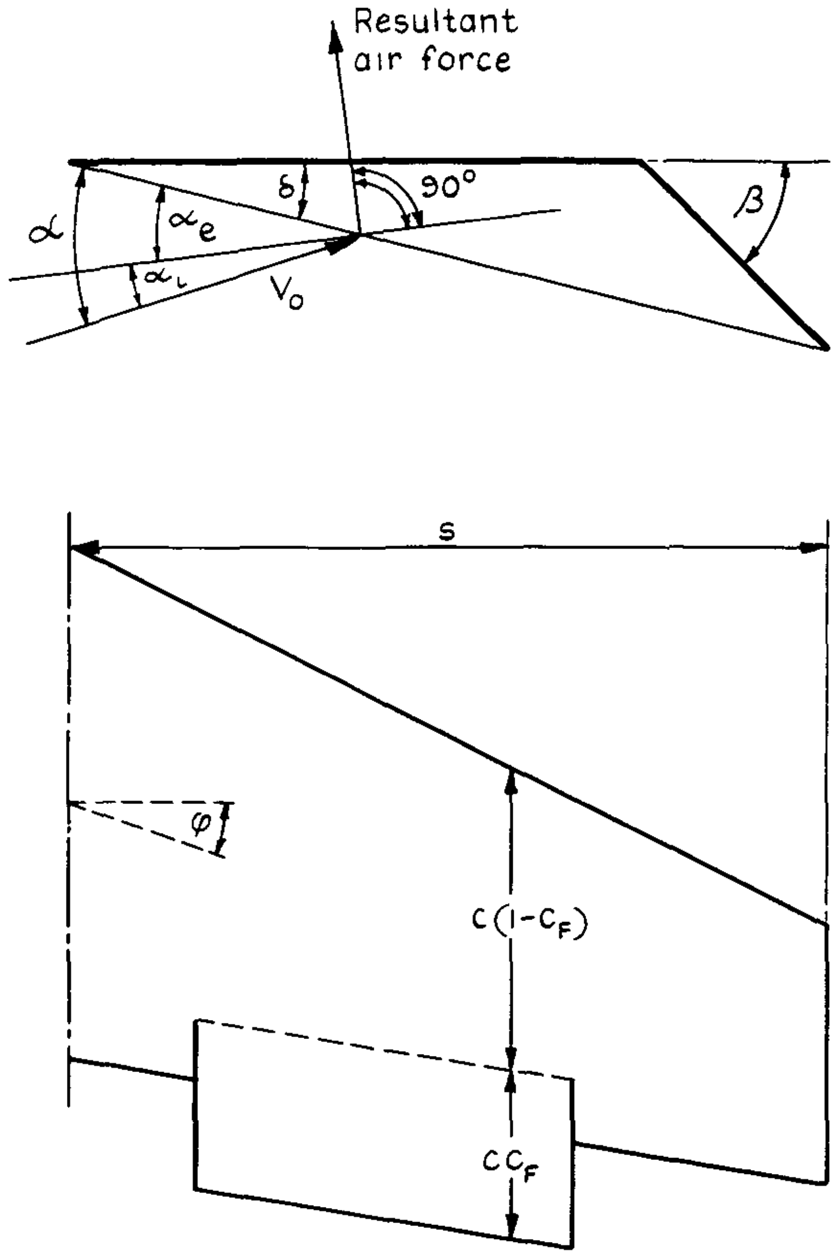

Fig. 1 Wing and flap geometry

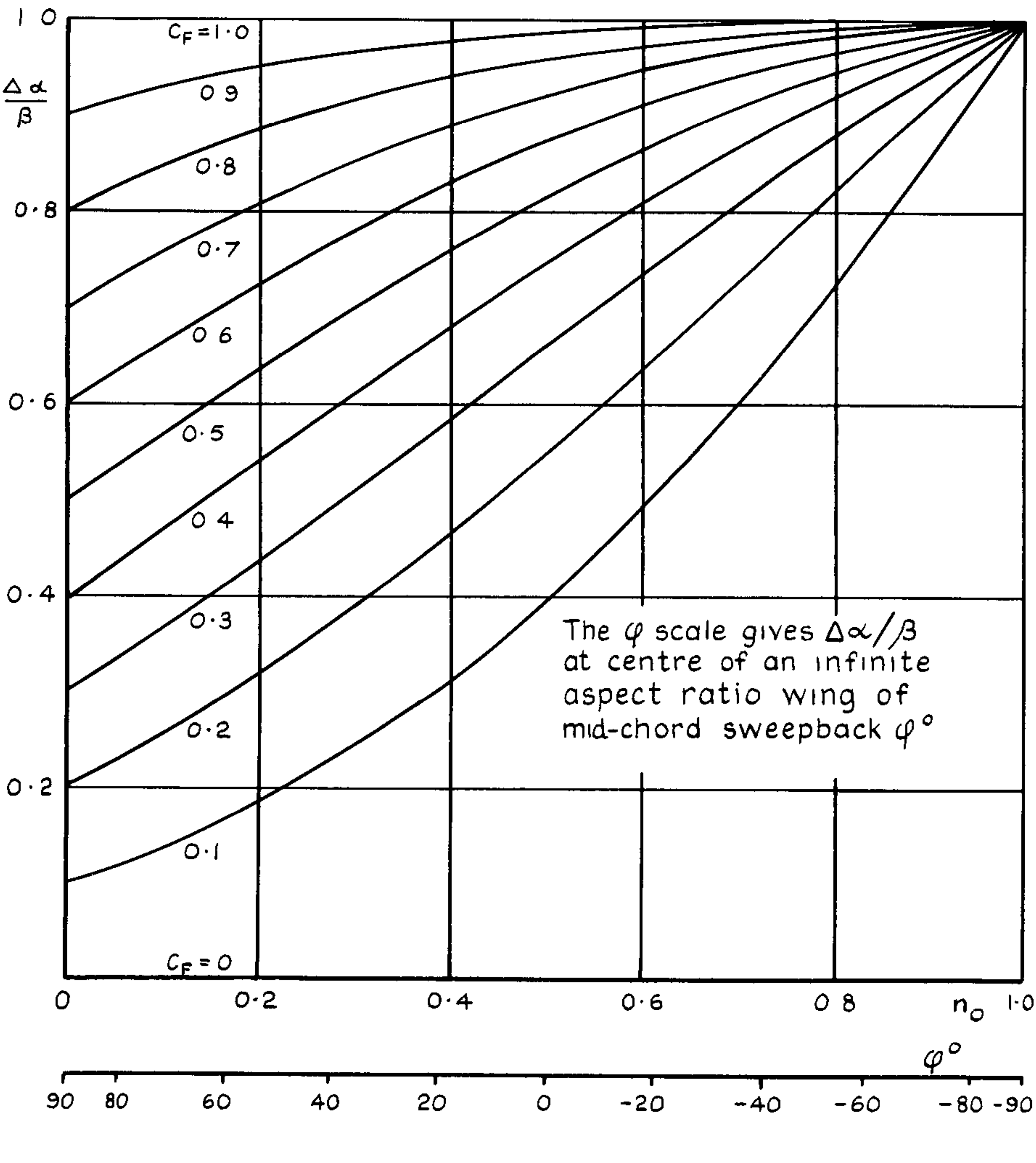

Fig. 2 Variation of equivalent incidence with C_F and n_o

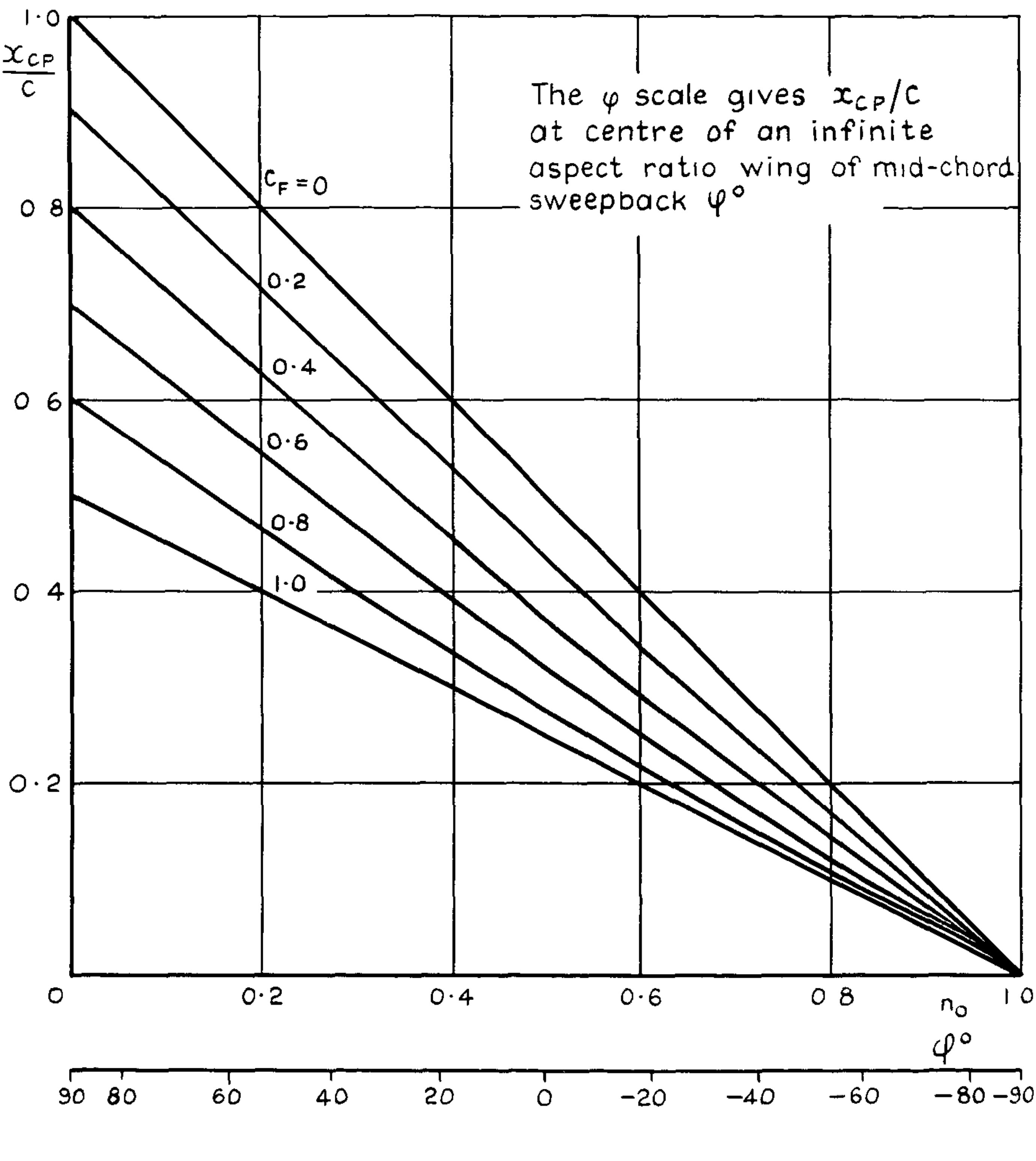

Fig.3 Variation of chordwise centre of pressure with C_F and n_o

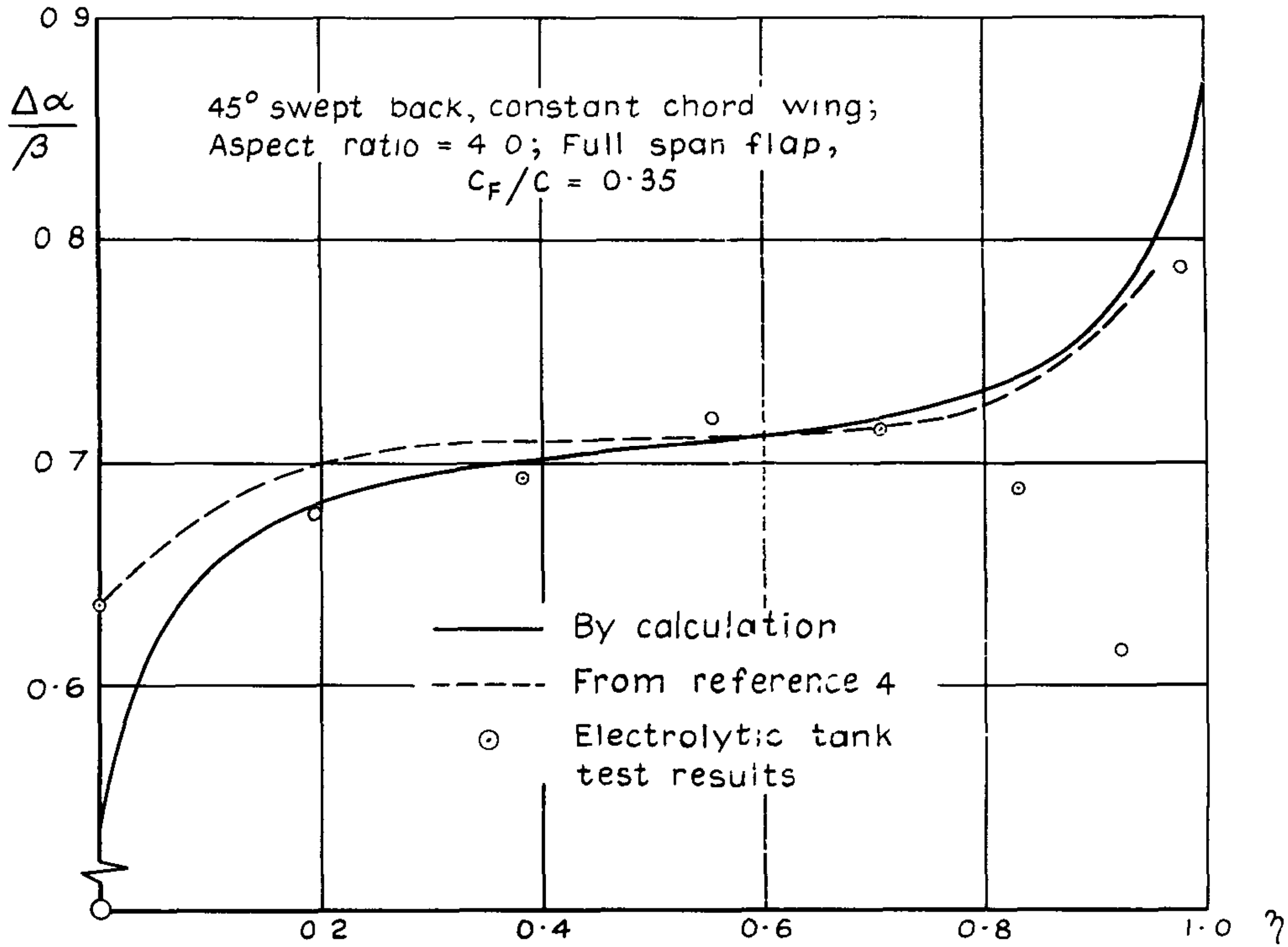

Fig. 4 Spanwise variation of equivalent incidence

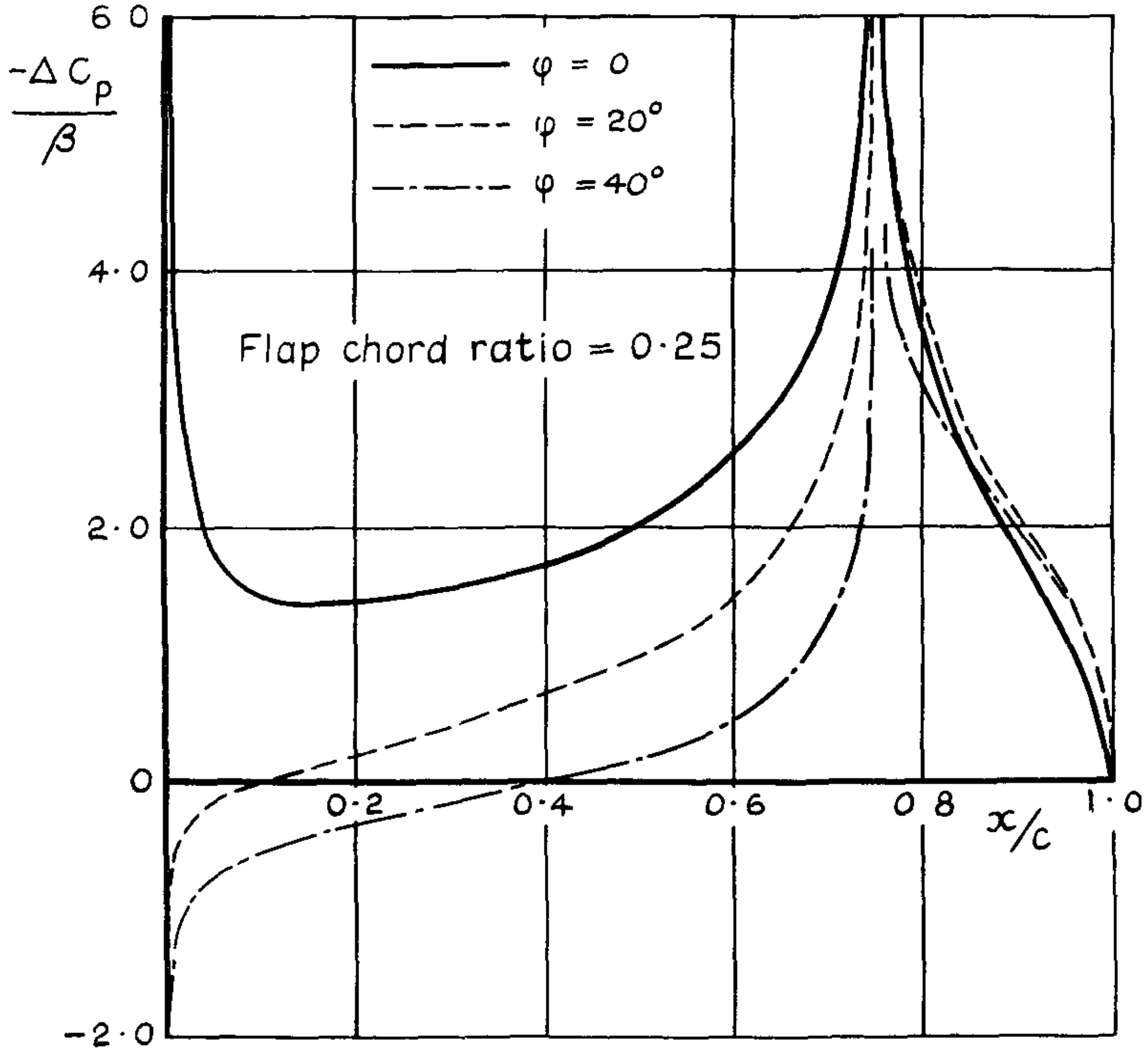

Fig. 5 Effect of sweepback on chordwise pressure distribution at centre of large aspect ratio wing

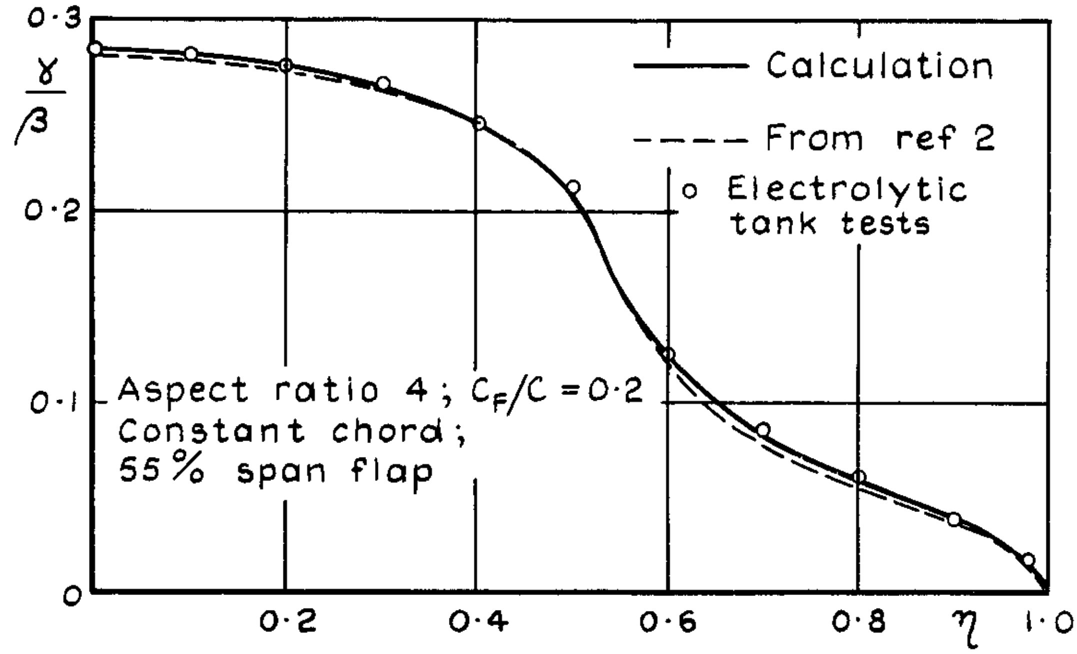

Fig. 6 Spanwise loading on an unswept wing with inboard flap

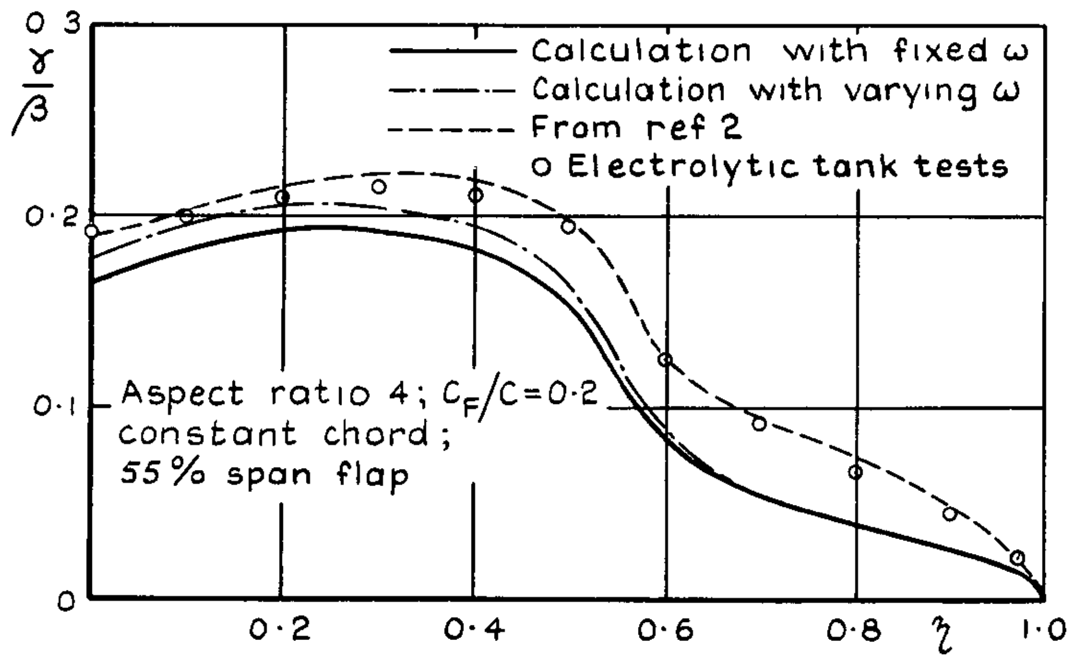

Fig. 7 Spanwise loading on a 45° sweptback wing with inboard flaps

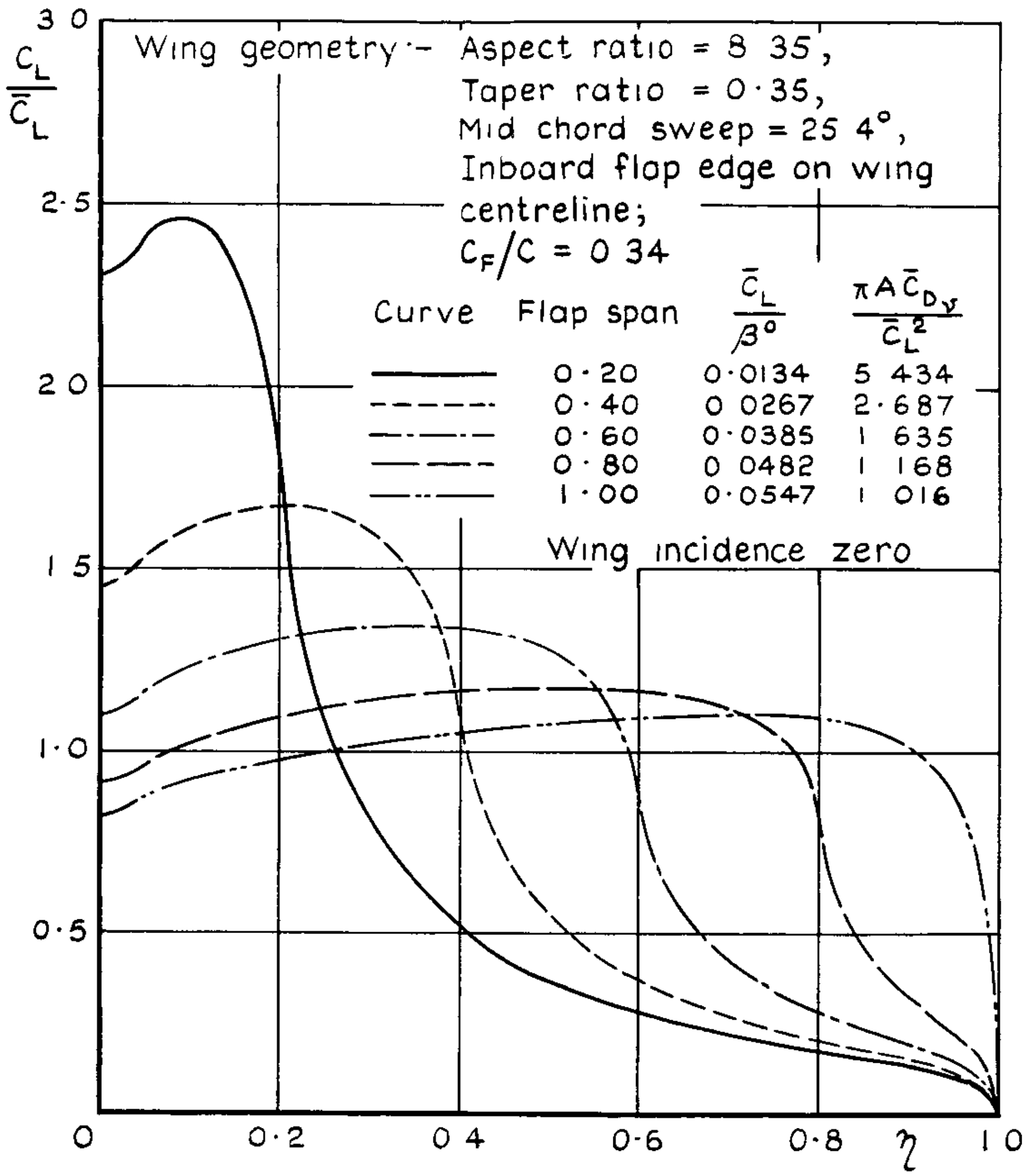

Fig. 8 Effect of flap span on spanwise load distributions

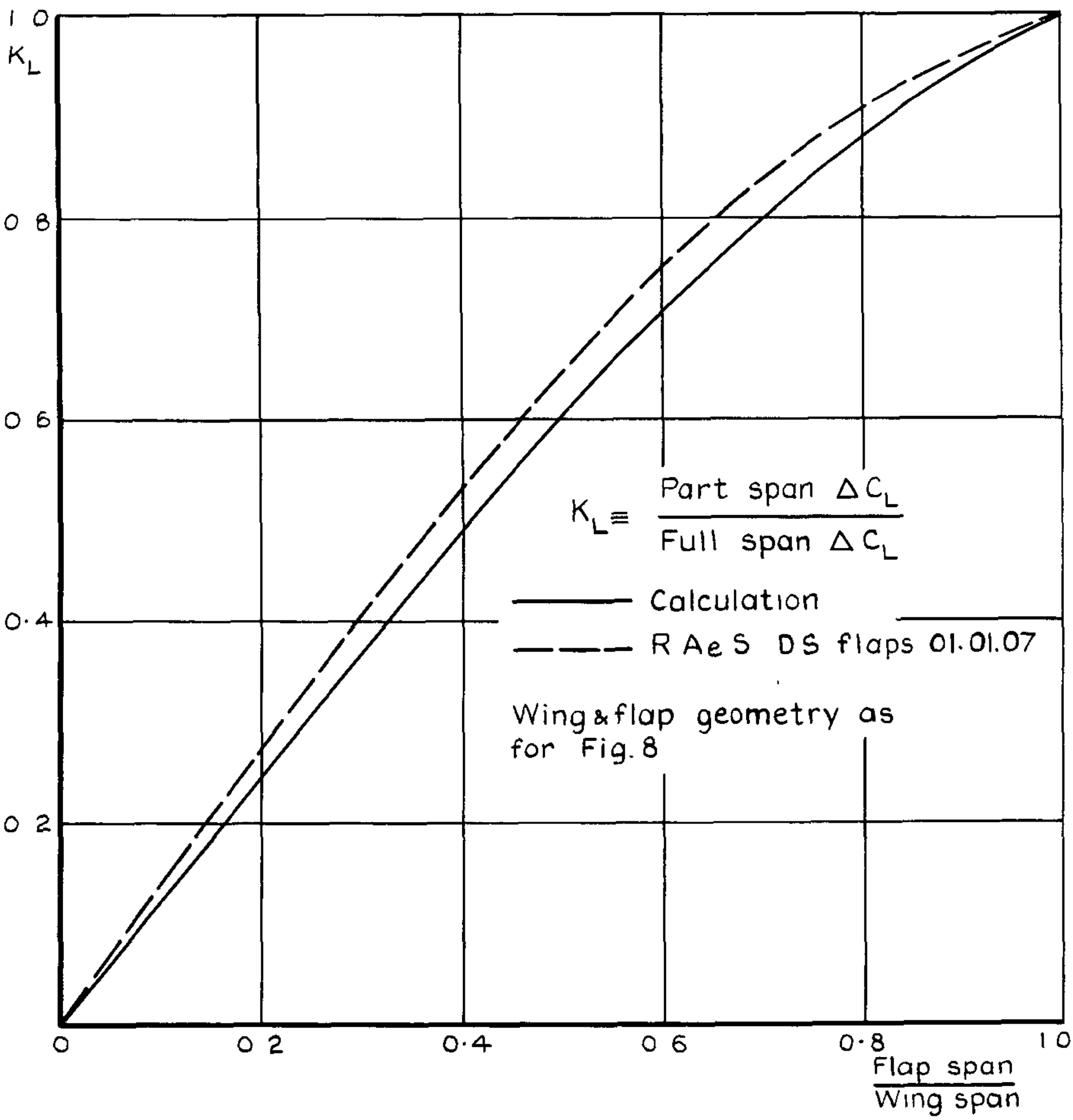

Fig. 9 Comparison of part span lift factors

Wing & flap geometry
as for Fig. 8

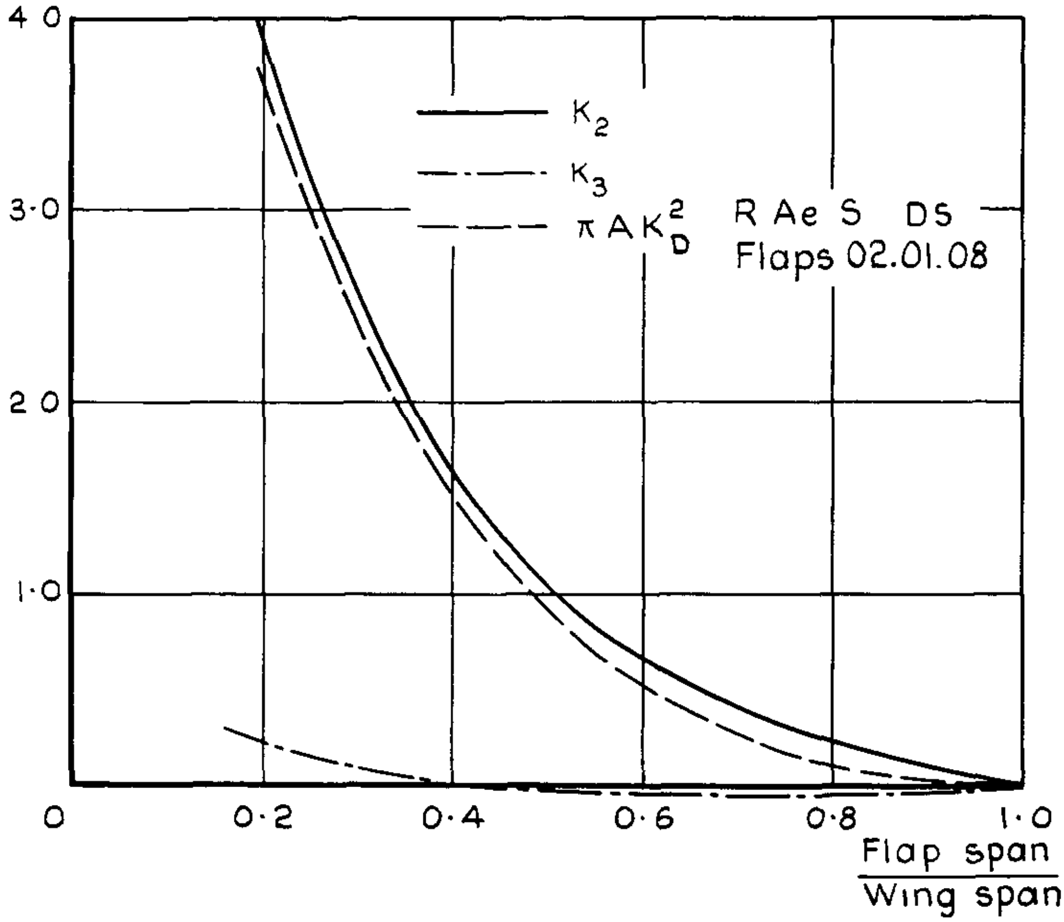

Fig. 10 Part span vortex drag factors

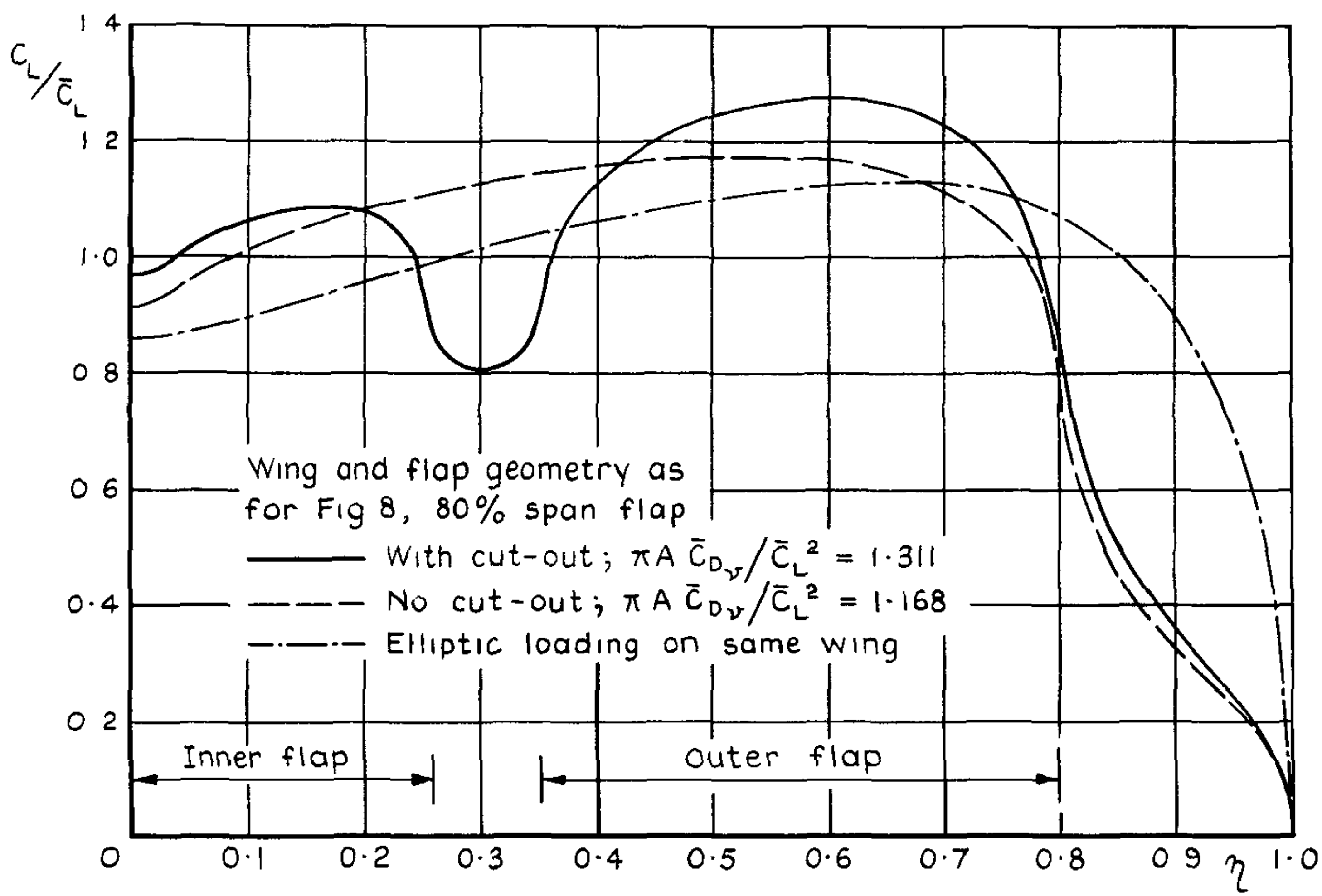

Fig.11 Effect of flap cut-out on spanwise loading at zero wing angle of incidence

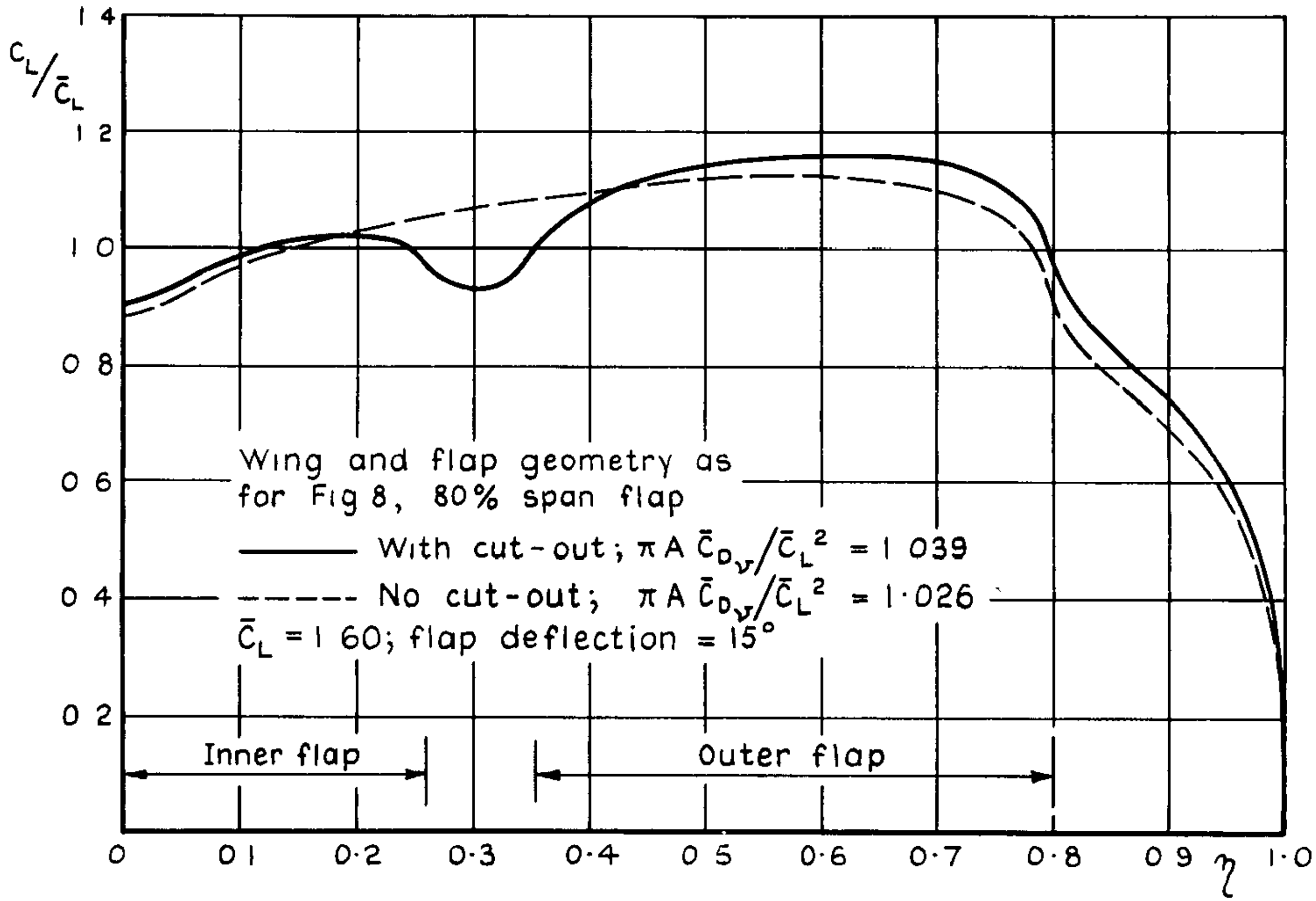

Fig.12 Effect of flap cut-out on spanwise loading with non-zero wing angle of incidence

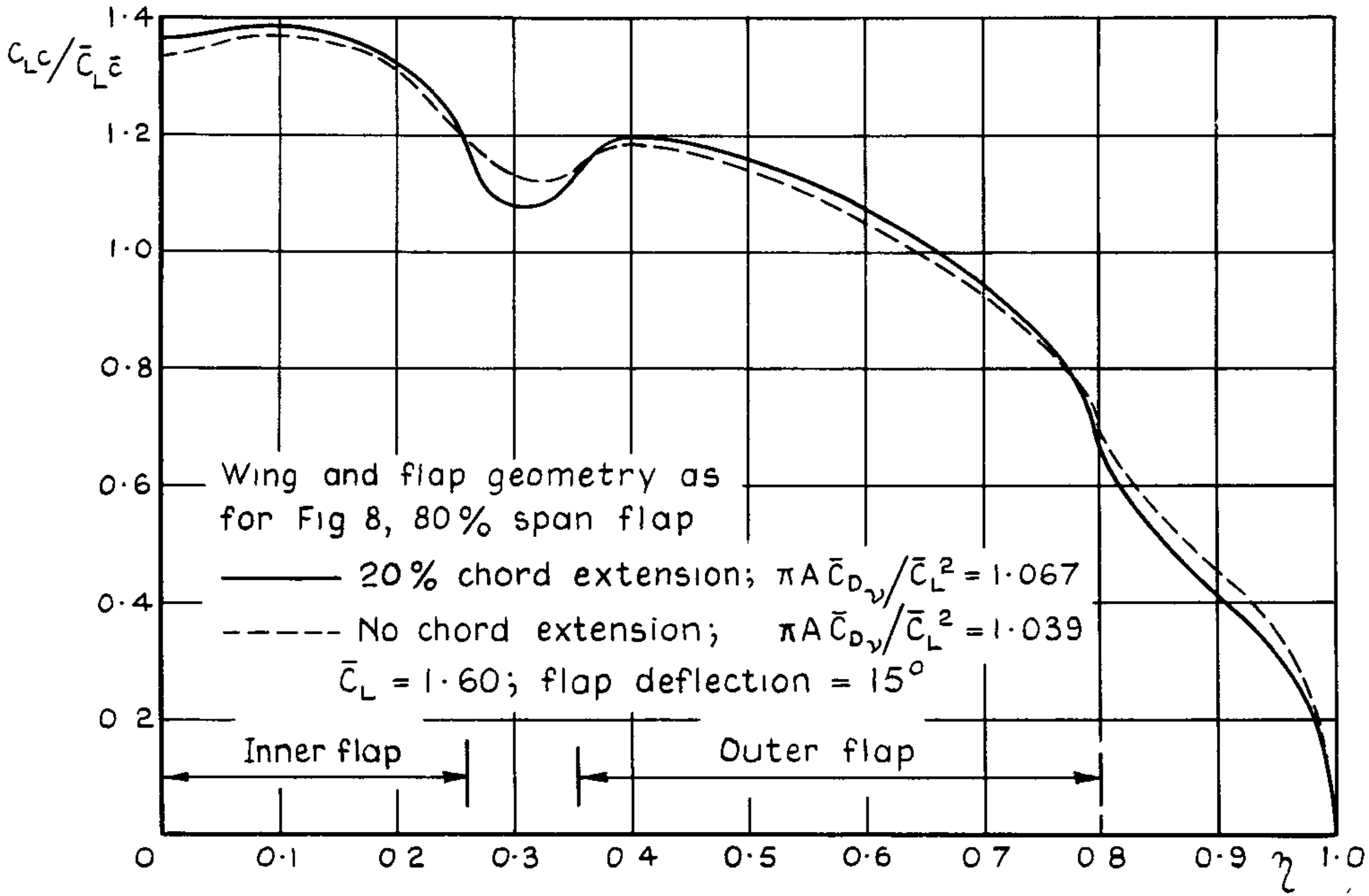

Fig. 13 Effect of extending chord flaps on spanwise loading

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